Deep Learning (CEE690.06) – Assignment 4

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Due: 11/6/2018

Problem 1: Variational Lower Bound

a) Show $D_{KL}(q(z|x)||p(z|x)) = 0$ if and only if $q(z|x) = p(z|x)$

We can start out with the following function:

$$D_{KL}(q(z|x)||p(z|x)) = \int q(z|x) log\left(\frac{q(z|x)}{p(z|x)}\right) dz$$

If $q(z|x) = p(z|x)$, then for all cases $log\left(\frac{q(z|x)}{p(z|x)}\right) = 0$ as $log(1) = 0$

Therefore $\int q(z|x) log\left(\frac{q(z|x)}{p(z|x)}\right) dz = \int q(z|x)(0) dz = 0 = D_{KL}(q(z|x)||p(z|x))$

For all cases where $\frac{q(z|x)}{p(z|x)} \neq 1$, $log\left(\frac{q(z|x)}{p(z|x)}\right) \neq 0$ and $\int q(z|x) log\left(\frac{q(z|x)}{p(z|x)}\right) dz \geq 0$

So if $q(z|x) \neq p(z|x)$, then $D_{KL}(q(z|x)||p(z|x)) \geq 0$

b) The equality above holds for $L(q;x) = \int q(z|x) log\left(\frac{p(x,z)}{q(z|x)}\right) dz$

The variational lower bound is described as:

$$L(q;x) = \int q(z|x) log\left(\frac{p(x,z)}{q(z|x)}\right) dz = \mathbb{E}_{q(x|z)}\left[-log\big(q(z|x)\big) + log(p(x,z))\right]$$

We want to make sure $log(p(x)) = D_{KL}(q(z|x)||p(z|x)) + L(q;x)$ holds true

$p(x,z) = p(x|z)p(z) = p(z|x)p(x)$ and $log(a) - log(b) = log\left(\frac{a}{b}\right)$

So $\mathbb{E}_{q(x|z)}\left[-log(q(z|x)) + log\big(p(x,z)\big)\right] = \mathbb{E}_{q(x|z)}\left[log\left(\frac{p(z|x)p(x)}{q(z|x)}\right)\right]$

If $q(z|x) = p(z|x)$, then $\mathbb{E}_{q(x|z)}\left[log\left(\frac{p(z|x)p(x)}{q(z|x)}\right)\right] = \mathbb{E}_{q(x|z)}\left[log\big(p(x)\big)\right]$

So $log(p(x)) = D_{KL}(q(z|x)||p(z|x)) + L(q;x) = (0) + \mathbb{E}_{q(x|z)}\left[log\big(p(x)\big)\right]$ *if* $q(z|x) = p(z|x)$

And $log(p(x)) = \mathbb{E}_{q(x|z)}\left[log\big(p(x)\big)\right]$, so our statement holds true

Problem 5: Bookkeeping

a) *How many hours did this assignment take you? (There is No correct answer here, this is just an information gathering exercise)*

Problem 1: 1.5 hours
Problem 2: 0.5 hours
Problem 3: 1.0 hours
Problem 4: 3.0 hours
Problem 5: 0.25 hours

Overall 6.25 hours. I got a little stuck on the conditional GAN because I didn't initially realize that I needed to include the label with the discriminator and was instead messing around with loss functions. Once I figured that out it didn't take much more time.

b) *Verify that you adhered to the Duke Community Standard in this assignment (https://studentaffairs.duke.edu/conduct/about-us/duke-community-standard), i.e., write "I adhered to the Duke Community Standard in the completion of this assignment"*

I adhered to the Duke Community Standard in the completion of this assignment