Three Lectures on Networks

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lecture 2: degrees, positions, and communities

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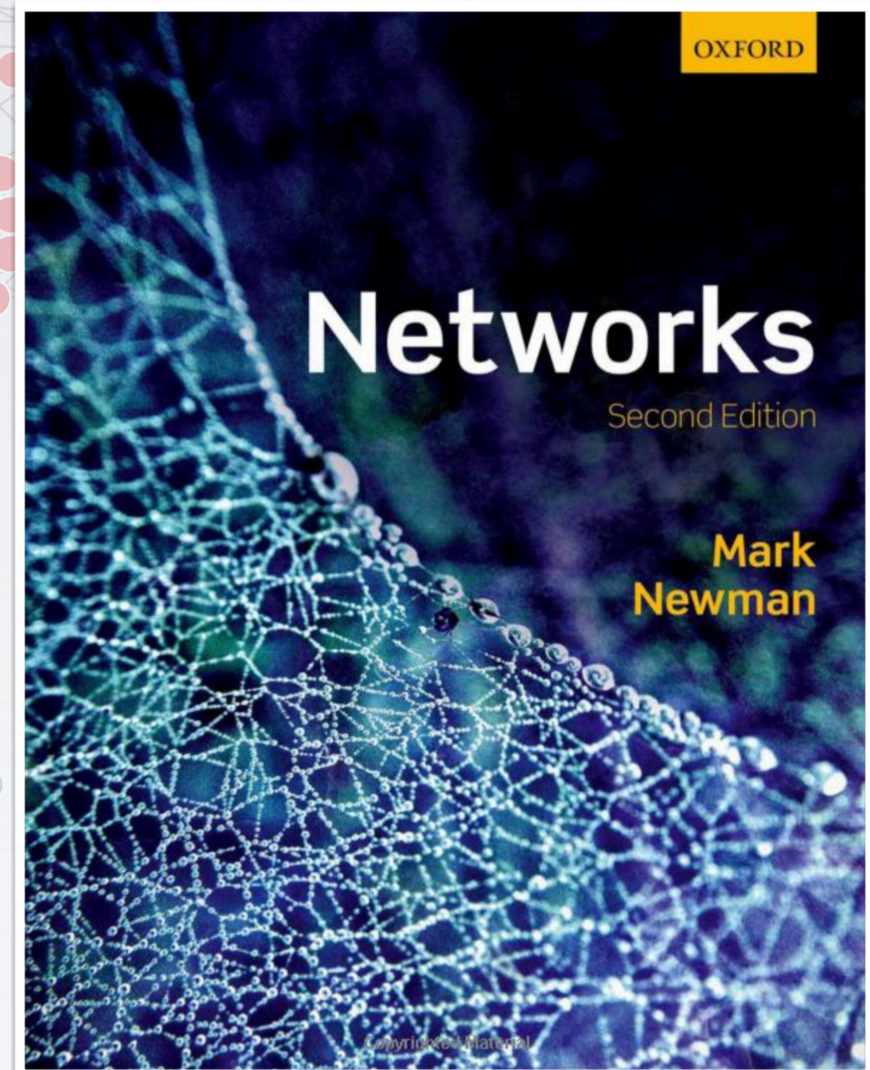
http://www-personal.umich.edu/~mejn/



University of Colorado **Boulder**

Network Analysis and Modeling

Instructor: Aaron Clauset *or* Daniel B. Larremore

This graduate-level course will examine modern techniques for analyzing and modeling the structure and dynamics of complex networks. The focus will be on statistical algorithms and methods, and both lectures and assignments will emphasize model interpretability and understanding the processes that generate real data. Applications will be drawn from computational biology and computational social science. No biological or social science training is required. (Note: this is not a scientific computing course, but there will be plenty of computing for science.)

Full lectures notes online (~150 pages in PDF)

https://aaronclauset.github.io/courses/5352/

University of Colorado **Boulder**

Biological Networks

Instructor: Aaron Clauset

This undergraduate-level course examines the computational representation and analysis of biological phenomena through the structure and dynamics of networks, from molecules to species. Attention focuses on algorithms for clustering network structures, predicting missing information, modeling flows, regulation, and spreading-process dynamics, examining the evolution of network structure, and developing intuition for how network structure and dynamics relate to biological phenomena.

Full lectures notes online (~150 pages in PDF)

https://aaronclauset.github.io/courses/3352/

Software

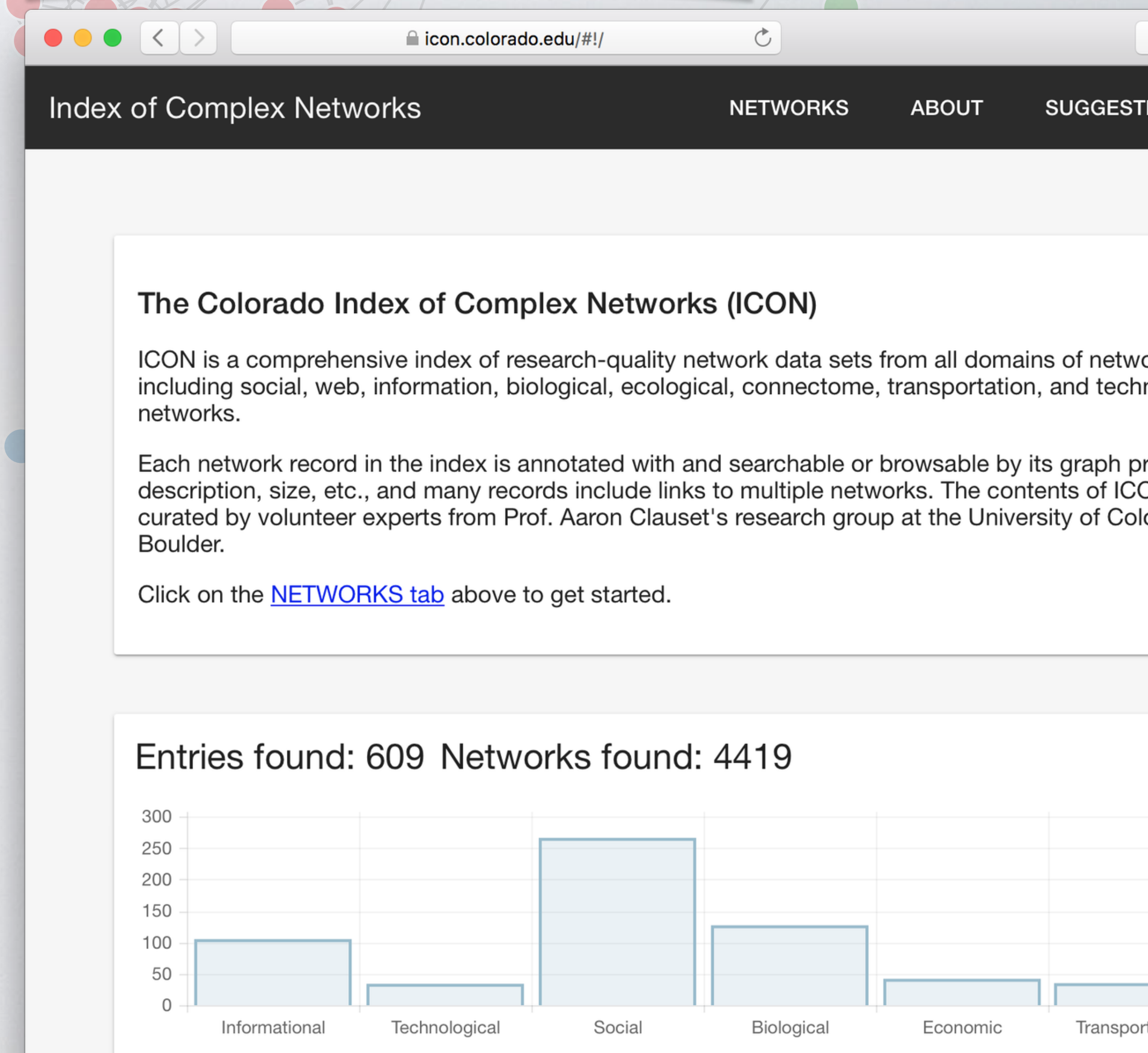
R
Python
Matlab
NetworkX [python]
igraph [python, R, c++]
graph-tool [python, c++]
GraphLab [python, c++]

Standalone editors

UCI-Net
NodeXL
Gephi
Pajek
Network Workbench
Cytoscape
yEd graph editor
Graphviz

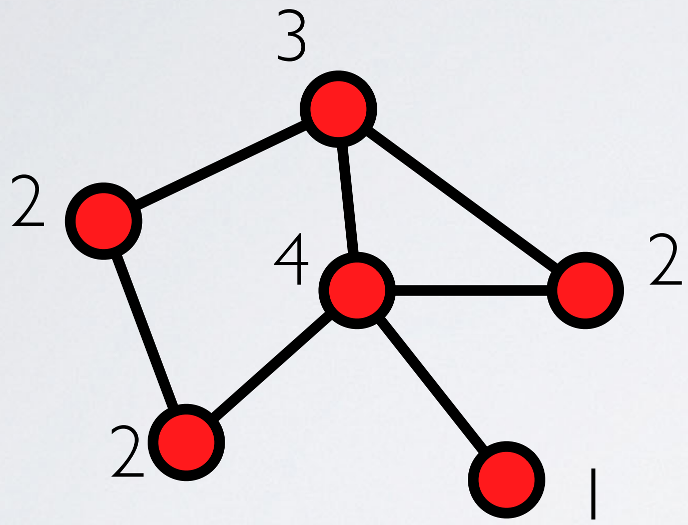
Network data sets

Colorado Index of Complex Networks



1. defining a network
2. **describing a network**
3. null models and statistical inference for networks

describing networks



degree:
number of connections k

$$k_i = \sum_j A_{ij}$$

when does node degree matter?

spreading processes on networks

network edges are the mechanism of transmission

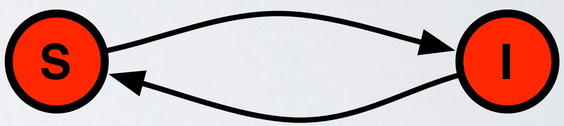
biological (diseases)

- SIS and SIR models

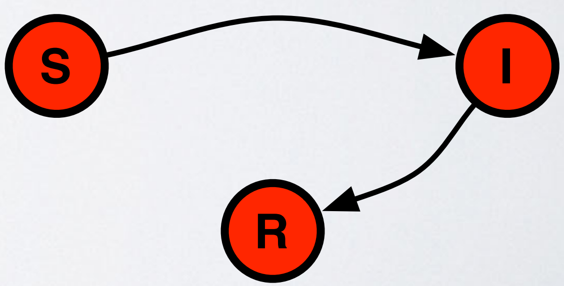
social (information)

- SIS, SIR models
- threshold models

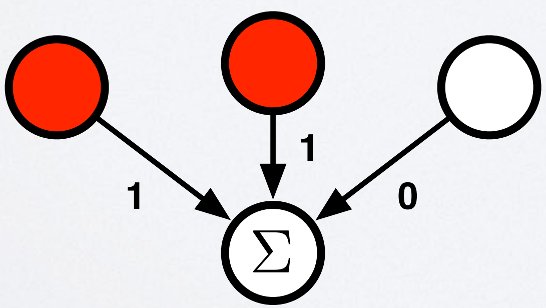
susceptible-infected-susceptible



susceptible-infected-recovered



threshold



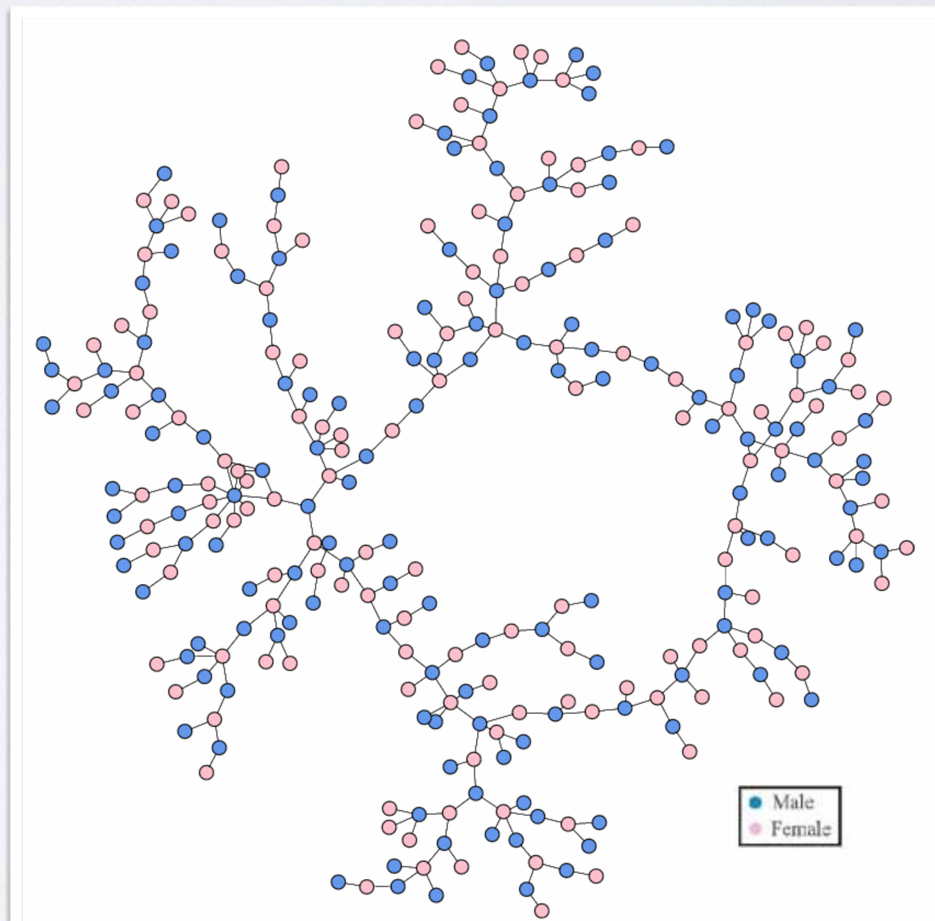
network degrees

Chains of Affection: The Structure of Adolescent Romantic and Sexual Networks

2004

Peter S. Bearman, *Columbia University*
James Moody, *Ohio State University*
Katherine Stovel, *University of Washington*

- relationship network in "Jefferson High"
- this subgraph is 52% of school
- who are most important disease spreaders?

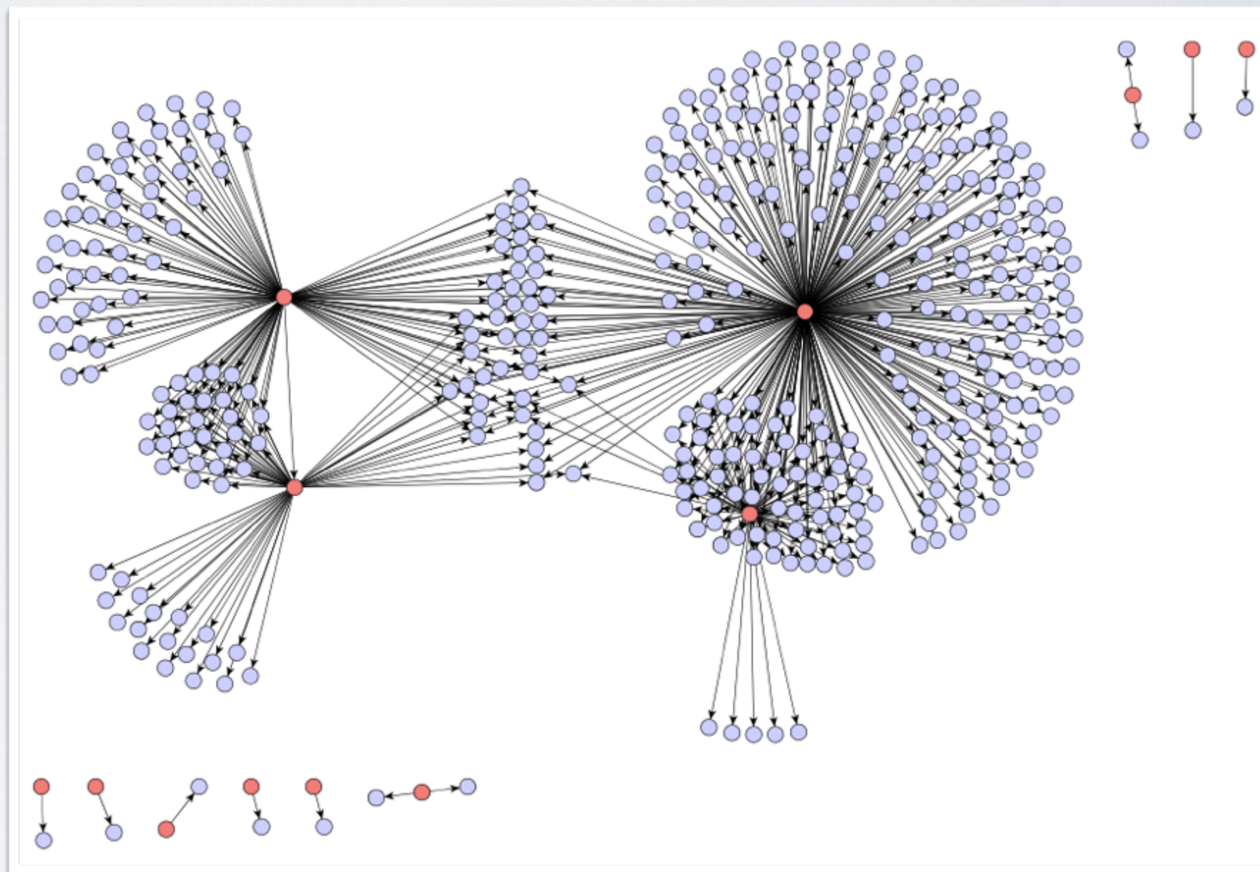


network degrees

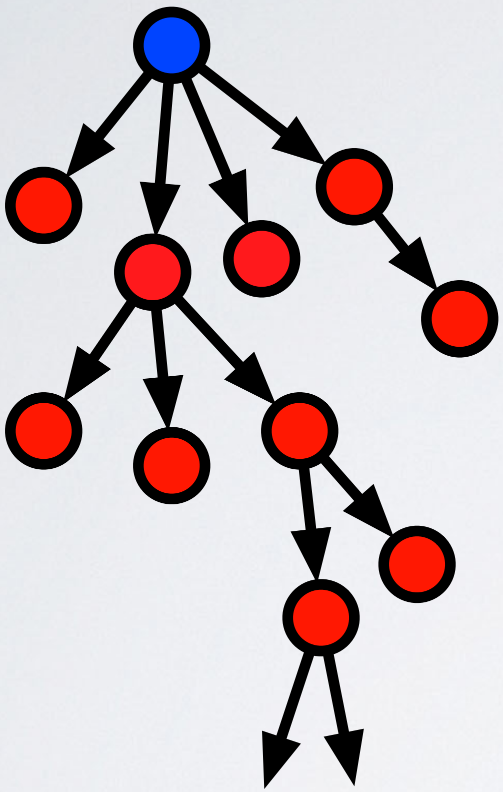
The Dynamics of Viral Marketing 2007

JURE LESKOVEC LADA A. ADAMIC BERNARDO A. HUBERMAN

- amazon.com viral marketing
- viral trace for "*Oh my Goddess!*" community
- very high degrees!
- most attempts to "influence" fail



network degrees



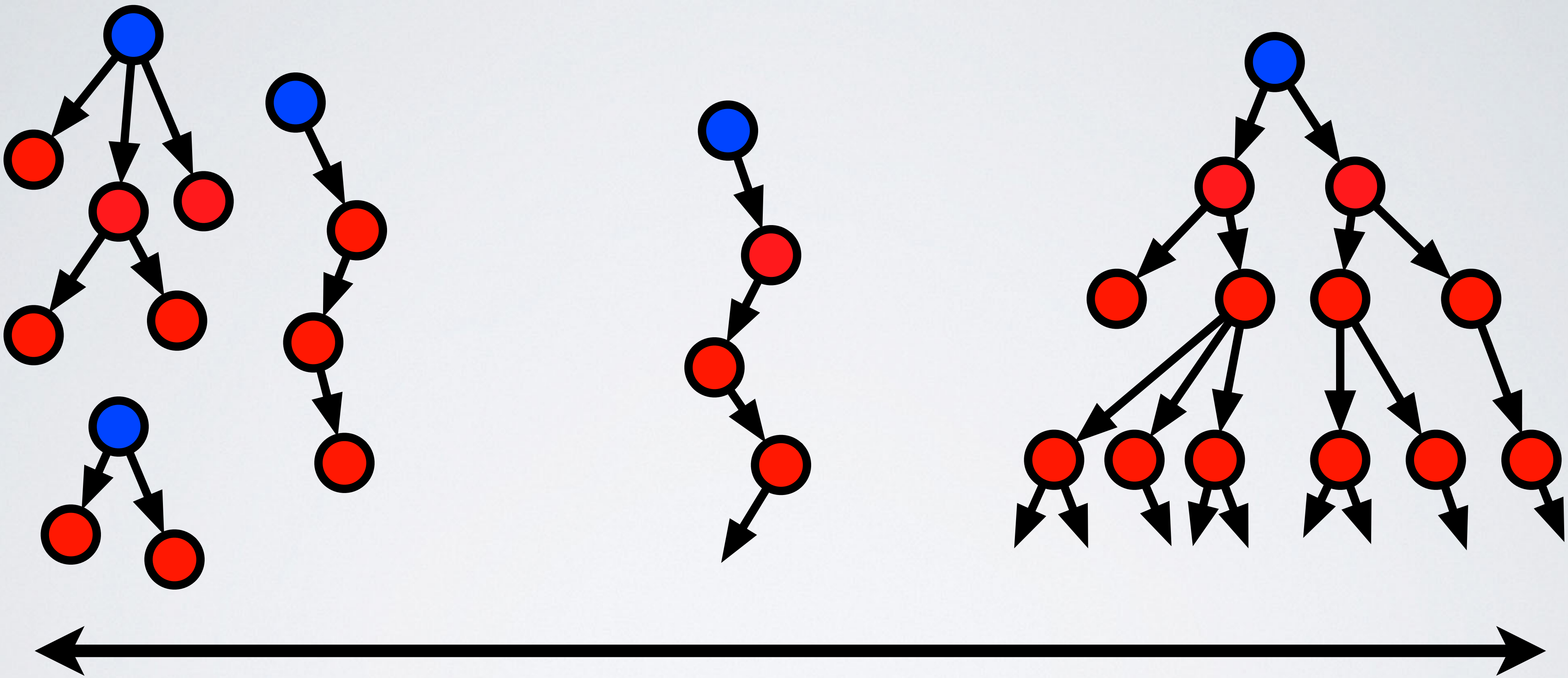
$R_0 = 0.923\ldots$

cascade
epidemic
branching process
spreading process

$$\begin{aligned} R_0 &= \text{net reproductive rate} \\ &= \text{average degree } \langle k \rangle \end{aligned}$$

caveat:
ignores network structure, dynamics, etc.

network degrees



$R_0 < 1$

"sub-critical"

small outbreaks

$R_0 = 1$

"critical"

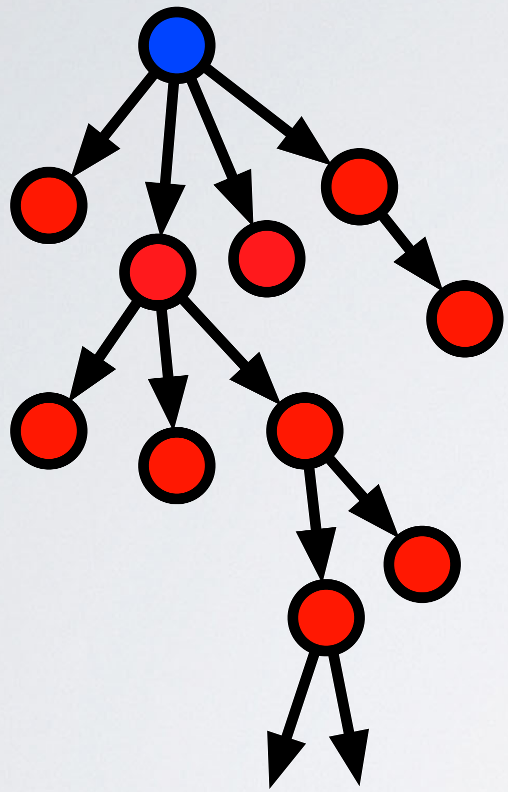
outbreaks of all sizes

$R_0 > 1$

"super-critical"

global epidemics

network degrees

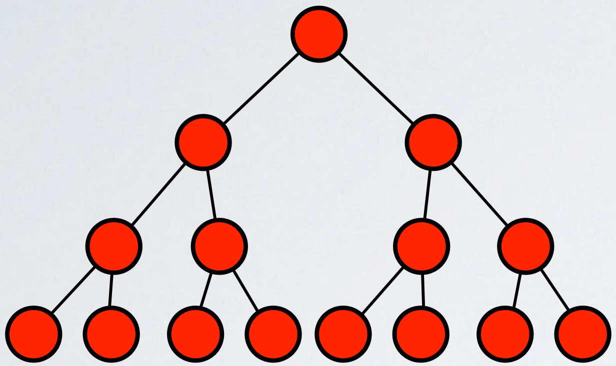


disease	R_0	transmission	vax.
measles	$12 - 18$	airborne	$90 - 95\%$
chickenpox	$7 - 12$		$85 - 90\%$
polio	$5 - 7$	fecal-oral route	$82 - 87\%$
small pox	$1.5 - 20+$	airborne droplet	$70 - 80\%$
H1N1 flu	$1 - 3$	airborne droplet	$\approx 67\%$
ebola	$1.5 - 2.5$	bodily fluids	
zika	2		
covid-19 (wildtype)	≈ 2.4	aerosols	$\approx 60\%$
covid-19 (alpha)	$4 - 5$	aerosols	$75 - 80\%$
covid-19 (delta)	$5 - 8$	aerosols	$80 - 88\%$
covid-19 (omicron)	$10 - 14$	aerosols	$90 - 93\%$

all super-critical

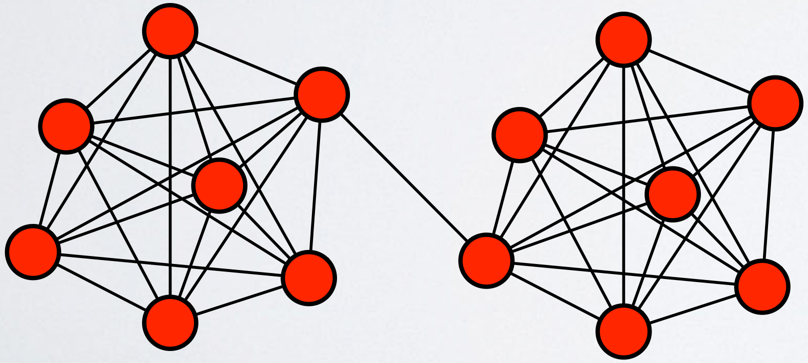
some numbers from Lauren Ancel Meyers (UT Austin)

network degrees



bigger cascades

- smaller overlap among neighbors
- more *expander*-like
  [more like a random graph]
- higher transmission probability
- lower activation threshold



smaller cascades

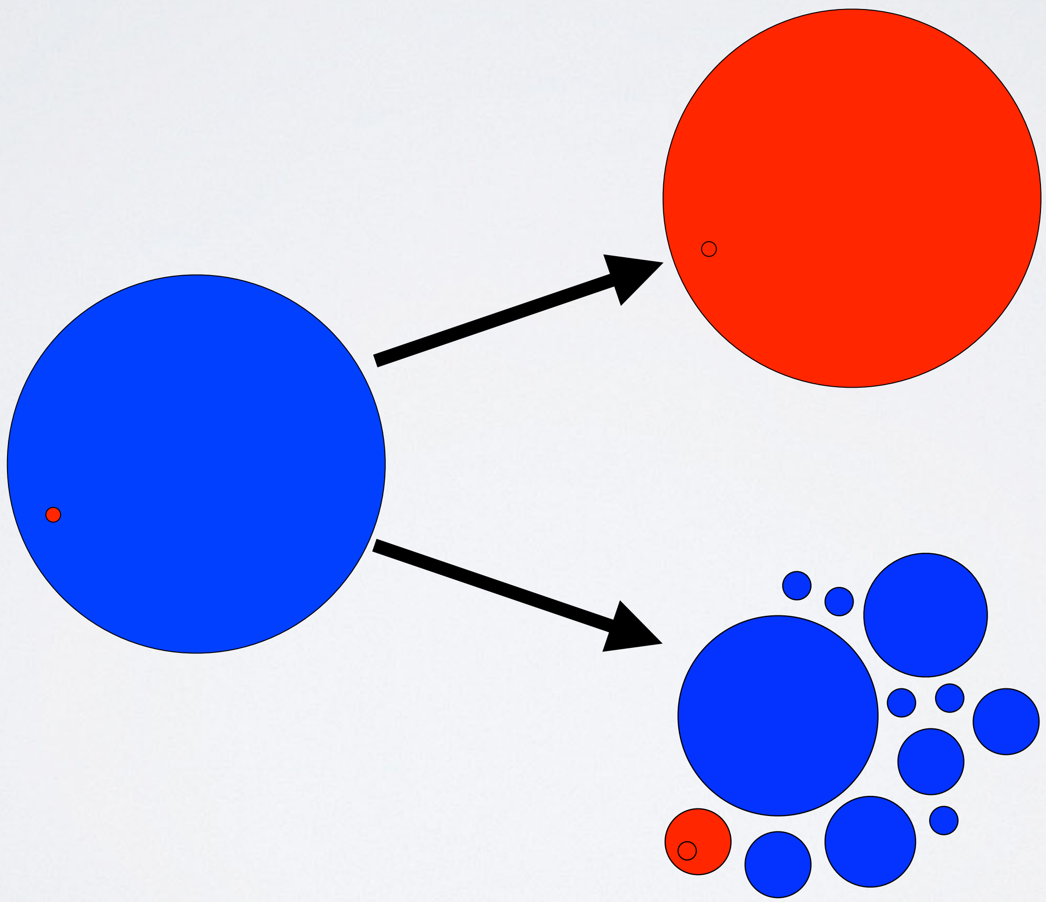
- larger overlap among neighbors
- more triangles
- smaller "communities"
- more spatial-like organization
- lower transmission probability
- higher activation threshold

Volz, J. Math. Bio. 56, 293–310 (2008)
Bansal et al., J. Royal Soc. Interface 4, 879–891 (2007)
Karrer and Newman, Phys. Rev. E 82, 016101 (2010)
Salathe and Jones, PLoS Comp. Bio. 6, e1000736 (2010)

network degrees

how could we halt the spread?

- break network into disconnected pieces



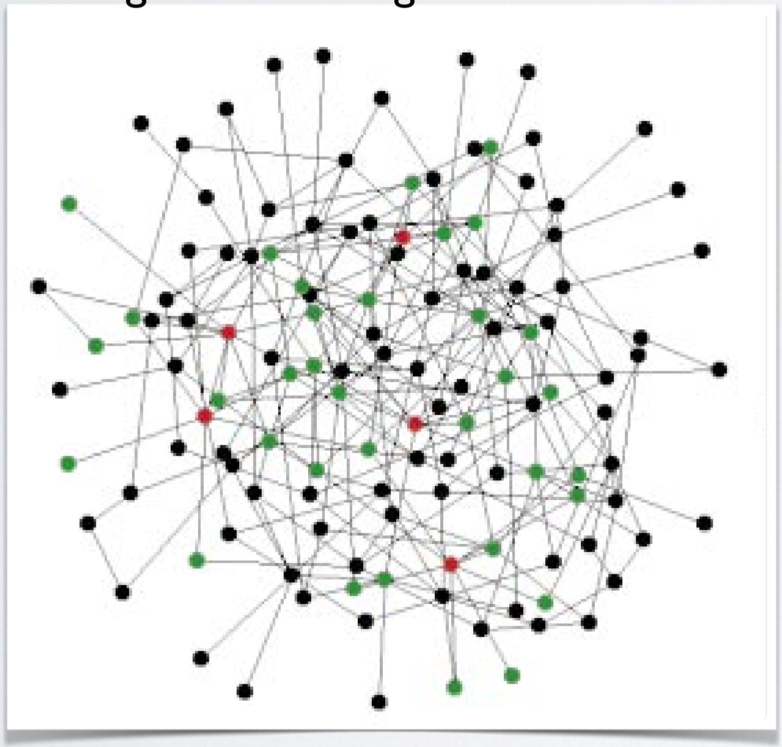
network degrees

two networks

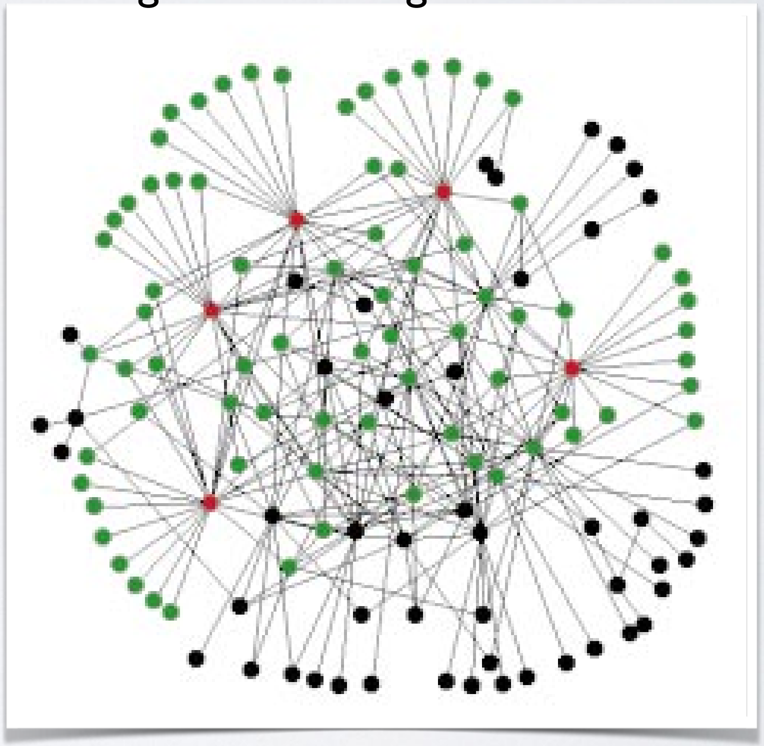
Error and attack tolerance of complex networks 2000

Réka Albert, Hawoong Jeong & Albert-László Barabási

homogeneous in degree

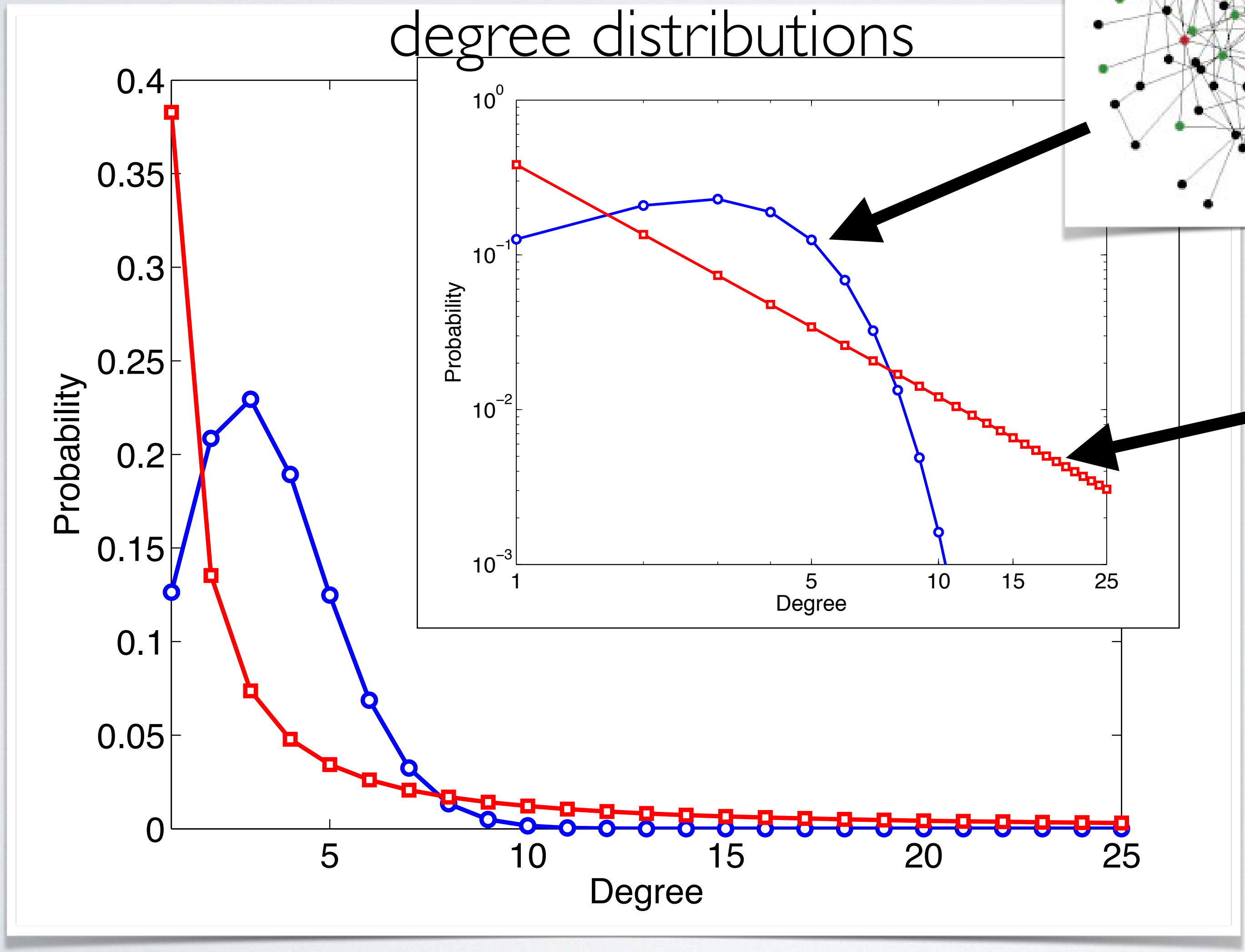


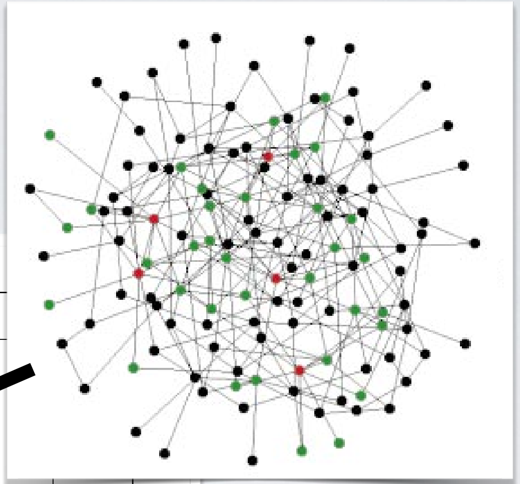
heterogeneous in degree

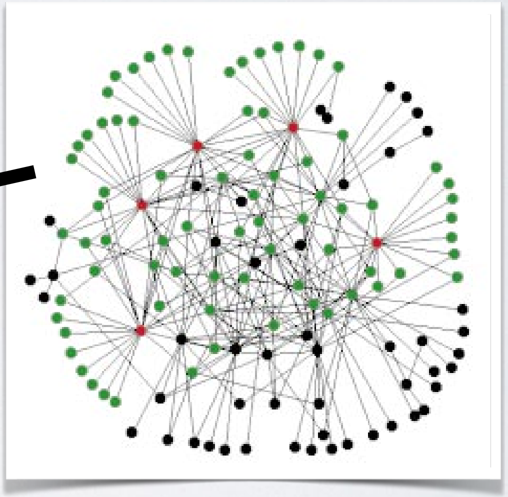


network degrees

two networks



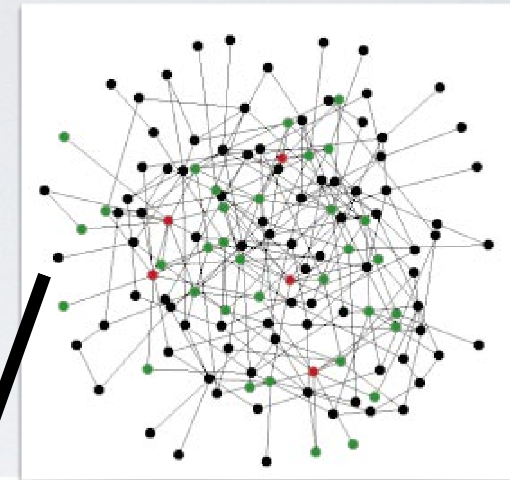


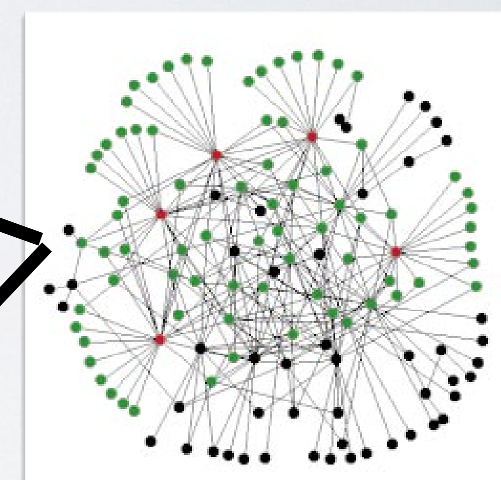


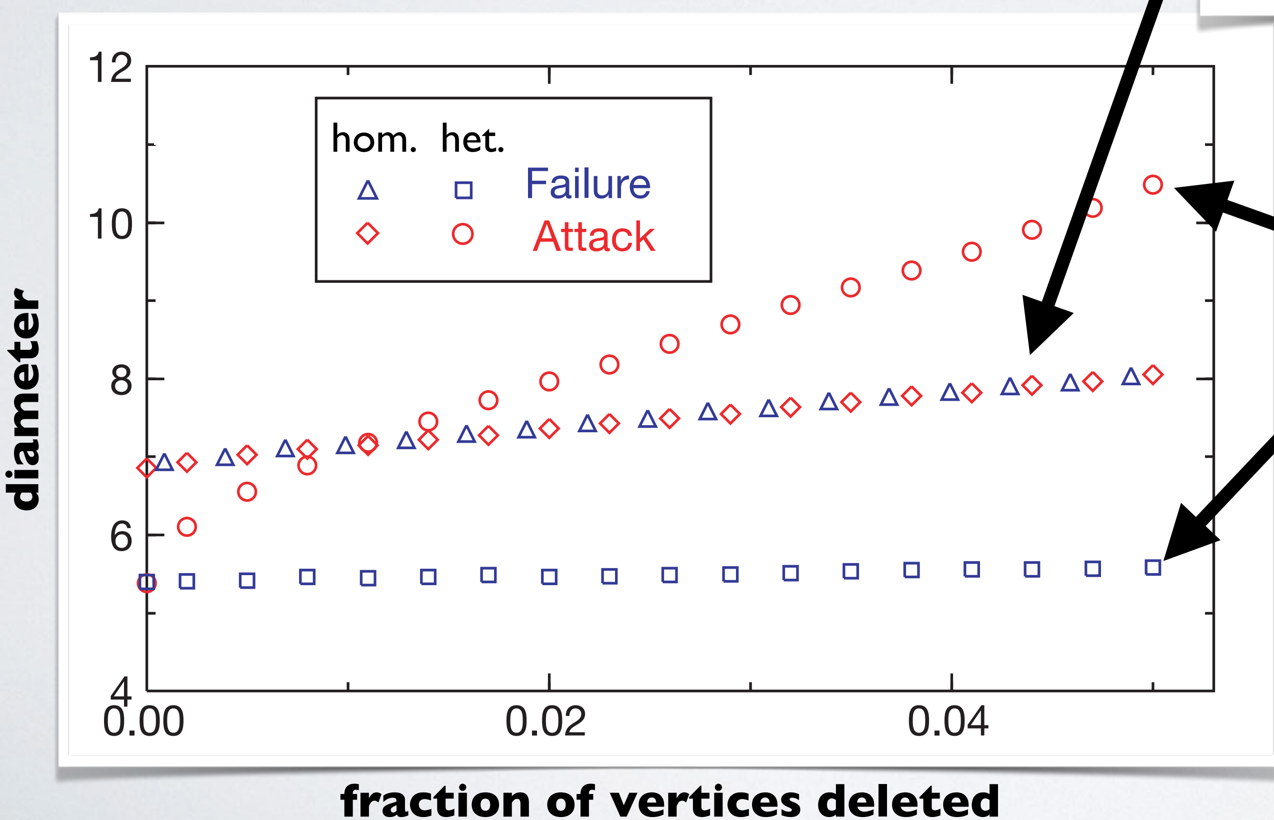
network degrees

strategy: delete vertices

1. uniformly at random ("failure")
2. in order of degree ("attack")





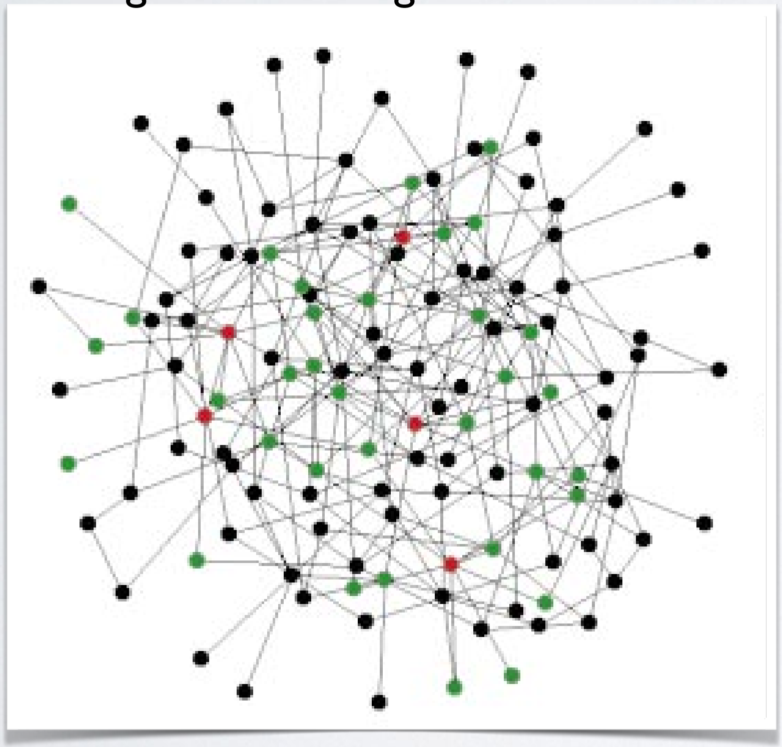


network degrees

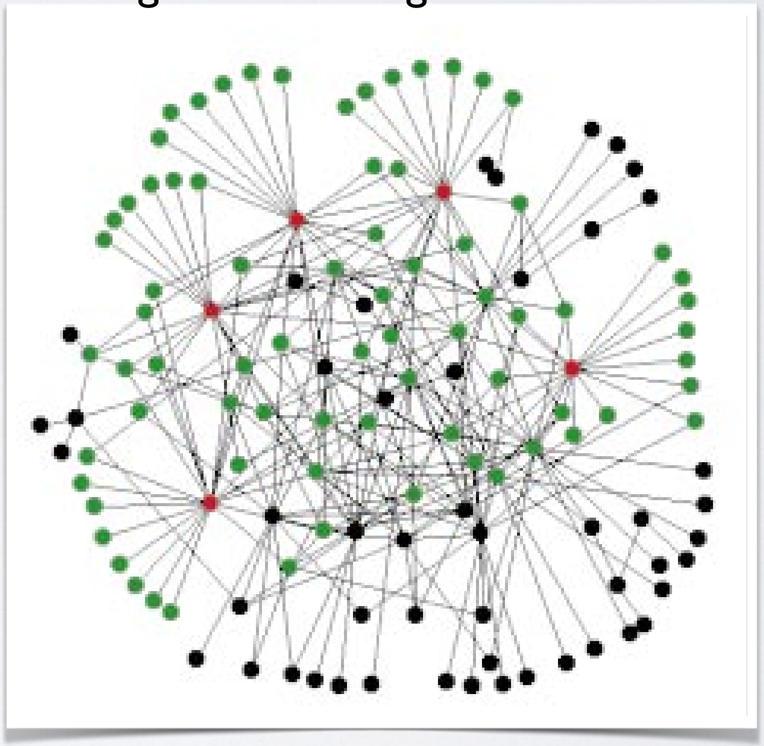
what promotes spreading?

- high-degree vertices*
- centrally-located vertices

homogeneous in degree



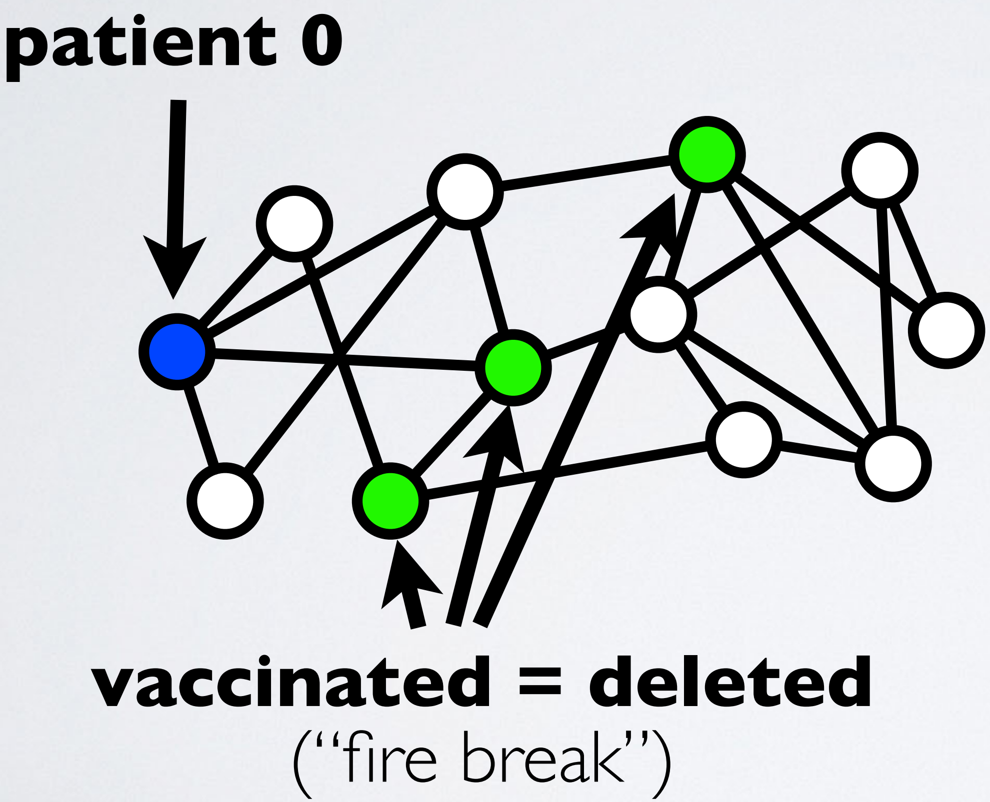
heterogeneous in degree



network degrees

strategy: delete vertices

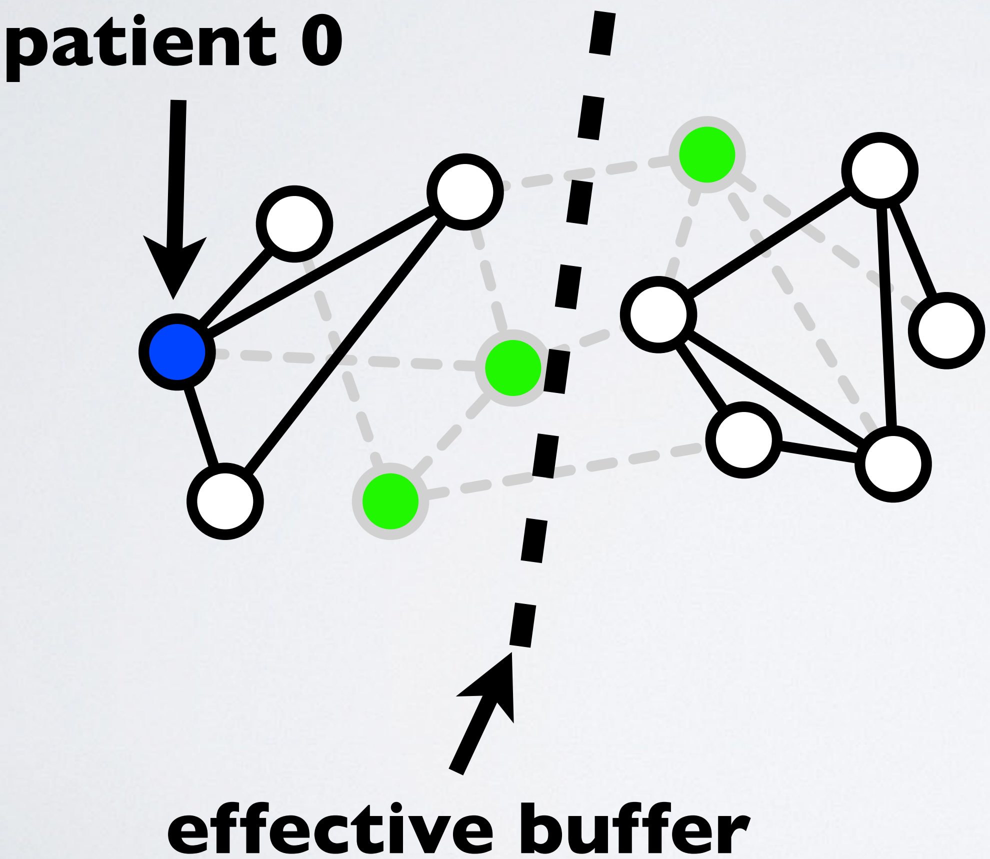
3. build "fire breaks"



software packages for simulating epidemics on networks

1. Epidemics on Networks (EoN) https://epidemicsonnetworks.readthedocs.io/en/latest/
2. SEIR+ Model https://github.com/ryansmcgee/seirsplus

network degrees



- **vaccination strategies**
  - the "front line" (hospitals)
  - high degree nodes
  - the vulnerable (old/young)

software packages for simulating epidemics on networks
1. Epidemics on Networks (EoN) https://epidemicsonnetworks.readthedocs.io/en/latest/
2. SEIR+ Model https://github.com/ryansmcgee/seirsplus

network degrees

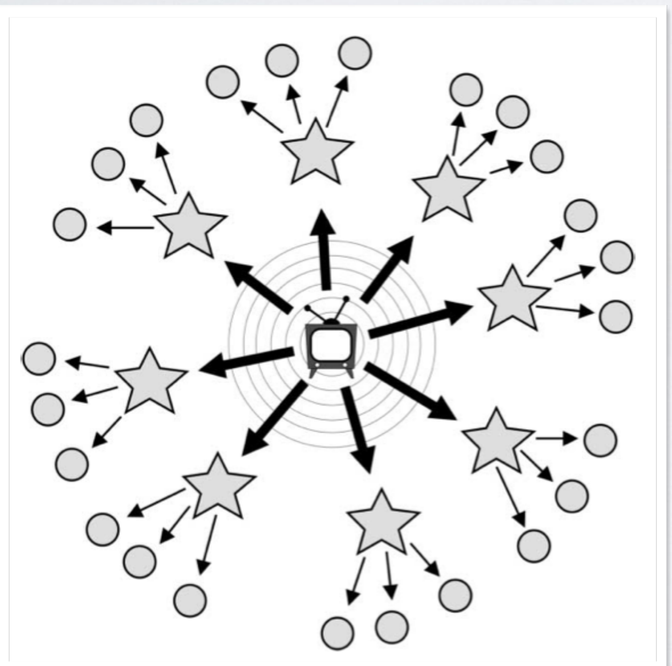
but, in social networks...

network degrees

Influentials, Networks, and Public Opinion Formation

DUNCAN J. WATTS
PETER SHERIDAN DODDS*

2007



broadcast influence

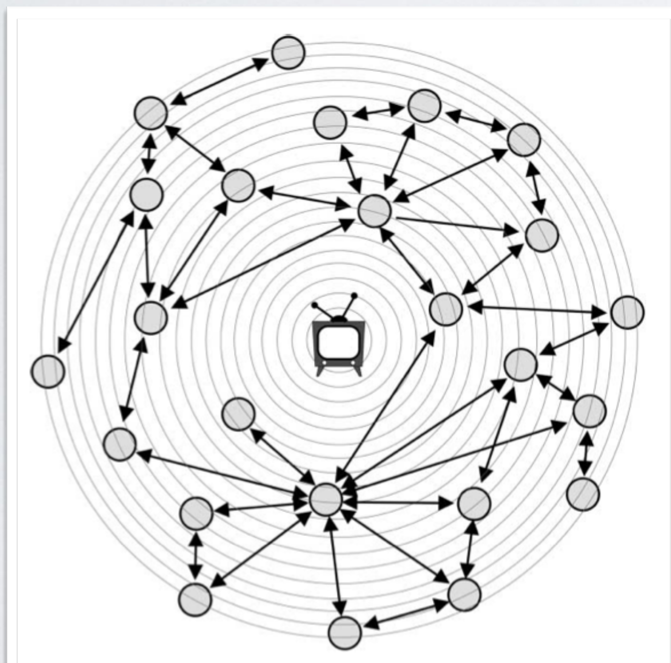
- classic information marketing
- message saturation
- **degree** is most important

network degrees

Influentials, Networks, and Public Opinion Formation

DUNCAN J. WATTS
PETER SHERIDAN DODDS*

2007



network influence

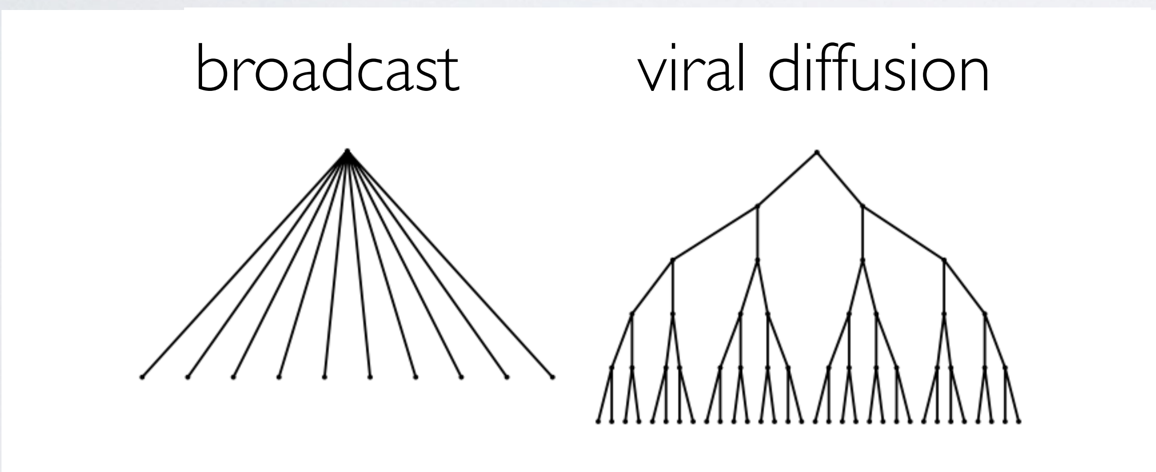
- "network" (decentralized) marketing
- high-degree = "opinion leader"
- high-degree alone = **irrelevant**
- a cascade requires a legion of *susceptibles* (a system-level property)

Watts & Dodds, Journal of Consumer Research 34 (2007)

network degrees

The Structural Virality of Online Diffusion

Sharad Goel, Ashton Anderson
Stanford University, Stanford, California, 94305 {scgoel@stanford.edu, ashton@cs.stanford.edu}
Jake Hofman, Duncan J. Watts
Microsoft Research, New York, New York 10016 {jmh@microsoft.com, duncan@microsoft.com}
2015

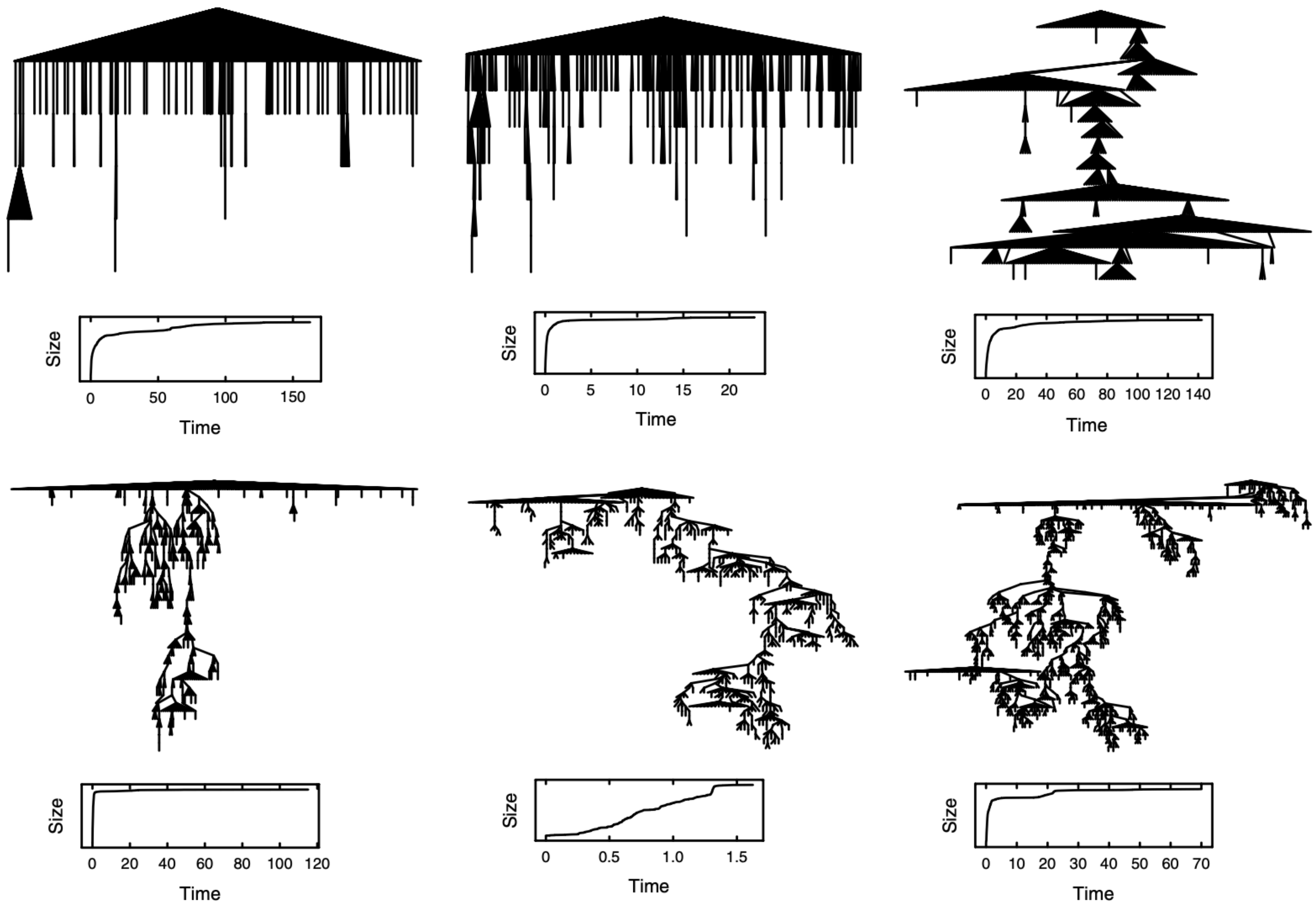


- 1 billion diffusion events, on twitter
- virality measure for each cascade
- cascade sizes are extremely high variance (maybe power law…)

Goel, et al. Management Science 62 (2015)

network degrees

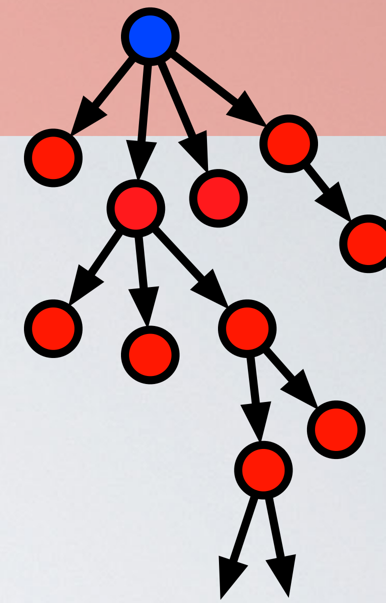
Figure 3 A Random Sample of Cascades Stratified and Ordered by Increasing Structural Virality, Ranging from 2 to 50



- enormous diversity of cascade shapes, depths

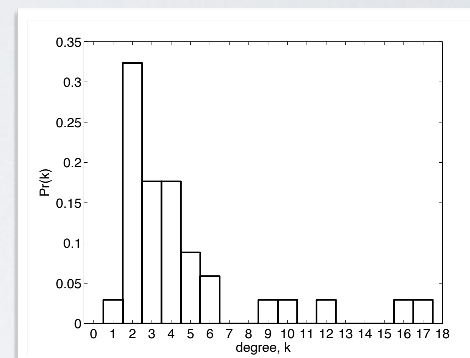
Goel, et al. Management Science 62 (2015)

network degrees



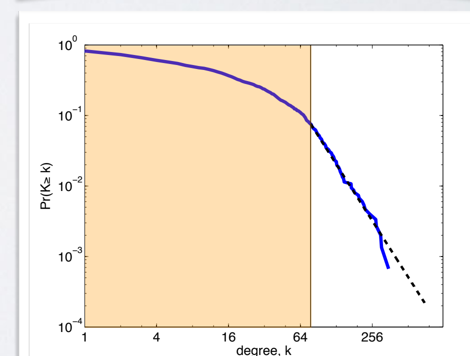
degrees:

- *first-order* description of network structure
- direct implications for spreading processes
- cascades require both susceptible population *and* spreaders

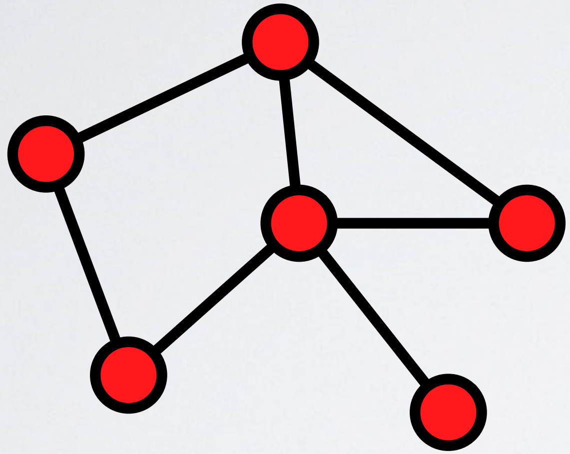


open questions:

- impact of degrees on other dynamics
- feedback from dynamics to degree
  [adaptive behaviors like self-quarantine, evangelism]
- when does degree *not* matter

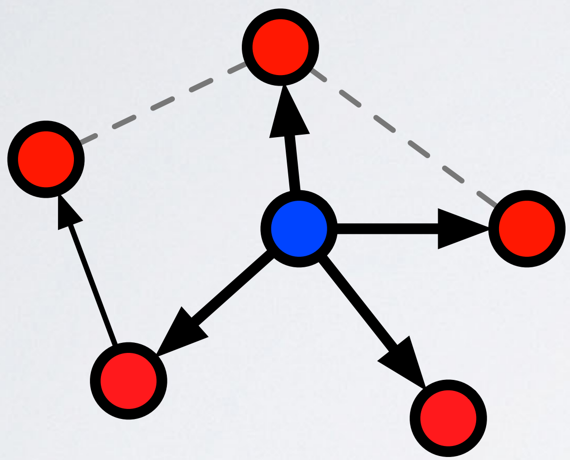


describing networks



position

describing networks



position = centrality:

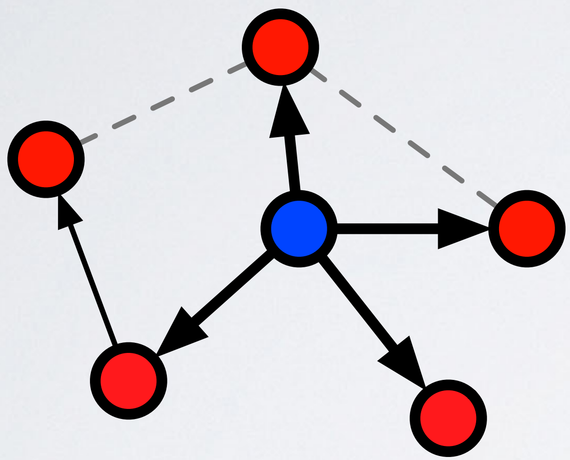
structural vs. dynamical importance

geometric	harmonic centrality
	closeness centrality
	betweenness centrality
connectivity	degree centrality
	eigenvector centrality
	PageRank
	Katz centrality
	many many more…

structural importance = cheap estimate of dynamical importance (aka "influence")

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, Social Networks 27, 55–71 (2005)

describing networks

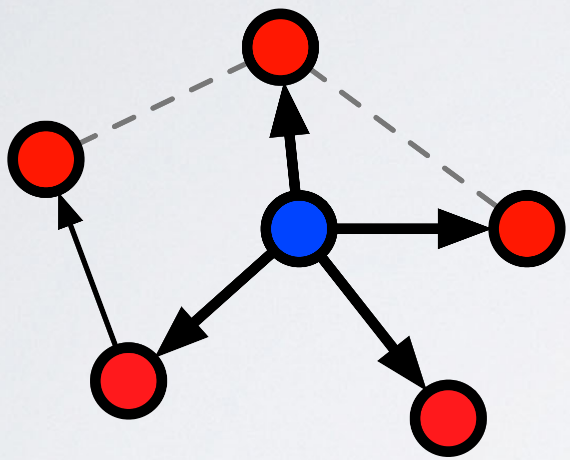


position = centrality:
structural vs. dynamical importance

centrality = unsupervised node ranking

$$f : G \to \overrightarrow{v}$$

describing networks



position = centrality:
harmonic, closeness centrality

importance = being in "center" of the network

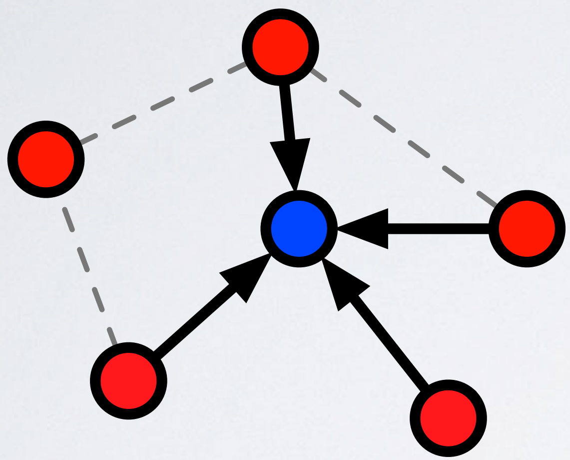
harmonic $$c_i = \frac{1}{n-1}\sum_{j\neq i}\frac{1}{d_{ij}}$$

length of shortest path

distance: $$d_{ij} = \begin{cases} \boxed{\ell_{ij}} & \text{if } j \text{ reachable from } i \\ \infty & \text{otherwise} \end{cases}$$

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, Social Networks 27, 55–71 (2005)

describing networks



position = centrality:
PageRank, Katz, eigenvector centrality

importance = sum of importances* of nodes that point at you

$$I_i = \sum_{j \to i} \frac{I_j}{k_j}$$

or, the right eigenvector of

$$\mathbf{A}\mathbf{x} = \lambda\,\mathbf{x}$$

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, Social Networks 27, 55–71 (2005)

*modulo several technical details

network position

an example

Giovanni de Medici

network position

Robust Action and the Rise of the Medici, 1400–1434[1]

John F. Padgett and Christopher K. Ansell

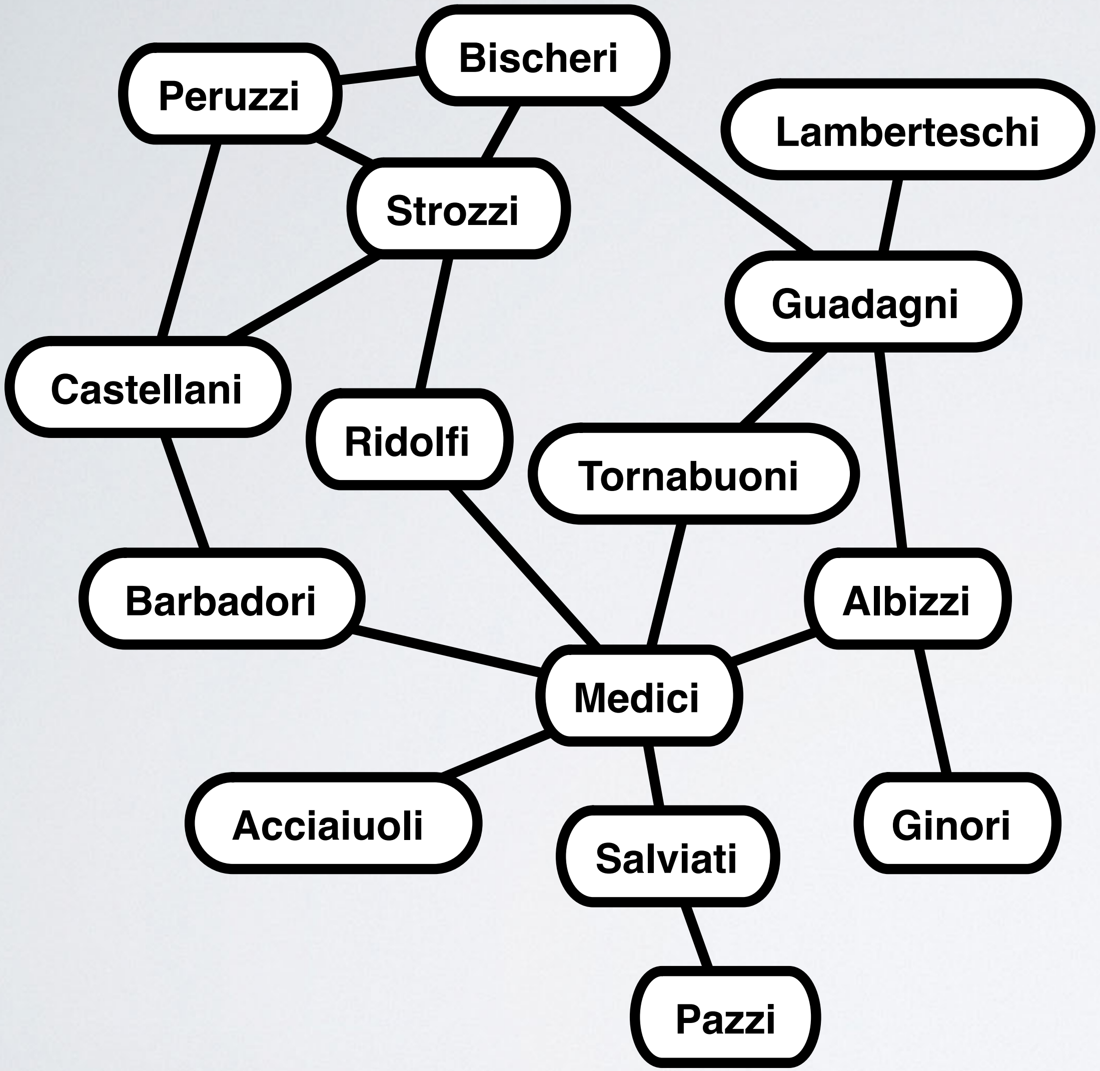
1993

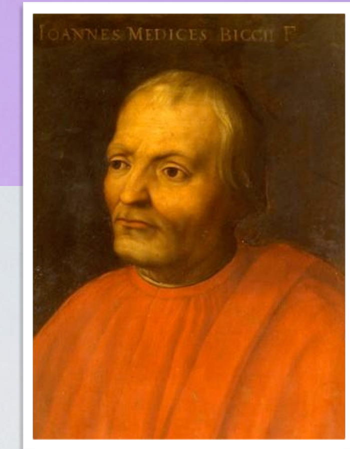
Duomo

Palazzo Medici

Giovanni de Medici

network position: harmonic

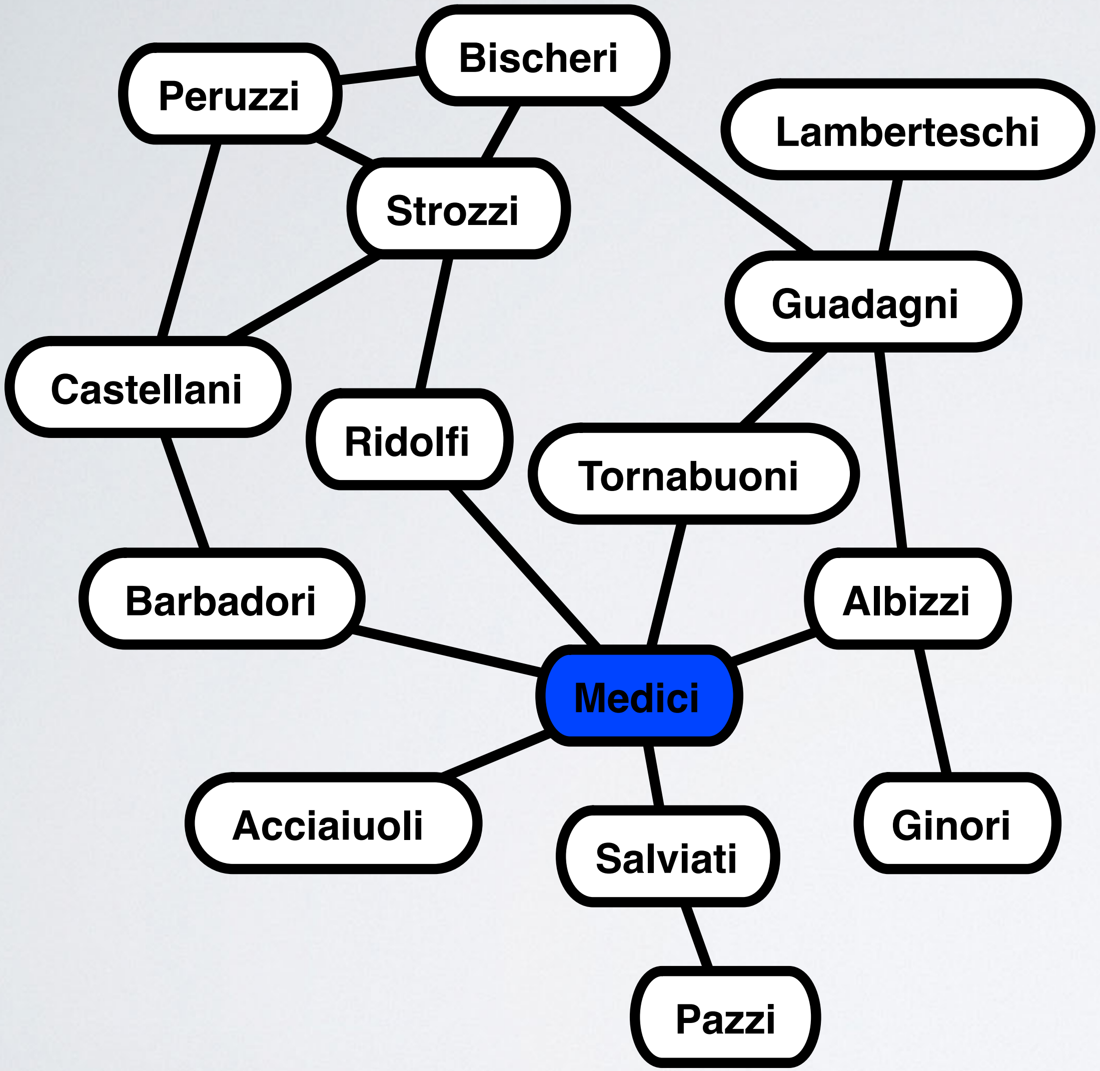


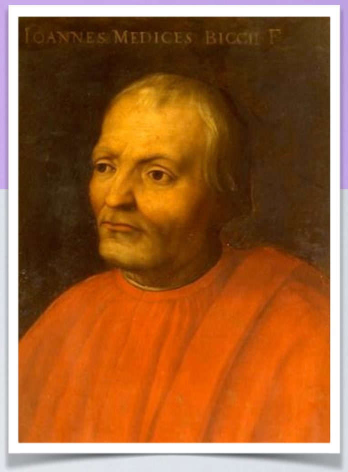


nodes: Florence families
edges: inter-family marriages

which family is most central?

network position: closeness

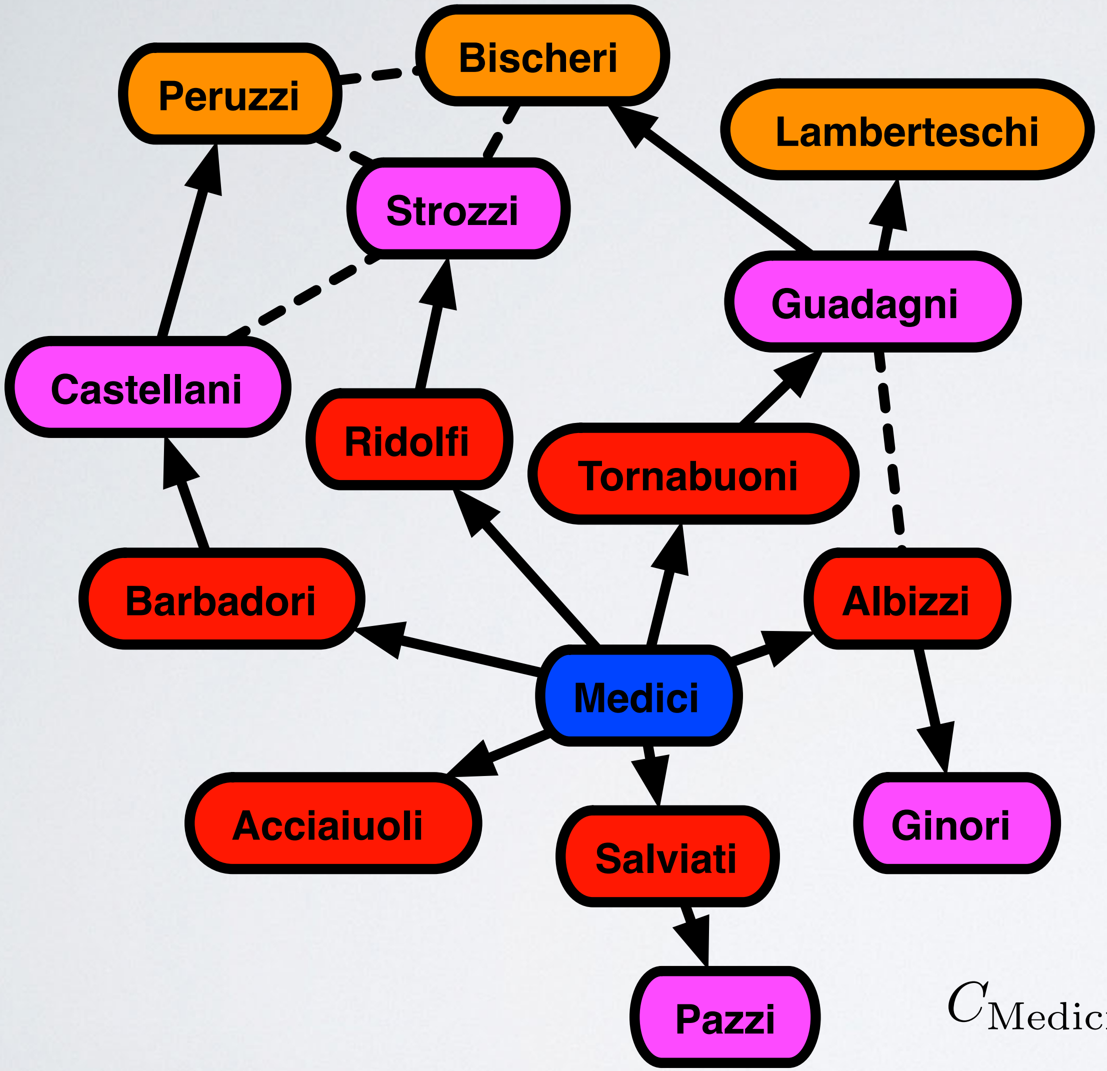


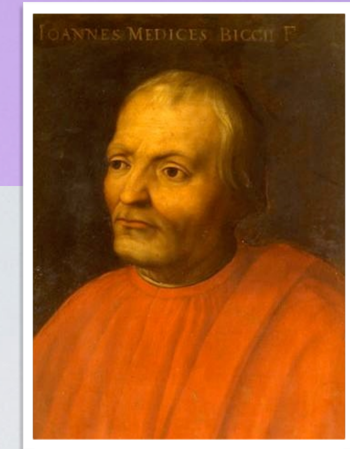


nodes: Florence families
edges: inter-family marriages

Medici?

network position: closeness



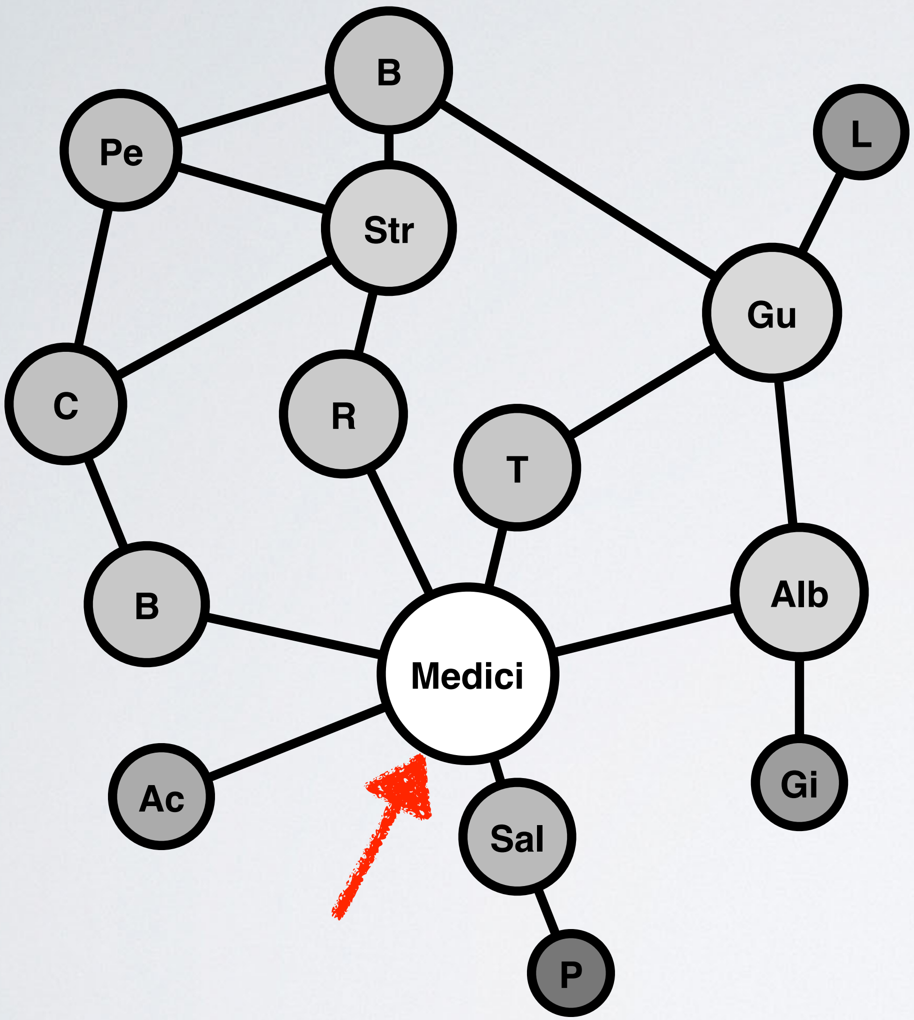


nodes: Florence families
edges: inter-family marriages

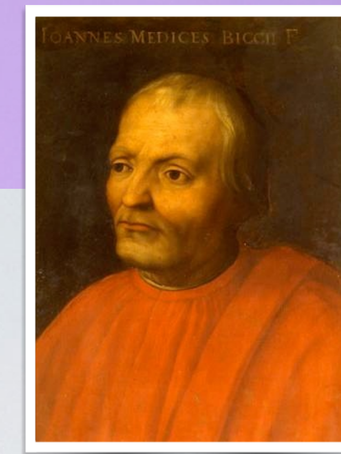
Medici.

$$C_{\text{Medici}} = 6\left(\frac{1}{1}\right) + 5\left(\frac{1}{2}\right) + 3\left(\frac{1}{3}\right)$$
$$= 9.5$$

network position: harmonic

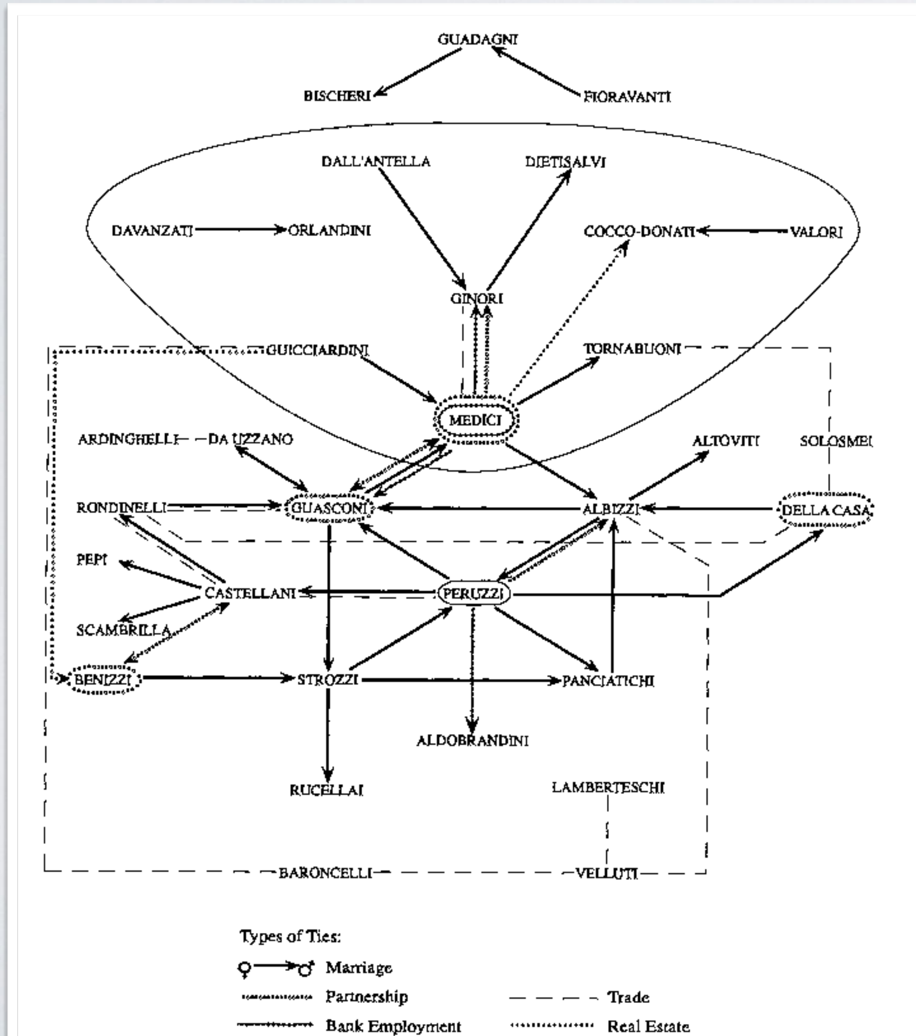


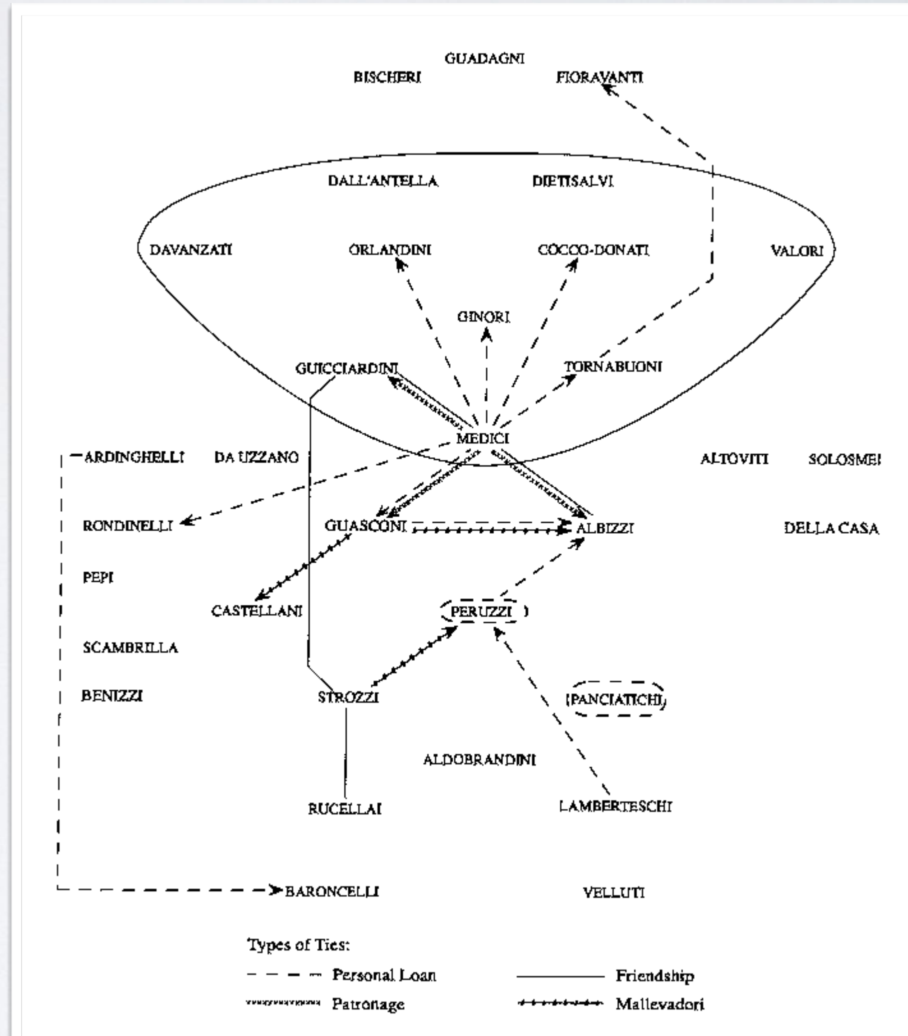
Medici	9.5
Guadagni	7.92
Albizzi	7.83
Strozzi	7.67
Ridolfi	7.25
Bischeri	7.2
Tornabuoni	7.17
Barbadori	7.08
Peruzzi	6.87
Castellani	6.87
Salviati	6.58
Acciaiuoli	5.92
Ginori	5.33
Lamberteschi	5.28
Pazzi	4.77



network position

actually, it's complicated...





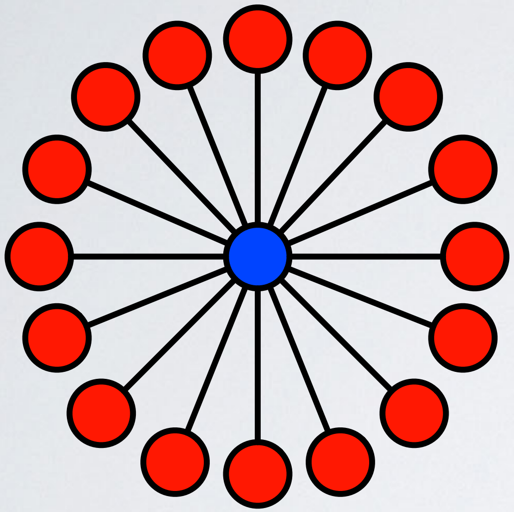
network position

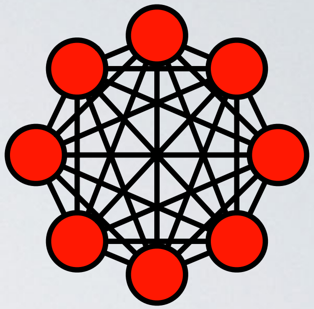
most
centralized

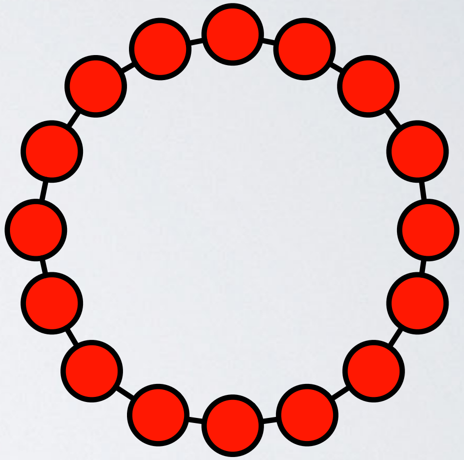
vast wilderness
of in-between

most
decentralized

network position





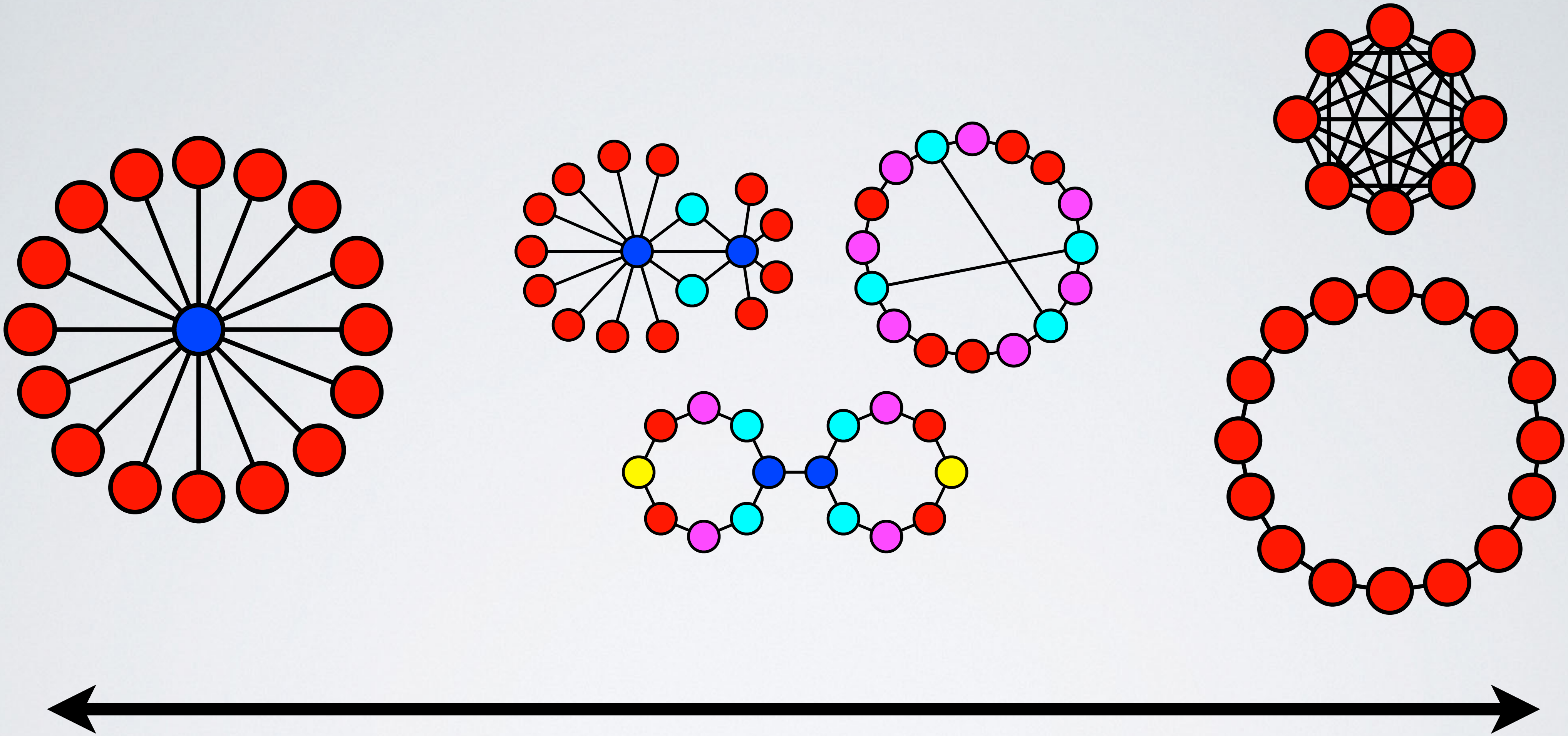


most centralized

vast wilderness of in-between

most decentralized

network position

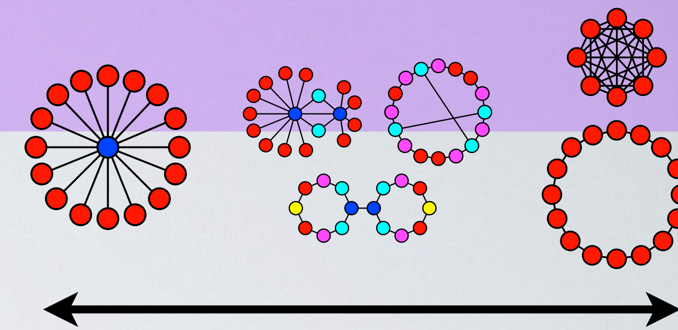


most centralized

vast wilderness of in-between

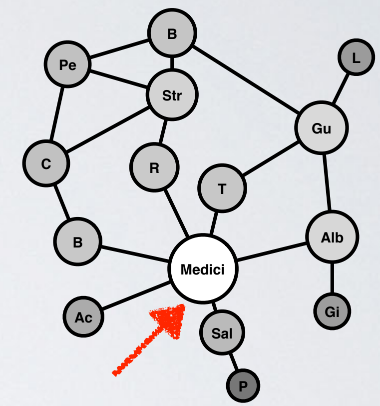
most decentralized

network position



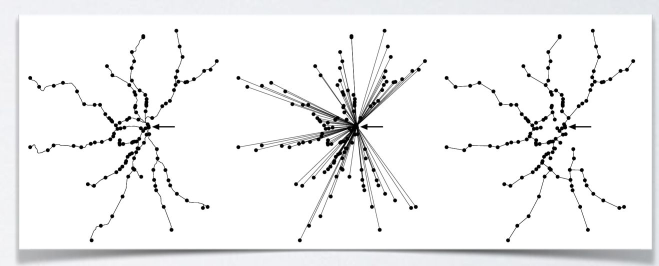
positions:

- *geometric* description of network structure
- core vs. periphery
- centrality = importance, influence
- *nearly all centrality scores highly correlated*



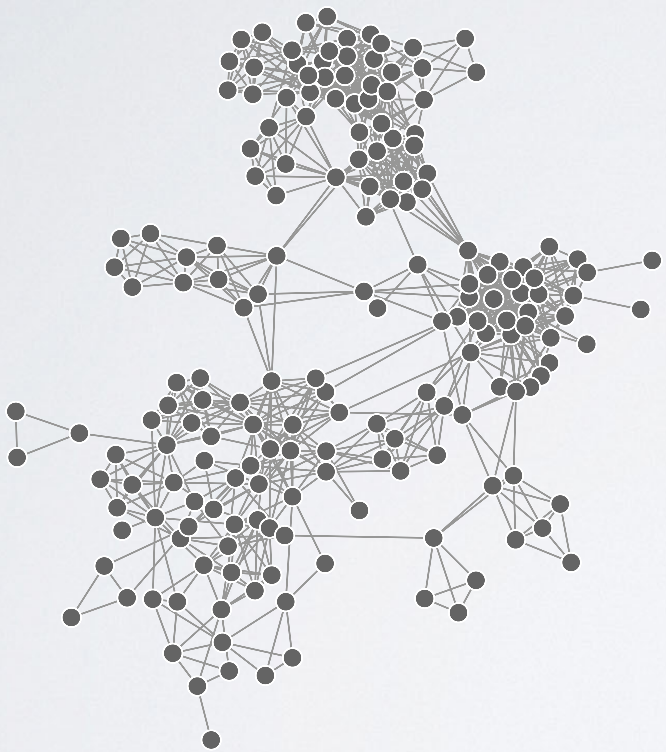
open questions:

- position and dynamics
- what does position predict?
- when does position *not* matter?

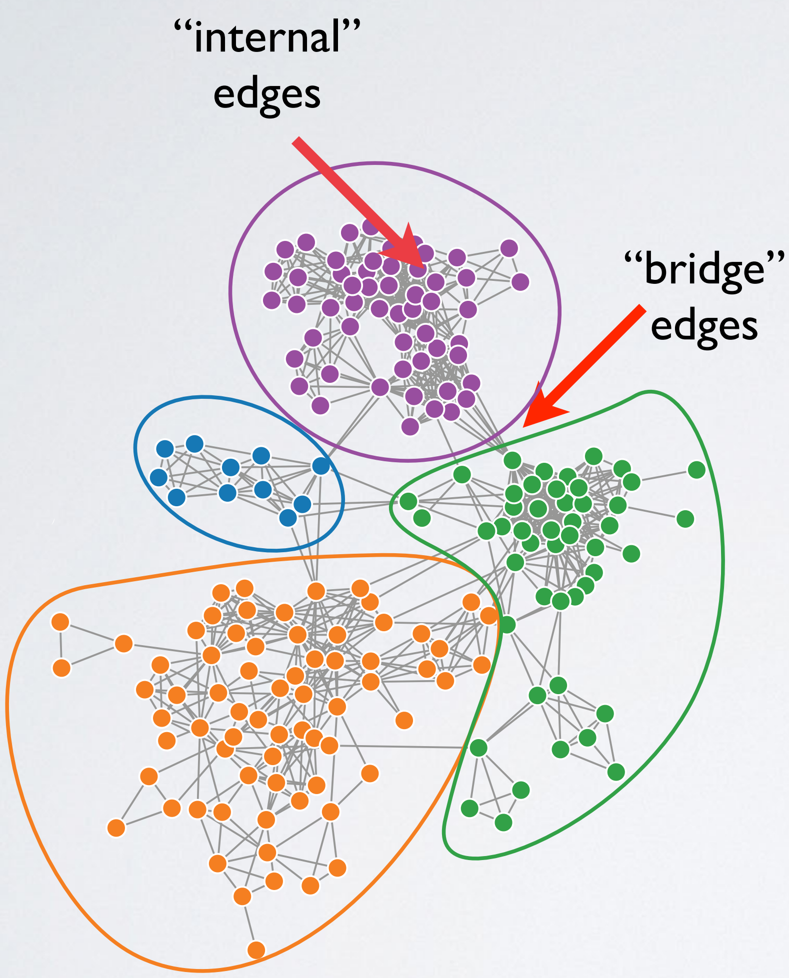


describing networks

community structure



community structure

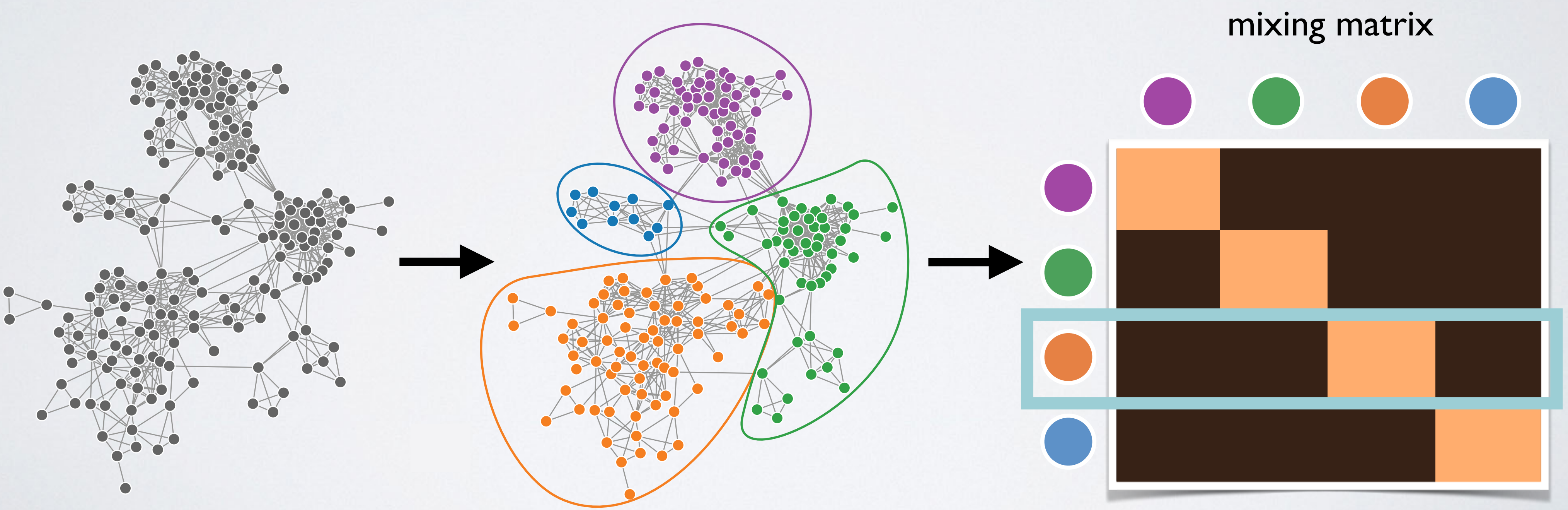


assortative community structure
(edges inside the groups)

community structure:
a *group of vertices* that
connect to *other groups* in
similar ways

community structure

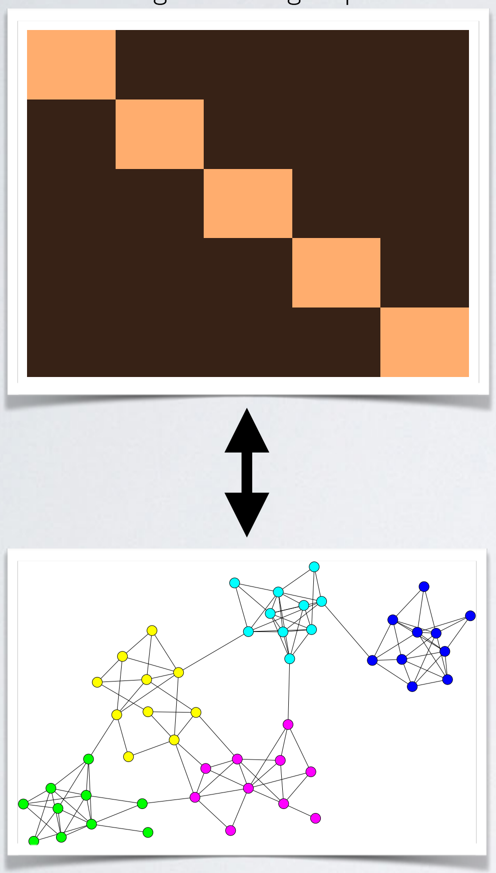
community structure:
a *group of vertices* that connect to *other groups* in similar ways



community structure

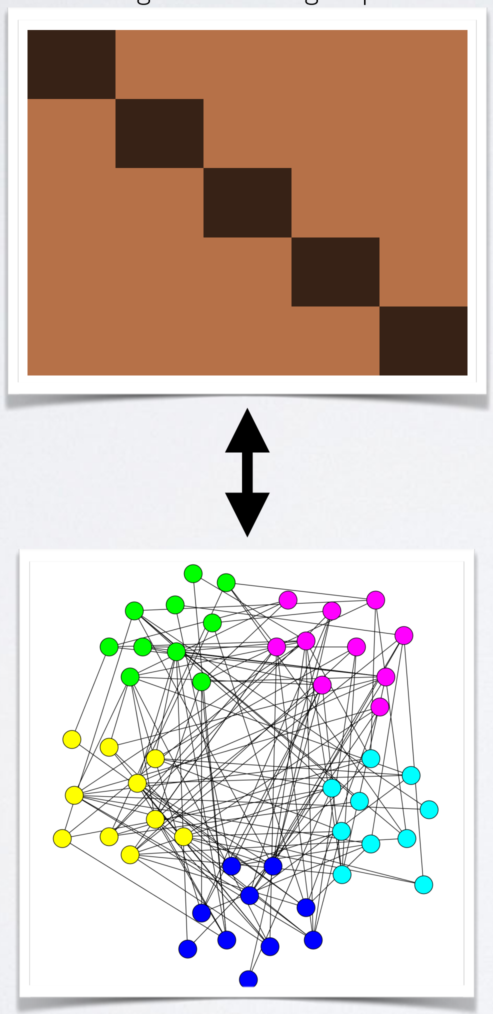
assortative

edges within groups



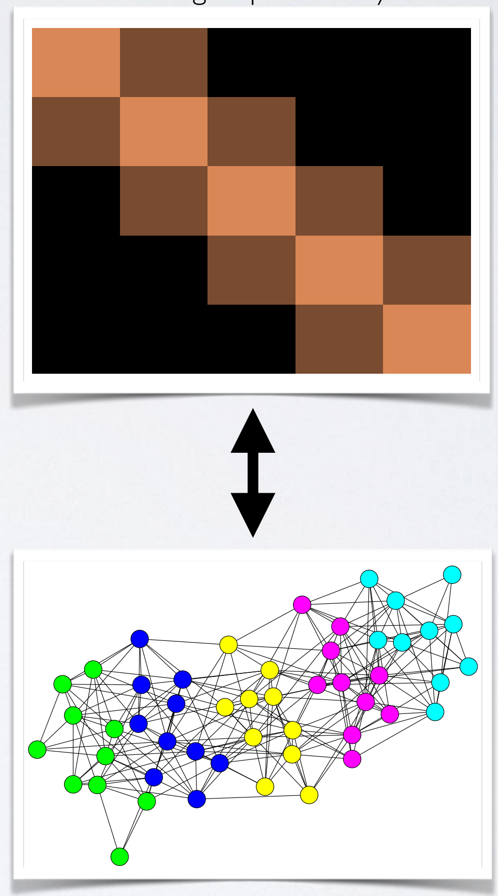
disassortative

edges between groups



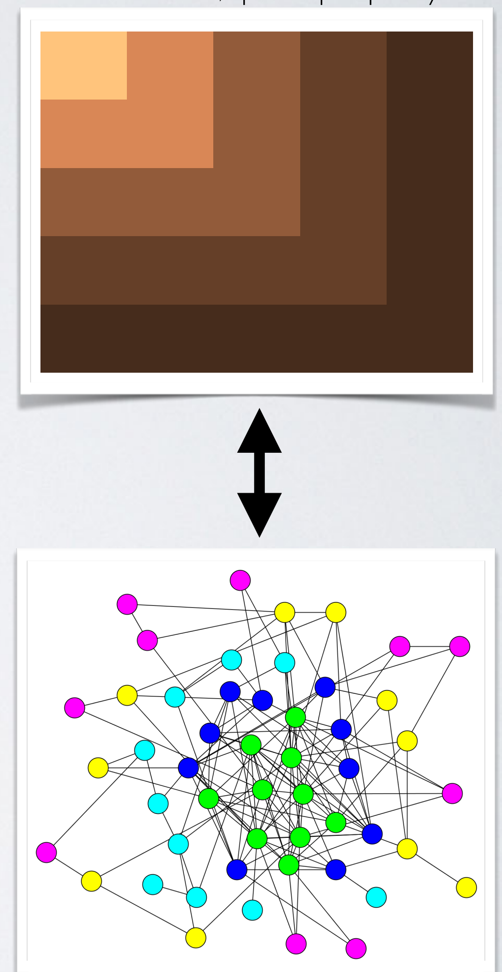
ordered

linear group hierarchy



core-periphery

dense core, sparse periphery



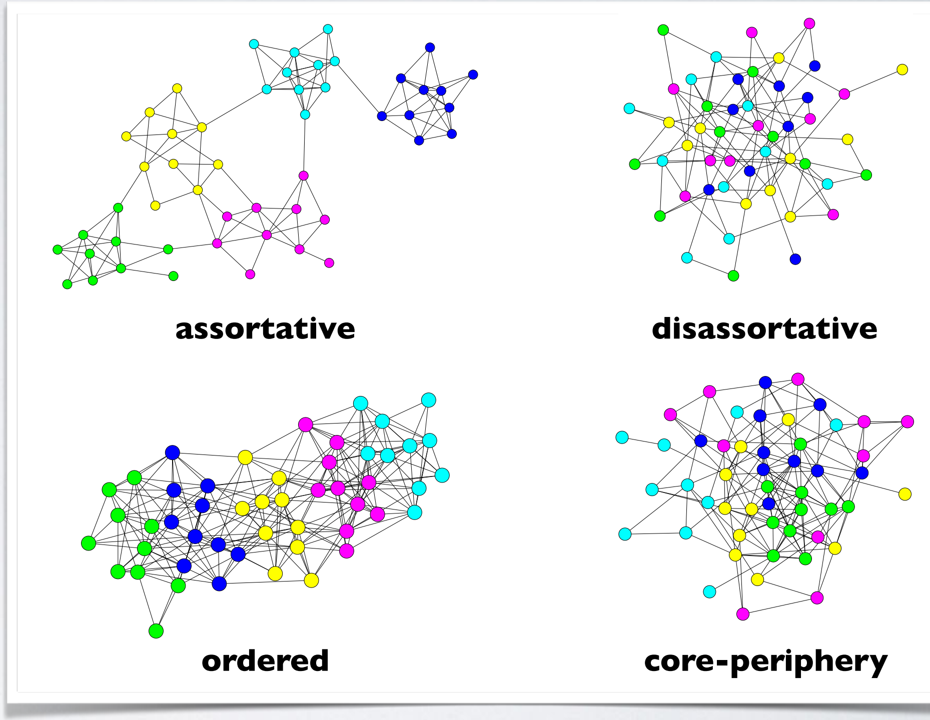
community structure

- enormous interest, especially since 2000
- dozens of algorithms for extracting various large-scale patterns
- hundreds of papers published
- spanning Physics, Computer Science, Statistics, Biology, Sociology, and more
- this was one of the first:

Community structure in social and biological networks

M. Girvan*†‡ and M. E. J. Newman*§ *PNAS* 2002

12,421+ citations on Google Scholar

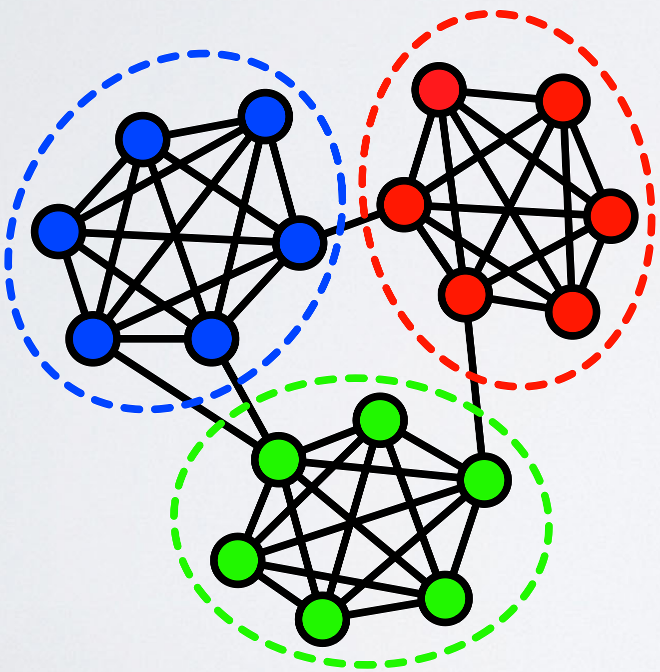


network communities

THE STRENGTH OF WEAK TIES:
A NETWORK THEORY REVISITED

1983

Mark Granovetter



most new job opportunities from "weak ties"

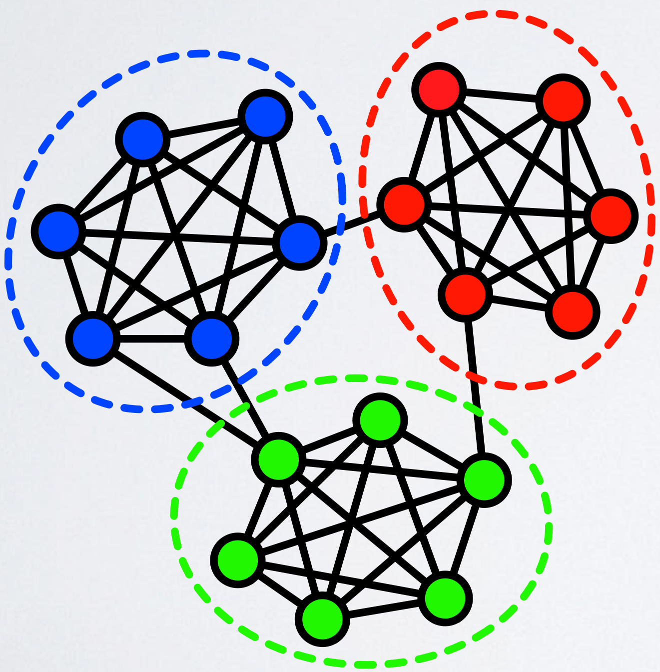
- within-community links = strong
- bridge links = weak

network communities

THE STRENGTH OF WEAK TIES: A NETWORK THEORY REVISITED

1983

Mark Granovetter



most new job opportunities from "weak ties"

- within-community links = strong
- bridge links = weak

why?

information propagates *quickly within* a community, but *slowly between* communities

network communities

Finding community structure in very large networks

Aaron Clauset, M. E. J. Newman, and Cristopher Moore 2004

amazon.com
co-purchasing network

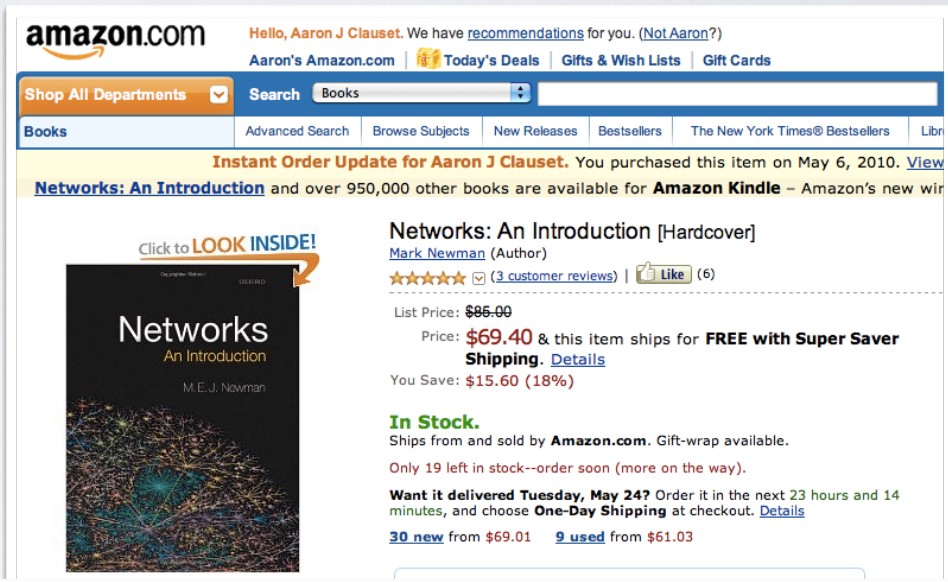
network communities

Finding community structure in very large networks

Aaron Clauset, M. E. J. Newman, and Cristopher Moore 2004

amazon.com
co-purchasing network
find partition that maximizes
modularity Q on those groups

$n = 409{,}687$ items
$m = 2{,}464{,}630$ edges

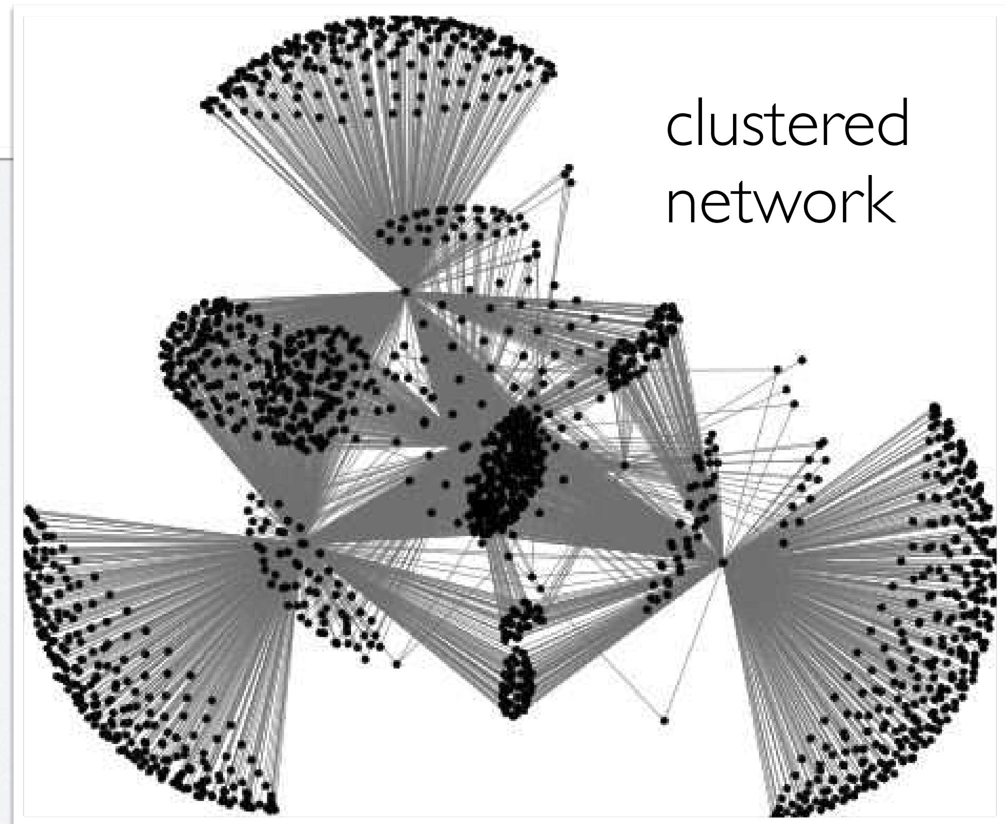


network communities

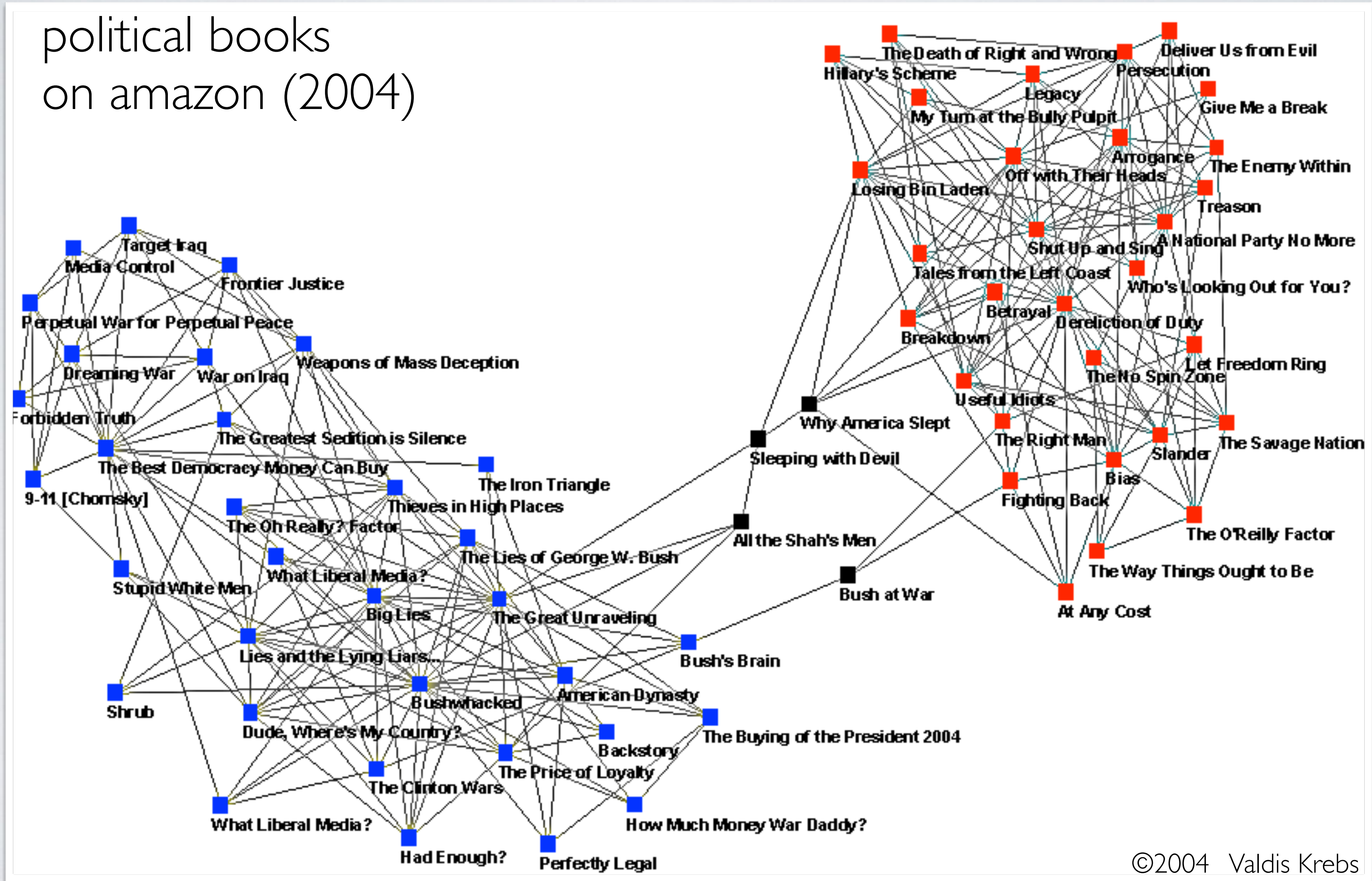
Rank	Size	Description
1	114538	General interest: politics; art/literature; general fiction; human nature; technical books; how things, people, computers, societies work, etc.
2	92276	The arts: videos, books, DVDs about the creative and performing arts
3	78661	Hobbies and interests I: self-help; self-education; popular science fiction, popular fantasy; leisure; etc.
4	54582	Hobbies and interests II: adventure books; video games/comics; some sports; some humor; some classic fiction; some western religious material; etc.
5	9872	classical music and related items
6	1904	children's videos, movies, music and books
7	1493	church/religious music; African-descent cultural books; homoerotic imagery
8	1101	pop horror; mystery/adventure fiction
9	1083	jazz; orchestral music; easy listening
10	947	engineering; practical fashion

purchases = interests

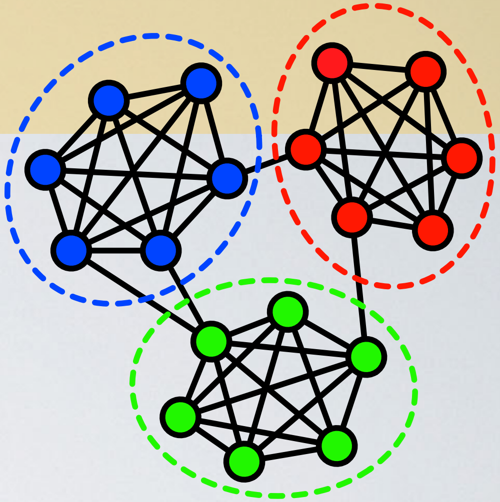
interests = clustered



network communities

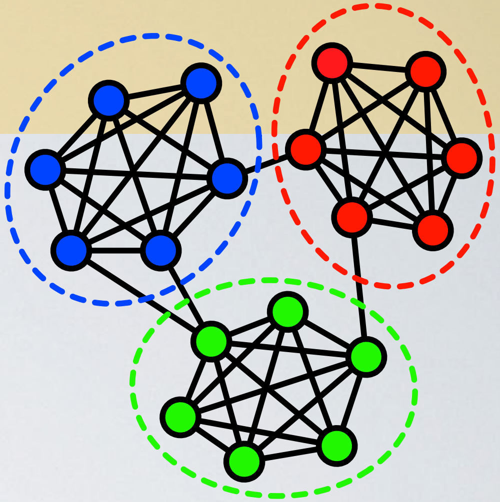


network communities



- *community = vertices with same pattern of inter-community connections*
- network macro-structure
- finding them like "network clustering"
  [there is *no best algorithm*, and there is No Free Lunch]
- allow us to *coarse grain* system structure
  [decompose heterogeneous structure into homogeneous blocks]
- constrains network synchronization, information flows, diffusion, influence

network communities



- *community = vertices with same pattern of inter-community connections*
- network macro-structure
- finding them like "network clustering"
  [there is *no best algorithm*, and there is No Free Lunch]
- allow us to *coarse grain* system structure
  [decompose heterogeneous structure into homogeneous blocks]
- constrains network synchronization, information flows, diffusion, influence

open questions:

- what processes generate communities?
- what impact on dynamics? network function?

describing networks

aka, summarizing a network's structure

$$f : G \rightarrow \underbrace{\{x_1, \ldots, x_k\}}_{\text{summary statistics}}$$

describing networks

aka, summarizing a network's structure

at the level of

nodes	meso	whole network
degree	group degree	size (num. nodes)
centrality (various)	group size	mean degree
reciprocity (local)	modularity	mean geodesic dist.
clustering coeff. (local)	mixing matrix	diameter
eccentricity	hierarchy	assortativity (degree)
…	motif counts	modularity
	…	reciprocity (global)
		clustering coeff. (global)
		…

describing networks

aka, summarizing a network's structure

- just counting things : $f : G \rightarrow \{x_1, \ldots, x_k\}$
- an infinite number of things you could count — which ones are meaningful to count?
- **warning** : *nearly all summary statistics correlate with degree*
- **things to ponder** : what is a node? what is an edge?
  - how do nodes **interact**?
  - what **causes** connections to change over time?
  - where is the **structure** : nodes? communities? network?
  - what is the role of node **degree** on dynamics?
  - what is role of node **position** on dynamics?

end of lecture 2

<u>lecture 3 : null models & inference for networks</u>

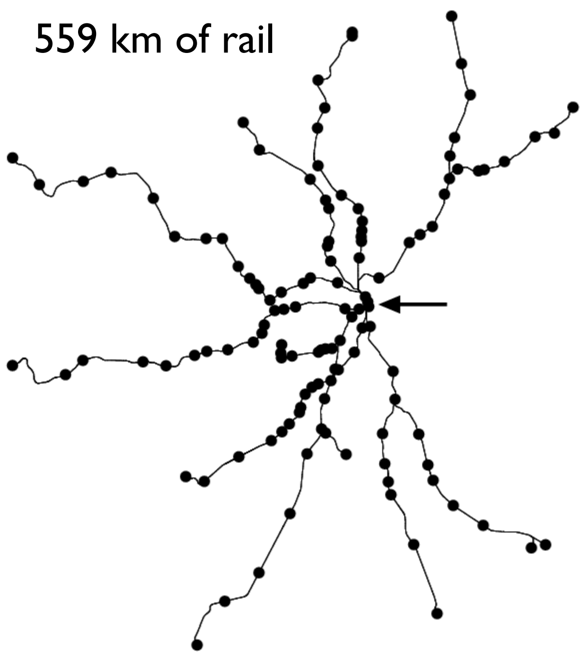
network position

an example

how does a network become centralized?

an example

optimizing paths for Boston commuter rail



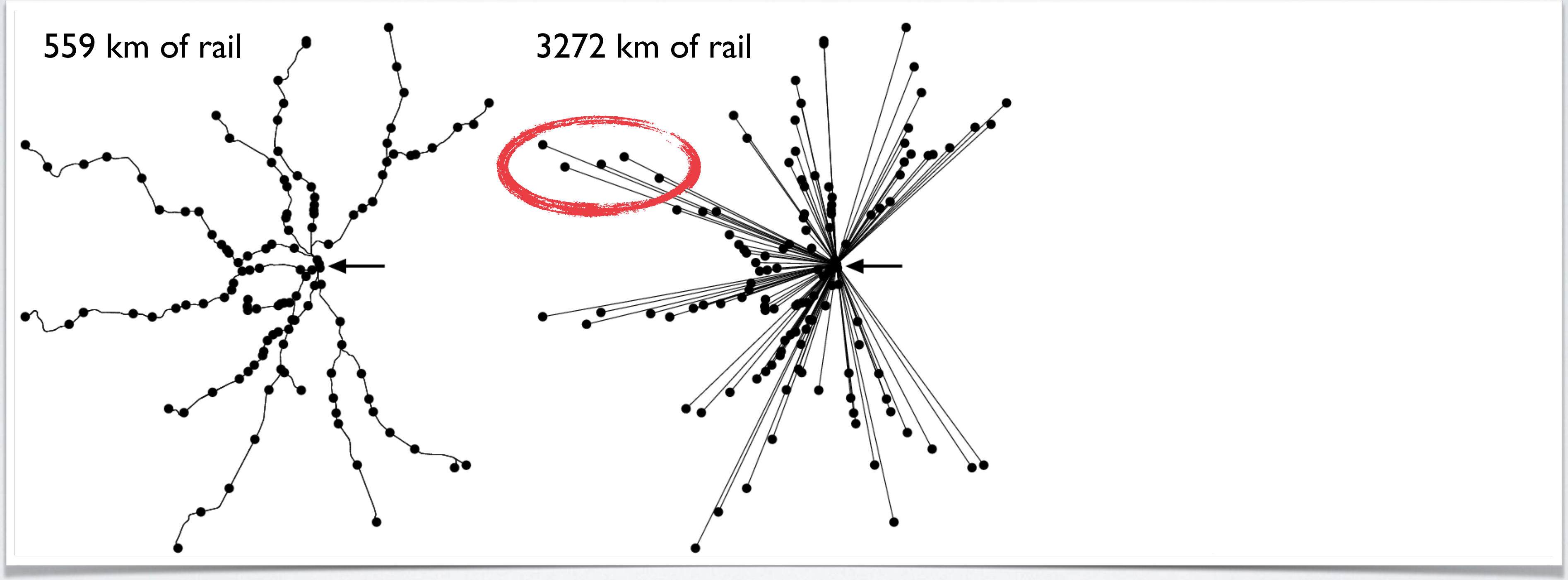
data

Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

network position

an example

optimizing paths for Boston commuter rail

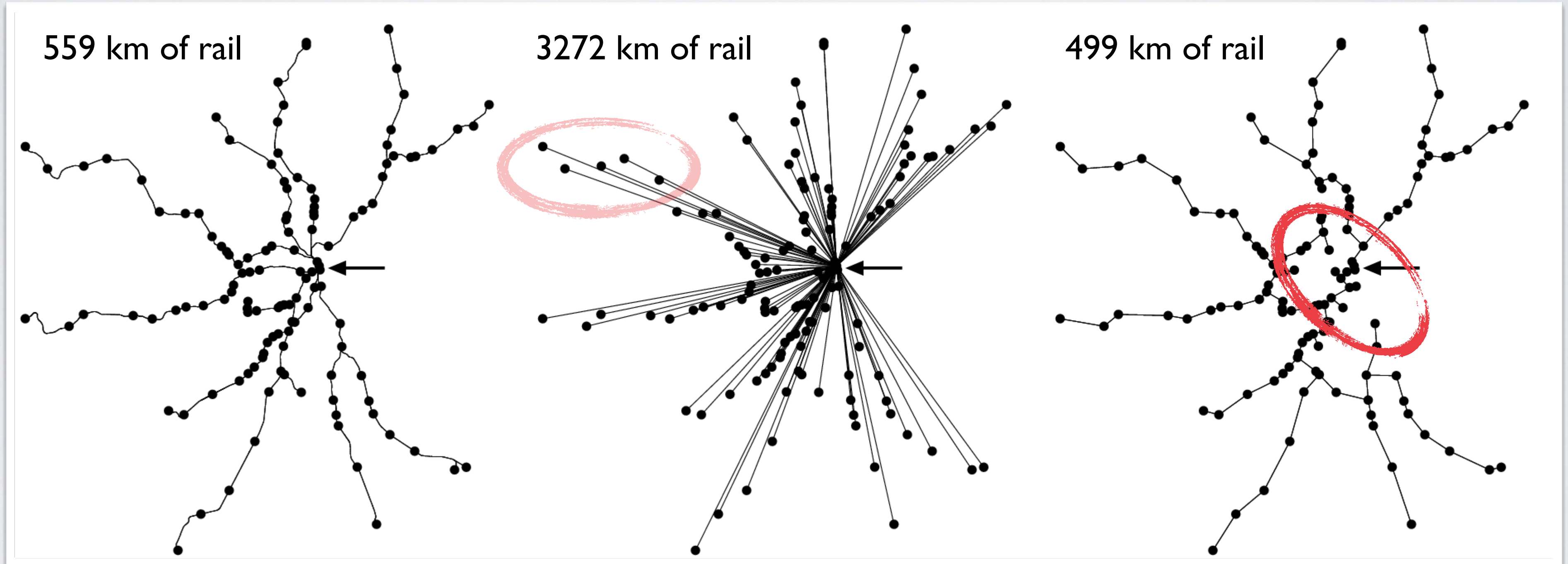


data minimum travel time

network position

an example

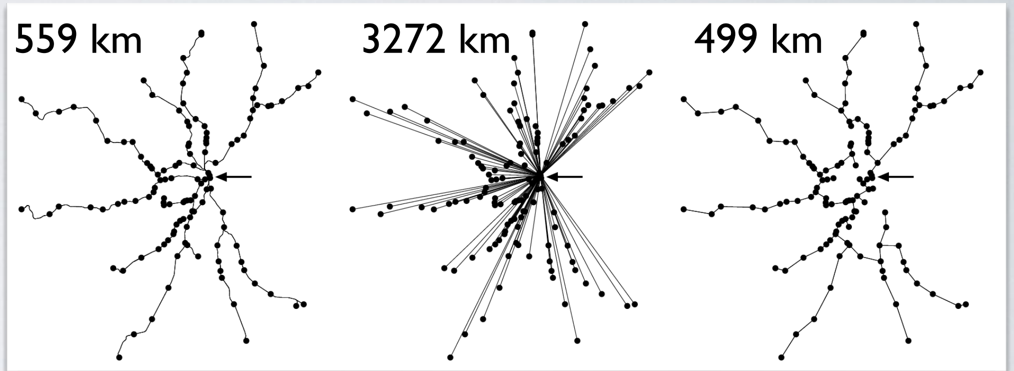
optimizing paths for Boston commuter rail



data minimum travel time minimum weight tree

Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

network position



$q = 1.14$ $q = 1$ $q = 1.61$

route factor

$$q = \frac{1}{n} \sum_{i=1}^{n} \frac{\ell_{i0}}{d_{i0}}$$

mean ratio of distance along edges ℓ_{i0} to direct Euclidean distance d_{i0} to root 0

Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

network position

a simple model

embed n vertices in a plane

until all vertices connected

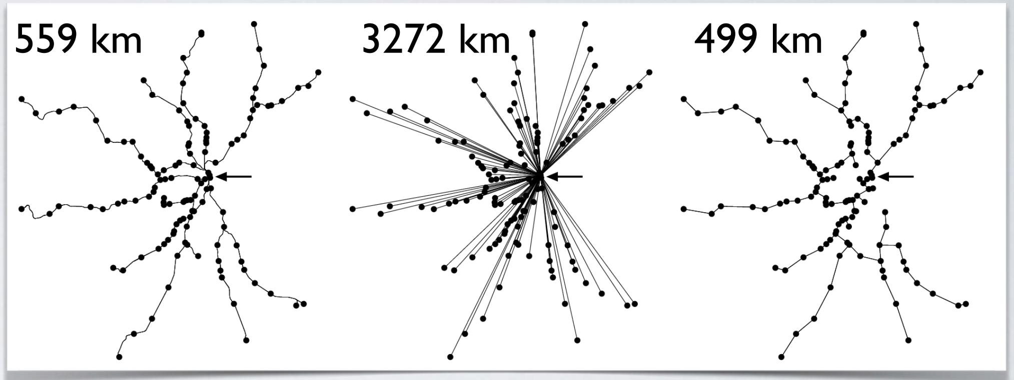
add edge (i, j) with minimum value for

$$w_{ij} = d_{ij} + \beta \ell_{j0}$$

distance from i to j

parameter

route length to root



$q = 1.14$ $q = 1$ $q = 1.61$

$\beta = 0$ ⟶ minimum spanning tree*

$\beta > 0$ ⟶ prefer shorter paths to root

Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

*this is exactly Prim's algorithm for MSTs

network position

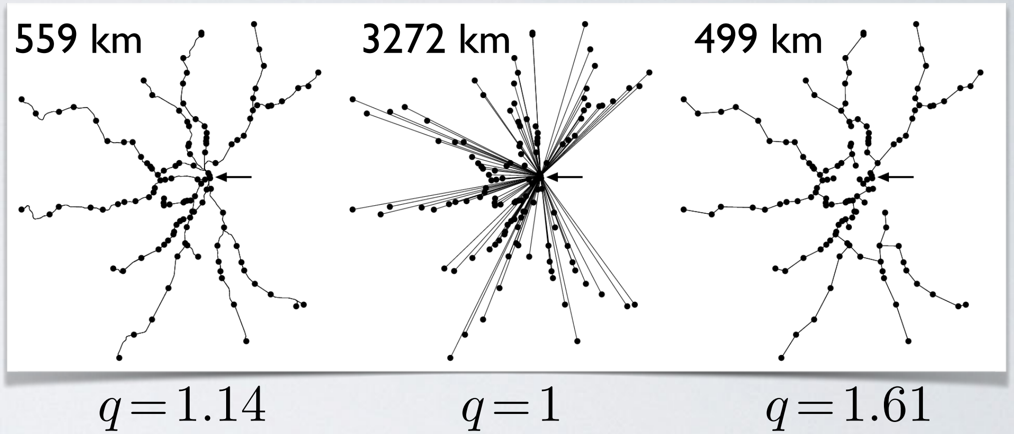
a simple model

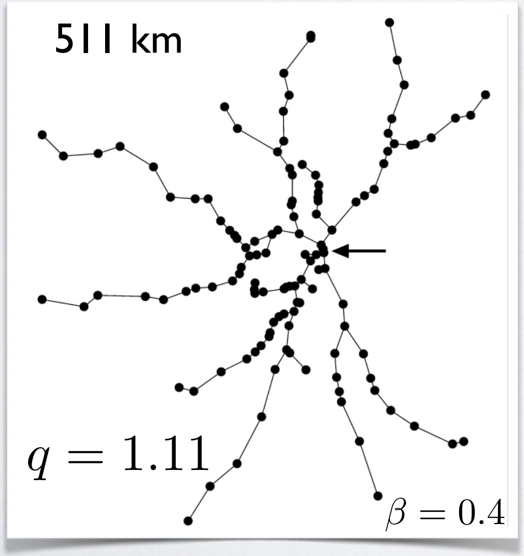
embed n vertices in a plane

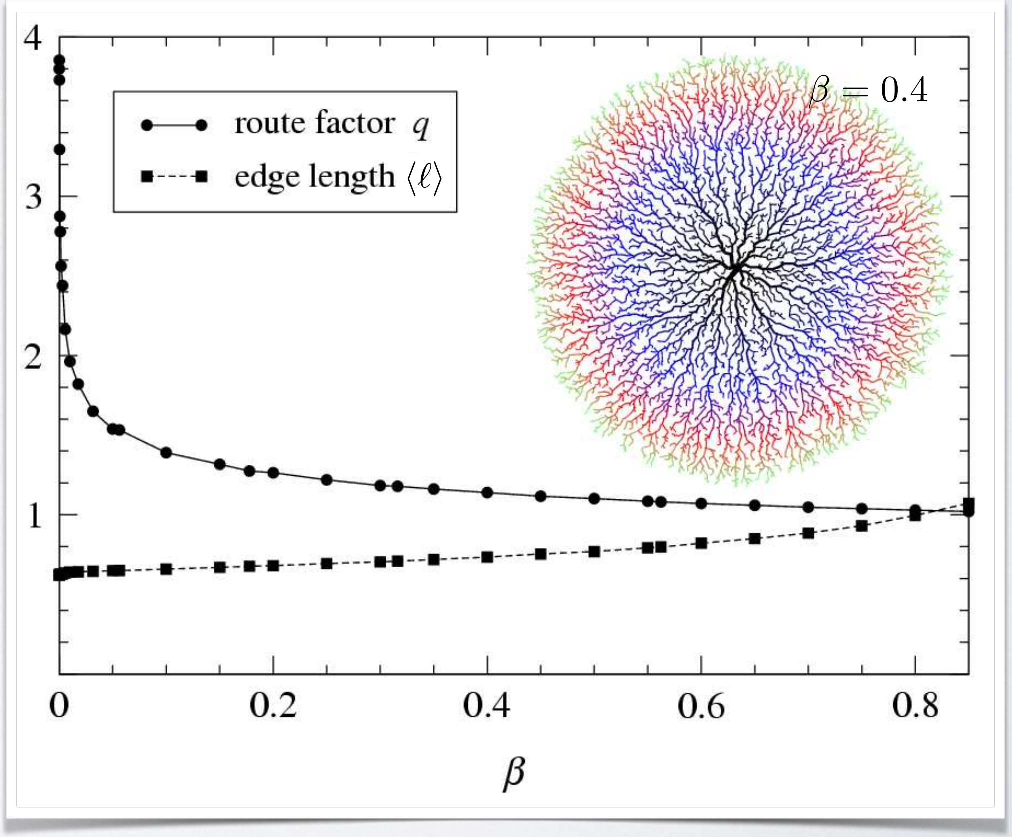
until all vertices connected

add edge (i, j) with minimum value for

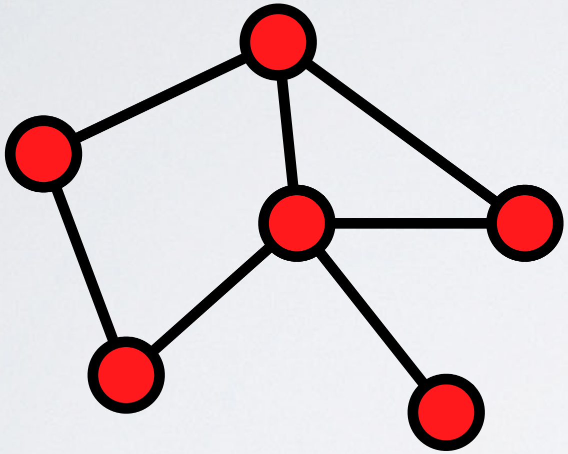
$$w_{ij} = d_{ij} + \beta \ell_{j0}$$







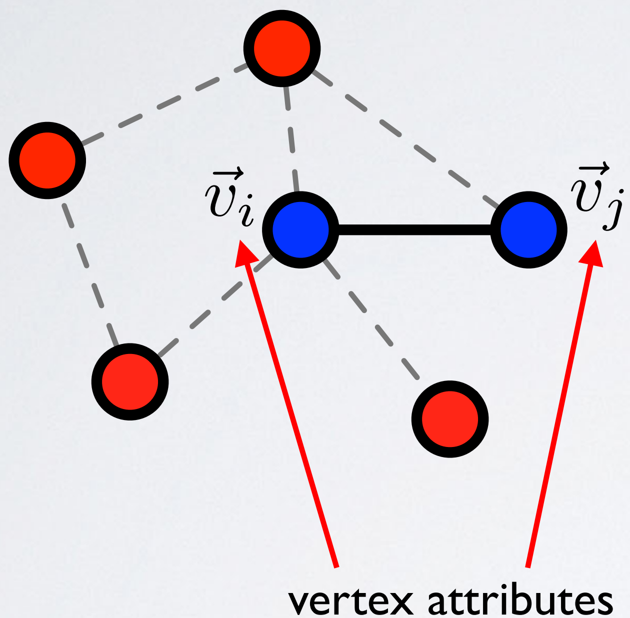
Gastner & Newman, *J. Stat. Mech.* P01015 (2006)



homophily and assortative mixing

like links with like

assortative mixing



homophily and assortative mixing

like links with like

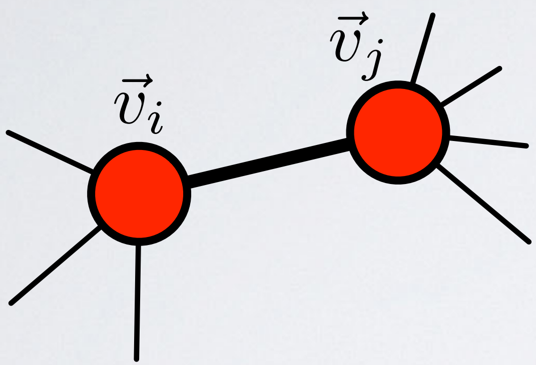
assortativity coefficient r quantifies homophily

three types:

- scalar attributes
- vertex degrees
- categorical variables

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

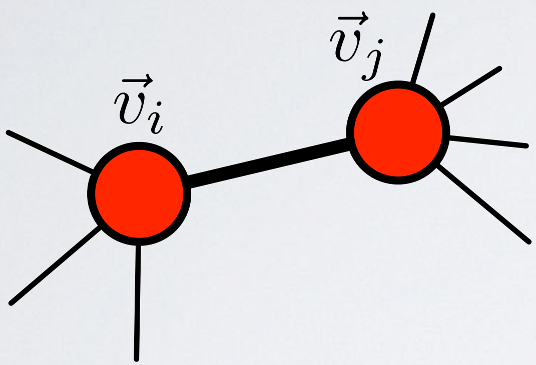
scalar attributes:
mean value across ties

$$\mu = \frac{1}{2m}\sum_i \sum_j A_{ij} v_i$$

$$\boxed{= \frac{1}{2m}\sum_i k_i v_i}$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

scalar attributes:
covariance across ties

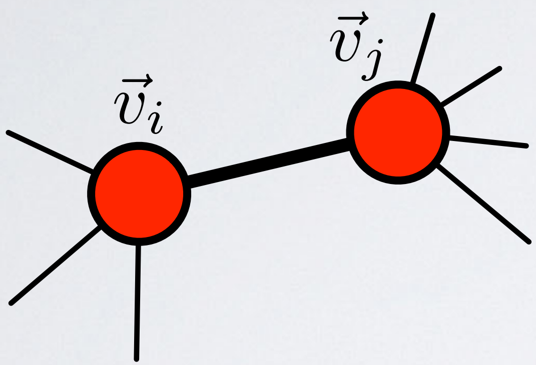
$$\mathrm{cov}(v_i, v_j) = \frac{\sum_{ij} A_{ij}(v_i - \mu)(v_j - \mu)}{\sum_{ij} A_{ij}}$$

$$\left(\mu = \frac{1}{2m}\sum_i k_i v_i\right) \qquad = \frac{1}{2m}\sum_{ij} A_{ij} v_i v_j - \mu^2$$

$$\boxed{= \frac{1}{2m}\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right) v_i v_j}$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

assortativity coefficient (scalar)

$$r = \frac{\mathrm{cov}(v_i, v_j)}{\mathrm{var}(v_i, v_j)}$$

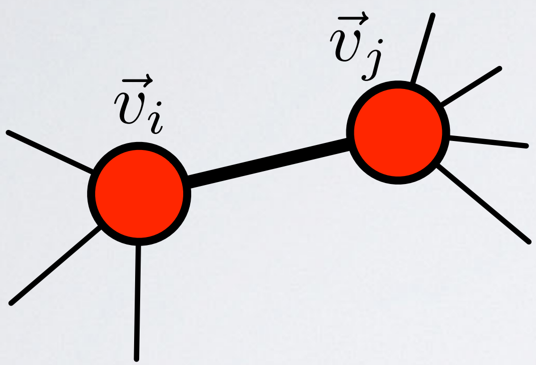
$$= \boxed{\frac{\sum_{ij}(A_{ij} - k_i k_j/2m)\, v_i v_j}{\sum_{ij} k_i \delta_{ij} - k_i k_j/2m}}$$

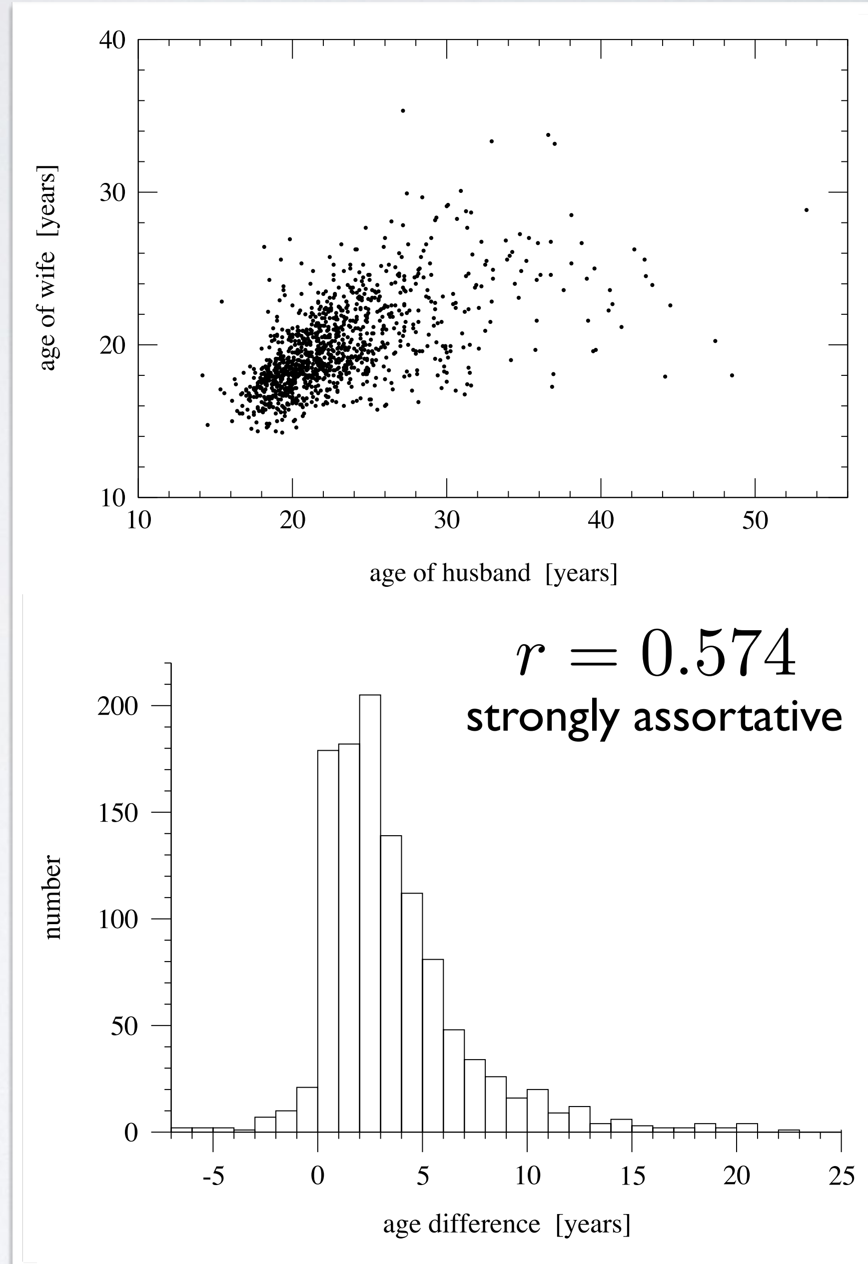
[this is just a Pearson correlation across edges]

$$-1 \leq r \leq 1$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing





$r = 0.574$

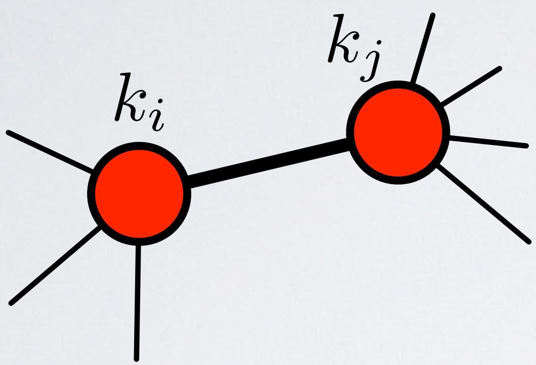
strongly assortative

(top) scatter plot of ages of 1141 married couples at time of marriage [1995 US National Survey of Family Growth]

(bottom) histogram of age differences (M-F) for same data

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

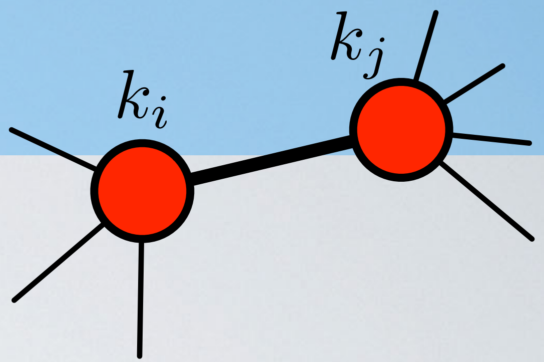
degree:

just another scalar *

Newman, *Phys. Rev. E* **67**, 026126 (2003).

* the assortativity coefficient formula simplifies somewhat in this case. see the Ref in the left corner for more details

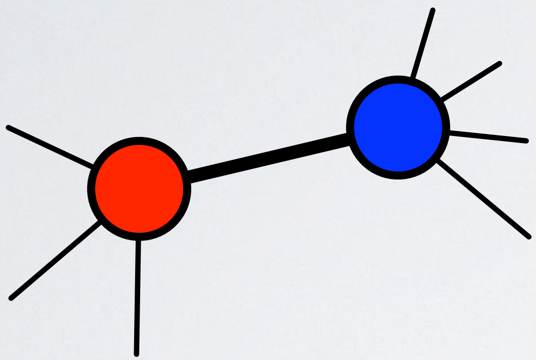
assortative mixing



	network	type	size n	degree assortativity r	error σ_r
social	physics coauthorship	undirected	52 909	0.363	0.002
	biology coauthorship	undirected	1 520 251	0.127	0.0004
	mathematics coauthorship	undirected	253 339	0.120	0.002
	film actor collaborations	undirected	449 913	0.208	0.0002
	company directors	undirected	7 673	0.276	0.004
	student relationships	undirected	573	−0.029	0.037
	email address books	directed	16 881	0.092	0.004
technological	power grid	undirected	4 941	−0.003	0.013
	Internet	undirected	10 697	−0.189	0.002
	World-Wide Web	directed	269 504	−0.067	0.0002
	software dependencies	directed	3 162	−0.016	0.020
biological	protein interactions	undirected	2 115	−0.156	0.010
	metabolic network	undirected	765	−0.240	0.007
	neural network	directed	307	−0.226	0.016
	marine food web	directed	134	−0.263	0.037
	freshwater food web	directed	92	−0.326	0.031

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

categorical variables:

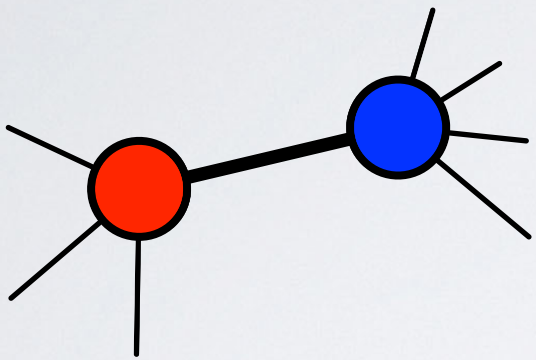
let e_{ij} be fraction of edges connecting vertices of type i to vertices of type j

matrix sum $$\sum_{ij} e_{ij} = 1$$

marginals $$\sum_{j} e_{ij} = a_i \qquad \sum_{i} e_{ij} = b_j$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

categorical variables:
assortativity coefficient*

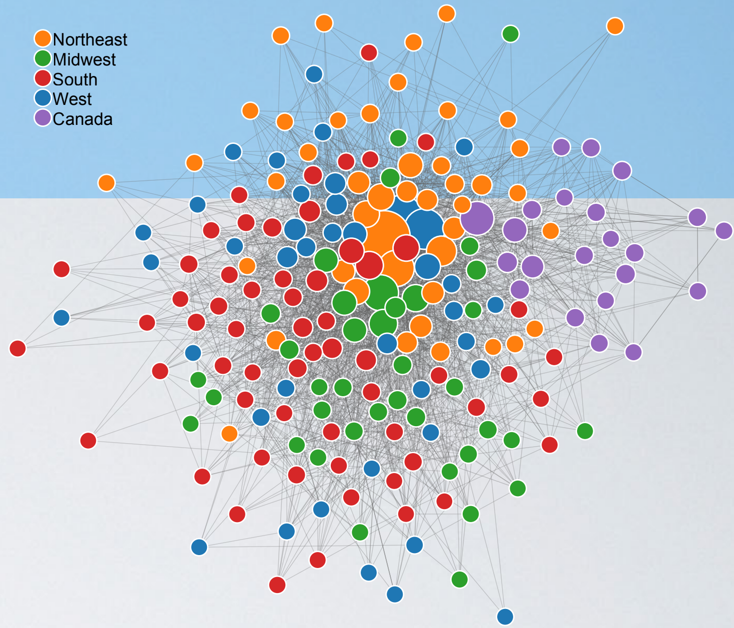
$$r = \frac{\sum_i e_{ii} - \sum_i a_i b_i}{1 - \sum_i a_i b_i}$$

$$= \frac{\mathrm{Tr}\,\mathbf{e} - ||\mathbf{e}^2||}{1 - ||\mathbf{e}^2||}$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

* this equation is equivalent to the popular *modularity* measure *Q* used to score the strength of community structure

assortative mixing



4388 Computer Science faculty
vertices are PhD granting institutions in North America
edge (u, v) means PhD at u and now faculty at v
labels are US census regions + Canada

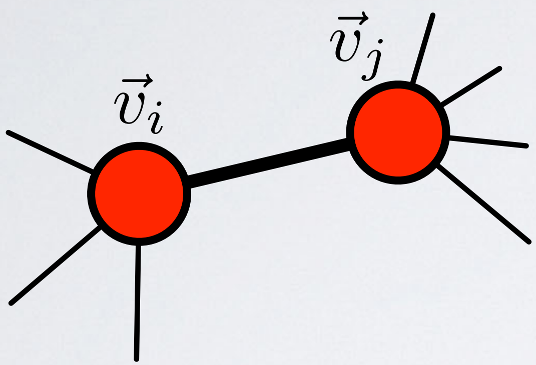
	Northeast	Midwest	South	West	Canada	a_u
Northeast	**0.237**	0.084	0.098	0.104	0.028	0.552
Midwest	0.084	**0.134**	0.088	0.059	0.016	0.381
South	0.098	0.088	**0.166**	0.068	0.012	0.432
West	0.104	0.059	0.068	**0.145**	0.017	0.393
Canada	0.028	0.016	0.012	0.017	**0.170**	0.242
a_u	0.552	0.381	0.432	0.393	0.242	

$$r = 0.215$$

moderately assortative

Clauset, Arbesman, Larremore, *Science Advances* **1**, e1400005 (2015).

assortative mixing



homophily and assortative mixing

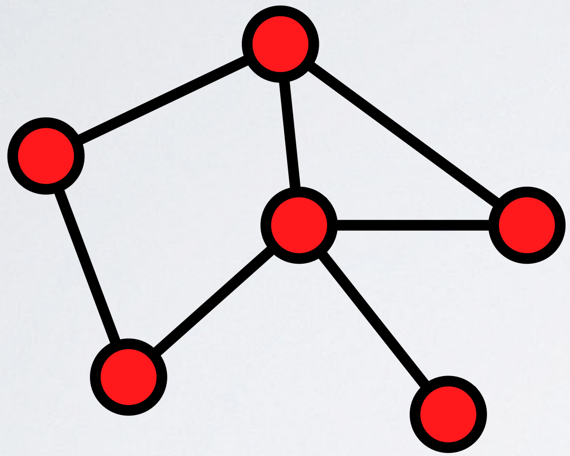
like links with like

- random graphs tend to be disassortative $r \leq 0$ because the mixing is uniform
- social networks (apparently) highly assortative, in every way (attribute, degree, category)
- extremal values $r \approx \{-1, 1\}$ suggest underlying mechanism on that variable

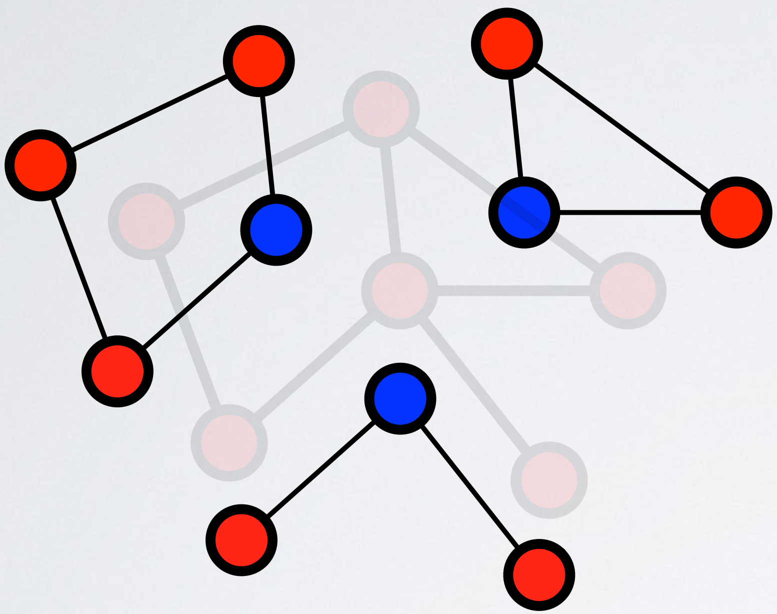
Newman, *Phys. Rev. E* **67**, 026126 (2003).

describing networks

motifs

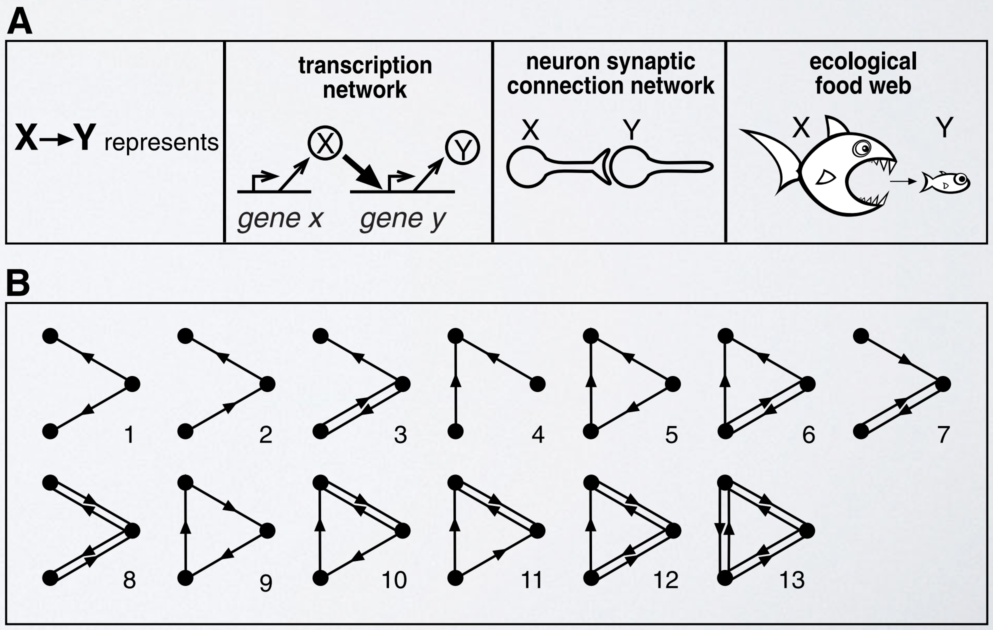


describing networks



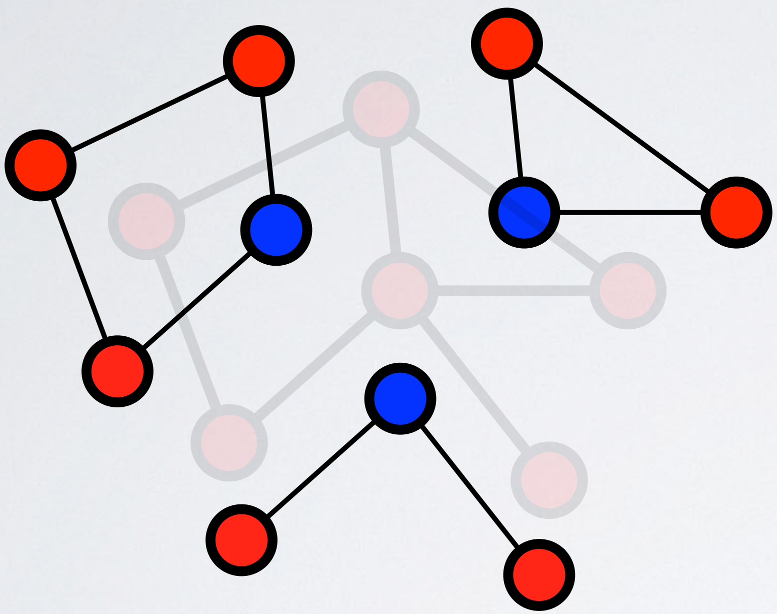
motifs:
small subgraphs (of interest), which we then count

compare counts against null model (random graph model)



Milo et al., *Science* **298**, 824-827 (2002).

describing networks



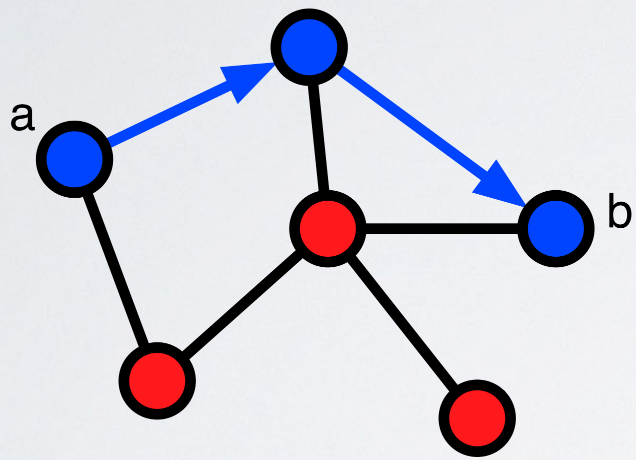
motifs:
small subgraphs (of interest), which we then count

compare counts against null model (random graph model)

- efficient counting is tricky (combinatorics + graph isomorphism)
- choice of null model key
- lots of work in this area, mainly in molecular biology and neuroscience
- see

Sporns and Kotter, *PLoS Biol.* **2**, e369 (2004)
Matias et al., *REVSTAT* **4**, 31-51 (2006)
Wong et al., Brief. in Bioinfo. **13**, 202-215 (2011)

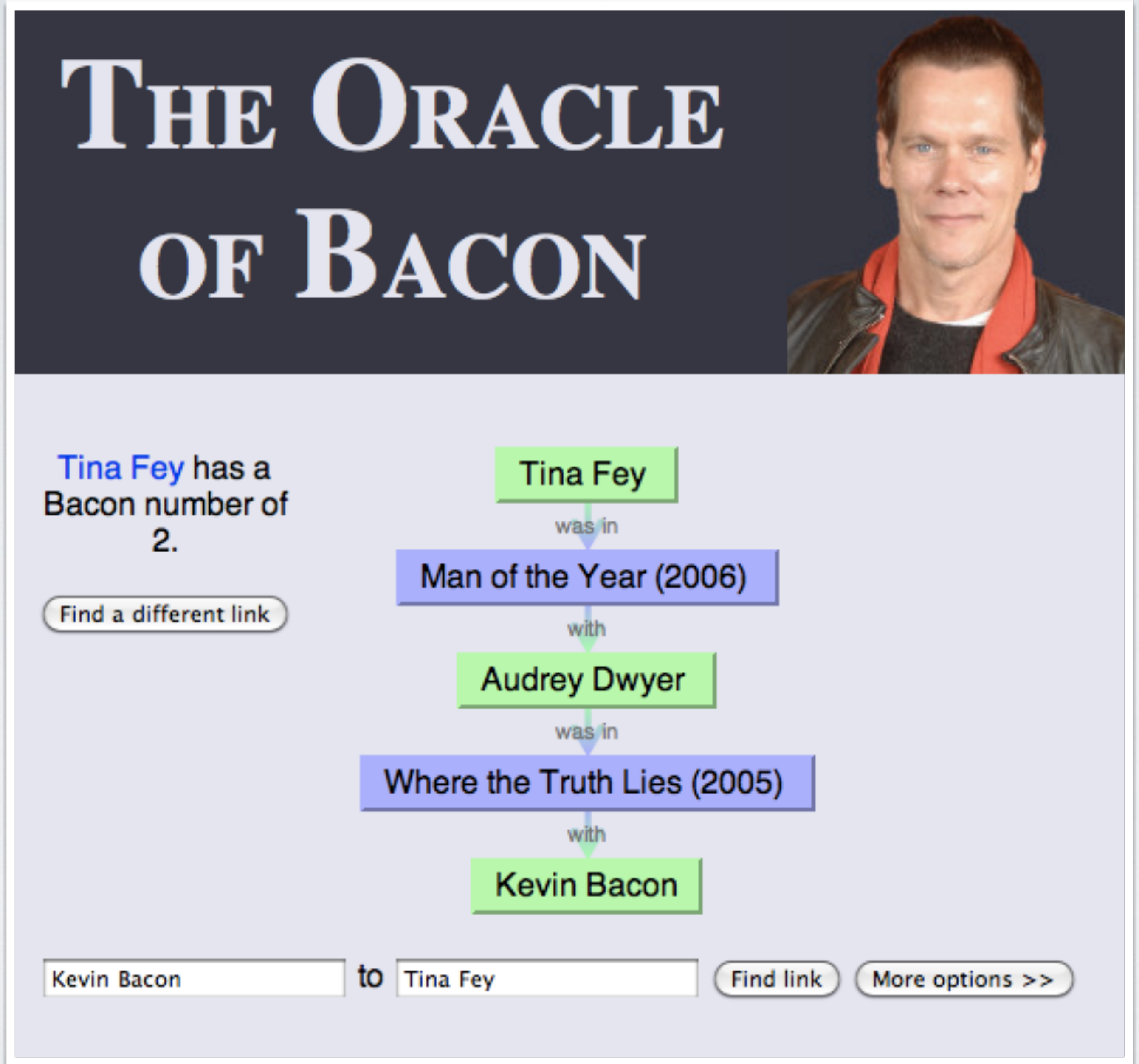
describing networks



path:
number of "hops" between two nodes

$$\ell_{a \to b} = 2$$

network paths



http://oracleofbacon.org

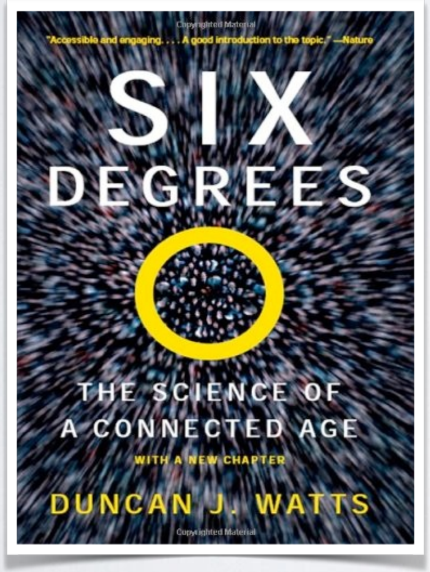
1994 - present

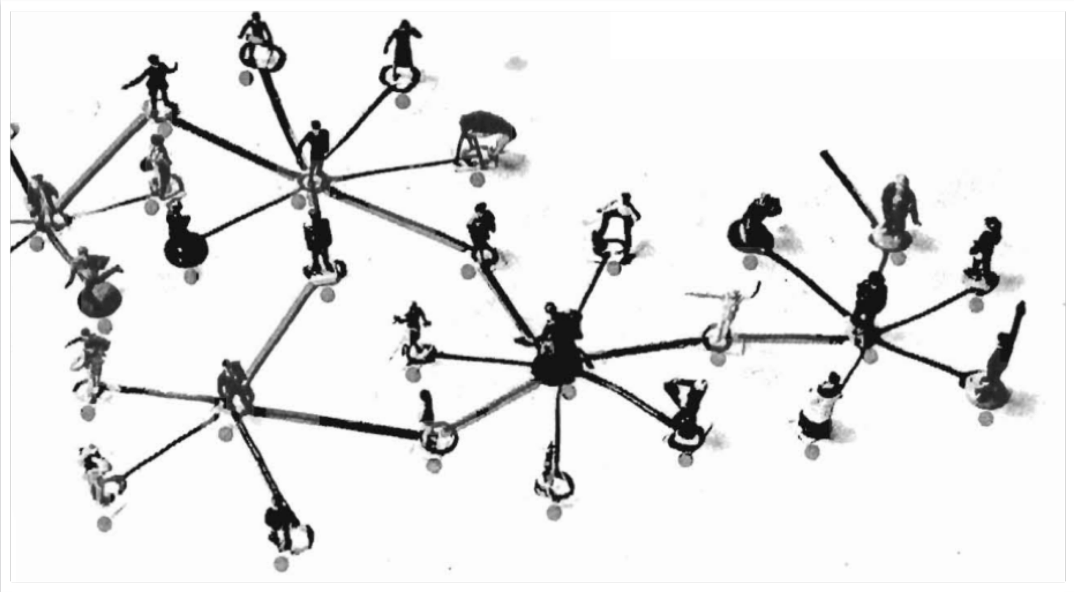
network paths

The Small-World Problem

By Stanley Milgram

1967





network paths

The Small-World Problem

By Stanley Milgram

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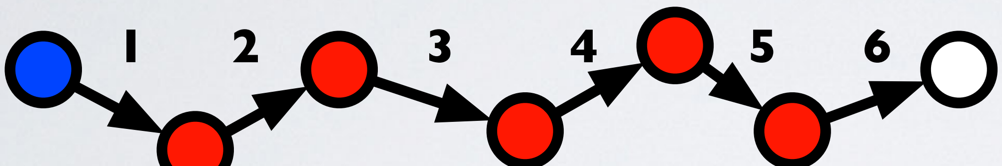
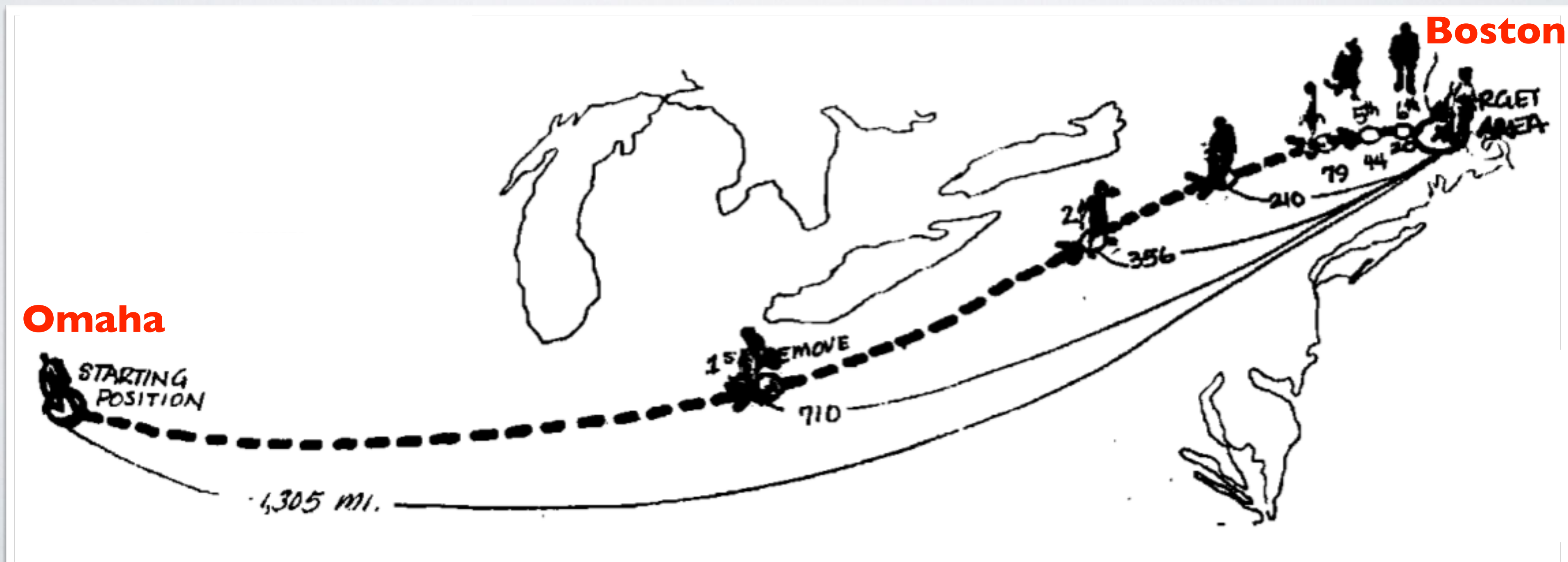


$$\langle \ell \rangle = 6$$

6-degrees of separation



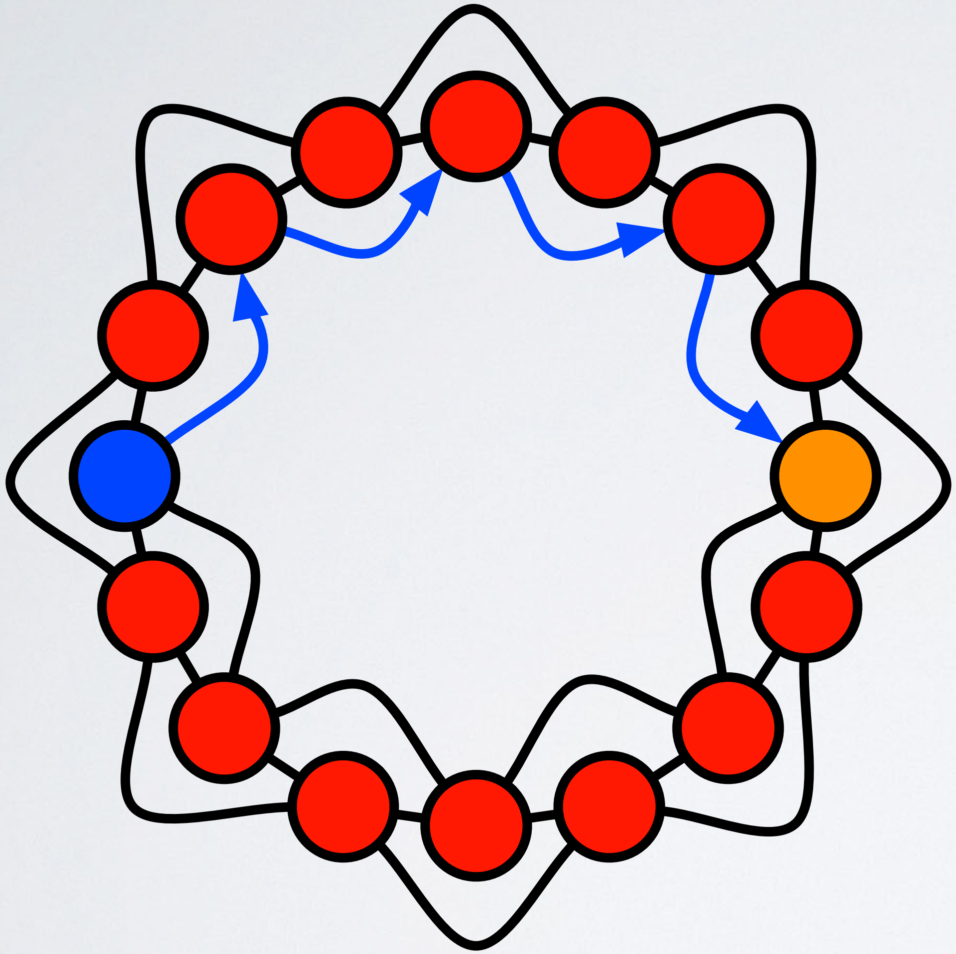
network paths

Collective dynamics of 'small-world' networks

Duncan J. Watts* & Steven H. Strogatz

1998

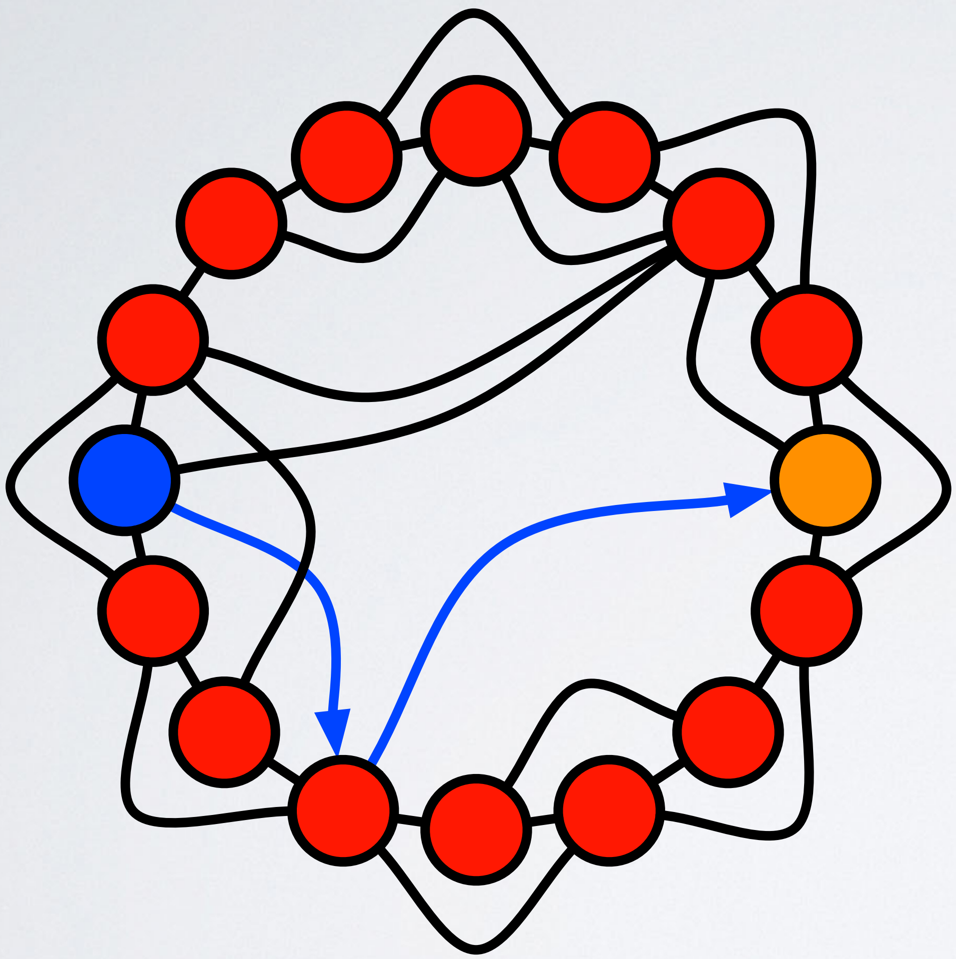
network paths



all links "local"

- most nodes far away
- high "clustering"

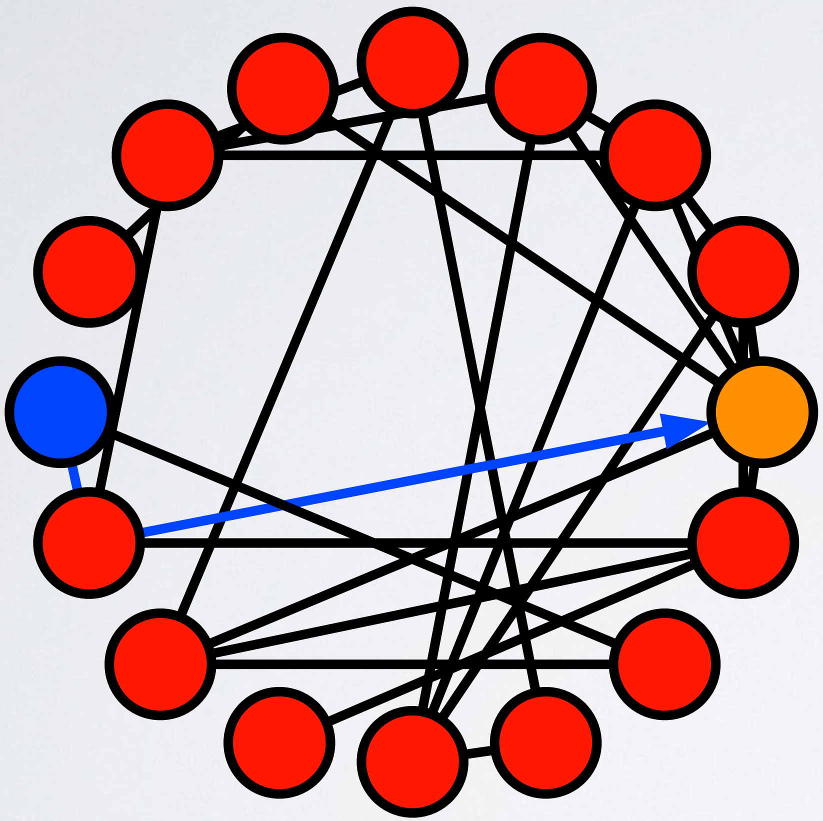
network paths



most links "local"
some links random

- most nodes near
- high "clustering"
- short paths can be found

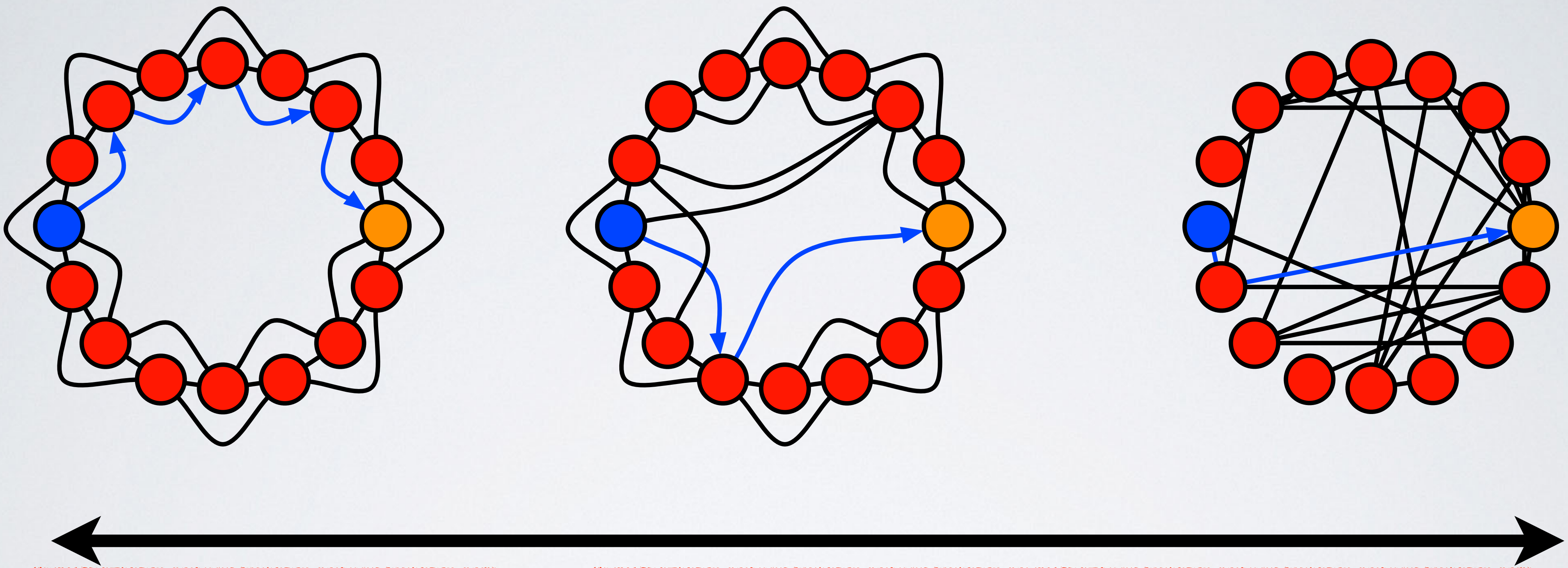
network paths



all links random

- Erdos-Renyi graph
- most nodes near
- short paths hard to find
- no "clustering"

it's a small world after all



big world
high clustering

small world
high clustering

small world
low clustering

it’s a small world after all

Geographic routing in social networks

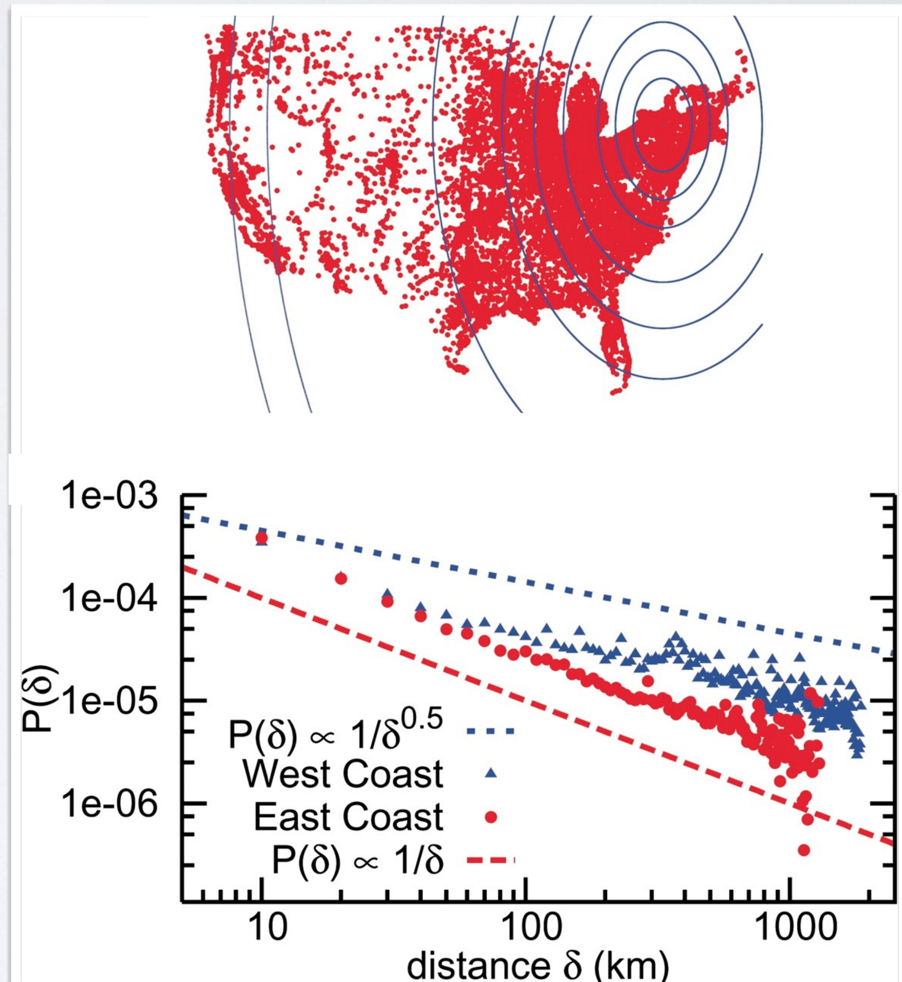
2005

David Liben-Nowell[*†‡§], Jasmine Novak[†], Ravi Kumar[†¶], Prabhakar Raghavan[¶‖], and Andrew Tomkins[†¶]

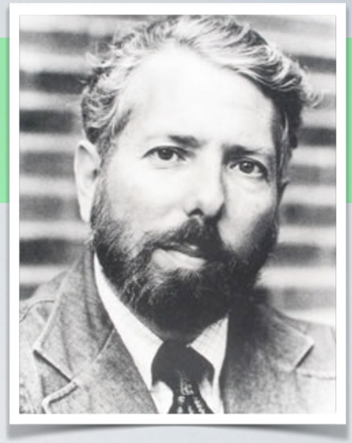
LiVEJOURNAL™

495,836 geo-located users

- most links “local”
- remaining links span all scales
- high clustering
- small “diameter”



network paths



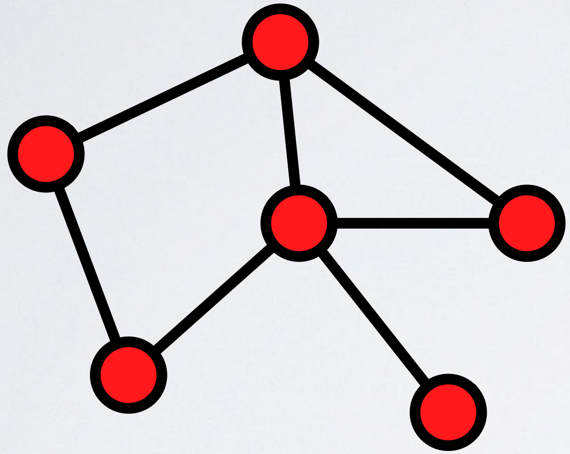
- path = sequence of edges $a \to \cdots \to b$
- many short paths = "small world"
- social world is surprisingly small, yet highly "clustered"
  (many locally dense groups)

open questions:

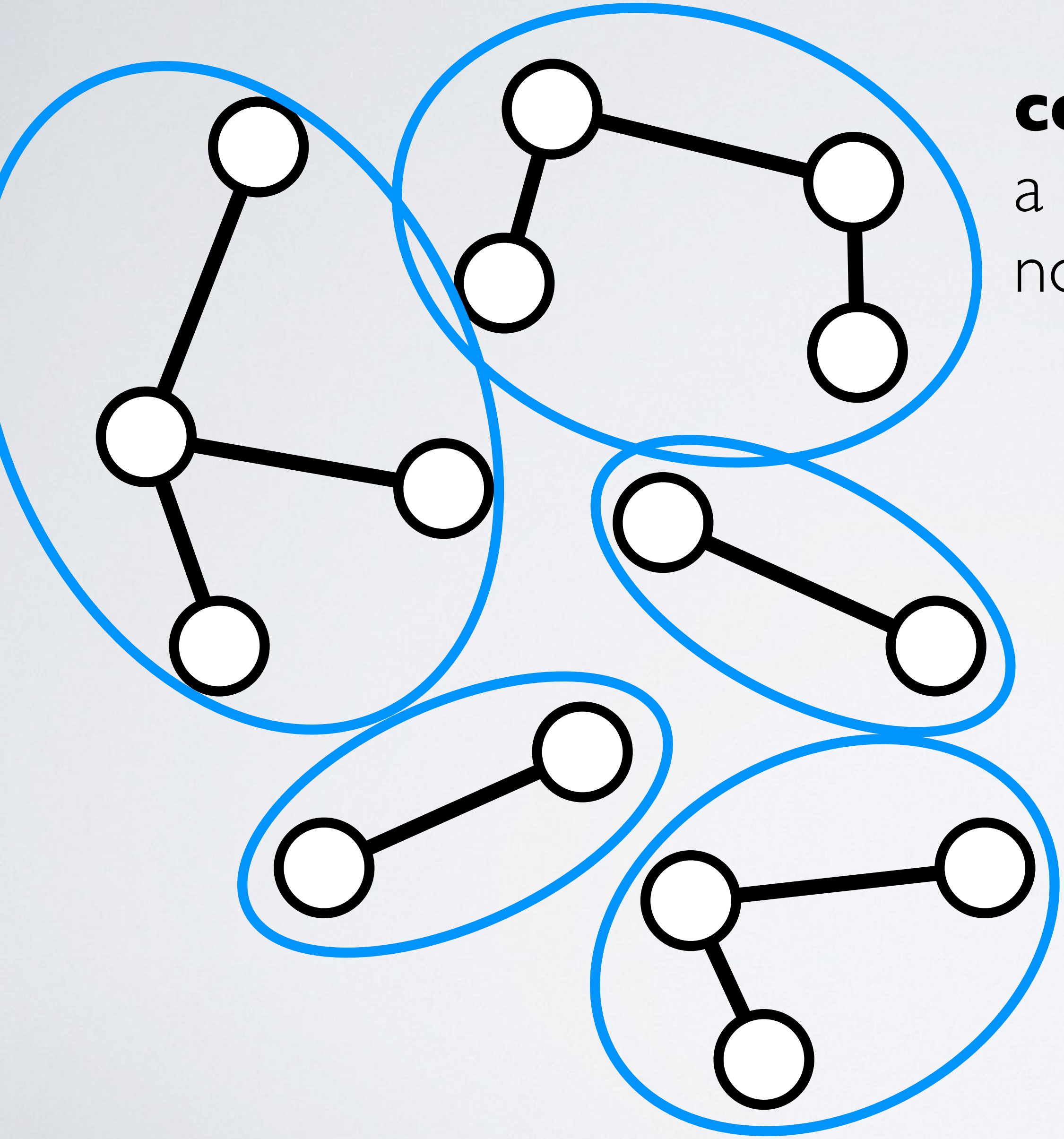
- how do big social networks self-organize?
- what processes shrink big worlds?
- social information filtering

describing networks

components

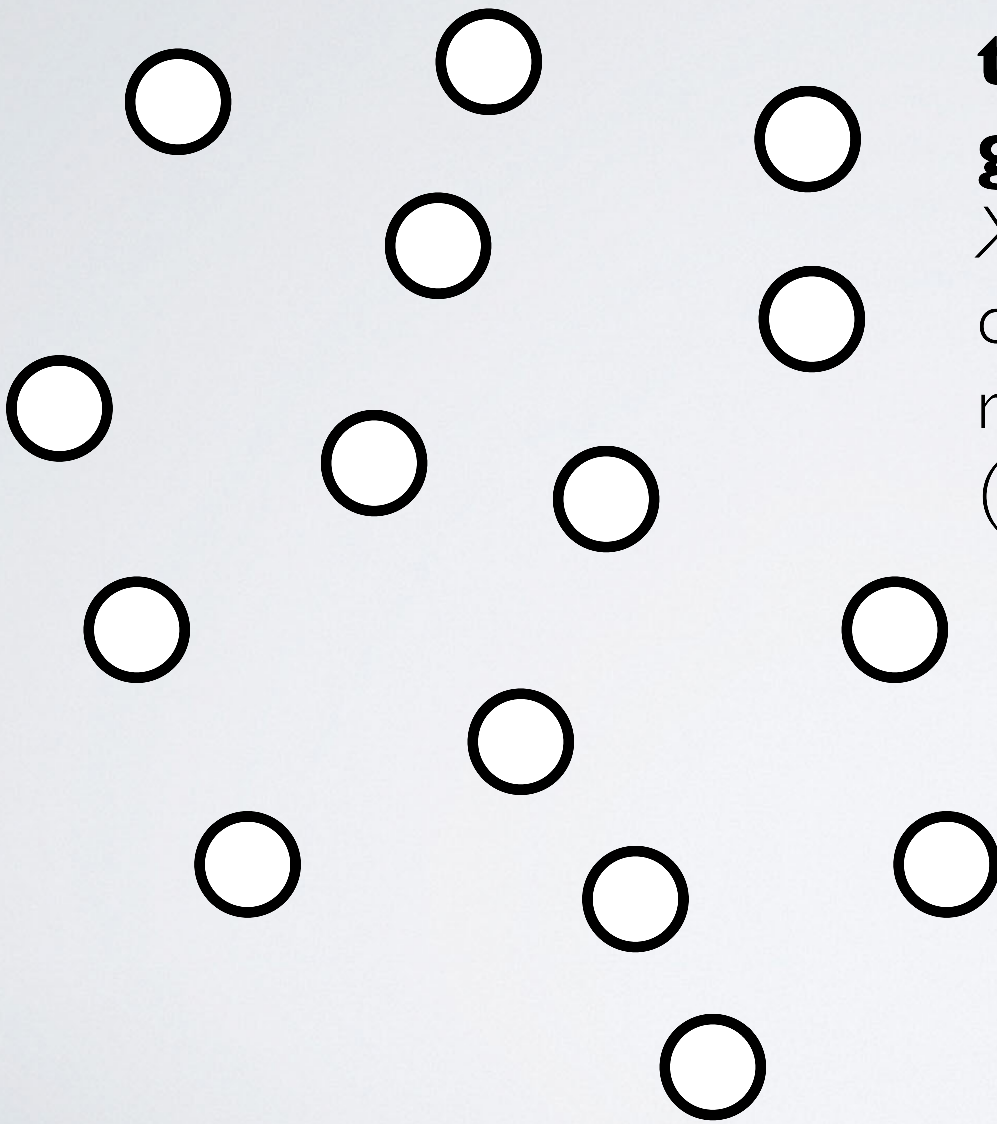


network terminology

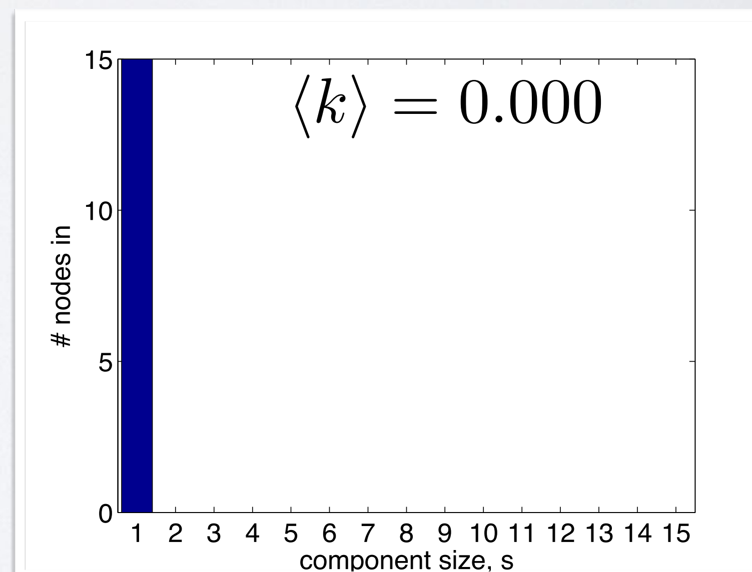


component:
a group of connected nodes

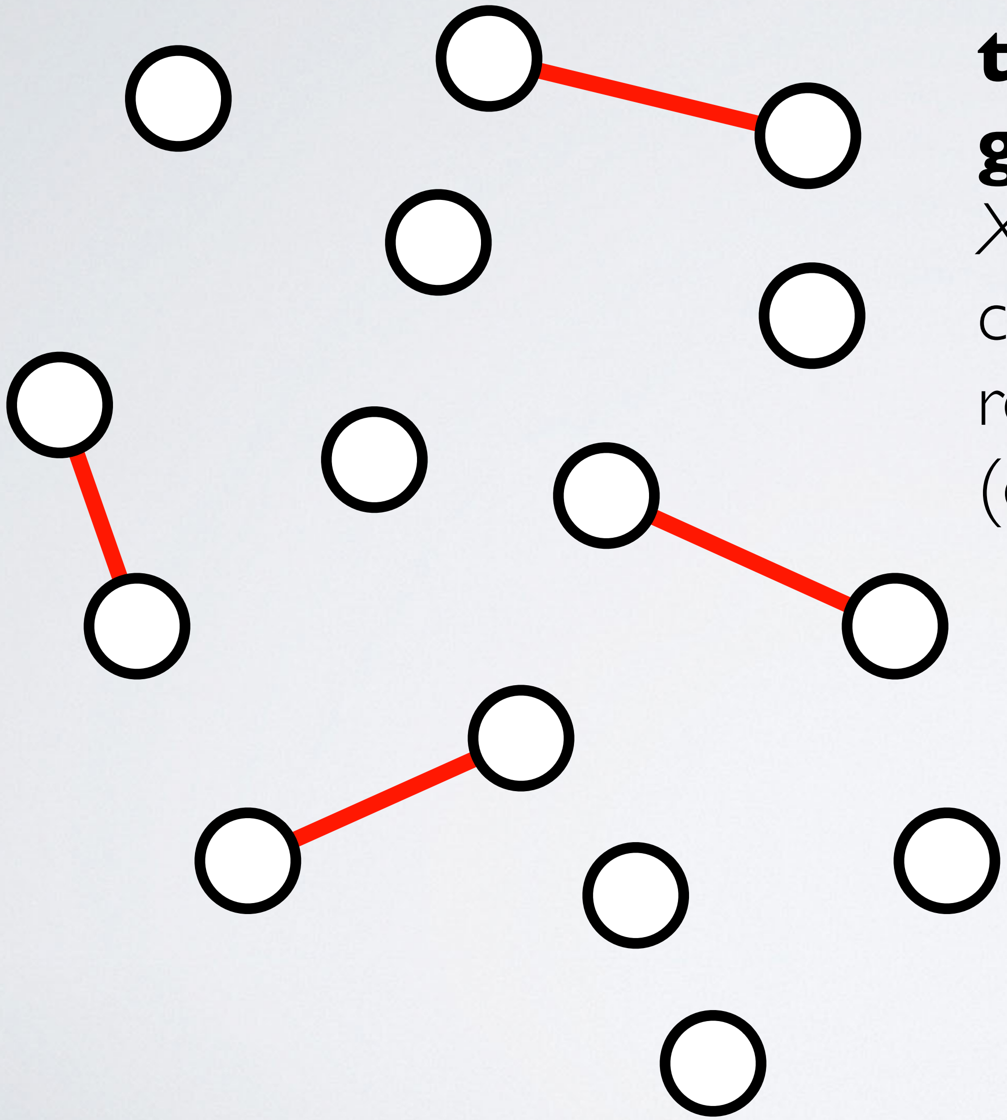
network components



the percolation game: choose X random pairs
connect them
repeat
(count components)



network components



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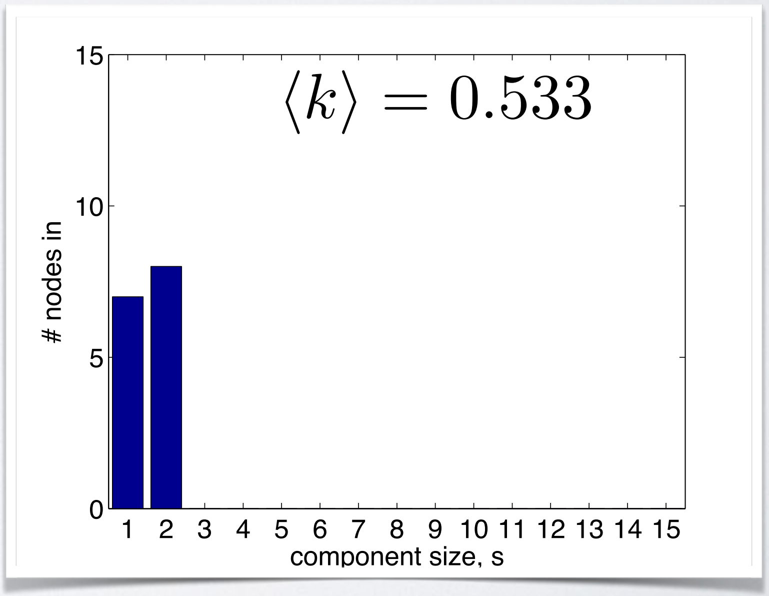

network components

the percolation game: choose X random pairs
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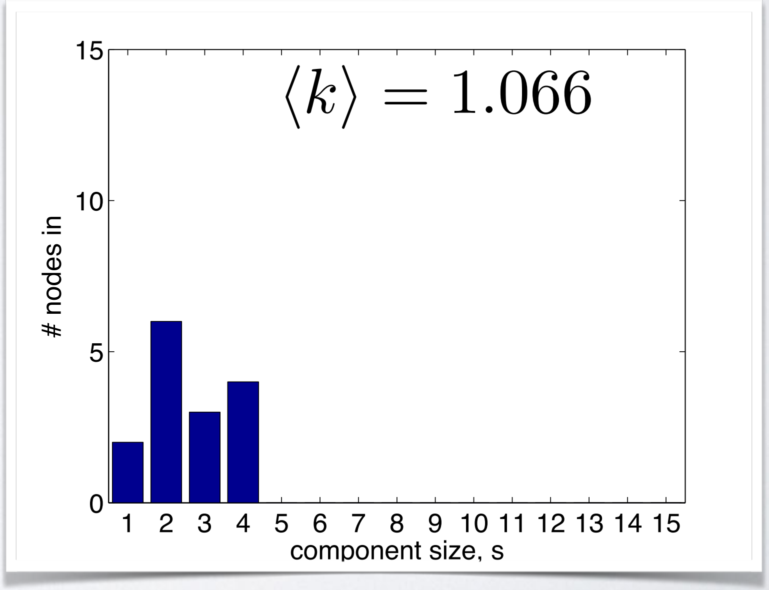
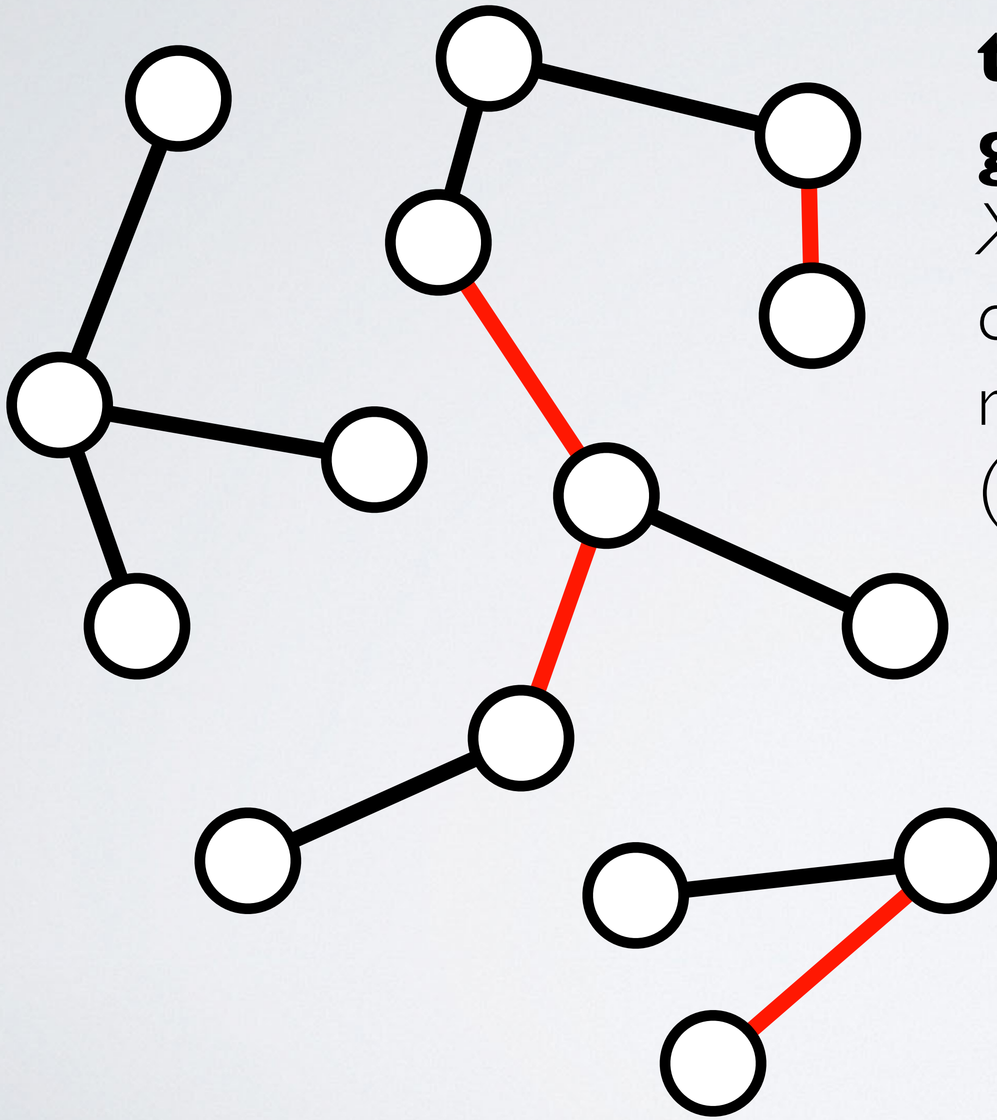


network components



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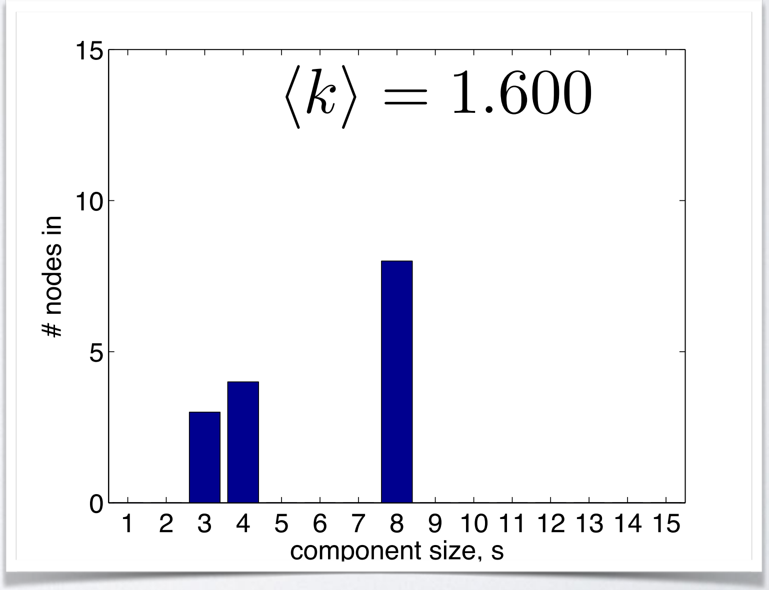
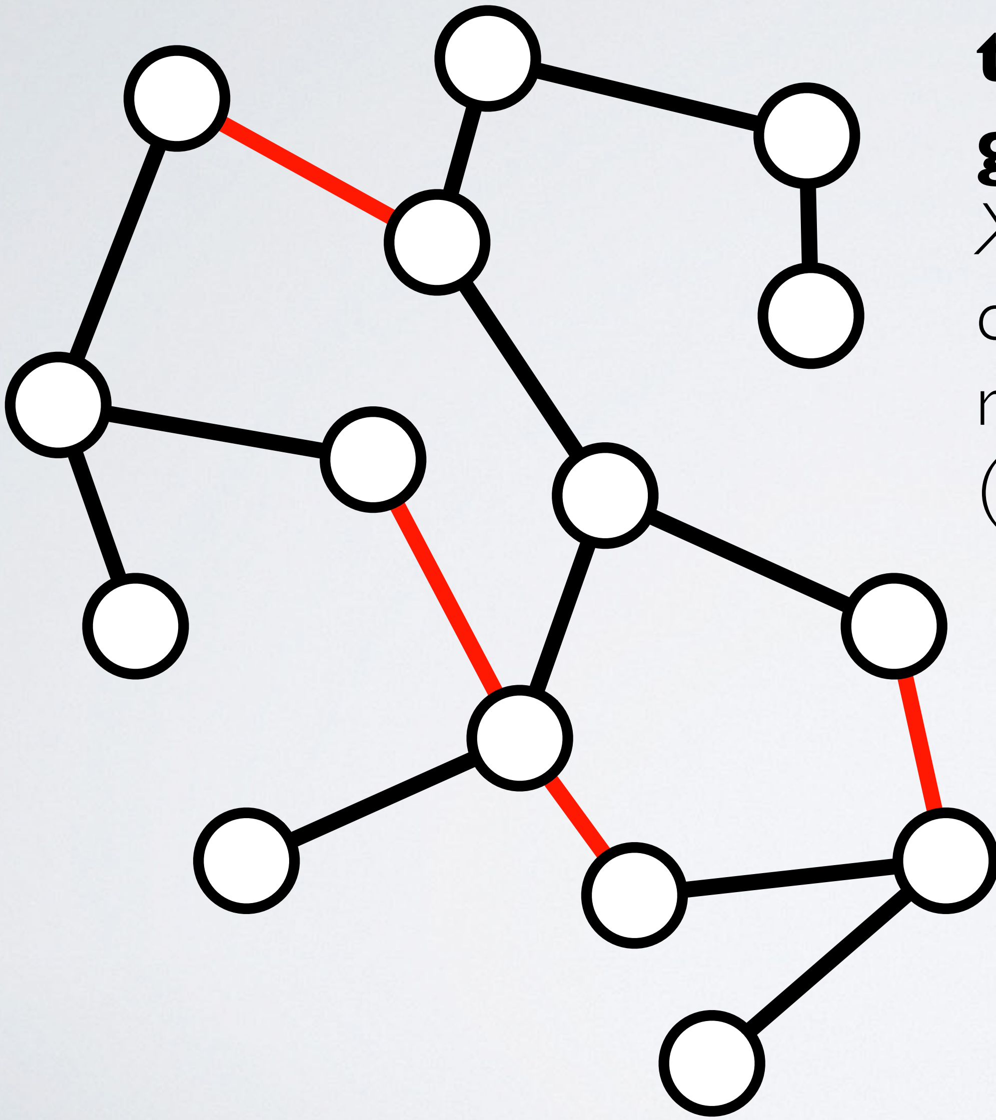


network components



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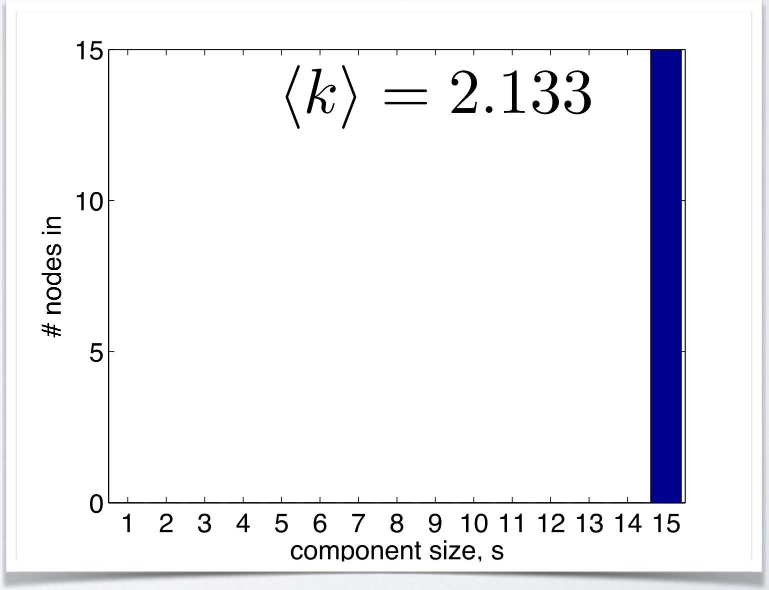
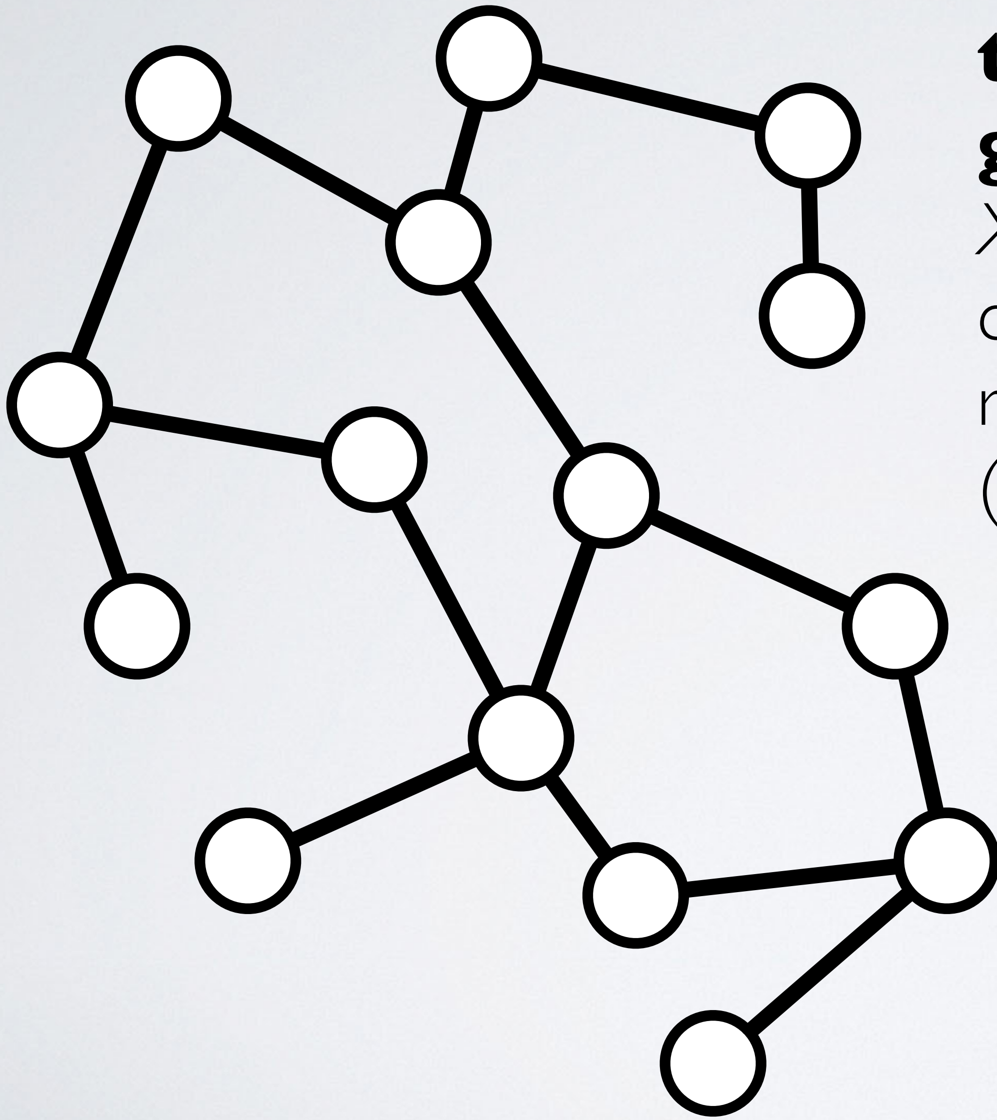


network components



the percolation game: choose X random pairs
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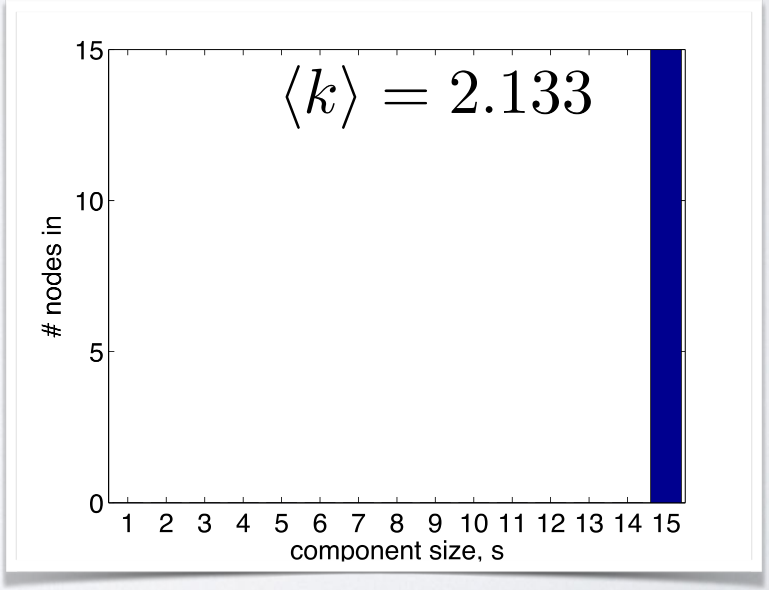
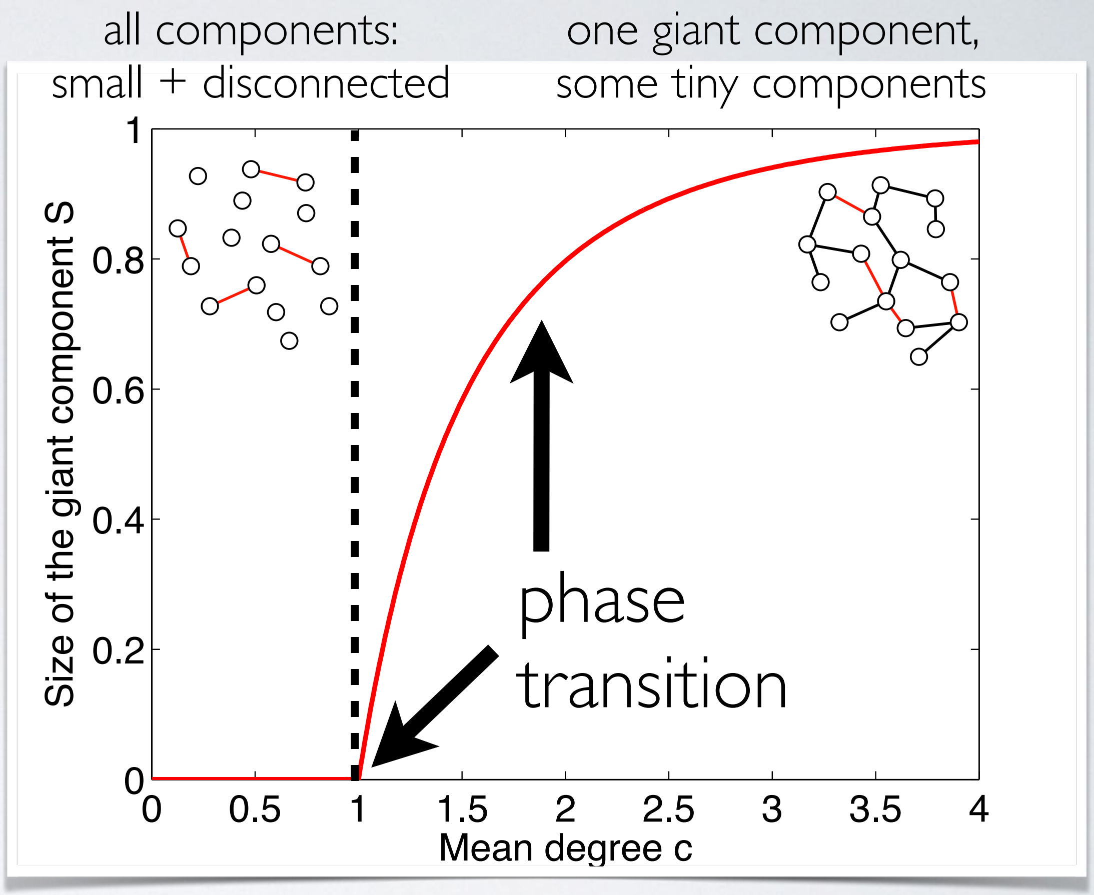
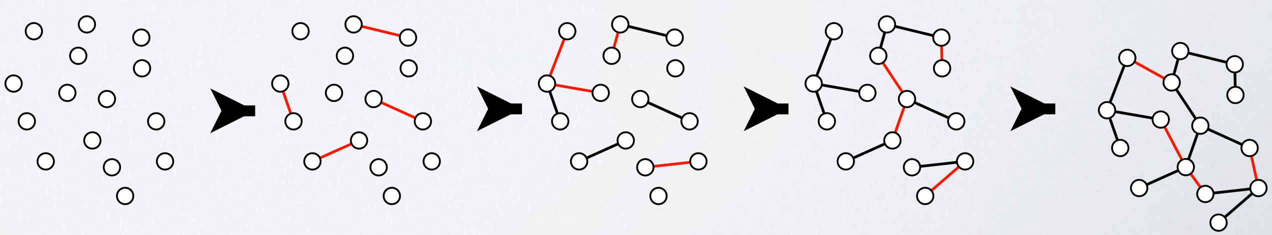


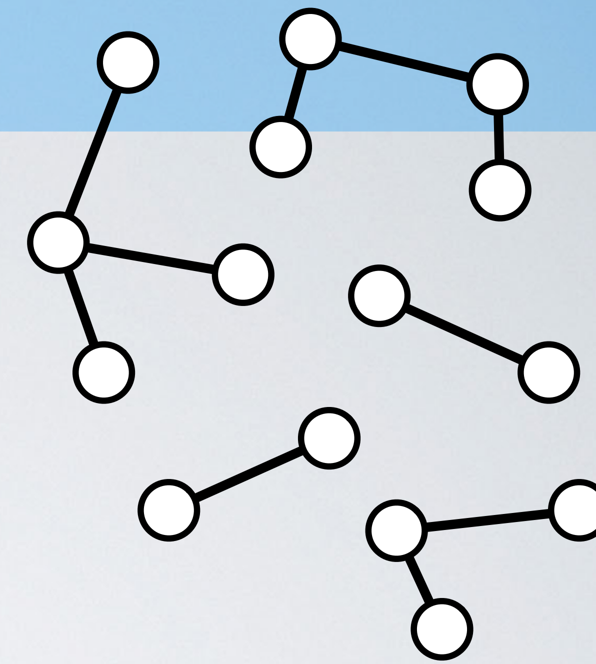
the "giant" component

- add edges randomly
- at first, components are small and disconnected
- at critical value, these components begin linking
- beyond, all nodes in single "giant" component





network components



- component = connected group
- component dynamics are independent (no information flow)
- *phase transition*: sudden emergence of new behavior (giant component)

open questions:

- other network properties + phase transitions
- adaptive wiring
- local vs. global connectivity rules