

# Introduction to Networks

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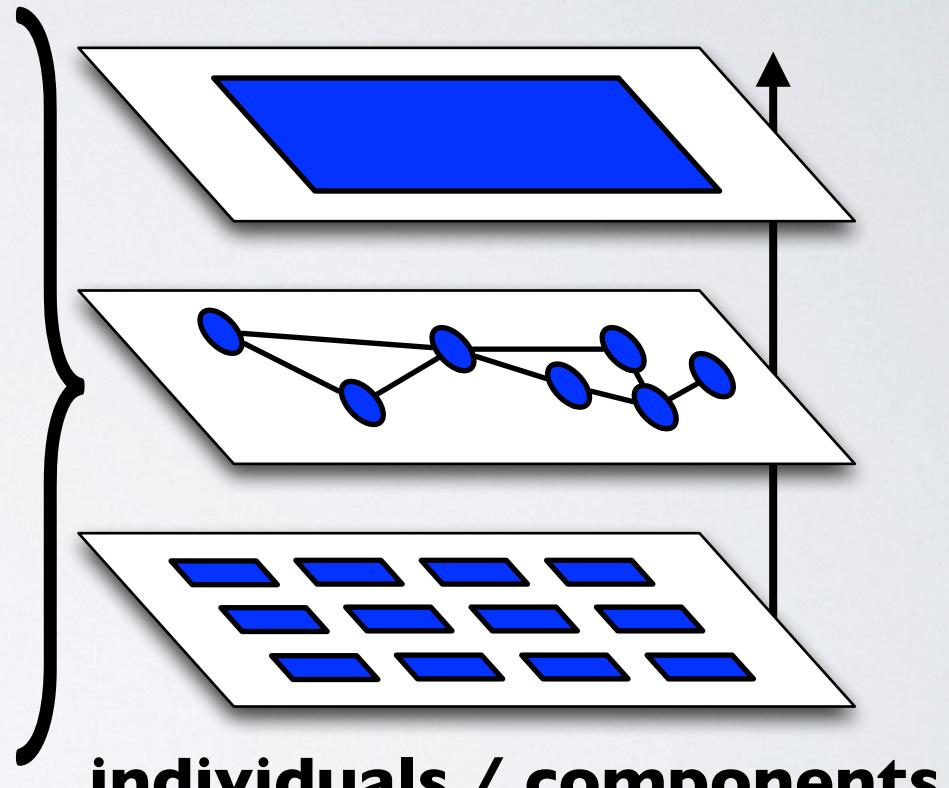
External Faculty, Santa Fe Institute

**what are networks?**

# **what are networks?**

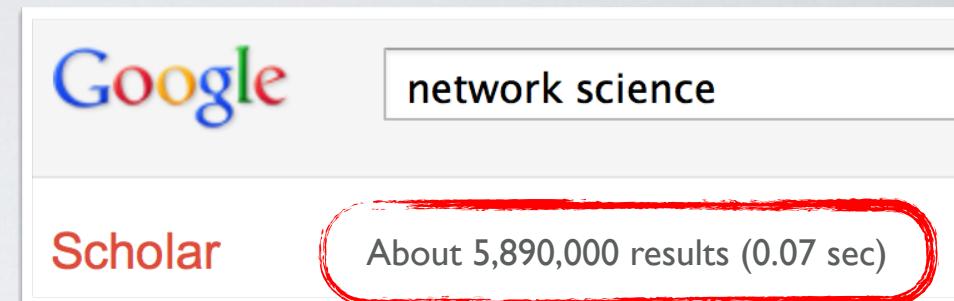
- an approach
- a mathematical representation
- provide structure to complexity
- *structure above* individuals / components
- *structure below* system / population

## **system / population**



# this lecture

- build intuition
- highlight a few concepts & questions
- provide some examples
- pointers to further study
- not a substitute for technical coursework



it's a big field now

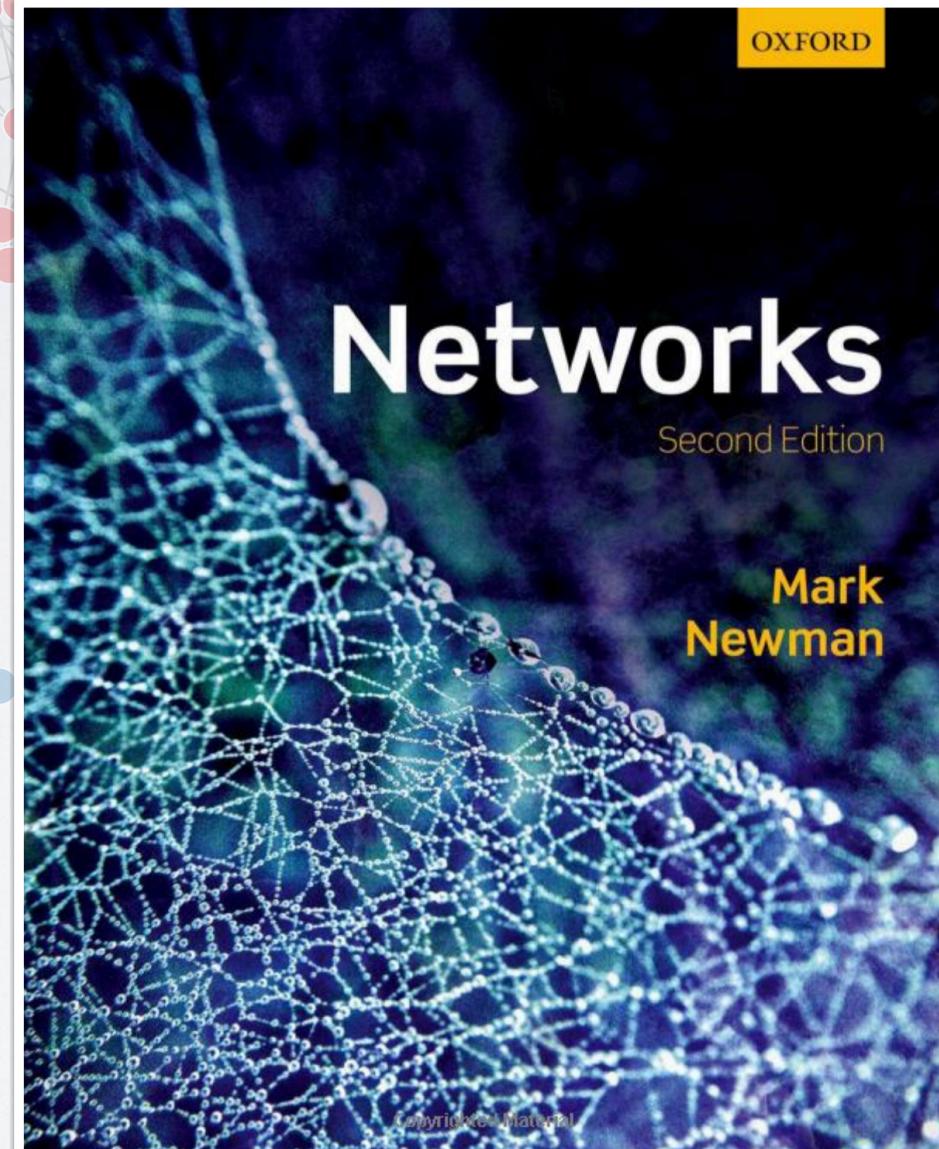


Mark Newman

Professor of Physics  
University of Michigan

External Faculty  
Santa Fe Institute

<http://www-personal.umich.edu/~mejn/>





University of Colorado **Boulder**

## **Network Analysis and Modeling**

Instructor: Aaron Clauset or Daniel B. Larremore

This graduate-level course will examine modern techniques for analyzing and modeling the structure and dynamics of complex networks. The focus will be on statistical algorithms and methods, and both lectures and assignments will emphasize model interpretability and understanding the processes that generate real data. Applications will be drawn from computational biology and computational social science. No biological or social science training is required. (Note: this is not a scientific computing course, but there will be plenty of computing for science.)

*Full lectures notes online (~150 pages in PDF)*

<https://aaronclauset.github.io/courses/5352/>



University of Colorado **Boulder**

## **Biological Networks**

Instructor: Aaron Clauset

This undergraduate-level course examines the computational representation and analysis of biological phenomena through the structure and dynamics of networks, from molecules to species. Attention focuses on algorithms for clustering network structures, predicting missing information, modeling flows, regulation, and spreading-process dynamics, examining the evolution of network structure, and developing intuition for how network structure and dynamics relate to biological phenomena.

*Full lectures notes online (~150 pages in PDF)*

<https://aaronclauset.github.io/courses/3352/>

## Software

R

Python

Matlab

★ NetworkX [python]

graph-tool [python, c++]

GraphLab [python, c++]

## Standalone editors

UCI-Net

NodeXL

Gephi

Pajek

Network Workbench

Cytoscape

yEd graph editor

Graphviz

## Network data sets



Colorado Index of Complex Networks

The screenshot shows a web browser window with the URL [icon.colorado.edu/#!/](http://icon.colorado.edu/#/). The page title is "Index of Complex Networks". There are three main navigation tabs: "NETWORKS" (which is highlighted in blue), "ABOUT", and "SUGGEST".

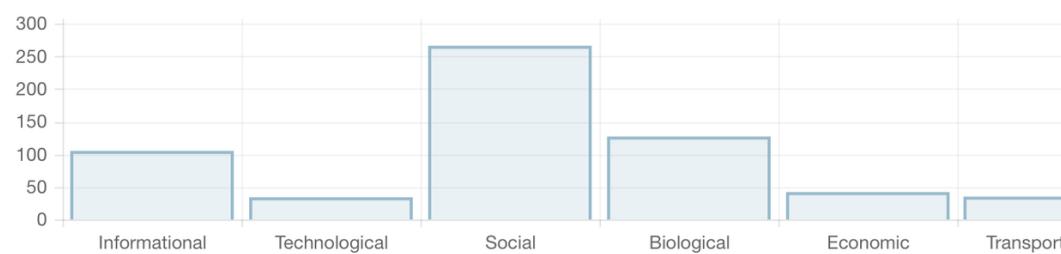
### The Colorado Index of Complex Networks (ICON)

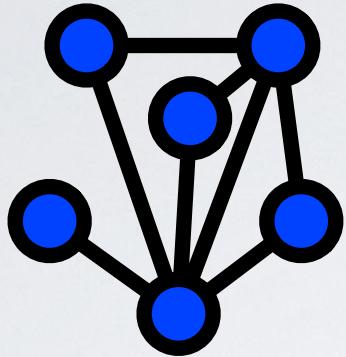
ICON is a comprehensive index of research-quality network data sets from all domains of networks, including social, web, information, biological, ecological, connectome, transportation, and technological networks.

Each network record in the index is annotated with and searchable or browsable by its graph properties, description, size, etc., and many records include links to multiple networks. The contents of ICON are curated by volunteer experts from Prof. Aaron Clauset's research group at the University of Colorado Boulder.

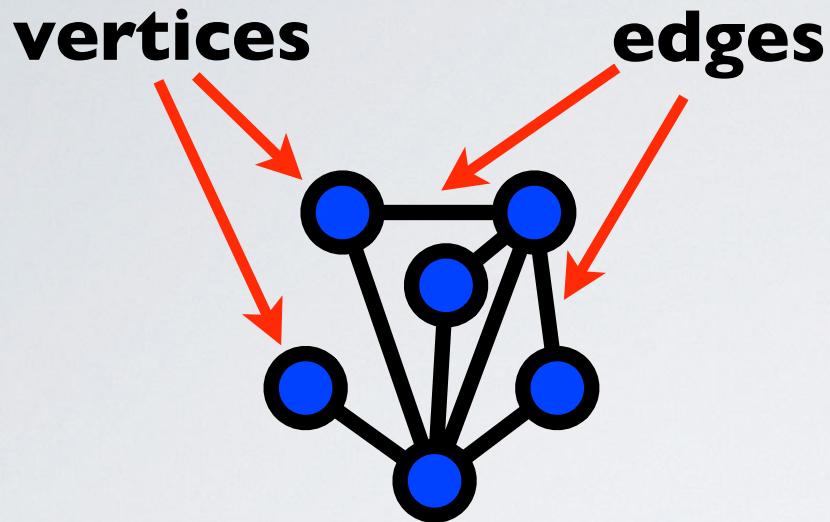
Click on the [NETWORKS tab](#) above to get started.

Entries found: 609 Networks found: 4419





🤔 **the two most fundamental  
questions in network science**



## **what is a vertex?**

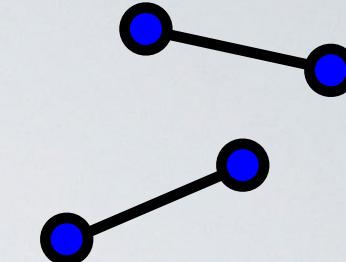
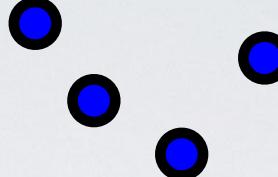
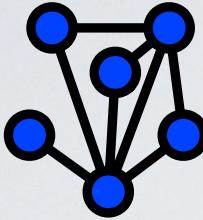
$V$  distinct objects (vertices / nodes / actors)

## **when are two vertices connected?**

$$E \subseteq V \times V$$

pairwise relations (edges / links / ties)

**informationaltelecommunications**



**transportation**

**network**

Internet(1)

Internet(2)

software

World Wide Web

documents

power grid transmission

rail system

road network(1)

road network(2)

airport network

friendship network

sexual network

metabolic network

protein-interaction network

gene regulatory network

neuronal network

food web

**vertex**

computer

autonomous system (ISP)

function

web page

article, patent, or legal case

generating or relay station

rail station

intersection

named road

airport

person

person

metabolite

protein

gene

neuron

species

**edge**

IP network adjacency

BGP connection

function call

hyperlink

citation

transmission line

railroad tracks

pavement

intersection

non-stop flight

friendship

intercourse

metabolic reaction

bonding

regulatory effect

synapse

predation or resource transfer

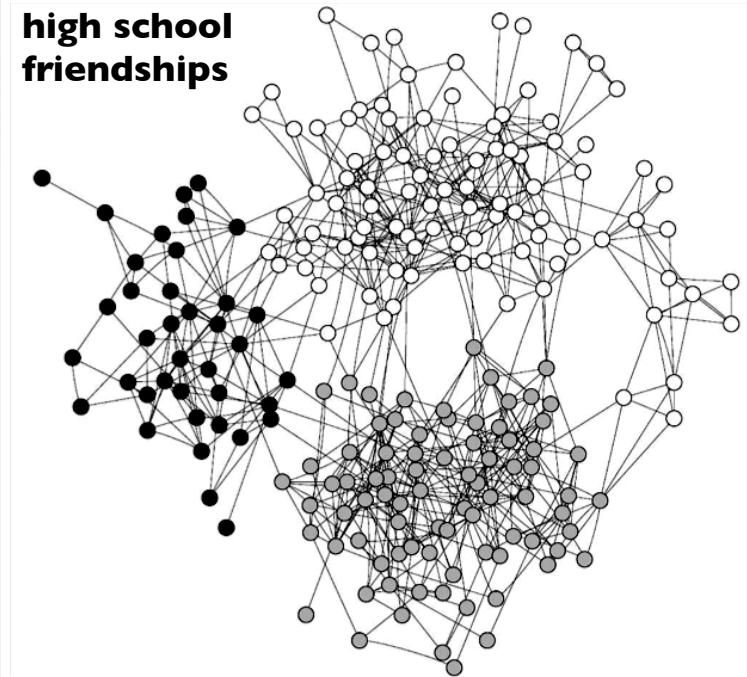
**social**

**biological**

# social networks

**vertex:** a person

**edge:** friendship, collaborations, sexual contacts, communication, authority, exchange, etc.

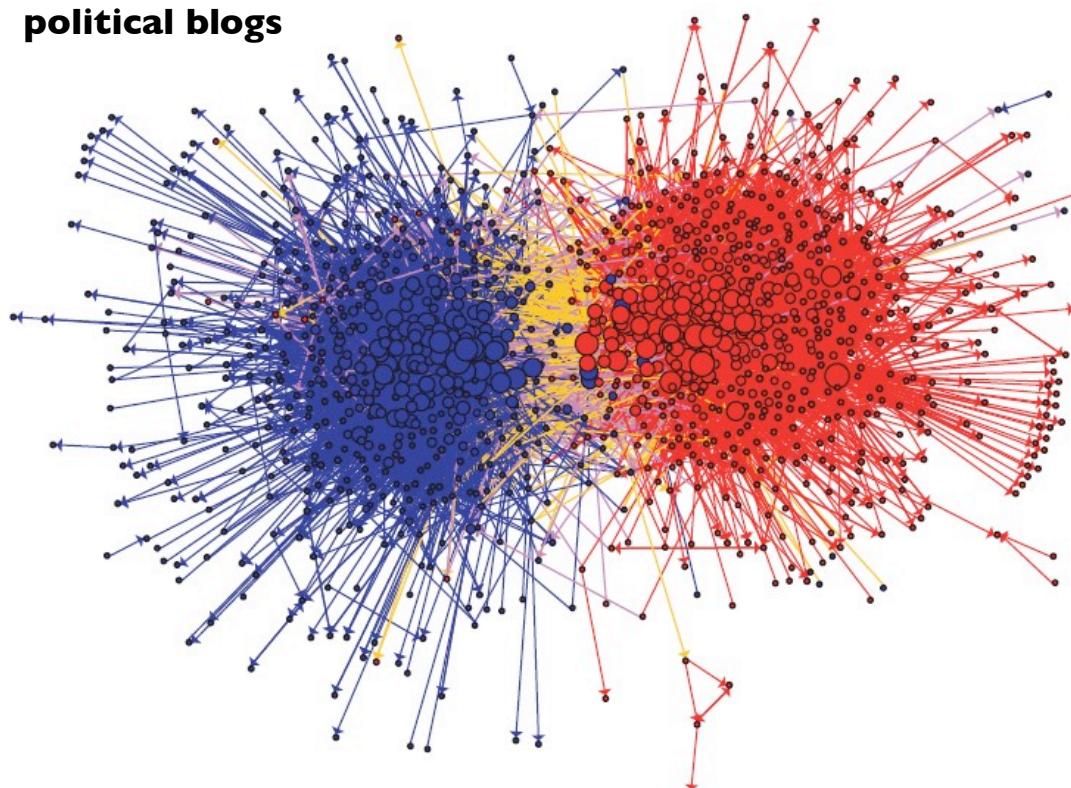


# information networks

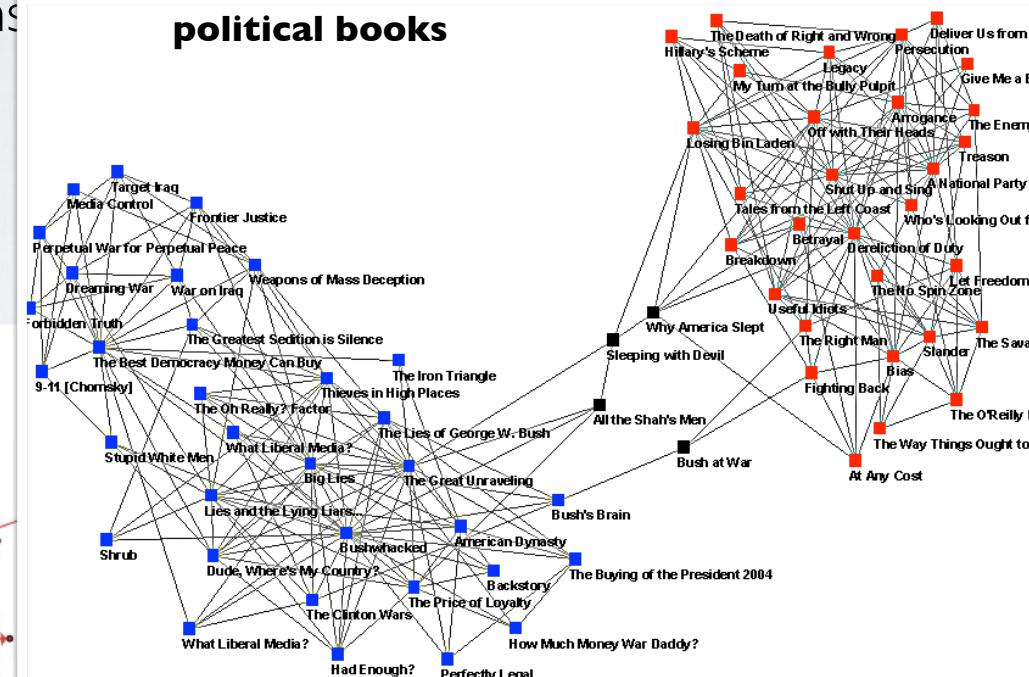
**vertex:** books, blogs, webpages, etc.

**edge:** citations, hyperlinks, recommendations  
similarity, etc.

political blogs



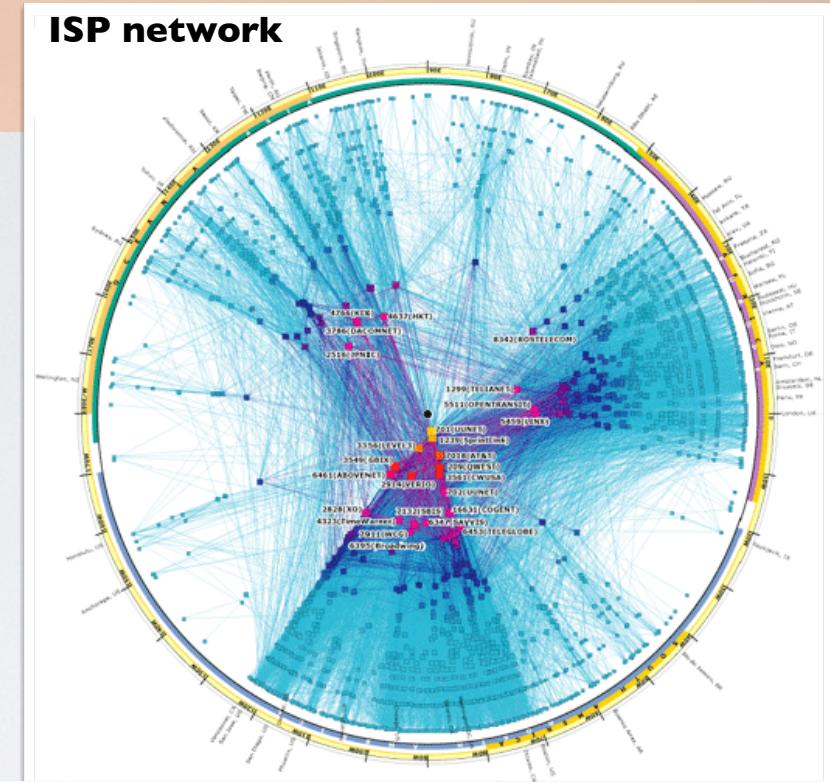
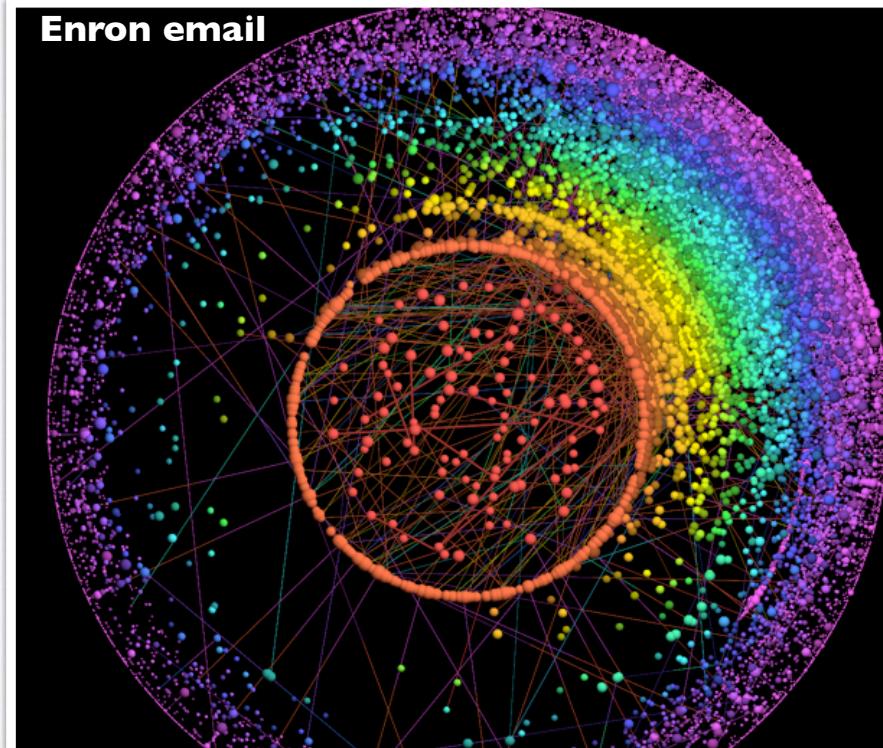
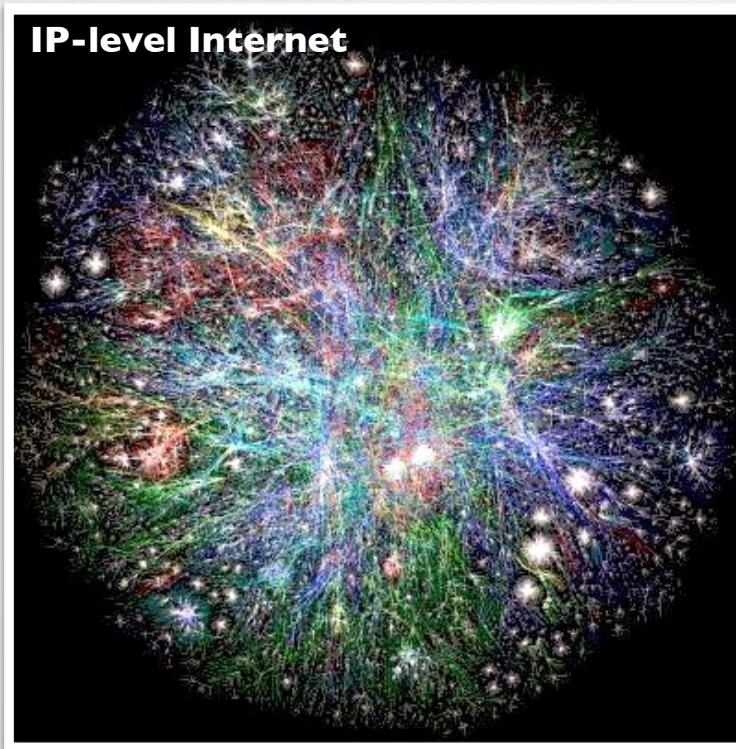
political books



# communication networks

**vertex:** network router, ISP, email address, mobile phone number, etc.

**edge:** exchange of information



# transportation networks

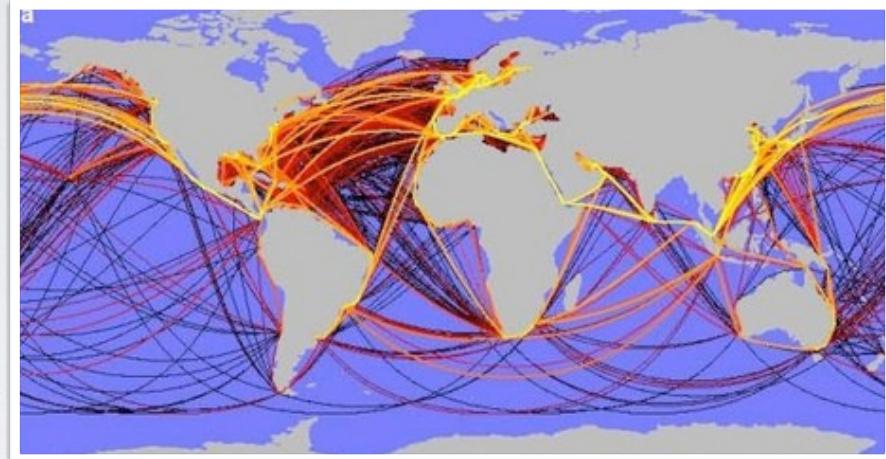
**vertex:** city, airport, junction, railway station, river confluence, etc.

**edge:** physical transportation of material



US Interstates

global shipping



global air traffic

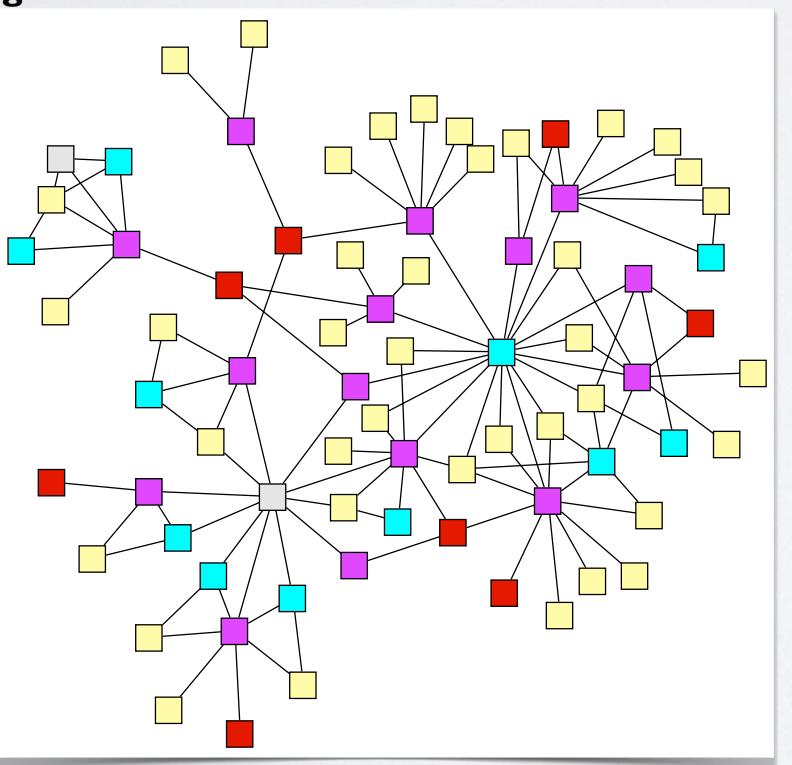


# **biological networks**

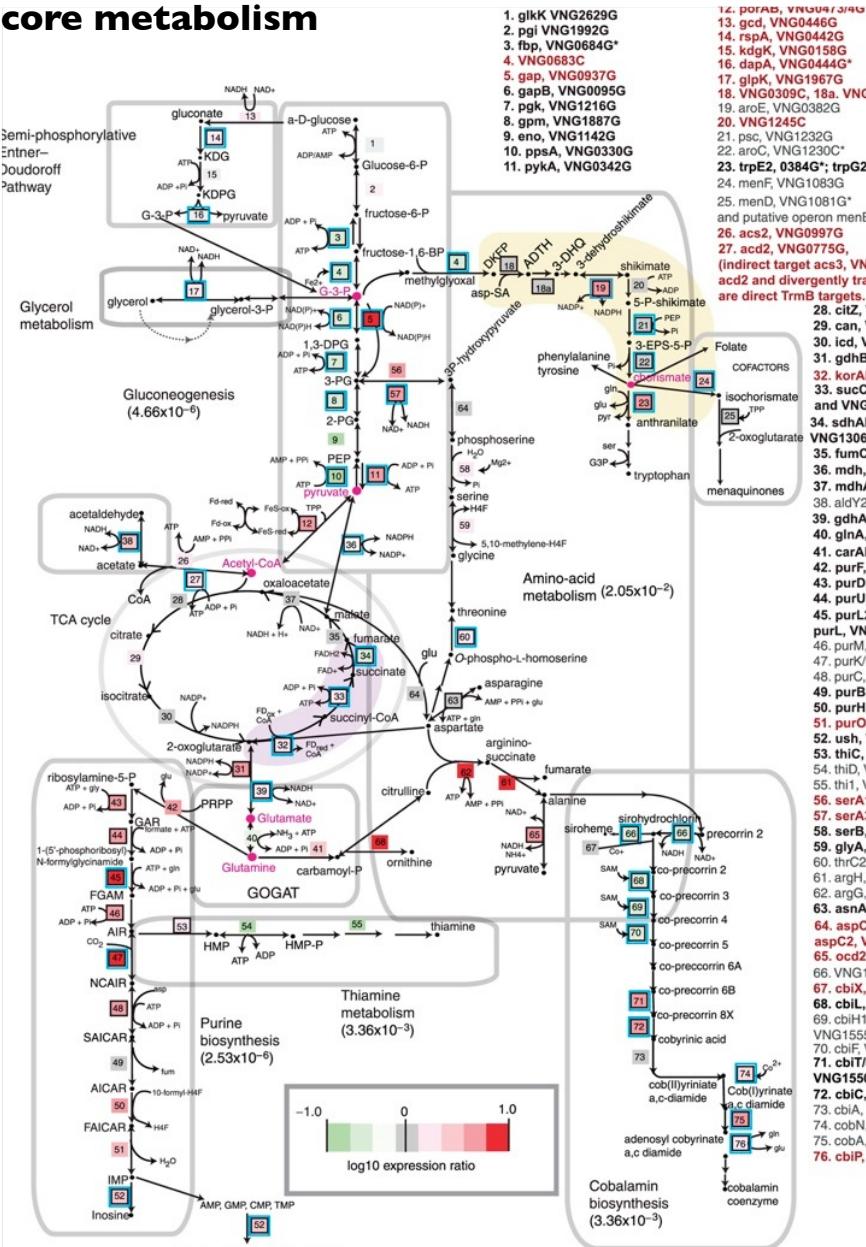
**vertex:** species, metabolic, protein, gene, neuron, etc.

**edge:** predation, chemical reaction, binding, regulation, activation, etc.

# **grassland foodweb**

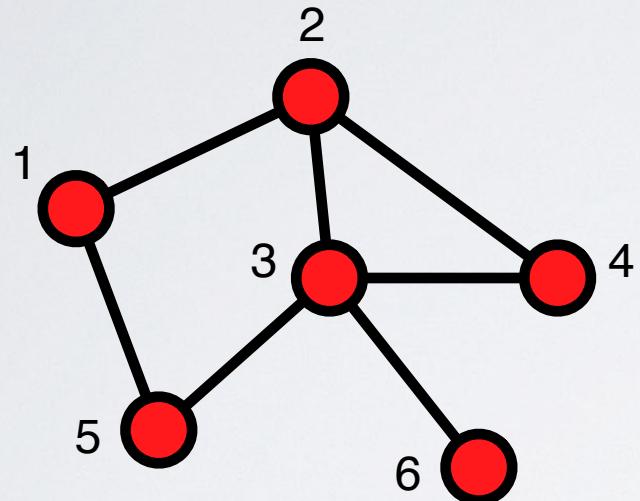


## core metabolism



# **representing networks**

# **a simple network**

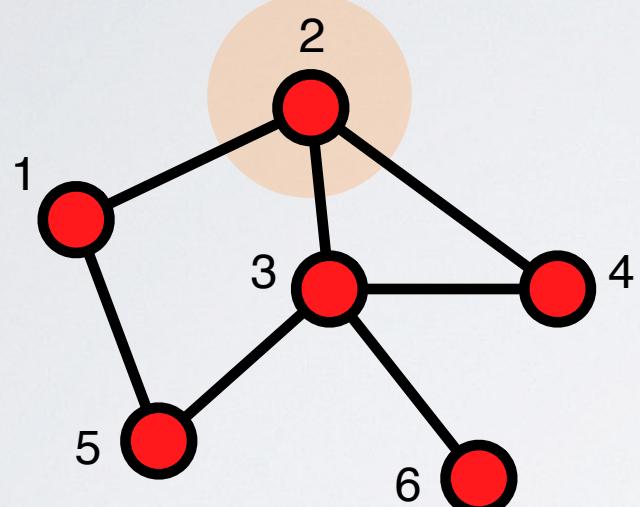


undirected

unweighted

no self-loops

# a *simple* network



undirected

unweighted

no self-loops

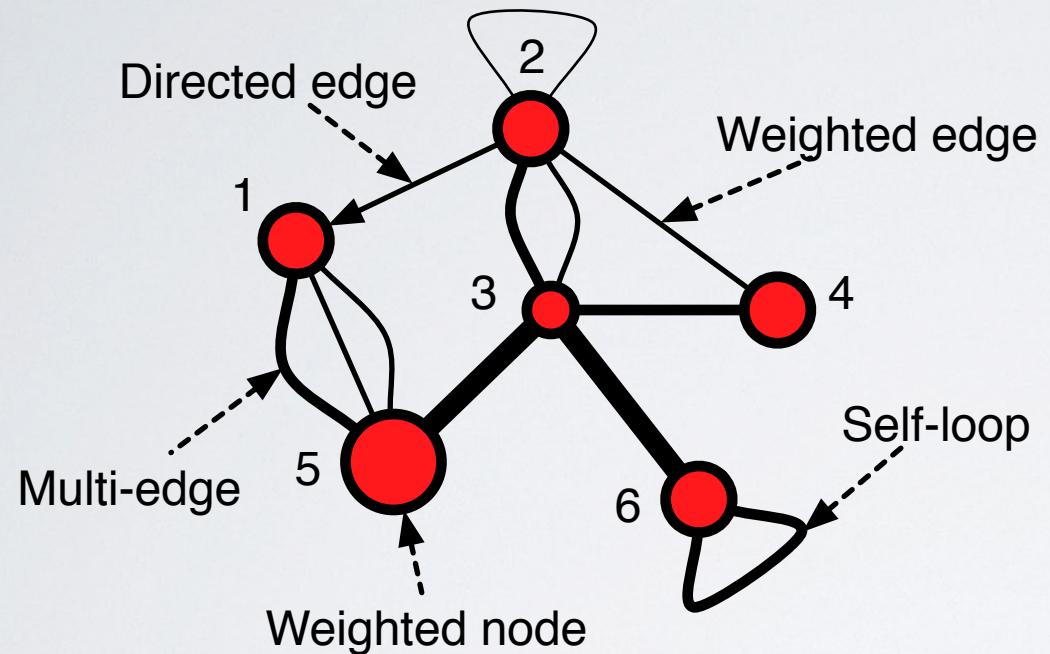
adjacency matrix

$A$	1	2	3	4	5	6
1	0	1	0	0	1	0
2	1	0	1	1	0	0
3	0	1	0	1	1	1
4	0	1	1	0	0	0
5	1	0	1	0	0	0
6	0	0	1	0	0	0

adjacency list

$A$
$1 \rightarrow \{2, 5\}$
$2 \rightarrow \{1, 3, 4\}$
$3 \rightarrow \{2, 4, 5, 6\}$
$4 \rightarrow \{2, 3\}$
$5 \rightarrow \{1, 3\}$
$6 \rightarrow \{3\}$

# a less simple network

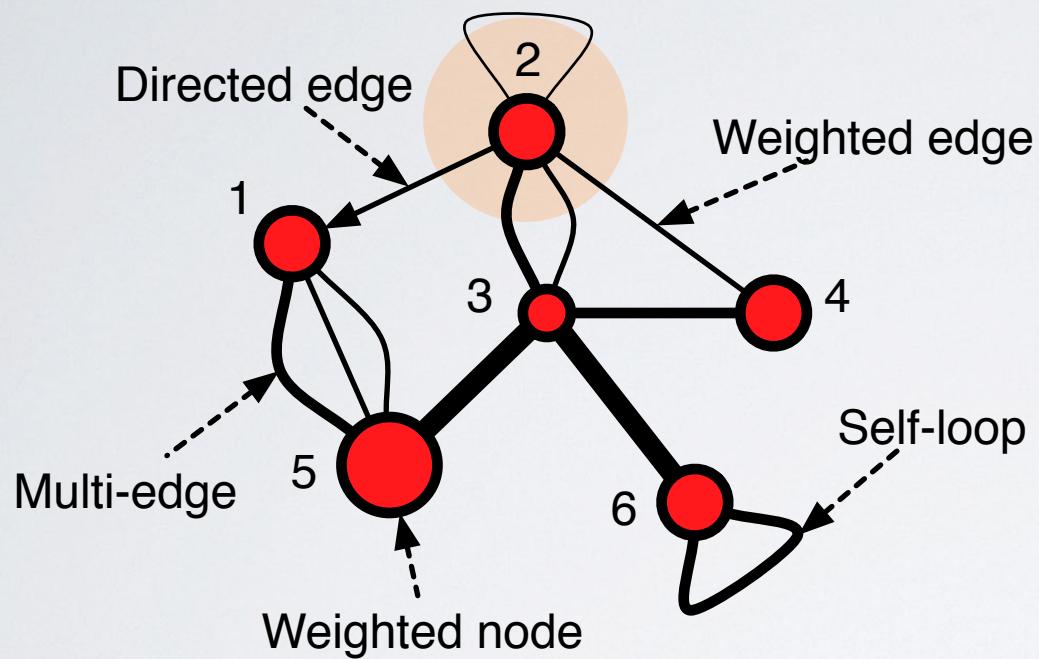


~~undirected~~

~~unweighted~~

~~no self loops~~

# a less simple network



adjacency matrix

$A$	1	2	3	4	5	6
1	0	0	0	0	{1, 1, 2}	0
2	1	$\frac{1}{2}$	{2, 1}	1	0	0
3	0	{2, 1}	0	2	4	4
4	0	1	2	0	0	0
5	{1, 1, 2}	0	4	0	0	0
6	0	0	4	0	0	2

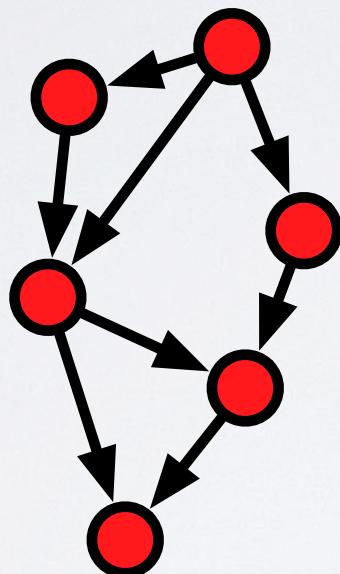
adjacency list

$A$
1 $\rightarrow \{(5, 1), (5, 1), (5, 2)\}$
2 $\rightarrow \{(1, 1), (2, \frac{1}{2}), (3, 2), (3, 1), (4, 1)\}$
3 $\rightarrow \{(2, 2), (2, 1), (4, 2), (5, 4), (6, 4)\}$
4 $\rightarrow \{(2, 1), (3, 2)\}$
5 $\rightarrow \{(1, 1), (1, 1), (1, 2), (3, 4)\}$
6 $\rightarrow \{(3, 4), (6, 2)\}$

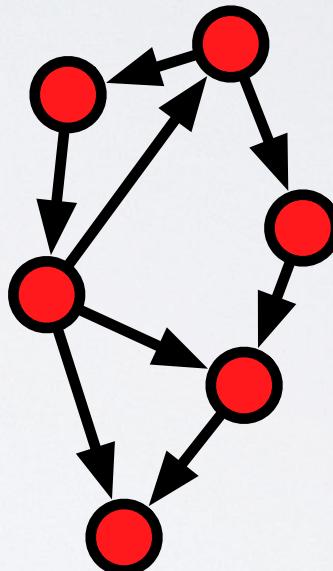
# directed networks

$$A_{ij} \neq A_{ji}$$

citation networks  
foodwebs\*  
epidemiological  
others?



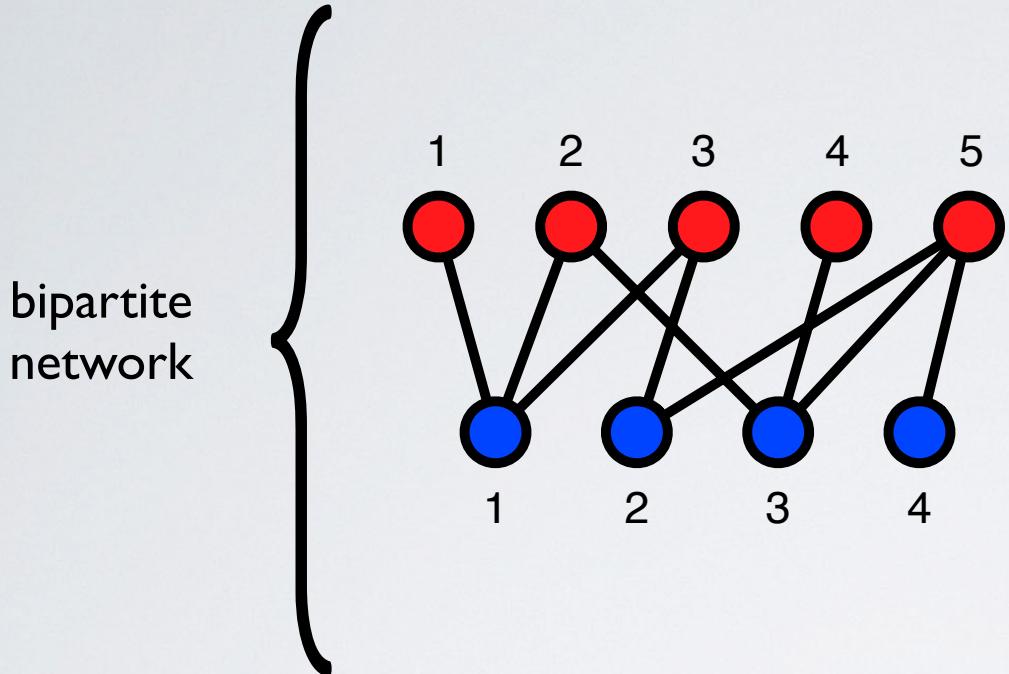
directed acyclic graph



directed graph

WWW  
friendship?  
flows of goods,  
information  
economic exchange  
dominance  
neuronal  
transcription  
time travelers

# bipartite networks



no within-type edges

authors & papers

actors & movies/scenes

musicians & albums

people & online groups

people & corporate boards

people & locations (checkins)

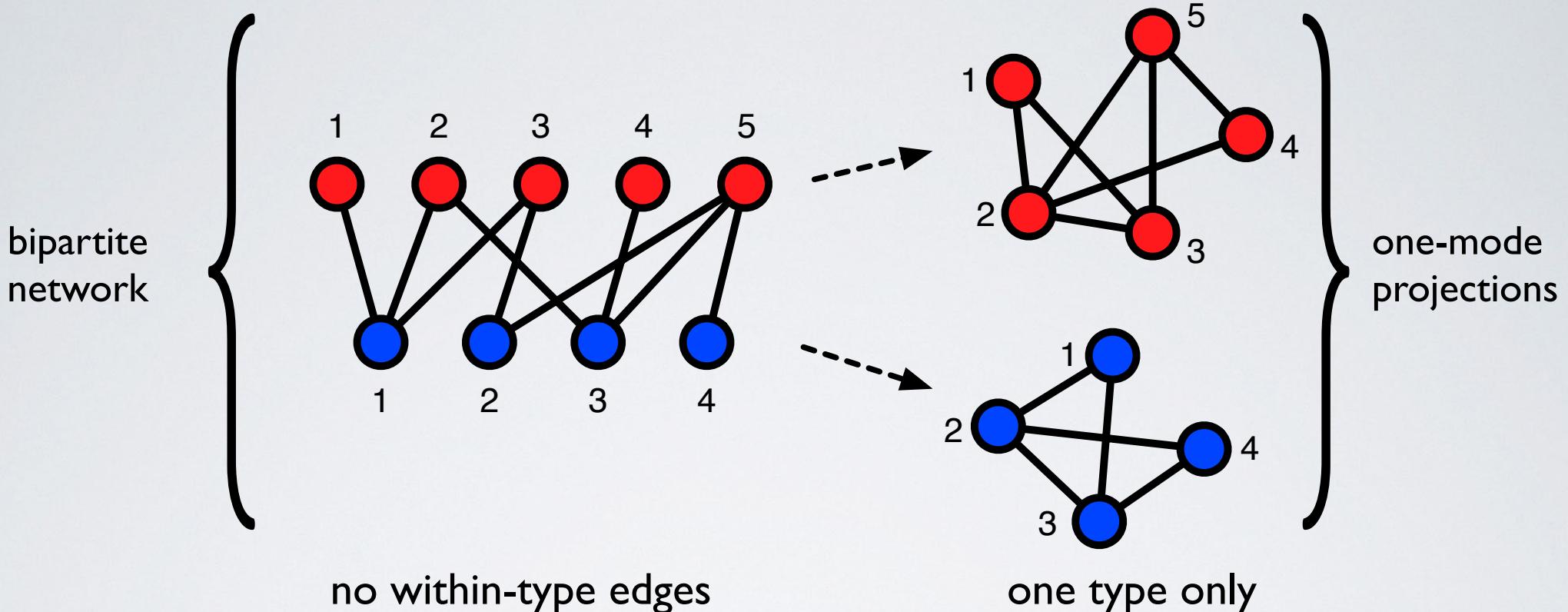
metabolites & reactions

genes & substrings

words & documents

plants & pollinators

# bipartite networks



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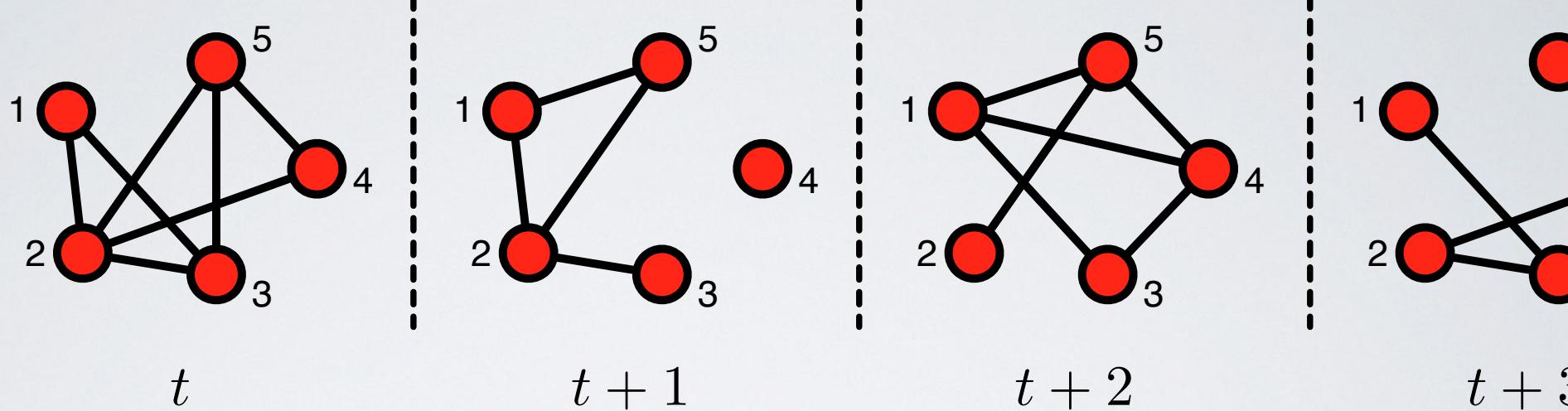
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# temporal networks



any network over time; comes in two flavors

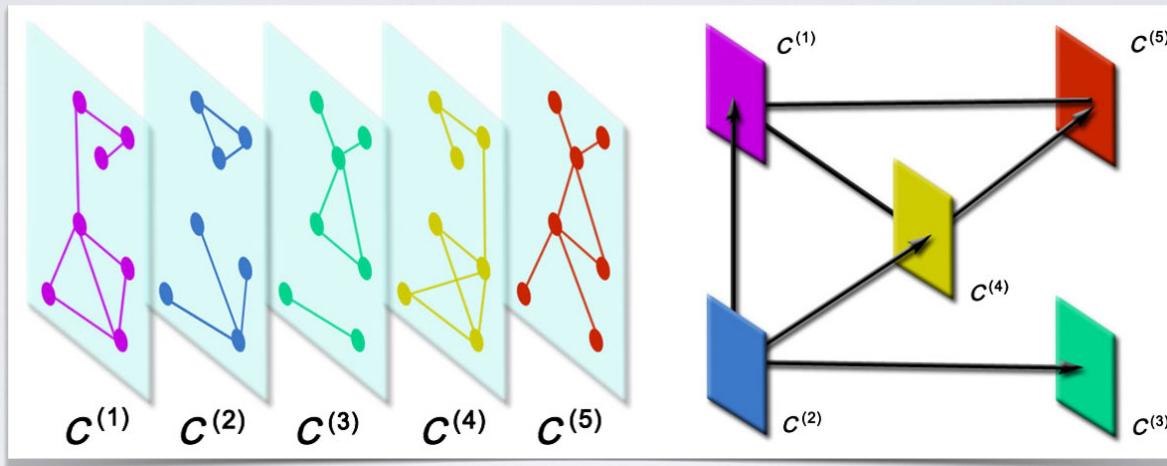
1. discrete time (snapshots), edges  $(i, j, t)$
2. continuous time, edges  $(i, j, t_s, \Delta t)$

physical proximity over time

transportation connections over time

social interactions over time

# multiplex or multilayer networks



multiple network "layers"

each layer has same set of nodes  $V$

but different sets of edges  $\{E_1, E_2, \dots, E_\ell\}$

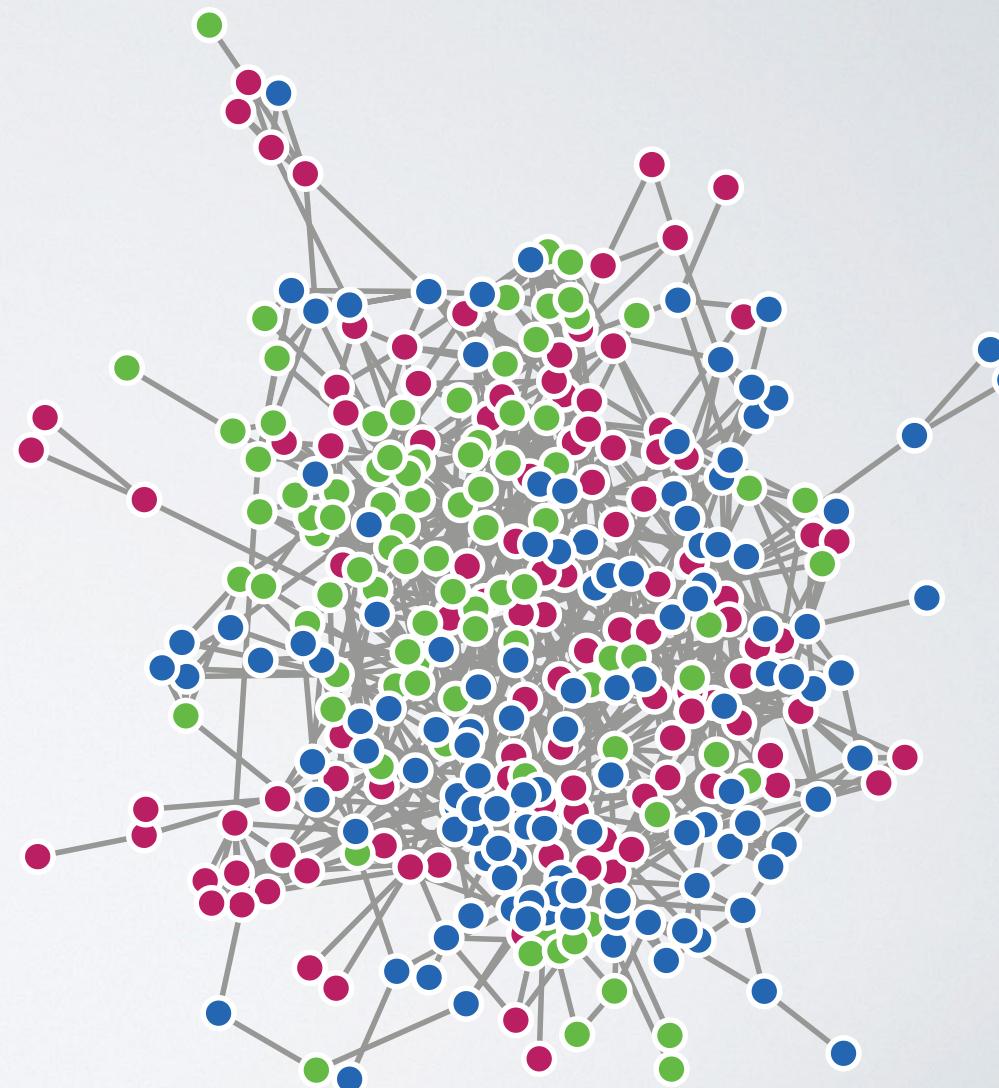
different types of transportation within a city

different types of social interactions (trust, socializing, co-located, etc.)

interactions on different social media platforms

# analyzing networks

what real networks look like...



# analyzing networks

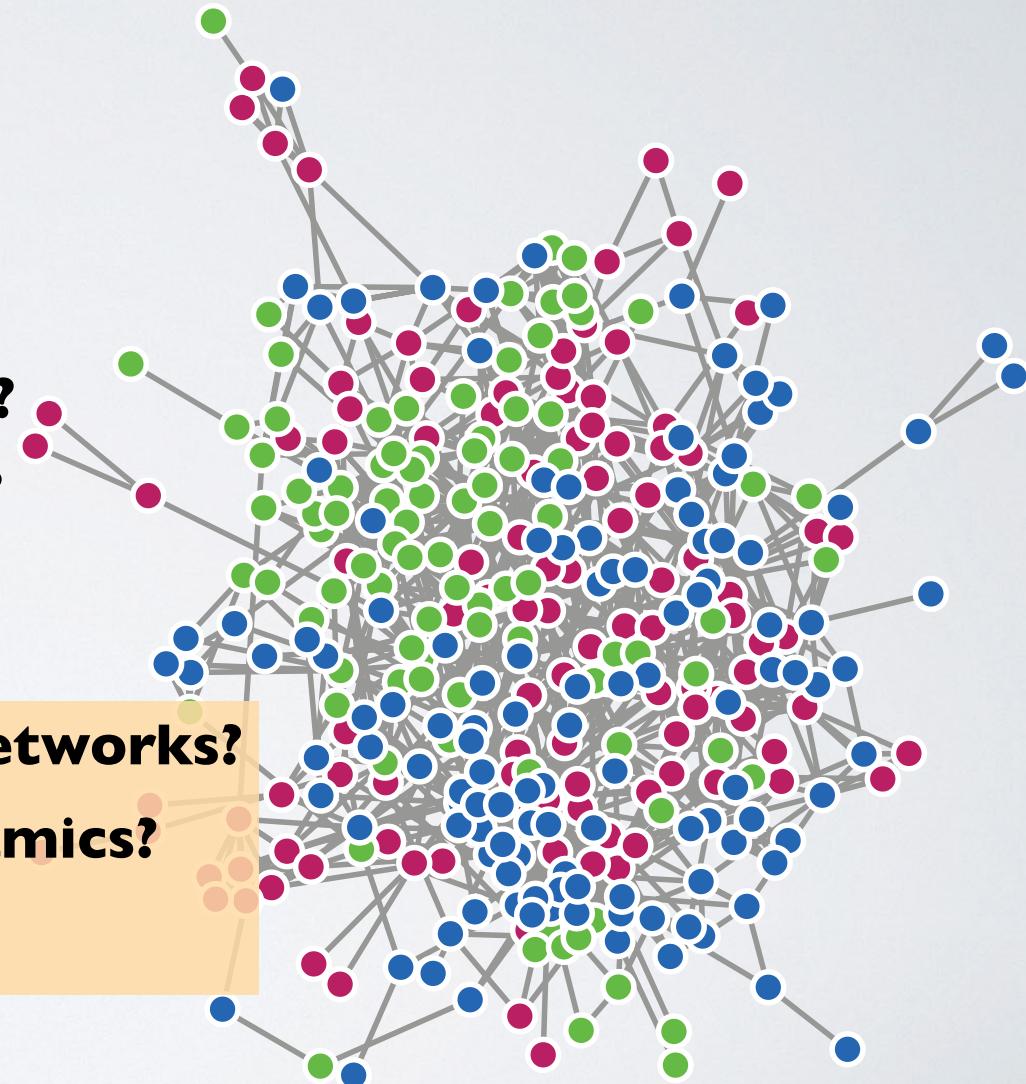
what real networks look like...

questions:

- **how are the edges organized?**
- **how do vertices differ?**
- **does network location matter?**
- **are there underlying patterns?**

what we want to know

- **what processes shape these networks?**
- **how does network shape dynamics?**
- **how can we tell?**



# analyzing networks

what we want : understand its structure

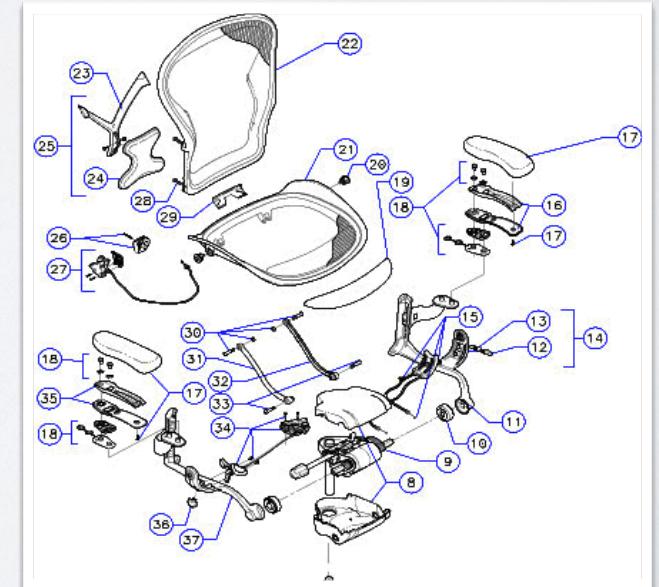
$$f : \text{object} \rightarrow \{\theta_1, \dots, \theta_k\}$$

- **what are the fundamental parts?**
- **how are these parts organized?**
- **where are the degrees of freedom  $\vec{\theta}$ ?**
- **how can we define an abstract class?**
- **structure — dynamics — function?**

what does **local-level structure** look like?

what does **large-scale structure** look like?

how does **structure constrain** function?



# analyzing networks

6 major approaches

- I. **exploratory data analysis:** count & compare all the things (degree distributions, centrality scores, community detection, etc.)

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# analyzing networks

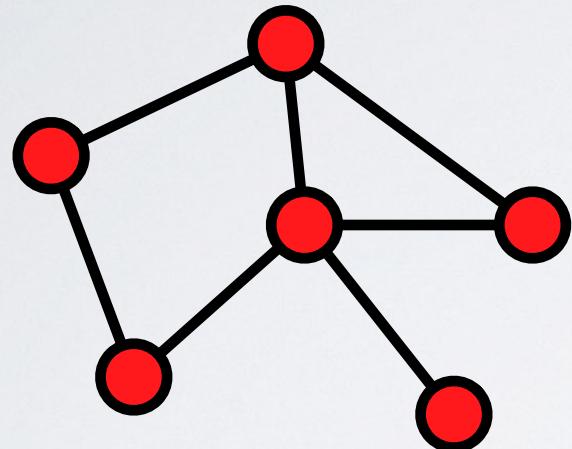
6 major approaches

- ★ 1. **exploratory data analysis:** count & compare all the things (degree distributions, centrality scores, community detection, etc.)
- 2. **simple regressions:** convert network structure into node-level features, and do traditional explanatory modeling
- ★ 3. **null models:** use some kind of random graph to identify non-random patterns as deviations from the null
- 4. **mechanisms / simulations:** explain structural or dynamical patterns as caused by specific process
- ★ 5. **predictive models:** fit parametric model of network structure & use it to predict missing or future data (edges, labels, etc.)
- 6. **network experiments:** manipulate structure and measure node-level or graph-level behavior as function of changes

# **analyzing networks**

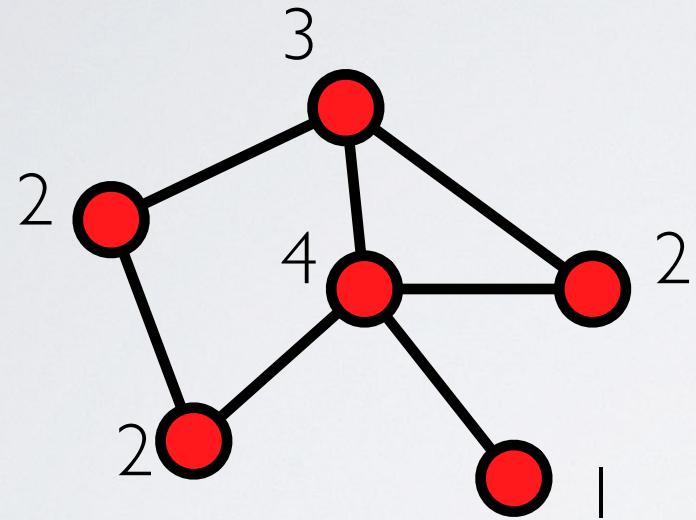
- degrees & distributions
- network position & centrality scores
- some applications

# **degree distributions**



**degree**

# degree distributions

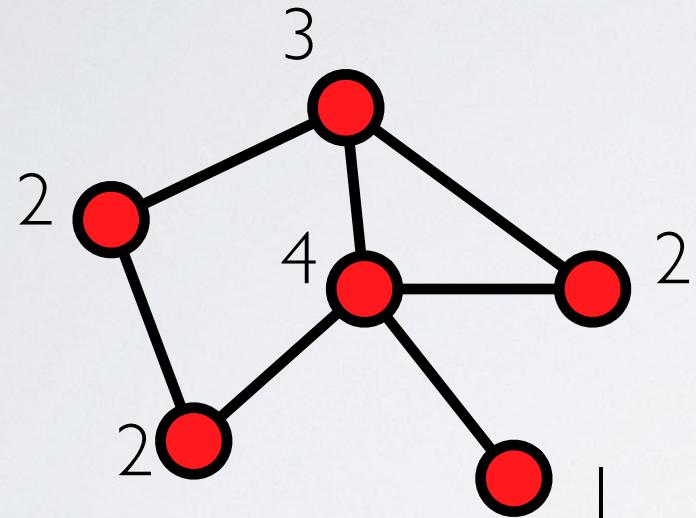


**degree:**

number of connections  $k$

$$k_i = \sum_j A_{ij}$$

# degree distributions



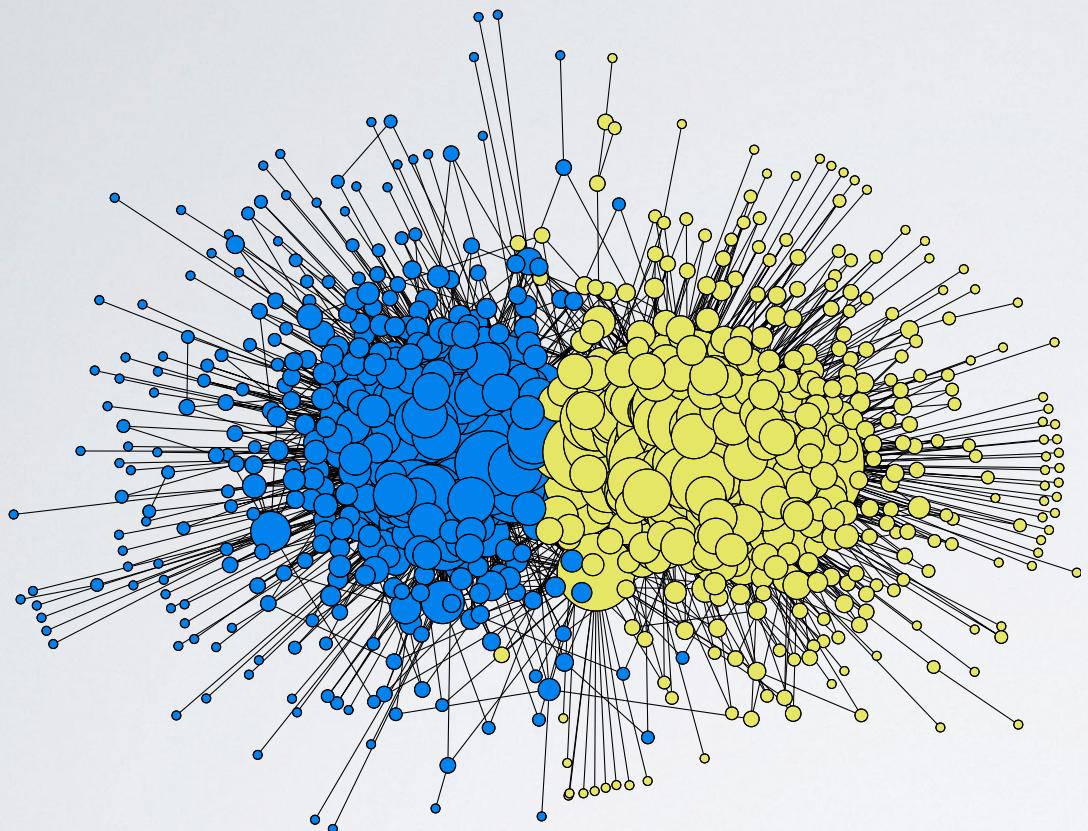
**degree:**  
number of connections  $k$

$$k_i = \sum_j A_{ij}$$

degree sequence :  $\{1, 2, 2, 2, 3, 4\}$

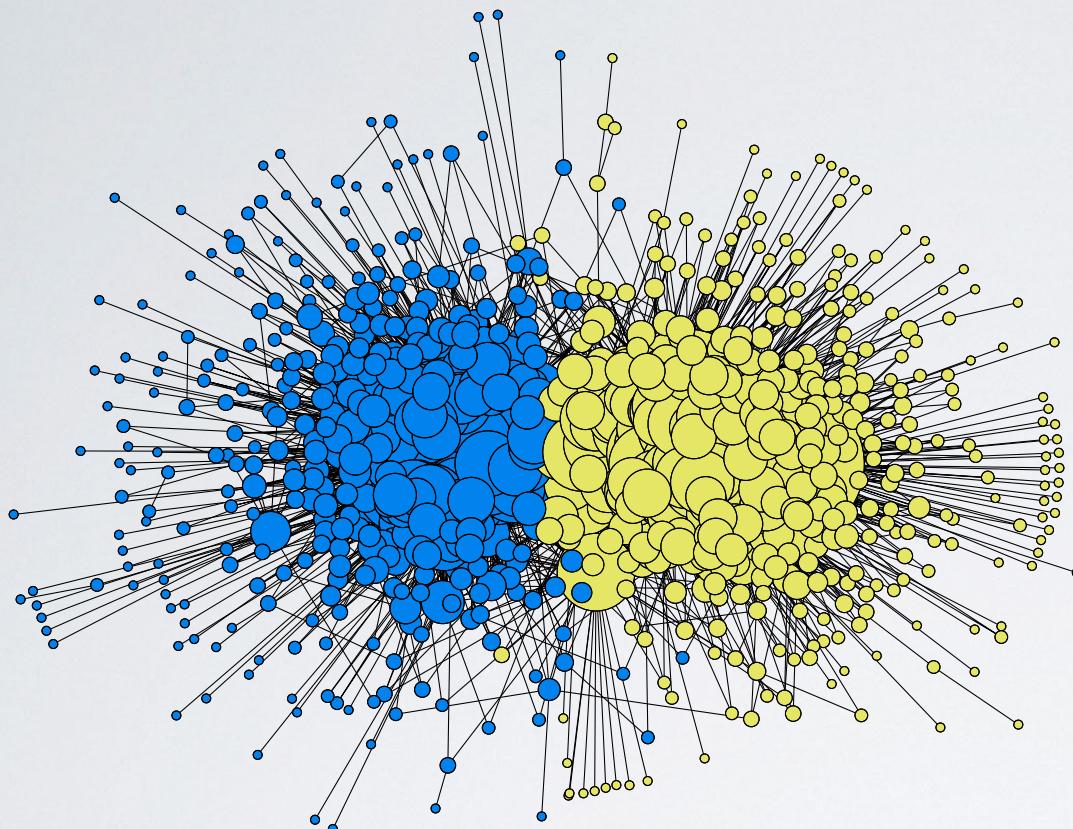
degree distribution :  $\Pr(k) = \left[ \left(1, \frac{1}{6}\right), \left(2, \frac{3}{6}\right), \left(3, \frac{1}{6}\right), \left(4, \frac{1}{6}\right) \right]$

# degree distributions

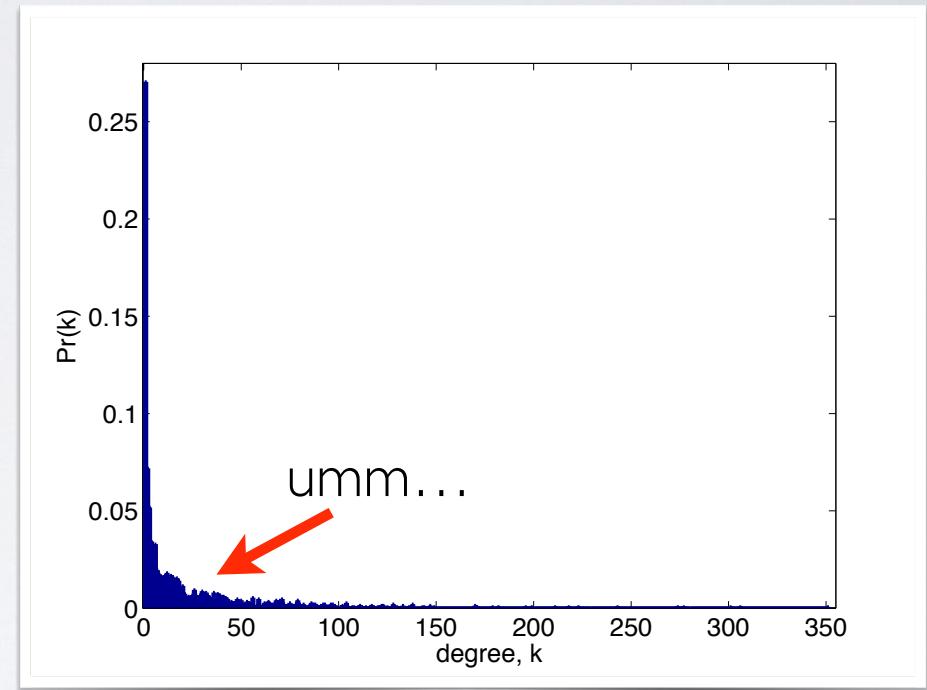


political blogs

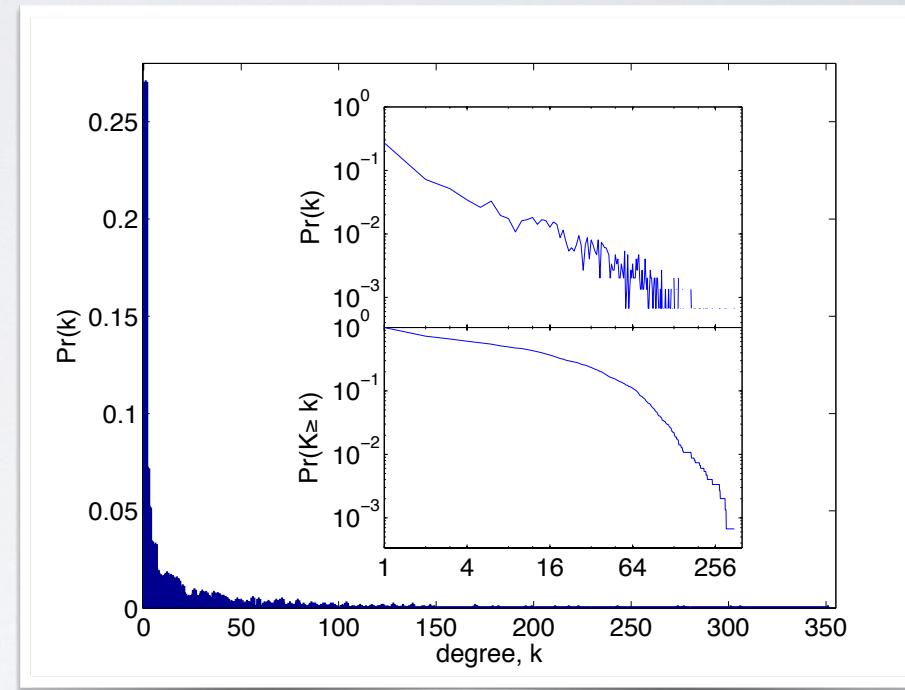
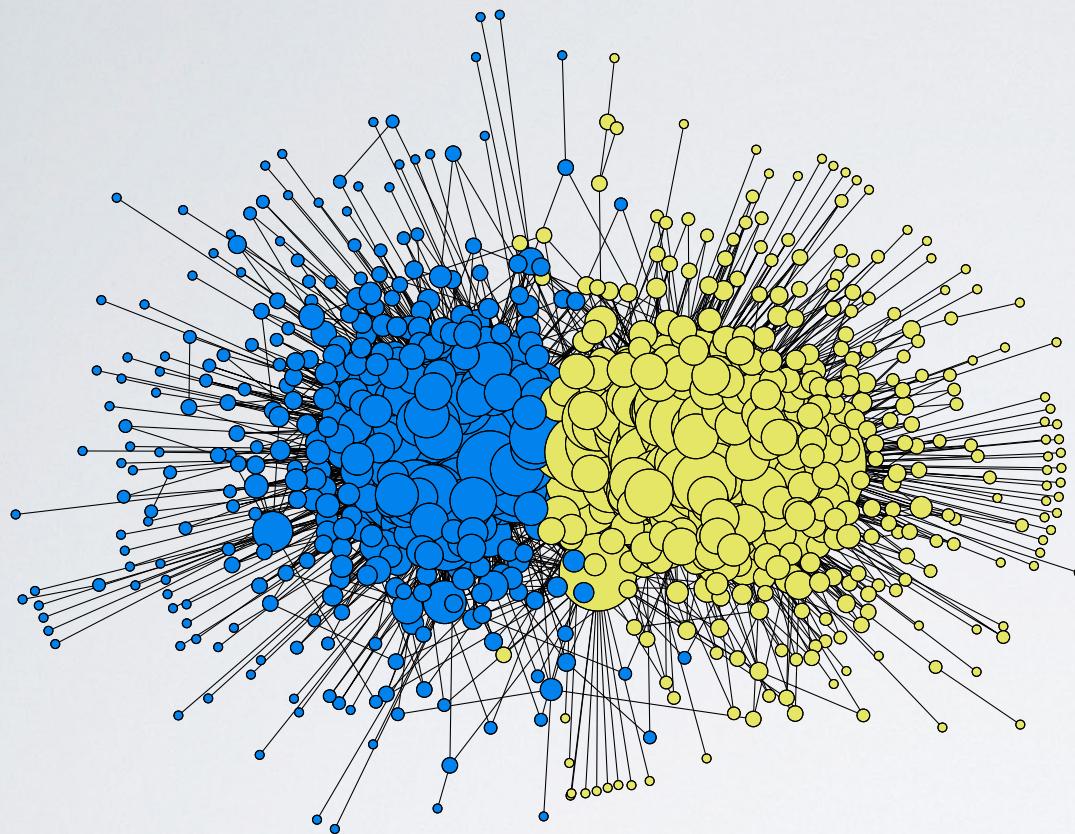
# degree distributions



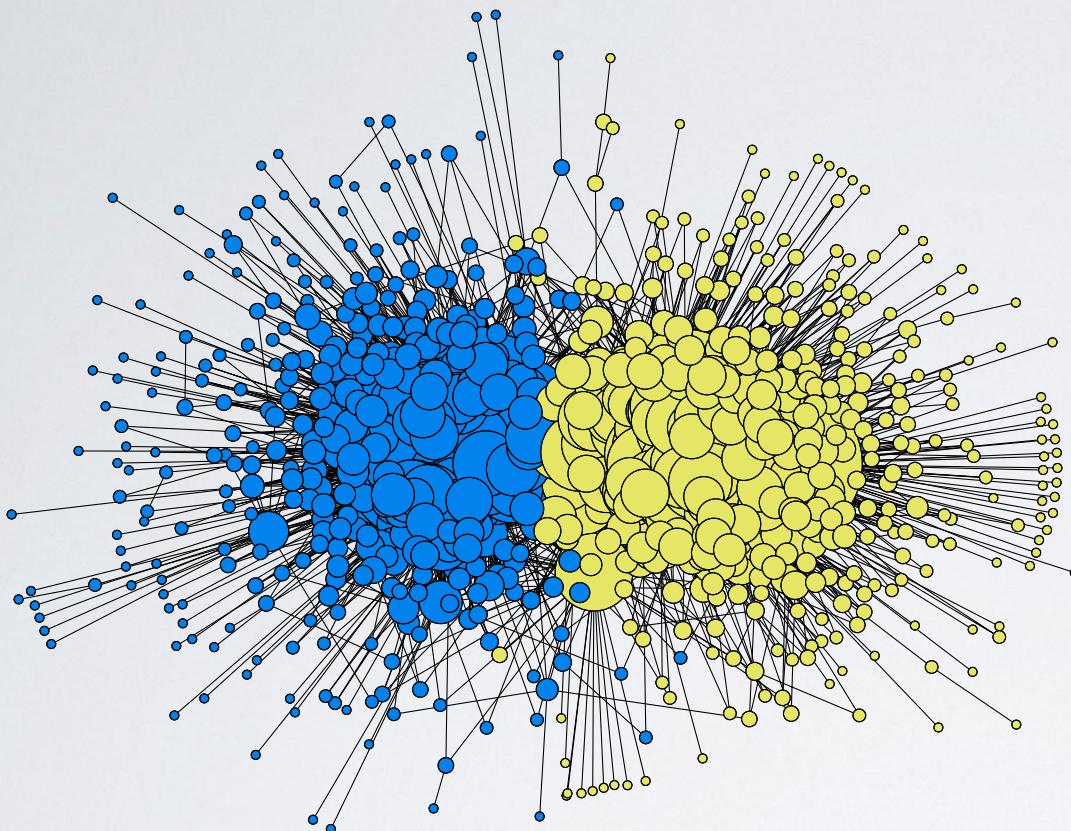
political blogs



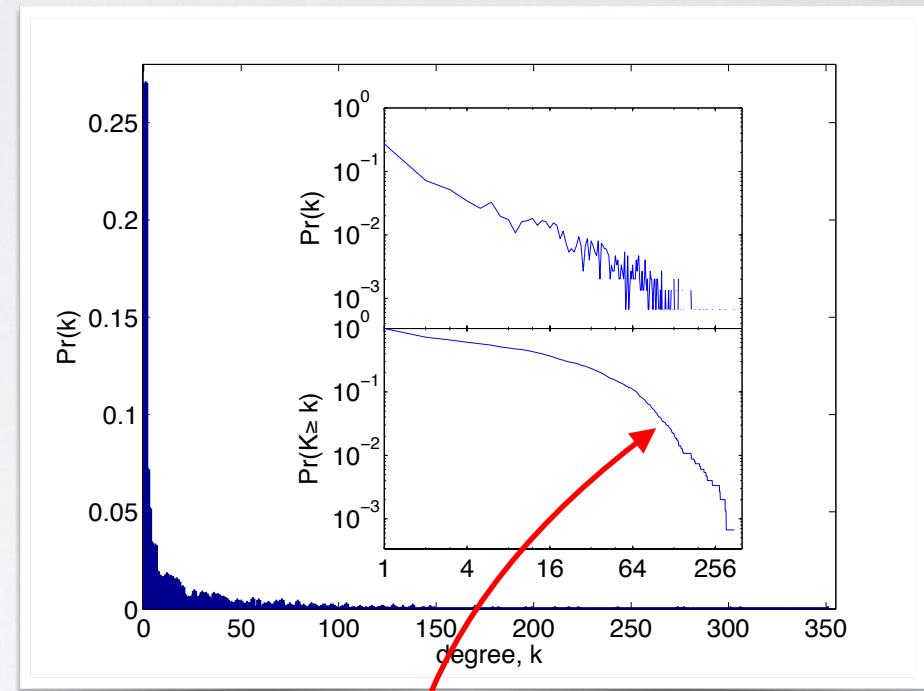
# degree distributions



# degree distributions



political blogs

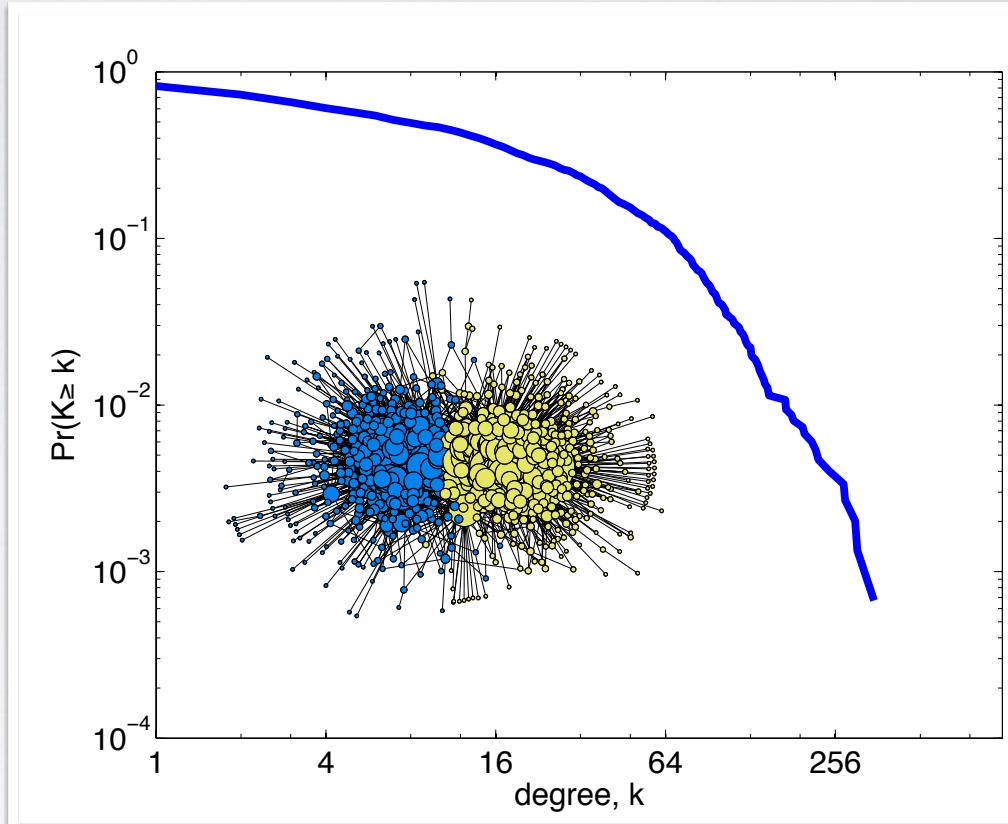


is this a power law?

# degree distributions

## 🤔 fun facts I:

- nearly all real networks exhibit a ***heavy-tailed degree distribution***
- **very few** networks exhibit perfect power-law degree distributions
- **some** distributions exhibit power-law tails
- power laws are cool!  
but identifying them in data  
(and not confusing them for other things) ***requires statistics***



# degree distributions

## 🤔 fun facts 2:

- **degree distribution** is the first-order description of network structure
- **degree heterogeneity** alone drives interesting phenomena ("friendship paradox", spreading dynamics, etc.)
- **degree heterogeneity** alone explains many other network patterns (various centralities, disassortativity, etc.)
- the **configuration model** is how to tell : random graph model with specified degree sequence



airport network & global epidemics

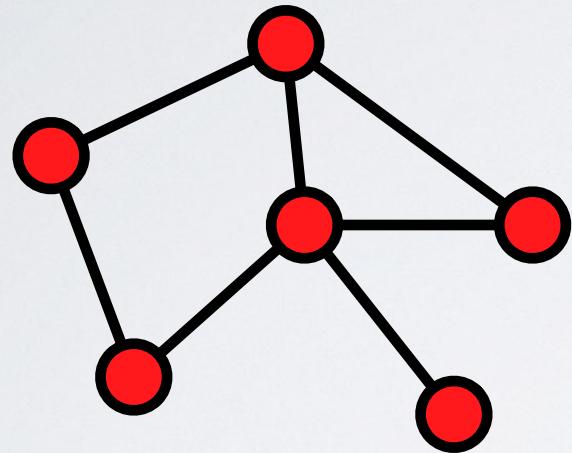
SIAM REVIEW  
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## Configuring Random Graph Models with Fixed Degree Sequences\*

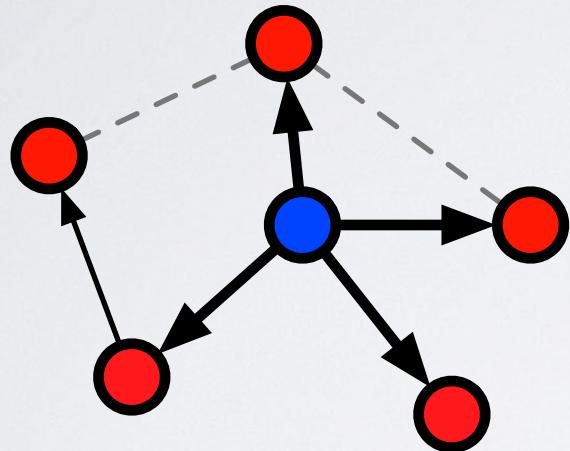
Bailey K. Fosdick<sup>†</sup>  
Daniel B. Larremore<sup>‡</sup>  
Joel Nishimura<sup>§</sup>  
Johan Ugander<sup>¶</sup>

# **network position**



# **position**

# network position



**position = centrality:**  
structural vs. dynamical importance

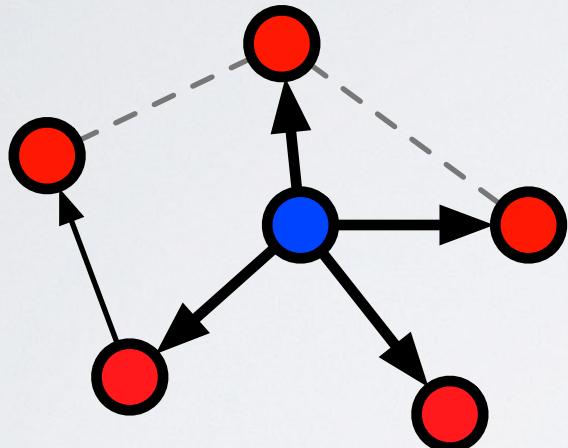
geometric  
connectivity

- harmonic centrality
- closeness centrality
- betweenness centrality
- degree centrality
- eigenvector centrality
- PageRank
- Katz centrality
- many many more...



structural importance = cheap  
estimate of dynamical importance  
(aka "influence")

# network position



**position = centrality:**

structural vs. dynamical importance

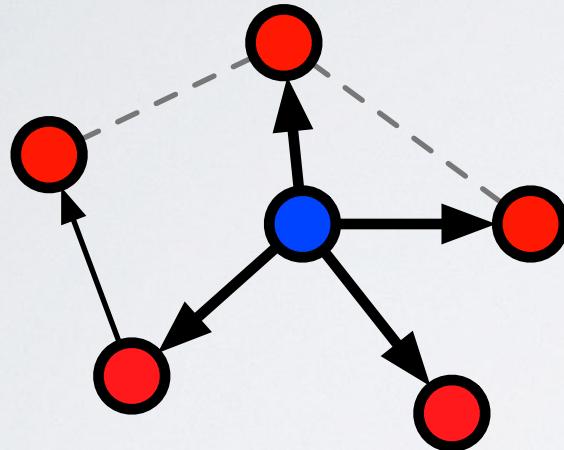
centrality = unsupervised node ranking

$$f: G \rightarrow \overrightarrow{v}$$



there are an infinite number of choices of  $f$ !  
most are correlated  
choose  $f$  that is most meaningful for downstream analysis

# network position



**position = centrality:**  
harmonic, closeness centrality

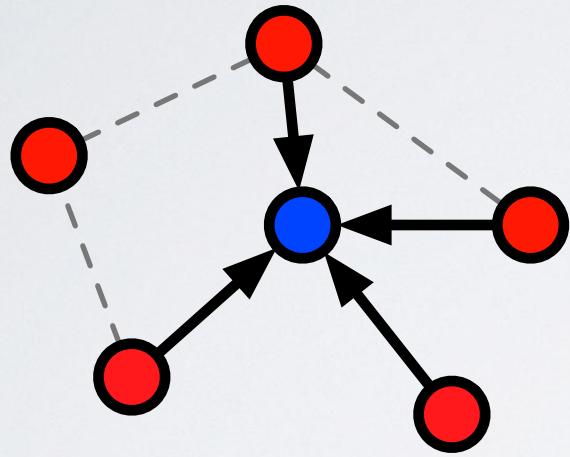
importance = being in “center” of the network

$$\text{harmonic} \quad c_i = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

length of shortest path

distance:  $d_{ij} = \begin{cases} \ell_{ij} & \text{if } j \text{ reachable from } i \\ \infty & \text{otherwise} \end{cases}$

# network position



## position = centrality:

PageRank, Katz, eigenvector centrality

importance = sum of importances of nodes that point at you\*

$$I_i = \sum_{j \rightarrow i} \frac{I_j}{k_j}$$

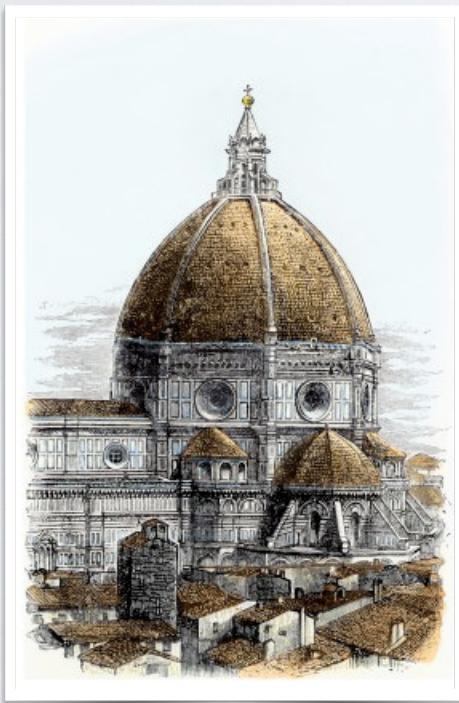
or, the right eigenvector of

$$\mathbf{Ax} = \lambda \mathbf{x}$$

# network position

## Robust Action and the Rise of the Medici, 1400–1434

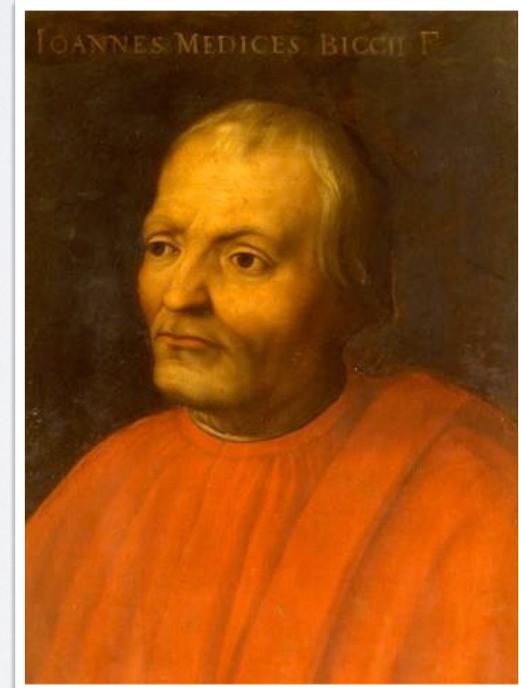
John F. Padgett and Christopher K. Ansell  
*University of Chicago*



Duomo

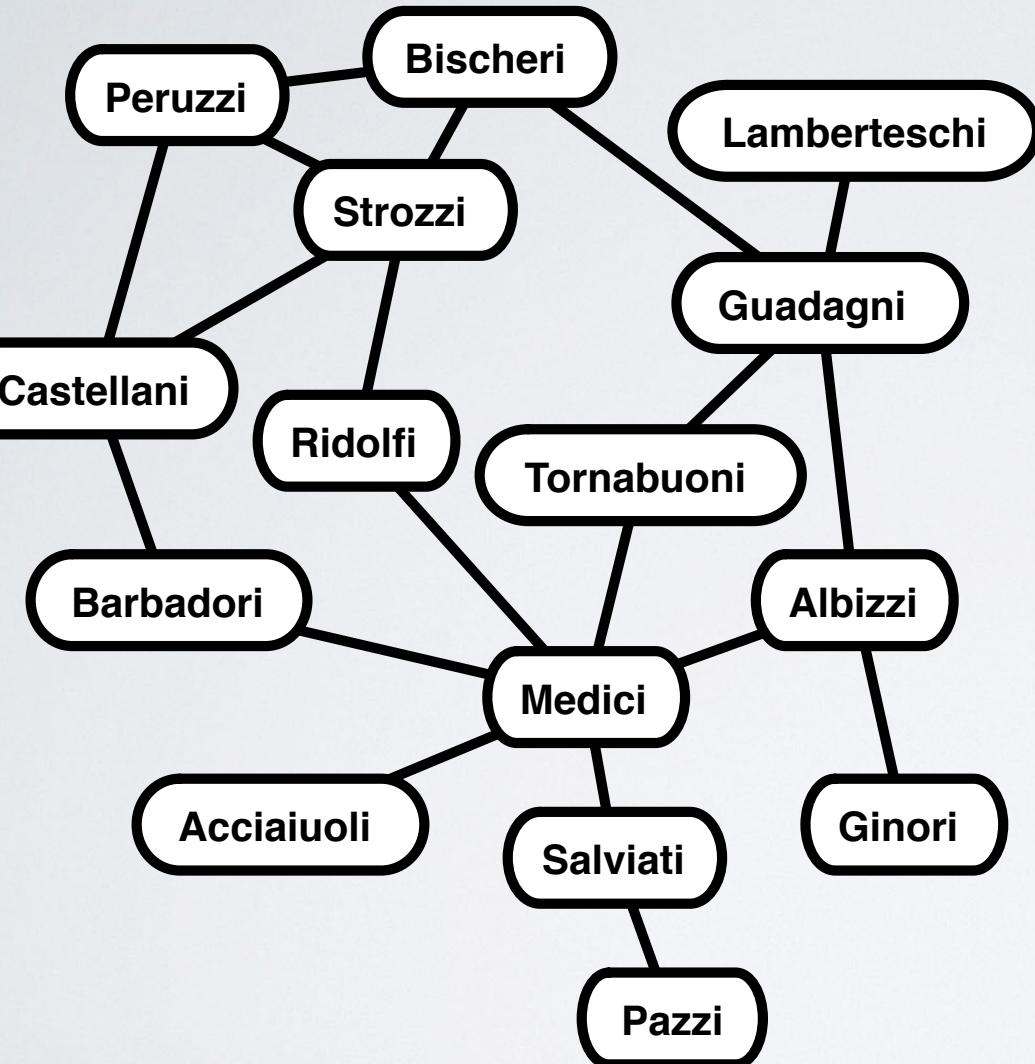


Palazzo Medici



Giovanni de Medici

# network position

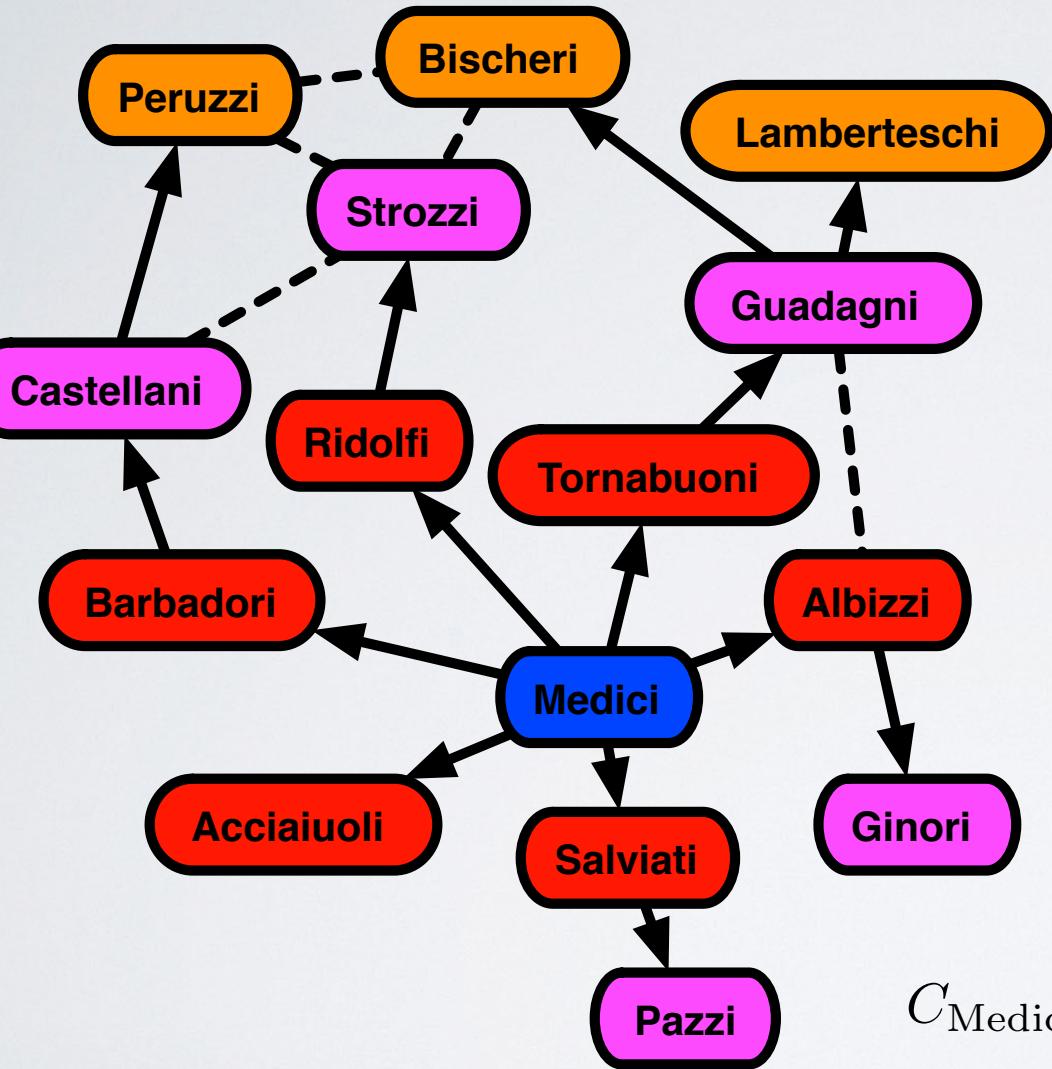


**nodes:** Florence families

**edges:** inter-family marriages

**which family is most central?**

# network position



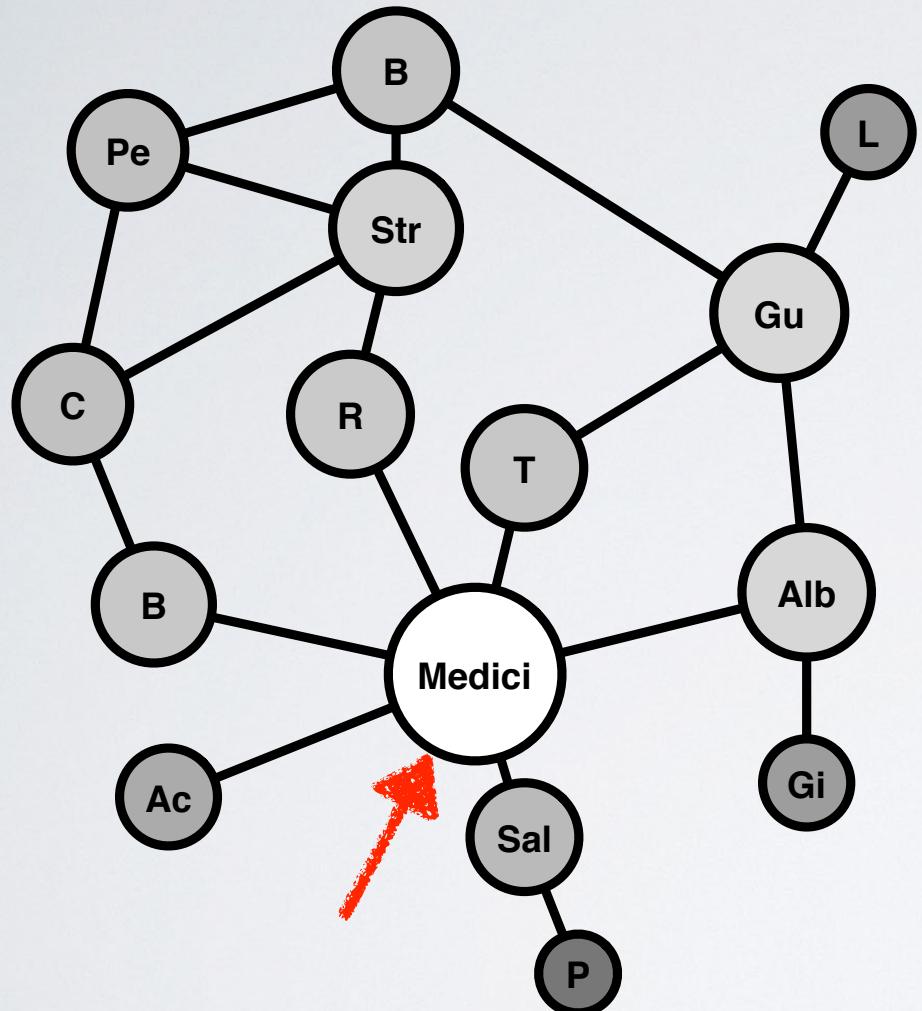
**nodes:** Florence families

**edges:** inter-family marriages

**which family is most central?**  
**Medici.**

$$C_{\text{Medici}} = 6 \left( \frac{1}{1} \right) + 5 \left( \frac{1}{2} \right) + 3 \left( \frac{1}{3} \right)$$
$$= 9.5$$

# network position

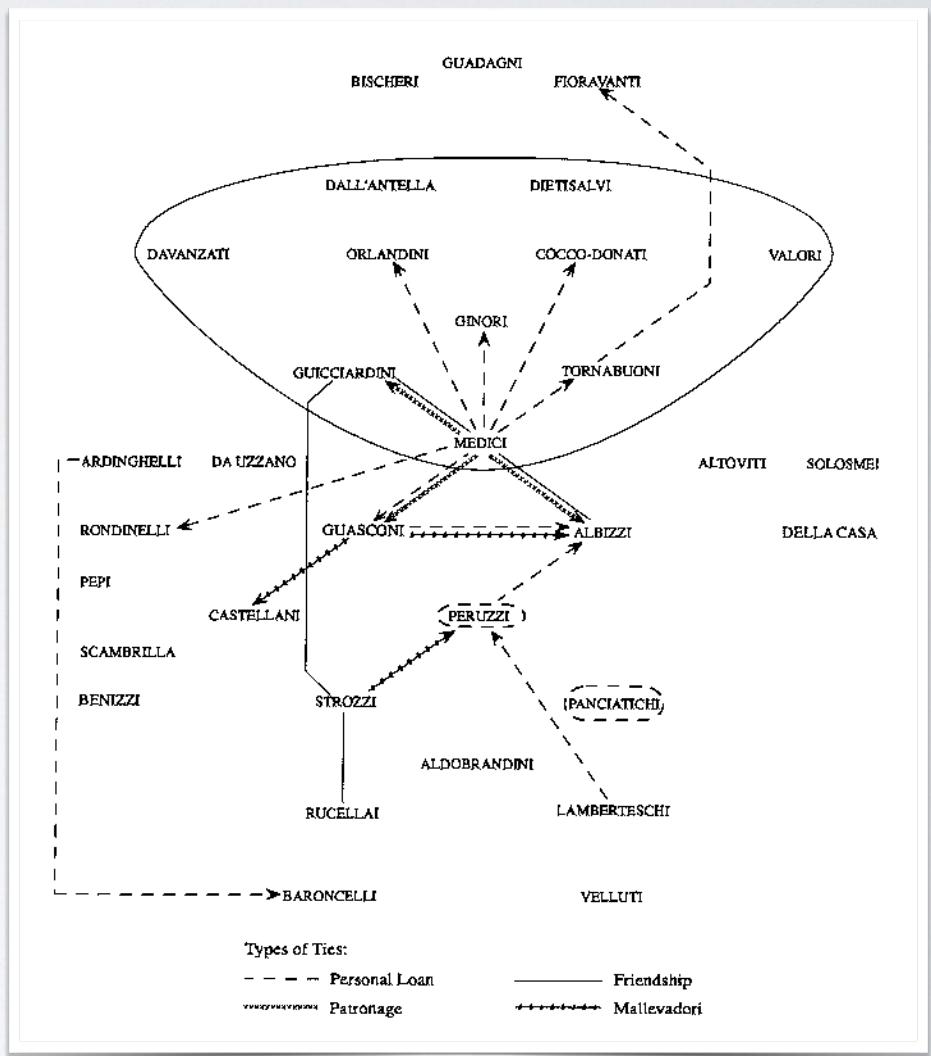
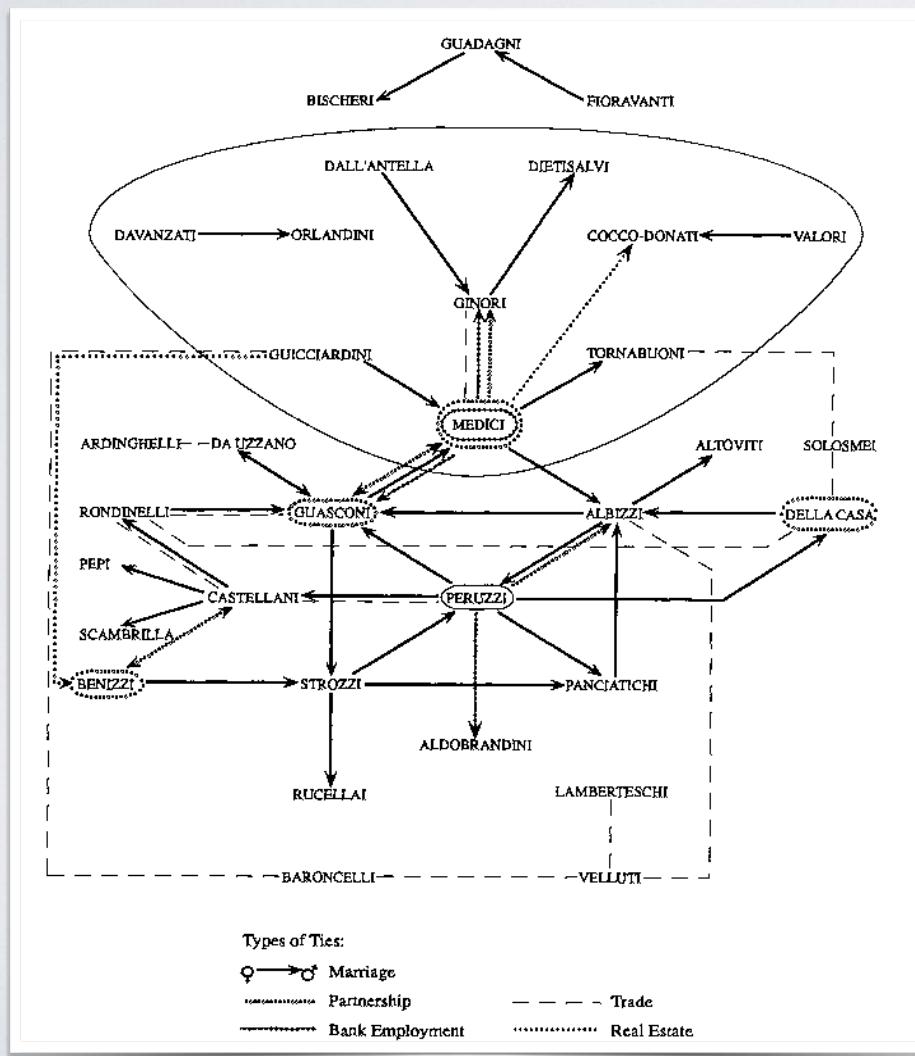


<b>Medici</b>	<b>9.5</b>
<b>Guadagni</b>	<b>7.92</b>
<b>Albizzi</b>	<b>7.83</b>
<b>Strozzi</b>	<b>7.67</b>
<b>Ridolfi</b>	<b>7.25</b>
<b>Bischeri</b>	<b>7.2</b>
<b>Tornabuoni</b>	<b>7.17</b>
<b>Barbadori</b>	<b>7.08</b>
<b>Peruzzi</b>	<b>6.87</b>
<b>Castellani</b>	<b>6.87</b>
<b>Salviati</b>	<b>6.58</b>
<b>Acciaiuoli</b>	<b>5.92</b>
<b>Ginori</b>	<b>5.33</b>
<b>Lamberteschi</b>	<b>5.28</b>
<b>Pazzi</b>	<b>4.77</b>



# network position

actually, it's complicated...



[1] Marriage edges were only one type of inter-family interaction; hence, centrality on them alone is a simplification, and the deeper questions are about dynamics (how did the full network assemble over time?) and about function (was network position causally related to Medici dominance?).

# let's apply these network concepts

RESEARCH ARTICLE

NETWORK SCIENCES

## Systematic inequality and hierarchy in faculty hiring networks

Aaron Clauset,<sup>1,2,3\*</sup> Samuel Arbesman,<sup>4</sup> Daniel B. Larremore<sup>5,6</sup>

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10.1126/sciadv.1400005

# let's apply these network concepts

RESEARCH ARTICLE

NETWORK SCIENCES

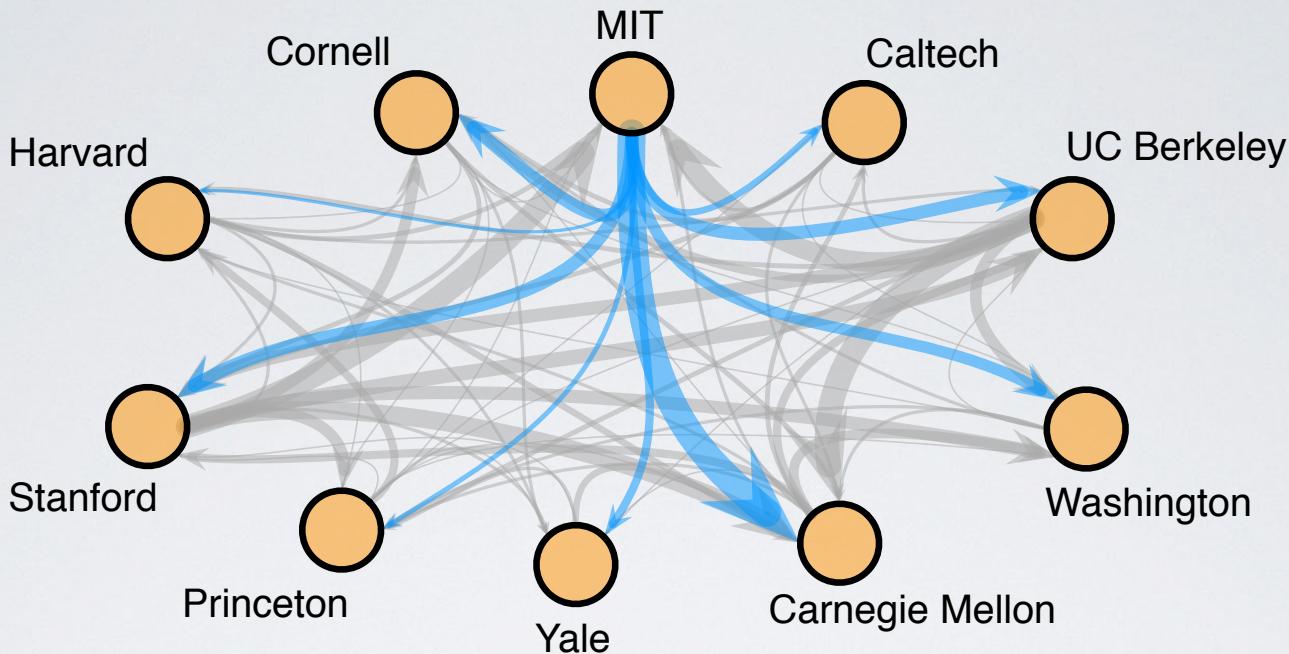
## Systematic inequality and hierarchy in faculty hiring networks

Aaron Clauset,<sup>1,2,3\*</sup> Samuel Arbesman,<sup>4</sup> Daniel B. Larremore<sup>5,6</sup>

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10.1126/sciadv.1400005

### faculty market is a *network*

- vertices are PhD-granting universities
- consumers  $\leftrightarrow$  producers
- $v$  hires from  $u$ , add an edge  $u \rightarrow v$



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## collecting the data

complete, hand-curated data for  
19,000 tenure-track faculty  
across 461 departments in

- Computer Science (205 depts)
- Business (112)
- History (144)

roughly 5000 hours of manual data  
collection

```
>>> record 1059
# facultyName : James H. Martin
# email      :
# sex        : M
# department : Computer Science
# place      : University of Colorado, Boulder
# current    : Full Professor
# [Education]
# degree     : BS
# place      : Columbia University
# field      : Computer Science
# years      : ????-1981
# [Education]
# degree     : PhD
# place      : UC Berkeley
# field      : Computer Science
# years      : ????-1988
# [Faculty]
# rank       : Assistant Professor
# place      : University of Colorado, Boulder
# years      : 1989-1995
# [Faculty]
# rank       : Associate Professor
# place      : University of Colorado, Boulder
# years      : 1995-2007
# [Faculty]
# rank       : Full Professor
# place      : University of Colorado, Boulder
# years      : 2007-2011
# recordDate : 7/4/2011
```

[1] CS data from 2011, Business schools from 2012, History from 2013

[2] all data from public sources, mainly faculty CVs and homepages

[3] data collected by a team of 12 students over 3 years, using a random 20% re-collection protocol, with script-based post-processing to detect errors & inconsistencies, which were then corrected by hand

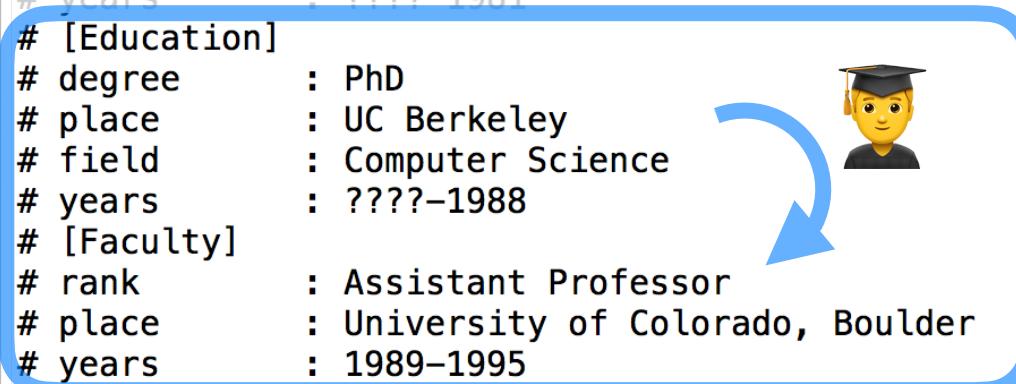
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## what's in the data

	Computer Science	Business	History
institutions	205	112	144
tenure-track faculty	5032	9336	4556
mean size	25	83	32

$\sum = 18,924$

## what's in the data

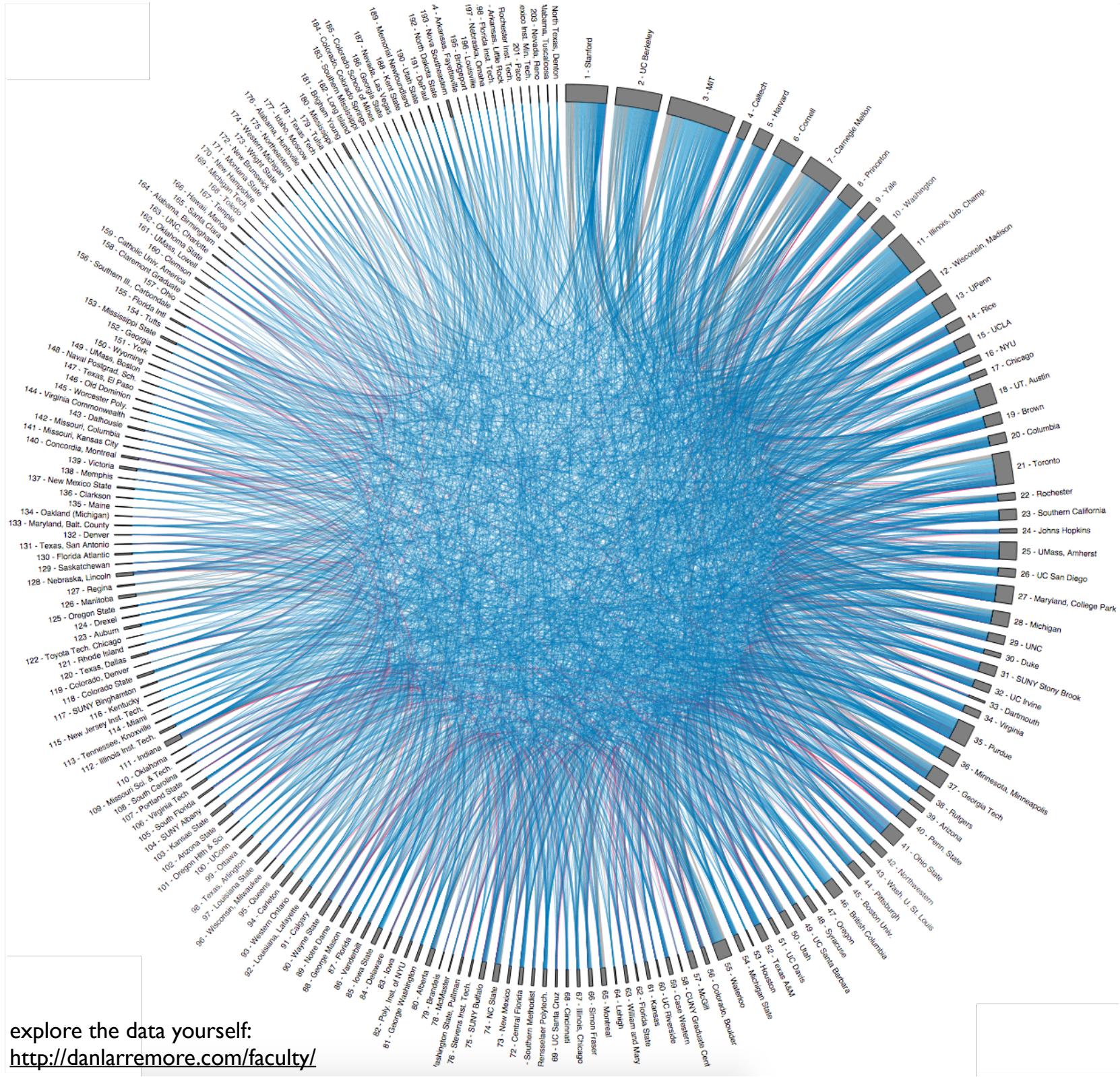
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mean size	25	83	32	
Full Professors	2400 (48%)	4294 (46%)	2097 (46%)	
Associate Prof.	1772 (35%)	2521 (27%)	1611 (35%)	
Assistant Prof.	860 (17%)	2521 (27%)	848 (19%)	
female	15%	22%	36%	

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female	15%	22%	36%	
PhDs in-sample	87%	84%	89%	



nearly closed hiring systems



explore the data yourself:  
<http://danlarremore.com/faculty/>

**huge inequalities in faculty production**

# huge inequalities in faculty production

## Gini coefficients (out-degree)

- 0.69, 0.62, 0.72

## 50% of faculty from

- 18, 16, 8 universities

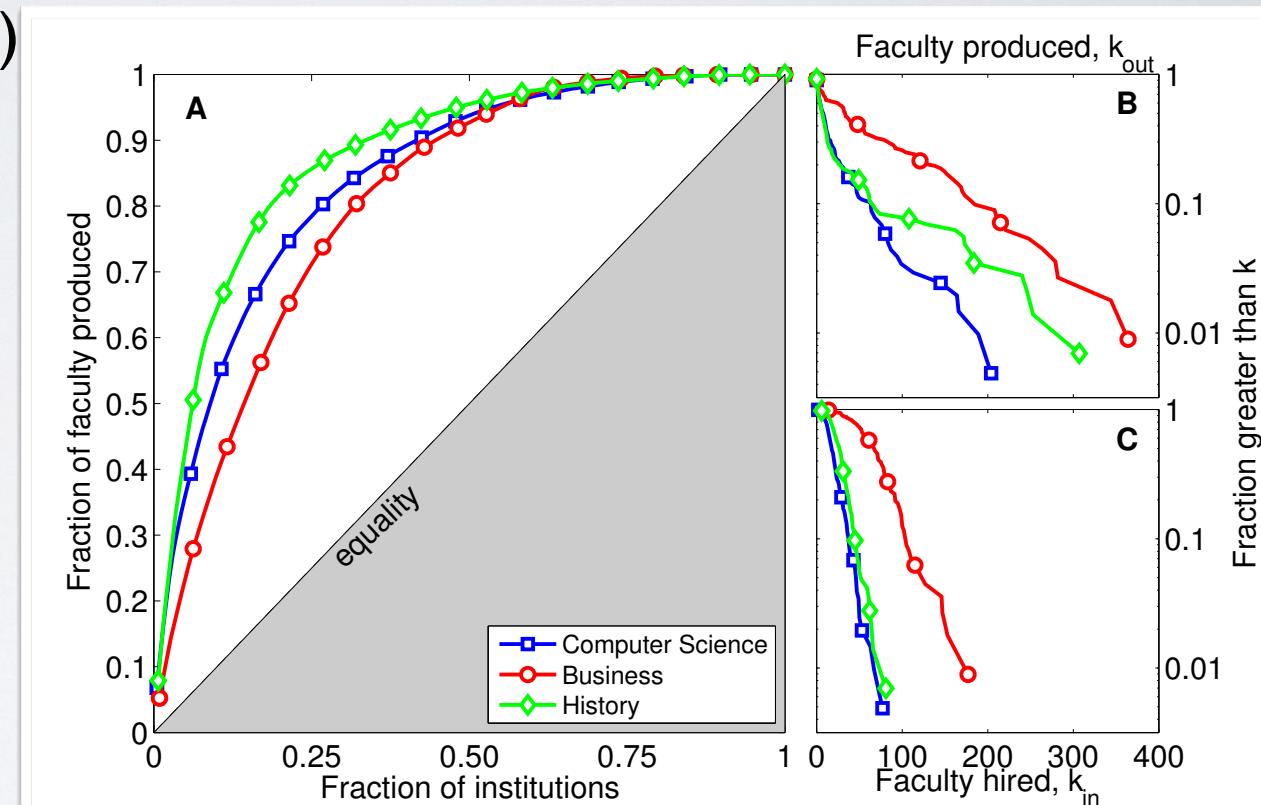
## net producers $k_{\text{out}}/k_{\text{in}} > 1$

- 24%, 36%, 18%

## I-10 producers vs.

- 11-20 : 1.6, 2.1, 3.0x more
- 21-30 : 3.1, 2.3, 5.6x more

CS   Business   History



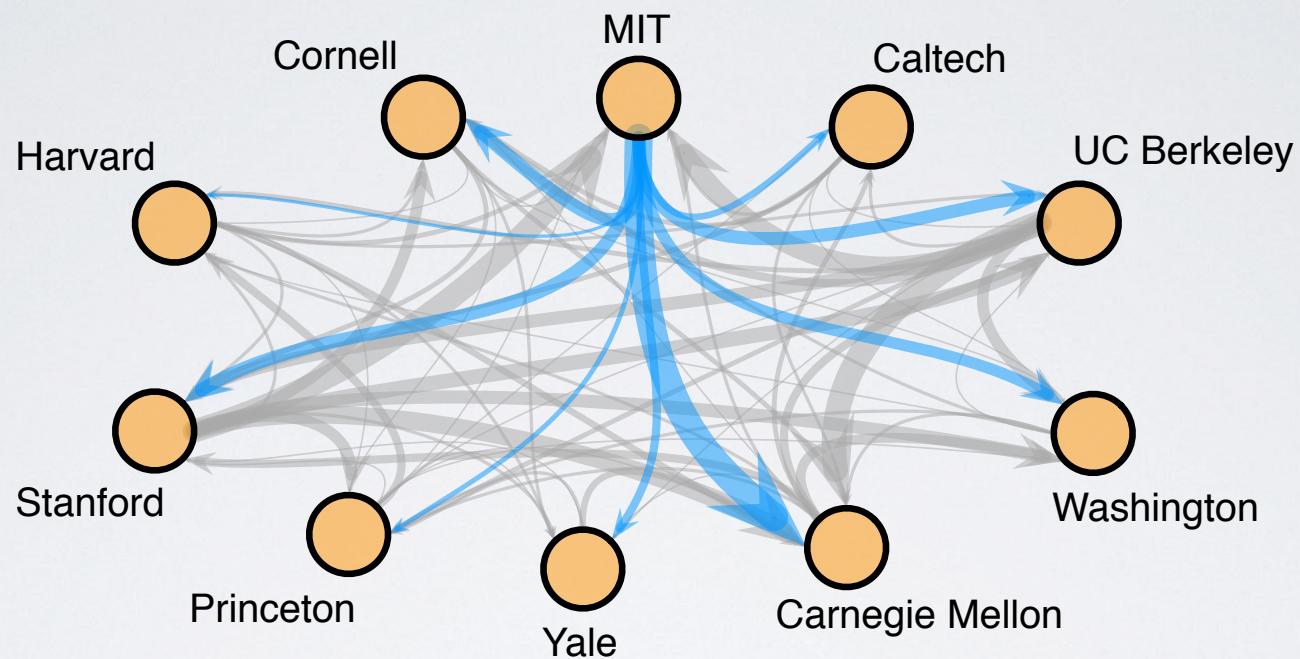
[1] order: CS, Business, History

[2] U.S. Income Gini coefficient = 0.45

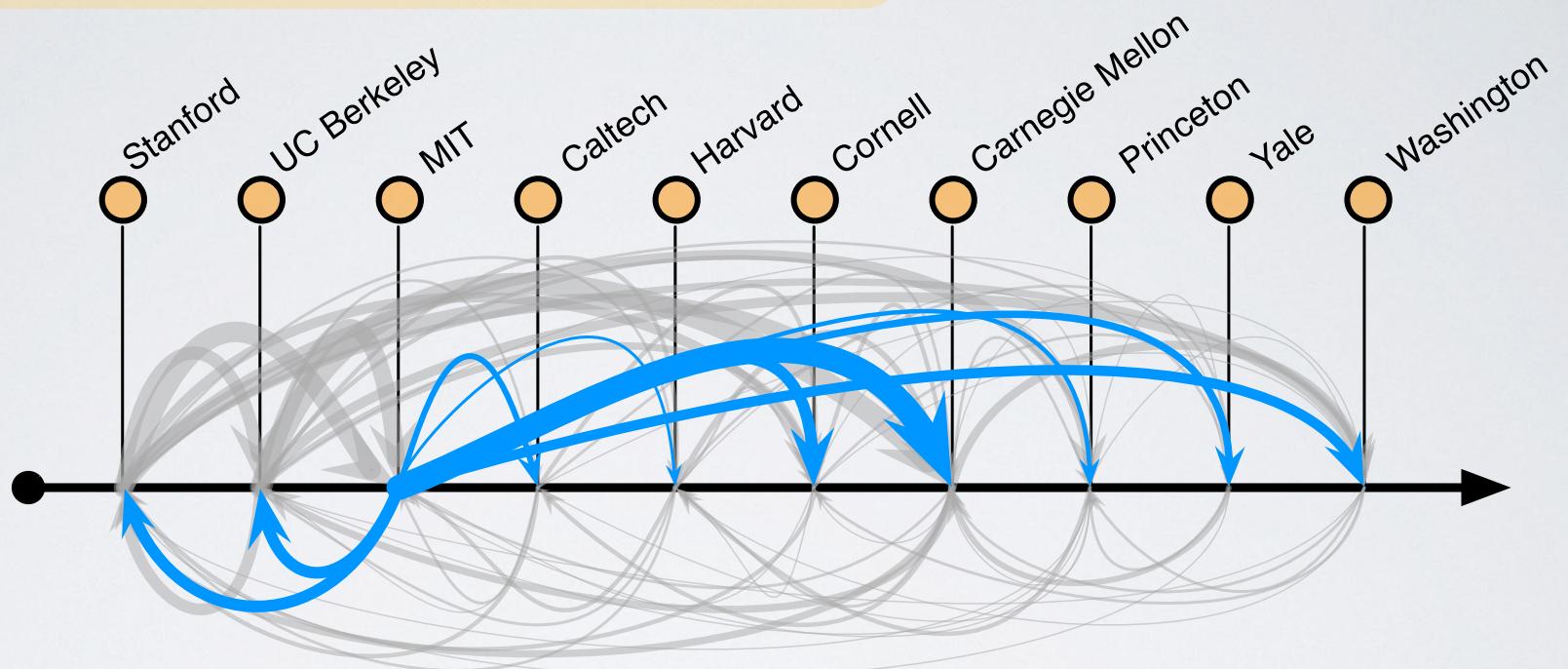
## **a prestige hierarchy**

- difficult to talk about inequalities in academia without talking about rankings (centralities!)
- let's extract a data-driven ranking from the network

## a prestige hierarchy



## a prestige hierarchy



- select permutation (a ranking)  $\pi$  that minimizes the number of "rank violations" : edges  $(u, v)$  where  $\pi_v < \pi_u$
- higher-ranked nodes have greater "placement power"
- equivalent to *minimum feedback arc set problem* (NP-hard)

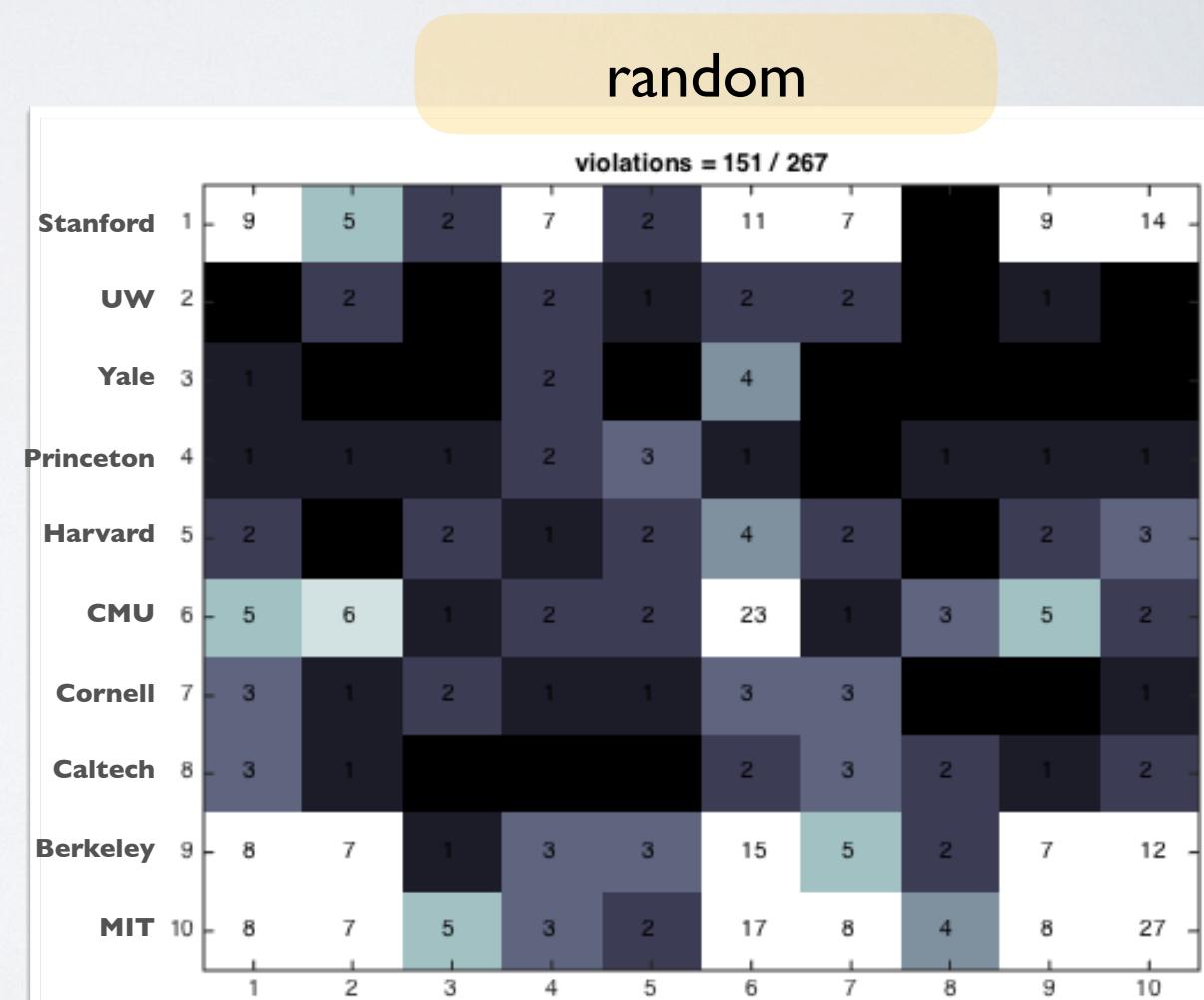
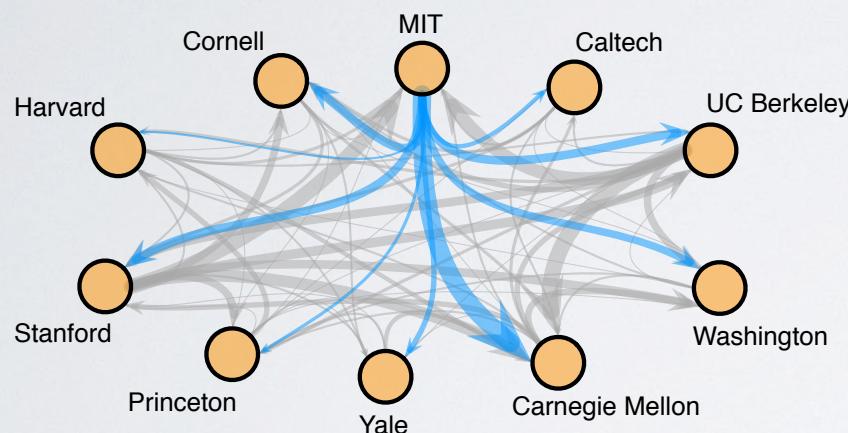
[1] these "MVR"s have a deep history in social theory for extracting dominance or prestige hierarchies from data, especially in animal behavior

[2] MFAS: find the set of arcs of minimum cardinality whose removal converts a directed graph  $G$  into a directed acyclic graph

[3] there are many equivalent MVRs for our network. we sample these using a zero-temp MCMC, and average across them to obtain  $\langle \pi \rangle$

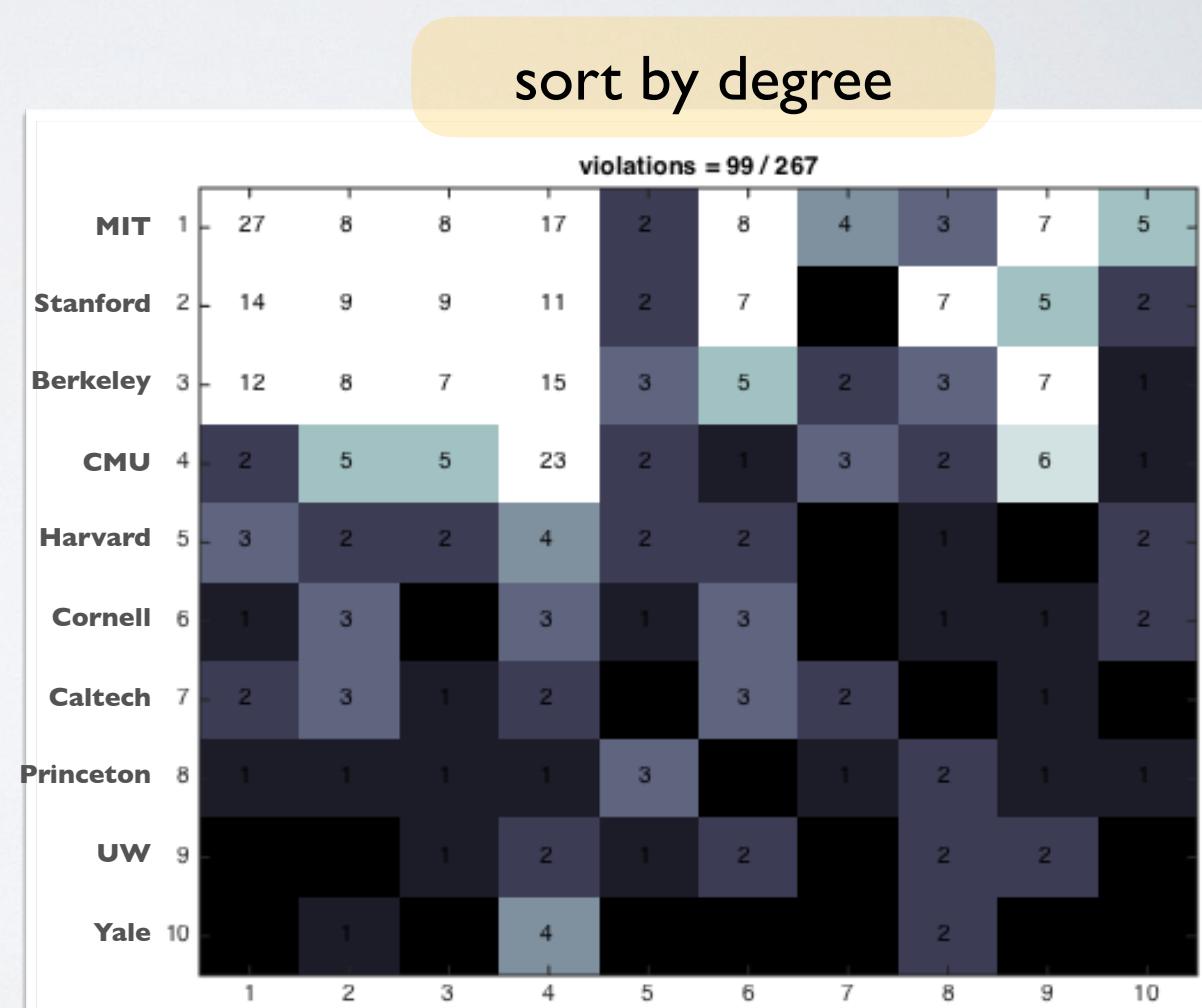
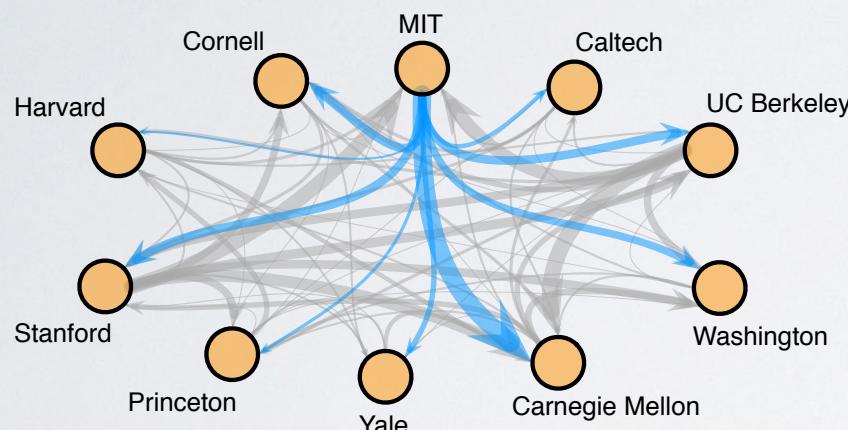
## simple zero-temperature MCMC sampler:

- given an ordering  $\pi$  with  $\psi(\pi, A)$  rank violations on network  $A$
- repeat *ad infinitum*: choose a pair  $(u, v)$ , swap their ranks  $\pi_u \leftrightarrow \pi_v$  to obtain  $\pi'$ , compute  $\psi(\pi', A)$ , accept change if  $\psi(\pi', A) \geq \psi(\pi, A)$
- for instance:



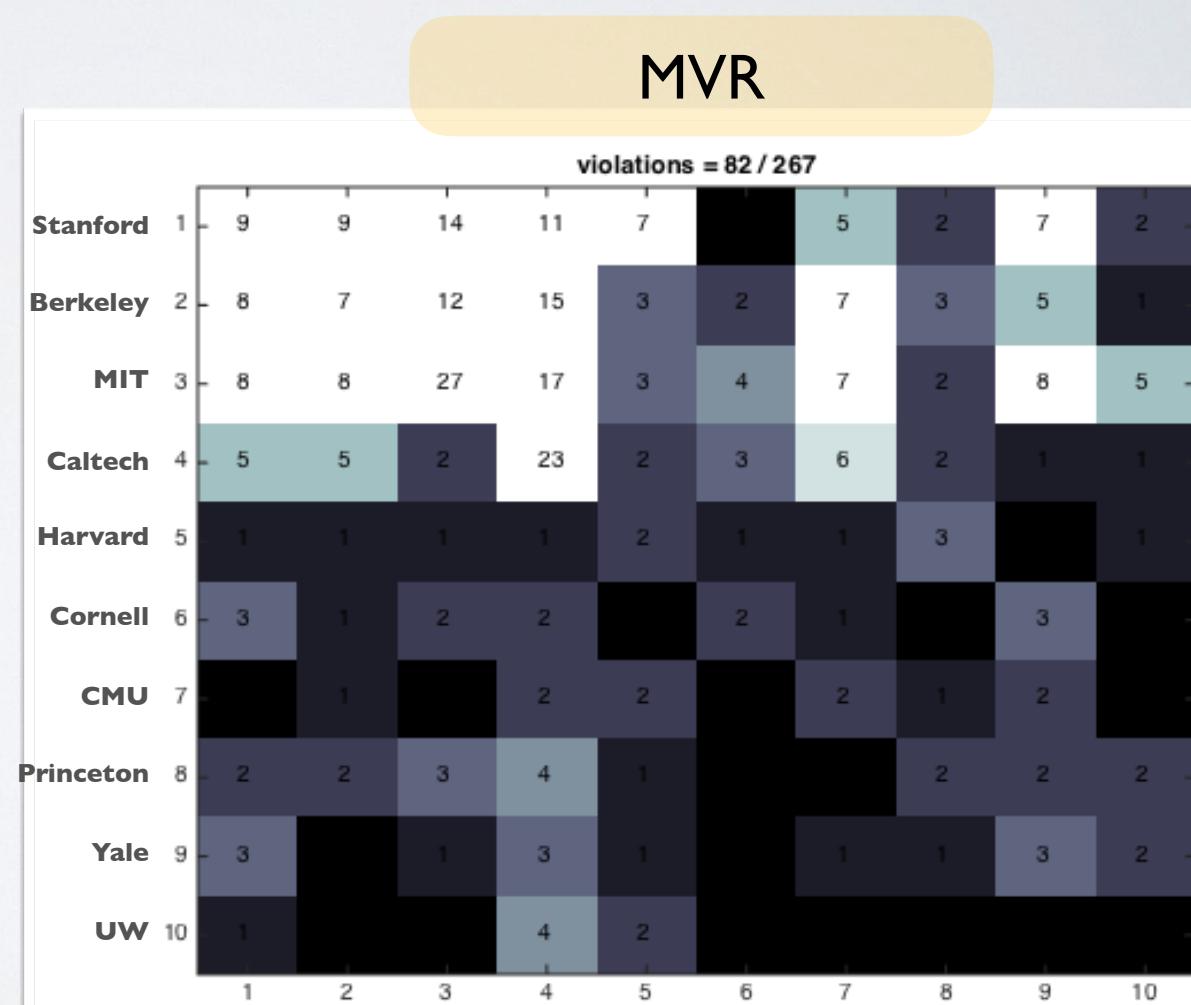
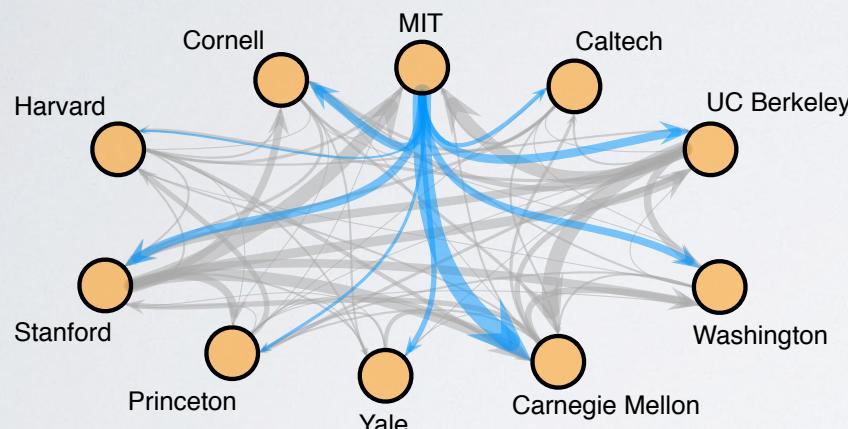
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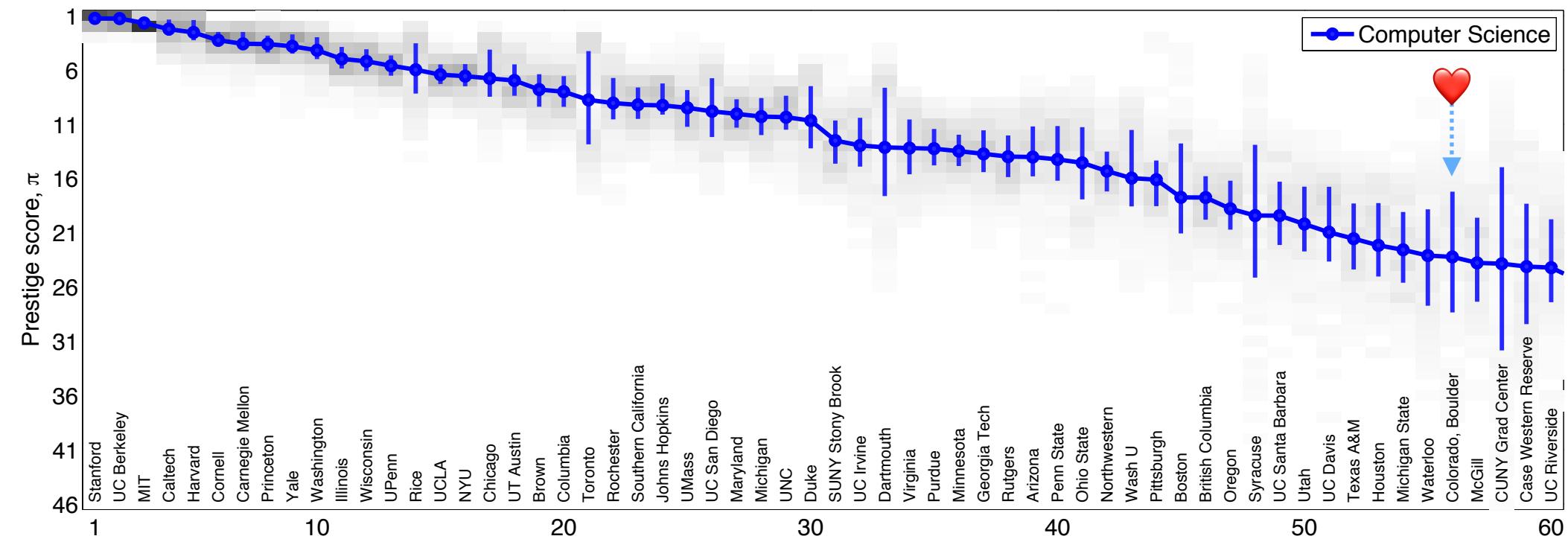
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## **a prestige hierarchy**

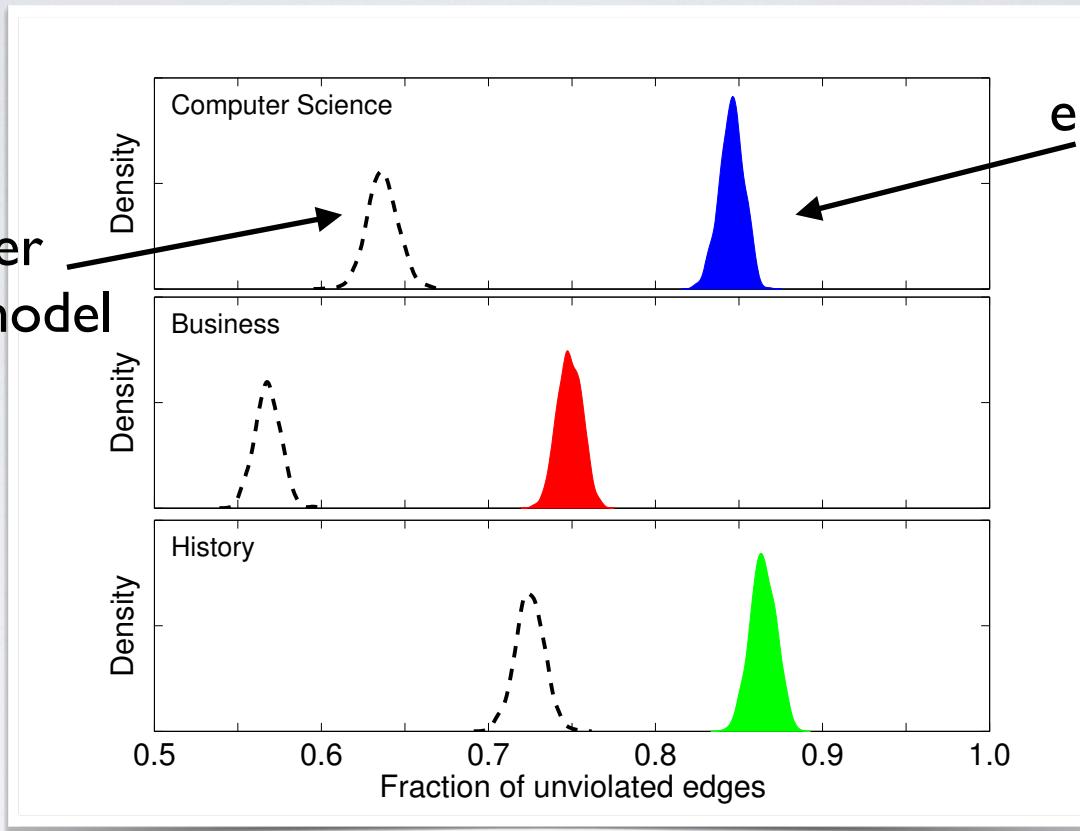
- what do these prestige hierarchies look like?
- what do they tell us about the structure of faculty hiring?
- what predicts placement?



## prestige rankings correlate with USNews and NRC

- here, prestige  $\pi$  quantifies *placement power*
- uncertainty increases as prestige decreases
- similar results, but different orderings for Business and History

fraction under configuration model



empirical fraction

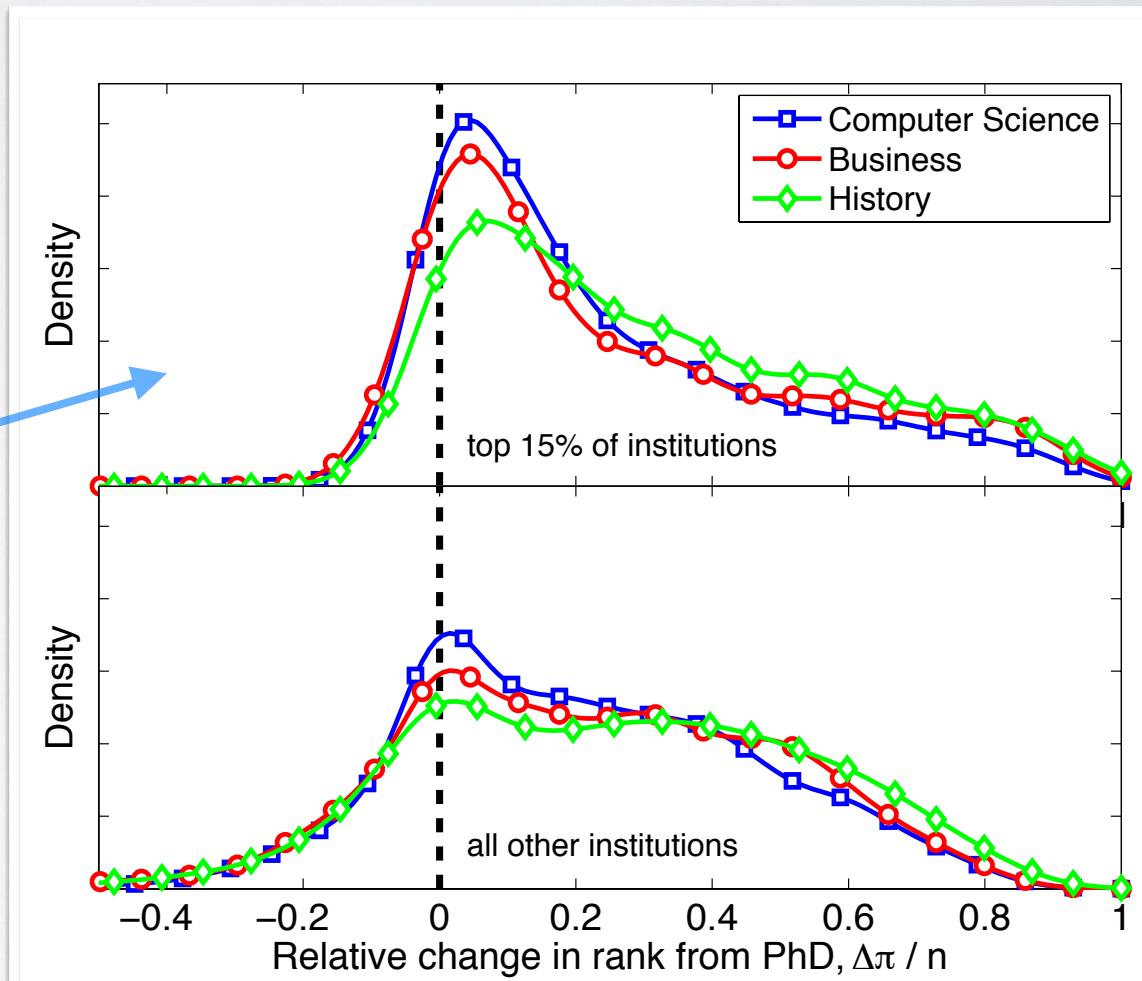
degrees alone do not explain all of the hierarchy

- use configuration model with *observed* degree sequence
- extract MVRs for random graphs & measure fraction of unviolated edges
- compare that null distribution to empirical fraction
- the gap is prestige effect beyond faculty production alone

**most placements are down the hierarchy**

## most placements are down the hierarchy

- down : 88%, 86%, 91%
- up : 12%, 14%, 9%
- $\langle \Delta\pi \rangle = 47, 27, 42$  steps down
- CS: top 15% of departments produce 68% of their own faculty  
and hire 7% from outside top 25% of departments



## **what predicts placement?**

- compare 10 node-level features ("importances"):

prestige

US News rank

NRC rank

out-degree

in-degree

out/in degree

eigenvector centrality

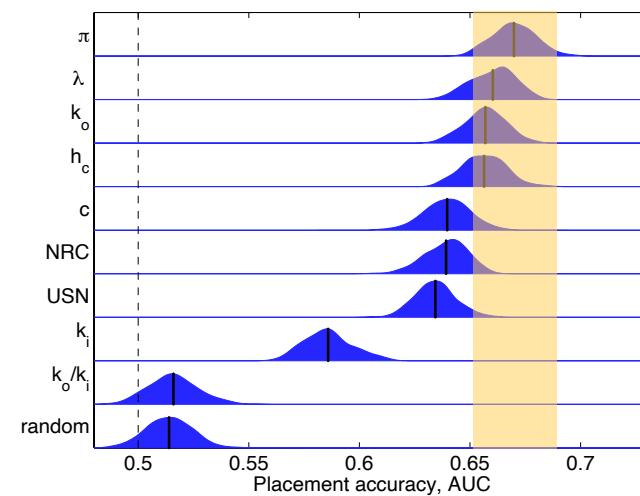
harmonic centrality

closeness centrality

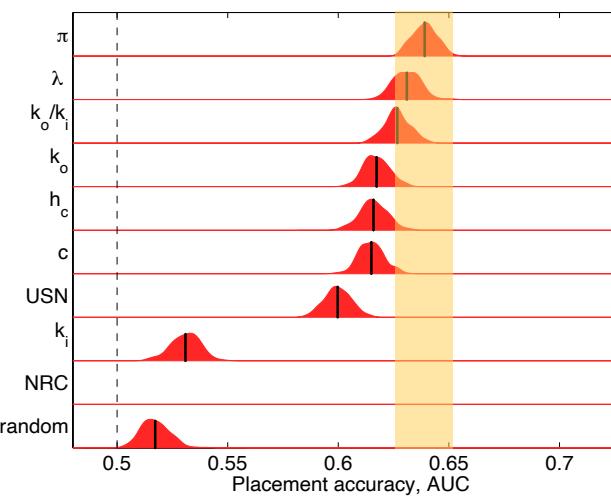
random

## what predicts placement?

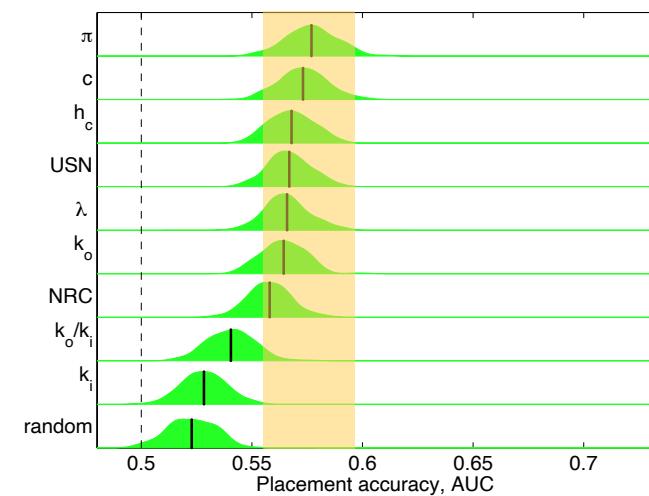
- prestige best *single* predictor in all 3 fields
- order of other features varies by field
- AUCs all below 0.67 = plenty of room for improvement



**CS**



**Business**



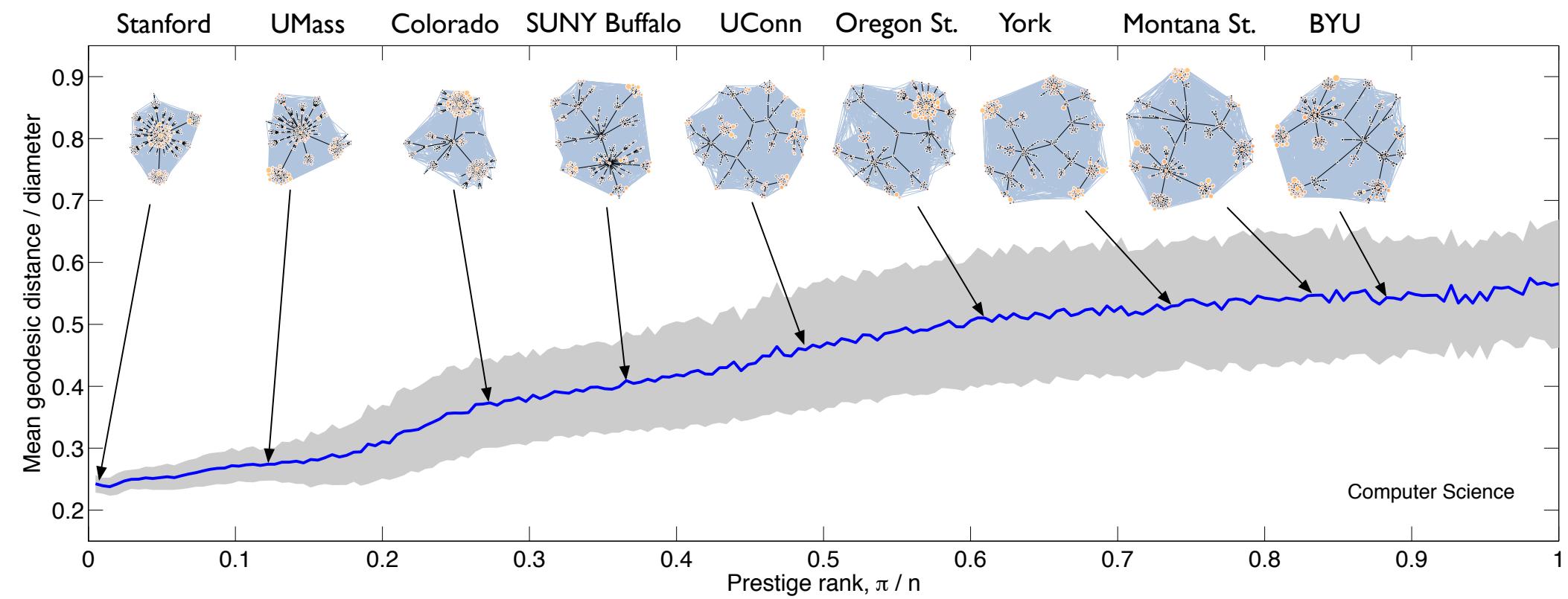
**History**

## **prestige correlates with network position**

- core and periphery

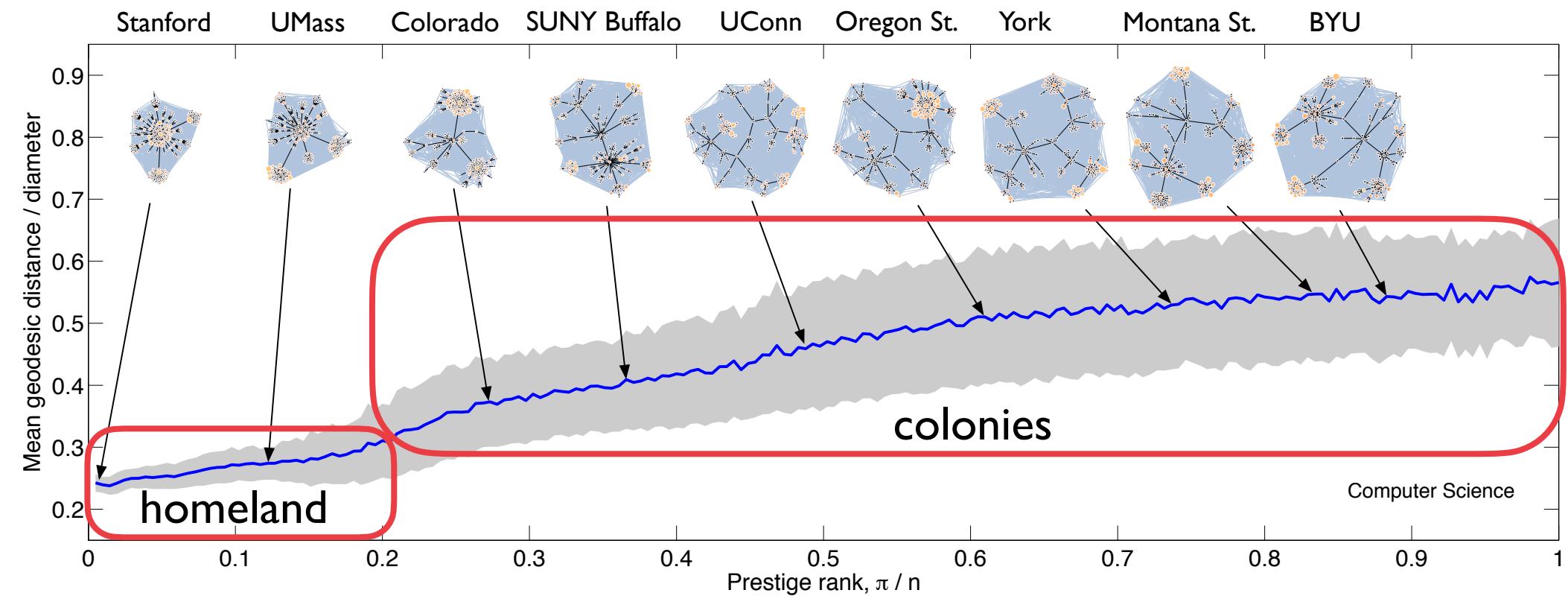
## prestige correlates with network position

- core and periphery



## prestige correlates with network position

- ~~core and periphery~~ homeland and colonies
- prestige is *influence*, via doctoral placement, over research agendas, research communities, and departmental norms across the discipline



## **inequality and prestige hierarchies**

- prestige is *influence*, encoded in faculty hiring network
- faculty flow out of network core, into periphery ("the colonies")
- small fraction stay inside core
- only ~10% of hires flow "upstream"

## **future work**

- how to measure cultural influence of core departments?
- what is different about "upstream" hires?
- what role for other inequalities : gender, ethnicity/race, SES, neighborhood effects, productivity, etc.?



## **conclusions and outlook**



## conclusions and outlook

🎉 networks are cool  
[ obviously, right? ]



# conclusions and outlook



networks are cool

[ obviously, right? ]



powerful window into structure of complex systems

[ structure + dynamics = function ]



network methods for exploiting rich data

[ connectivity + node annotations + edge weights + temporal information | link or label prediction | etc. ]



abundance of interesting science applications

[ dynamics of social influence | emergence of hierarchy | online social network assembly | etc. ]



but be careful: network methods have fundamental limits:

- networks are themselves a *model* of underlying system
- centralities and community detection typically *unsupervised*
- some supervision & auxiliary data = *better* inferences / predictions
- formulate your mechanism in terms of nodes, edges, and attributes
- have fun!



University of Colorado **Boulder**

more material here A yellow hand emoji pointing to the right, indicating where to click for more material.

## Network Analysis and Modeling

Instructor: Aaron Clauset or Daniel Larremore

This graduate-level course will examine modern techniques for analyzing and modeling the structure and dynamics of complex networks. The focus will be on statistical algorithms and methods, and both lectures and assignments will emphasize model interpretability and understanding the processes that generate real data. Applications will be drawn from computational biology and computational social science. No biological or social science training is required. (Note: this is not a scientific computing course, but there will be plenty of computing for science.)

*Full lectures notes online (~150 pages in PDF)*

<http://santafe.edu/~aarond/courses/5352/>