

NEW WASHINGTON SYSTEM CCD STANDARD FIELDS

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ABSTRACT

Sixty-three new high-quality Washington system standard stars have been established in eight fields suitable for observation with a CCD. Each field includes stars with a wide color range sufficiently faint to be observed with large telescopes. The fields are equatorial (with one exception) and spaced approximately every 3 hr in right ascension. Five of the fields contain stars which are already Washington standards, thus facilitating the creation of new standards. Virtually all observations were obtained with a CCD as detector. The R (Kron-Cousins) filter is shown to accurately reproduce T_1 photometry with an efficiency which is three times greater. The mean number of observations per star is 16, and the mean colors and magnitudes are determined to about 0.005 mag. The stars range in magnitude from $10 \leq T_1 \leq 16$, with most lying between 11.5 and 14.5. The stars range in color from $-1 \leq (C - T_1) \leq 4$, with most lying between 0.9 and 3.0, including the full range of astrophysical interest for the system. We also derive transformation equations between some Washington and $UBVRI$ indices for standard stars in common. © 1996 American Astronomical Society.

1. INTRODUCTION

The Washington photometric system (Canterna 1976) was developed along the guidelines drawn by Wallerstein & Helfer (1966) for an optimum broadband system for deriving temperatures and abundances for late-type giants. In this regard, the system has proven useful for determining abundances of giants in a variety of applications, including Galactic globular clusters (e.g., Canterna 1975; Geisler *et al.* 1992b), open clusters (Canterna *et al.* 1986; Geisler *et al.* 1992a), the Galactic bulge population (Tyson 1991; Geisler & Friel 1992), Cepheids (Harris 1981, 1983), Magellanic Cloud clusters (Gascoigne *et al.* 1981; Geisler 1987) and dwarf spheroidals (Canterna 1975; Canterna & Schommer 1978). The latest metallicity calibration using the traditional two-color diagram method (Geisler *et al.* 1991) confirmed that the system offers a unique combination of efficiency and accuracy for determining metallicity in late-type giants over the full range of stellar abundances, although the system has a reduced sensitivity for the coolest metal-poor stars. Geisler (1994) has shown that a new technique, using only the $(C - T_1)$ color of the giant branch, is very promising for deriving precise and accurate metal abundances, even for the most metal-poor clusters. Gonzalez & Piché (1992) have re-

cently shown that the system also has significant advantages for the study of G and K dwarfs. In addition, a number of studies have used the high metallicity sensitivity of the integrated $(C - T_1)$ color to study the metallicity distribution of large samples of globular clusters in distant galaxies (e.g., Geisler & Forte 1990; Secker *et al.* 1995).

As the system finds application to studies of more distant objects using larger telescopes with more efficient detectors, the need for fainter standards has also grown. This need was partially fulfilled by the creation of four standard fields by Geisler (1990, hereafter referred to as G90) using traditional photoelectric photometry. These fields contain an average of 6 stars each (22 total) from $11 \leq T_1 \leq 14$ with a good color range within a small field suitable for observation with a CCD. However, the number of fields is a bare minimum and most of the stars are too bright to be observed with CCDs on 4 m class telescopes except with very short and/or out-of-focus exposures which degrade the photometric accuracy. Thus, it is important to rectify this situation by manufacturing additional, fainter standards in the existing fields and by establishing new fields. The manufacture of good standard stars for the calibration of a photometric system is a very time-consuming, painstaking, distinctly unglamorous, and rather unrewarding task. Nevertheless, it is also an essential one.

This paper then presents the results of a several-year campaign to establish a large number of new Washington system standards in a variety of fields amenable to observation with a CCD on a large (4 m class) telescope. The observations and reductions are described in Sec. 2 and the results given in Sec. 3.

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TABLE 1. New Washington CCD standard fields.

Field	RA(2000)	Dec(2000)	Number of Stars	Color Range ($C - T_1$)	Magnitude Range (T_1)
SA 92	0 ^h 55 ^m 22.0 ^s	+00 ^d 40'30"	8(10)	0.71–2.36	11.2–14.7
PG0231+051	2 ^h 33 ^m 39.7 ^s	+05 ^d 19'05"	9	-0.84–3.00	9.8–16.3
SA98	6 ^h 52 ^m 04.1 ^s	-00 ^d 23'58"	7(15)	0.23–3.84	9.5–14.5
SA101	9 ^h 57 ^m 28.2 ^s	-00 ^d 19'38"	9	0.89–2.64	12.4–14.5
NGC 3680	11 ^h 25 ^m 32.9 ^s	-43 ^d 17'20"	8(14)	0.74–3.27	10.2–13.9
SA107	15 ^h 39 ^m 29.2 ^s	-00 ^d 14'36"	9	1.08–3.04	9.9–14.2
SA110	18 ^h 43 ^m 13.7 ^s	+00 ^d 32'27"	6(11)	0.97–4.63	11.0–13.7
SA114	22 ^h 41 ^m 38.8 ^s	+01 ^d 12'14"	7(12)	-0.21–2.43	10.5–14.8

2. CANDIDATE SELECTION, OBSERVATIONS, AND REDUCTIONS

The fields and stars to be observed were selected with several criteria in mind. The basic guidelines of G90 were followed, but with several improvements; viz. the fields should ideally satisfy the following requirements: (1) Approximately eight stars in a single CCD frame ($\sim 100''$). (2) A large color range [≥ 1.5 in $(C - T_1)$ or 0.8 in $(B - V)$], bracketing the range of astrophysical interest [$1 \leq (C - T_1)_0 \leq 3$]. (3) A relatively small range in brightness (\sim a factor of 10), so that one can obtain sufficient signal from the faintest star without saturating the brightest. The magnitude range should be such that the stars are bright enough to be observed with a CCD within a reasonable time (≤ 100 s) on a 1 m class telescope and faint enough to not saturate with a 4 m telescope in the minimum exposure time (~ 3 s) required to prevent significant shutter timing and scintillation errors. Previous experience suggested a good range to be $T_1 \sim 12$ –15. (4) The stars should be as uncrowded as possible. In practice, this meant that no significant contamination fell within $\sim 10''$ of the candidate standard star, so that aperture photometry (NOT PSF photometry) could be carried out with a diameter of $\sim 15''$, comparable to that used to set up the original photoelectric standards. Again, clusters were avoided for this reason, with the exception of NGC 3680, a southern open cluster which is one of the G90 fields and which contains a sufficient number of clean stars for standardization. (5) The fields should be equatorial, with a total of eight fields approximately equally spaced in right ascension. This ensures that at least two fields will always be observable at a reasonable air mass (< 1.5) from either hemisphere.

With these guidelines in mind, the eight fields described in Table 1 were chosen. Note that SA indicates the Harvard Selected Areas. The four G90 fields were the first fields selected, as they had previously been chosen for satisfying many of these criteria. These fields are SA 98, 110, and 114, and NGC 3680. Since local photoelectric standards already exist, manufacturing new standards in these fields is relatively simply and accurately achieved. These existing standard fields also provide a wealth of fainter, isolated stars ripe for standardization. For example, examination of the Landolt (1992) *UBVRI* standards reveals no less than 20 stars with $13 \leq V \leq 15$ within a $15'$ field in Selected Area 98. Of course, there are also additional suitable stars in these Areas. I also selected three excellent new fields from Landolt's (1992) work: the PG 0231+051 field and Selected Areas 101 and 107. Examination of his standards reveals some 6 to 12 stars

per field of suitable magnitude, and with a wide color range, bracketing the range of astrophysical interest, with additional stars on the frames suitable for standardization as well. Note that these three new fields do not contain any prior Washington standards. The last field chosen was Selected Area 92, in which two original Washington photoelectric standards (Harris & Canterna 1979) already exist. These consist of a sufficiently faint red-blue pair and thus are a perfect core around which to build additional standards.

The stars eventually selected within each field for standardization met the above requirements as well as possible, as indicated in Table 1. The stars were either selected on the basis of the Landolt (1992) *UBVRI* photometry and/or after CCD exposures in the Washington filters indicated the optimum convolution of field size, color and magnitude. The eight fields and the new standards, as well as the previous Washington standards that fall within the same fields, are identified in Fig. 1(a)–1(h). A total of 63 new Washington standards has been established. Note that the Number of Stars column in Table 1 gives both the number of new standards and the total that fall within the field (in parentheses), including previous standards. The color and magnitude ranges also include all standards in the field. Note that two previous Washington standards from G90, SA 98 stars 648 and 696, were found to be significantly contaminated by neighbors during this study and thus should no longer be considered as standards. Note that not all of the above criteria are completely satisfied by the selected fields and stars. However, they represent an excellent compromise with reality.

The data were taken with the CTIO 4, 1, and 0.9 m, the KPNO 4 and 0.9 m, and the CFHT 3.6 m telescopes over a total of 20 photometric nights from 1993 February to 1994 December (excepting a single night of data which was obtained in 1990 October). The observations were obtained with a CCD as detector (generally a Tektronix 2048² pixel device) except for two nights for which a GaAs photomultiplier was used. The photoelectric observations were obtained with the same equipment, techniques, and filters as used to define previous standards (Canterna 1976; Harris & Canterna 1979; G90). For the CCD observations, the filters used were the "standard" Washington (or RI) CCD filters available at CTIO and KPNO (an identical set was purchased for the CFHT observations). These filters are constructed as follows, using the author's prescription as determined at the beginning of this investigation:

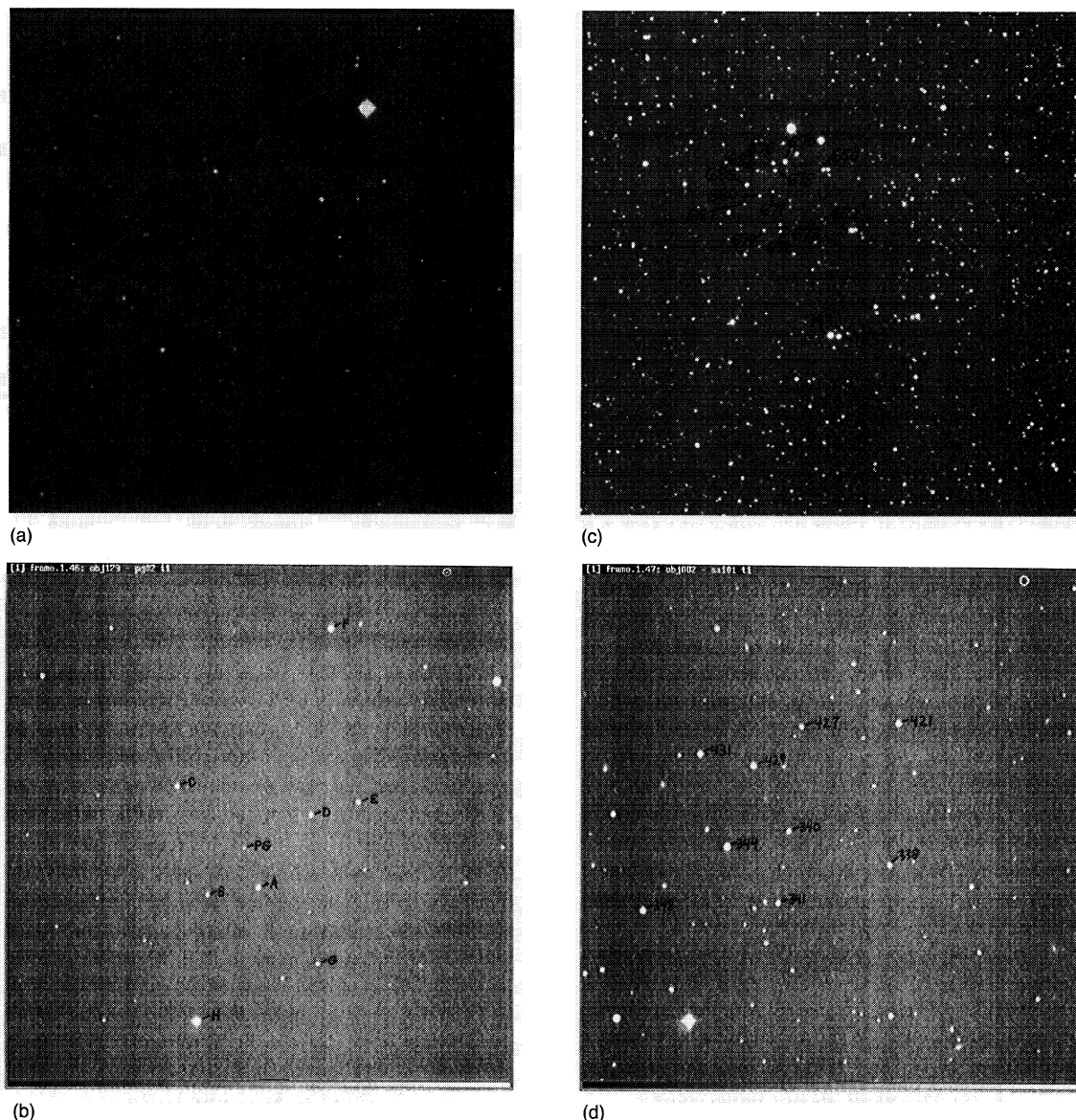


FIG. 1. Finding charts for the new standard stars, as well as any previous standards that lie within the field. N is up and E is to the left. All of the frames were taken with the T_1 filter on the CTIO 0.9 m telescope, where the field size is $13.5'$ on a side, except for the SA 92 and SA 98 frames, which were taken with the T_1 filter on the KPN 0.9 m at $f/7.5$ and have a field size of $23.2'$ on a side. The centers of the displayed figures are very close to the coordinates given in Table 1. (a) SA 92, (b) PG 0231+051, (c) SA 98, (d) SA 101, (e) NGC 3680, (f) SA 107, (g) SA 110, (h) SA 114.

$C=4$ mm Corning CS 7-59+2 mm Schott
BG39+2 mm Schott WG280,

$M=5$ mm Corning CS 4-96+3 mm
Schott GG455,

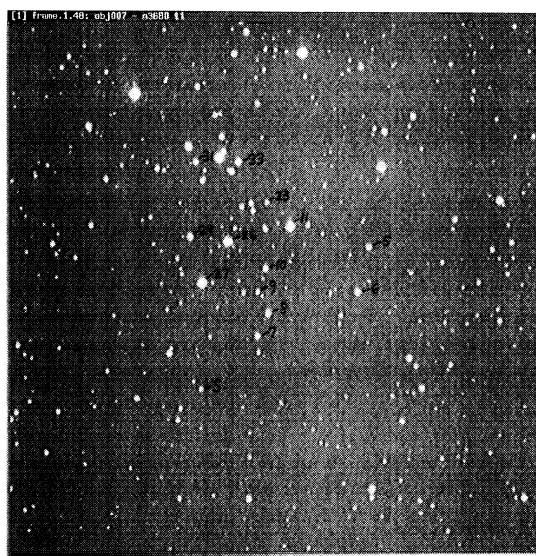
$T_1=3$ mm Schott OG590+2 mm
Schott BG38+3 mm Schott WG280

OR standard "Harris" R(Kron-Cousins),
=2 mm OG570+2 mm KG3

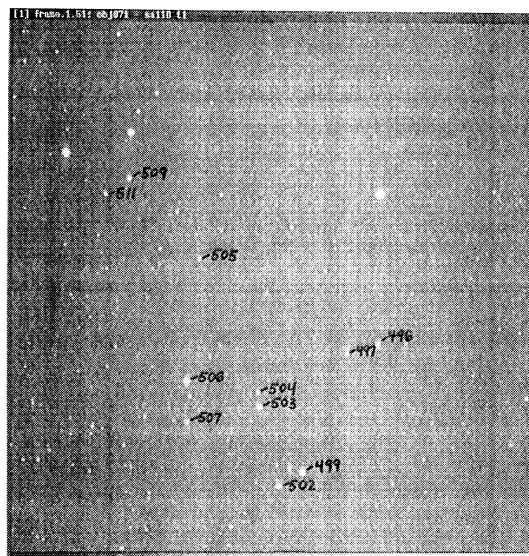
T_2 =standard I (Kron-Cousins),

All filters are glass (except for the $T_2(I)$ filter, which is an interference filter), and part of the standard sets available at CTIO, KPNO, and CFHT. The glass filter prescriptions represent improvements over those defined in Geisler (1987). Note that no photometry was obtained in the DDO 51 filter, which was added to the Washington system by Geisler (1984). This filter is used to separate dwarfs from giants and only requires *relative* photometry within a field.

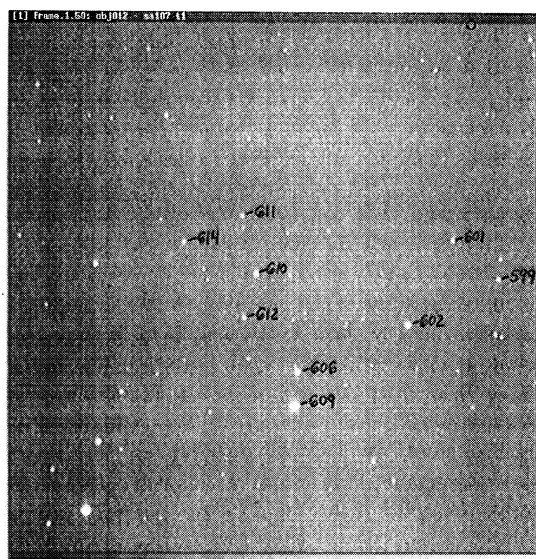
The CCD filters described above are generally the same as those used to define the standard (photoelectric) system, with several exceptions: The solid CuSO_4 filter used in C to prevent red leak has been replaced with 2 mm of BG39



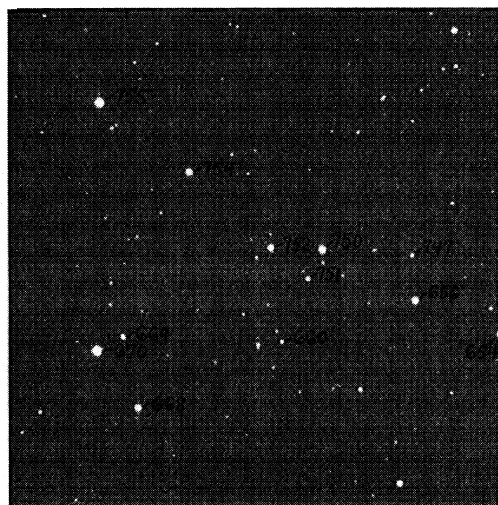
(e)



(g)



(f)



(h)

FIG. 1. (continued)

which is a very close glass equivalent and therefore much cheaper (and much less susceptible to thermal and pressure effects). In addition, an extra 0.5 mm of BG38 has been added to the T_1 filter to minimize red leak beyond $1\ \mu\text{m}$, where the GaAs photomultiplier has lost all sensitivity but where a CCD still retains some. Nevertheless, a small red leak of the order of 0.1%–1% remains for $\lambda \geq 1\ \mu\text{m}$ in T_1 ; however, the CCD response is typically very small in this region, \lesssim a few %, and overall light leak should be virtually negligible except for very cool and/or highly reddened stars. The WG280 filters of the appropriate thickness were cemented to C and T_1 so that they are of the same total thickness as (and thus parfocal with) M . These filters pass essentially all light redward of $2800\ \text{\AA}$ and thus do not change the response of the band pass. Additional comments concerning

the substitution of the R_{KC} filter for T_1 are given below. The T_2 filter has long been known to be identical to the I_{KC} filter (Canterna 1976; Bessell 1979). Since most observatories already possess this filter, it was deemed financially appropriate to take advantage of this fact. Users seeking to make their own version should bear in mind that this filter should avoid any red leak beyond $\sim 9000\ \text{\AA}$, which is the red cutoff of the GaAs photomultiplier.

We also recommend that future Washington CCD sets use the following prescriptions for the C and M filters:

$C = 3\ \text{mm Schott BG3} + 2\ \text{mm Schott BG40}$,

$M = 3\ \text{mm Schott GG455} + 2\ \text{mm}$

Schott BG39 + 2 mm Schott S861⁷

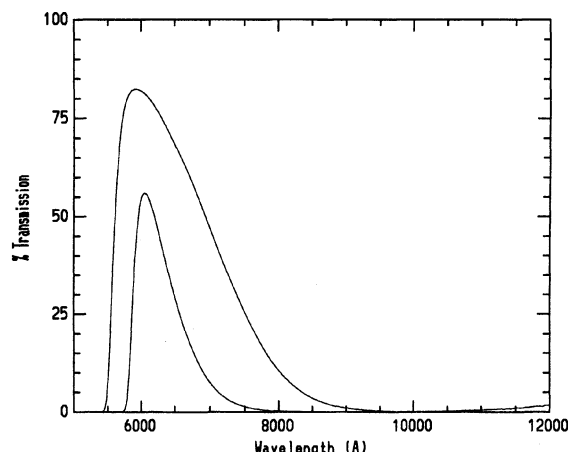


FIG. 2. Transmission of the R (upper curve) and T_1 (lower curve) filters, as taken from the measurements of the 4" filters at KPNO. Note the very similar spectral responses but the much greater throughput of the R filter. Also note the small red-leak in the T_1 filter beyond $1\ \mu\text{m}$, which is not present in the R filter.

(plus appropriate thicknesses of Schott WG280 added to each filter to make them of equal thickness with the other Washington filters used). Here we have substituted Schott filters for the Corning filters, since the latter do not guarantee the optical quality of their filters. We have searched for the optimum combination of Schott filters which reproduces the original response curves of the Washington system, as given by Canterna & Harris (1979), and have arrived at the above solutions. In the determination, we have also included the response of a blue sensitive Tektronix CCD such as those in current use at CTIO and KPNO, since the detailed response of the detector, especially in the UV, is an important criteria in matching the standard system. The C filter prescribed here has a peak sensitivity some 14% higher than that of the current filter and the total bandpass is $\sim 20\%$ more efficient. Thus, this filter should significantly speed up most Washington programs, which are generally limited by exposure times in this filter. This filter combination does have a small red leak between ~ 6700 and $7700\ \text{\AA}$ but it is only 0.03% or less and should be insignificant. We also note that the effective wavelength of this new filter is somewhat bluer than the current filter and that observations taken with it should therefore produce a smaller (and more desirable) color term (see below). Note that this new prescription is taken from Bessell (1990) but that a number of other combinations were investigated. The new M filter recommended here is NOT the one advocated by Bessell (1990), as that filter is a rather poor match and also has a significant red leak beyond $1\ \mu\text{m}$. The new filter is a very close match to the current filter at all wavelengths and photometry should be virtually identical.

During the course of this study, it was realized that the standard R_{KC} filter available at most observatories, which is well known to be a close match to the T_1 filter (Canterna 1976; Bessell 1979) as shown by Fig. 2, has a much higher peak response ($\sim 80\%$) than that of the T_1 filter ($\sim 55\%$), which requires a fairly thick BG38 filter to minimize redleak, which unfortunately also depresses the response throughout the entire region of the filter's sensitivity. The R filter also

avoids any red-leak problems. In addition, the "Harris" R_{KC} filter generally used for CCD observations is about twice as broad as T_1 , FWHM ~ 1500 vs $700\ \text{\AA}$ (although they have very similar effective wavelengths, $\sim 6300\ \text{\AA}$). The major difference in the detailed response of the two filters, other than increased transmission at all wavelengths for the R_{KC} filter and broader response, is that the R_{KC} filter has a long red tail, from ~ 8000 to $9500\ \text{\AA}$, that does not occur in the T_1 filter. However, the response is quite low over this interval ($<10\%$ of peak) and this region contains very few significant spectral features in G and K stars. The most important are the Ca triplet lines near $8500\ \text{\AA}$, but these fall near the tail where the sensitivity is very low. At the same time, the author was developing programs which required deep T_1 photometry, which would benefit greatly from use of a filter with much higher throughput.

Thus, it was decided to see how well the R_{KC} filter could reproduce T_1 . In order to test this as carefully as possible, photometry was obtained in both the R and T_1 filters, as well as the other Washington filters, during two consecutive photometric nights in 1994 June on the CTIO 0.9 m telescope (which were also used to manufacture standards). The R filter used was from the CTIO Harris 3" filter set No. 2. Care was taken to observe stars over as large a color range as possible in order to check color effects. A total of about 40 previous standards was observed each night. The results are presented in Tables 2(a) and 2(b) for the two nights, where the total number of standards remaining, after discarding the number indicated, is given for each index using both the T_1 and R filters. Also given are the respective rms values from the transformation of the instrumental to the standard system and the value of the color term [listed in Table 2(a) only since the mean of the two nights was adopted]. It is seen that using the R filter typically results in somewhat poorer photometry (larger rms values, with more stars discarded) but the differences are generally not large. The color terms are also comparable and indeed somewhat closer to the ideal values expected using the R filter (see below for a more complete discussion of color terms).

Figure 3 shows how the difference $\Delta(C - T_1)$ between the observed value using the R filter and the standard value varies as a function of the standard $(C - T_1)$ value for these two nights of observations. No stars have been discarded. One can see that the scatter is rather large but that over the range $-0.2 \leq (C - T_1) \leq 3.3$ there is no significant systematic color effect. However, note that beyond this upper limit a discrepancy sets in, becoming very significant (~ 0.3 mag) by $(C - T_1) \sim 4.6$. The coolest K stars have intrinsic colors of $2.7 \leq (C - T_1)_0 \leq 3.2$. Thus, substituting the R filter provides good photometry over the original range of astrophysical interest of the system, i.e., G and K stars. Care must be taken, however, when observing either highly reddened G and K stars or M stars. For example, Geisler (1994) advocates a new metallicity calibration of the system which involves the $(C - T_1)$ color of the giant branch. Giant branch tip stars in very metal-rich and/or reddened populations may fall outside this limit. However, it would require very high reddening values [$E(B - V) \sim 0.5$] to move the integrated color of even the most metal-rich globular clusters, $(C - T_1)_0 \sim 2.2$, beyond

TABLE 2. Reproducing T_1 photometry with the R_{KC} filter.
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Index	N_{Total}	$N_{Discarded}$	RMS	Color Term
$C - T_1(T_1)$	32	4	0.027	1.189
$C - T_1(R)$	31	4	0.035	1.157
$M - T_1(T_1)$	36	0	0.012	1.063
$M - T_1(R)$	32	3	0.014	0.902
$T_1 - T_2(T_1)$	36	0	0.008	0.881
$T_1 - T_2(R)$	35	0	0.013	1.100
$T_1(MandT_1)$	35	1	0.013	-0.109
$T_1(MandR)$	32	3	0.017	0.057
$T_1(CandT_1)$	35	1	0.012	-0.047
$T_1(CandR)$	32	3	0.018	0.027

Index	N_{Total}	$N_{Discarded}$	RMS
$C - T_1(T_1)$	33	3	0.028
$C - T_1(R)$	27	2	0.027
$M - T_1(T_1)$	35	1	0.011
$M - T_1(R)$	28	1	0.014
$T_1 - T_2(T_1)$	41	0	0.010
$T_1 - T_2(R)$	32	2	0.012
$T_1(MandT_1)$	38	3	0.014
$T_1(MandR)$	28	1	0.015
$T_1(CandT_1)$	39	2	0.017
$T_1(CandR)$	27	2	0.016

this limit, so that the integrated technique of Geisler & Forte (1990) for determining cluster metallicities should be completely unaffected by substituting the R filter.

The real advantage of the R filter over the T_1 filter is shown by a comparison of the throughputs as graphically illustrated in Fig. 2. It was noted at the telescope that the R flat fields yielded ~ 3 times as many photons as T_1 . The zero points obtained in the transformation confirmed this: the mean zero point in T_1 was 22.22, while in R it was 23.42 (using the Tek2048 No. 4 CCD and converting to $1 e^-/ADU$). Thus, the R filter is a full 1.2 mag faster than T_1 , passing three times as much light. The above zero points correspond to count rates of 7.7 and $23.3 e^-/s$, respectively, on a 0.9 m telescope for a star with $T_1=20$. Using this equipment, 1% photometry can be achieved for a star with $T_1=20$ in 2200 s using the T_1 filter and in half this time using the R filter (assuming $5 e^-$ read noise, $1''$ seeing and a new moon with sky brightness=20.9 mag/□" in R and 22.1 in T_1). A limiting exposure of, say, 1 hr with the T_1 filter

gives 1% photometry for a star with $T_1=20.4$, while the R filter will achieve this for a star almost 0.5 mag fainter.

These results then indicate that the R filter is a more efficient substitute for T_1 and can accurately reproduce the standard system for stars within the normal range of astrophysical interest of the system, with $0 \leq (C - T_1) \leq 3.2$. It should be emphasized that, for the most accurate photometry, the T_1 filter should be used. However, for photometry of faint objects within this color range where photons are at a premium, photometry of higher precision can be obtained for fainter objects using the R filter for a small sacrifice in accuracy. The R filter was then employed for this program on a total of six nights; however, data were not obtained for any star redder than $(C - T_1)=3.0$.

The above selection of filters, then, should adequately reproduce the overall responses of the standard (photoelectric) Washington filters; however, some differences in detail will be present due to the distinct CCD vs photomultiplier responses. Also differences due to the varying responses of CCDs should be expected, particularly in the C filter, where the CCD blue sensitivity cutoff is a strong function of CCD thickness, illumination technique, coating, etc.

Flat-field exposures generally consisted of dome flats for the redder T_1 and T_2 filters and twilight flats for the bluer C and M filters. All exposures equalled or exceeded 3 s so as to minimize shutter timing and scintillation errors; in practice, virtually all exposures were >8 s. Almost all of the observations of the new standards were obtained at an air mass <1.5 , and all observations of a given field were obtained within a few minutes. Multiple observations were obtained of some of the new standards on some nights. Care was taken to observe a large number of previous standards each night in order to determine both the extinction and, especially, accurate transformation to the standard system. On average, 2.5 observations over an air mass range >0.3 of each of 9 previous standards were obtained from which to determine the extinction. These nightly values were used if they differed significantly from the mean values for that run. The determination of the transformation coefficients was

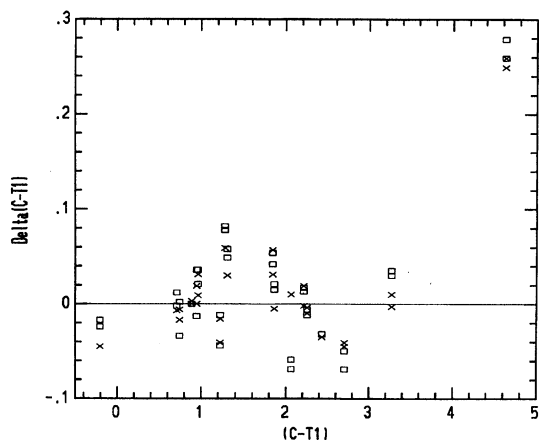


FIG. 3. The difference between the observed and standard $(C - T_1)$ values as a function of standard $(C - T_1)$ value for standard stars observed on 1994 June 24 (squares) and 1994 June 25 (X's) using an R filter. Note the lack of any significant systematic error for stars with $(C - T_1) < 3.3$ but the large error for the reddest object.

based on an average of 28 standard star observations each night.

The data were reduced with the APPHOT task in the DIGIPHOT package in IRAF,⁴ as the stars were deliberately selected to be very clean. Any star whose peak intensity was within $\sim 10\%$ of saturation was discarded. The mean of the pixels in the sky annulus was used to determine the sky level, as prescribed by Walker *et al.* (1992) in order to avoid quantization problems. Aperture photometry was derived in a variety of apertures for all standards in each field. Since there were a large number of stars per field (typically eight), one can take advantage of this to derive a mean aperture growth curve that is more accurately determined than that for an individual star, especially a faint one. The IRAF routine GROWTHCURVE in the CTIO package was utilized, which was kindly written for the author by P. Gigoux. This routine plots the growth curves for each star in a field together and allows one to discard any outliers, then determines the mean aperture correction between the user-defined inner and outer radii, and finally applies this correction to the magnitude measured at the inner radius to yield the final magnitude at the outer radius. An outer diameter corresponding to $\sim 15''$ was used to reproduce as closely as possible the observations defining the standard system.

The FITPARAMS task in the PHOTCAL package, also developed by P. Gigoux, was used to reduce the photometry to the standard system. Extinction and transformation coefficients were solved for simultaneously. The $M-T_1$, T_1-T_2 , and T_1 indices included only a first-order extinction, zero-point, and linear color terms with the form:

$$M-T_1 = a + b \cdot (m-t_1) + b \cdot c \cdot X.$$

The symbols here have their usual meaning, with $M-T_1$ the color index transformed to the standard system, $(m-t_1)$ the instrumental color, X is the airmass (taken as the mean in the case of two filters), a is the zero point, b is the color term, and c is the extinction coefficient (in the case of T_1 , the color term b was removed from the extinction term and the index employed in the color term was $m-t_1$). For the $(C-M)$ and $(C-T_1)$ indices, an additional second-order extinction term was added and occasionally a second-order color term, depending on the severity of the CCD sensitivity drop in the C filter. Note that solving for colors instead of separate magnitudes is appropriate because all observations of a field were obtained within a few minutes.

It was also found that a nonlinear term of the form $d \cdot T_1$ was required to improve the T_1 photometry depending on which detector was used and when. All data taken with either the Tek2048 No. 3 or No. 4 CCDs at CTIO between the period 1993 February 6 and 1994 June 25 (a total of eight nights, all using the standard T_1 filter) were found to be affected, including data taken using both the old VEB and the new ARCON CCD controllers. For these eight nights, the mean value of the nonlinear coefficient d was found to be

$0.987 \pm 0.006(\sigma)$. For the other ten nights of this investigation which employed CCDs (using both the T_1 and R filters), including a night with the Tek 512 No. 4 CCD at CTIO, three nights in 1994 November to December with the CTIO Tek2048 No. 4, three nights at KPNO (using both T2KA and T2KB) and three nights at the CFHT using the Loral 3 chip, the mean value of d was 1.002 ± 0.002 , indicating linearity. It should be noted that a mean of 29 standards over a large magnitude range (~ 5 mag) was used to determine the nightly value of d . Indeed, adding the new standards for which mean magnitudes were already well determined at the time of reduction in order to enlarge the sample size (to ~ 50 – 100 observations) and magnitude range confirmed these values. Note that nonlinearities in the Tek2048 No. 3 chip on the order of 2% per magnitude have been reported elsewhere (Walker 1993), but the problem was addressed and the fix reported to be good to $\sim 0.1\%$ after 1993 April. Independent evidence supports this (Walker 1994). The origin of this discrepancy is uncertain; however, the number and magnitude range of standards employed, the coincidence of the d values obtained, the presence of the effect for CTIO data and the absence of the effect in KPNO data taken within three weeks using virtually the same set of standards, all convinced us that the nonlinearity was significant and we applied this nonlinear term to the appropriate data.

The magnitude of the color term b is a direct measure of how well the instrumental system matches the standard system. Ideally, $b \sim 1 \pm 0.1$ for color indices and $\sim 0 \pm 0.1$ for magnitudes. The b values determined during this study generally satisfied these criteria in the case of the $M-T_1$, T_1-T_2 , and T_1 indices. The observed ranges for these values were 1.02–1.07 for $(M-T_1)$ (using the T_1 filter), and 0.90–0.93 (using the R filter). For (T_1-T_2) , the ranges were 0.86–0.94 and 0.95–1.10 for the T_1 and R filters, respectively. For T_1 using the T_1 filter, the observed range was $-0.12 \leq b \leq -0.03$, while the R filter yielded values from 0.01 to 0.06. However, for the indices involving the blue-ultraviolet C filter, the b values had much larger ranges and frequently exceeded the above limits. The observed b values for $(C-M)$ had a very large range, from 0.7 to 1.5, indicating very unsatisfactory matching to the standard system. The range observed in $(C-T_1)$, however, was much smaller and the values were generally much closer to unity: 1.04–1.29 (using T_1) and 1.10–1.16 (using R).

Thus, these results illustrate that $(M-T_1)$, (T_1-T_2) , and T_1 photometry with the CCD filter combinations used are good matches to the standard system, $(C-T_1)$ photometry is less so but still acceptable, but that the $(C-M)$ natural system is quite removed from the standard system and requires an uncomfortably large color term. Recall, however, that use of the new C filter recommended above should decrease this color term. Also note that the color terms using the R filter were generally closer to the ideal than those obtained with the T_1 filter. Part of this reason may be the small red leak still present with the T_1 filter beyond 1μ . This strengthens our confidence in using the R filter for T_1 .

The present study, together with previous CCD Washington observations, comprise a reasonably large dataset of CCD photometry of bright stars in a single photometric sys-

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tem (~ 30 stars observed on each of 24 nights). The author has also obtained many nights of Washington photometry using a photoelectric photomultiplier as detector (a total of 82 over his professional career, with a similar number of standards observed per night), and it is of some interest to compare how well these two techniques perform in the basic test of photometric capabilities, i.e., how precisely one can reproduce the standard system. I have determined the mean nightly standard deviation (error of one observation) from the transformation to the standard system for both types of photometry. The mean values found for all nights using a photomultiplier as detector are: $(C-M)=0.0104 \pm 0.0028(\sigma)$, $(M-T_1)=0.0077 \pm 0.0023$, $(T_1-T_2)=0.0098 \pm 0.0023$, and $T_1=0.0164 \pm 0.0055$. The respective values using a CCD as detector are: 0.0266 ± 0.0068 , 0.0109 ± 0.0020 , 0.0119 ± 0.0038 , and 0.0161 ± 0.0043 . (The value for $(C-T_1)$ using a CCD is 0.0275 ± 0.0074 ; $(C-T_1)$ photoelectric photometry was generally not obtained). It is clear that photoelectric Washington photometry reproduces the standard system (which of course was set up with a photomultiplier) more precisely than the CCD technique in all three color indices but that magnitudes are approximately equally precisely measured with both techniques. It should be said that the standards observed with a photomultiplier were in general somewhat brighter than those observed with a CCD, accounting for some of this difference in the case of the color indices but indicating that magnitudes are actually then generally somewhat more precisely measured with a CCD, due to the higher detector quantum efficiency. The differences in $(M-T_1)$ and (T_1-T_2) are small and actually within the errors but probably real; however, note the very large and significant difference in $(C-M)$. In this index, the photomultiplier wins handily over a CCD, evidencing the difficulty involved in obtaining accurate CCD photometry in blue-ultraviolet filters, where the chip response can change very dramatically over short wavelength intervals. In contrast a GaAs photomultiplier has a very constant response in this region. Undoubtedly, flat-fielding errors in this region are also partly to blame. We stress, then, that CCD photometry in the Washington system is as good as, or better than, photoelectric photometry in determining T_1 magnitudes, is somewhat worse but still quite good in measuring the redder indices $(M-T_1)$ and (T_1-T_2) and is substantially poorer in obtaining the blue-ultraviolet index $(C-M)$ [and undoubtedly $(C-T_1)$].

The mean nightly standard deviation from the transformation to the standard system obtained during this study (two nights of photoelectric photometry, the rest using a CCD) was 26, 11, 12, 15, and 28 mmag (0.001 mag) for $C-M$, $M-T_1$, T_1-T_2 , T_1 , and $C-T_1$, respectively. These values are very close to the mean values given above for all CCD nights, as expected.

3. RESULTS

The final mean colors and magnitudes, their standard errors of the mean ($\text{SEM}=\sigma/N^{1/2}$, in units of 0.1 mmag), and the number of observations N are given in Table 3 for a total

of 63 new Washington system standards. The PG 0231+051 designations are from Landolt (1992) except for stars F–H, which were named in this study, and the NGC 3680 numbering scheme is that of Eggen (1969). The mean number of observations per star is 16. Before deriving the final means, any datum falling $>3\sigma$ from the preliminary mean was discarded. Some 0.8% of the observations were thus eliminated. An additional 2.3% of remaining outliers was discarded by hand. Many of the discarded observations had been tagged as having some potential problem, such as occurring at high air mass or falling on a bad column, at the time of observation or reduction. For the remaining data, the mean standard deviation of one observation is 23, 17, 16, 18, and 26 mmag in $(C-M)$, $(M-T_1)$, (T_1-T_2) , T_1 and $(C-T_1)$, respectively. The mean standard error of the mean is 6.3, 4.6, 4.5, 4.9, and 7.0 mmag, respectively. The above values are very comparable to the corresponding ones in G90, as well as those in Landolt (1992) for recent *UBVRI* standardization work. Indeed, $(C-M)$ and $(C-T_1)$ CCD photometry of these faint stars is significantly more precise than the $(U-B)$ photoelectric photometry of Landolt's (1992) standards, demonstrating the much greater ease of C observations over those in the U filter. These stars are thus now firmly established as high quality Washington standards.

Figure 4 shows the variation of the standard deviation in each color or magnitude as a function of the corresponding index for the new standards. Figures 4(a) and 4(e) for $(C-M)$ and $(C-T_1)$ indicate that the bluest star, PG 0231+051, has an exceedingly large error, not unexpected given that it is the faintest star in the sample (by >1 mag in T_1) and also the bluest (by ~ 1.5 mag), lying well to the blue of the bluest previous Washington standard. Thus, the "standard" values involving the C indices for this object should be regarded as very uncertain. However, the errors in the other colors for this star are not excessive. The figures show no other trend with color for the sample. In particular, the reddest stars are well-behaved, with the exception of the reddest star in $(C-T_1)$, SA 110–509. However, the other red stars, with $(C-T_1)$ colors up to ~ 3 , show no increased error. Combined with the results discussed in the last section, we emphasize further that $(C-T_1)$ photometry to this limit, using either the T_1 or R filter, should be accurate, but accuracy will be increasingly compromised beyond this limit, especially with the R filter. Fortunately, virtually all objects of astrophysical interest for the system fall within this range. Note that, although the bulk of the stars fall within an apparently narrow color range, this range again includes the full range of interest. Also note that each field indeed includes a wide range of colors, generally encompassing those typical of program objects likely to be observed with the Washington system, thus facilitating the determination of transformation color terms.

Figure 4(d) shows the variation of the T_1 error with magnitude. This shows a slow increase for the fainter stars, as expected from photon statistics, especially for PG 0231+051. In fact, this is the only star whose $\sigma_{T_1} > 2(\langle\sigma_{T_1}\rangle)$. Thus, we see no strong evidence for any variables in our sample (the discarded data were limited to only one or two observations per star and were most likely due to observa-

TABLE 3. New Washington standards.

Star	C-M	SEM	N	$M - T_1$	SEM	N	$T_1 - T_2$	SEM	N	T_1	SEM	N	$C - T_1$	SEM	N
SA 92-253	1.326	44	12	1.010	37	14	0.636	35	14	13.376	38	14	2.359	56	13
SA 92-254	0.951	53	16	0.686	45	16	0.436	27	15	13.234	40	16	1.636	81	16
SA 92-255	0.870	51	15	0.637	47	16	0.432	52	16	13.929	72	16	1.514	59	16
SA 92-259	0.626	45	14	0.535	64	15	0.382	52	15	14.671	83	16	1.155	73	15
SA 92-268	0.590	44	4	0.558	100	4	0.373	136	4	13.203	106	4	1.150	116	4
SA 92-335	0.696	57	16	0.544	36	16	0.348	35	16	12.151	43	16	1.240	44	16
SA 92-340	0.933	57	16	0.693	40	16	0.480	51	16	13.146	27	16	1.628	48	16
SA 92-348	0.555	44	16	0.501	32	16	0.335	35	16	11.776	33	16	1.063	55	16
PG0231+051	-0.624	233	13	-0.213	70	14	-0.178	47	14	16.228	103	14	-0.839	197	12
PG0231+051-A	0.732	60	14	0.585	39	14	0.404	45	14	12.389	40	13	1.319	42	14
PG0231+051-B	1.659	85	13	1.298	49	14	1.081	32	14	13.849	53	13	3.000	84	13
PG0231+051-C	0.659	63	14	0.567	45	14	0.409	48	14	13.333	47	13	1.232	56	14
PG0231+051-D	1.299	76	14	0.945	48	14	0.635	65	14	13.389	56	13	2.244	18	12
PG0231+051-E	0.672	72	14	0.558	28	14	0.386	47	14	13.436	30	13	1.234	51	14
PG0231+051-F	0.506	97	13	0.460	49	13	0.314	54	12	11.561	69	12	0.977	73	12
PG0231+051-G	0.517	49	14	0.504	42	14	0.360	29	13	14.125	36	13	1.032	49	14
PG0231+051-H	1.390	117	8	0.897	41	8	0.619	50	3	9.830	40	7	2.280	62	8
SA 98-652	0.422	60	23	0.470	51	23	0.355	53	24	14.518	47	24	0.901	63	24
SA 98-672	1.150	81	21	0.852	56	23	0.606	41	23	13.896	35	23	2.007	93	22
SA 98-673	0.308	74	22	0.331	35	21	0.268	32	24	13.616	29	22	0.641	85	24
SA 98-677	0.693	71	24	0.651	55	23	0.521	59	24	14.173	39	22	1.354	75	24
SA 98-679	0.592	61	21	0.523	47	22	0.361	31	24	13.324	34	23	1.123	43	23
SA 98-680	1.234	59	21	0.886	39	23	0.623	36	24	13.544	32	24	2.129	44	21
SA 98-687	0.547	79	24	0.532	41	22	0.430	38	21	14.444	35	21	1.080	75	23
SA 101-338	0.595	56	20	0.535	29	20	0.364	30	20	13.422	61	18	1.130	65	21
SA 101-340	0.877	41	16	0.671	39	17	0.441	23	15	13.704	51	17	1.548	61	17
SA 101-341	0.538	51	16	0.492	42	17	0.322	38	16	14.010	65	17	1.027	51	17
SA 101-344	0.652	41	16	0.569	32	17	0.391	47	16	12.351	47	17	1.222	49	17
SA 101-348	0.800	53	16	0.614	39	17	0.386	41	16	12.998	57	17	1.412	77	17
SA 101-421	0.435	51	21	0.453	37	21	0.313	50	20	12.865	44	19	0.892	52	21
SA 101-427	0.847	71	17	0.656	34	17	0.439	44	17	14.519	48	17	1.508	76	17
SA 101-429	1.161	44	20	0.872	28	20	0.554	30	19	12.876	48	19	2.043	54	21
SA 101-431	1.485	49	20	1.125	34	21	0.752	32	19	12.882	44	19	2.641	48	20
NGC 3680-5	0.356	48	20	0.401	39	21	0.284	18	21	12.569	31	21	0.753	42	20
NGC 3680-11	1.230	38	19	0.814	25	21	0.533	16	21	10.386	29	21	2.047	45	19
NGC 3680-23	0.563	40	20	0.518	20	21	0.359	19	21	13.627	36	21	1.084	29	20
NGC 3680-26	1.343	43	19	0.877	27	21	0.560	27	21	10.385	32	21	2.223	37	19
NGC 3680-29	0.406	44	18	0.429	27	21	0.295	18	21	11.687	30	21	0.829	39	19
NGC 3680-31	0.386	44	19	0.422	21	21	0.292	22	21	12.835	33	21	0.802	29	20
NGC 3680-33	0.417	52	19	0.433	15	21	0.302	17	21	11.621	36	21	0.846	33	20
NGC 3680-S	0.836	61	19	0.627	29	21	0.425	32	21	13.937	47	21	1.470	63	19
SA 107-599	0.751	55	12	0.615	73	12	0.435	83	13	14.222	47	13	1.377	139	12
SA 107-601	1.695	104	11	1.303	38	11	0.907	56	13	13.762	53	13	3.037	108	12
SA 107-602	1.093	46	16	0.794	22	14	0.561	37	16	11.586	36	15	1.896	64	14
SA 107-606	0.990	64	13	0.713	58	13	0.454	39	13	12.192	52	13	1.715	85	12
SA 107-609	1.311	46	12	0.865	78	12	0.572	111	12	9.923	76	12	2.185	98	11
SA 107-610	1.243	58	13	0.892	54	12	0.573	50	13	12.704	54	12	2.156	58	11
SA 107-611	0.983	65	16	0.739	43	15	0.495	66	15	13.763	62	14	1.736	64	14
SA 107-612	0.921	102	15	0.766	65	16	0.581	53	15	13.739	67	15	1.720	95	15
SA 107-614	0.551	55	16	0.521	46	16	0.389	41	16	13.575	35	13	1.080	65	15
SA 110-496	1.043	79	7	0.897	91	6	0.745	44	7	12.445	51	8	1.970	71	7
SA 110-497	1.099	98	7	0.897	88	6	0.682	56	7	13.573	45	12	2.005	78	11
SA 110-504	1.527	77	7	1.138	59	6	0.730	36	7	13.245	42	13	2.696	89	12
SA 110-505	1.154	66	6	0.910	60	6	0.676	49	7	13.635	48	8	2.078	100	6
SA 110-509	2.461	70	7	1.680	53	6	1.269	42	7	12.060	58	8	4.134	193	8
SA 110-511	1.349	80	6	1.008	41	6	0.729	41	7	13.718	53	8	2.367	109	8
SA 114-654	0.638	67	9	0.527	51	9	0.355	27	9	11.484	57	9	1.170	37	9
SA 114-660	0.518	30	10	0.508	38	10	0.385	50	11	14.759	74	11	1.041	49	11
SA 114-668	0.831	53	9	0.626	56	9	0.416	57	9	12.146	30	9	1.458	58	9
SA 114-669	1.126	54	9	0.860	46	9	0.571	51	9	13.348	34	9	2.007	50	8
SA 114-747	1.320	50	9	0.964	52	11	0.604	63	11	14.372	84	12	2.294	105	12
SA 114-754	0.830	49	9	0.624	56	9	0.418	37	9	12.034	40	9	1.455	48	9
SA 114-755	0.518	45	9	0.454	78	9	0.288	92	9	10.616	95	9	0.976	106	9

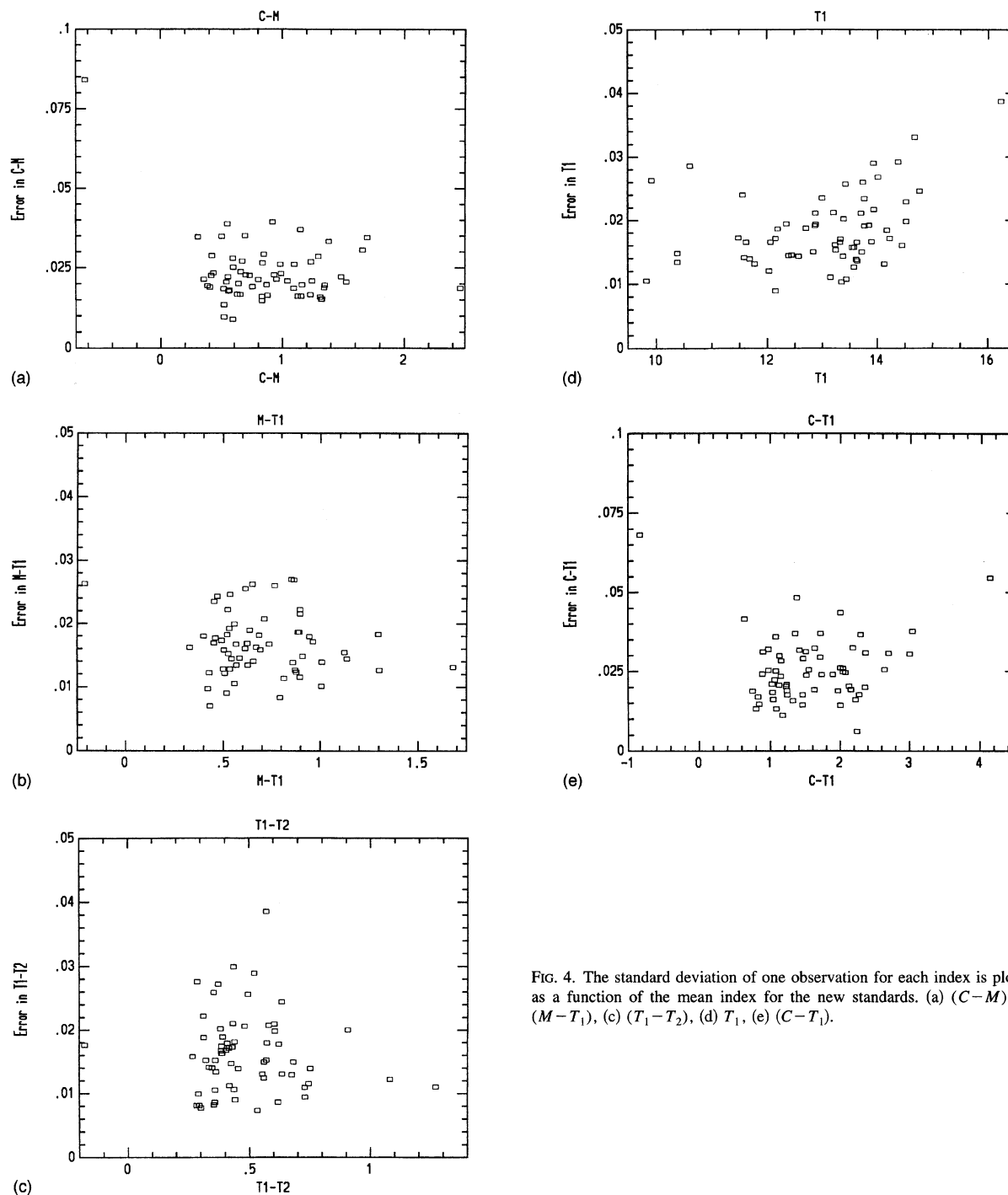


FIG. 4. The standard deviation of one observation for each index is plotted as a function of the mean index for the new standards. (a) $(C-M)$, (b) $(M-T_1)$, (c) (T_1-T_2) , (d) T_1 , (e) $(C-T_1)$.

tional errors or problems). However, there are two stars near the bright end whose errors are surprisingly large, SA 107–609 and SA 114–755. Both of these stars also have unusually large errors in all other indices. SA 107–609 was observed only twice by Landolt (1973) and it is possible that this object is a variable but it is also possible that some of the current observations were affected by saturation. SA 114–755 has been observed extensively by Landolt (1983) and no evidence for variation was found so it is unlikely this

object is a variable. In all, we have four program stars in common with Landolt (1973), one in common with Landolt (1983) and 27 in common with Landolt (1992) and none of these objects are noted as variable or possible variable in any of his studies.

The new standards generally lie in the range $T_1 \sim 11.5\text{--}14.5$. Some are bright enough in various filters that care should be taken to prevent saturation on large telescopes. We recommend taking both a short and a long expo-

TABLE 4. Reproducing *UBVRI* photometry with *CT₁* photometry.

Index	RMS	b1	b2
<i>R</i>	0.02	0.003	-0.017
<i>U - V</i>	0.15	-0.372	1.072
<i>B - V</i>	0.03	0.076	0.475
<i>V - R</i>	0.03	0.049	0.273
<i>V - I</i>	0.08	0.115	0.514

sure for each field and each filter in order to ensure adequate statistics for accurate measurement of both the brighter and fainter stars. As a rough guide, one can expect the following count rates in e^-/s for a $C=M=T_1=T_2=20$ mag star with an 0.9 m telescope and Tektronix CCD: $C=8$ (the value corresponding to the new C filter should be ~ 10), $M=15$, $T_1=8$ (T_1 filter) or 23 (R filter), and $T_2=10$.

It is of interest to investigate how well Washington photometry can reproduce indices on the standard *UBVRI* system. It is often useful to know at what level of accuracy a given color index can be transformed into another one and what the approximate equivalent of a photometric color in the Washington system is on the *UBVRI* system. There now exist a total of 53 stars that are high quality standards in both the *UBVRI* and *CMT₁T₂* systems which are ideal for performing this comparison. These stars have a very wide color range, with $-0.84 \leq (C-T_1) \leq 4.63$ and $-0.33 \leq (B-V) \leq 2.33$. To this purpose, we present Table 4. Here we have solved equations of the form:

$$U-V = b1 + b2 * (C - T_1).$$

In each case we have used the $(C-T_1)$ index for the Washington color, and have only investigated color indices involving the V filter. For the magnitude comparison, we have chosen the R and T_1 filters, and solved the equation:

$$R = b1 + T_1 + b2 * (C - T_1).$$

The table gives the rms value of the fit and the computed coefficients (generally two stars were discarded in each fit). The errors in these coefficients are typically 0.005–0.02. A comparison of the (T_1-T_2) index with $V-R$ and $R-I$ can be found in Taylor (1986).

It is seen that several *UBVRI* indices are very well reproduced by corresponding Washington indices. In particular, T_1 reproduces R magnitudes very well indeed, to a level of accuracy as well as can be expected from the errors in the respective indices. This is also true of $B-V$ and $V-R$: a linear transformation of the $(C-T_1)$ color reproduces these colors exceedingly well. We note that a corresponding fit using $(M-T_1)$ as the independent variable reproduces $B-V$ with an rms of only 0.01. Thus, it is quite reasonable to translate *CMT₁* photometry into R , $B-V$, and $V-R$ at these levels of accuracy. We take the small errors, zero point and color term for the $R: T_1$ transformation as further evidence for their compatibility and the suitability of using an R filter to reproduce T_1 .

On the other hand, $(C-T_1)$ does not reproduce well the $U-V$ or $V-I$ indices. The resulting errors very significantly exceed the combination of the observational errors. Using the $(M-T_1)$ index to reproduce $V-I$ still results in an rms of almost 0.04. It is therefore recommended that transformations involving these indices be avoided.

We finally point out the presence of an extremely red star in the SA 110 field. This is the relatively bright star $\sim 3'W$ of star 502. In one measurement, we obtained values of $T_1=14.20$, $(T_1-T_2)=3.25$. This is equivalent to $R-I \sim 3.2$, making it one of the reddest objects known.

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