# ProblemSet4

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## 1 Math 210

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#### 1.1.1 Problem Set 4

#### 1.1.2 3/11/21

### 1.2 Problem 1.

[2]:
```
def print_tableau(a,indep_names,dep_names):
#
# Given matrix "a" and lists of variables names "indep_names" and "dep_names",
# this function prints the matrix and labels in standard tableau format
# (including adding the -1, the minus signs in the last column, and labeling␣
 ↪the lower-right as obj)
#
# First, check the inputs: indep_names should be one shorter than the number of␣
 ↪columns of A
#                          dep_names should be one shorter than the number of␣
 ↪rows of A
#
    nrows = a.shape[0]    # use the shape function to determine number of rows␣
 ↪and cols in A
    ncols = a.shape[1]
    nindep = len(indep_names)
    ndep = len(dep_names)
    if nindep != ncols-1:
        print("WARNING: # of indep vbles should be one fewer than # columns of␣
 ↪matrix")
    if ndep != nrows-1:
        print("WARNING: # of dep vbles should be one fewer than # rows of␣
 ↪matrix")
# Now do the printing (uses a variety of formatting techniques in Python)      ␣
 ↪
    for j in range(ncols-1):                    # Print the independent␣
 ↪variables in the first row
        print(indep_names[j].rjust(10),end="")  # rjust(10) makes fields 10␣
 ↪wide and right-justifies;
```

```
                                                #    the end command prevents␣
↪newline)
    print("        -1")                         # Tack on the -1 at the end of␣
↪the first row
    for i in range(nrows-1):
        for j in range(ncols):                  # Print all but the last row of␣
↪the matrix
            print("%10.3f" % a[i][j],end="") # The syntax prints in a field 10␣
↪wide, showing 3 decimal points
        lab = "= -" + dep_names[i]
        print(lab.rjust(10))
    for j in range(ncols):
        print("%10.3f" % a[nrows-1][j],end="")  # Print the last row of the␣
↪matrix, with label "obj" at end
    lab = "= obj"
    print(lab.rjust(10))
    print(" ")    # Put blank line at bottom
```

[3]:
```
def pivot(a,pivrow,pivcol,indep_names,dep_names) :
#
# Given matrix "a", a row number "pivrow" and column number "pivcol",
#  and lists of variable names "indep_names" and "dep_names", this
#  function does three things:
#    (1) outputs the new version of the matrix after a pivot,
#    (2) updates the lists of variable names post-pivot
#    (3) prints the new matrix, including labels showing the variable names
#
# First, check the inputs: indep_names should be one shorter than the number of␣
↪columns of A
#                          dep_names should be one shorter than the number of␣
↪rows of A
#                          you should not be pivoting on the last row or last␣
↪column
#
    a = a.astype(float)    # make sure entries are treated as floating point␣
↪numbers
    nrows = a.shape[0]     # use the shape function to determine number of rows␣
↪and cols in A
    ncols = a.shape[1]
    nindep = len(indep_names)
    ndep = len(dep_names)
    if nindep != ncols-1:
        print("WARNING: # of indep vbles should be one fewer than # columns of␣
↪matrix")
    if ndep != nrows-1:
```

```
        print("WARNING: # of dep vbles should be one fewer than # rows of matrix")
    if pivrow > nrows-1 or pivcol > ncols-1:
        print("WARNING: should not pivot on last row or column")
    newa = a.copy()       # make a copy of A, to be filled in below with result of pivot
    p = a[pivrow-1][pivcol-1]    # identify pivot element
    newa[pivrow-1][pivcol-1] = 1/p    # set new value of pivot element
    # Set entries in p's row
    for j in range(ncols):
        if j != pivcol-1:
            newa[pivrow-1][j]=a[pivrow-1][j]/p;
    # Set entries in p's column
    for i in range(nrows):
        if i != pivrow-1:
            newa[i][pivcol-1]=-a[i][pivcol-1]/p;
    # Set all other entries
    for i in range(nrows):
        for j in range(ncols):
            if i != pivrow-1 and j != pivcol-1:
                r = a[i][pivcol-1]
                q = a[pivrow-1][j]
                s = a[i][j]
                newa[i][j]=(p*s-q*r)/p
    # Now swap the variable names
    temp = indep_names[pivcol-1]
    indep_names[pivcol-1]=dep_names[pivrow-1]
    dep_names[pivrow-1]=temp
    print_tableau(newa,indep_names,dep_names) # Print the matrix with updated labels
    return newa;
```

[4]:
```
def target(a) :
    nrows = a.shape[0]     # use the shape function to determine number of rows and cols in "a"
    ncols = a.shape[1]
    import numpy as np
    v = np.empty(ncols-1)
    for i in range(ncols-1):
        v[i]=a[nrows-1,i]
    biggest_c = np.max(v)
    where_is_biggest_c = np.argmax(v)+1
    if biggest_c > 0 :
        return where_is_biggest_c
    else :
        return -1
```

[5]:
```
def select(a,pivcolnum) :
    nrows = a.shape[0]    # use the shape function to determine number of rows␣
 ↪and cols in A
    ncols = a.shape[1]
# First task: work down the column and record the b/a ratios in a vector v
#     except record -1 if a is negative or zero
    import numpy as np
    v = np.zeros(nrows-1)
    for i in range(nrows-1):
        if a[i,pivcolnum-1]>0 :
            v[i] = a[i,ncols-1]/a[i,pivcolnum-1]
        else :
            v[i] = -1
# Second task: if max b/a > -1, find min b/a by hand (ignoring zero entries in␣
 ↪v)
    if np.max(v) > -1 :
        min_so_far = np.max(v)+1  # Initialize variable to be for-sure bigger␣
 ↪than the min
        for i in range(nrows-1):
            if v[i] > -1 and v[i] < min_so_far :
                min_so_far = v[i]
                where_is_min = i+1   # Add 1 to use human numbering
        return where_is_min      # Once we've scanned v for min, we can return␣
 ↪result
    else :
        return -1
```

[6]:
```
# Create Simplex BF
def SimplexBF(a,indep_names,dep_names):
  nrows, ncols = a.shape
  a_new = a
  print_tableau(a_new,indep_names,dep_names)
  while np.max(a_new[nrows-1,:-1])>0:
    pivcol=target(a_new)
    pivrow=select(a_new,pivcol)
    if pivrow == -1:
      return("Unbounded")
    else:
      a_new=pivot(a_new,pivrow,pivcol,indep_names,dep_names)
      print_tableau(a_new,indep_names,dep_names)
```

[7]:
```
import numpy as np
np.set_printoptions(suppress=True)
a = np.array([[-1,-1,1],
              [-2,-4,7],
             [1,1,0]])
indep_names = ["x1","x2"]
```

```
dep_names = ["t1","t2"]
print_tableau(a,indep_names,dep_names)
```

```
      x1        x2        -1
-1.000    -1.000     1.000     = -t1
-2.000    -4.000     7.000     = -t2
 1.000     1.000     0.000     = obj
```

[8]:
```
SimplexBF(a,indep_names,dep_names)
```

```
      x1        x2        -1
-1.000    -1.000     1.000     = -t1
-2.000    -4.000     7.000     = -t2
 1.000     1.000     0.000     = obj
```

[8]:
```
'Unbounded'
```

### 1.3 Problem 2.

### 1.4 a.

When $b_2/b_{23} > b_3/b_{33}$, we will pivot on $b_{33}$, so let's think about what $b_2, b_3, b_4$ become. Using the pivot algorithm, these become:

$$b_{2,new} = \frac{a_{33}b_2 - a_{23}b_3}{a_{33}} = b_2 - \frac{a_{23}b_3}{a_{33}}$$
$$b_{3,new} = \frac{b_3}{a_{33}}$$
$$b_{4,new} = \frac{a_{33}b_4 - a_{43}b_3}{a_{33}} = b_4 - \frac{a_{43}b_3}{a_{33}}$$

We are interested in the condition for which these are greater than zero. Let's implement those conditions and do some algebra:

$$b_{2,new} = b_2 - \frac{a_{23}b_3}{a_{33}} > 0 \iff \frac{b_2}{a_{23}} > \frac{b_3}{a_{33}}$$
$$b_{3,new} = \frac{b_3}{a_{33}} > 0$$
$$b_{4,new} = b_4 - \frac{a_{43}b_3}{a_{33}} > 0 \iff \frac{b_4}{a_{43}} < \frac{b_3}{a_{33}}$$

The first of those conditions is given. The second condition is true because we are given that both $a_{33}$ and $b_3$ are greater than zero, so their quotient is positive. For the final expression the sign flipped because we are given that $a_{43} < 0$. We are given that $b_4 > 0$, so their quotient is negative. Furthermore, since both $a_{33}$ and $b_3$ are greater than zero, their quotient is positive. A negative is always less than a positive, the condition is satisfied.

### 1.4.1 b.

Now assuming $b_2/a_{23} < b_3/a_{33}$ we would instead pivot on $a_{23}$ giving the following expressions for $b_2, b_3, b_4$:

$$\begin{aligned} b_{2,new} &= \frac{b_2}{a_{23}} \\ b_{3,new} &= \frac{a_{23}b_3 - a_{33}b_2}{a_{23}} = b_3 - \frac{a_{33}b_2}{a_{23}} \\ b_{4,new} &= \frac{a_{23}b_4 - a_{43}b_2}{a_{23}} = b_4 - \frac{a_{43}b_2}{a_{23}} \end{aligned}$$

Again finding where these expressions are greater than zero:

$$\begin{aligned} b_{2,new} &= \frac{b_2}{a_{23}} > 0 \\ b_{3,new} &= b_3 - \frac{a_{33}b_2}{a_{23}} > 0 \iff \frac{b_3}{a_{33}} > \frac{b_2}{a_{23}} \\ b_{4,new} &= b_4 - \frac{a_{43}b_3}{a_{33}} > 0 \iff \frac{b_4}{a_{43}} < \frac{b_3}{a_{33}} \end{aligned}$$

The first expression is greater than zero because both $b_2$ and $a_{23}$ are negative so their quotient is positive. The second expression holds by assumption. The third equation holds because the LHS is negative as argued above while the RHS is positive.

## 1.5 Problem 3.

### 1.5.1 a.

We have the following tableau to apply SimplexNBF to:

[9]:
```
a=np.array([
            [-2,-10,3,-20,8],
            [-1,1,0,-3,-6],
            [1,4,-1,8,-1],
            [1,1,1,1,0]])
indep_names=["x","y","z","w"]
dep_names=["t1","t2","t3"]
print_tableau(a,indep_names,dep_names)
```

```
       x         y         z         w        -1
  -2.000   -10.000     3.000   -20.000     8.000     = -t1
  -1.000     1.000     0.000    -3.000    -6.000     = -t2
   1.000     4.000    -1.000     8.000    -1.000     = -t3
   1.000     1.000     1.000     1.000     0.000     = obj
```

We pivot on the second to last row (the last constraint row). The only negative option is $a_{33}$, so we pivot:

[10]:
```
a2=pivot(a,3,3,indep_names,dep_names)
```

```
       x         y        t3         w        -1
   1.000     2.000     3.000     4.000     5.000     = -t1
  -1.000     1.000     0.000    -3.000    -6.000     = -t2
  -1.000    -4.000    -1.000    -8.000     1.000      = -z
   2.000     5.000     1.000     9.000    -1.000     = obj
```

We target row 2 and the only options are $a_{21}$ and $a_{24}$, we pivot on the smaller ratio, $a_{24}$. There are no ratios below those two numbers that are positive.

[11]:
```
a3=pivot(a2,2,4,indep_names,dep_names)
#we then pivot on 2,1 because it is below a candidate
a3=pivot(a2,2,1,indep_names,dep_names)
```

```
       x         y        t3         t2        -1
  -0.333     3.333     3.000     1.333    -3.000     = -t1
   0.333    -0.333    -0.000    -0.333     2.000      = -w
   1.667    -6.667    -1.000    -2.667    17.000      = -z
  -1.000     8.000     1.000     3.000   -19.000     = obj

       w         y        t3         t2        -1
   1.000     3.000     3.000     1.000    -1.000     = -t1
  -1.000    -1.000    -0.000     3.000     6.000      = -x
  -1.000    -5.000    -1.000    -5.000     7.000      = -z
   2.000     7.000     1.000     3.000   -13.000     = obj
```

Nice, well not nice for the objective function, because we should target the top row and then when looking for a candidate we see no positive entries. We apply the theorem to say there is no answer.

### 1.5.2 b.

Now modifying the matrix slightly and pivoting:

[12]:
```
a=np.array([
            [-2,-10,3,-20,8],
            [-1,1,0,-3,-6],
            [-1,-4,1,-8,1],
            [1,1,1,1,0]])
indep_names=["x","y","z","w"]
dep_names=["t1","t2","t3"]
print_tableau(a,indep_names,dep_names)
a2=pivot(a,2,4,indep_names,dep_names)
SimplexBF(a2,indep_names,dep_names)
```

```
     x         y         z         w        -1
-2.000   -10.000     3.000   -20.000     8.000     = -t1
-1.000     1.000     0.000    -3.000    -6.000     = -t2
-1.000    -4.000     1.000    -8.000     1.000     = -t3
 1.000     1.000     1.000     1.000     0.000     = obj

     x         y         z        t2        -1
 4.667   -16.667     3.000    -6.667    48.000     = -t1
 0.333    -0.333    -0.000    -0.333     2.000      = -w
 1.667    -6.667     1.000    -2.667    17.000     = -t3
 0.667     1.333     1.000     0.333    -2.000     = obj

     x         y         z        t2        -1
 4.667   -16.667     3.000    -6.667    48.000     = -t1
 0.333    -0.333    -0.000    -0.333     2.000      = -w
 1.667    -6.667     1.000    -2.667    17.000     = -t3
 0.667     1.333     1.000     0.333    -2.000     = obj
```

[12]: `'Unbounded'`

The problem is unbounded which is kind of no solution, but for application purposes, we do have a solution.

## 1.6 Problem 4.

### 1.6.1 a.

We're really only one manipulation away from having a canonical min problem. Doing some algebra gives: minimize $x + 2y + z$ such that

$$3x + 6y + 2z \geq 12 \tag{1}$$
$$2x + 1y + 3z \geq 10 \tag{2}$$
$$1x + 2y - z \geq -2 \tag{3}$$
$$x, y, z \geq 0 \tag{4}$$

### 1.6.2 b.

Instead of minimizing $x + 2y + z$ we can maximize $-x - 2y - z$ such that:

$$-3x - 6y - 2z \leq -12 \tag{5}$$
$$-2x - 1y - 3z \leq -10 \tag{6}$$
$$-1x - 2y + z \leq 2 \tag{7}$$
$$x, y, z \geq 0 \tag{8}$$

We of course have to use SimplexNBF, but first let's set up the tableau:

[13]:
```
a=np.array([
            [-3,-6,-2,-12],
            [-2,-1,-3,-10],
            [-1,-2,1,2],
            [-1,-2,-1,0]])
indep_names=["x","y","z"]
dep_names=["t1","t2","t3"]
print_tableau(a,indep_names,dep_names)
```

```
      x         y         z        -1
 -3.000    -6.000    -2.000   -12.000     = -t1
 -2.000    -1.000    -3.000   -10.000     = -t2
 -1.000    -2.000     1.000     2.000     = -t3
 -1.000    -2.000    -1.000     0.000     = obj
```

I pivot on $a_{33}$ because it has the smallest ratio of any allowable entry at or below a candidate.

[14]:
```
a2=pivot(a,3,3,indep_names,dep_names)
```

```
      x         y        t3        -1
 -5.000   -10.000     2.000    -8.000     = -t1
 -5.000    -7.000     3.000    -4.000     = -t2
 -1.000    -2.000     1.000     2.000      = -z
 -2.000    -4.000     1.000     2.000     = obj
```

Next we pivot on $a_{22}$.

[15]:
```
a3=pivot(a2,2,2,indep_names,dep_names)
# 3,1 is the obvious choice for a pivot
a4=pivot(a3,1,3,indep_names,dep_names)
SimplexBF(a4,indep_names,dep_names)
```

```
      x         t2        t3        -1
  2.143    -1.429    -2.286    -2.286     = -t1
  0.714    -0.143    -0.429     0.571      = -y
  0.429    -0.286     0.143     3.143      = -z
  0.857    -0.571    -0.714     4.286     = obj

      x         t2        t1        -1
 -0.938     0.625    -0.438     1.000     = -t3
  0.313     0.125    -0.188     1.000      = -y
  0.562    -0.375     0.062     3.000      = -z
  0.187    -0.125    -0.312     5.000     = obj

      x         t2        t1        -1
 -0.938     0.625    -0.438     1.000     = -t3
  0.313     0.125    -0.188     1.000      = -y
```

```
 0.562    -0.375     0.062     3.000      = -z
 0.187    -0.125    -0.312     5.000     = obj

     y        t2        t1        -1
 3.000     1.000    -1.000     4.000     = -t3
 3.200     0.400    -0.600     3.200      = -x
-1.800    -0.600     0.400     1.200      = -z
-0.600    -0.200    -0.200     4.400     = obj

     y        t2        t1        -1
 3.000     1.000    -1.000     4.000     = -t3
 3.200     0.400    -0.600     3.200      = -x
-1.800    -0.600     0.400     1.200      = -z
-0.600    -0.200    -0.200     4.400     = obj
```

That actually only pivoted one more time, and we found that the minimum value is attained at $y = 0$ and $x = 3.2, z = 1.2$. And the objective function reads that it attains a value of $-4.4$, but we need to convert this back, so multiplying by $-1$ gives that the min value is 4.4.

### 1.7 Problem 5.

We know that we need at least 300 towels in the first period. So $x \geq 300$. In the next period we need 200 towels, we have $x - 300$ new unused and $y$ towels sent in for one day service. So $x - 300 + y \geq 200$. Finally, we have $(x - 300 + y) - 200$ leftover from period 2. And then we have $z$ sent for two day on day one and $w$ sent for one day on day two. That means we have: $(x - 300 + y) - 200 + w + z \geq 400$.

We then have the conditions that prevent washing more towels than used. For day one: $y + z \leq 300$. Then for day two: $w \leq 200$.

Cleaning these up a bit we have:

$$x \geq 300 \tag{9}$$
$$x + y \geq 500 \tag{10}$$
$$x + y + w + z \geq 900 \tag{11}$$
$$y + z \leq 300 \tag{12}$$
$$w \leq 200 \tag{13}$$

Converting this to canonical max constraints gives:

$$-x \leq -300 \tag{14}$$
$$-x - y \leq -500 \tag{15}$$
$$-x - y - w - z \leq -900 \tag{16}$$
$$y + z \leq 300 \tag{17}$$
$$w \leq 200 \tag{18}$$

Tableauing this gives:

[15]:

## 1.8 Problem 6.

### 1.8.1 a.

The first step is to figure out which patterns produce what quantities of what cuts. The following table shows that relationship:

Knowing how many of each pattern we need, we can convert that table into the following set of constraints:

$$p_1 + p_2 \geq 15 \tag{19}$$
$$2p_3 + 2p_4 + p_5 + p_6 \geq 50 \tag{20}$$
$$2p_7 + 2p_8 \geq 50 \tag{21}$$
$$p_7 + 2p_9 \geq 40 \tag{22}$$
$$2p_5 + 2p_6 + p_9 \geq 40 \tag{23}$$
$$2p_1 + 4p_2 + 2p_3 + 4p_4 + 2p_5 + 4p_6 + 4p_9 \geq 245 \tag{24}$$
$$p_1 + p_3 + p_5 + p_7 + 4p_8 \geq 100 \tag{25}$$
$$\forall i \in \{1, \ldots, 9\}, p_i \geq 0 \tag{26}$$

However, we now have something in the form of canonical min, which is not what we are trying to solve. So we can multiply each of these constraints by $-1$ to flip the signs should get us into the world of canonical max. That gives:

$$-p_1 - p_2 \leq -15 \tag{27}$$
$$-2p_3 - 2p_4 - p_5 - p_6 \leq -50 \tag{28}$$
$$-2p_7 - 2p_8 \leq -50 \tag{29}$$
$$-p_7 - 2p_9 \leq -40 \tag{30}$$
$$-2p_5 - 2p_6 - p_9 \leq -40 \tag{31}$$
$$-2p_1 - 4p_2 - 2p_3 - 4p_4 - 2p_5 - 4p_6 - 4p_9 \leq -245 \tag{32}$$
$$-p_1 - p_3 - p_5 - p_7 - 4p_8 \leq -100 \tag{33}$$
$$\forall i \in \{1, \ldots, 9\}, p_i \geq 0 \tag{34}$$

The final step is to convert the objective function (which we are trying to minimize) to its negative, so we can maximize the negative:

$$-\sum_{i=1}^{9} p_i$$

Okay with all of that we can put the pieces together into the tableau.

[16]:
```
a= -1*np.array([
    [1,1,0,0,0,0,0,0,0,15],
    [0,0,2,2,1,1,0,0,0,50],
    [0,0,0,0,0,0,2,2,0,50],
    [0,0,0,0,0,0,1,0,2,40],
    [0,0,0,0,2,2,0,0,1,40],
    [2,4,2,4,2,4,0,0,4,245],
    [1,0,1,0,1,0,1,4,0,100],
    [1,1,1,1,1,1,1,1,1,0]
  ])

indep_names = ["p1","p2","p3","p4","p5","p6","p7","p8","p9"]

dep_names = ["4x6","2x6","2x5","3x3","1x6","1x2","1x3"]

print_tableau(a,indep_names,dep_names)
```

```
        p1        p2        p3        p4        p5        p6        p7        p8
p9        -1
    -1.000    -1.000     0.000     0.000     0.000     0.000     0.000     0.000
0.000   -15.000    = -4x6
     0.000     0.000    -2.000    -2.000    -1.000    -1.000     0.000     0.000
0.000   -50.000    = -2x6
     0.000     0.000     0.000     0.000     0.000     0.000    -2.000    -2.000
0.000   -50.000    = -2x5
     0.000     0.000     0.000     0.000     0.000     0.000    -1.000     0.000
-2.000   -40.000    = -3x3
     0.000     0.000     0.000     0.000    -2.000    -2.000     0.000     0.000
-1.000   -40.000    = -1x6
    -2.000    -4.000    -2.000    -4.000    -2.000    -4.000     0.000     0.000
-4.000  -245.000    = -1x2
    -1.000     0.000    -1.000     0.000    -1.000     0.000    -1.000    -4.000
0.000  -100.000    = -1x3
    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000
-1.000     0.000     = obj
```

The tableau above has a negative value for every row in the $-1$ column, so I select the lowest one and compute ratios from that lowest row and select the smallest. By inspection we will select $a_{78}$ because it is smallest. Doing that pivot gives:

[17]:
```
a2=pivot(a,7,8,indep_names,dep_names)
```

```
        p1        p2        p3        p4        p5        p6        p7       1x3
p9        -1
    -1.000    -1.000    -0.000     0.000    -0.000     0.000    -0.000     0.000
0.000   -15.000    = -4x6
    -0.000     0.000    -2.000    -2.000    -1.000    -1.000    -0.000     0.000
```

```
0.000   -50.000    = -2x6
     0.500    -0.000     0.500    -0.000     0.500    -0.000    -1.500    -0.500
-0.000    -0.000    = -2x5
    -0.000     0.000    -0.000     0.000    -0.000     0.000    -1.000     0.000
-2.000   -40.000    = -3x3
    -0.000     0.000    -0.000     0.000    -2.000    -2.000    -0.000     0.000
-1.000   -40.000    = -1x6
    -2.000    -4.000    -2.000    -4.000    -2.000    -4.000    -0.000     0.000
-4.000  -245.000    = -1x2
     0.250    -0.000     0.250    -0.000     0.250    -0.000     0.250    -0.250
-0.000    25.000     = -p8
    -0.750    -1.000    -0.750    -1.000    -0.750    -1.000    -0.750    -0.250
-1.000    25.000     = obj
```

Here we select the 6th row because it is the last negative row, computing the ratios (and the few in the 7th column) we pivot on $a_{62}$ $a_{64}$ or $a_{66}$ or $a_{69}$. I select the first case, giving:

[18]:
```
a3=pivot(a2,6,2,indep_names,dep_names)
```

```
       p1       1x2        p3        p4        p5        p6        p7       1x3
p9        -1
    -0.500    -0.250     0.500     1.000     0.500     1.000    -0.000    -0.000
1.000    46.250    = -4x6
    -0.000     0.000    -2.000    -2.000    -1.000    -1.000    -0.000     0.000
-0.000   -50.000    = -2x6
     0.500    -0.000     0.500    -0.000     0.500    -0.000    -1.500    -0.500
-0.000    -0.000    = -2x5
    -0.000     0.000    -0.000    -0.000    -0.000    -0.000    -1.000     0.000
-2.000   -40.000    = -3x3
    -0.000     0.000    -0.000    -0.000    -2.000    -2.000    -0.000     0.000
-1.000   -40.000    = -1x6
     0.500    -0.250     0.500     1.000     0.500     1.000     0.000    -0.000
1.000    61.250     = -p2
     0.250    -0.000     0.250    -0.000     0.250    -0.000     0.250    -0.250
-0.000    25.000     = -p8
    -0.250    -0.250    -0.250    -0.000    -0.250    -0.000    -0.750    -0.250
-0.000    86.250     = obj
```

I identify the next pivot to be at $a_{55}$ giving:

[19]:
```
a4=pivot(a3,5,5,indep_names,dep_names)
```

```
       p1       1x2        p3        p4       1x6        p6        p7       1x3
p9        -1
    -0.500    -0.250     0.500     1.000     0.250     0.500    -0.000    -0.000
0.750    36.250    = -4x6
    -0.000    -0.000    -2.000    -2.000    -0.500    -0.000    -0.000    -0.000
```

```
0.500   -30.000    = -2x6
     0.500    -0.000     0.500    -0.000     0.250    -0.500    -1.500    -0.500
-0.250   -10.000    = -2x5
    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -1.000    -0.000
-2.000   -40.000    = -3x3
     0.000    -0.000     0.000     0.000    -0.500     1.000     0.000    -0.000
0.500    20.000     = -p5
     0.500    -0.250     0.500     1.000     0.250     0.500    -0.000    -0.000
0.750    51.250     = -p2
     0.250    -0.000     0.250    -0.000     0.125    -0.250     0.250    -0.250
-0.125    20.000     = -p8
    -0.250    -0.250    -0.250    -0.000    -0.125     0.250    -0.750    -0.250
0.125    91.250     = obj
```

Our next pivot is $a_{49}$ giving:

[20]:
```
a5=pivot(a4,4,9,indep_names,dep_names)
```

```
        p1        1x2         p3         p4        1x6         p6         p7        1x3
3x3         -1
    -0.500    -0.250     0.500     1.000     0.250     0.500    -0.375    -0.000
0.375    21.250    = -4x6
    -0.000    -0.000    -2.000    -2.000    -0.500    -0.000    -0.250    -0.000
0.250   -40.000    = -2x6
     0.500    -0.000     0.500    -0.000     0.250    -0.500    -1.375    -0.500
-0.125    -5.000    = -2x5
     0.000     0.000     0.000     0.000     0.000     0.000     0.500     0.000
-0.500    20.000     = -p9
    -0.000    -0.000    -0.000    -0.000    -0.500     1.000    -0.250    -0.000
0.250    10.000     = -p5
     0.500    -0.250     0.500     1.000     0.250     0.500    -0.375    -0.000
0.375    36.250     = -p2
     0.250    -0.000     0.250    -0.000     0.125    -0.250     0.312    -0.250
-0.062    22.500     = -p8
    -0.250    -0.250    -0.250    -0.000    -0.125     0.250    -0.812    -0.250
0.062    88.750     = obj
```

Our next pivot would be $a_{37}$

[21]:
```
a6=pivot(a5,3,7,indep_names,dep_names)
```

```
        p1        1x2         p3         p4        1x6         p6        2x5        1x3
3x3         -1
    -0.636    -0.250     0.364     1.000     0.182     0.636    -0.273     0.136
0.409    22.614    = -4x6
    -0.091    -0.000    -2.091    -2.000    -0.545     0.091    -0.182     0.091
0.273   -39.091    = -2x6
```

```
    -0.364     0.000    -0.364     0.000    -0.182     0.364    -0.727     0.364
0.091     3.636     = -p7
     0.182    -0.000     0.182    -0.000     0.091    -0.182     0.364    -0.182
-0.545    18.182     = -p9
    -0.091    -0.000    -0.091    -0.000    -0.545     1.091    -0.182     0.091
0.273    10.909     = -p5
     0.364    -0.250     0.364     1.000     0.182     0.636    -0.273     0.136
0.409    37.614     = -p2
     0.364    -0.000     0.364    -0.000     0.182    -0.364     0.227    -0.364
-0.091    21.364     = -p8
    -0.545    -0.250    -0.545    -0.000    -0.273     0.545    -0.591     0.045
0.136    91.705     = obj
```

Our final NBF pivot is $a_{23}$

[22]:
```
a7=pivot(a6,2,3,indep_names,dep_names)
```

```
        p1       1x2       2x6        p4       1x6        p6       2x5       1x3
3x3        -1
    -0.652    -0.250     0.174     0.652     0.087     0.652    -0.304     0.152
0.457    15.815    = -4x6
     0.043     0.000    -0.478     0.957     0.261    -0.043     0.087    -0.043
-0.130    18.696     = -p3
    -0.348     0.000    -0.174     0.348    -0.087     0.348    -0.696     0.348
0.043    10.435     = -p7
     0.174    -0.000     0.087    -0.174     0.043    -0.174     0.348    -0.174
-0.522    14.783     = -p9
    -0.087    -0.000    -0.043     0.087    -0.522     1.087    -0.174     0.087
0.261    12.609     = -p5
     0.348    -0.250     0.174     0.652     0.087     0.652    -0.304     0.152
0.457    30.815     = -p2
     0.348    -0.000     0.174    -0.348     0.087    -0.348     0.196    -0.348
-0.043    14.565     = -p8
    -0.522    -0.250    -0.261     0.522    -0.130     0.522    -0.543     0.022
0.065   101.902     = obj
```

We now have a basic feasible problem so I run SimplexBF to get the solution

[23]:
```
SimplexBF(a7,indep_names,dep_names)
```

```
        p1       1x2       2x6        p4       1x6        p6       2x5       1x3
3x3        -1
    -0.652    -0.250     0.174     0.652     0.087     0.652    -0.304     0.152
0.457    15.815    = -4x6
     0.043     0.000    -0.478     0.957     0.261    -0.043     0.087    -0.043
-0.130    18.696     = -p3
    -0.348     0.000    -0.174     0.348    -0.087     0.348    -0.696     0.348
```

```
0.043    10.435     = -p7
     0.174    -0.000     0.087    -0.174     0.043    -0.174     0.348    -0.174
-0.522    14.783     = -p9
    -0.087    -0.000    -0.043     0.087    -0.522     1.087    -0.174     0.087
0.261    12.609     = -p5
     0.348    -0.250     0.174     0.652     0.087     0.652    -0.304     0.152
0.457    30.815     = -p2
     0.348    -0.000     0.174    -0.348     0.087    -0.348     0.196    -0.348
-0.043    14.565     = -p8
    -0.522    -0.250    -0.261     0.522    -0.130     0.522    -0.543     0.022
0.065   101.902     = obj

        p1       1x2       2x6        p3       1x6        p6       2x5       1x3
3x3        -1
    -0.682    -0.250     0.500    -0.682    -0.091     0.682    -0.364     0.182
0.545     3.068    = -4x6
     0.045     0.000    -0.500     1.045     0.273    -0.045     0.091    -0.045
-0.136    19.545     = -p4
    -0.364     0.000     0.000    -0.364    -0.182     0.364    -0.727     0.364
0.091     3.636     = -p7
     0.182     0.000     0.000     0.182     0.091    -0.182     0.364    -0.182
-0.545    18.182     = -p9
    -0.091    -0.000     0.000    -0.091    -0.545     1.091    -0.182     0.091
0.273    10.909     = -p5
     0.318    -0.250     0.500    -0.682    -0.091     0.682    -0.364     0.182
0.545    18.068     = -p2
     0.364     0.000     0.000     0.364     0.182    -0.364     0.227    -0.364
-0.091    21.364     = -p8
    -0.545    -0.250     0.000    -0.545    -0.273     0.545    -0.591     0.045
0.136    91.705     = obj

        p1       1x2       2x6        p3       1x6        p6       2x5       1x3
3x3        -1
    -0.682    -0.250     0.500    -0.682    -0.091     0.682    -0.364     0.182
0.545     3.068    = -4x6
     0.045     0.000    -0.500     1.045     0.273    -0.045     0.091    -0.045
-0.136    19.545     = -p4
    -0.364     0.000     0.000    -0.364    -0.182     0.364    -0.727     0.364
0.091     3.636     = -p7
     0.182     0.000     0.000     0.182     0.091    -0.182     0.364    -0.182
-0.545    18.182     = -p9
    -0.091    -0.000     0.000    -0.091    -0.545     1.091    -0.182     0.091
0.273    10.909     = -p5
     0.318    -0.250     0.500    -0.682    -0.091     0.682    -0.364     0.182
0.545    18.068     = -p2
     0.364     0.000     0.000     0.364     0.182    -0.364     0.227    -0.364
-0.091    21.364     = -p8
    -0.545    -0.250     0.000    -0.545    -0.273     0.545    -0.591     0.045
```

```
0.136    91.705     = obj

        p1       1x2       2x6        p3       1x6       4x6       2x5       1x3
3x3        -1
    -1.000    -0.367     0.733    -1.000    -0.133     1.467    -0.533     0.267
0.800     4.500     = -p6
     0.000    -0.017    -0.467     1.000     0.267     0.067     0.067    -0.033
-0.100    19.750     = -p4
     0.000     0.133    -0.267    -0.000    -0.133    -0.533    -0.533     0.267
-0.200     2.000     = -p7
     0.000    -0.067     0.133     0.000     0.067     0.267     0.267    -0.133
-0.400    19.000     = -p9
     1.000     0.400    -0.800     1.000    -0.400    -1.600     0.400    -0.200
-0.600     6.000     = -p5
     1.000     0.000     0.000     0.000     0.000    -1.000     0.000     0.000
0.000    15.000     = -p2
     0.000    -0.133     0.267     0.000     0.133     0.533     0.033    -0.267
0.200    23.000     = -p8
     0.000    -0.050    -0.400    -0.000    -0.200    -0.800    -0.300    -0.100
-0.300    89.250     = obj

        p1       1x2       2x6        p3       1x6       4x6       2x5       1x3
3x3        -1
    -1.000    -0.367     0.733    -1.000    -0.133     1.467    -0.533     0.267
0.800     4.500     = -p6
     0.000    -0.017    -0.467     1.000     0.267     0.067     0.067    -0.033
-0.100    19.750     = -p4
     0.000     0.133    -0.267    -0.000    -0.133    -0.533    -0.533     0.267
-0.200     2.000     = -p7
     0.000    -0.067     0.133     0.000     0.067     0.267     0.267    -0.133
-0.400    19.000     = -p9
     1.000     0.400    -0.800     1.000    -0.400    -1.600     0.400    -0.200
-0.600     6.000     = -p5
     1.000     0.000     0.000     0.000     0.000    -1.000     0.000     0.000
0.000    15.000     = -p2
     0.000    -0.133     0.267     0.000     0.133     0.533     0.033    -0.267
0.200    23.000     = -p8
     0.000    -0.050    -0.400    -0.000    -0.200    -0.800    -0.300    -0.100
-0.300    89.250     = obj
```

Reading the solution off of this we have the following:

$$p_1 = 0 \tag{35}$$
$$p_2 = 15 \tag{36}$$
$$p_3 = 0 \tag{37}$$
$$p_4 = 19.75 \tag{38}$$
$$p_5 = 6 \tag{39}$$
$$p_6 = 4.5 \tag{40}$$
$$p_7 = 2 \tag{41}$$
$$p_8 = 23 \tag{42}$$
$$p_9 = 19 \tag{43}$$
$$\tag{44}$$

And we use a total of 89.25 stencils

### 1.8.2 b.

We can solve this problem by adding the following set of constraints to the constraints of part a.

$$\forall i, \sum_{j \neq i} 0p_j + p_i \leq 15$$

Since we have 9 stencils, we need 9 new constraints. We can create these new constraints in the following way:

[24]:
```
new_constraints=np.append(np.identity(9),np.transpose(np.array([np.
 ↪repeat(15,9)])),axis=1)

b=np.append(new_constraints,a,axis=0)

indep_names = ["p1","p2","p3","p4","p5","p6","p7","p8","p9"]

dep_names =␣
 ↪["p1_15","p2_15","p3_15","p4_15","p5_15","p6_15","p7_15","p8_15","p9_15","4x6","2x6","2x5",

print_tableau(b,indep_names,dep_names)
```

```
        p1         p2         p3         p4         p5         p6         p7         p8
p9         -1
     1.000      0.000      0.000      0.000      0.000      0.000      0.000      0.000
0.000     15.000  = -p1_15
     0.000      1.000      0.000      0.000      0.000      0.000      0.000      0.000
0.000     15.000  = -p2_15
     0.000      0.000      1.000      0.000      0.000      0.000      0.000      0.000
0.000     15.000  = -p3_15
     0.000      0.000      0.000      1.000      0.000      0.000      0.000      0.000
```

```
0.000    15.000  = -p4_15
     0.000     0.000     0.000     0.000     1.000     0.000     0.000     0.000
0.000    15.000  = -p5_15
     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000
0.000    15.000  = -p6_15
     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
0.000    15.000  = -p7_15
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     1.000
0.000    15.000  = -p8_15
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
1.000    15.000  = -p9_15
    -1.000    -1.000     0.000     0.000     0.000     0.000     0.000     0.000
0.000   -15.000    = -4x6
     0.000     0.000    -2.000    -2.000    -1.000    -1.000     0.000     0.000
0.000   -50.000    = -2x6
     0.000     0.000     0.000     0.000     0.000     0.000    -2.000    -2.000
0.000   -50.000    = -2x5
     0.000     0.000     0.000     0.000     0.000     0.000    -1.000     0.000
-2.000   -40.000    = -3x3
     0.000     0.000     0.000     0.000    -2.000    -2.000     0.000     0.000
-1.000   -40.000    = -1x6
    -2.000    -4.000    -2.000    -4.000    -2.000    -4.000     0.000     0.000
-4.000  -245.000    = -1x2
    -1.000     0.000    -1.000     0.000    -1.000     0.000    -1.000    -4.000
0.000  -100.000    = -1x3
    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000    -1.000
-1.000     0.000     = obj
```

The only things worthy of note there is that I had to make a 9 long vector of 15s and then convert it to a $1 \times 9$ array, then transpose it to make it a $9 \times 1$ array. Then I was able to append it to the identity. Finally, note that the $\leq 15$ constraints aren't multiplied by negative one because they are in canonical max form already ($\leq$ constraints).

Here's the shadow of a proof for why we can begin with all the same pivots (just translating to their new spots in the matrix).

Note that we have a matrix of the form:

$$B = \begin{bmatrix} L \\ A \end{bmatrix}$$

Where $L$ is the $\leq 15$ constraints and $A$ is the same matrix as in part $a$. Note that we would never pivot on any value in $L$ until we have removed all of the negative values in the $-1$ column for $A$ (because $L$ is above $A$). Okay, so that means for the first step we are finding our pivot in $A$, but we already did that in part $A$, so we know we are going to pivot on $a_{78}$, but that no longer has the same position in the $B$ matrix. We have added nine rows on top and there are no new columns, so $a_{78}$ is at $a_{16,8}$. Note that since our pivot is in $A$, the other values in $A$ are only affected by values in $A$. i.e. one of the rectangles in the pivot algorithm for the entries in $A$ will never go into $L$ unless

the pivot is in $L$. But we have already ruled out the pivot being in $L$. The same is not true for $L$. The entries in $L$ are affected by $L$ and $A$, but that makes no difference to this proof.

What I have suggested is that after pivoting on $b_{16,8}$ the new block matrix will be exactly what it was in the part $a$ just with some more matrices on top. This proof could be carried on by induction to show that we can just use the same pivots until there are no negative values in $A$ (which is where SimplexBF started). Okay let's do all of those first six pivots then.

[25]:
```
b2=pivot(b,7+9,8,indep_names,dep_names)
b3=pivot(b2,6+9,2,indep_names,dep_names)
b4=pivot(b3,5+9,5,indep_names,dep_names)
b5=pivot(b4,4+9,9,indep_names,dep_names)
b6=pivot(b5,3+9,7,indep_names,dep_names)
b7=pivot(b6,2+9,3,indep_names,dep_names)
```

```
        p1        p2        p3        p4        p5        p6        p7        1x3
p9        -1
     1.000     0.000    -0.000     0.000    -0.000     0.000    -0.000     0.000
0.000    15.000  = -p1_15
    -0.000     1.000    -0.000     0.000    -0.000     0.000    -0.000     0.000
0.000    15.000  = -p2_15
    -0.000     0.000     1.000     0.000    -0.000     0.000    -0.000     0.000
0.000    15.000  = -p3_15
    -0.000     0.000    -0.000     1.000    -0.000     0.000    -0.000     0.000
0.000    15.000  = -p4_15
    -0.000     0.000    -0.000     0.000     1.000     0.000    -0.000     0.000
0.000    15.000  = -p5_15
    -0.000     0.000    -0.000     0.000    -0.000     1.000    -0.000     0.000
0.000    15.000  = -p6_15
    -0.000     0.000    -0.000     0.000    -0.000     0.000     1.000     0.000
0.000    15.000  = -p7_15
    -0.250     0.000    -0.250     0.000    -0.250     0.000    -0.250     0.250
0.000   -10.000  = -p8_15
    -0.000     0.000    -0.000     0.000    -0.000     0.000    -0.000     0.000
1.000    15.000  = -p9_15
    -1.000    -1.000    -0.000     0.000    -0.000     0.000    -0.000     0.000
0.000   -15.000    = -4x6
    -0.000     0.000    -2.000    -2.000    -1.000    -1.000    -0.000     0.000
0.000   -50.000    = -2x6
     0.500    -0.000     0.500    -0.000     0.500    -0.000    -1.500    -0.500
-0.000    -0.000    = -2x5
    -0.000     0.000    -0.000     0.000    -0.000     0.000    -1.000     0.000
-2.000   -40.000    = -3x3
    -0.000     0.000    -0.000     0.000    -2.000    -2.000    -0.000     0.000
-1.000   -40.000    = -1x6
    -2.000    -4.000    -2.000    -4.000    -2.000    -4.000    -0.000     0.000
-4.000  -245.000    = -1x2
     0.250    -0.000     0.250    -0.000     0.250    -0.000     0.250    -0.250
```

```
-0.000    25.000     = -p8
    -0.750    -1.000    -0.750    -1.000    -0.750    -1.000    -0.750    -0.250
-1.000    25.000     = obj

        p1       1x2        p3        p4        p5        p6        p7       1x3
p9        -1
     1.000     0.000    -0.000    -0.000    -0.000    -0.000    -0.000     0.000
-0.000    15.000  = -p1_15
    -0.500     0.250    -0.500    -1.000    -0.500    -1.000    -0.000     0.000
-1.000   -46.250  = -p2_15
    -0.000     0.000     1.000    -0.000    -0.000    -0.000    -0.000     0.000
-0.000    15.000  = -p3_15
    -0.000     0.000    -0.000     1.000    -0.000    -0.000    -0.000     0.000
-0.000    15.000  = -p4_15
    -0.000     0.000    -0.000    -0.000     1.000    -0.000    -0.000     0.000
-0.000    15.000  = -p5_15
    -0.000     0.000    -0.000    -0.000    -0.000     1.000    -0.000     0.000
-0.000    15.000  = -p6_15
    -0.000     0.000    -0.000    -0.000    -0.000    -0.000     1.000     0.000
-0.000    15.000  = -p7_15
    -0.250     0.000    -0.250    -0.000    -0.250    -0.000    -0.250     0.250
-0.000   -10.000  = -p8_15
    -0.000     0.000    -0.000    -0.000    -0.000    -0.000    -0.000     0.000
1.000    15.000  = -p9_15
    -0.500    -0.250     0.500     1.000     0.500     1.000    -0.000    -0.000
1.000    46.250    = -4x6
    -0.000     0.000    -2.000    -2.000    -1.000    -1.000    -0.000     0.000
-0.000   -50.000    = -2x6
     0.500    -0.000     0.500    -0.000     0.500    -0.000    -1.500    -0.500
-0.000    -0.000    = -2x5
    -0.000     0.000    -0.000    -0.000    -0.000    -0.000    -1.000     0.000
-2.000   -40.000    = -3x3
    -0.000     0.000    -0.000    -0.000    -2.000    -2.000    -0.000     0.000
-1.000   -40.000    = -1x6
     0.500    -0.250     0.500     1.000     0.500     1.000     0.000    -0.000
1.000    61.250     = -p2
     0.250    -0.000     0.250    -0.000     0.250    -0.000     0.250    -0.250
-0.000    25.000     = -p8
    -0.250    -0.250    -0.250    -0.000    -0.250    -0.000    -0.750    -0.250
-0.000    86.250     = obj

        p1       1x2        p3        p4       1x6        p6        p7       1x3
p9        -1
     1.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p1_15
    -0.500     0.250    -0.500    -1.000    -0.250    -0.500    -0.000    -0.000
-0.750   -36.250  = -p2_15
    -0.000    -0.000     1.000    -0.000    -0.000    -0.000    -0.000    -0.000
```

```
-0.000    15.000  = -p3_15
    -0.000    -0.000    -0.000     1.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p4_15
    -0.000     0.000    -0.000    -0.000     0.500    -1.000    -0.000     0.000
-0.500    -5.000  = -p5_15
    -0.000    -0.000    -0.000    -0.000    -0.000     1.000    -0.000    -0.000
-0.000    15.000  = -p6_15
    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000     1.000    -0.000
-0.000    15.000  = -p7_15
    -0.250    -0.000    -0.250    -0.000    -0.125     0.250    -0.250     0.250
0.125    -5.000  = -p8_15
    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000
1.000    15.000  = -p9_15
    -0.500    -0.250     0.500     1.000     0.250     0.500    -0.000    -0.000
0.750    36.250    = -4x6
    -0.000    -0.000    -2.000    -2.000    -0.500    -0.000    -0.000    -0.000
0.500   -30.000    = -2x6
     0.500    -0.000     0.500    -0.000     0.250    -0.500    -1.500    -0.500
-0.250   -10.000    = -2x5
    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -1.000    -0.000
-2.000   -40.000    = -3x3
     0.000    -0.000     0.000     0.000    -0.500     1.000     0.000    -0.000
0.500    20.000     = -p5
     0.500    -0.250     0.500     1.000     0.250     0.500    -0.000    -0.000
0.750    51.250     = -p2
     0.250    -0.000     0.250    -0.000     0.125    -0.250     0.250    -0.250
-0.125    20.000     = -p8
    -0.250    -0.250    -0.250    -0.000    -0.125     0.250    -0.750    -0.250
0.125    91.250     = obj

        p1        1x2         p3         p4        1x6         p6         p7        1x3
3x3         -1
     1.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p1_15
    -0.500     0.250    -0.500    -1.000    -0.250    -0.500     0.375    -0.000
-0.375   -21.250  = -p2_15
    -0.000    -0.000     1.000    -0.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p3_15
    -0.000    -0.000    -0.000     1.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p4_15
    -0.000     0.000    -0.000    -0.000     0.500    -1.000     0.250     0.000
-0.250     5.000  = -p5_15
    -0.000    -0.000    -0.000    -0.000    -0.000     1.000    -0.000    -0.000
-0.000    15.000  = -p6_15
    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000     1.000    -0.000
-0.000    15.000  = -p7_15
    -0.250    -0.000    -0.250    -0.000    -0.125     0.250    -0.312     0.250
0.062    -7.500  = -p8_15
```

```
    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.500    -0.000
0.500    -5.000  = -p9_15
    -0.500    -0.250     0.500     1.000     0.250     0.500    -0.375    -0.000
0.375    21.250    = -4x6
    -0.000    -0.000    -2.000    -2.000    -0.500    -0.000    -0.250    -0.000
0.250   -40.000    = -2x6
     0.500    -0.000     0.500    -0.000     0.250    -0.500    -1.375    -0.500
-0.125    -5.000    = -2x5
     0.000     0.000     0.000     0.000     0.000     0.000     0.500     0.000
-0.500    20.000     = -p9
    -0.000    -0.000    -0.000    -0.000    -0.500     1.000    -0.250    -0.000
0.250    10.000     = -p5
     0.500    -0.250     0.500     1.000     0.250     0.500    -0.375    -0.000
0.375    36.250     = -p2
     0.250    -0.000     0.250    -0.000     0.125    -0.250     0.312    -0.250
-0.062    22.500     = -p8
    -0.250    -0.250    -0.250    -0.000    -0.125     0.250    -0.812    -0.250
0.062    88.750     = obj

        p1       1x2        p3        p4       1x6        p6       2x5       1x3
3x3        -1
     1.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p1_15
    -0.364     0.250    -0.364    -1.000    -0.182    -0.636     0.273    -0.136
-0.409   -22.614  = -p2_15
    -0.000    -0.000     1.000    -0.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p3_15
    -0.000    -0.000    -0.000     1.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p4_15
     0.091    -0.000     0.091    -0.000     0.545    -1.091     0.182    -0.091
-0.273     4.091  = -p5_15
    -0.000    -0.000    -0.000    -0.000    -0.000     1.000    -0.000    -0.000
-0.000    15.000  = -p6_15
     0.364    -0.000     0.364    -0.000     0.182    -0.364     0.727    -0.364
-0.091    11.364  = -p7_15
    -0.364    -0.000    -0.364    -0.000    -0.182     0.364    -0.227     0.364
0.091    -6.364  = -p8_15
    -0.182    -0.000    -0.182    -0.000    -0.091     0.182    -0.364     0.182
0.545    -3.182  = -p9_15
    -0.636    -0.250     0.364     1.000     0.182     0.636    -0.273     0.136
0.409    22.614    = -4x6
    -0.091    -0.000    -2.091    -2.000    -0.545     0.091    -0.182     0.091
0.273   -39.091    = -2x6
    -0.364     0.000    -0.364     0.000    -0.182     0.364    -0.727     0.364
0.091     3.636     = -p7
     0.182    -0.000     0.182    -0.000     0.091    -0.182     0.364    -0.182
-0.545    18.182     = -p9
    -0.091    -0.000    -0.091    -0.000    -0.545     1.091    -0.182     0.091
```

```
0.273    10.909     = -p5
     0.364    -0.250     0.364     1.000     0.182     0.636    -0.273     0.136
0.409    37.614     = -p2
     0.364    -0.000     0.364    -0.000     0.182    -0.364     0.227    -0.364
-0.091    21.364     = -p8
    -0.545    -0.250    -0.545    -0.000    -0.273     0.545    -0.591     0.045
0.136    91.705     = obj

        p1       1x2       2x6        p4       1x6        p6       2x5       1x3
3x3        -1
     1.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p1_15
    -0.348     0.250    -0.174    -0.652    -0.087    -0.652     0.304    -0.152
-0.457   -15.815  = -p2_15
    -0.043    -0.000     0.478    -0.957    -0.261     0.043    -0.087     0.043
0.130    -3.696  = -p3_15
    -0.000    -0.000    -0.000     1.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000  = -p4_15
     0.087    -0.000     0.043    -0.087     0.522    -1.087     0.174    -0.087
-0.261     2.391  = -p5_15
    -0.000    -0.000    -0.000    -0.000    -0.000     1.000    -0.000    -0.000
-0.000    15.000  = -p6_15
     0.348    -0.000     0.174    -0.348     0.087    -0.348     0.696    -0.348
-0.043     4.565  = -p7_15
    -0.348    -0.000    -0.174     0.348    -0.087     0.348    -0.196     0.348
0.043     0.435  = -p8_15
    -0.174    -0.000    -0.087     0.174    -0.043     0.174    -0.348     0.174
0.522     0.217  = -p9_15
    -0.652    -0.250     0.174     0.652     0.087     0.652    -0.304     0.152
0.457    15.815    = -4x6
     0.043     0.000    -0.478     0.957     0.261    -0.043     0.087    -0.043
-0.130    18.696     = -p3
    -0.348     0.000    -0.174     0.348    -0.087     0.348    -0.696     0.348
0.043    10.435     = -p7
     0.174    -0.000     0.087    -0.174     0.043    -0.174     0.348    -0.174
-0.522    14.783     = -p9
    -0.087    -0.000    -0.043     0.087    -0.522     1.087    -0.174     0.087
0.261    12.609     = -p5
     0.348    -0.250     0.174     0.652     0.087     0.652    -0.304     0.152
0.457    30.815     = -p2
     0.348    -0.000     0.174    -0.348     0.087    -0.348     0.196    -0.348
-0.043    14.565     = -p8
    -0.522    -0.250    -0.261     0.522    -0.130     0.522    -0.543     0.022
0.065   101.902     = obj
```

So unfortunately, we are not yet in basic feasible, so let's run a few steps of NBF. Being a bit loose with things, let's pivot on $b_{24}$ giving:

[30]:
```
b8=pivot(b7,9,4,indep_names,dep_names)
b9=pivot(b8,8,7,indep_names,dep_names)
b10=pivot(b9,3,4,indep_names,dep_names)
b11=pivot(b10,3,1,indep_names,dep_names)
b12=pivot(b11,8,7,indep_names,dep_names)
b13=pivot(b12,7,1,indep_names,dep_names)
b14=pivot(b13,7,9,indep_names,dep_names)
b15=pivot(b14,2,4,indep_names,dep_names)
b16=pivot(b15,4,1,indep_names,dep_names)
b17=pivot(b16,7,6,indep_names,dep_names)
```

```
        p1       1x2       2x6     p9_15       1x6        p6       2x5       1x3
3x3        -1
     1.000    -0.000    -0.000     0.000    -0.000     0.000    -0.000     0.000
0.000    15.000  = -p1_15
    -1.000     0.250    -0.500     3.750    -0.250    -0.000    -1.000     0.500
1.500   -15.000  = -p2_15
    -1.000    -0.000     0.000     5.500    -0.500     1.000    -2.000     1.000
3.000    -2.500  = -p3_15
     1.000     0.000     0.500    -5.750     0.250    -1.000     2.000    -1.000
-3.000    13.750  = -p4_15
     0.000    -0.000     0.000     0.500     0.500    -1.000     0.000     0.000
0.000     2.500  = -p5_15
    -0.000    -0.000    -0.000     0.000    -0.000     1.000    -0.000     0.000
0.000    15.000  = -p6_15
     0.000    -0.000     0.000     2.000     0.000     0.000     0.000     0.000
1.000     5.000  = -p7_15
     0.000     0.000     0.000    -2.000     0.000     0.000     0.500     0.000
-1.000     0.000  = -p8_15
    -1.000    -0.000    -0.500     5.750    -0.250     1.000    -2.000     1.000
3.000     1.250     = -p4
    -0.000    -0.250     0.500    -3.750     0.250     0.000     1.000    -0.500
-1.500    15.000    = -4x6
     1.000     0.000     0.000    -5.500     0.500    -1.000     2.000    -1.000
-3.000    17.500     = -p3
     0.000     0.000     0.000    -2.000     0.000     0.000     0.000     0.000
-1.000    10.000     = -p7
     0.000    -0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p9
     0.000     0.000     0.000    -0.500    -0.500     1.000     0.000     0.000
0.000    12.500     = -p5
     1.000    -0.250     0.500    -3.750     0.250     0.000     1.000    -0.500
-1.500    30.000     = -p2
     0.000    -0.000     0.000     2.000     0.000     0.000    -0.500     0.000
1.000    15.000     = -p8
     0.000    -0.250     0.000    -3.000    -0.000     0.000     0.500    -0.500
-1.500   101.250     = obj
```

```
        p1       1x2       2x6     p9_15       1x6        p6     p8_15       1x3
3x3        -1
     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
0.000    15.000  = -p1_15
    -1.000     0.250    -0.500    -0.250    -0.250    -0.000     2.000     0.500
-0.500   -15.000  = -p2_15
    -1.000     0.000     0.000    -2.500    -0.500     1.000     4.000     1.000
-1.000    -2.500  = -p3_15
     1.000     0.000     0.500     2.250     0.250    -1.000    -4.000    -1.000
1.000    13.750  = -p4_15
     0.000    -0.000     0.000     0.500     0.500    -1.000    -0.000     0.000
0.000     2.500  = -p5_15
     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000
0.000    15.000  = -p6_15
     0.000    -0.000     0.000     2.000     0.000     0.000    -0.000     0.000
1.000     5.000  = -p7_15
     0.000     0.000     0.000    -4.000     0.000     0.000     2.000     0.000
-2.000     0.000    = -2x5
    -1.000     0.000    -0.500    -2.250    -0.250     1.000     4.000     1.000
-1.000     1.250     = -p4
    -0.000    -0.250     0.500     0.250     0.250     0.000    -2.000    -0.500
0.500    15.000    = -4x6
     1.000     0.000     0.000     2.500     0.500    -1.000    -4.000    -1.000
1.000    17.500     = -p3
     0.000     0.000     0.000    -2.000     0.000     0.000    -0.000     0.000
-1.000    10.000     = -p7
     0.000    -0.000     0.000     1.000     0.000     0.000    -0.000     0.000
0.000    15.000     = -p9
     0.000     0.000     0.000    -0.500    -0.500     1.000    -0.000     0.000
0.000    12.500     = -p5
     1.000    -0.250     0.500     0.250     0.250     0.000    -2.000    -0.500
0.500    30.000     = -p2
     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
0.000    15.000     = -p8
     0.000    -0.250     0.000    -1.000    -0.000     0.000    -1.000    -0.500
-0.500   101.250     = obj

        p1       1x2       2x6     p3_15       1x6        p6     p8_15       1x3
3x3        -1
     1.000     0.000     0.000     0.000    -0.000     0.000     0.000     0.000
-0.000    15.000  = -p1_15
    -0.900     0.250    -0.500    -0.100    -0.200    -0.100     1.600     0.400
-0.400   -14.750  = -p2_15
     0.400    -0.000    -0.000    -0.400     0.200    -0.400    -1.600    -0.400
0.400     1.000  = -p9_15
     0.100     0.000     0.500     0.900    -0.200    -0.100    -0.400    -0.100
0.100    11.500  = -p4_15
```

```
    -0.200    -0.000     0.000     0.200     0.400    -0.800     0.800     0.200
-0.200     2.000  = -p5_15
    -0.000     0.000     0.000     0.000    -0.000     1.000     0.000     0.000
-0.000    15.000  = -p6_15
    -0.800    -0.000     0.000     0.800    -0.400     0.800     3.200     0.800
0.200     3.000  = -p7_15
     1.600    -0.000    -0.000    -1.600     0.800    -1.600    -4.400    -1.600
-0.400     4.000    = -2x5
    -0.100    -0.000    -0.500    -0.900     0.200     0.100     0.400     0.100
-0.100     3.500     = -p4
    -0.100    -0.250     0.500     0.100     0.200     0.100    -1.600    -0.400
0.400    14.750    = -4x6
    -0.000     0.000     0.000     1.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000     = -p3
     0.800    -0.000    -0.000    -0.800     0.400    -0.800    -3.200    -0.800
-0.200    12.000     = -p7
    -0.400    -0.000     0.000     0.400    -0.200     0.400     1.600     0.400
-0.400    14.000     = -p9
     0.200    -0.000    -0.000    -0.200    -0.400     0.800    -0.800    -0.200
0.200    13.000     = -p5
     0.900    -0.250     0.500     0.100     0.200     0.100    -1.600    -0.400
0.400    29.750     = -p2
    -0.000     0.000     0.000     0.000    -0.000     0.000     1.000     0.000
-0.000    15.000     = -p8
     0.400    -0.250    -0.000    -0.400     0.200    -0.400    -2.600    -0.900
-0.100   102.250     = obj

     p9_15       1x2       2x6     p3_15       1x6        p6     p8_15       1x3
3x3        -1
    -2.500     0.000     0.000     1.000    -0.500     1.000     4.000     1.000
-1.000    12.500  = -p1_15
     2.250     0.250    -0.500    -1.000     0.250    -1.000    -2.000    -0.500
0.500   -12.500  = -p2_15
     2.500    -0.000    -0.000    -1.000     0.500    -1.000    -4.000    -1.000
1.000     2.500     = -p1
    -0.250     0.000     0.500     1.000    -0.250     0.000    -0.000     0.000
-0.000    11.250  = -p4_15
     0.500    -0.000     0.000     0.000     0.500    -1.000     0.000     0.000
0.000     2.500  = -p5_15
     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000
0.000    15.000  = -p6_15
     2.000    -0.000     0.000     0.000     0.000     0.000     0.000     0.000
1.000     5.000  = -p7_15
    -4.000     0.000     0.000     0.000     0.000     0.000     2.000     0.000
-2.000     0.000    = -2x5
     0.250    -0.000    -0.500    -1.000     0.250     0.000     0.000     0.000
0.000     3.750     = -p4
     0.250    -0.250     0.500    -0.000     0.250     0.000    -2.000    -0.500
```

```
0.500    15.000    = -4x6
     0.000     0.000     0.000     1.000     0.000    -0.000    -0.000    -0.000
0.000    15.000     = -p3
    -2.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
-1.000    10.000     = -p7
     1.000    -0.000     0.000     0.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p9
    -0.500     0.000     0.000     0.000    -0.500     1.000     0.000     0.000
0.000    12.500     = -p5
    -2.250    -0.250     0.500     1.000    -0.250     1.000     2.000     0.500
-0.500    27.500     = -p2
     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
0.000    15.000     = -p8
    -1.000    -0.250     0.000     0.000     0.000     0.000    -1.000    -0.500
-0.500   101.250     = obj

     p9_15       1x2       2x6     p3_15       1x6        p6       2x5       1x3
3x3        -1
     5.500     0.000     0.000     1.000    -0.500     1.000    -2.000     1.000
3.000    12.500  = -p1_15
    -1.750     0.250    -0.500    -1.000     0.250    -1.000     1.000    -0.500
-1.500   -12.500  = -p2_15
    -5.500     0.000     0.000    -1.000     0.500    -1.000     2.000    -1.000
-3.000     2.500     = -p1
    -0.250     0.000     0.500     1.000    -0.250     0.000     0.000     0.000
-0.000    11.250  = -p4_15
     0.500    -0.000     0.000     0.000     0.500    -1.000    -0.000     0.000
0.000     2.500  = -p5_15
     0.000     0.000     0.000     0.000     0.000     1.000    -0.000     0.000
0.000    15.000  = -p6_15
     2.000    -0.000     0.000     0.000     0.000     0.000    -0.000     0.000
1.000     5.000  = -p7_15
    -2.000     0.000     0.000     0.000     0.000     0.000     0.500     0.000
-1.000     0.000  = -p8_15
     0.250    -0.000    -0.500    -1.000     0.250     0.000    -0.000     0.000
0.000     3.750     = -p4
    -3.750    -0.250     0.500    -0.000     0.250     0.000     1.000    -0.500
-1.500    15.000    = -4x6
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p3
    -2.000     0.000     0.000     0.000     0.000     0.000    -0.000     0.000
-1.000    10.000     = -p7
     1.000    -0.000     0.000     0.000     0.000     0.000    -0.000     0.000
0.000    15.000     = -p9
    -0.500     0.000     0.000     0.000    -0.500     1.000    -0.000     0.000
0.000    12.500     = -p5
     1.750    -0.250     0.500     1.000    -0.250     1.000    -1.000     0.500
1.500    27.500     = -p2
```

```
     2.000     0.000     0.000     0.000    -0.000     0.000    -0.500     0.000
1.000    15.000     = -p8
    -3.000    -0.250     0.000     0.000     0.000     0.000     0.500    -0.500
-1.500   101.250     = obj

     p7_15       1x2       2x6     p3_15       1x6        p6       2x5       1x3
3x3        -1
    -2.750     0.000     0.000     1.000    -0.500     1.000    -2.000     1.000
0.250    -1.250  = -p1_15
     0.875     0.250    -0.500    -1.000     0.250    -1.000     1.000    -0.500
-0.625    -8.125  = -p2_15
     2.750     0.000     0.000    -1.000     0.500    -1.000     2.000    -1.000
-0.250    16.250     = -p1
     0.125     0.000     0.500     1.000    -0.250     0.000     0.000     0.000
0.125    11.875  = -p4_15
    -0.250     0.000     0.000     0.000     0.500    -1.000     0.000     0.000
-0.250     1.250  = -p5_15
    -0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000
0.000    15.000  = -p6_15
     0.500    -0.000     0.000     0.000     0.000     0.000    -0.000     0.000
0.500     2.500  = -p9_15
     1.000     0.000     0.000     0.000     0.000     0.000     0.500     0.000
0.000     5.000  = -p8_15
    -0.125     0.000    -0.500    -1.000     0.250     0.000    -0.000     0.000
-0.125     3.125     = -p4
     1.875    -0.250     0.500    -0.000     0.250     0.000     1.000    -0.500
0.375    24.375    = -4x6
    -0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p3
     1.000     0.000     0.000     0.000     0.000     0.000    -0.000     0.000
0.000    15.000     = -p7
    -0.500     0.000     0.000     0.000     0.000     0.000     0.000     0.000
-0.500    12.500     = -p9
     0.250     0.000     0.000     0.000    -0.500     1.000    -0.000     0.000
0.250    13.750     = -p5
    -0.875    -0.250     0.500     1.000    -0.250     1.000    -1.000     0.500
0.625    23.125     = -p2
    -1.000     0.000     0.000     0.000    -0.000     0.000    -0.500     0.000
0.000    10.000     = -p8
     1.500    -0.250     0.000     0.000     0.000     0.000     0.500    -0.500
0.000   108.750     = obj

     p7_15       1x2       2x6     p3_15       1x6        p6       2x5       1x3
p9_15        -1
    -3.000     0.000     0.000     1.000    -0.500     1.000    -2.000     1.000
-0.500    -2.500  = -p1_15
     1.500     0.250    -0.500    -1.000     0.250    -1.000     1.000    -0.500
1.250    -5.000  = -p2_15
```

```
     3.000     0.000     0.000    -1.000     0.500    -1.000     2.000    -1.000
0.500    17.500     = -p1
     0.000     0.000     0.500     1.000    -0.250     0.000     0.000     0.000
-0.250    11.250  = -p4_15
     0.000     0.000     0.000     0.000     0.500    -1.000     0.000     0.000
0.500     2.500  = -p5_15
    -0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000
-0.000    15.000  = -p6_15
     1.000    -0.000     0.000     0.000     0.000     0.000    -0.000     0.000
2.000     5.000    = -3x3
     1.000     0.000     0.000     0.000     0.000     0.000     0.500     0.000
-0.000     5.000  = -p8_15
    -0.000     0.000    -0.500    -1.000     0.250     0.000    -0.000     0.000
0.250     3.750     = -p4
     1.500    -0.250     0.500    -0.000     0.250     0.000     1.000    -0.500
-0.750    22.500    = -4x6
    -0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
-0.000    15.000     = -p3
     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
-0.000    15.000     = -p7
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p9
     0.000     0.000     0.000     0.000    -0.500     1.000     0.000     0.000
-0.500    12.500     = -p5
    -1.500    -0.250     0.500     1.000    -0.250     1.000    -1.000     0.500
-1.250    20.000     = -p2
    -1.000     0.000     0.000     0.000    -0.000     0.000    -0.500     0.000
-0.000    10.000     = -p8
     1.500    -0.250     0.000     0.000     0.000     0.000     0.500    -0.500
-0.000   108.750     = obj

     p7_15       1x2       2x6     p2_15       1x6        p6       2x5       1x3
p9_15        -1
    -1.500     0.250    -0.500     1.000    -0.250    -0.000    -1.000     0.500
0.750    -7.500  = -p1_15
    -1.500    -0.250     0.500    -1.000    -0.250     1.000    -1.000     0.500
-1.250     5.000  = -p3_15
     1.500    -0.250     0.500    -1.000     0.250     0.000     1.000    -0.500
-0.750    22.500     = -p1
     1.500     0.250    -0.000     1.000     0.000    -1.000     1.000    -0.500
1.000     6.250  = -p4_15
     0.000     0.000    -0.000     0.000     0.500    -1.000     0.000    -0.000
0.500     2.500  = -p5_15
    -0.000     0.000    -0.000     0.000     0.000     1.000     0.000    -0.000
-0.000    15.000  = -p6_15
     1.000    -0.000    -0.000     0.000     0.000    -0.000    -0.000    -0.000
2.000     5.000    = -3x3
     1.000     0.000    -0.000     0.000     0.000    -0.000     0.500    -0.000
```

```
-0.000     5.000  = -p8_15
    -1.500    -0.250     0.000    -1.000    -0.000     1.000    -1.000     0.500
-1.000     8.750     = -p4
     1.500    -0.250     0.500    -0.000     0.250     0.000     1.000    -0.500
-0.750    22.500    = -4x6
     1.500     0.250    -0.500     1.000     0.250    -1.000     1.000    -0.500
1.250    10.000     = -p3
     1.000     0.000    -0.000     0.000     0.000    -0.000     0.000    -0.000
-0.000    15.000     = -p7
     0.000     0.000    -0.000     0.000     0.000    -0.000     0.000    -0.000
1.000    15.000     = -p9
     0.000     0.000    -0.000     0.000    -0.500     1.000     0.000    -0.000
-0.500    12.500     = -p5
    -0.000    -0.000    -0.000     1.000    -0.000    -0.000    -0.000    -0.000
-0.000    15.000     = -p2
    -1.000     0.000    -0.000     0.000    -0.000    -0.000    -0.500    -0.000
-0.000    10.000     = -p8
     1.500    -0.250    -0.000     0.000     0.000    -0.000     0.500    -0.500
-0.000   108.750     = obj

     p4_15       1x2       2x6     p2_15       1x6        p6       2x5        1x3
p9_15        -1
     1.000     0.500    -0.500     2.000    -0.250    -1.000    -0.000     0.000
1.750    -1.250  = -p1_15
     1.000    -0.000     0.500    -0.000    -0.250     0.000    -0.000     0.000
-0.250    11.250  = -p3_15
    -1.000    -0.500     0.500    -2.000     0.250     1.000     0.000    -0.000
-1.750    16.250     = -p1
     0.667     0.167    -0.000     0.667     0.000    -0.667     0.667    -0.333
0.667     4.167  = -p7_15
    -0.000     0.000     0.000     0.000     0.500    -1.000     0.000     0.000
0.500     2.500  = -p5_15
     0.000     0.000    -0.000     0.000     0.000     1.000     0.000    -0.000
0.000    15.000  = -p6_15
    -0.667    -0.167     0.000    -0.667    -0.000     0.667    -0.667     0.333
1.333     0.833    = -3x3
    -0.667    -0.167     0.000    -0.667    -0.000     0.667    -0.167     0.333
-0.667     0.833  = -p8_15
     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p4
    -1.000    -0.500     0.500    -1.000     0.250     1.000     0.000    -0.000
-1.750    16.250    = -4x6
    -1.000     0.000    -0.500     0.000     0.250    -0.000     0.000    -0.000
0.250     3.750     = -p3
    -0.667    -0.167     0.000    -0.667    -0.000     0.667    -0.667     0.333
-0.667    10.833     = -p7
    -0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p9
```

```
    -0.000     0.000     0.000     0.000    -0.500     1.000     0.000     0.000
-0.500    12.500     = -p5
     0.000     0.000    -0.000     1.000     0.000    -0.000     0.000    -0.000
0.000    15.000     = -p2
     0.667     0.167    -0.000     0.667     0.000    -0.667     0.167    -0.333
0.667    14.167     = -p8
    -1.000    -0.500     0.000    -1.000    -0.000     1.000    -0.500    -0.000
-1.000   102.500     = obj

     p4_15       1x2       2x6     p2_15       1x6       3x3       2x5       1x3
p9_15        -1
    -0.000     0.250    -0.500     1.000    -0.250     1.500    -1.000     0.500
3.750    -0.000  = -p1_15
     1.000     0.000     0.500     0.000    -0.250    -0.000     0.000    -0.000
-0.250    11.250  = -p3_15
     0.000    -0.250     0.500    -1.000     0.250    -1.500     1.000    -0.500
-3.750    15.000     = -p1
     0.000     0.000     0.000     0.000    -0.000     1.000     0.000     0.000
2.000     5.000  = -p7_15
    -1.000    -0.250     0.000    -1.000     0.500     1.500    -1.000     0.500
2.500     3.750  = -p5_15
     1.000     0.250    -0.000     1.000     0.000    -1.500     1.000    -0.500
-2.000    13.750  = -p6_15
    -1.000    -0.250     0.000    -1.000    -0.000     1.500    -1.000     0.500
2.000     1.250     = -p6
     0.000     0.000     0.000     0.000     0.000    -1.000     0.500     0.000
-2.000     0.000  = -p8_15
     1.000     0.000     0.000     0.000     0.000    -0.000     0.000     0.000
0.000    15.000     = -p4
     0.000    -0.250     0.500     0.000     0.250    -1.500     1.000    -0.500
-3.750    15.000    = -4x6
    -1.000     0.000    -0.500     0.000     0.250     0.000    -0.000     0.000
0.250     3.750     = -p3
     0.000     0.000    -0.000     0.000     0.000    -1.000     0.000    -0.000
-2.000    10.000     = -p7
     0.000     0.000     0.000     0.000     0.000    -0.000     0.000     0.000
1.000    15.000     = -p9
     1.000     0.250    -0.000     1.000    -0.500    -1.500     1.000    -0.500
-2.500    11.250     = -p5
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p2
     0.000     0.000     0.000     0.000    -0.000     1.000    -0.500     0.000
2.000    15.000     = -p8
     0.000    -0.250    -0.000     0.000     0.000    -1.500     0.500    -0.500
-3.000   101.250     = obj
```

At long last we have a basic feasible tableau, running simplexBF gives:

[31]:
```
SimplexBF(b17,indep_names,dep_names)
```

```
     p4_15       1x2       2x6     p2_15       1x6       3x3       2x5       1x3
p9_15        -1
    -0.000     0.250    -0.500     1.000    -0.250     1.500    -1.000     0.500
3.750    -0.000  = -p1_15
     1.000     0.000     0.500     0.000    -0.250    -0.000     0.000    -0.000
-0.250    11.250  = -p3_15
     0.000    -0.250     0.500    -1.000     0.250    -1.500     1.000    -0.500
-3.750    15.000     = -p1
     0.000     0.000     0.000     0.000    -0.000     1.000     0.000     0.000
2.000     5.000  = -p7_15
    -1.000    -0.250     0.000    -1.000     0.500     1.500    -1.000     0.500
2.500     3.750  = -p5_15
     1.000     0.250    -0.000     1.000     0.000    -1.500     1.000    -0.500
-2.000    13.750  = -p6_15
    -1.000    -0.250     0.000    -1.000    -0.000     1.500    -1.000     0.500
2.000     1.250     = -p6
     0.000     0.000     0.000     0.000     0.000    -1.000     0.500     0.000
-2.000     0.000  = -p8_15
     1.000     0.000     0.000     0.000     0.000    -0.000     0.000     0.000
0.000    15.000     = -p4
     0.000    -0.250     0.500     0.000     0.250    -1.500     1.000    -0.500
-3.750    15.000    = -4x6
    -1.000     0.000    -0.500     0.000     0.250     0.000    -0.000     0.000
0.250     3.750     = -p3
     0.000     0.000    -0.000     0.000     0.000    -1.000     0.000    -0.000
-2.000    10.000     = -p7
     0.000     0.000     0.000     0.000     0.000    -0.000     0.000     0.000
1.000    15.000     = -p9
     1.000     0.250    -0.000     1.000    -0.500    -1.500     1.000    -0.500
-2.500    11.250     = -p5
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p2
     0.000     0.000     0.000     0.000    -0.000     1.000    -0.500     0.000
2.000    15.000     = -p8
     0.000    -0.250    -0.000     0.000     0.000    -1.500     0.500    -0.500
-3.000   101.250     = obj

     p4_15       1x2       2x6     p2_15       1x6       3x3     p8_15       1x3
p9_15        -1
    -0.000     0.250    -0.500     1.000    -0.250    -0.500     2.000     0.500
-0.250    -0.000  = -p1_15
     1.000     0.000     0.500     0.000    -0.250    -0.000    -0.000    -0.000
-0.250    11.250  = -p3_15
     0.000    -0.250     0.500    -1.000     0.250     0.500    -2.000    -0.500
0.250    15.000     = -p1
```

```
     0.000     0.000     0.000     0.000    -0.000     1.000    -0.000     0.000
2.000     5.000  = -p7_15
    -1.000    -0.250     0.000    -1.000     0.500    -0.500     2.000     0.500
-1.500     3.750  = -p5_15
     1.000     0.250    -0.000     1.000     0.000     0.500    -2.000    -0.500
2.000    13.750  = -p6_15
    -1.000    -0.250     0.000    -1.000    -0.000    -0.500     2.000     0.500
-2.000     1.250     = -p6
     0.000     0.000     0.000     0.000     0.000    -2.000     2.000     0.000
-4.000     0.000    = -2x5
     1.000     0.000     0.000     0.000     0.000     0.000    -0.000     0.000
0.000    15.000     = -p4
     0.000    -0.250     0.500     0.000     0.250     0.500    -2.000    -0.500
0.250    15.000    = -4x6
    -1.000     0.000    -0.500     0.000     0.250     0.000     0.000     0.000
0.250     3.750     = -p3
     0.000     0.000    -0.000     0.000     0.000    -1.000    -0.000    -0.000
-2.000    10.000     = -p7
     0.000     0.000     0.000     0.000     0.000     0.000    -0.000     0.000
1.000    15.000     = -p9
     1.000     0.250    -0.000     1.000    -0.500     0.500    -2.000    -0.500
1.500    11.250     = -p5
     0.000     0.000     0.000     1.000     0.000     0.000    -0.000     0.000
0.000    15.000     = -p2
     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
0.000    15.000     = -p8
     0.000    -0.250    -0.000     0.000     0.000    -0.500    -1.000    -0.500
-1.000   101.250     = obj

     p4_15       1x2       2x6     p2_15       1x6       3x3     p8_15       1x3
p9_15        -1
    -0.000     0.250    -0.500     1.000    -0.250    -0.500     2.000     0.500
-0.250    -0.000  = -p1_15
     1.000     0.000     0.500     0.000    -0.250    -0.000    -0.000    -0.000
-0.250    11.250  = -p3_15
     0.000    -0.250     0.500    -1.000     0.250     0.500    -2.000    -0.500
0.250    15.000     = -p1
     0.000     0.000     0.000     0.000    -0.000     1.000    -0.000     0.000
2.000     5.000  = -p7_15
    -1.000    -0.250     0.000    -1.000     0.500    -0.500     2.000     0.500
-1.500     3.750  = -p5_15
     1.000     0.250    -0.000     1.000     0.000     0.500    -2.000    -0.500
2.000    13.750  = -p6_15
    -1.000    -0.250     0.000    -1.000    -0.000    -0.500     2.000     0.500
-2.000     1.250     = -p6
     0.000     0.000     0.000     0.000     0.000    -2.000     2.000     0.000
-4.000     0.000    = -2x5
     1.000     0.000     0.000     0.000     0.000     0.000    -0.000     0.000
```

```
0.000    15.000     = -p4
     0.000    -0.250     0.500     0.000     0.250     0.500    -2.000    -0.500
0.250    15.000    = -4x6
    -1.000     0.000    -0.500     0.000     0.250     0.000     0.000     0.000
0.250     3.750     = -p3
     0.000     0.000    -0.000     0.000     0.000    -1.000    -0.000    -0.000
-2.000    10.000     = -p7
     0.000     0.000     0.000     0.000     0.000     0.000    -0.000     0.000
1.000    15.000     = -p9
     1.000     0.250    -0.000     1.000    -0.500     0.500    -2.000    -0.500
1.500    11.250     = -p5
     0.000     0.000     0.000     1.000     0.000     0.000    -0.000     0.000
0.000    15.000     = -p2
     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
0.000    15.000     = -p8
     0.000    -0.250    -0.000     0.000     0.000    -0.500    -1.000    -0.500
-1.000   101.250     = obj

     p4_15       1x2       2x6     p2_15       2x5       3x3     p8_15       1x3
p9_15        -1
    -0.000     0.250    -0.500
1.000180143985094819 8.750-3602879701896396.500360287970189639 7.500
0.500-7205759403792795.000    -0.000  = -p1_15
     1.000     0.000     0.500
0.000180143985094819 8.500-3602879701896395.000360287970189639 5.000
-0.000-7205759403792794.000    11.250  = -p3_15
     0.000    -0.250     0.500
-1.000-180143985094819 8.750360287970189639 6.500-3602879701896397.500
-0.500720575940379279 5.000    15.000     = -p1
     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000
2.000     5.000  = -p7_15
    -1.000    -0.250     0.000
-1.000-3602879701896396.000720575940379278 8.000-7205759403792787.000
0.500144115188075855 82.000     3.750  = -p5_15
     1.000     0.250    -0.000     1.000    -0.200     0.900    -2.400    -0.500
2.800    13.750  = -p6_15
    -1.000    -0.250     0.000    -1.000     0.200    -0.900     2.400     0.500
-2.800     1.250     = -p6
     0.000     0.000     0.000
0.000720575940379279 4.000-14411518807585580.000144115188075855 80.000
0.000-28823037615171176.000     0.000    = -1x6
     1.000     0.000     0.000     0.000    -0.000     0.000    -0.000     0.000
0.000    15.000     = -p4
     0.000    -0.250     0.500
0.000-180143985094819 8.750360287970189639 6.500-3602879701896398.500
-0.500720575940379279 5.000    15.000    = -4x6
    -1.000     0.000    -0.500
0.000-180143985094819 8.500360287970189639 5.000-3602879701896395.000
```

```
0.0007205759403792794.000     3.750     = -p3
     0.000     0.000    -0.000     0.000    -0.000    -1.000    -0.000    -0.000
-2.000    10.000     = -p7
     0.000     0.000     0.000     0.000    -0.000     0.000    -0.000     0.000
1.000    15.000     = -p9
     1.000     0.250    -0.000
1.0003602879701896396.000-7205759403792788.0007205759403792787.000
-0.500-14411518807585582.000    11.250     = -p5
     0.000     0.000     0.000     1.000    -0.000     0.000    -0.000     0.000
0.000    15.000     = -p2
     0.000     0.000     0.000     0.000    -0.000     0.000     1.000     0.000
0.000    15.000     = -p8
     0.000    -0.250    -0.000     0.000    -0.250     0.000    -1.500    -0.500
0.000   101.250     = obj

     p4_15       1x2       2x6     p2_15       2x5       3x3     p8_15       1x3
p9_15        -1
    -0.000     0.250    -0.500
1.0001801439850948198.750-3602879701896396.5003602879701896397.500
0.500-7205759403792795.000    -0.000  = -p1_15
     1.000     0.000     0.500
0.0001801439850948198.500-3602879701896395.0003602879701896395.000
-0.000-7205759403792794.000    11.250  = -p3_15
     0.000    -0.250     0.500
-1.000-1801439850948198.7503602879701896396.500-3602879701896397.500
-0.5007205759403792795.000    15.000     = -p1
     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000
2.000     5.000  = -p7_15
    -1.000    -0.250     0.000
-1.000-3602879701896396.0007205759403792788.000-7205759403792787.000
0.50014411518807585582.000     3.750  = -p5_15
     1.000     0.250    -0.000     1.000    -0.200     0.900    -2.400    -0.500
2.800    13.750  = -p6_15
    -1.000    -0.250     0.000    -1.000     0.200    -0.900     2.400     0.500
-2.800     1.250     = -p6
     0.000     0.000     0.000
0.0007205759403792794.000-14411518807585580.0001441151880758558O.000
0.000-28823037615171176.000     0.000    = -1x6
     1.000     0.000     0.000     0.000    -0.000     0.000    -0.000     0.000
0.000    15.000     = -p4
     0.000    -0.250     0.500
0.000-1801439850948198.7503602879701896396.500-3602879701896398.500
-0.5007205759403792795.000    15.000    = -4x6
    -1.000     0.000    -0.500
0.000-1801439850948198.5003602879701896395.000-3602879701896395.000
0.0007205759403792794.000     3.750     = -p3
     0.000     0.000    -0.000     0.000    -0.000    -1.000    -0.000    -0.000
-2.000    10.000     = -p7
```

```
     0.000     0.000     0.000     0.000    -0.000     0.000    -0.000     0.000
1.000    15.000     = -p9
     1.000     0.250    -0.000
1.0003602879701896396.000-7205759403792788.0007205759403792787.000
-0.500-14411518807585582.000    11.250     = -p5
     0.000     0.000     0.000     1.000    -0.000     0.000    -0.000     0.000
0.000    15.000     = -p2
     0.000     0.000     0.000     0.000    -0.000     0.000     1.000     0.000
0.000    15.000     = -p8
     0.000    -0.250    -0.000     0.000    -0.250     0.000    -1.500    -0.500
0.000   101.250     = obj

     p4_15       1x2       2x6     p2_15       2x5       3x3     p8_15       1x3
p5_15        -1
    -0.500     0.125    -0.500     0.500    -0.313     0.000     1.875     0.750
0.500     1.875  = -p1_15
     0.500    -0.125     0.500    -0.500    -0.313     0.625     0.000     0.250
0.500    13.125  = -p3_15
     0.500    -0.125     0.500    -0.500     0.313     0.000    -1.875    -0.750
-0.500    13.125     = -p1
     0.000     0.000    -0.000     0.000     0.500     0.000     1.000     0.000
-0.000     5.000  = -p7_15
    -0.000    -0.000     0.000    -0.000    -0.250     0.500    -0.500     0.000
0.000     0.000  = -p9_15
     1.000     0.250    -0.000     1.000     0.500    -0.500    -1.000    -0.500
-0.000    13.750  = -p6_15
    -1.000    -0.250     0.000    -1.000    -0.500     0.500     1.000     0.500
0.000     1.250     = -p6
    -2.000    -0.500     0.000    -2.000    -1.250     2.500     0.000     1.000
2.000     7.500    = -1x6
     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
-0.000    15.000     = -p4
     0.500    -0.125     0.500     0.500     0.313     0.000    -3.125    -0.750
-0.500    13.125    = -4x6
    -0.500     0.125    -0.500     0.500     0.313    -0.625     0.000    -0.250
-0.500     1.875     = -p3
    -0.000    -0.000     0.000    -0.000    -0.500    -0.000    -1.000    -0.000
0.000    10.000     = -p7
     0.000     0.000    -0.000     0.000     0.250    -0.500     0.500    -0.000
-0.000    15.000     = -p9
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p5
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
-0.000    15.000     = -p2
     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
-0.000    15.000     = -p8
     0.000    -0.250    -0.000     0.000    -0.250    -0.000    -1.500    -0.500
-0.000   101.250     = obj
```

```
     p4_15       1x2       2x6     p2_15       2x5       3x3     p8_15       1x3
p5_15        -1
    -0.500     0.125    -0.500     0.500    -0.313     0.000     1.875     0.750
0.500     1.875  = -p1_15
     0.500    -0.125     0.500    -0.500    -0.313     0.625     0.000     0.250
0.500    13.125  = -p3_15
     0.500    -0.125     0.500    -0.500     0.313     0.000    -1.875    -0.750
-0.500    13.125     = -p1
     0.000     0.000    -0.000     0.000     0.500     0.000     1.000     0.000
-0.000     5.000  = -p7_15
    -0.000    -0.000     0.000    -0.000    -0.250     0.500    -0.500     0.000
0.000     0.000  = -p9_15
     1.000     0.250    -0.000     1.000     0.500    -0.500    -1.000    -0.500
-0.000    13.750  = -p6_15
    -1.000    -0.250     0.000    -1.000    -0.500     0.500     1.000     0.500
0.000     1.250     = -p6
    -2.000    -0.500     0.000    -2.000    -1.250     2.500     0.000     1.000
2.000     7.500    = -1x6
     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
-0.000    15.000     = -p4
     0.500    -0.125     0.500     0.500     0.313     0.000    -3.125    -0.750
-0.500    13.125    = -4x6
    -0.500     0.125    -0.500     0.500     0.313    -0.625     0.000    -0.250
-0.500     1.875     = -p3
    -0.000    -0.000     0.000    -0.000    -0.500    -0.000    -1.000    -0.000
0.000    10.000     = -p7
     0.000     0.000    -0.000     0.000     0.250    -0.500     0.500    -0.000
-0.000    15.000     = -p9
     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p5
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
-0.000    15.000     = -p2
     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
-0.000    15.000     = -p8
     0.000    -0.250    -0.000     0.000    -0.250    -0.000    -1.500    -0.500
-0.000   101.250     = obj

     p4_15       1x2       2x6     p1_15       2x5       3x3     p8_15       1x3
p5_15        -1
    -1.000     0.250    -1.000     2.000    -0.625     0.000     3.750     1.500
1.000     3.750  = -p2_15
    -0.000     0.000    -0.000     1.000    -0.625     0.625     1.875     1.000
1.000    15.000  = -p3_15
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p1
     0.000     0.000     0.000    -0.000     0.500     0.000     1.000    -0.000
-0.000     5.000  = -p7_15
```

```
    -0.000     0.000    -0.000     0.000    -0.250     0.500    -0.500     0.000
0.000     0.000  = -p9_15
     2.000     0.000     1.000    -2.000     1.125    -0.500    -4.750    -2.000
-1.000    10.000  = -p6_15
    -2.000     0.000    -1.000     2.000    -1.125     0.500     4.750     2.000
1.000     5.000     = -p6
    -4.000     0.000    -2.000     4.000    -2.500     2.500     7.500     4.000
4.000    15.000    = -1x6
     1.000     0.000     0.000    -0.000     0.000     0.000     0.000     0.000
-0.000    15.000     = -p4
     1.000    -0.250     1.000    -1.000     0.625     0.000    -5.000    -1.500
-1.000    11.250    = -4x6
     0.000     0.000     0.000    -1.000     0.625    -0.625    -1.875    -1.000
-1.000     0.000     = -p3
    -0.000     0.000    -0.000     0.000    -0.500    -0.000    -1.000     0.000
0.000    10.000     = -p7
     0.000     0.000     0.000    -0.000     0.250    -0.500     0.500    -0.000
-0.000    15.000     = -p9
     0.000     0.000     0.000    -0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p5
     1.000    -0.250     1.000    -2.000     0.625     0.000    -3.750    -1.500
-1.000    11.250     = -p2
     0.000     0.000     0.000    -0.000     0.000     0.000     1.000     0.000
-0.000    15.000     = -p8
     0.000    -0.250     0.000    -0.000    -0.250    -0.000    -1.500    -0.500
-0.000   101.250     = obj

     p4_15       1x2       2x6     p1_15       2x5       3x3     p8_15       1x3
p5_15        -1
    -1.000     0.250    -1.000     2.000    -0.625     0.000     3.750     1.500
1.000     3.750  = -p2_15
    -0.000     0.000    -0.000     1.000    -0.625     0.625     1.875     1.000
1.000    15.000  = -p3_15
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p1
     0.000     0.000     0.000    -0.000     0.500     0.000     1.000    -0.000
-0.000     5.000  = -p7_15
    -0.000     0.000    -0.000     0.000    -0.250     0.500    -0.500     0.000
0.000     0.000  = -p9_15
     2.000     0.000     1.000    -2.000     1.125    -0.500    -4.750    -2.000
-1.000    10.000  = -p6_15
    -2.000     0.000    -1.000     2.000    -1.125     0.500     4.750     2.000
1.000     5.000     = -p6
    -4.000     0.000    -2.000     4.000    -2.500     2.500     7.500     4.000
4.000    15.000    = -1x6
     1.000     0.000     0.000    -0.000     0.000     0.000     0.000     0.000
-0.000    15.000     = -p4
     1.000    -0.250     1.000    -1.000     0.625     0.000    -5.000    -1.500
```

```
-1.000    11.250    = -4x6
     0.000     0.000     0.000    -1.000     0.625    -0.625    -1.875    -1.000
-1.000     0.000     = -p3
    -0.000     0.000    -0.000     0.000    -0.500    -0.000    -1.000     0.000
0.000    10.000     = -p7
     0.000     0.000     0.000    -0.000     0.250    -0.500     0.500    -0.000
-0.000    15.000     = -p9
     0.000     0.000     0.000    -0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p5
     1.000    -0.250     1.000    -2.000     0.625     0.000    -3.750    -1.500
-1.000    11.250     = -p2
     0.000     0.000     0.000    -0.000     0.000     0.000     1.000     0.000
-0.000    15.000     = -p8
     0.000    -0.250     0.000    -0.000    -0.250    -0.000    -1.500    -0.500
-0.000   101.250     = obj

        p3       1x2       2x6     p1_15       2x5       3x3     p8_15       1x3
p5_15       -1
1200959900632132.500     0.250    -0.400-1200959900632131.750750599937895082.875
-750599937895083.125-2251799813685248.500-1200959900632132.250-1200959900632133.
000     6.950  = -p2_15
     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
0.000    15.000  = -p3_15
    -0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p1
    -0.333     0.000    -0.000     0.333     0.292     0.208     1.625     0.333
0.333     5.000  = -p7_15
     0.167     0.000     0.000    -0.167    -0.146     0.396    -0.812    -0.167
-0.167     0.000  = -p9_15
-2401919801264264.000     0.000    -0.2002401919801264264.500-1501199875790165.5
00150119987579016 5.250450359962737049 7.000240191980126426 4.500240191980126426 6.0
00     3.600  = -p6_15
2401919801264264.000     0.000     0.200-2401919801264264.500150119987579016 5.50
0-1501199875790165.250-4503599627370497.000-2401919801264264.500-240191980126426
6.000    11.400     = -p6
4803839602528529.000     0.000     0.400-4803839602528530.000300239975158033 1.00
0-3002399751580329.500-9007199254740998.000-4803839602528530.000-480383960252853
0.000    27.800    = -1x6
-1200959900632131.500     0.000    -0.600120095990063213 3.000-750599937895082.87
5750599937895082.500225179981368525 0.000120095990063213 3.000120095990063213 .000
11.800     = -p4
-1200959900632132.500    -0.250     0.400120095990063213 3.000-750599937895082.87
5750599937895083.125225179981368524 7.000120095990063213 2.250120095990063213 .000
8.050    = -4x6
1200959900632131.500     0.000     0.600-1200959900632133.000750599937895082.875
-750599937895082.500-2251799813685250.000-1200959900632133.000-1200959900632133.
000     3.200  = -p4_15
     0.333     0.000     0.000    -0.333    -0.292    -0.208    -1.625    -0.333
```

```
-0.333    10.000     = -p7
    -0.167     0.000    -0.000     0.167     0.146    -0.396     0.812     0.167
0.167    15.000     = -p9
    -0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p5
-1200959900632132.500    -0.250     0.4001200959900632131.750-750599937895082.87
5750599937895083.1252251799813685248.5001200959900632132.2501200959900632133.000
8.050     = -p2
    -0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
0.000    15.000     = -p8
    -0.000    -0.250    -0.000     0.000    -0.250    -0.000    -1.500    -0.500
0.000   101.250     = obj

        p3       1x2       2x6     p1_15       2x5       3x3     p8_15       1x3
p5_15       -1
1200959900632132.500     0.250    -0.400-1200959900632131.750750599937895082.875
-750599937895083.125-2251799813685248.500-1200959900632132.250-1200959900632133.
000     6.950  = -p2_15
     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
0.000    15.000  = -p3_15
    -0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p1
    -0.333     0.000    -0.000     0.333     0.292     0.208     1.625     0.333
0.333     5.000  = -p7_15
     0.167     0.000     0.000    -0.167    -0.146     0.396    -0.812    -0.167
-0.167     0.000  = -p9_15
-2401919801264264.000     0.000    -0.2002401919801264264.500-1501199875790165.5
0015011998757901 65.2504503599627370497.0002401919801264264.5002401919801264266.0
00     3.600  = -p6_15
2401919801264264.000     0.000     0.200-2401919801264264.5001501199875790165.50
0-1501199875790165.250-4503599627370497.000-2401919801264264.500-240191980126426
6.000    11.400     = -p6
4803839602528529.000     0.000     0.400-4803839602528530.0003002399751580331.00
0-3002399751580329.500-9007199254740998.000-4803839602528530.000-480383960252853
0.000    27.800    = -1x6
-1200959900632131.500     0.000    -0.6001200959900632133.000-750599937895082.87
5750599937895082.5002251799813685250.0001200959900632133.0001200959900632133.000
11.800     = -p4
-1200959900632132.500    -0.250     0.4001200959900632133.000-750599937895082.87
5750599937895083.1252251799813685247.0001200959900632132.2501200959900632133.000
8.050    = -4x6
1200959900632131.500     0.000     0.600-1200959900632133.000750599937895082.875
-750599937895082.500-2251799813685250.000-1200959900632133.000-1200959900632133.
000     3.200  = -p4_15
     0.333     0.000     0.000    -0.333    -0.292    -0.208    -1.625    -0.333
-0.333    10.000     = -p7
    -0.167     0.000    -0.000     0.167     0.146    -0.396     0.812     0.167
0.167    15.000     = -p9
```

```
    -0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p5
-1200959900632132.500    -0.250     0.4001200959900632131.750-750599937895082.87
5750599937895083.1252251799813685248.5001200959900632132.2501200959900632133.000
8.050     = -p2
    -0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000
0.000    15.000     = -p8
    -0.000    -0.250    -0.000     0.000    -0.250    -0.000    -1.500    -0.500
0.000   101.250     = obj

        p3       1x2       2x6     p6_15       2x5       3x3     p8_15       1x3
p5_15        -1
     0.938     0.250    -0.500     0.500     0.352    -0.820    -0.938    -0.469
-0.469     8.750  = -p2_15
     1.000     0.000     0.000    -0.000     0.000     0.000     0.000     0.000
0.000    15.000  = -p3_15
     1.000     0.000     0.000    -0.000     0.625    -0.625    -1.875    -1.000
-1.000    15.000     = -p1
     0.000     0.000     0.000    -0.000     0.500     0.000     1.000     0.000
-0.000     5.000  = -p7_15
     0.000     0.000     0.000     0.000    -0.250     0.500    -0.500     0.000
0.000     0.000  = -p9_15
    -1.000     0.000    -0.000     0.000    -0.625     0.625     1.875     1.000
1.000     0.000  = -p1_15
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p6
     0.000     0.000     0.000     2.000    -0.938     1.875    -1.875     0.000
2.812    35.000    = -1x6
     1.172     0.000    -0.500    -0.500     0.352    -0.586     0.000     0.000
-0.703    10.000     = -p4
     0.234    -0.250     0.500    -0.500     0.352     0.117    -2.812    -0.703
-0.703     6.250    = -4x6
    -1.172     0.000     0.500     0.500    -0.352     0.586     0.000     0.000
0.703     5.000  = -p4_15
     0.000     0.000    -0.000     0.000    -0.500    -0.000    -1.000    -0.000
0.000    10.000     = -p7
     0.000     0.000    -0.000    -0.000     0.250    -0.500     0.500     0.000
-0.000    15.000     = -p9
     0.000     0.000     0.000    -0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p5
    -0.938    -0.250     0.500    -0.500    -0.352     0.820     0.938     0.469
0.469     6.250     = -p2
     0.000     0.000     0.000    -0.000     0.000     0.000     1.000     0.000
0.000    15.000     = -p8
     0.000    -0.250    -0.000    -0.000    -0.250    -0.000    -1.500    -0.500
-0.000   101.250     = obj

        p3       1x2       2x6     p6_15       2x5       3x3     p8_15       1x3
```

```
p5_15        -1
     0.938     0.250    -0.500     0.500     0.352    -0.820    -0.938    -0.469
-0.469     8.750  = -p2_15
     1.000     0.000     0.000    -0.000     0.000     0.000     0.000     0.000
0.000    15.000  = -p3_15
     1.000     0.000     0.000    -0.000     0.625    -0.625    -1.875    -1.000
-1.000    15.000     = -p1
     0.000     0.000     0.000    -0.000     0.500     0.000     1.000     0.000
-0.000     5.000  = -p7_15
     0.000     0.000     0.000     0.000    -0.250     0.500    -0.500     0.000
0.000     0.000  = -p9_15
    -1.000     0.000    -0.000     0.000    -0.625     0.625     1.875     1.000
1.000     0.000  = -p1_15
     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000
0.000    15.000     = -p6
     0.000     0.000     0.000     2.000    -0.938     1.875    -1.875     0.000
2.812    35.000    = -1x6
     1.172     0.000    -0.500    -0.500     0.352    -0.586     0.000     0.000
-0.703    10.000     = -p4
     0.234    -0.250     0.500    -0.500     0.352     0.117    -2.812    -0.703
-0.703     6.250    = -4x6
    -1.172     0.000     0.500     0.500    -0.352     0.586     0.000     0.000
0.703     5.000  = -p4_15
     0.000     0.000    -0.000     0.000    -0.500    -0.000    -1.000    -0.000
0.000    10.000     = -p7
     0.000     0.000    -0.000    -0.000     0.250    -0.500     0.500     0.000
-0.000    15.000     = -p9
     0.000     0.000     0.000    -0.000     0.000     0.000     0.000     0.000
1.000    15.000     = -p5
    -0.938    -0.250     0.500    -0.500    -0.352     0.820     0.938     0.469
0.469     6.250     = -p2
     0.000     0.000     0.000    -0.000     0.000     0.000     1.000     0.000
0.000    15.000     = -p8
     0.000    -0.250    -0.000    -0.000    -0.250    -0.000    -1.500    -0.500
-0.000   101.250     = obj
```

Reading off the solution, we have that we produce the following setup:

$$p_1 = 15 \tag{45}$$
$$p_2 = 6.25 \tag{46}$$
$$p_3 = 0 \tag{47}$$
$$p_4 = 10 \tag{48}$$
$$p_5 = 15 \tag{49}$$
$$p_6 = 15 \tag{50}$$
$$p_7 = 10 \tag{51}$$
$$p_8 = 15 \tag{52}$$
$$p_1 = 15 \tag{53}$$
$$\tag{54}$$

Producing a total of 101.25 sheets of paper while satisfying the necessary constraints.