Math/CS/Econ 210 Spring 2021 —HW # 2 Assignment Due Thu. Feb. 25, 5 PM

Please submit by uploading a single PDF document to the Moodle link for HW2.

Reading: Strayer Sections 2.1–2.4 (see also the lecture slides/recordings for Fri 2/19 through Wed 2/24)

Reminder of HW collaboration rules (from syllabus): I encourage you to discuss homework problems with other students in the class and/or with me. What you turn in should reflect your personal understanding of the problems, so you must write the solutions yourself without referring to notes from your collaborative work. For coding problems (there are none on this HW), this means you may not cut-and-paste (or anything of that nature) except from sample code that I provide or from your own previous work.

1: Here's a variant of the Bakery Problem that only needs one "variable swap" to get to the solution:

$$\begin{aligned} \text{Maximize } 30x_1 + 12x_2 & & \text{(sales revenue)} \\ \text{subject to } 10x_1 + 5x_2 &\le 150 & \text{(flour constraint)} \\ 4x_1 + 3x_2 &\le 80 & \text{(sugar constraint)} \\ x_1, x_2 &\ge 0 \end{aligned}$$

Following the approach taken in this past Friday's lab:

(a) Reformulate this problem using slack variables.

(b) Solve the first constraint for x_1 and use it to rewrite the problem to have the objective function expressed in terms of t_1 and x_2, and the constraints having t_1, x_2 on the left-hand sides. Use the equation notation we used in Friday's lab, not Tucker tableaus.

(c) Based on your result in (b), determine the solution: what is the maximum value of the objective function? What are the values of x_1, x_2, t_1, t_2 where that maximum occurs?

(d) You should find that one t_j is zero and the other is not. Interpret the values of t_1 and t_2 by writing a sentence that starts "For the solution that maximizes sales, the bakery. . . " and then contains either "flour" or "sugar".

2: Consider the Tucker tableau below (it's for our standard Bakery Problem):

$$\begin{array}{|ccc|l} d & c & -1 & \\ \hline 10 & 5 & 150 & = -t_1 \\ 4 & 3 & 80 & = -t_2 \\ \hline 30 & 20 & 0 & = obj \\ \hline \end{array}$$

Compute the new table if you pivot on the (2,1) entry (where the 4 is located). Do the pivot by hand (just this once...) and show the details of your computation. (You are welcome to *check* your result using my pivot code.)

For the remaining problems, any time you need to compute a pivot, you are encouraged to use my Python code that you can download from Moodle. It should run either on `colab.research.google.com` or on our Jupyterhub.

3: Consider the problem to maximize $2x + 3y$ on the set defined by the constraints:

$$\begin{aligned} 2y &\geq x + 2 \\ y &\geq 7 - x \\ y &\leq 2x + 4 \\ 2y &\leq 18 - x \\ x, y &\geq 0 \end{aligned}$$

(a) Draw the constraint set. Extend the lines past the constraint set so that you can also see the "non-corners" (intersections that lie outside the constraint set).

(b) Express the problem in Canonical Max form.

(c) Write the problem as a Tucker tableau.

(d) Pivot on the (1,1) entry. **If** we were done at this point, such that setting the independent variables to zero was the solution, what solution would be predicted? This should be an intersection point in your picture for (a), perhaps a corner, or perhaps a non-corner.

(e) From your result in (d), pivot on the (4,2) entry. Now we **are** done. Based on the final tableau, what is the solution (maximum value and location where the max occurs).

You can think of this algorithm as creating a path from intersection-point to intersection-point, starting at the origin and ending at the solution (in this case, there is one intermediate intersection-point, which you found in (d)).

The remaining three problems reference an algorithm I call `SimplexBF`. You can find it spelled out: (1) on p. 42 of Section 2.5 of Strayer, or (2) in the in-class activity on Mon. Feb. 22, or (3) in the lecture slides for Wed. Feb. 24.

4. For each tableau below, if we target the 6 in the bottom row, which pivot should be chosen if we use the Candidate/Select rule of `SimplexBF`? (No need to compute any new tableaus; just say what the pivot should be in each case.)

$$\begin{array}{|ccc|l} x & y & -1 & \\ \hline 1 & 2 & 4 & = -t_1 \\ 2 & 1 & 5 & = -t_2 \\ 5 & 2 & 3 & = -t_3 \\ 3 & 2 & 3 & = -t_4 \\ \hline 6 & 2 & 0 & = obj \\ \hline \end{array} \qquad \begin{array}{|ccc|l} x & y & -1 & \\ \hline 1 & 2 & 4 & = -t_1 \\ 2 & 1 & 5 & = -t_2 \\ 5 & 2 & 3 & = -t_3 \\ 3 & 2 & 0 & = -t_4 \\ \hline 6 & 2 & 0 & = obj \\ \hline \end{array} \qquad \begin{array}{|ccc|l} x & y & -1 & \\ \hline 1 & 2 & 4 & = -t_1 \\ 2 & 1 & 5 & = -t_2 \\ -5 & 2 & 3 & = -t_3 \\ -3 & 2 & 3 & = -t_4 \\ \hline 6 & 2 & 0 & = obj \\ \hline \end{array}$$

5. Consider the following variant of the Bakery Problem with three variables and three constraints:

Maximize $15x_1 + 20x_2 + 12x_3$ subject to:

$$\begin{aligned} 2x_1 + 4x_2 + 3x_3 &\leq 200 \\ x_1 + 1.5x_2 + 2x_3 &\leq 150 \\ x_1 + 0.25x_2 + 0.5x_3 &\leq 50 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

The variables represent how many batches of cookies (x_1), cakes (x_2), or batches of brownies (x_3) we make. The three constraints represent our flour (in cups), sugar (in cups), and butter (in sticks) available. The objective function is the sales revenue (in dollars).

Use the `SimplexBF` algorithm to solve this problem. Show the result of each step, giving the tableau and what pivot you used.

6 (Strayer Ch. 2, # 9a, with some guidance): A nut company makes three different mixtures of nuts having the following compositions and profits per pound:

	Peanuts	Cashews	Pecans	Profit
Mixture 1	100%	0%	0%	\$ 2
Mixture 2	80%	15%	5%	\$ 1.50
Mixture 3	60%	30%	10%	\$ 1

The management of the company decides that it wants to produce at least twice as much of mixture 3 as of mixture 2 and at least twice as much of mixture 2 as of mixture 1. The company has 500 pounds of peanuts, 250 pounds of cashews, and 100 pounds of pecans available. If the company can sell everything that it produces, how many pounds of each mixture should be produced so as to maximize profits.

(a) Set this up as a canonical maximization problem and write down the corresponding tableau.

(b) Solve by the `SimplexBF` algorithm. Show each intermediate tableau and indicate each pivot used.

(c) How much of each mixture should the company produce, and what is the maximal profit?

(d) Which of the main constraints are "achieved" (that means the inequality is actually an equality)? You can answer this question without further computation by looking at the values of the slack variables.