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CECS 622
Simulation and Modeling of Discrete Engineering Systems

Assignment # 4

I. Chi- Squared Test

a. **The following 20 numbers are generated using the Excel uniformly distributed over the interval 0 to 1**.

0.21	0.88	0.37	0.06	0.98	0.61	0.89	0.28	0.70	0.94
0.46	0.92	0.34	0.08	0.79	0.82	0.36	0.62	0.27	0.10

Random Numbers

- Compute the histogram over the 10 subintervals: [0, 0.1), [0.1, 0.2) … [0.9, 1.0).
- Perform the Chi-square Goodness-of-fit test with alpha = 0.05
- For extra credit
- Perform the Kolmogrov-Smirnov Goodness-of-fit test with alpha = 0.05

b. The grade distribution from two semesters is given below

	This Year	Last year
a	20	10
b	22	19
c	13	25
d	2	4
f	2	1

- Compare the grades using Chi-squared test to decide whether the overall distributions are statistically different or not.

Note: Chi-squared critical values are provided in the Appendix for your convenience

II. Queue Analysis

The following data provides the arrival times and service times that each customer will require, for the first 13 customers at a single server system. Upon arrival, a customer either enters service if the server is free or joins the waiting line. When the server completes work on a customer, the next one in line (i.e. the one who has been waiting the longest) enters service. Assume that the system is clear at time 0 and customer #13 is the last customer to be serviced

Arrival Times:	12	31	63	95	99	154	198	221	304	346	411	455	537
Service Times:	40	32	55	48	18	50	47	18	28	54	40	72	12

Queuing Data

a) Determine the system departure and response times of these 13 customers
b) What is the percent server utilization?
c) Repeat (a) and (b) using two servers and a customer can be served by server #1 or by the trainee server #2 if the main server #1 is busy.

Note: You can write a program or do this problem manually.

III. Probability Functions

Suppose that X has a piecewise probability density function which is defined as following:

$$f(x) = \begin{cases} c_1; & 0 \leq x < 2, \\ c_2; & 2 \leq x < 3, \\ c_1; & 3 \leq x < 5; \end{cases}$$

c1and c_2 are constants

(a) Sketch f(x) and find c_1 and c_2 assuming that the probabilities over the intervals [0, 2), [2, 3), and [3, 5) are the same and equal to 1/3 for each.

(b) Determine E(X) and Var(X)

(c) P {1< X < 3}

(d) P{X > 3 | X > 1}

(e) Is X memoryless? Please elaborate.

APPENDIX

Critical values of the Chi-square distribution with *d* degrees of freedom

	Probability of exceeding the critical value						
d	0.05	0.01	0.001	*d*	0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	7.815	11.345	16.266	13	22.362	27.688	34.528
4	9.488	13.277	18.467	14	23.685	29.141	36.123
5	11.070	15.086	20.515	15	24.996	30.578	37.697
6	12.592	16.812	22.458	16	26.296	32.000	39.252
7	14.067	18.475	24.322	17	27.587	33.409	40.790
8	15.507	20.090	26.125	18	28.869	34.805	42.312
9	16.919	21.666	27.877	19	30.144	36.191	43.820
10	18.307	23.209	29.588	20	31.410	37.566	45.315

***INTRODUCTION TO POPULATION GENETICS*, Table D.1**

Critical values of the Chi-square distribution at p = 0.05, 0.01, & 0.001 for d = 1 - 20 degrees of freedom