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Longevity forecasting by socio-economic groups using compositional data analysis

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Summary. Several Organisation for Economic Co-operation and Development countries have recently implemented an automatic link between the statutory retirement age and life expectancy for the total population to ensure sustainability in their pension systems due to increasing life expectancy. As significant mortality differentials are observed across socio-economic groups, future changes in these differentials will determine whether some socio-economic groups drive increases in the retirement age, leaving other groups with fewer pensionable years. We forecast life expectancy by socio-economic groups and compare the forecast performance of competing models by using Danish mortality data and find that the most accurate model assumes a common mortality trend. Life expectancy forecasts are used to analyse the consequences of a pension system where the statutory retirement age is increased when total life expectancy is increasing.

Keywords: Compositional data; Forecasting; Longevity; Pension; Socio-economic groups

1. Introduction

Recently, several Organisation for Economic Co-operation and Development countries have established an automatic link between their pension systems and increases in life expectancy: examples are Finland, Denmark, Portugal, Italy, the Netherlands, the Slovak Republic and Sweden (Organisation for Economic Co-operation and Development, 2017). The link is either between pension payments and life expectancy, as in Sweden and Italy, or between the statutory retirement age and life expectancy, as in Denmark and the Netherlands (Organisation for Economic Co-operation and Development, 2012). In the latter case, life expectancy for the total population is used to regulate the pension system, such that the statutory retirement age will be increased if life expectancy for the whole population increases. Thus, the pace by which life

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expectancies change in different socio-economic groups (SEGs) will have implications for the number of pensionable years. This paper analyses the consequences of such a pension system on socio-economic inequalities by forecasting life expectancy by SEG. The aim is to identify the most accurate model for forecasting mortality by SEG by comparing different models, basing the selection on their inclusion of dependence among the SEGs. Forecasts are used to measure the implication of the current pension scheme in Denmark in terms of expected years with pension. Our empirical setting focuses on Denmark but, as mortality differs significantly by SEG for almost all developed countries (Mackenbach *et al.*, 2003), the outline of the results that are presented are relevant for all countries which link life expectancy changes with their pension systems.

A successful forecast of mortality differentials between socio-economic subpopulations relies on the model's ability to capture different aspects of the differentials. Villegas and Haberman (2014) showed that changes in mortality differentials can be modelled successfully by using multipopulation mortality models and the analysis that is presented in this paper uses this approach. Multipopulation models aim at coherently modelling and forecasting mortality data from several populations or subpopulations.

Mortality forecasts are, currently, almost exclusively performed by using models which decompose age-specific mortality rates into age, period and sometimes also cohort effects, inspired by the Lee and Carter (LC) (1992) model, which is the most popular mortality forecasting model in countries with data of high quality (Booth, 2006; Cairns *et al.*, 2009; Coelho and Nunes, 2011; Enchev *et al.*, 2017). One major limitation with the LC type of models is that they generally underestimate improvements in mortality as a result of assuming constant age-specific and relative improvements (Bergeron-Boucher *et al.*, 2017; Booth and Tickle, 2008). As the LC model is fitted to historical data, the model gives a high weight to improvements in mortality for relatively young ages when fitted to data from developed countries as mortality has declined most in these age groups. When this pattern is imposed in forecasts the model fails to capture improvements at higher ages.

Oeppen (2008) and later Bergeron-Boucher *et al.* (2017) suggested the use of life table deaths to forecast mortality based on compositional data analysis (CODA) to alleviate this limitation in the LC models. CODA mortality models shift deaths from younger ages towards older ages because the covariance structure in compositional data is utilized when analysing life table deaths with CODA. For example, a decreasing number of deaths at young ages implies that more deaths occur at older ages. Thus, the use of life table deaths has an advantage, compared with models that use mortality rates, as it allows the rate of mortality improvements to change. CODA mortality models are especially useful when forecasting mortality for populations where the mortality patterns are changing, e.g. if life expectancy has been stagnating and then begins to experience improving mortality again. This is the case for the Danish population because Denmark in the 1980s and the first part of the 1990s experienced a stagnation in the improvement of mortality (Jarner *et al.*, 2008). Other countries have experienced similar stagnation periods, e.g. the Netherlands and the USA from around 1984 to 2000 (Meslé and Vallin, 2006). The LC type of models (e.g. the Lee–Li (LL) model) do not capture the shifting patterns which lead to less accurate life expectancy forecasts. In contrast CODA models allow more interactive dynamics in the observed mortality trajectories, and more accurate forecasts are often found when these models are fitted to data (Bergeron-Boucher *et al.*, 2017). Therefore, the CODA models offer an attractive alternative to LC-type models when forecasting mortality by SEG.

Several CODA mortality models have been suggested (Oeppen, 2008; Bergeron-Boucher *et al.*, 2017, 2018; Kjærgaard *et al.*, 2019) and this paper discusses the suitability of using CODA models to forecast mortality in SEGs by comparing existing models as well as proposing a new model. The CODA models were developed in a multipopulation framework but their performance has not been tested in a socio-economic setting. The analysis could have included other multipopulation models formulated on death rates such as the models that were presented in Villegas and Haberman (2014) or alternative models using the probability of death such as Cairns *et al.* (2006) or Cairns *et al.* (2009). We do not consider these models because they have a tendency to produce lower life expectancy forecasts with constant age-specific improvements without redistribution of deaths as in the CODA mortality models; see Bergeron-Boucher *et al.* (2019) for further details and discussions.

For the total population in a country, independent modelling of the populations might be reasonable, but for subpopulations within the same country it is likely that factors such as healthcare, public policy and technology affect all subpopulations. Factors which affect all subpopulations could be incorrectly specified by the independence model and could lead to implausible forecasts with possibly unbounded divergence between the subgroups (Villegas and Haberman, 2014). Hence, modelling the dependence between SEGs is important but it is not straightforward to determine how and to what extent dependence should be included to obtain the most accurate forecasts. A key contribution of this paper is in the selection of models which include different levels of dependence, imposed in both the time and the age structures of mortality. The models that are used in the analysis are selected on the basis of their inclusion of time- and age-dependent parameters so that different degrees of flexibility with respect to time and age dimensions are allowed. The inclusion of dependence spans from on one side an independent treatment (including six parameter vectors for both time and age dimensions), i.e. the independent CODA (called 'Inde-CoDA') and LC models. On the other side, we include models with one parameter vector for both the time and the age dimension, i.e. the relative CODA ('Rela-CoDA'), three-dimensional CODA ('3D-CoDA') and LL models. The LL model is an extension of the LC model to multiple populations. The new model that is suggested in this paper, dynamic factor CODA ('Dynam-CoDA'), places itself in between these extremes by having one age dimension and multiple time dimensions. Further, by comparing CODA mortality and models that are formulated on death rates (LC and LL) we check whether it is necessary to include the death redistribution that is imposed in the CODA models.

We find that, out of six different models, CODA mortality models that model mortality changes for each SEG proportional to a common trend provide the most accurate life expectancy forecasts for Danish males and females. Thus, models allowing for multiple trends and an individual treatment of the SEG are less suitable when forecasting, indicating a high degree of homogeneity among the socio-economic-specific mortality trends. Further, models based on death rates (the LC and LL models) provided in general the lowest life expectancy forecasts and less accurate forecasts compared with the CODA models assuming a common trend. Hence, these models could not capture the mortality development that is observed in a developed country with shifting mortality patterns. By measuring life expectancy forecasts at the statutory retirement age from 2016 to 2030 we find large socio-economic differences in the number of pensionable years. These differences are expected to persist until 2030, meaning that changing retirement ages in the Danish pension system are predicted to have the same relative effect by SEG.

The data that are analysed in the paper and the programs that were used to analyse them can be obtained from

`https://rss.onlinelibrary.wiley.com/hub/journal/1467985x/series-a-datasets`.

2. Danish pension system and socio-economic groups

In 2007, Denmark implemented a pension scheme that gradually increases the pension age in line with the increase in life expectancy, targeting receipt of pension for 14.5 years. The scheme was implemented to finance an increasing number of retirees from large birth cohorts while also taking into account increasing lifetimes in the entire population. The exact rules are complicated (Finansministeriet, 2017) but basically the pension age will be increased if life expectancy exceeds 14.5 years at the statutory retirement age. Hence, the pension age will increase if life expectancy increases regardless of the population subgroups experiencing mortality improvements. Widening life expectancy differentials across SEG would therefore not only imply larger inequality in lifespan but also a larger difference in the number of years that people can expect to receive labour market and public pensions. For example, if the highest SEG is experiencing a decline in mortality and the other groups experience no change, the pension age will increase, leaving the lower SEG with fewer expected years with a pension. Analysing and forecasting life expectancy across SEG is therefore highly relevant when studying the distributional consequences of a pension system.

SEGs are, in this study, based on an individual's income and wealth and not educational status, as is more common, because we want to capture mortality trends that measure the underlying lifespan inequality in the population over time. This is not well captured by analysing mortality by education if the population experiences large changes in the national educational level as well as compositional changes over recent decades (Brønnum-Hansen and Baadsgaard, 2012). Mortality trends by education thus include a selection effect where, in particular, people with very low or no education comprise a small selected group, in addition to mortality differences caused by different health conditions (Colardyn and Baltzer, 2008). This selection effect is referred to as the downward bias in mortality from education (Hendi, 2015). Using income and wealth for measuring SEG, groups of approximately equal size can be found because income and wealth are continuous variables, constituting an individual ranking basis. Time consistent mortality trends can thereby be calculated. Cairns *et al.* (2019) showed that it is important to consider both income and wealth as both high income and high wealth are associated with low mortality. For example, an individual person can be well off with a low or medium level of income if he enjoys a sufficiently high accumulation of wealth.

2.1. Data for Danish socio-economic groups

The Danish population was divided into five gender-specific SEGs, of (almost) equally large size, based on an affluence index following the procedure that was suggested by Cairns *et al.* (2019), which is found to produce a consistent and relevant classification of SEGs in relation to life expectancy in each subgroup. Cairns *et al.* (2019) defined SEG by weighting individuals' gross annual income by a factor of 15, compared with their net wealth, i.e. $A = W + KY$, where A is the affluence index, W is net wealth, $K = 15$ is the weighting factor and Y is gross annual income. Cairns *et al.* (2019) found that migration between SEGs should be allowed until age 67 years, after which the groups are fixed. Individual information about income, wealth and marital status was obtained from the Danish central registers. Data are only available from 1985 because reliable information about income at an individual level does not exist for the whole study population before 1985. Further details about the socio-economic measure and data can be found in Cairns *et al.* (2019).

The socio-economic data are available from age 50 years and grouped at 100 years and older. Mortality is measured by the death rates m_x, life table deaths distributions d_x or life expectancy e_x, all calculated by using standard life table techniques following Preston *et al.*

(2001). All variables are measured by single age $x=(50,\ldots,X)$ and single year $t=1985,\ldots,T$, and $g=(1,\ldots,G)$ is used to denote a socio-economic group constituting a subpopulation, where group 1 is the economically most well-off group. To avoid a problem of artificial compression in the life table forecasts we estimate single-year age-specific deaths and exposures after the age 100 years, when fitting the CODA models (Bergeron-Boucher *et al.*, 2017). We use a penalized composite link model for this as suggested by Rizzi *et al.* (2015) and calculate single yearly age mortality until age 105 years.

Other studies have, like Cairns *et al.* (2019), used income to define SEG, e.g. Tarkiainen *et al.* (2012), who used taxable income and found widening life expectancy differentials for the lowest SEG compared with the others in the Finnish population. Mortality trends by SEG, similar to those presented in this study, were found for Danish males and females by Brønnum-Hansen and Baadsgaard (2012). A steep increase in life expectancy for the lowest socio-economic group during the first part of the data period was found by Brønnum-Hansen and Baadsgaard (2012), suggesting that the large improvements for the lowest group are due to changing conditions in the labour market and the flexibility model of the Danish labour market. The socio-economic classification that was used by Brønnum-Hansen and Baadsgaard (2012) differs from that suggested by Cairns *et al.* (2019) by using disposable income, missing the inclusion of any wealth measure, and by the number of groups.

3. Methods

The LC model is included in the analysis as a benchmark and compared with the CODA models proposed. For comparison, all models use similar notation for the time and age index.

3.1. The Lee–Carter model

The LC model is a single-population model and treats subpopulations independently. The model decomposes age-specific mortality rates $m_{t,x,g}$ by singular value decomposition (SVD), using only the first rank, after having subtracted the average level of mortality, i.e.

$$\log(m_{t,x,g})=\alpha_{x,g}+\beta_{x,g}k_{t,g}+\epsilon_{t,x,g}, \tag{1}$$

where $\alpha_{x,g}=(1/T)\Sigma_{t=1}^{T}\log(m_{t,x,g})$ is the temporal arithmetic average measuring the general mortality age pattern, $k_{t,g}$ is an index over time of the general level of mortality, $\beta_{x,g}$ are age-specific responses to the index and $\epsilon_{t,x,g}$ denotes the independent and identically distributed (IID) error term. Mortality forecasts are calculated with the LC model by extrapolating $k_{t,g}$ using an auto-regressive integrated moving average (ARIMA) model to produce mortality rate forecasts.

3.2. The Li–Lee model

The CODA models are also compared with the multipopulation extension of the LC model: the LL model. The LL model (Li and Lee, 2005) estimates a common factor term for the average population by applying an LC model to the average death rates. Next, the common factor is subtracted from each socio-economic subpopulation and SVD is used to decompose deviations by subpopulation-specific age and time parameters. The LL model can be written as

$$\log(m_{t,x,g})=\alpha_{x,g}+B_xK_t+\beta_{x,g}k_{t,g}+\epsilon_{t,x,g}, \tag{2}$$

where K_t is an index of the general level of mortality for the average population, B_x is the age-specific response to changes in the index and $\epsilon_{t,x,g}$ the IID error term. The group-specific parameters are interpreted similarly to the parameters in the LC model but relative to changes in the common factor term. Following Li and Lee (2005), we assume that the $k_{t,g}$ follow an auto-regressive AR(1) model which is used to calculate life expectancy forecasts, and changes in subpopulation-specific mortality thus converge to the national level described by the common factor.

3.3. Compositional data methods

We present and analyse four different CODA models and forecast mortality for each subpopulation. Three of the models have already been used to forecast mortality in different settings whereas the dynamic factor CODA model is proposed for the first time. The CODA models differ from traditional LC modelling by using life table death distributions instead of death rates and by applying CODA techniques to introduce redistribution of deaths.

Compositional data are defined as a composition with only positive entries summing to a fixed constant and life table deaths are densities summing to 1 in each year, if rescaled to the life table radix, and therefore contain only relative information. The radix of a life table is the assumed number of new born individuals in the life table: also called the root of the life table. The number can be set arbitrarily as it has only a relative meaning (Preston *et al.*, 2001). Aitchison (1982) showed that traditional decomposition methods, e.g. SVD, do not apply to compositional data as data co-ordinates cannot vary freely but are constrained to vary between 0 and a constant. Instead, it is necessary to transform the data so that they can vary freely and to backtransform after the decomposition has been carried out (Pawlowsky-Glahn and Buccianti, 2011).

The analysis that is presented in this paper uses the centred log-ratio transformation clr which is the log-ratio of the composition of life table deaths divided by its geometric mean, $(g_t = d_{t,1} \cdot d_{t,2} \cdots d_{t,X})^X$, i.e.

$$\mathrm{clr}(d_{t,x}) = \ln\left(\frac{d_{t,x}}{g_t}\right). \tag{3}$$

All the models except Inde-CoDA and the LC model account for dependence between subpopulations but differ in their assumptions on the nature of dependence. Dynam-CoDA restricts the age dimension but allows different mortality time trends and by including Dynam-CoDA in the analysis we explore whether flexibility in the time dimension is important when forecasting mortality by SEG. Rela-CoDA constrains both age and time dimensions to a national trend for all subpopulations by modelling deviations from a national common trend. 3D-CoDA also restricts the subgroups to follow the same common age and time factors but instead of modelling the residuals, as with Rela-CoDA, a third dimension, related to the population-specific pace of mortality, is introduced. Summing up, we analyse whether dependence between SEGs is most useful for forecasting when it is introduced in the time dimension (Dynam-CoDA), by a common national trend (Rela-CoDA) or by the structure in the age and time dimensions (3D-CoDA). It is important to analyse different levels of flexibility to determine the most suitable forecasting model. A very complex model might provide a very accurate fit of the observed mortality but could lack ability to forecast mortality because of bias–variance trade-off (Hastie *et al.*, 2008). By analysing several models that provide different forecast methods, we test for alternative ways that mortality by SEG can be related.

All models capture level differentials by calculating subpopulation-specific means $a_{x,g}$ but mortality improvement differentials are captured differently in the models; in Dynam-CoDA, g mortality time indices measuring time changes, $k_{t,g}$; in Rela-CoDA by both age-specific dynamics $b_{x,g}$ and $k_{t,g}$; and in 3D-CoDA by an additional third-dimensional parameter vector related to population-specific pace of mortality. Life table deaths could also be modelled by using other statistical methods than CODA as long as the constraint in the data is fulfilled so that the death sums to the total. For example, in a study by Basellini and Camarda (2019) life table deaths were modelled by using the segmented transformation age-at-death distributions model. The models that are used and developed in this paper have the strength of identifying age and time dimensions of the data, which makes it possible to answer questions about trend differences between SEGs. The CODA methodology makes it possible to apply these age- and time-factorized models to life table deaths.

The CODA models cannot be estimated if the population experiences 0 death counts as the logarithm of the life table is calculated (Bergeron-Boucher *et al.*, 2017). However, this was not a problem for the data that are used in this paper. Section H of the on-line supplementary material presents common imputation procedures that can be applied in the case of 0 death counts.

Next we introduce the CODA models that are used in the study. Fig. 1 summarizes the models by illustrating the parameter structure.

3.4. *Independent (Inde-CoDA)*

The simplest way to analyse socio-economic subpopulations is to treat each subpopulation independently, similarly to the LC model but in a CODA setting. Using CODA, that means applying the model that was suggested by Oeppen (2008) to each subpopulation. Inde-CoDA centres life table deaths for each subpopulation ($d_{t,x,g}$) by differencing out the age-specific geometric mean. The model uses the operator '$\ominus$' which is the subtraction operator in CODA. For d_x and α_x in a specific year and subgroup,

$$d_x \ominus \alpha_x = C\left[\frac{d_1}{\alpha_1}, \frac{d_2}{\alpha_2}, \ldots, \frac{d_X}{\alpha_X}\right].$$

The closing operator '$C[\cdot]$' is defined as

$$C[d_x] = \left[\frac{d_1}{\sum d_i}, \frac{d_2}{\sum d_i}, \ldots, \frac{d_X}{\sum d_i}\right]\bar{K},$$

where $\bar{K}$ is a constant equal to the sum (Pawlowsky-Glahn and Buccianti, 2011). The centred $d_{t,x,g}$ are approximated by SVD, i.e.

$$\mathrm{clr}(d_{t,x,g} \ominus \alpha_{x,g}) = b^1_{x,g} k^1_{t,g} + \ldots + b^p_{x,g} k^p_{t,g} + \epsilon_{t,x,g}, \tag{4}$$

where $k^p_{t,g}$ is an index representing the overall mortality development over time for rank p approximation, $b^p_{x,g}$ the age-specific response changes in $k^p_{t,g}$ and $\epsilon_{t,x,g}$ the IID error term. The changes in mortality are thus decomposed into an age and time dimension. $b^p_{x,g}$ displays how deaths are redistributed in the forecasts, fulfilling the restriction that the total number of deaths needs to be maintained. For positive $k^p_{t,g}$-values, which is the case for all forecast years, deaths are redistributed from ages with negative $b^p_{x,g}$-values towards those with positive $b^p_{x,g}$-values. Mortality forecasts are calculated by extrapolating $k^p_{t,g}$ by using ARIMA models. Estimates and forecasts of $d_{t,x,g}$ are transformed back by using the inverse of the clr-transformation and estimates of $\alpha_{x,g}$ are added compositionally. The inverse procedure of clr ensures that the initial life table constraint is fulfilled so that deaths sum to the radix in each year.

3.5. *Dynamic factor compositional data model (Dynam-CoDA)*

One way to incorporate dependence between subpopulations is to estimate k_t time trends for each subgroup and to forecast these in a system estimating dependence between time trends but to use the same β_x for all subgroups. To do so, we propose to stack the life table deaths for each subpopulation vertically and to perform an analysis similarly to Inde-CoDA. A matrix of size

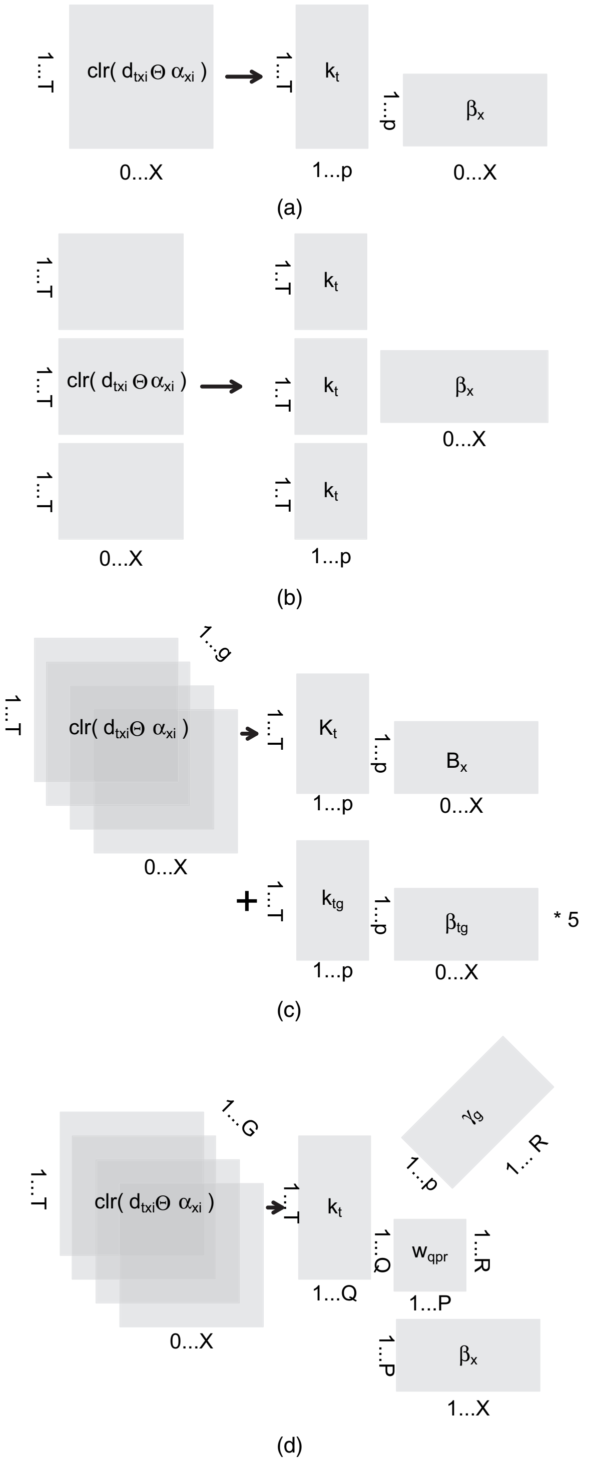

Fig. 1. Graphical representation of the CODA models: (a) subpopulations independent (Inde-CoDA); (b) dynamic factor CODA (Dynam-CoDA); (c) relative model (Rela-CoDA); (d) three-dimensional model (3D-CoDA)

$TG \times X$ of life table deaths is first centred and transformed and, from an SVD, g group-specific time parameter vectors and one age parameter vector are calculated, i.e.

$$\mathrm{clr}(d_{t,x,g} \ominus \alpha_{x,g}) = b_x^1 k_{t,g}^1 + \ldots + b_x^p k_{t,g}^p + \epsilon_{t,x,g}. \tag{5}$$

Dynam-CoDA is thus assuming g time indices describing the time dimension and one age parameter vector common to all subpopulations measuring the age dimension, for each rank approximation. $\epsilon_{t,x,g}$ is the IID error term.

If the subpopulation mortality trends move together, it is possible that they share common time trends which can be modelled together. We use a multilevel dynamic factor model here to model $k_{t,g}$ jointly. The multilevel dynamic factor model determines a factor that is common to all subgroups and factors which are shared only by one or more of the subgroups. More specifically the $k_{t,g}^1$ are factorized by using

$$k_{t,g}^1 = \gamma_g' P_t + \lambda_g' R_t + \epsilon_{t,g}, \qquad g = 1, \ldots, G, \tag{6}$$

where P_t is the vector of factors that pervade all groups, R_t is the vector of factors that pervade only a subset of groups and γ_g and λ_g are the corresponding loadings. Using the selection method that was suggested by Hallin and Liska (2007), one P_t-factor is estimated for Danish males and females, two R_t-factors for Danish females and one R_t-factor for Danish males. Dynam-CoDA is thereby incorporating dependence between subgroups by estimating a common factor for all groups but also factors (R_t) which are shared only by some of the subgroups. Details about the multilevel dynamic factor model are presented in the on-line supplementary material section A.

Forecasts of P_t and R_t are calculated by using ARIMA procedures. Finally, $k_{t,g}^1$-forecasts are perturbed on β_x and forecasts of the life table deaths are calculated by backtransforming and centring the life table deaths forecast.

3.6. Relative model (Rela-CoDA)

The third model forecasts the mortality for each subpopulation in relation to the national mortality and is a variation of the model that was suggested by Bergeron-Boucher *et al.* (2017).

Rela-CoDA is estimated in two steps. In step 1, a simple CODA mortality model, using rank 1 SVD, is fitted to the national life table deaths and national forecasts of age-specific responses and a mortality index are produced. The subscript g is left out in the first step and a superscript N is added when denoting the national mortality $d_{t,x}^N$:

$$\mathrm{clr}(d_{t,x}^N \ominus \alpha_x) = B_x K_t + \epsilon_{t,x}, \tag{7}$$

In the second step each subpopulation is considered after subtracting the geometric means of national deaths from the subpopulation-specific geometrical means by using CODA perturbation. Both the subpopulation-specific and national life table deaths are first rescaled so that each row sums to 1. A rank 1 SVD is used to calculate subpopulation-specific estimates of b_x and k_t, and an $\epsilon_{t,x,g}$ IID error term, i.e.

$$\mathrm{clr}(d_{t,x,g} \ominus \alpha_{x,g} \ominus d_{t,x}^N) = b_{x,g} k_{t,g} + \epsilon_{t,x,g}. \tag{8}$$

Rela-CoDA is thus a variation of the Li and Lee (2005) model within a CODA framework. Forecasts of K_t and $k_{t,g}$ are calculated with an ARIMA model and an auto-regressive moving average model respectively, and the specific auto-regressive and moving average terms are selected by using the Akaike information criterion (AIC). Rela-CoDA thus assumes stationary $k_{t,g}$-parameters such that the change in mortality will converge for the subpopulations towards the national level, similarly to what is normally assumed in the LL model. Forecasts of the life table deaths in each subpopulation are calculated by backtransforming and centring the life table deaths forecast according to the initial data constraint. The difference between this model and that of Bergeron-Boucher *et al.* (2017) is that they used $B_x K_t$-estimates from an average population instead of the national deaths. Arguably, in our approach, more of the national variation is accounted for because of the use of national deaths.

3.7. Three-dimensional model (3D-CoDA)

The fourth CODA model that we consider introduces a third dimension to capture subpopulation differentials. The model was suggested by Bergeron-Boucher *et al.* (2018) and applied to Canadian regions. Bergeron-Boucher *et al.* (2018) suggested, similarly to the other CODA models, first to centre and transform the life table deaths but, instead of an SVD approximation, 3D-CoDA uses a three-way principal component analysis (Tucker, 1966). The model can be written as

$$\mathrm{clr}(d_{t,x,g} \ominus \alpha_{x,g}) = \sum_{q=1}^{Q} \sum_{p=1}^{P} \sum_{r=1}^{R} w_{qpr}(k_{t,q}\beta_{x,p}\gamma_{g,r}) + \epsilon_{t,x,g}, \tag{9}$$

where the w_{qpr} are elements in a weighting array containing weights between the loading matrices $\boldsymbol{\beta}$, $\mathbf{k}$ and $\boldsymbol{\gamma}$.

Thus, the model assumes that all subpopulations, for each rank, share the same mortality index $k_{t,g}$ and the same age responses $b_{x,p}$ but that each subpopulation experiences changes in mortality at a different pace measured by the parameter vector $\gamma_{g,r}$. $\epsilon_{t,x,g}$ is an IID error term. A high degree of similarity in the time and age structures of the mortality development in each subpopulation is therefore assumed. In line with the analysis by Bergeron-Boucher *et al.* (2018) we consider only equal elements of Q, P and R for 3D-CoDA but unequal elements are analysed with the more general population value decomposition which is described in the on-line supplementary material section C together with comparative results. The population value decomposition model did not provide more accurate forecasts than 3D-CoDA. (None of the models include any covariates such as gross domestic product, smoking prevalences or other relevant factors. The main problem of including covariates is that many of these covariates are often more difficult to forecast than the mortality patterns themselves. Thus, forecasts including covariates are often found to be less reliable (Cairns *et al.*, 2011). A few studies have included covariates in the LC model, e.g. French and O'Hare (2014), but it is beyond the scope of this paper to include covariates in the CODA mortality model framework. These extensions are left for further research.)

4. Results

4.1. Estimation findings

We show parameter estimates for the rank 1 SVD in this section, fitting the models to the whole data period for Danish males (parameter estimates for Danish females are shown in the on-line supplementary material in Figs A3–A7). Inde-CoDA and 3D-CoDA ($P=R=Q=2$) estimated by using two ranks and parameter estimates for the second rank are shown in the supplementary material Figs A8 and A9. A rank 1 approximation was found to be sufficient for Dynam-CoDA as higher rank approximations did not improve the forecast accuracy for this model. Mortality forecasts are calculated by extrapolating $k_{t,g}^{p}$ using ARIMA models. A random walk with drift is generally used as a suitable model similarly to other extrapolative mortality models as for example the LC model (Booth and Tickle, 2008).

Fig. 2 shows estimates for α_x for the CODA models and for the LC or LL models. α_x for the CODA models is bell shaped as it describes the general death distribution whereas α_x is

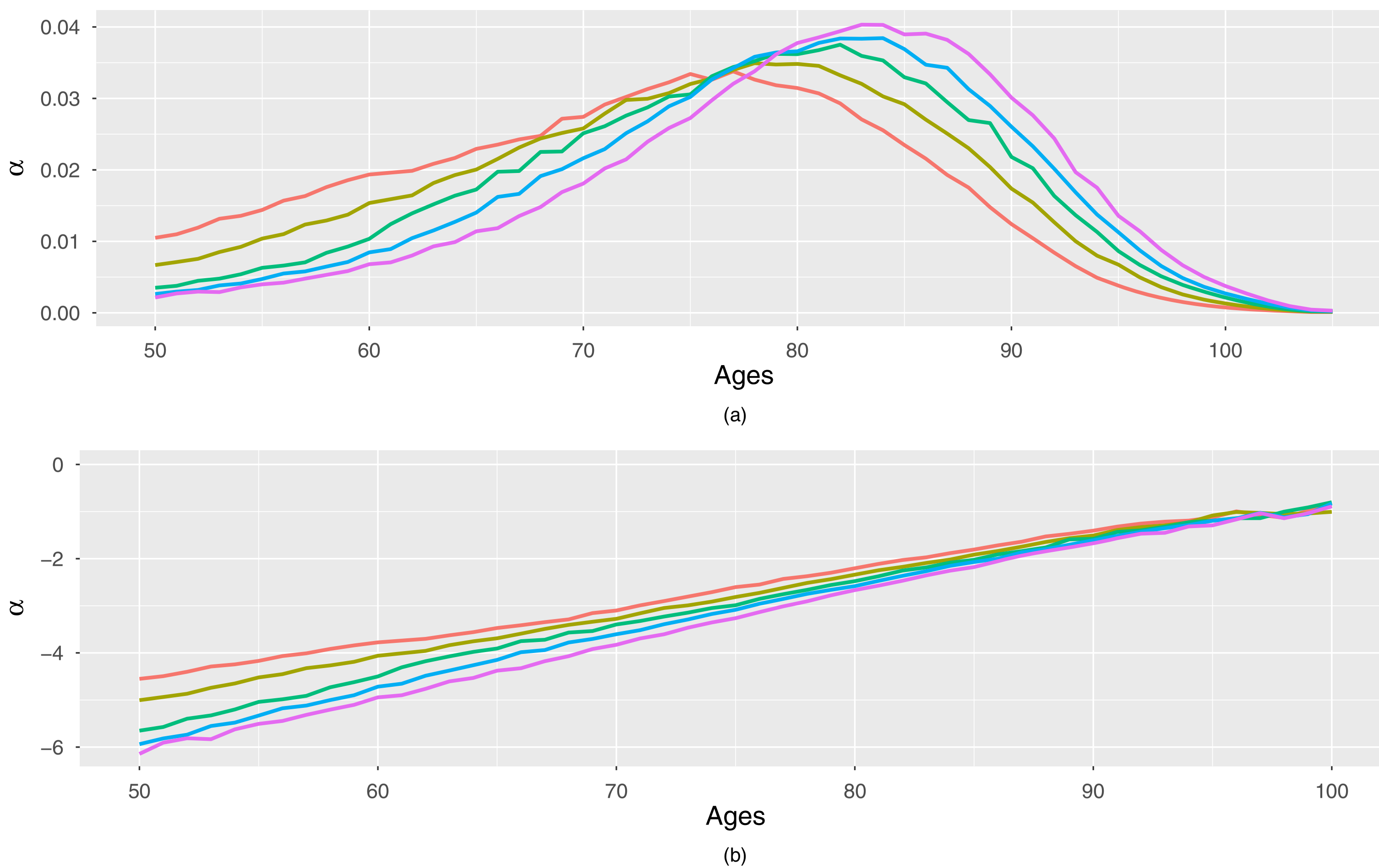

Fig. 2. α-estimates for (a) the CODA and (b) LC models for Danish males (α_x-estimates are the same for all CODA models and the same for the LC and LL models; thus, the estimates are shown only once): , SEG G1; , SEG G2; , SEG G3; , SEG G4; , SEG G5

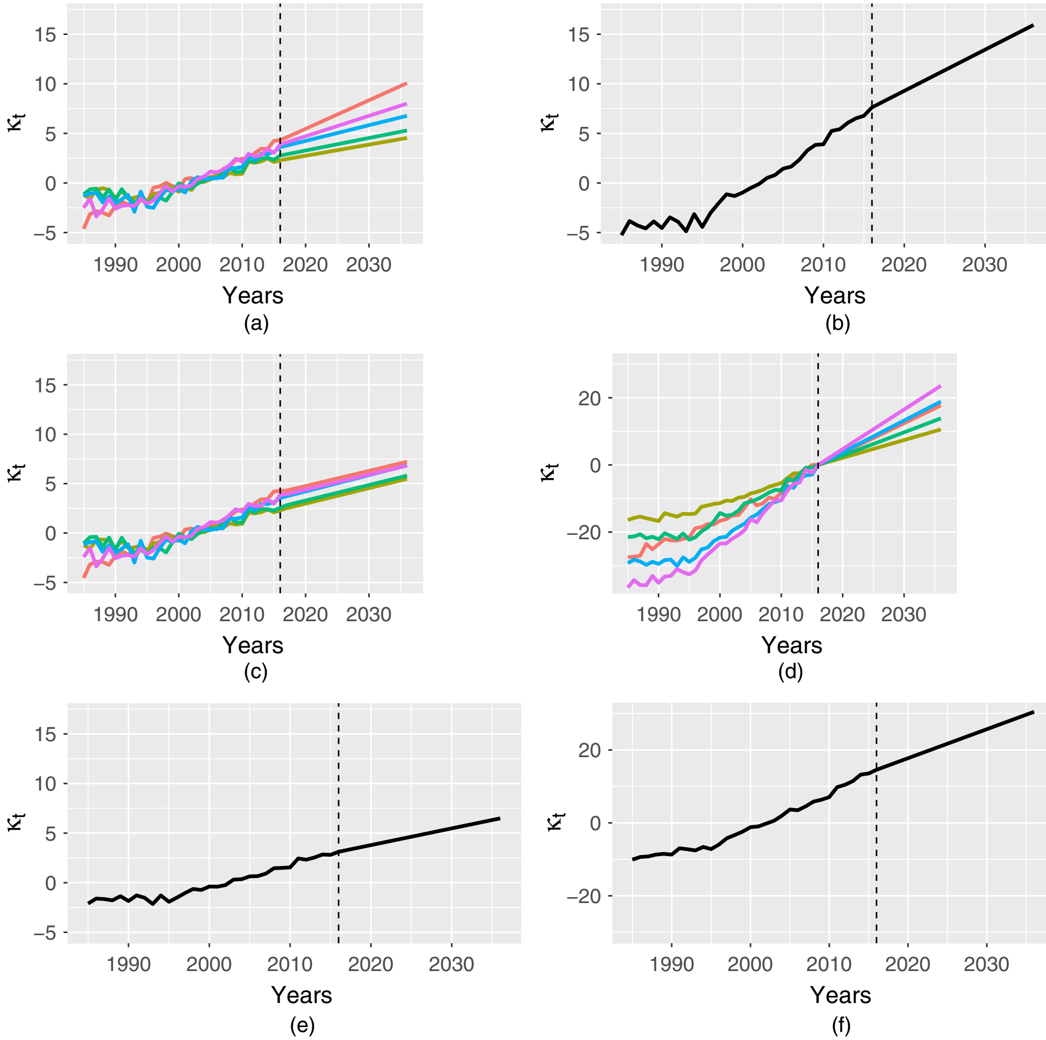

Fig. 3. First-rank κ_t-estimates and forecasts by using different models for Danish males (the LC and LL parameter estimates are plotted on a different scale compared with the CODA models for visibility) (—, SEG G1; —, SEG G2; —, SEG G3; —, SEG G4; —, SEG G5): (a) Inde-CoDA; (b) 3D-CoDA; (c) Dynam-CoDA; (d) LC; (e) Rela-CoDA; (f) LL

increasing for the LC or LL models as these models use log-death-rates for modelling, where the rate of mortality is increasing with age. Differences in α_x by socio-economic status follow the ordering of the SEG with a higher mortality at almost all ages for group G1 compared with group G5. Hence, the part of the mortality differentials between the SEG that is related to differences in the level of mortality follows the ordering of the SEG. Further, because α_x is assumed to be stable over time, the ordering in the level differentials component will persist in the forecasts (Villegas and Haberman, 2014).

Fig. 3 shows k_t^1-estimates and forecasts for all the models which capture the overall mortality development over time in an index. The time indices are increasing for the CODA models and for LC and the LL models, meaning that mortality has been declining over time from 1985 to 2016 for all models. Parameters for the LC and the LL models are plotted on a different scale

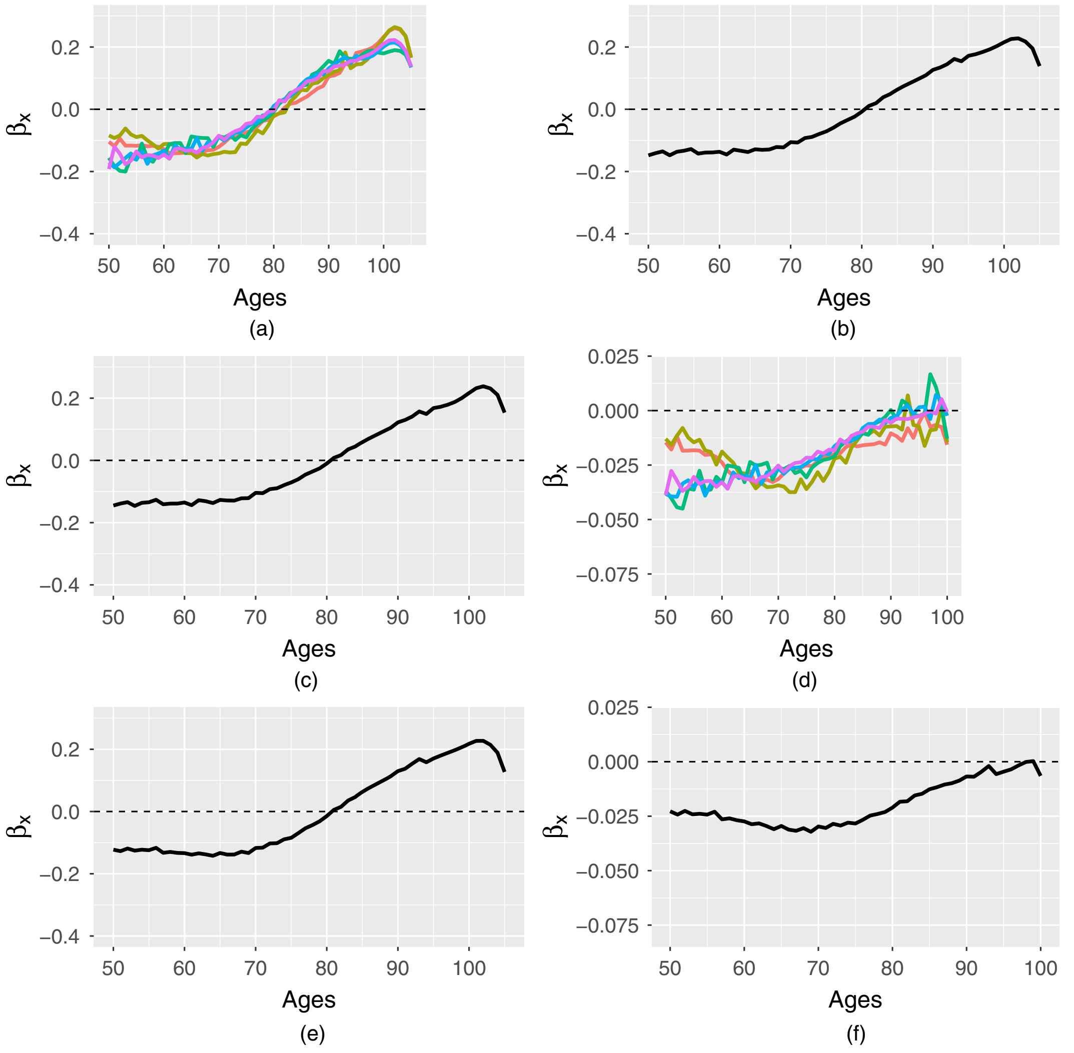

Fig. 4. First-rank β_x-estimates and forecasts by using different models for Danish males (the LC and LL parameter estimates are plotted on a different scale compared with the CODA models for visibility) (, SEG G1; , SEG G2; , SEG G3; , SEG G4; , SEG G5): (a) Inde-CoDA; (b) 3D-CoDA; (c) Dynam-CoDA; (d) LC; (e) Rela-CoDA; (f) LL

for visibility. The scale differences are a consequence of the LC and LL models being based on death rates, as with the β_x-estimates.

Inde-CoDA estimates for k_t show differences across SEG with the steepest increase for the lowest SEG, meaning that this group experiences the largest improvements in mortality. The second-lowest SEG in Inde-CoDA is predicted to have the lowest future improvements in mortality. Similar k_t^1-patterns are estimated with Dynam-CoDA, but different forecasts are produced when dependence between subgroups is taken into account by the multilevel dynamic factor model. The multilevel dynamic factor model estimates a global common factor, identifying dependence between the SEG in the mortality index k_t^1. The five SEGs are predicted to have a similar increase in k_t^1, meaning that they are expected to exhibit similar mortality improvements. The common factor is therefore the dominating part of the Dynam-CoDA forecasts, implying only small trend differences for the SEG. The CODA models with common factor terms, 3D-CoDA and Rela-CoDA, both identify an upward sloping time trend in both estimates and forecasts.

β_x-estimates for the models are shown in Fig. 4. All the CODA models follow a similar pattern which can be described by looking at the β_x-estimates from Inde-CoDA. Over age, the generally increasing pattern in β_x means that, when $k_{t,g}$ is increasing and becomes positive, deaths are shifted from ages with negative β_x towards older ages where β_x is positive. For Inde-CoDA β_x is, for groups G1 and G2, decreasing at ages 50–70 years and increasing over age until around age 102 years, followed by a decrease. For groups G3–G5 no decrease is found for ages 50–70 years. Groups G1 and G2 have the highest β_x-estimates for ages 50–60 years but lowest from age 60 to 95 years compared with other groups. The relatively high β_x-values at ages 50–60 years and subsequent low values at ages 60–96 years for the G1 and G2 groups imply that fewer deaths are transferred to higher ages for the same shift in $k_{t,g}$-values: these groups will experience a lower improvement in life expectancy for the same increase in $k_{t,g}$ compared with the other groups.

β_x parameter estimates for models assuming a common β_x-vector generally follow patterns that were observed for groups G3–G5. Assuming a common β_x-parameter thus provides a better fit for these groups than for groups G1 and G2. Similar patterns are found for the LC or LL models with relatively high values for groups G1 and G2 at ages 50–60 years. Note that these models do not directly imply transfer of deaths as with the CODA models.

Fig. 5 shows the group-specific parameter estimates for Rela-CoDA and the LL model, i.e. the group-specific adjustments to the common trend identified in Rela-CoDA and the LL models. Because the group-specific $k_{t,g}$-terms are assumed to be stationary, and thus mean reverting, all SEGs are assumed to follow the same long-run mortality trend dominated by the common K_t-term, as seen in Fig. 5(a). No particular trending pattern is found for $k_{t,g}$-parameters in Rela-CoDA, meaning that the stationarity and thus mean reverting pattern fits Rela-CoDA well. In Fig. 5(c) the $k_{t,g}$-parameters in the LL model display more upward and downward trending patterns, making it more difficult to forecast and less suitable to assume a mean reverting pattern. This makes the Rela-CoDA formulation more attractive for forecasting as the patterns are easier to predict. The different patterns in Rela-CoDA and the LL model are a consequence of Rela-CoDA subtracting $d^N_{t,g}$ instead of parameter estimates $K_t B_x$ as in the LL model. The age- and group-specific age responses $\beta_{x,g}$ are similar for all age groups and describe how each age group responds to changes in $k_{t,g}$. Note that $\beta_{x,g}$, in Fig. 5(d), for Rela-CoDA shows large differences by age and less by SEG and, thus, plotted close together.

The γ_g-estimates for 3D-CoDA show how the common factor terms are scaled for each SEG (Fig. 6). Higher γ_g-estimates are found for the lower SEG, meaning that, in the view of 3D-CoDA, mortality is decreasing more slowly for those groups compared with higher SEG. The only exception is group G1 where the fastest decline is found.

Figs A12–A16 in the on-line supplementary material show standardized residuals for all the models studied across all SEGs. None of the residuals shows any particular pattern and thus we do not include specific cohort terms in the models. (Cohort terms can be included in the models but cannot be identified uniquely because of the exact relationship between age, calendar year and cohort birth year (cohort = calendar year − age).)

4.2. Out-of-sample comparison and selection of forecasting model

To determine which mortality model is most suitable for forecasting mortality we compare the out-of-sample forecast performance of the various models. Data are available from 1985 to 2016. To have a sufficient number of years for fitting the models, we consider forecasts with a length of

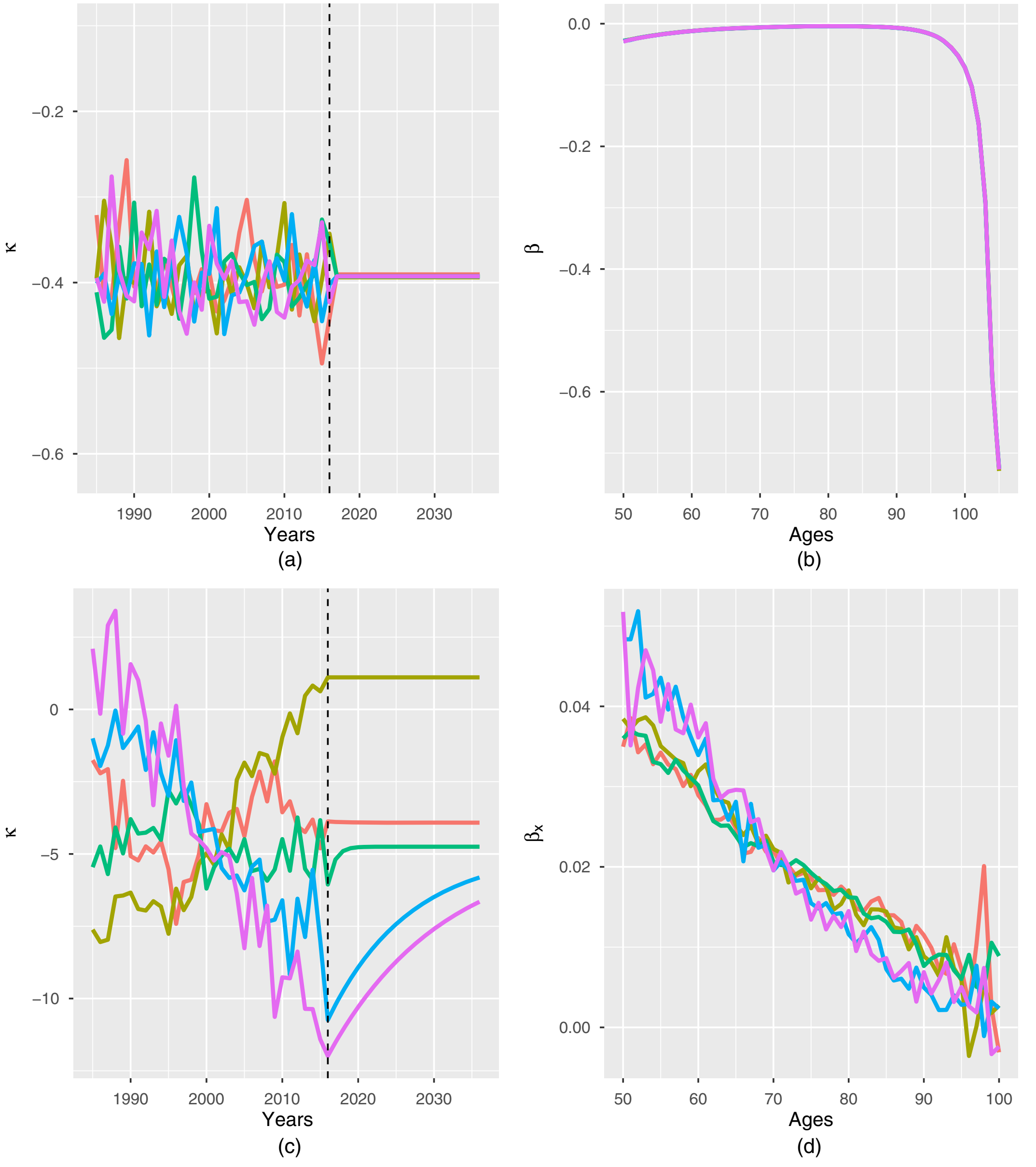

Fig. 5. Group-specific κ_t- and β-estimates for the LL and Rela-CoDA models for Danish males (—, SEG G1; —, SEG G2; —, SEG G3; —, SEG G4; —, SEG G5): (a) group-specific κ_t, Rela-CoDA; (b) group-specific β_x, Rela-CoDA; (c) group-specific κ_t, LL; (d) group specific β_x, LL

5–11 years calculated by rolling the onset of the forecasts (Shang, 2015), i.e. for the first forecast we use a fitting period from 1985 to 2005 and the period from 2005 to 2016 for validation. For the second forecast, 1 year is added to the fitting period by reducing the validation period by 1 year. From this a minimum of two-thirds of the data period is used to fit the models and a third for validation. Forecast errors are measured by the root-mean-square error (RMSE) comparing observed and forecast life expectancy at age 50 years:

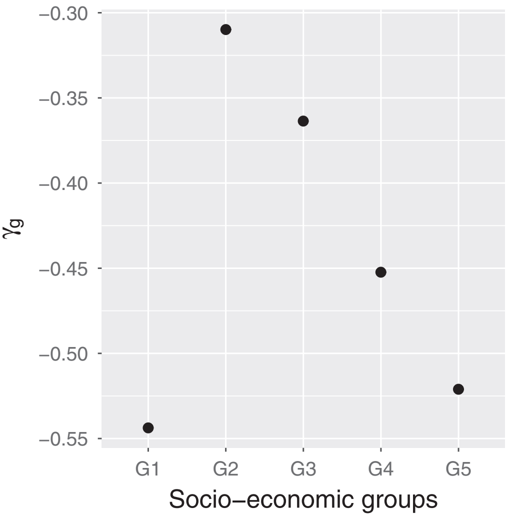

Fig. 6. γ_g-estimates for Danish males by using 3D-CoDA

$$\text{RMSE} = \sqrt{\left\{ \frac{\sum_{h=1}^{H} (e_{h,50} - \tilde{e}_{h,50})^2}{H} \right\}} \tag{10}$$

where $h \in (1, 2, \ldots, H)$ is the number of forecast years and e_{50} the observed life expectancy at age 50 years and $\tilde{e}_{50}$ the corresponding forecast. The average RMSE is calculated by averaging over the different forecast horizons and is used to compare the models. We use life expectancy at age, so, for comparison, it is calculated by using mortality information for all the age groups considered. A good in-sample fit can be achieved by introducing a large number of parameters but this will not guarantee a good forecast. Because forecasting is the objective of this paper, only the out-of-sample performance of the models is considered when selecting the most suitable model.

RMSEs are shown in Tables 1 and 2 for Danish males and females respectively. In-sample fit measures can be found in the on-line supplementary material section F.

For the Danish males, Rela-CoDA, 3D-CoDA and Inde-CoDA provide the lowest forecast errors for the various SEGs. For Danish females, Rela-CoDA and 3D-CoDA are the most accurate. Rela-CoDA provides the lowest forecast error on average for all the groups for Danish males and thus we conclude that this model is the most accurate model for forecasting mortality by SEG. Similarly, 3D-CoDA provides the lowest forecast error for females, but the accuracy is just slightly higher for Rela-CoDA. A good fit for Rela-CoDA and 3D-CoDA indicates a high degree of homogeneity in the Danish mortality trends across SEG and a model based on a national trend is thus useful when forecasting. Further, it also shows the value of modelling the dependence between the subgroup-specific mortality trends. As Dynam-CoDA did not provide better forecasts, in general, it does not seem relevant to allow for different patterns in the time trends. It is sufficient to have the same time and age pattern for each group and to adjust by modelling the residuals from a national pattern as in Rela-CoDA or by allowing for a different pace as in 3D-CoDA. A model based on death rates (LC or LL) did not provide the lowest forecast error for any of the groups. Hence, better forecasts are obtained for Danish mortality data by using life table deaths when forecasting life expectancy. (We test the significance of forecast performance differences of the seven models by using a Clark–West test (Clark and West, 2006) in the

Table 1. RMSE of e_{50}, averaged over seven forecast horizons for Danish males†

Model	*Results for the following groups:*					*Average*
	SEG G1	*SEG G2*	*SEG G3*	*SEG G4*	*SEG G5*	
Inde-CoDA	*0.4511*	0.8159	0.8970	0.8657	0.5136	0.7087
Dynam-CoDA	0.7190	0.6016	0.7286	0.8333	0.6419	0.7049
Rela-CoDA	0.7088	*0.1980*	*0.3615*	0.5626	0.5884	*0.4838*
3D-CoDA	0.7200	0.5040	0.4641	*0.5117*	*0.4110*	0.5221
LC	0.6637	0.5284	0.5487	0.6442	0.4878	0.5746
LL	0.7459	0.3212	0.5617	0.8514	0.9750	0.6910

†The lowest RMSE forecast error is indicated in italics.

Table 2. RMSE of e_{50}, averaged over seven forecast horizons for Danish females†

Model	*Results for the following groups:*					*Average*
	SEG G1	*SEG G2*	*SEG G3*	*SEG G4*	*SEG G5*	
Inde-CoDA	0.3243	0.8037	0.8056	0.8551	1.2521	0.8081
Dynam-CoDA	0.8396	0.8327	0.7752	0.9767	0.9456	0.8740
Rela-CoDA	0.6475	*0.6366*	*0.5778*	*0.7384*	0.8240	0.6848
3D-CoDA	*0.2797*	0.7305	0.7581	0.8308	*0.7846*	*0.6767*
LC	0.3460	0.8033	0.7827	0.8298	0.9123	0.7348
LL	0.7968	0.6709	0.6927	0.8805	0.9745	0.8030

†The lowest RMSE forecast error is indicated in italics.

medium horizon. Details about the Clark–West test are presented in the on-line supplementary material section D. We test the forecast differences for each SEG towards the best performing model for each sex and find that a large majority of the forecasts are significantly different. Results of the Clark–West test are presented in the on-line supplementary material Table A3.)

5. Implication for the pension age and its developments

Having identified the CODA models with a common trend as the most suitable models for forecasting life expectancy for Danish SEGs, these forecasts and remaining life expectancy at the pension age are reported for both Danish males and Danish females in Figs 7(c) and 7(d) until 2030. Life expectancy forecasts are calculated by using Rela-CoDA for each sex as this model provided the lowest RMSE forecast error for males and just slightly higher forecast errors for females compared with 3D-CoDA. This ensures coherence in the analysis of the consequences for the pension system as the same model is used for both sexes. Fig. 7(a) shows the statutory retirement age in Denmark until 2030 and, finally, Fig. 7(b) shows life expectancy forecasts for Danish males at age 50 years for completeness.

At age 50 years, life expectancy for the lowest SEG is converging towards the other groups at a decreasing pace from 1985 to 2026. Mortality differentials for the other groups stay roughly the same during the data period. Similar trends are observed for Danish females and are shown in the on-line supplementary material.

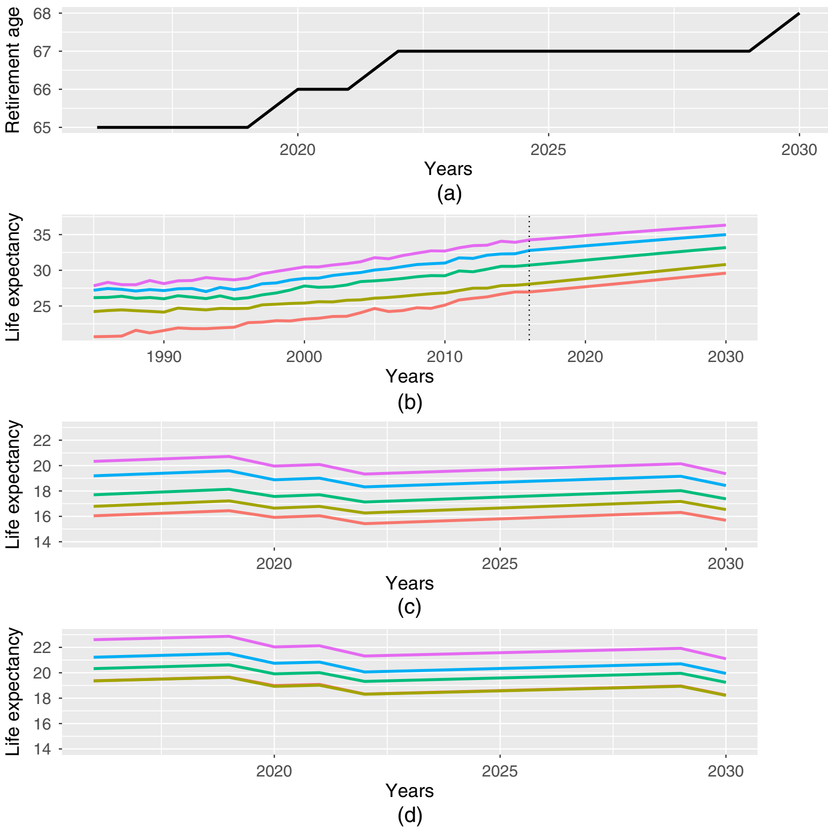

Fig. 7. Threshold ages for statutory retirement age and remaining life expectancy at the statutory retirement age by using Rela-CoDA for forecasting (—, SEG G1; —, SEG G2; —, SEG G3; —, SEG G4; —, SEG G5): (a) statutory retirement age by year; (b) life expectancy for Danish males by using Rela-CoDA (⋮, start of the forecast); (c) remaining life expectancy at the statutory retirement age, males; (d) remaining life expectancy at the statutory retirement age, females

In Fig. 7(c), Danish males in the lowest SEG would have a remaining life expectancy at pension age of around 16 years in 2016, which is 4.5 years less than in the highest group. The other groups fall between with remaining life expectancy from around 17 to 19 years in 2016. Danish males are, for all groups, forecast to have slightly falling life expectancy at pension age, meaning that pension age is expected to increase faster than life expectancy until 2030. For Danish females in Fig. 7(d), groups G1 and G2 have the same remaining life expectancy at pension age at around 19 years in 2016 and the similarity remains in the forecast. The other SEGs follow with the highest life expectancy for group G5 at around 23 years. Note that females in groups G1 and G2 have almost the same remaining life expectancy and thus their life expectancies are plotted close together. All groups, for both males and females, will have a life expectancy more than 14.5 years, which is the desired long-term goal in the current pension scheme, despite the social inequality of pensionable years. Thus, all SEGs can expect to receive a public pension in more years than the politically desired number of years if they retire at the statutory retirement age.

Future mortality differentials by SEG are highly relevant for determining the consequences of the current pension system in Denmark. Life expectancy forecasts show that relatively large differentials are expected 14 years ahead between SEG. As a strong common mortality trend was found across SEG, increases in the pension age are predicted to have a similar effect across SEG. No particular group is expected to drive future changes in the total life expectancy. Another implication of the strong common trend is that mortality differentials between the groups persist. Thus, the Danish pension reform in 2007 does not introduce further inequalities than already observed in the predicted number of life expectancy years after the statutory retirement age.

6. Concluding remarks

This study provides life expectancy forecasts for the Danish population by SEG. SEGs were measured with an affluence index constructed by weighting income and wealth. Two models using death rates and five models using life table deaths were compared and, on the basis of the models' out-of-sample performance, Rela-CoDA and 3D-CoDA were found to forecast life expectancy most accurately for males and females respectively. Both models use a common trend for all subgroups to forecast mortality. The six models differed by their method for including dependence between the SEGs: Inde-CoDA and the LC model treat the SEG independently, the LL model and Rela-CoDA in relation to a common mortality level, Dynam-CoDA by relating multiple mortality trends and 3D-CoDA by scaling common age and time structures. That Rela-CoDA and 3D-CoDA provided the most accurate forecast indicates the existence of a high degree of homogeneity in the mortality trends between the Danish SEGs, so common k_t- and b_x-parameters could be assumed for all SEGs when forecasting. Despite the similarity in the trend by which mortality changes, large mortality differentials were observed throughout the data period. Models formulated on death rates did not provide the most accurate forecast, indicating that these models did not capture changes in life expectancy in the validation period.

Populations in many Organisation for Economic Co-operation and Development countries are expected to age in the years ahead, increasing the cost of pensions and healthcare (European Commission, 2018). As a consequence several countries in the Organisation for Economic Co-operation and Development have linked changes in their pension system to changes in life expectancy for the whole population (Organisation for Economic Co-operation and Development, 2017). In this, the relatively large mortality differentials between SEGs also constitute a distributional issue because the lower SEGs have less private pension and lower capital income from savings, making them more dependent on the public pension (Pensionskommissionen, 2015). The lower groups are thus affected more when the statutory retirement age is increased because they do not have sufficient wealth to retire before the statutory retirement age. The consequences of future mortality differentials are therefore larger today than before.

The mortality differentials are also important for the public support of the pension reforms as mortality improvements in the future could be distributed unequally and thus implicitly new reforms will have unanticipated consequences. However, the results for Denmark indicate that the inequalities are not increased further for Denmark, but this might not be so in other countries. Further research should include data from other countries but maintain the focus on forecasting mortality by SEG at retirement ages.

Forecasts of mortality in general and of mortality differentials are not only relevant from an individual and public point of view but also for private pension companies (Richards, 2008) that could experience a mismatch in the composition of SEGs between their insured population and the national population. Determination of possible mismatch is also highly relevant for the risk management of pension liabilities as it can ensure a better hedge of the risk that a pensions fund's customers live longer than anticipated—longevity risk (Cairns *et al.*, 2019). Forecasts of the mortality differential could be used to inform pension companies about their longevity risk and improve the hedging of the longevity risk.

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Supporting information

Additional 'supporting information' may be found in the on-line version of this article:

'Supplementary material for: Longevity forecasting by Danish socio-economic groups using compositional data analysis'.