Money For Nothing, Clicks for Free

Aaron Graybill

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Introduction

The advent of social media and streaming services like Spotify, Youtube, and Instagram have given artists of all kinds unprecendented ability to connect their work to their audience (SOURCE?). However, the increased supply of artists makes it harder for each to cut through the noise and reach financial freedom from their craft (SOURCE?).

Throughout history, artists have had to earn the accolades of their fan. Yatta yatta, that's still true, but this paper explores how the introduction of internet media changes the economic incentives and decisions of artists.

This paper develops and analyzes a model of how artists build and maintain a reputation. This paper pulls from the cultural economics of (Rosen, 1981) and the branding literature (Klein & Leffler, 1981; Shapiro, 1983).

Model

Overview

I will now outline a model of how artistic reputation develops over time. To do this, I will first examine the consumer's problem in which she decides to consume a quantity of different artists' work taking as given the price and current reputation of the firm. The consumer's consumption choice will specify a demand function relating quantity consumed to price and current reputation. The artist (firm) takes the demand function and their current reputation as given and sets a price of their work which determines present profit and next period's reputation. The artist chooses prices in every period that maximize their discounted lifetime profits.

The Consumer's Problem

I begin by assuming that there exists a representative consumer who has per-period income I. She derives per-period utility according to $U(a, g)$ where g is a numeraire aggregate consumption good, and a is the value of artistic consumption. I assume monotonicity in both goods: $U_a > 0$ and $U_g > 0$. I further assume that he consumer has decreasing marginal utility from each additional unit of consumption: $U_{aa} < 0$ and $U_{gg} < 0$. These assumptions are consistent with standard economic assumptions of consumer behavior. Finally, I assume that the consumer's utility is quasilinear such that $U(g, a) = u(a) + v(g)$. I assume that the consumer is only concerned with consumption in each period and does not consider future utilities.

Now I assume that the value of art, a, does not come from a single good but from the portfolio of art that the consumer purchases. In fact, I assume that there exists a continuum of partially substitutable artists indexed by $i \in [0, N]$, where N parametrizes the total number of artists. The consumer derives utility from the amount of art she consumes from each seller, denoted $n(i)$ as well as the reputation of the artist, $r(i)$, that she consumes from. Following (Rosen, 1981), I assume that quantity and reputation are strongly substitutable and can be modeled as a single unit. In addition I add an idiosyncratic and exogenous consumer preference parameter for each firm, $\alpha(i)$. These assumptios are conducive to the now-canonical monopolistic competition model presented in (Dixit & Stiglitz, 1977). As such I let:

$$v(a) = \left(\int_0^N \alpha(i) r(i) n(i)^\rho di \right)^{\frac{1}{\rho}}$$

We assume that consumers take the price for a certain artist's work as given, denoted $p(i)$.

As such, the consumer's only choice variables are the number of units to consume from each artist, $n(i)$, and the number of non-art goods she consumes g. She chooses this bundle to maximize her utility subject to her budget constraint: $g + \int_0^N n(i)p(i)di \leq I$.

If the consumer commits to spending E_a on artistic goods, then the Marshallian demand for units of any artist i becomes:

$$n(i) = \left(P \frac{\alpha(i) r(i)}{p(i)} \right)^\sigma \frac{E_a}{P}$$

Where the elasticity of substitution is given by $\sigma = \frac{1}{1-\rho}$, and the aggregate reputation-adjusted price level is $P = \left[\int_0^N \left(\frac{\alpha(\omega) r(\omega)}{p(\omega)} \right)^\sigma p(\omega) d\omega \right]^{\frac{1}{1-\sigma}}$. Since there are a continuum of firms, price changes from a finite or countably infinite number of firms has no affect on the aggregate price level, P.

Then the consumer chooses how to allocate their income I towards artistic goods E_a and consumption goods g. The consumer's optimal choice of artistic consumption for firm i becomes:

$$n(i) = \left(P \frac{\alpha(i) r(i)}{p(i)} \right)^\sigma \frac{I - [v']^{-1} \left(\frac{1}{P} \right)}{P}$$

The expression above gives the Marshallian demand for art from artist i in terms of the price that artist i sets and the reputation they have in the current period. The artist will use this demand function when deciding the optimal price level to set.

Consumer Comparative Statics First note that finite or countably infinite changes to idiosyncratic preferences, reputation, or prices have no affect on the aggregate price level P because the integral weighs an uncountably infinite number of artists. As such, for an artist i finite or countably infinite changes in tastes, preferences, and prices of *other* artists have no effect on how much quantity is demanded for artist i.

Some of the results below depend on whether or not the constant elasticity, $\sigma \lesseqgtr 1$. If the elasticity of substitution is greater than 1, then small changes to the price of one good causes disproportionately large changes tp the demand for other goods. This sort of behavior is likely inconsistent with the art market where we should expect price effects to dissipate over the continuum of sellers, not amplify. With that, for the purposes of this analysis, we assume that $\sigma < 1$.

Beginning with comparative statics, for any artist i, increases to the taste parameter satisfy $n_\alpha > 0$. Additionally, increases to reputation increase demand for a good because $n_r > 0$. Increases to an individual artist's price decrease consumption as expected, so $n_p < 0$. Further note that increases to income increase consumption as expected, $n_I > 0$.

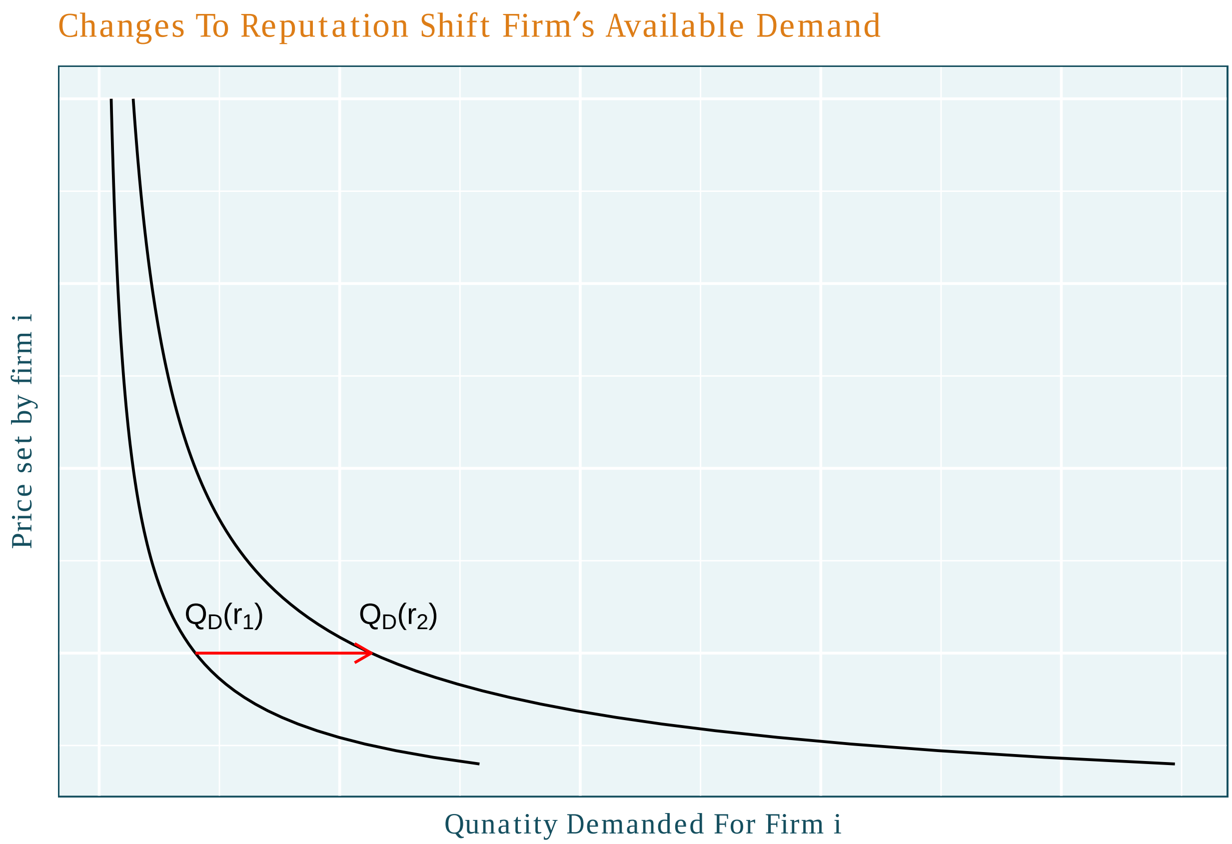
THE FOLLOWING RESULTS MAY REQUIRE SOME INADA-TYPE THINGS, AND SHOULD ALSO MAYBE BE $\geq$ AND NOT $>$, SHOULD WORK THROUGH TOGETHER

In terms of the overall price level, $n_P > 0$. This result stems from our assumption that $v''(g) < 0$, which implies that $[v']^{-1}$ is also decreasing. Under the assumption that $\sigma < 1$, $n_P > 0$, so increases to the overall price level increase consumption of each artistic good. This is equivalent to saying that the substitution effect dominates the income effect.

Finally, increasing the number of firms, the N in the upper bound of the integral, also satisfies $n_N > 0$, so holding other things equal more competition generates more consumer demand for each individual artist. One might interpret this result as the effect of genres of art benefiting from having more artists in that genre.

The Artist's Problem

We model the artist's problem dynamically. Let the artist have underlying artistic talent κ (it can be viewed as a relative of capital). The market is assumed to be monopolistically competitive, so every artist has full control of their price in every period, but price decisions may cause consumers to substitute to other products. The artist takes their current reputation as given. This assumption is used to simplify analysis. Allowing a flexible reputation in the current period obfuscates how reputation evolves by allowing producers to increase there reputation more quickly. As argued above, consumer demand is a function of both price and reputation. However, with the fixed reputation, the artist only has access to a single demand curve in each period. Changes to reputation in future periods shift the artist's available demand curve. This can be represented graphically in the following way.



The artist's current revenue is the product of the price they set, p, and the demand induced by that price choice $n(i, p, r, \alpha)$ **This is terrible notation, help**. In addition to revenue, the artist's choice of price affects the number of goods they must produce which incurs some cost $C(n(p, r, \alpha))$. **ADD ASSUMPTIONS ABOUT CONVEXITY IF WE'RE DOING THAT**. The current period's profit is given by $p \cdot n(p, r, \alpha) - C(n(p, r, \alpha))$.

The artist's problem is not just to maximize current profits, but to maximize the discounted stream of all profits. Unlike traditional growth models, the link tying one period to another is not capital accumulation, but reputation accumulation. I assume that next period's reputation, r_{t+1}, is a function of the current period's reputation, the number of units sold, and the artist's underlying talent. Denote this reputation update function $\mathcal{R}$ such that $r_{t+1} = \mathcal{R}(n_t, r_t, \kappa)$. The functional form of $\mathcal{R}$ is integral to this analysis and multiple specifications will be examined and connected to real world scenarios.

We can formulate the artist's problem two ways. First, the artist can choose a p_t and r_{t+1} in every period. I can represent this as:

$$\max_{p_t, r_{t+1}} \left\{ \sum_{t=0}^{\infty} \beta^t \left[p_t \cdot n(p_t, r_t) - C(n(p_t, r_t)) - \lambda_t (r_{t+1} - \mathcal{R}(n(p_t, r_t), r_t, \kappa)) \right] \right\}$$

Alternately, firm can optimize recursively such the agent's maximal discounted lifetime profit is their profits in this period plus their discounted maximal lifetime from next period onward. The state variable that links one period to the next is their reputation, r. As such The Bellman equation is the function $V(r)$ satisfying

$$V(r) = \max_p \{pn(p,r) - C(n(p,r)) + \beta V(r') \quad s.t. \quad r' = \mathcal{R}(m, r, \kappa)\}$$

Exploration of First Order Conditions

Using the every-period summation model, the first order conditions are that:

$$\begin{aligned}
\frac{\partial V}{\partial p} &: \beta^t \left[n(\cdot) + p_t n_p(\cdot) - C'(n(\cdot)) n_p(\cdot) + \lambda_t \mathcal{R}_n(n(\cdot), r_t, \kappa) n_p(\cdot) \right] = 0 \\
\frac{\partial V}{\partial r_{t+1}} &: \ -\beta^t \lambda_t + \beta^{t+1} \left[n_r(\cdot) p_{t+1} - C'(n(\cdot)) n_r(\cdot) + \lambda_{t+1} \left(\mathcal{R}_n(n(\cdot), r_{t+1}) n_r(\cdot) + \mathcal{R}_r(n(\cdot), r_{t+1}) \right) \right] = 0 \\
\frac{\partial V}{\partial \lambda_t} &: \ r_{t+1} = \mathcal{R}(n(\cdot), r_t)
\end{aligned}$$

Before simplifying those expressions, I will briefly interpret them in the context of the traditional theory of the firm by explaining where marginal benefit equals marginal cost. The derivative of lifetime utility with respect to price is akin to the firm's price setting problem. The artist will continue to raise the price of their work until the marginal benefits of that price increase equal the marginal cost of that increase in price. The marginal benefit has two channels. First, increases to price have a direct effect on revenue, the $n(\cdot) + p_t n_p(\cdot)$ term in the above first order condition. The overall effect of increasing price on marginal revenue is generically ambiguous, while the demand and price are non-negative, quantity demanded is decreasing in price, so the sum of the aforementioned terms is ambiguous. However, in this case so long as $\sigma < 1$, increases to price increase revenue despite lower quantity demanded. I will later argue that $\sigma < 1$

In addition to the present pecuniary value of increasing prices, changing price has an affect on the present quantity demanded which has an effect on the artist's next period's reputation. The reputational benefit is given by: $\lambda_t \mathcal{R}_n(n(\cdot), r_t, \kappa) n_p(\cdot)$. The marginal increase in reputation from increased changes in quantity adjusted by the cost of violating the constraint at time t combine to make the marginal reputational benefit. The sign of this benefit is abiguous and depends on the sign of $\mathcal{R}_n$. In some markets, we might expect more exposure (higher n) to increase reputation and in others, we might see more scarcity (lower n) increase reputation. In the case of more exposure giving more reputation, increasing price decreases next period's reputation because fewer people buy more expensive products.

The cost of increasing price only enters explicitly in the $C'(n(\cdot))n_p(\cdot)$ term. Since the cost function is increasing in n and the quantity demanded is decreasing in price, the $C'(n(\cdot))n_p(\cdot)$ must be negative. As such the overall contribution of increasing price on utility is positive due to suppressed demand allowing to incur fewer production costs.

While the artist cannot directly choose their next period's reputation, r_{t+1}, this first order condition is also worth framing in the marginal benefit equals marginal cost context. Note that λ_t is the shadow cost of violating the constraint in period t. As such, we can say that this shadow cost for period t plus the discounted costs of production next period, $\beta C'(n)n_r$, combine and must equal the marginal benefit. The marginal benefit in this case is the discounted marginal revenue generated from increased reputation in period $t+1$, $\beta n_r()p$, plus the reputational benefit $\mathcal{R}_n(n(\cdot), r_{t+1})n_r(\cdot) + \mathcal{R}_r(n(\cdot), r_{t+1})$ scaled by next period's shadow cost, λ_{t+1}. Reputational benefits gain value relative to per period profits in situations where violating the constraint is more costly. The sign of $\mathcal{R}_n(n(\cdot), r_{t+1})n_r(\cdot) + \mathcal{R}_r(n(\cdot), r_{t+1})$ is ambiguous and dependent on how strongly increased quantity affects reputation. If increased quantity strongly negatively affects reputation, then increases to costs next period tighten the constraint, and make the artist produce less. However, if the reputational gain from increased quantity is less important than your previous reputation (reputations are sticky), then increasing the shadow cost next period loosens the constraint and allow the artist to produce more.

Alternate Formulation, Maximizing over Quantity Instead of Price

To reiterate, the artist is a price setter, but they know that their price decisions perfectly inform the quantity of art demanded for each firm. This implies that we can instead think of the artist as choosing an ideal quantity to produce and then deducing which price they must charge to induce their desired demand. As such, I rewrite the artist's problem as follows: **WHAT GOES UNDER MAX HERE**

$$\max_{\{\lambda\}_t, \{n\}_t, \{r\}_{t+1}} \left\{ \sum_{t=0}^{\infty} \beta^t \left[p(n_t, r_t, \alpha_t) n_t - C(n_t) - \lambda_t (r_{t+1} - \mathcal{R}(n_t, r_t)) \right] \right\}$$

This formulation has the benefit of minimizing the number of chain rules in the first order conditions.

The first order conditions become:

$$\begin{aligned}
\frac{\partial V}{\partial n_t} &: p(n_t, r_t, \alpha_t) + n_t p_n(n_t, r_t, \alpha_t) - C'(n_t) + \lambda_t \mathcal{R}_n(n_t, r_t, \kappa) = 0 \\
\frac{\partial V}{\partial r_{t+1}} &: \; -\beta^t \lambda_t + \beta^{t+1} \left[n_{t+1} p_r(n_{t+1}, r_{t+1}, \alpha_{t+1}) + \lambda_{t+1} \mathcal{R}_r(n_{t+1}, r_{t+1}, \kappa) \right] = 0 \\
\frac{\partial V}{\partial \lambda_t} &: \; r_{t+1} = \mathcal{R}(n_t, r_t, \kappa)
\end{aligned}$$

From the n_t first order condition, we can simplify to:

$$\lambda_t = \frac{C'(n_t) + p(n_t, r_t, \alpha_t)(\frac{1}{\sigma} - 1)}{\mathcal{R}_n(n_t, r_t, \kappa)}$$

Using the previous result, we can simplify the reputation first order condition to:

$$\lambda_t = \beta \left[n_{t+1} p_r^{t+1}(\cdot) + \frac{C'(n_{t+1}) + (\frac{1}{\sigma} - 1) p^{t+1}(\cdot)}{\mathcal{R}_n^{t+1}(\cdot)} \mathcal{R}_r^{t+1} \right]$$

We can implicitly define the optimal policy function n_{t+1} in terms of n_t by substituting in our expression for λ_t which yields the following:

$$\frac{C_t'(\cdot) + (\frac{1}{\sigma} - 1) p^t(\cdot)}{\mathcal{R}_n^t(\cdot)} = \beta \left[n_{t+1} p_r^{t+1}(\cdot) + \frac{C_{t+1}'(\cdot) + (\frac{1}{\sigma} - 1) p^{t+1}(\cdot)}{\mathcal{R}_n^{t+1}(\cdot) / \mathcal{R}_r^{t+1}(\cdot)} \right]$$

However, we cannot progess any further because there are n_{t+1}s inside most of the functions, and without functional forms, we cannot isolate n_{t+1} solely in terms of factors uncontrollable to the artist.

Functional Form Assumptions

It is difficult to draw additional insight from the model without making functional form assumptions to simplify the algebra. As such, I assume that $C(n) = \frac{c}{2} n^2$, so cost is increasing and convex in the number of units produced, n. This functional form is intended to capture an artist's finite time and increasing difficulty of producing additional quality work. The c parameter measures the convexity of the cost function and gives the constant rate at which marginal cost increases. **NEED TO JUSTIFY WHY EXPONENT 2**

I will use $\mathcal{R}(n, r, \kappa) = (1-\delta)\kappa\left(\frac{r}{n}\right)^{\gamma}$. The δ term, similar to the neoclassical growth framework, parametrizes the depreciation of reputation. The κ term in this case is akin to technology in the AK model. Holding other factors equal, increases to *kappa* increase the total reputation by a constant amount. The γ term in the above functional form is used to parametrize the importance of the temporal variables n and r in reputation production. Low values of γ prioritize innate talent κ, but high values of γ imply that increasing reputation of quanity produced have larger effects on next period's reputation. Further note that $R(n, r, \kappa)$ is homogenous of degree zero in n and r, scaling up both by the same amount has no net effect on reputation. This specification is intentionally weak as we would expect that prior reputation has a larger affect on current reputation than past level of sales. As such, this functional forms understates the importance of investment in reputation, so results should hopefully be strong enough to support reputation investment empirically.

Under these functional form specifications, we can simplify the first order conditions from above to:

$$\begin{aligned}
\lambda_t &= \beta\left[n_{t+1}p_r^{t+1}(\cdot) + \frac{C'_{t+1}(\cdot) + (\frac{1}{\sigma}-1)p^{t+1}(\cdot)}{\mathcal{R}_n^{t+1}(\cdot)/\mathcal{R}_r^{t+1}(\cdot)}\right] \\
&= \beta\left[n_{t+1}p_r^{t+1}(\cdot) + \frac{C'_{t+1}(\cdot) + (\frac{1}{\sigma}-1)p^{t+1}(\cdot)}{-r_{t+1}/n_{t+1}}\right] \\
&= \beta n_{t+1}\left[p_r^{t+1}(\cdot) + \frac{C'_{t+1}(\cdot) + (\frac{1}{\sigma}-1)p^{t+1}(\cdot)}{-r_{t+1}}\right] \\
&= \beta n_{t+1}\left[p_r^{t+1}(\cdot) - \frac{C'_{t+1}(\cdot)}{r_{t+1}} - \left(\frac{1}{\sigma}-1\right)\frac{p^{t+1}(\cdot)}{r_{t+1}}\right] \\
&= \beta n_{t+1}\left[p_r^{t+1}(\cdot) - \frac{C'_{t+1}(\cdot)}{r_{t+1}} - \left(\frac{1}{\sigma}-1\right)p_r^{t+1}(\cdot)\right] \\
&= \beta n_{t+1}\left[-\frac{C'_{t+1}(\cdot)}{r_{t+1}} + \left(2-\frac{1}{\sigma}\right)p_r^{t+1}(\cdot)\right] \\
&= \beta n_{t+1}\left[\left(2-\frac{1}{\sigma}\right)p_r^{t+1}(\cdot) - \frac{c}{r_{t+1}}n_{t+1}\right]
\end{aligned}$$

Then we can use the expression for λ_t from the first order condition for quantity, n to find n solely in terms of r and exogenous parameters.

Doing so gives:

$$\begin{aligned}
\frac{cn_t + p^t(\cdot)(\frac{1}{\sigma}-1)}{\mathcal{R}_n^t(\cdot)} &= \beta n_{t+1}\left[\left(2-\frac{1}{\sigma}\right)p_r^{t+1}(\cdot) - \frac{c}{r_{t+1}}n_{t+1}\right] \\
\frac{cn_t + p^t(\cdot)(\frac{1}{\sigma}-1)}{-\gamma n_t^{-1}\mathcal{R}^t(\cdot)} &= \beta n_{t+1}\left[\left(2-\frac{1}{\sigma}\right)p_r^{t+1}(\cdot) - \frac{c}{r_{t+1}}n_{t+1}\right] \\
\frac{cn_t - p^t(\cdot)(\frac{1}{\sigma}-1)}{-\gamma n_t^{-1}r_{t+1}} &= \beta n_{t+1}\left[\left(2-\frac{1}{\sigma}\right)p_r^{t+1}(\cdot) - \frac{c}{r_{t+1}}n_{t+1}\right] \\
\left[cn_t + p^t(\cdot)(\frac{1}{\sigma}-1)\right]n_t &= -\beta\gamma n_{t+1}\left[\left(2-\frac{1}{\sigma}\right)p_r^{t+1}(\cdot)r_{t+1} - \frac{c}{r_{t+1}}n_{t+1}r_{t+1}\right] \\
\left[cn_t + p^t(\cdot)(\frac{1}{\sigma}-1)\right]n_t &= -\beta\gamma n_{t+1}\left[\left(2-\frac{1}{\sigma}\right)p^{t+1}(\cdot) - cn_{t+1}\right] \\
\left[cn_t + p^t(\cdot)(\frac{1}{\sigma}-1)\right]n_t &= \beta\gamma n_{t+1}\left[cn_{t+1} - \left(2-\frac{1}{\sigma}\right)p^{t+1}(\cdot)\right]
\end{aligned}$$

Steady State Analysis

The above derivation is hard to take any further, but in the steady state equilibrium we can drop the time subscripts and solve for n. This gives:

$$
\begin{aligned}
\left[cn + p(\frac{1}{\sigma} - 1)\right] n &= \beta\gamma n \left[cn - \left(2 - \frac{1}{\sigma}\right) p\right] \\
cn + p(\frac{1}{\sigma} - 1) &= \beta\gamma \left[cn - \left(2 - \frac{1}{\sigma}\right) p\right] \\
(1 - \beta\gamma)cn &= -\beta\gamma \left(2 - \frac{1}{\sigma}\right) p - p\left(\frac{1}{\sigma} - 1\right) \\
(1 - \beta\gamma)cn &= \beta\gamma \left(\frac{1}{\sigma} - 2\right) p - p\left(\frac{1}{\sigma} - 1\right) \\
(1 - \beta\gamma)cn/p &= \beta\gamma \left(\frac{1}{\sigma} - 2\right) - \left(\frac{1}{\sigma} - 1\right) \\
(1 - \beta\gamma)cn/p &= (\beta\gamma - 1)\frac{1}{\sigma} - 2\beta\gamma + 1 \\
(1 - \beta\gamma)cn/p &= (\beta\gamma - 1)\frac{1}{\sigma} + (1 - \beta\gamma) - \beta\gamma \\
cn/p &= -\frac{1}{\sigma} + 1 - \frac{\beta\gamma}{(1 - \beta\gamma)} \\
cn/p &= 1 - \frac{1}{\sigma} - \frac{\beta\gamma}{(1 - \beta\gamma)}
\end{aligned}
$$

HOWEVER, THE TERM ABOVE ON THE RHS IS LESS THAN ZERO BC $\sigma < 1$ SO EITHER I'VE MADE A MISTAKE OR THERE IS NO STEADY STATE

And it's actually less than zero not because of the convexity of cost in n (in fact any positive exponent on n will still give this undesirable result.) Actually, you can even get this result with only an increasing cost function, so no functional form required. Also changing $\mathcal{R}_n$ such that increases to quantity increase reputation still has this terrible negative quantity produced business. That seems to indicate to me that there just may not be a steady state.

Market Equilibrium

Any equilibrium requires that I find a $p(r)$ and series of m and n over time such that market's clear $(m = n)$

Perturbation Exploration (reputational Shocks)

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