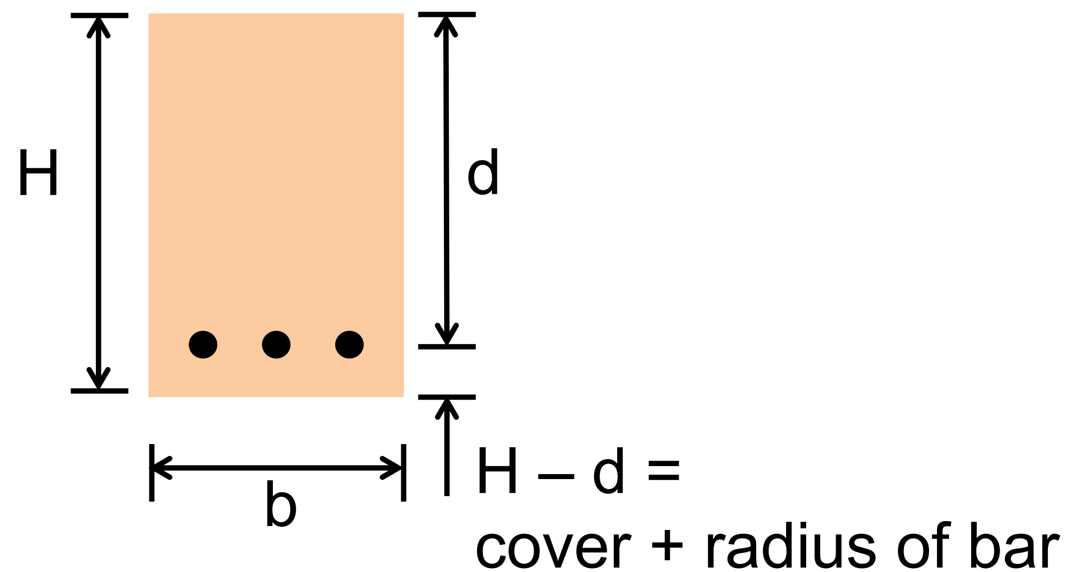
Sample Problem Walkthrough and Beam Analysis

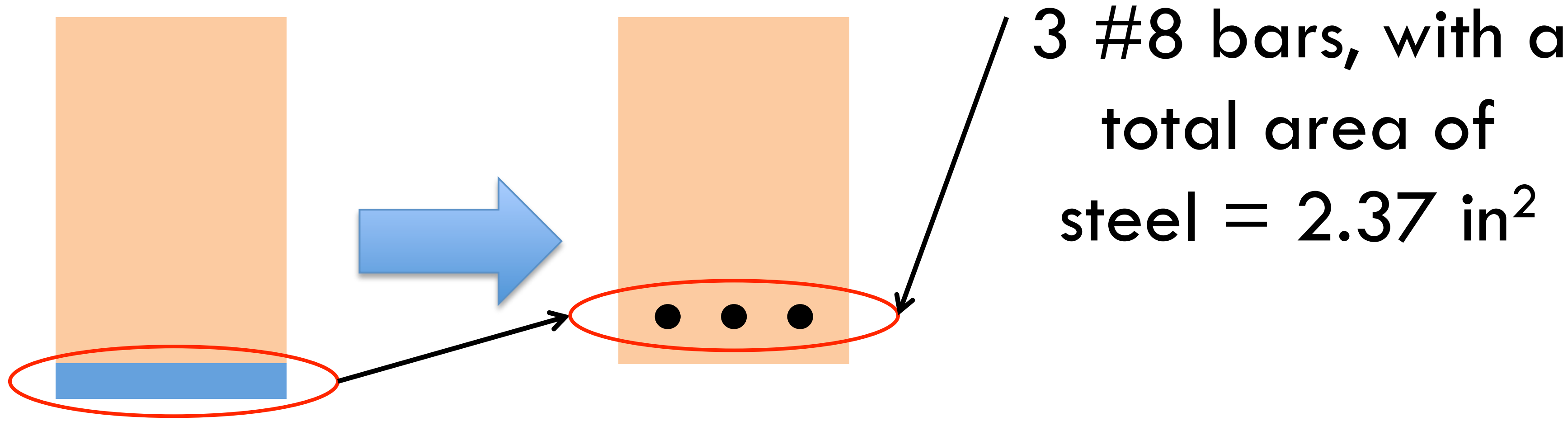


CIVIL, ENVIRONMENTAL AND SUSTAINABLE ENGINEERING
SCHOOL OF SUSTAINABLE ENGINEERING AND THE BUILT ENVIRONMENT
ARIZONA STATE UNIVERSITY

CEE 421: Concrete Structures

Fall 2017

Moving to Reinforced Concrete (RC) beams



We stated in class that, although reinforcing the tension side of a beam with a metal plate is great from a mechanics point of view, it is not good from a material point of view because the steel plate can rust and eventually loose it's load-bearing capacity. It also prevents us from seeing any developing cracks. Therefore, we move the steel inside the concrete to help keep it protected and so we can see any cracks on the tension side of the beam.

Logic Flow Behind Analysis

If given an existing section and asked to evaluate it's ability to handle a factored moment (or loads that give rise to a factored moment)

Check if the beam cracks:

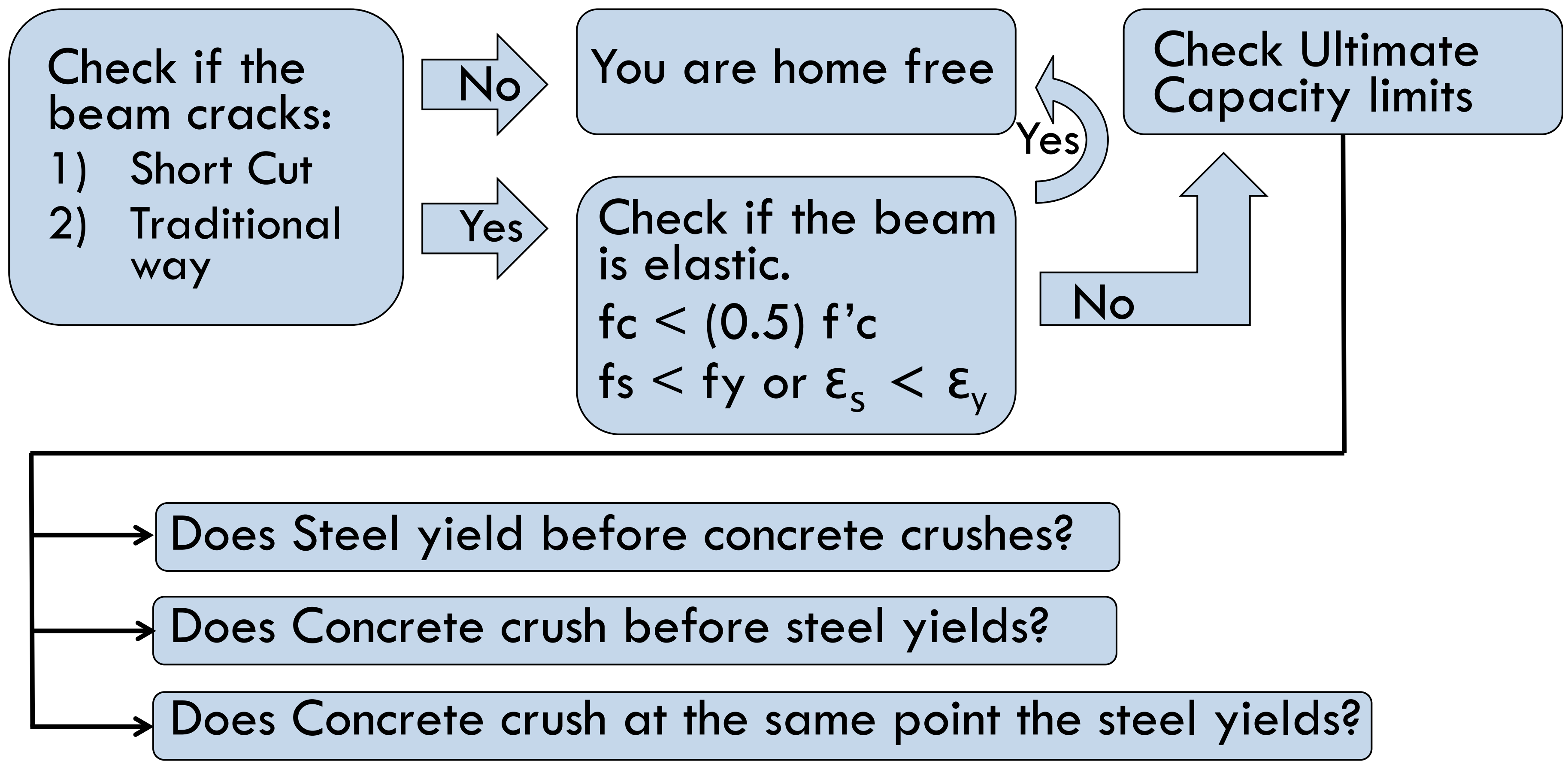
1) Short Cut
2) Traditional way

No → You are home free

Yes → ?

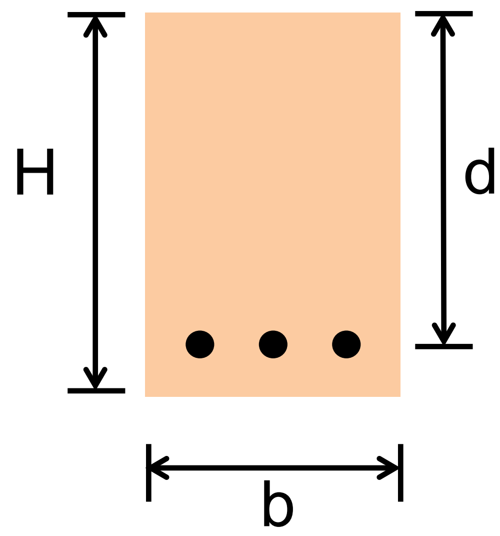
Logic Flow Behind Analysis

If given an existing section and asked to evaluate it's ability to handle a factored moment (or loads that give rise to a factored moment)



Ultimate Capacity

Sample problem



Givens:

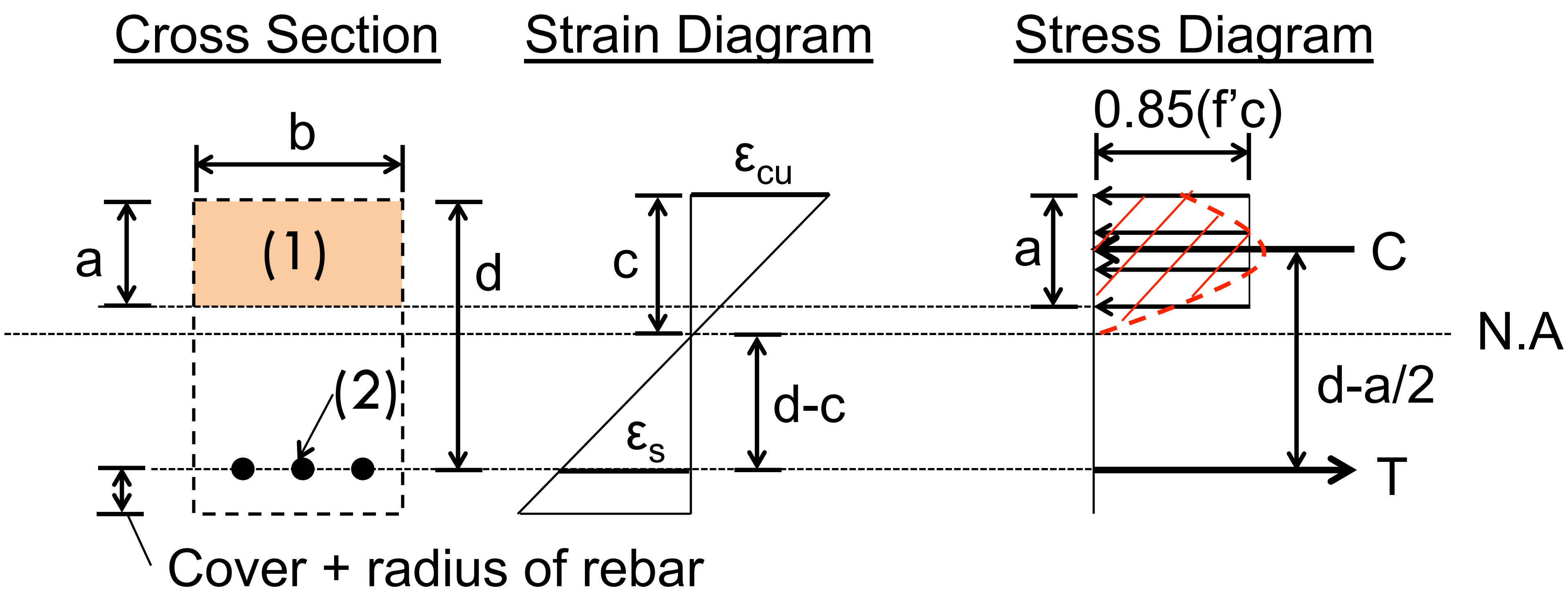
1) H = 25 in
2) b = 10 in
3) d = 23 in
4) Steel bars = 3 #8 bars. From ACI tables, A_s = 2.37 in^2
5) f_y = 60 ksi
6) E_s = 29000 ksi, E_c = 3640 ksi, n ~ 8.
7) f'c = 4000 psi & fr = 475 psi

Introduce the Reinforcement Ratio: $\rho = \dfrac{A_s}{bd}$

The problem statement is "Imagine you have this existing cross section of a beam in a stadium and you are asked to find the ultimate moment capacity of this section

Ultimate Capacity

<u>Question at hand:</u> find M_u of the section?



Almost always, the easiest one to check first is to assume the beam is under-reinforced, which means the steel yields before the concrete crushes, but this <u>MUST</u> be checked later making sure $\varepsilon_s \geq \varepsilon_y$! Also, in ultimate capacity, $M_u = \phi M_n$

Equations: $a = \beta_1 c$, $C = (0.85)(f'_c)(a)(b)$ & $T = A_s f_s$

Ultimate Capacity

<u>Back to the question at hand:</u> find M_u of the section?

If the steel yields first, then $f_s = f_y$. Recalling the equations on the previous slide:

$$T = A_s f_s \Rightarrow A_s f_y \qquad C = (0.85)(f_c')(a)(b)$$

$$C = T \Rightarrow a = \frac{A_s f_y}{(0.85)(f_c')(b)} = \frac{(2.35\ \text{in}^{\cancel{2}})(60\ \cancel{\text{ksi}})}{(0.85)(4\ \cancel{\text{ksi}})(10\ \cancel{\text{in}})} = 4.15\ \text{in}$$

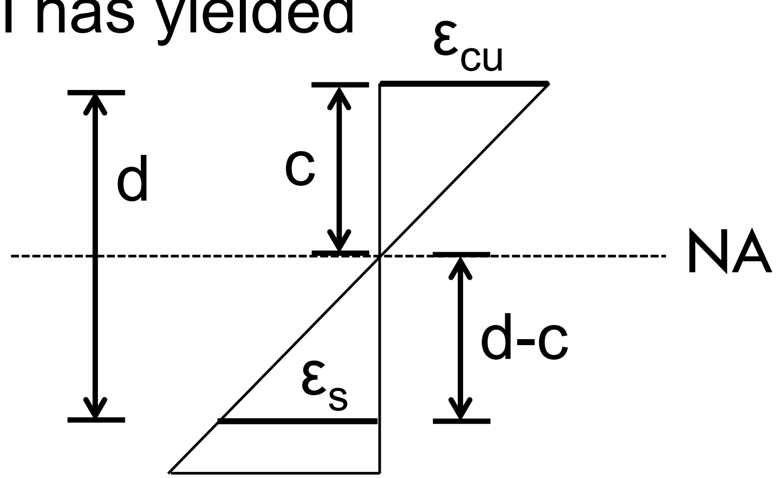
$$\therefore c = \frac{a}{\beta_1} = \frac{(4.15\ \text{in})}{(0.85)} = 4.88\ \text{in}$$

This is for f'c = 4000 psi concrete, see Table ACI 22.2.2.4.3

Ultimate Capacity

<u>Back to the question at hand:</u> find M_u of the section?

Now, go back to the strain diagram and use similar triangles to ensure the steel has yielded



$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_s}{d-c} \Rightarrow \varepsilon_s = \frac{\varepsilon_{cu}(d-c)}{c} = \frac{(0.003)(23\text{ in} - 4.88\text{ in})}{(4.88\text{ in})} = 0.011$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60\ \cancel{\text{ksi}}}{29{,}000\ \cancel{\text{ksi}}} = 0.0021 << 0.011$$

So, the steel has yielded and we can set $f_s = f_y$!

Ultimate Capacity

<u>Back to the question at hand:</u> find M_u of the section?

Therefore, since $\varepsilon_s >> \varepsilon_y$, $f_s = f_y$

$$\therefore M_n = (A_s)(f_y)\left(d - \frac{a}{2}\right) = \frac{(2.35\ \text{in}^2)(60\ \text{ksi})(23 - 4.15/2\ \text{in})}{(12\ \text{in/ft})} = 246\ \text{kip ft}$$

But! Remember, $M_u = \phi M_n \therefore M_u = (0.9)(246\ \text{kip ft}) = 221.4\ \text{kip ft}$

Note: $\phi = 0.9$ for a tension controlled beam.

Now, in this example, the steel yielded much earlier than the concrete would crush. What would have changed if we found out that the concrete would have crushed before the steel yielded?

Ultimate Capacity

Back to the question at hand: now find M_u of the section assuming compression failure?

If, during the previous analysis, we found that the strain in the steel was less than the yield strain, we would still be able to calculate the capacity, but the math gets a little "messy".

Since the concrete still crushes, we use the strain in the concrete as 0.003, but now the stress in the steel is f_s, not f_y.

From equilibrium, we still have: $C = T = A_s f_s = (0.85)(f_c')(a)(b)$

Since steel has not yielded, it still obeys Hooke's Law: $f_s = E_s \varepsilon_s$

And we can still use similar triangles from the strain diagram:

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_s}{d-c} \Rightarrow \varepsilon_s = \frac{\varepsilon_{cu}(d-c)}{c}$$

Now, let's combine equations:

Ultimate Capacity

Back to the question at hand: now find M_u of the section assuming compression failure?

Now, let's plug in all we already know:

$$C = T = A_s \underbrace{f_s}_{f_s = E_s \varepsilon_s} = (0.85)(f_c')(\overbrace{a}^{a=\beta_1 c})(b)$$

$$f_s = E_s \varepsilon_s \leftarrow \varepsilon_s = \frac{\varepsilon_{cu}(d-c)}{c}$$

Now, we get:

$$(0.85)(f_c')(\beta_1)(b)(c^2) = A_s E_s \varepsilon_{cu}(d-c) \Rightarrow$$

$$\underbrace{(0.85)(f_c')(\beta_1)(b)}_{\text{A}}(c^2) + \underbrace{(A_s)(E_s)(\varepsilon_{cu})}_{\text{B}}(c) - \underbrace{(A_s)(E_s)(\varepsilon_{cu})(d)}_{\text{C}} = 0$$

This is a quadratic equation for c, using the quadratic formula, and again only the positive root makes sense, therefore we find c:

$$c = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \Rightarrow \frac{-B + \sqrt{B^2 + 4AC}}{2A}$$

I get c = <u>9.73 in</u>

Ultimate Capacity

Back to the question at hand: now find M_u of the section assuming compression failure?

From knowing c, you can get a:

$$a = \beta_1 c = 8.27\text{ in}$$

Now, we plug back into our moment equations :

$$M_n = A_s f_s \left(d - \frac{a}{2}\right) = (0.85)(f'_c)(a)(b)\left(d - \frac{a}{2}\right) \Rightarrow$$

$$M_n = \frac{(0.85)(4\ \cancel{\text{ksi}})(8.27\ \cancel{\text{in}})(10\ \cancel{\text{in}})(23\ \cancel{\text{in}} - 8.27/2\ \cancel{\text{in}})}{(12\ \cancel{\text{in}}/\text{ft})} = 442.1\text{ k x ft}$$

This is why we always must go back and check to make sure the steel yielded in the first example of ultimate analysis! If the steel doesn't yield, we must do this!

Is this realistic? Let's find out!

Ultimate Capacity

Back to the question at hand: now find M_u of the section assuming compression failure?

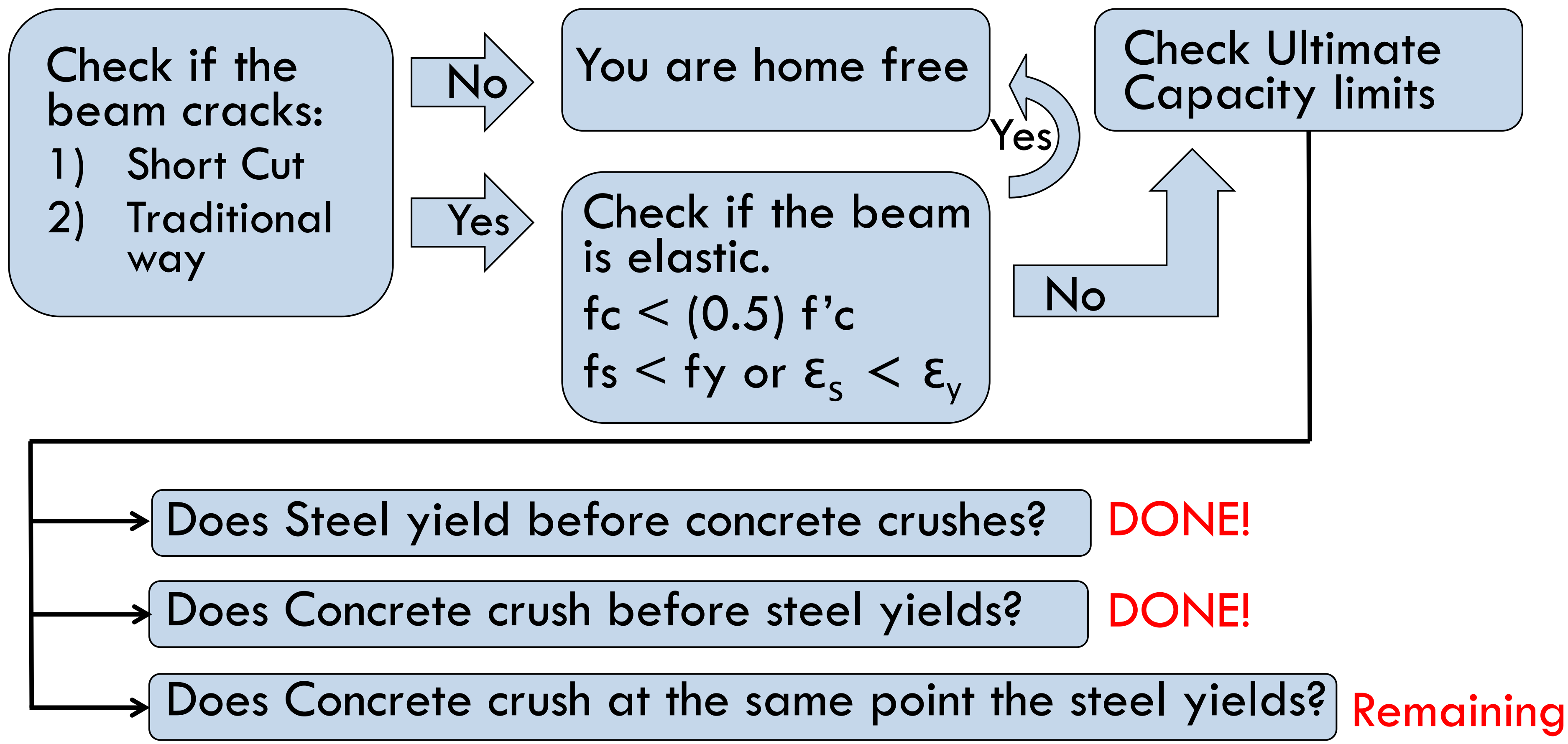
Now, let's find the stress in the steel (since we know it should obey Hooke's Law):

$$f_s = (E_s)\frac{\varepsilon_{cu}(d-c)}{c} = \frac{(29000\text{ ksi})(0.003)(23\text{ in} - 9.73\text{ in})}{(9.73\text{ in})} = 118.7\text{ ksi}$$

So, since the assumption of concrete crushing before steel yields assumes that the steel still obey's Hooke's law. If this condition were true, then the yield strength of the steel would have to be greater than 118 ksi! So in our example, we were using 60 ksi steel, which is much less than 118 ksi, so therefore it is not possible that the concrete is crushing before the steel.

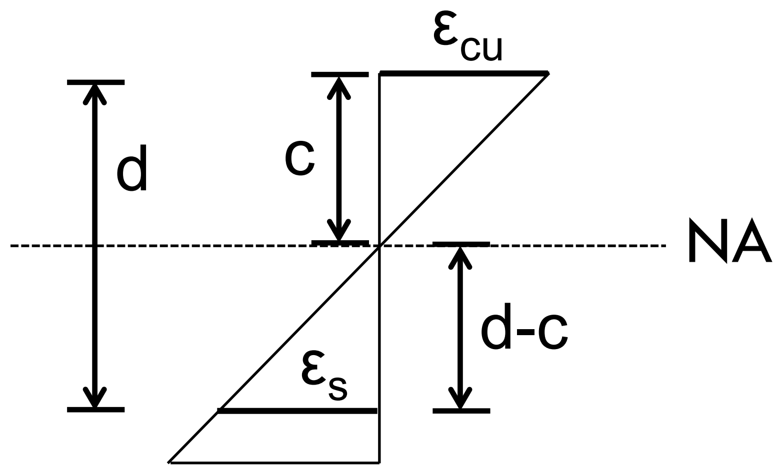
Logic Flow Behind Analysis

If given an existing section and asked to evaluate it's ability to handle a factored moment (or loads that give rise to a factored moment)



Ultimate Capacity

<u>When the concrete crushes at the same instance that the steel is yielding, this is called Balanced Condition:</u> now find M_u of the section assuming Balanced Condition?



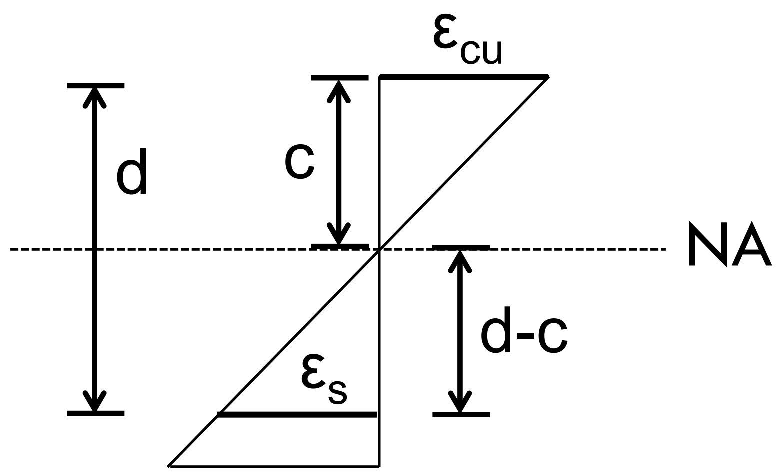
Since both the concrete is crushing and the steel is yielding, $\varepsilon_s = \varepsilon_y$, and $\varepsilon_{cu} = 0.003$

$$\frac{c}{\varepsilon_{cu}} = \frac{d-c}{\varepsilon_s} \Rightarrow c\varepsilon_s = \varepsilon_{cu}\left(d-c\right) \Rightarrow d\left(\varepsilon_{cu}\right) = c\left(\varepsilon_s + \varepsilon_{cu}\right) \Rightarrow c_{bal} = \frac{d\left(\varepsilon_{cu}\right)}{\left(\varepsilon_s + \varepsilon_{cu}\right)}$$

Doing this, I get c = 13.6 inches.

Ultimate Capacity

<u>Now, again, let's check the stresses in the steel:</u> now find M_u of the section assuming Balanced Condition?



So, all we need is to find 'a', then plug into our moment equations:

$$a = \beta_1 c = (0.85)13.6 = 11.56 \text{ in}$$

$$\therefore M_n = (A_s)(f_y)\left(d - \frac{a}{2}\right) = \frac{(2.35 \text{ in}^2)(60 \text{ ksi})(23 - 11.56/2 \text{ in})}{(12 \text{ in/ft})} = 202.3 \text{ kip ft}$$

Our example was not balanced though, the steel yielded long before the concrete crushed.

Ultimate Capacity

<u>Was our section balanced?</u>

No because we saw it had greater moment capacity when the steel was such that $\varepsilon_s > \varepsilon_y$ and the concrete was just about to crush.

However, before we close out these three scenarios, let's look a little more into the balanced condition because it will be very useful to design in the next chapters.

Ultimate Capacity

<u>Balanced Section:</u>

Recall: $c_{bal} = \dfrac{(d)(\varepsilon_{cu})}{\underbrace{(\varepsilon_s + \varepsilon_{cu})}}$ & from equilibrium $C = T = A_s f_y = (0.85)(f_c')(\overbrace{a}^{a=\beta_1 c})(b)$

$$\therefore A_{s(bal)} = \frac{(0.85)(f_c')(\beta_1)(c_{bal})(b)}{f_y} \Rightarrow$$

Now, plug in ε_{cu} = 0.003 and that E_s = 29000000 psi to get

$$\rho_{bal} = \frac{(0.85)(f_c')(\beta_1)}{f_y}\left(\frac{\varepsilon_{cu}}{\varepsilon_s + \varepsilon_{cu}}\right) = \rho_b = \frac{(0.85)(f_c')(\beta_1)}{f_y}\left(\frac{87000}{87000 + f_y}\right)$$

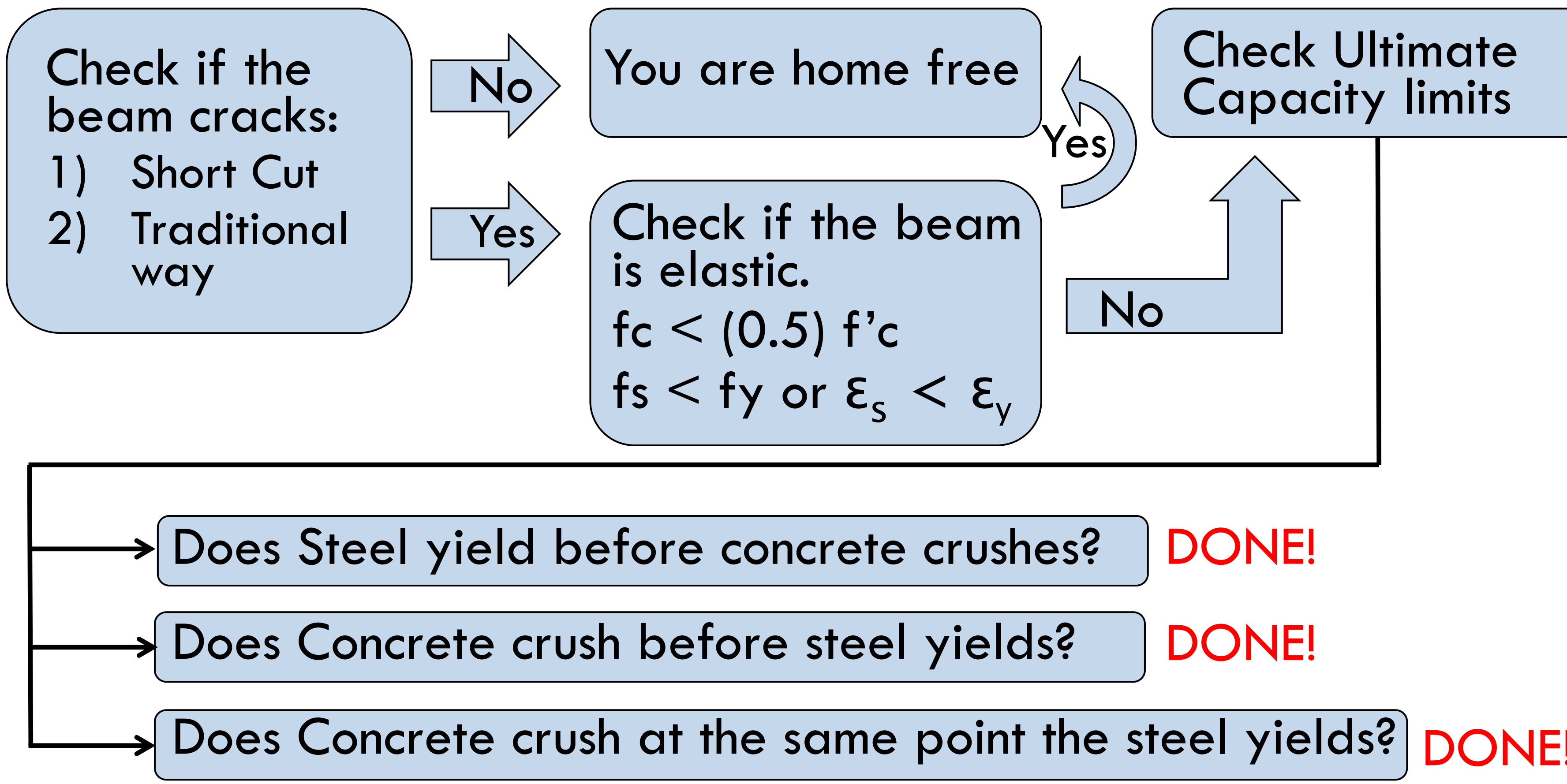
psi

Notice that we do not need <u>ANY</u> information about the geometry of the beam or applied loads to get the balanced reinforcement ratio. This is a very powerful design tool because it only depends on the material properties!

Let’s summarize Ultimate Capacity!

Logic Flow Behind Analysis

If given an existing section and asked to evaluate it's ability to handle a factored moment (or loads that give rise to a factored moment)



Analysis Summary

<u>What *really* did/do we do in beam analysis?:</u>

We:

1) Are given an existing section to work with, where all dimensions and material properties are known.
2) In terms of questions, we are 'typically' either asked to find:
   a) If the beam can sustain a moment calculated from factored loading <u>*or*</u>
   b) The ultimate capacity (maximum design moment) of an existing section. (Remember the factored moment $M_u = \Phi M_n$!
3) To do the first question, we have to check if the beam
   a) Has cracked
   b) If cracked, does it still obey Hooke's law
   c) If not, which ultimate state does it follow?

<u>But, in all of these steps, what did we *actually* do?</u>

Analysis Summary

<u>What *really* did/do we do in beam analysis?:</u>

To determine which analysis process to use, we calculated stresses in the materials (steel and concrete) and compared them to limits from ACI. Then we then found the capacity by calculating M_n from the limit of the type of analysis that we did.

Analysis Summary

<u>Let's do a sample problem:</u>

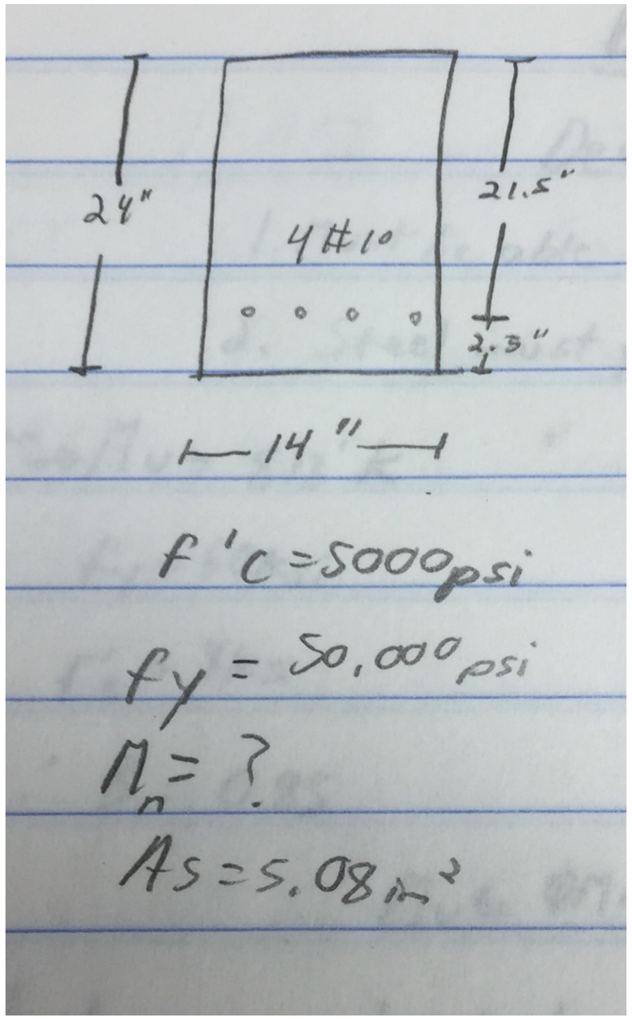
Find the ultimate capacity of a beam on the next slide, assume the steel yields before the concrete crushes:

You have now been shown 2 ways of handling this:

1) First, we assumed that fs = fy, found 'a', then calculated 'c', then had to check that $\varepsilon_s >> \varepsilon_y$ and therefore $f_s = f_y$
2) You can also calculate ρ and compare it to ρ_{bal}. If $\rho < \rho_{bal}$, the steel is yielding, now set $f_s = f_y$, find a and M_n. (In fact, ACI sets limits on ρ when doing design, we will see this next).

Analysis Summary

Let's do a sample problem:



Try option 2 first:

$$\rho_b = \frac{(0.85)(f_c')(\beta_1)}{f_y}\left(\frac{87000}{87000+f_y}\right) =$$ psi

$$\rho_b = \frac{(0.85)(f_c')(\beta_1)}{f_y}\left(\frac{87}{87+f_y}\right)$$ ksi

$$a = \frac{A_s f_y}{(0.85)(f_c')(b)} \qquad M_n = (A_s)(f_y)\left(d - \frac{a}{2}\right)$$