Money For Nothing, Clicks for Free

Aaron Graybill

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Introduction

$$\mathbb{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$$

$$\Gamma\Delta\Theta\Lambda\Xi\Sigma$$

The advent of social media and streaming services like Spotify, Youtube, and Instagram have given artists of all kinds unprecendented ability to connect their work to their audience (SOURCE?). However, the increased supply of artists makes it harder for each to cut through the noise and reach financial freedom from their craft (SOURCE?).

Throughout history, artists have had to earn the accolades of their fan. Yatta yatta, that's still true, but this paper explores how the introduction of internet media changes the economic incentives and decisions of artists.

This paper develops and analyzes a model of how artists build and maintain a reputation. This paper pulls from the cultural economics of (Rosen, 1981) and the branding literature (Klein & Leffler, 1981; Shapiro, 1983).

Literature Review

Outline:

1. Landscape of Literature

- Cultural Economics of Art Supply and Demand
- Branding literature
- Signalling
-

2. Gaps in literature
3. Applications of Literature Convexity of profit in talent

Actual Draft

Modeling an artist's optimal behavior in the media streaming market connects multiple broader fields of research. The first group is cultural economics which applies economic ideas to the markets for cultural goods. Relevant to the media streaming economy is

Model

Overview

I will now outline a model of how artistic reputation develops over time. To do this, I will first examine the consumer's problem in which she decides to consume a quantity of different artists' work taking as given the price and current reputation of the firm. The consumer's consumption choice will specify a demand function relating quantity consumed to price and current reputation. The artist (firm) takes the demand function and their current reputation as given and sets a price of their work which determines present profit and next period's reputation. The artist chooses prices in every period that maximize their discounted lifetime profits.

The Consumer's Problem

I begin by assuming that there exists a representative consumer who spends an amount E on artistic goods in every period. I assume that the value of art, does not come from a single good but from the portfolio of art that the consumer purchases. I additionally assume that the value of art fully depreciates after each period **THIS ASSUMPTION IS UBER-DUBIOUS BUT FIXING IT WOULD LIKELY BE INTRACTABLE**. **I GUESS IF OUR MOTIVATING EXAMPLE IS GOING TO CONCERTS THEN IT'S ALMOST OKAY BUT THAT MIGHT REQUIRE A DISCRETE CHOICE MODEL** I assume that there exists a continuum of partially substitutable artists indexed by $i \in [0, N]$, where N parametrizes the total number of artists. The consumer derives utility from the amount of art she consumes from artist i, denoted $n(i)$ as well as the reputation of the artist, $r(i)$, that she consumes from. In addition, I add an idiosyncratic and exogenous consumer preference parameter for each firm, $\alpha(i)$. I enter these three preference parameters multiplicatively following (Rosen, 1981) which gives them a high degree of substitutability. These assumptions are conducive to the now-canonical monopolistic competition model presented in (Dixit & Stiglitz, 1977). As such I let consumer utility be given by:

$$U = \left(\int_0^N \alpha(i) r(i) n(i)^\rho di \right)^{\frac{1}{\rho}}$$

We assume that consumers take the price for a certain artist's work as given, denoted $p(i)$.

As such, the consumer's only choice variable is number of units to consume from each artist, $n(i)$. She chooses this bundle to maximize her utility subject to her budget constraint: $\int_0^N n(i)p(i)di \leq E$.

The Marshallian demand under the Dixit-Stiglitz formulation is:

$$n^*(i) = \left(P \frac{\alpha(i) r(i)}{p(i)} \right)^\sigma \frac{E}{P}$$

Where the elasticity of substitution is given by $\sigma = \frac{1}{1-\rho}$, and the aggregate reputation-adjusted price level is $P = \left[\int_0^N \left(\frac{\alpha(\omega) r(\omega)}{p(\omega)} \right)^\sigma p(\omega) d\omega \right]^{\frac{1}{1-\sigma}}$. Since there are a continuum of firms, price changes from a finite or countably infinite number of firms has no affect on the aggregate price level, P.

The expression above gives the Marshallian demand for art from artist i in terms of the price that artist i sets and the reputation they have in the current period. The artist will use this demand function when deciding the optimal price level to set.

Consumer Comparative Statics NEEDS UPDATING First note that finite or countably infinite changes to idiosyncratic preferences, reputation, or prices have no affect on the aggregate price level P because the integral weighs an uncountably infinite number of artists. As such, for an artist i finite or countably infinite changes in tastes, preferences, and prices of *other* artists have no effect on how much quantity is demanded for artist i.

Some of the results below depend on whether or not the constant elasticity, $\sigma \lesseqgtr 1$. If the elasticity of substitution is greater than 1, then small changes to the price of one good causes disproportionately large changes tp the demand for other goods. This sort of behavior is likely inconsistent with the art market where we should expect price effects to dissipate over the continuum of sellers, not amplify. With that, for the purposes of this analysis, we assume that $\sigma < 1$.

Beginning with comparative statics, for any artist i, increases to the taste parameter satisfy $n_\alpha > 0$. Additionally, increases to reputation increase demand for a good because $n_r > 0$. Increases to an individual artist's price decrease consumption as expected, so $n_p < 0$. Further note that increases to income increase consumption as expected, $n_I > 0$.

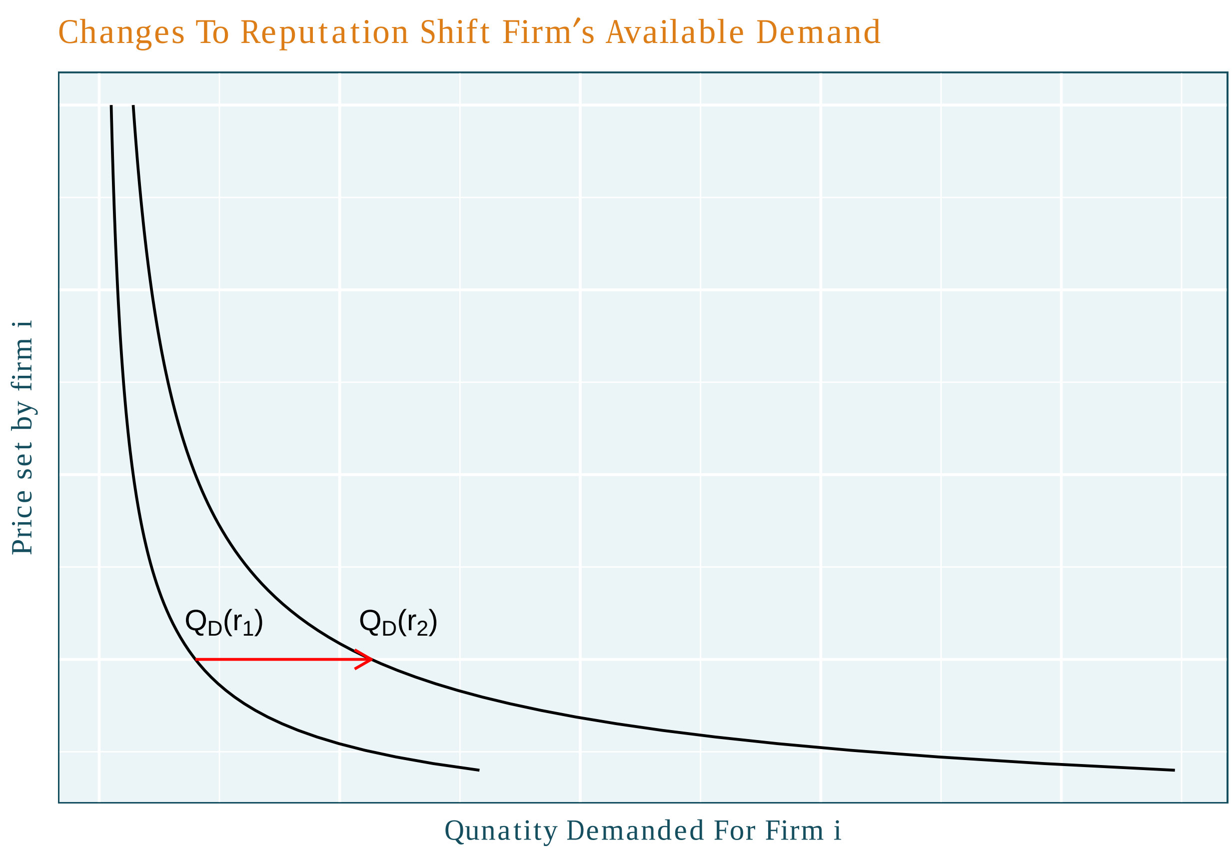
THE FOLLOWING RESULTS MAY REQUIRE SOME INADA-TYPE THINGS, AND SHOULD ALSO MAYBE BE $\geq$ AND NOT $>$, SHOULD WORK THROUGH TOGETHER

In terms of the overall price level, $n_P > 0$. This result stems from our assumption that $v''(g) < 0$, which implies that $[v']^{-1}$ is also decreasing. Under the assumption that $\sigma < 1$, $n_P > 0$, so increases to the overall price level increase consumption of each artistic good. This is equivalent to saying that the substitution effect dominates the income effect.

Finally, increasing the number of firms, the N in the upper bound of the integral, also satisfies $n_N > 0$, so holding other things equal more competition generates more consumer demand for each individual artist. One might interpret this result as the effect of genres of art benefiting from having more artists in that genre.

The Artist's Problem

We model the artist's problem dynamically. Let the artist have underlying artistic talent κ (it can be viewed as a relative of capital). The market is assumed to be monopolistically competitive, so every artist has full control of their price in every period, but price decisions may cause consumers to substitute to other products. The artist takes their current reputation as given. This assumption is used to simplify analysis. Allowing a flexible reputation in the current period obfuscates how reputation evolves by allowing producers to increase there reputation more quickly. As argued above, consumer demand is a function of both price and reputation. However, with the fixed reputation, the artist only has access to a single demand curve in each period. Changes to reputation in future periods shift the artist's available demand curve. This can be represented graphically in the following way.



To begin, I will take a two period case. We begin by assuming that the artist starts with reputation r_0. In period one, the artist chooses an amount to produce, n_1, and an amount to invest, I in their next period's reputation. We assume that investment is transformed into reputation according to $f(I)$ with next period's reputation, r', given by $r' = \kappa(1-\delta)r_0 + f(I)$.

In period 2, the artist inherits their reputation r' and chooses a quantity to produce n_2. The artist's intertemporal problem is to maximize their discounted profits over the two periods. This discounted profit function is given by:

$$\Pi = p(n_1, r_0)n_1 - C(n_1) - I + \beta(p(n_2, r')n_2 - C(n_2)) \; s.t. r' = \kappa(1-\delta)r_0 + f(I)$$

We can treat this an unconstrained multivariable maximization problem by substituting in for r'. Doing so yields the three following first order conditions:

$$\begin{aligned}
&\frac{\partial \Pi}{\partial n_1} : p_n(n_1, r_0)n_1 + p(n_1, r_0) - C'(n_1) = 0 \\
&\frac{\partial \Pi}{\partial n_2} : p_n(n_2, \kappa(1-\delta)r_0 + f(I))n_2 + p(n_2, \kappa(1-\delta)r_0 + f(I)) - C'(n_2) = 0 \\
&\frac{\partial \Pi}{\partial I} - 1 + \beta p_r(n_2, \kappa(1-\delta)r_0 + f(I)) \cdot f'(I) \cdot n_2 = 0
\end{aligned}$$

First note that the first order condition for n_1 is decoupled from all other first order conditions, so we can solve for n_1^* in isolation.

The other two first order conditions we have to work a bit harder for. Using the fact that $np_n(\cdot) = -\frac{1}{\sigma}p(\cdot)$, we can simply the first order conditions to:

$$\begin{aligned}
\left(1 - \frac{1}{\sigma}\right) p(n_2, \kappa(1-\delta)r_0 + f(I)) - C'(n_2) = 0 \\
-1 + \frac{\beta f'(I)n_2}{\kappa(1-\delta)r_0 + f(I)} p(n_2, \kappa(1-\delta)r_0 + f(I)) = 0
\end{aligned}$$

Without functional form assumptions on the cost function and the investment productivity funciton, we can't go any further, but we still can discuss comparative statics given some weak assumptions on the behavior of cost and investment functions respectively.

Production Comparative Statics

Entering into comparative statics, we need a few assumptions. Beginning with σ, the elasticity of substitution. I will assume that $\sigma < 1$ becuase we should expect the effect of price changes in one good to disapate over the other goods, not amplify.

I will further assume that both $C'(n)$ and $C''(n)$ are positive, so increasing production increases cost and at an increasing rate.

With that, the effect of increasing κ the talent parameter is ambiguous and depends on the sign of $\left(1 - \frac{1}{\sigma}\right) p_n(n_2, r') - C''(n_2)$ at the optimum.

So long as reputation is increasing in investment, we have that increasing talent κ decreases investment. Similarly, increasing reputation depreciation has an ambiguous effect on production, n_2, but increases reputation investment. Finally increasing initial reputation has an ambiguous effect on quantity produced and an negative effect on investment.

Steady State Analysis

The above derivation is hard to take any further, but in the steady state equilibrium we can drop the time subscripts and solve for n. This gives:

$$
\begin{aligned}
\left[cn + p(\frac{1}{\sigma} - 1)\right] n &= \beta\gamma n \left[cn - \left(2 - \frac{1}{\sigma}\right) p\right] \\
cn + p(\frac{1}{\sigma} - 1) &= \beta\gamma \left[cn - \left(2 - \frac{1}{\sigma}\right) p\right] \\
(1-\beta\gamma)cn &= -\beta\gamma \left(2 - \frac{1}{\sigma}\right) p - p\left(\frac{1}{\sigma} - 1\right) \\
(1-\beta\gamma)cn &= \beta\gamma \left(\frac{1}{\sigma} - 2\right) p - p\left(\frac{1}{\sigma} - 1\right) \\
(1-\beta\gamma)cn/p &= \beta\gamma \left(\frac{1}{\sigma} - 2\right) - \left(\frac{1}{\sigma} - 1\right) \\
(1-\beta\gamma)cn/p &= (\beta\gamma - 1)\frac{1}{\sigma} - 2\beta\gamma + 1 \\
(1-\beta\gamma)cn/p &= (\beta\gamma - 1)\frac{1}{\sigma} + (1-\beta\gamma) - \beta\gamma \\
cn/p &= -\frac{1}{\sigma} + 1 - \frac{\beta\gamma}{(1-\beta\gamma)} \\
cn/p &= 1 - \frac{1}{\sigma} - \frac{\beta\gamma}{(1-\beta\gamma)}
\end{aligned}
$$

HOWEVER, THE TERM ABOVE ON THE RHS IS LESS THAN ZERO BC $\sigma < 1$ SO EITHER I'VE MADE A MISTAKE OR THERE IS NO STEADY STATE

And it's actually less than zero not because of the convexity of cost in n (in fact any positive exponent on n will still give this undesirable result.) Actually, you can even get this result with only an increasing cost function, so no functional form required. Also changing $\mathcal{R}_n$ such that increases to quantity increase reputation still has this terrible negative quantity produced business. That seems to indicate to me that there just may not be a steady state.

Market Equilibrium

Any equilibrium requires that I find a $p(r)$ and series of m and n over time such that market's clear ($m = n$)

Perturbation Exploration (reputational Shocks)

Bibliography

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