Open in Colab

Math 210

Aaron Graybill

Problem Set 6

4/8/21

In [2]:
```
import numpy as np
from scipy.optimize import linprog
import pandas as pd
```

Problem 1.

a.

I convert the problem to a single source problem:

Edges: $\mathcal{E} = \{(0,1),(0,2),(0,4),(1,2),(1,3),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5),(4,6),(5,4),(5,6)\}$.

Fix the flow on $(0,1),(0,2),(0,4)$ to $30, 6$, and 4 respectively.

Let the currently unspecified constraint set be $\mathcal{C}$.

Maximize the flow into vertex 6, so $\max_{x_\mathcal{E} \in \mathcal{C}} \{x_{46} + x_{56}\}$

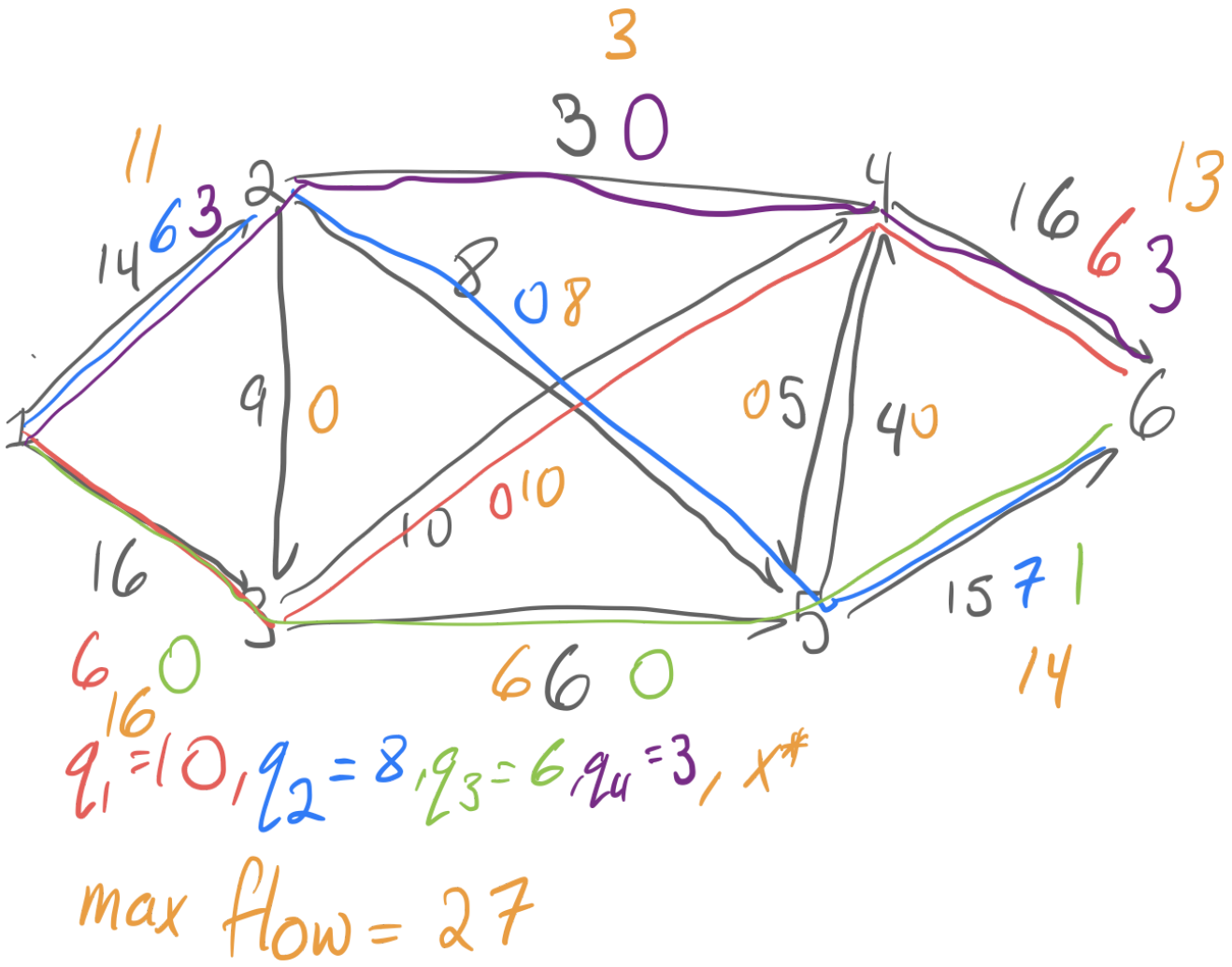
We have the following constraint set:

$$\begin{aligned} x_{12} + x_{13} &= x_{01} \\ x_{23} + x_{24} + x_{25} &= x_{02} + x_{12} \\ x_{34} + x_{35} &= x_{13} + x_{23} \\ x_{45} + x_{46} &= x_{24} + x_{34} + x_{54} \\ x_{54} + x_{56} &= x_{25} + x_{35} + x_{45} \end{aligned}$$

In [3]:
```
a_eq=np.array([
            [1,0,-1,-1,-1,0,0,0,0,0,0],
            [0,1,1,0,0,-1,-1,0,0,0,0],
            [0,0,0,1,0,1,0,-1,-1,1,0],
            [0,0,0,0,1,0,1,1,0,-1,-1]])
print(a_eq)

b_eq=np.zeros((4))
print(b_eq)
a_leq=np.identity(11)
b_leq=[14,16,9,3,8,10,6,5,16,4,15]

c=[0,0,0,0,0,0,0,0,-1,0,-1]

linprog(c,A_ub=a_leq,b_ub=b_leq,A_eq=a_eq,b_eq=b_eq,method="simplex")
```

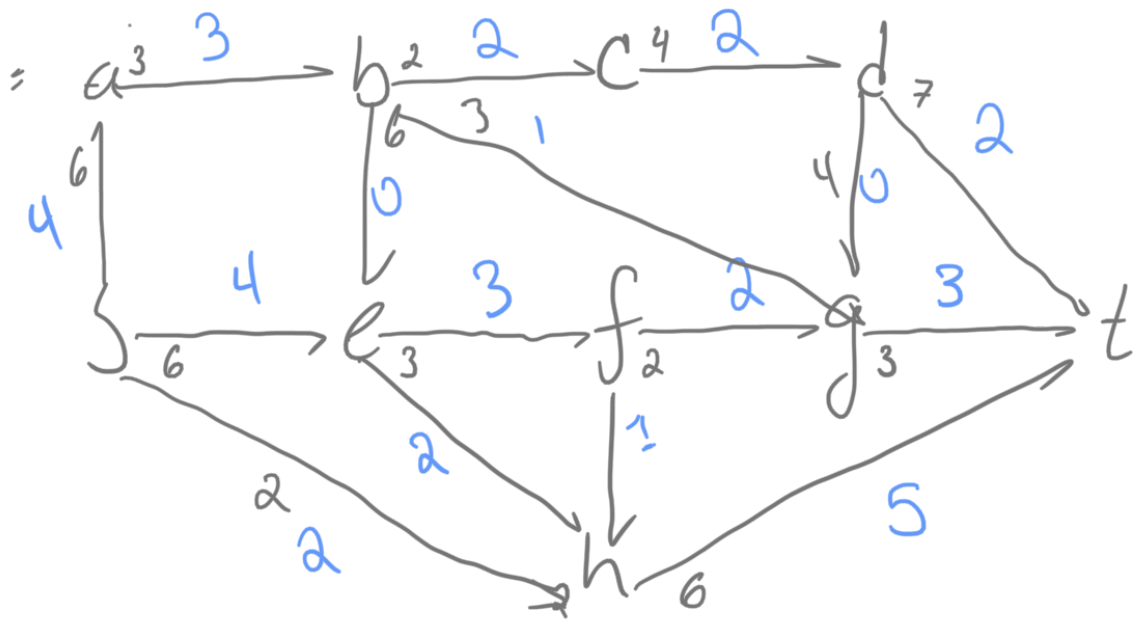
```
[[ 1  0 -1 -1 -1  0  0  0  0  0  0]
 [ 0  1  1  0  0 -1 -1  0  0  0  0]
 [ 0  0  0  1  0  1  0 -1 -1  1  0]
 [ 0  0  0  0  1  0  1  1  0 -1 -1]]
[0. 0. 0. 0.]
```

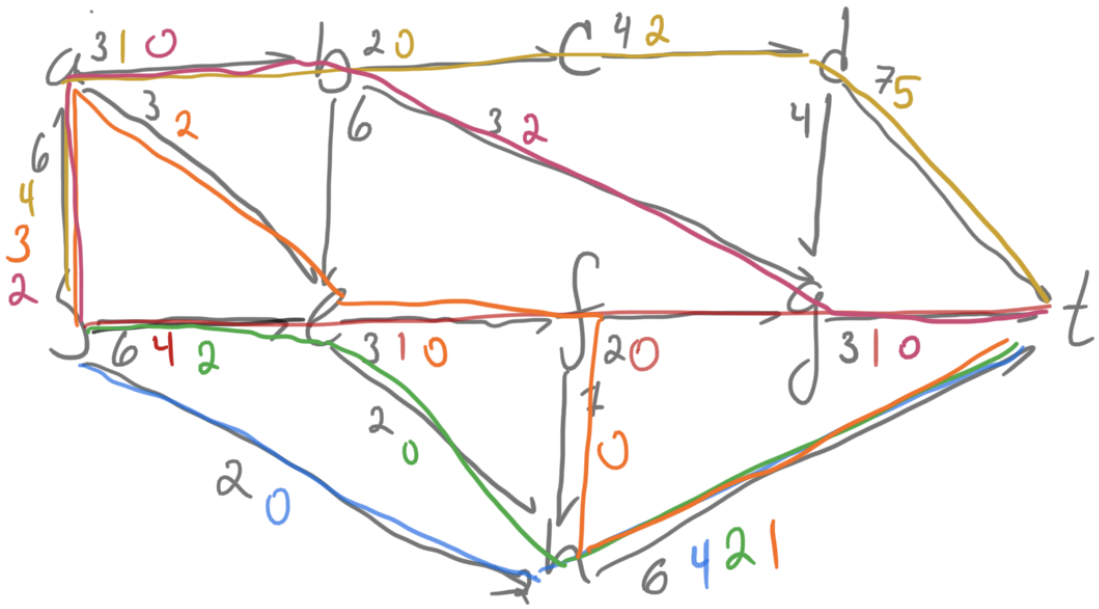
Out[3]:
```
     con: array([0., 0., 0., 0.])
     fun: -27.0
 message: 'Optimization terminated successfully.'
     nit: 15
   slack: array([3., 0., 9., 0., 0., 0., 0., 4., 0., 0., 4.])
  status: 0
 success: True
       x: array([11., 16.,  0.,  3.,  8., 10.,  6.,  1., 16.,  4., 11.])
```


3
30
11
14
63
2
4
16
63
13
8
08
9
0
05
40
6
10
010
16
3
5
157
1
60
660
14
16
$q_1=10, q_2=8, q_3=6, q_4=3, x^*$
max flow = 27

Problem 2.

The optimal flows are in the first chart and the process is in the second:





$$q_1 = 2, q_2 = 2, q_3 = 2, q_4 = 2, q_5 = 1, q_6 = 1$$

Problem 3.

In [4]:
```
#used to make label names:
  #source: geeksforgeeks.org/python-insert-the-string-at-the-beginning-of-all-items-in-a-list/
def prepend(list, str):

  # Using '% s'
  str += '% s'
  list =  [str % i for i in list]
  return(list)
```

In [5]:
```
import itertools

def create_eq_matrix(nvert,edgemat):

  A=np.zeros((nvert,(nvert**2)))  #too large will shrink later,
                                  #but makes for easy indexing
  c=np.zeros(nvert**2) # ditto

  #loop over the edgemat, taking each path as an iterand
  for count, value in enumerate(edgemat[:,0:2]):

    col_num=(nvert)*(value[0])+(value[1]) # n columns from each vertex

    if (value[0]==0): #coming from source
      #Don't have to worry about constraint on sources
      A[value[1],col_num]=1
    elif (value[1]==(nvert-1)): #going to sink
      #dont have to worry about constraint on sink
      A[value[0],col_num]=-1
      c[col_num]=-1#Hey this is an important column, but it in the objective
    else: #intermediate
      A[value[1],col_num]=1
      A[value[0],col_num]=-1

  A=np.append(A,np.array([c]),axis=0) #add the row of cs

  #Name appropriately
  col_names=[] #create vector of names
  for i in itertools.product(np.arange(nvert),repeat=2): #all combos of indexes
    col_names += [str(i[0])+","+str(i[1])]
  row_names=prepend(np.arange(nvert),"V")
  row_names+="c" #add one final row name

  A=pd.DataFrame(A,index=row_names,columns=col_names)# put things into pd data

  # Remove unused rows and columns
  A=A.loc[(A != 0).any(axis=1), (A != 0).any(axis=0)]

  #separate A and c again
  c=A.iloc[-1:,:]
  A=A.iloc[:-1,:]

  #create easy matrices and vectors
  b_eq=np.zeros(A.shape[0])
  A_leq=np.identity(edgemat.shape[0])
  b_leq=edgemat[:,2]

  #do optimization
  optimal=linprog(c,A_eq=A,A_ub=A_leq,b_eq=b_eq,b_ub=b_leq,method="simplex")
  flows=np.array(np.array([optimal.x]))

  #generateoutput
  summary=pd.DataFrame(flows,index=["flow"],columns=A.columns)
  max="The maximum is: " +str(-optimal.fun)

  #print output and return
  print(max)
  return (summary,A)
```

Problem 1 Confirmation

In [6]:
```
create_eq_matrix(6,edgemat=np.array([
                                    [0,1,14],
                                    [0,2,16],
                                    [1,2,9],
                                    [1,3,3],
                                    [1,4,8],
                                    [2,3,10],
                                    [2,4,6],
                                    [3,4,5],
                                    [3,5,16],
                                    [4,3,4],
                                    [4,5,15]]))[0]
```

```
The maximum is: 27.0
```

Out[6]:

	0,1	0,2	1,2	1,3	1,4	2,3	2,4	3,4	3,5	4,3	4,5
flow	11.0	16.0	0.0	3.0	8.0	10.0	6.0	1.0	16.0	4.0	11.0

Problem 2 Confirmation.

In [7]:
```
create_eq_matrix(10,
                 np.array([
                           [0,1,6],
                           [0,5,6],
                           [0,8,2],
                           [1,2,3],
                           [1,5,3],
                           [2,3,2],
                           [2,5,6],
                           [2,7,3],
                           [3,4,4],
                           [4,7,4],
                           [4,9,7],
                           [5,6,3],
                           [5,8,2],
                           [6,7,2],
                           [6,8,1],
                           [7,9,3],
                           [8,9,6]
                           ]))[0]
```

```
The maximum is: 10.0
```

Out[7]:

	0,1	0,5	0,8	1,2	1,5	2,3	2,5	2,7	3,4	4,7	4,9	5,6	5,8	6,7	6,8	7,9	8,9
flow	6.0	2.0	2.0	3.0	3.0	2.0	0.0	1.0	2.0	0.0	2.0	3.0	2.0	2.0	1.0	3.0	5.0

Problem 4.

a.

In [8]:
```
A_leq=np.identity(9)
A_eq=np.array([
               [1,0,-1,-1,0,0,0,0,0],
               [0,0,1,0,-1,-1,1,0,0],
               [0,1,0,1,0,0,-1,-1,0],
               [0,0,0,0,1,0,0,1,-1]
])

b_eq=np.zeros(4)
b_leq=[5,6,4,1,2,4,3,3,6]

tab=np.concatenate((A_eq,A_leq),axis=0)
b_full=np.transpose(np.array([np.append(b_eq,b_leq)]))

c=np.array([[0,0,0,0,0,1,0,0,1,0]])

tab=np.concatenate((tab,b_full),axis=1)
tab=np.concatenate((tab,c),axis=0)

print(tab)

indep_names=["x01","x03","x12","x13","x24","x25","x32","x34","x45"]
dep_names=["0","0","0","0","c01","c03","c12","c13","c24","c25","c32","c34","c45"]

indep_names_dual=["mu1","mu2","mu3","mu4","y01","y03","y12","y13","y24","y25","y32","y34","y45"]
dep_names_dual=["s01","s03","s12","s13","s24","s25","s32","s34","s45"]
```

```
[[ 1.  0. -1. -1.  0.  0.  0.  0.  0.  0.]
 [ 0.  0.  1.  0. -1. -1.  1.  0.  0.  0.]
 [ 0.  1.  0.  1.  0.  0. -1. -1.  0.  0.]
 [ 0.  0.  0.  0.  1.  0.  0.  1. -1.  0.]
 [ 1.  0.  0.  0.  0.  0.  0.  0.  0.  5.]
 [ 0.  1.  0.  0.  0.  0.  0.  0.  0.  6.]
 [ 0.  0.  1.  0.  0.  0.  0.  0.  0.  4.]
 [ 0.  0.  0.  1.  0.  0.  0.  0.  0.  1.]
 [ 0.  0.  0.  0.  1.  0.  0.  0.  0.  2.]
 [ 0.  0.  0.  0.  0.  1.  0.  0.  0.  4.]
 [ 0.  0.  0.  0.  0.  0.  1.  0.  0.  3.]
 [ 0.  0.  0.  0.  0.  0.  0.  1.  0.  3.]
 [ 0.  0.  0.  0.  0.  0.  0.  0.  1.  6.]
 [ 0.  0.  0.  0.  0.  1.  0.  0.  1.  0.]]
```

In [9]:
```
def print_tableau(a,indep_names,dep_names,indep_names_dual,dep_names_dual):
#
# Given matrix "a" and lists of variables names "indep_names" and "dep_names",
#   and (for the dual) "indep_names_dual" and "dep_names_dual",
# this function prints the matrix and labels in standard tableau format
# (including adding the -1, the minus signs in the last column, and labeling the lower-right as obj)
```

```
#
# First, check the inputs: indep_names and dep_names_dual should be one shorter than the number of columns of A
#                          dep_names and indep_names_dual should be one shorter than the number of rows of A
#
    nrows = a.shape[0]    # use the shape function to determine number of rows and cols in A
    ncols = a.shape[1]
    nindep = len(indep_names)
    nindep_dual = len(indep_names_dual)
    ndep = len(dep_names)
    ndep_dual = len(dep_names_dual)
    if nindep != ncols-1:
        print("WARNING: # of indep vbles should be one fewer than # columns of matrix")
    if ndep != nrows-1:
        print("WARNING: # of dep vbles should be one fewer than # rows of matrix")
    if nindep_dual != nrows-1:
        print("WARNING: # of indep dual vbles should be one fewer than # rows of matrix")
    if ndep_dual != ncols-1:
        print("WARNING: # of dep dual vbles should be one fewer than # columns of matrix")
# Now do the printing (uses a variety of formatting techniques in Python)
    print("          ",end="")       # On first line, leave blank space so we can fit in dual labels lower down
    for j in range(ncols-1):                    # Print the independent variables in the first row
        print(indep_names[j].rjust(10),end="")  # rjust(10) makes fields 10 wide and right-justifies;
                                                #    the end command prevents newline)
    print("        -1")                         # Tack on the -1 at the end of the first row
    for i in range(nrows-1):
        print(indep_names_dual[i].rjust(10),end="")
        for j in range(ncols):                  # Print all but the last row of the matrix
                print("%10.3f" % a[i][j],end="") # The syntax prints in a field 10 wide, showing 3 decimal point
        lab = "= -" + dep_names[i]
        print(lab.rjust(10))
    print("        -1",end="")
    for j in range(ncols):
        print("%10.3f" % a[nrows-1][j],end="")  # Print the last row of the matrix, with label "obj" at end
    lab = "= obj"
    print(lab.rjust(10))
    print("          ",end="")
    for j in range(ncols-1):
        lab = "=" + dep_names_dual[j]
        print(lab.rjust(10),end="")
    print("  =dualobj")
    print(" ")    # Put blank line at bottom
```

In [10]:

```
def pivot(a,pivrow,pivcol,indep_names,dep_names,indep_names_dual,dep_names_dual) :
#
# Given matrix "a", a row number "pivrow" and column number "pivcol",
#  and lists of variable names "indep_names" and "dep_names", this
#  function does three things:
#    (1) outputs the new version of the matrix after a pivot,
#    (2) updates the lists of variable names post-pivot
#    (3) prints the new matrix, including labels showing the variable names
#
# First, check the inputs: indep_names should be one shorter than the number of columns of A
#                          dep_names should be one shorter than the number of rows of A
#                          you should not be pivoting on the last row or last column
#
    nrows = a.shape[0]    # use the shape function to determine number of rows and cols in A
    ncols = a.shape[1]
    nindep = len(indep_names)
    nindep_dual = len(indep_names_dual)
    ndep = len(dep_names)
    ndep_dual = len(dep_names_dual)
    if nindep != ncols-1:
        print("WARNING: # of indep vbles should be one fewer than # columns of matrix")
    if ndep != nrows-1:
        print("WARNING: # of dep vbles should be one fewer than # rows of matrix")
    if nindep_dual != nrows-1:
        print("WARNING: # of indep dual vbles should be one fewer than # rows of matrix")
    if ndep_dual != ncols-1:
        print("WARNING: # of dep dual vbles should be one fewer than # columns of matrix")
    if pivrow > nrows-1 or pivcol > ncols-1:
        print("WARNING: should not pivot on last row or column")
    newa = a.copy()       # make a copy of A, to be filled in below with result of pivot
    p = a[pivrow-1][pivcol-1]   # identify pivot element
    newa[pivrow-1][pivcol-1] = 1/p   # set new value of pivot element
    # Set entries in p's row
    for j in range(ncols):
        if j != pivcol-1:
            newa[pivrow-1][j]=a[pivrow-1][j]/p;
    # Set entries in p's column
    for i in range(nrows):
        if i != pivrow-1:
```

```
            newa[i][pivcol-1]=-a[i][pivcol-1]/p;
    # Set all other entries
    for i in range(nrows):
        for j in range(ncols):
            if i != pivrow-1 and j != pivcol-1:
                r = a[i][pivcol-1]
                q = a[pivrow-1][j]
                s = a[i][j]
                newa[i][j]=(p*s-q*r)/p
    # Now transfer the new tableau into a
    for i in range(nrows) :
        for j in range(ncols) :
            a[i][j] = newa[i][j]
    # Now swap the variable names
    temp = indep_names[pivcol-1]
    indep_names[pivcol-1]=dep_names[pivrow-1]
    dep_names[pivrow-1]=temp
    temp = indep_names_dual[pivrow-1]
    indep_names_dual[pivrow-1]=dep_names_dual[pivcol-1]
    dep_names_dual[pivcol-1]=temp
    print_tableau(newa,indep_names,dep_names,indep_names_dual,dep_names_dual) # Print the matrix with updated la
    return 0;
```

In [11]:
```
def column_delete(a,col_to_remove,indep_names,dep_names,indep_names_dual,dep_names_dual) :
    import numpy as np
    anew = np.delete(a,col_to_remove-1,axis=1)
    del indep_names[col_to_remove-1]
    del dep_names_dual[col_to_remove-1]
    print_tableau(anew,indep_names,dep_names,indep_names_dual,dep_names_dual)
    return anew
```

In [12]:
```
def row_delete(a,row_to_remove,indep_names,dep_names,indep_names_dual,dep_names_dual) :
    import numpy as np
    anew = np.delete(a,row_to_remove-1,axis=0)
    del dep_names[row_to_remove-1]
    del indep_names_dual[row_to_remove-1]
    print_tableau(anew,indep_names,dep_names,indep_names_dual,dep_names_dual)
    return anew
```

In [13]:
```
def target(a) :
    nrows = a.shape[0]    # use the shape function to determine number of rows and cols in "a"
    ncols = a.shape[1]
    import numpy as np
    v = np.empty(ncols-1)
    for i in range(ncols-1):
        v[i]=a[nrows-1,i]
    biggest_c = np.max(v)
    where_is_biggest_c = np.argmax(v)+1
    if biggest_c > 0 :
        return where_is_biggest_c
    else :
        return -1
```

In [14]:
```
def select(a,pivcolnum) :
    nrows = a.shape[0]    # use the shape function to determine number of rows and cols in A
    ncols = a.shape[1]
# First task: work down the column and record the b/a ratios in a vector v
#     except record -1 if a is negative or zero
    import numpy as np
    v = np.zeros(nrows-1)
    for i in range(nrows-1):
        if a[i,pivcolnum-1]>0 :
            v[i] = a[i,ncols-1]/a[i,pivcolnum-1]
        else :
            v[i] = -1
# Second task: if max b/a > -1, find min b/a by hand (ignoring zero entries in v)
    if np.max(v) > -1 :
        min_so_far = np.max(v)+1  # Initialize min to be for-sure bigger than the min
        for i in range(nrows-1):
            if v[i] > -1 and v[i] < min_so_far :
                min_so_far = v[i]
                where_is_min = i+1   # Add 1 to use human numbering
        return where_is_min      # Once we've scanned v for min, we can return result

    else :          # Otherwise, we find the m
        return -1
```

In [15]:
```
def simplexbf(a,indep_names,dep_names,dual_indep_names,dual_dep_names):
# Run the simplexbf algorithm
```

```
# Inputs: np.array "a" (assumed to be basic feasible)
#         lists of variable names indep_names and dep_names (pivot will catch if they're wrong size)
# Output: -1 if we stop because problem is unbounded, 0 if we continue to a solution
#         -9 if we take too many steps
    nrows = a.shape[0]    # use shape to find # of rows and cols in A
    ncols = a.shape[1]
    print("Starting SimplexBF (will do nothing if solution can already be determined)")
    pivcol = target(a)
    nsteps = 0
    while pivcol > -1 and nsteps < 50:  # Repeat until either solution found or 50 pivots completed
        pivrow = select(a,pivcol)
        if pivrow == -1 :
            return -1    # If select reports -1, problem is unbounded, so exit this function
        else :
            pivot(a,pivrow,pivcol,indep_names,dep_names,dual_indep_names,dual_dep_names)
            nsteps=nsteps+1
            pivcol = target(a)
    if nsteps >= 50:
        return -9    # we took too many pivots
    else:
        return 0
```

In [16]:

```
def targetnbf(a):
    nrows = a.shape[0]
    ncols = a.shape[1]
    import numpy as np
    checkrow = nrows-2
    while a[checkrow,ncols-1] >= -0.00000001 :
        if checkrow == 0 :   # if still in the "while" and at the top,
            return -1        # all the b's were >= 0, so return -1
        else :
            checkrow = checkrow-1
    return checkrow+1    # if we exit the "while", we found a negative
#                          b, so return current row # (in human numbering)
```

In [17]:

```
def selectnbf(a,targetrow) :
#  Given inputs "a" (tableau as an np.array, numbers only, no labels)
#       and "targetrow" (a row that has a negative b; start-at-1 numbering assumed),
#  computes a pivot that could be chosen by SimplexNBF and
#  outputs "pivrow" and "pivcol", the row and column (start-at-1 numbering) of that pivot
#  If the targeted row has no negative aij, returns -2 for both pivrow and pivcol
    nrows = a.shape[0]
    ncols = a.shape[1]
    import numpy as np
    targetrow = targetrow-1   # convert to start-at-0
    pivcol = ncols-2     # column index of last aij
    while a[targetrow,pivcol] >= 0 :
        if pivcol == 0 :   # if pivcol makes it to zero, all aij
            return [-2,-2]   # in this row were >= 0, so problem infeasible
        else :
            pivcol = pivcol-1
    minsofar = a[targetrow,ncols-1]/a[targetrow,pivcol] # we found a negative aij
    pivrow = targetrow
    for i in range(targetrow+1,nrows-2):   # now check below it for a smaller bi/aij with aij>0
        if a[i,pivcol]>0 and a[i,ncols-1]/a[i,pivcol] < minsofar :
            minsofar = a[i,ncols-1]/a[i,pivcol]
            pivrow = i
    return [pivrow+1,pivcol+1]   # Return result (shifted to start-at-1 numbering)
```

In [18]:

```
def simplexnbf(a,indep_names,dep_names,dual_indep_names,dual_dep_names):
# Run the simplexnbf algorithm
# Inputs: np.array "a"
#         lists of variable names indep_names and dep_names (pivot will catch if they're wrong size)
# Output: -2 if we stop because problem is infeasible, 0 if we stop at a basic feasible tableau
#.        -9 if we take too many pivots
#    (Also, the tableau "a" and variable-lists are updated with each pivot)
    nrows = a.shape[0]
    ncols = a.shape[1]
    print("Starting SimplexNBF (will do nothing if already basic feasible)")
    nsteps = 0
    targetrow = targetnbf(a)
    while targetrow > -1 and nsteps < 50:  # Repeat until either basic feasible tableau produced or 50 pivots co
        [pivrow,pivcol] = selectnbf(a,targetrow)
        if pivrow == -2 :
            return -2    # If selectnbf reports -2, problem is infeasible, so exit this function
        else :
            pivot(a,pivrow,pivcol,indep_names,dep_names,dual_indep_names,dual_dep_names)
            nsteps=nsteps+1
            targetrow = targetnbf(a)
    if nsteps >= 50:
```

```
        return -9    # took too many pivots
    else:
        return 0
```

In [19]:
```
def simplex(a,indep_names,dep_names,dual_indep_names,dual_dep_names) :
# Runs the simplex algorithm (doing NBF if needed, then BF)
# Inputs: np.array "a"
#         lists of variable names indep_names and dep_names (pivot will catch if they're wrong size)
# Output: -2 if problem is infeasible
#         -1 if problem is unbounded
#         0 if problem has a solution
#    (Also, the tableau "a" and variable-lists are updated with each pivot)
    nrows = a.shape[0]
    ncols = a.shape[1]
    print("Initial tableau")
    print_tableau(a,indep_names,dep_names,dual_indep_names,dual_dep_names)
    code = simplexnbf(a,indep_names,dep_names,dual_indep_names,dual_dep_names)
    if code == -2 :
        print("Problem is infeasible")
        return -2
    elif code == -9 :
        print("SimplexNBF took too many pivots")
    else :
        code = simplexbf(a,indep_names,dep_names,dual_indep_names,dual_dep_names)
        if code == -1 :
            print("Problem is unbounded")
            return -1
        elif code == -9 :
            print("SimplexBF took too many pivots")
        else :
            print("Problem has solution, final tableau is shown above")
            return 0
```

In [20]:
```
def simplexeq(a,k,indep_names,dep_names,dual_indep_names,dual_dep_names) :
# Specialized function to do the "pre-simplex" step to handle tableaus where the
#   first k rows correspond to equality constraints.
    nrows = a.shape[0]
    ncols = a.shape[1]
    print_tableau(a,indep_names,dep_names,dual_indep_names,dual_dep_names)
    for i in range(k) :
        j=0
        pivcol=-1
        for j in range(ncols-1) :
            if abs(a[i,j]) > 0.000001 :
                pivrow=i+1
                pivcol=j+1
                break
        if pivcol == -1 :
            return -3
        else :
            pivot(a,pivrow,pivcol,indep_names,dep_names,dual_indep_names,dual_dep_names)
            a=column_delete(a,pivcol,indep_names,dep_names,dual_indep_names,dual_dep_names)
            ncols=ncols-1
    code = simplex(a,indep_names,dep_names,dual_indep_names,dual_dep_names)
    return code
```

In [21]:
```
tab,4,indep_names,dep_names,indep_names_dual,dep_names_dual
```

Out[21]:
```
(array([[ 1.,  0., -1., -1.,  0.,  0.,  0.,  0.,  0.,  0.],
        [ 0.,  0.,  1.,  0., -1., -1.,  1.,  0.,  0.,  0.],
        [ 0.,  1.,  0.,  1.,  0.,  0., -1., -1.,  0.,  0.],
        [ 0.,  0.,  0.,  0.,  1.,  0.,  0.,  1., -1.,  0.],
        [ 1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  5.],
        [ 0.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  6.],
        [ 0.,  0.,  1.,  0.,  0.,  0.,  0.,  0.,  0.,  4.],
        [ 0.,  0.,  0.,  1.,  0.,  0.,  0.,  0.,  0.,  1.],
        [ 0.,  0.,  0.,  0.,  1.,  0.,  0.,  0.,  0.,  2.],
        [ 0.,  0.,  0.,  0.,  0.,  1.,  0.,  0.,  0.,  4.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  1.,  0.,  0.,  3.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  1.,  0.,  3.],
        [ 0.,  0.,  0.,  0.,  0.,  0.,  0.,  0.,  1.,  6.],
        [ 0.,  0.,  0.,  0.,  0.,  1.,  0.,  0.,  1.,  0.]]),
 4,
 ['x01', 'x03', 'x12', 'x13', 'x24', 'x25', 'x32', 'x34', 'x45'],
 ['0',
  '0',
  '0',
  '0',
  'c01',
  'c03',
  'c12',
  'c13',
```

```
  'c24',
  'c25',
  'c32',
  'c34',
  'c45'],
 ['mu1',
  'mu2',
  'mu3',
  'mu4',
  'y01',
  'y03',
  'y12',
  'y13',
  'y24',
  'y25',
  'y32',
  'y34',
  'y45'],
 ['s01', 's03', 's12', 's13', 's24', 's25', 's32', 's34', 's45'])
```

In [22]:
```
simplexeq(tab,4,indep_names,dep_names,indep_names_dual,dep_names_dual)
```

```
                x01       x03       x12       x13       x24       x25       x32       x34       x45        -1
      mu1     1.000     0.000    -1.000    -1.000     0.000     0.000     0.000     0.000     0.000     0.000
= -0
      mu2     0.000     0.000     1.000     0.000    -1.000    -1.000     1.000     0.000     0.000     0.000
= -0
      mu3     0.000     1.000     0.000     1.000     0.000     0.000    -1.000    -1.000     0.000     0.000
= -0
      mu4     0.000     0.000     0.000     0.000     1.000     0.000     0.000     1.000    -1.000     0.000
= -0
      y01     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000     5.000
= -c01
      y03     0.000     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000     6.000
= -c03
      y12     0.000     0.000     1.000     0.000     0.000     0.000     0.000     0.000     0.000     4.000
= -c12
      y13     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000     0.000     1.000
= -c13
      y24     0.000     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000     2.000
= -c24
      y25     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000     0.000     4.000
= -c25
      y32     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000     3.000
= -c32
      y34     0.000     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000     3.000
= -c34
      y45     0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000     1.000     6.000
= -c45
       -1     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000     1.000     0.000
= obj
               =s01      =s03      =s12      =s13      =s24      =s25      =s32      =s34      =s45  =dualobj

                  0       x03       x12       x13       x24       x25       x32       x34       x45        -1
      s01     1.000     0.000    -1.000    -1.000     0.000     0.000     0.000     0.000     0.000     0.000
= -x01
      mu2    -0.000     0.000     1.000     0.000    -1.000    -1.000     1.000     0.000     0.000     0.000
= -0
      mu3    -0.000     1.000     0.000     1.000     0.000     0.000    -1.000    -1.000     0.000     0.000
= -0
      mu4    -0.000     0.000     0.000     0.000     1.000     0.000     0.000     1.000    -1.000     0.000
= -0
      y01    -1.000     0.000     1.000     1.000     0.000     0.000     0.000     0.000     0.000     5.000
= -c01
      y03    -0.000     1.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000     6.000
= -c03
      y12    -0.000     0.000     1.000     0.000     0.000     0.000     0.000     0.000     0.000     4.000
= -c12
      y13    -0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000     0.000     1.000
= -c13
      y24    -0.000     0.000     0.000     0.000     1.000     0.000     0.000     0.000     0.000     2.000
= -c24
      y25    -0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000     0.000     4.000
= -c25
      y32    -0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000     3.000
= -c32
      y34    -0.000     0.000     0.000     0.000     0.000     0.000     0.000     1.000     0.000     3.000
= -c34
      y45    -0.000     0.000     0.000     0.000     0.000     0.000     0.000     0.000     1.000     6.000
= -c45
       -1    -0.000     0.000     0.000     0.000     0.000     1.000     0.000     0.000     1.000     0.000
= obj
               =mu1      =s03      =s12      =s13      =s24      =s25      =s32      =s34      =s45  =dualobj

                x03       x12       x13       x24       x25       x32       x34       x45        -1
      s01     0.000    -1.000    -1.000     0.000     0.000     0.000     0.000     0.000     0.000    = -x01
      mu2     0.000     1.000     0.000    -1.000    -1.000     1.000     0.000     0.000     0.000      = -0
```

mu3	1.000	0.000	1.000	0.000	0.000	-1.000	-1.000	0.000	0.000	= -0
mu4	0.000	0.000	0.000	1.000	0.000	0.000	1.000	-1.000	0.000	= -0
y01	0.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	5.000	= -c01
y03	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.000	= -c03
y12	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	4.000	= -c12
y13	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	= -c13
y24	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	2.000	= -c24
y25	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	4.000	= -c25
y32	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	3.000	= -c32
y34	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	3.000	= -c34
y45	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	6.000	= -c45
-1	0.000	0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	= obj
	=s03	=s12	=s13	=s24	=s25	=s32	=s34	=s45	=dualobj	

	x03	0	x13	x24	x25	x32	x34	x45	-1	
s01	0.000	1.000	-1.000	-1.000	-1.000	1.000	0.000	0.000	0.000	= -x01
s12	0.000	1.000	0.000	-1.000	-1.000	1.000	0.000	0.000	0.000	= -x12
mu3	1.000	-0.000	1.000	0.000	0.000	-1.000	-1.000	0.000	0.000	= -0
mu4	0.000	-0.000	0.000	1.000	0.000	0.000	1.000	-1.000	0.000	= -0
y01	0.000	-1.000	1.000	1.000	1.000	-1.000	0.000	0.000	5.000	= -c01
y03	1.000	-0.000	0.000	0.000	0.000	0.000	0.000	0.000	6.000	= -c03
y12	0.000	-1.000	0.000	1.000	1.000	-1.000	0.000	0.000	4.000	= -c12
y13	0.000	-0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	= -c13
y24	0.000	-0.000	0.000	1.000	0.000	0.000	0.000	0.000	2.000	= -c24
y25	0.000	-0.000	0.000	0.000	1.000	0.000	0.000	0.000	4.000	= -c25
y32	0.000	-0.000	0.000	0.000	0.000	1.000	0.000	0.000	3.000	= -c32
y34	0.000	-0.000	0.000	0.000	0.000	0.000	1.000	0.000	3.000	= -c34
y45	0.000	-0.000	0.000	0.000	0.000	0.000	0.000	1.000	6.000	= -c45
-1	0.000	-0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	= obj
	=s03	=mu2	=s13	=s24	=s25	=s32	=s34	=s45	=dualobj	

	x03	x13	x24	x25	x32	x34	x45	-1	
s01	0.000	-1.000	-1.000	-1.000	1.000	0.000	0.000	0.000	= -x01
s12	0.000	0.000	-1.000	-1.000	1.000	0.000	0.000	0.000	= -x12
mu3	1.000	1.000	0.000	0.000	-1.000	-1.000	0.000	0.000	= -0
mu4	0.000	0.000	1.000	0.000	0.000	1.000	-1.000	0.000	= -0
y01	0.000	1.000	1.000	1.000	-1.000	0.000	0.000	5.000	= -c01
y03	1.000	0.000	0.000	0.000	0.000	0.000	0.000	6.000	= -c03
y12	0.000	0.000	1.000	1.000	-1.000	0.000	0.000	4.000	= -c12
y13	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	= -c13
y24	0.000	0.000	1.000	0.000	0.000	0.000	0.000	2.000	= -c24
y25	0.000	0.000	0.000	1.000	0.000	0.000	0.000	4.000	= -c25
y32	0.000	0.000	0.000	0.000	1.000	0.000	0.000	3.000	= -c32
y34	0.000	0.000	0.000	0.000	0.000	1.000	0.000	3.000	= -c34
y45	0.000	0.000	0.000	0.000	0.000	0.000	1.000	6.000	= -c45
-1	0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	= obj
	=s03	=s13	=s24	=s25	=s32	=s34	=s45	=dualobj	

	0	x13	x24	x25	x32	x34	x45	-1	
s01	-0.000	-1.000	-1.000	-1.000	1.000	0.000	0.000	0.000	= -x01
s12	-0.000	0.000	-1.000	-1.000	1.000	0.000	0.000	0.000	= -x12
s03	1.000	1.000	0.000	0.000	-1.000	-1.000	0.000	0.000	= -x03
mu4	-0.000	0.000	1.000	0.000	0.000	1.000	-1.000	0.000	= -0
y01	-0.000	1.000	1.000	1.000	-1.000	0.000	0.000	5.000	= -c01
y03	-1.000	-1.000	0.000	0.000	1.000	1.000	0.000	6.000	= -c03
y12	-0.000	0.000	1.000	1.000	-1.000	0.000	0.000	4.000	= -c12
y13	-0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	= -c13
y24	-0.000	0.000	1.000	0.000	0.000	0.000	0.000	2.000	= -c24
y25	-0.000	0.000	0.000	1.000	0.000	0.000	0.000	4.000	= -c25
y32	-0.000	0.000	0.000	0.000	1.000	0.000	0.000	3.000	= -c32
y34	-0.000	0.000	0.000	0.000	0.000	1.000	0.000	3.000	= -c34
y45	-0.000	0.000	0.000	0.000	0.000	0.000	1.000	6.000	= -c45
-1	-0.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	= obj
	=mu3	=s13	=s24	=s25	=s32	=s34	=s45	=dualobj	

	x13	x24	x25	x32	x34	x45	-1	
s01	-1.000	-1.000	-1.000	1.000	0.000	0.000	0.000	= -x01
s12	0.000	-1.000	-1.000	1.000	0.000	0.000	0.000	= -x12
s03	1.000	0.000	0.000	-1.000	-1.000	0.000	0.000	= -x03
mu4	0.000	1.000	0.000	0.000	1.000	-1.000	0.000	= -0
y01	1.000	1.000	1.000	-1.000	0.000	0.000	5.000	= -c01
y03	-1.000	0.000	0.000	1.000	1.000	0.000	6.000	= -c03
y12	0.000	1.000	1.000	-1.000	0.000	0.000	4.000	= -c12
y13	1.000	0.000	0.000	0.000	0.000	0.000	1.000	= -c13
y24	0.000	1.000	0.000	0.000	0.000	0.000	2.000	= -c24
y25	0.000	0.000	1.000	0.000	0.000	0.000	4.000	= -c25
y32	0.000	0.000	0.000	1.000	0.000	0.000	3.000	= -c32
y34	0.000	0.000	0.000	0.000	1.000	0.000	3.000	= -c34
y45	0.000	0.000	0.000	0.000	0.000	1.000	6.000	= -c45
-1	0.000	0.000	1.000	0.000	0.000	1.000	0.000	= obj
	=s13	=s24	=s25	=s32	=s34	=s45	=dualobj	

	x13	0	x25	x32	x34	x45	-1	
s01	-1.000	1.000	-1.000	1.000	1.000	-1.000	0.000	= -x01
s12	0.000	1.000	-1.000	1.000	1.000	-1.000	0.000	= -x12
s03	1.000	-0.000	0.000	-1.000	-1.000	0.000	0.000	= -x03
s24	0.000	1.000	0.000	0.000	1.000	-1.000	0.000	= -x24

```
      y01     1.000    -1.000     1.000    -1.000    -1.000     1.000     5.000    = -c01
      y03    -1.000    -0.000     0.000     1.000     1.000     0.000     6.000    = -c03
      y12     0.000    -1.000     1.000    -1.000    -1.000     1.000     4.000    = -c12
      y13     1.000    -0.000     0.000     0.000     0.000     0.000     1.000    = -c13
      y24     0.000    -1.000     0.000     0.000    -1.000     1.000     2.000    = -c24
      y25     0.000    -0.000     1.000     0.000     0.000     0.000     4.000    = -c25
      y32     0.000    -0.000     0.000     1.000     0.000     0.000     3.000    = -c32
      y34     0.000    -0.000     0.000     0.000     1.000     0.000     3.000    = -c34
      y45     0.000    -0.000     0.000     0.000     0.000     1.000     6.000    = -c45
       -1     0.000    -0.000     1.000     0.000     0.000     1.000     0.000     = obj
              =s13      =mu4      =s25      =s32      =s34      =s45   =dualobj

                x13       x25       x32       x34       x45        -1
      s01    -1.000    -1.000     1.000     1.000    -1.000     0.000    = -x01
      s12     0.000    -1.000     1.000     1.000    -1.000     0.000    = -x12
      s03     1.000     0.000    -1.000    -1.000     0.000     0.000    = -x03
      s24     0.000     0.000     0.000     1.000    -1.000     0.000    = -x24
      y01     1.000     1.000    -1.000    -1.000     1.000     5.000    = -c01
      y03    -1.000     0.000     1.000     1.000     0.000     6.000    = -c03
      y12     0.000     1.000    -1.000    -1.000     1.000     4.000    = -c12
      y13     1.000     0.000     0.000     0.000     0.000     1.000    = -c13
      y24     0.000     0.000     0.000    -1.000     1.000     2.000    = -c24
      y25     0.000     1.000     0.000     0.000     0.000     4.000    = -c25
      y32     0.000     0.000     1.000     0.000     0.000     3.000    = -c32
      y34     0.000     0.000     0.000     1.000     0.000     3.000    = -c34
      y45     0.000     0.000     0.000     0.000     1.000     6.000    = -c45
       -1     0.000     1.000     0.000     0.000     1.000     0.000     = obj
              =s13      =s25      =s32      =s34      =s45   =dualobj

Initial tableau
                x13       x25       x32       x34       x45        -1
      s01    -1.000    -1.000     1.000     1.000    -1.000     0.000    = -x01
      s12     0.000    -1.000     1.000     1.000    -1.000     0.000    = -x12
      s03     1.000     0.000    -1.000    -1.000     0.000     0.000    = -x03
      s24     0.000     0.000     0.000     1.000    -1.000     0.000    = -x24
      y01     1.000     1.000    -1.000    -1.000     1.000     5.000    = -c01
      y03    -1.000     0.000     1.000     1.000     0.000     6.000    = -c03
      y12     0.000     1.000    -1.000    -1.000     1.000     4.000    = -c12
      y13     1.000     0.000     0.000     0.000     0.000     1.000    = -c13
      y24     0.000     0.000     0.000    -1.000     1.000     2.000    = -c24
      y25     0.000     1.000     0.000     0.000     0.000     4.000    = -c25
      y32     0.000     0.000     1.000     0.000     0.000     3.000    = -c32
      y34     0.000     0.000     0.000     1.000     0.000     3.000    = -c34
      y45     0.000     0.000     0.000     0.000     1.000     6.000    = -c45
       -1     0.000     1.000     0.000     0.000     1.000     0.000     = obj
              =s13      =s25      =s32      =s34      =s45   =dualobj

Starting SimplexNBF (will do nothing if already basic feasible)
Starting SimplexBF (will do nothing if solution can already be determined)
                x13       c12       x32       x34       x45        -1
      s01    -1.000     1.000     0.000     0.000     0.000     4.000    = -x01
      s12     0.000     1.000     0.000     0.000     0.000     4.000    = -x12
      s03     1.000    -0.000    -1.000    -1.000     0.000     0.000    = -x03
      s24     0.000    -0.000     0.000     1.000    -1.000     0.000    = -x24
      y01     1.000    -1.000     0.000     0.000     0.000     1.000    = -c01
      y03    -1.000    -0.000     1.000     1.000     0.000     6.000    = -c03
      s25     0.000     1.000    -1.000    -1.000     1.000     4.000    = -x25
      y13     1.000    -0.000     0.000     0.000     0.000     1.000    = -c13
      y24     0.000    -0.000     0.000    -1.000     1.000     2.000    = -c24
      y25     0.000    -1.000     1.000     1.000    -1.000     0.000    = -c25
      y32     0.000    -0.000     1.000     0.000     0.000     3.000    = -c32
      y34     0.000    -0.000     0.000     1.000     0.000     3.000    = -c34
      y45     0.000    -0.000     0.000     0.000     1.000     6.000    = -c45
       -1     0.000    -1.000     1.000     1.000     0.000    -4.000     = obj
              =s13      =y12      =s32      =s34      =s45   =dualobj

                x13       c12       c25       x34       x45        -1
      s01    -1.000     1.000    -0.000     0.000     0.000     4.000    = -x01
      s12     0.000     1.000    -0.000     0.000     0.000     4.000    = -x12
      s03     1.000    -1.000     1.000     0.000    -1.000     0.000    = -x03
      s24     0.000     0.000    -0.000     1.000    -1.000     0.000    = -x24
      y01     1.000    -1.000    -0.000     0.000     0.000     1.000    = -c01
      y03    -1.000     1.000    -1.000     0.000     1.000     6.000    = -c03
      s25     0.000     0.000     1.000     0.000     0.000     4.000    = -x25
      y13     1.000     0.000    -0.000     0.000     0.000     1.000    = -c13
      y24     0.000     0.000    -0.000    -1.000     1.000     2.000    = -c24
      s32     0.000    -1.000     1.000     1.000    -1.000     0.000    = -x32
      y32     0.000     1.000    -1.000    -1.000     1.000     3.000    = -c32
      y34     0.000     0.000    -0.000     1.000     0.000     3.000    = -c34
      y45     0.000     0.000    -0.000     0.000     1.000     6.000    = -c45
       -1     0.000     0.000    -1.000     0.000     1.000    -4.000     = obj
              =s13      =y12      =y25      =s34      =s45   =dualobj

                x13       c12       c25       x34       c24        -1
      s01    -1.000     1.000     0.000     0.000    -0.000     4.000    = -x01
      s12     0.000     1.000     0.000     0.000    -0.000     4.000    = -x12
      s03     1.000    -1.000     1.000    -1.000     1.000     2.000    = -x03
```

```
s24     0.000     0.000    -0.000     0.000     1.000     2.000    = -x24
y01     1.000    -1.000     0.000     0.000    -0.000     1.000    = -c01
y03    -1.000     1.000    -1.000     1.000    -1.000     4.000    = -c03
s25     0.000     0.000     1.000     0.000    -0.000     4.000    = -x25
y13     1.000     0.000     0.000     0.000    -0.000     1.000    = -c13
s45     0.000     0.000    -0.000    -1.000     1.000     2.000    = -x45
s32     0.000    -1.000     1.000     0.000     1.000     2.000    = -x32
y32     0.000     1.000    -1.000     0.000    -1.000     1.000    = -c32
y34     0.000     0.000     0.000     1.000    -0.000     3.000    = -c34
y45     0.000     0.000     0.000     1.000    -1.000     4.000    = -c45
 -1     0.000     0.000    -1.000     1.000    -1.000    -6.000     = obj
         =s13      =y12      =y25      =s34      =y24   =dualobj

          x13       c12       c25       c34       c24        -1
s01    -1.000     1.000     0.000    -0.000     0.000     4.000    = -x01
s12     0.000     1.000     0.000    -0.000     0.000     4.000    = -x12
s03     1.000    -1.000     1.000     1.000     1.000     5.000    = -x03
s24     0.000     0.000    -0.000    -0.000     1.000     2.000    = -x24
y01     1.000    -1.000     0.000    -0.000     0.000     1.000    = -c01
y03    -1.000     1.000    -1.000    -1.000    -1.000     1.000    = -c03
s25     0.000     0.000     1.000    -0.000     0.000     4.000    = -x25
y13     1.000     0.000     0.000    -0.000     0.000     1.000    = -c13
s45     0.000     0.000     0.000     1.000     1.000     5.000    = -x45
s32     0.000    -1.000     1.000    -0.000     1.000     2.000    = -x32
y32     0.000     1.000    -1.000    -0.000    -1.000     1.000    = -c32
s34     0.000     0.000     0.000     1.000    -0.000     3.000    = -x34
y45     0.000     0.000     0.000    -1.000    -1.000     1.000    = -c45
 -1     0.000     0.000    -1.000    -1.000    -1.000    -9.000     = obj
         =s13      =y12      =y25      =y34      =y24   =dualobj

Problem has solution, final tableau is shown above
```

Out[22]: 0

Reading off of the output above, the maximium flow is nine. With flow as indicated in the table. They are:

$$x01 = 4$$
$$x03 = 5$$
$$x12 = 4$$
$$x13 = 0$$
$$x24 = 2$$
$$x25 = 4$$
$$x32 = 2$$
$$x34 = 3$$
$$x45 = 5$$

We can tell that there is an edge between cuts between $(1,2), (2,5), (3,4), \& \ (2,4)$. This implies that $V_1 = 0, 1, 2, 3$ and $V_2 = 4, 5$.