arithmetic-tables.md

Arithmetic Tables

Addition Tables

0 plus	1 plus	2 plus
0 + 0 = 0	0 + 1 = 1	0 + 2 = 2
1 + 0 = 1	1 + 1 = 2	1 + 2 = 3
2 + 0 = 2	2 + 1 = 3	2 + 2 = 4
3 + 0 = 3	3 + 1 = 4	3 + 2 = 5
4 + 0 = 4	4 + 1 = 5	4 + 2 = 6
5 + 0 = 5	5 + 1 = 6	5 + 2 = 7
6 + 0 = 6	6 + 1 = 7	6 + 2 = 8
7 + 0 = 7	7 + 1 = 8	7 + 2 = 9
8 + 0 = 8	8 + 1 = 9	8 + 2 = 10
9 + 0 = 9	9 + 1 = 10	9 + 2 = 11
10 + 0 = 10	10 + 1 = 11	10 + 2 = 12
11 + 0 = 11	11 + 1 = 12	11 + 2 = 13
12 + 0 = 12	12 + 1 = 13	12 + 2 = 14

3 plus	4 plus	5 plus
0 + 3 = 3	0 + 4 = 4	0 + 5 = 5
1 + 3 = 4	1 + 4 = 5	1 + 5 = 6
2 + 3 = 5	2 + 4 = 6	2 + 5 = 7

3 plus	4 plus	5 plus
3 + 3 = 6	3 + 4 = 7	3 + 5 = 8
4 + 3 = 7	4 + 4 = 8	4 + 5 = 9
5 + 3 = 8	5 + 4 = 9	5 + 5 = 10
6 + 3 = 9	6 + 4 = 10	6 + 5 = 11
7 + 3 = 10	7 + 4 = 11	7 + 5 = 12
8 + 3 = 11	8 + 4 = 12	8 + 5 = 13
9 + 3 = 12	9 + 4 = 13	9 + 5 = 14
10 + 3 = 13	10 + 4 = 14	10 + 5 = 15
11 + 3 = 14	11 + 4 = 15	11 + 5 = 16
12 + 3 = 15	12 + 4 = 16	12 + 5 = 17

6 plus	7 plus	8 plus
0 + 6 = 6	0 + 7 = 7	0 + 8 = 8
1 + 6 = 7	1 + 7 = 8	1 + 8 = 9
2 + 6 = 8	2 + 7 = 9	2 + 8 = 10
3 + 6 = 9	3 + 7 = 10	3 + 8 = 11
4 + 6 = 10	4 + 7 = 11	4 + 8 = 12
5 + 6 = 11	5 + 7 = 12	5 + 8 = 13
6 + 6 = 12	6 + 7 = 13	6 + 8 = 14
7 + 6 = 13	7 + 7 = 14	7 + 8 = 15
8 + 6 = 14	8 + 7 = 15	8 + 8 = 16
9 + 6 = 15	9 + 7 = 16	9 + 8 = 17
10 + 6 = 16	10 + 7 = 17	10 + 8 = 18
11 + 6 = 17	11 + 7 = 18	11 + 8 = 19
12 + 6 = 18	12 + 7 = 19	12 + 8 = 20

9 plus	10 plus	11 plus
0 + 9 = 9	0 + 10 = 10	0 + 11 = 11
1 + 9 = 10	1 + 10 = 11	1 + 11 = 12
2 + 9 = 11	2 + 10 = 12	2 + 11 = 13
3 + 9 = 12	3 + 10 = 13	3 + 11 = 14
4 + 9 = 13	4 + 10 = 14	4 + 11 = 15
5 + 9 = 14	5 + 10 = 15	5 + 11 = 16
6 + 9 = 15	6 + 10 = 16	6 + 11 = 17
7 + 9 = 16	7 + 10 = 17	7 + 11 = 18
8 + 9 = 17	8 + 10 = 18	8 + 11 = 19
9 + 9 = 18	9 + 10 = 19	9 + 11 = 20
10 + 9 = 19	10 + 10 = 20	10 + 11 = 21
11 + 9 = 20	11 + 10 = 21	11 + 11 = 22
12 + 9 = 21	12 + 10 = 22	12 + 11 = 23

12 plus
0 + 12 = 12
1 + 12 = 13
2 + 12 = 14
3 + 12 = 15
4 + 12 = 16
5 + 12 = 17
6 + 12 = 18
7 + 12 = 19
8 + 12 = 20
9 + 12 = 21
10 + 12 = 22

12 plus
11 + 12 = 23
12 + 12 = 24

Multiplication Tables

0 times	1 times	2 times
0 × 0 = 0	0 × 1 = 0	0 × 2 = 0
1 × 0 = 0	1 × 1 = 1	1 × 2 = 2
2 × 0 = 0	2 × 1 = 2	2 × 2 = 4
3 × 0 = 0	3 × 1 = 3	3 × 2 = 6
4 × 0 = 0	4 × 1 = 4	4 × 2 = 8
5 × 0 = 0	5 × 1 = 5	5 × 2 = 10
6 × 0 = 0	6 × 1 = 6	6 × 2 = 12
7 × 0 = 0	7 × 1 = 7	7 × 2 = 14
8 × 0 = 0	8 × 1 = 8	8 × 2 = 16
9 × 0 = 0	9 × 1 = 9	9 × 2 = 18
10 × 0 = 0	10 × 1 = 10	10 × 2 = 20
11 × 0 = 0	11 × 1 = 11	11 × 2 = 22
12 × 0 = 0	12 × 1 = 12	12 × 2 = 24

3 times	4 times	5 times
0 × 3 = 0	0 × 4 = 0	0 × 5 = 0
1 × 3 = 3	1 × 4 = 4	1 × 5 = 5
2 × 3 = 6	2 × 4 = 8	2 × 5 = 10
3 × 3 = 9	3 × 4 = 12	3 × 5 = 15
4 × 3 = 12	4 × 4 = 16	4 × 5 = 20

3 times	4 times	5 times
5 × 3 = 15	5 × 4 = 20	5 × 5 = 25
6 × 3 = 18	6 × 4 = 24	6 × 5 = 30
7 × 3 = 21	7 × 4 = 28	7 × 5 = 35
8 × 3 = 24	8 × 4 = 32	8 × 5 = 40
9 × 3 = 27	9 × 4 = 36	9 × 5 = 45
10 × 3 = 30	10 × 4 = 40	10 × 5 = 50
11 × 3 = 33	11 × 4 = 44	11 × 5 = 55
12 × 3 = 36	12 × 4 = 48	12 × 5 = 60

6 times	7 times	8 times
0 × 6 = 0	0 × 7 = 0	0 × 8 = 0
1 × 6 = 6	1 × 7 = 7	1 × 8 = 8
2 × 6 = 12	2 × 7 = 14	2 × 8 = 16
3 × 6 = 18	3 × 7 = 21	3 × 8 = 24
4 × 6 = 24	4 × 7 = 28	4 × 8 = 32
5 × 6 = 30	5 × 7 = 35	5 × 8 = 40
6 × 6 = 36	6 × 7 = 42	6 × 8 = 48
7 × 6 = 42	7 × 7 = 49	7 × 8 = 56
8 × 6 = 48	8 × 7 = 56	8 × 8 = 64
9 × 6 = 54	9 × 7 = 63	9 × 8 = 72
10 × 6 = 60	10 × 7 = 70	10 × 8 = 80
11 × 6 = 66	11 × 7 = 77	11 × 8 = 88
12 × 6 = 72	12 × 7 = 84	12 × 8 = 96

9 times	10 times	11 times
0 × 9 = 0	0 × 10 = 0	0 × 11 = 0

9 times	10 times	11 times
1 × 9 = 9	1 × 10 = 10	1 × 11 = 11
2 × 9 = 18	2 × 10 = 20	2 × 11 = 22
3 × 9 = 27	3 × 10 = 30	3 × 11 = 33
4 × 9 = 36	4 × 10 = 40	4 × 11 = 44
5 × 9 = 45	5 × 10 = 50	5 × 11 = 55
6 × 9 = 54	6 × 10 = 60	6 × 11 = 66
7 × 9 = 63	7 × 10 = 70	7 × 11 = 77
8 × 9 = 72	8 × 10 = 80	8 × 11 = 88
9 × 9 = 81	9 × 10 = 90	9 × 11 = 99
10 × 9 = 90	10 × 10 = 100	10 × 11 = 110
11 × 9 = 99	11 × 10 = 110	11 × 11 = 121
12 × 9 = 108	12 × 10 = 120	12 × 11 = 132

12 times
0 × 12 = 0
1 × 12 = 12
2 × 12 = 24
3 × 12 = 36
4 × 12 = 48
5 × 12 = 60
6 × 12 = 72
7 × 12 = 84
8 × 12 = 96
9 × 12 = 108
10 × 12 = 120
11 × 12 = 132

12 times
$12 \times 12 = 144$

Table of Squares of 1-20

Squares of 1-10	Squares of 11-20
$1^2 = 1 \times 1 = 1$	$11^2 = 11 \times 11 = 121$
$2^2 = 2 \times 2 = 4$	$12^2 = 12 \times 12 = 144$
$3^2 = 3 \times 3 = 9$	$13^2 = 13 \times 13 = 169$
$4^2 = 4 \times 4 = 16$	$14^2 = 14 \times 14 = 196$
$5^2 = 5 \times 5 = 25$	$15^2 = 15 \times 15 = 225$
$6^2 = 6 \times 6 = 36$	$16^2 = 16 \times 16 = 256$
$7^2 = 7 \times 7 = 49$	$17^2 = 17 \times 17 = 289$
$8^2 = 8 \times 8 = 64$	$18^2 = 18 \times 18 = 324$
$9^2 = 9 \times 9 = 81$	$19^2 = 19 \times 19 = 361$
$10^2 = 10 \times 10 = 100$	$20^2 = 20 \times 20 = 400$

Table of Cubes of 1-10

Cubes
$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 = 3 \times 3 \times 3 = 27$
$4^3 = 4 \times 4 \times 4 = 64$
$5^3 = 5 \times 5 \times 5 = 125$
$6^3 = 6 \times 6 \times 6 = 216$
$7^3 = 7 \times 7 \times 7 = 343$
$8^3 = 8 \times 8 \times 8 = 512$

Cubes
$9^3 = 9 \times 9 \times 9 = 729$
$10^3 = 10 \times 10 \times 10 = 1000$

Divisibility Rules

Divisibility of a number by another number means that when the first number is divided by the second number, no remainder will be left. In that case the first number is said to be *divisible* by the second number. The purpose of divisibility rules is to be able to avoid having to perform an actual division to determine whether a number can be divided by another number without leaving a remainder in certain situations. An important use of such rules is in the simplification of fractions. Note that these rules are meant to be used together with multiplication tables and table of factorizations, to determine divisibility without having to perform a division.

Dividing with 2

A number is divisible by 2 if and only if its last digit is divisible by 2. In other words, if a number can be divided by 2 without leaving a remainder, then it has to mean that its last digit is one of 0, 2, 4, 6 or 8.

E.g. Out of the numbers 43, 58, 75, 90, 1027 and 2934; it is 58, 90 and 2934 that are divisible by 2, because their last digits are 8, 0 and 4 respectively.

NB: If a number is divisible by 2 it is called an *even* number; else it is called an *odd* number

Dividing with 3

A number is divisible by 3 if and only if the sum of its digits is divisible by 3. In other words, if a number can be divided by 3 without leaving a remainder, then it has to mean that when you add its digits together, you will get a number which is divisible by 3. And you can repeat the rule on each successive sum of digits obtained, until you get one of the multiples of 3 in the multiplication table, or until you get a single digit which will be 0, 3, 6 or 9 if and only if the starting number is divisible by 3.

E.g. Out of the numbers 24, 43, 58, 75, 90, 1027 and 2934; it is 24, 75, 90 and 2934 that are divisible by 3.

1. Taking 24, 2 + 4 = 6; and so getting 6 means 24 is divisible by 3 (could have avoided test by recognizing that 24 is a known multiple of 3)

2. Taking 43, 4 + 3 = 7, which is not 0, 3, 6 or 9; so 43 is not divisible by 3
3. Taking 58, 5 + 8 = 13. Then taking 13, 1 + 3 = 4, which is not 0, 3, 6 or 9; so 58 is not divisible by 3 (could have stopped test upon getting sum of 13)
4. Taking 75, 7 + 5 = 12. Then taking 12, 1 + 2 = 3; and so getting 3 means 75 is divisible by 3 (could have stopped test upon getting sum of 12)
5. Taking 90, 9 + 0 = 9; and so getting 9 means 90 is divisible by 3 (could have avoided test by ignoring the trailing zeros, and recognizing that 9 is a known multiple of 3)
6. Taking 1027, 1 + 0 + 2 + 7 = 10, which is known not to be a multiple of 3; so 1027 is not divisible by 3
7. Taking 2934, 2 + 9 + 3 + 4 = 18, which is a known multiple of 3; so 2934 is divisible by 3

Dividing with 5

A number is divisible by 5 if and only if its last digit is divisible by 5. In other words, if a number can be divided by 5 without leaving a remainder, then it has to mean that its last digit is either 0 or 5.

E.g. Out of the numbers 43, 58, 75, 90, 1027 and 2934; it is 75 and 90 that are divisible by 5, because their last digits are 5 and 0 respectively.

Dividing with 7

A number is divisible by 7 if and only if the difference between double of its last digit and the rest is divisible by 7. In other words, if a number can be divided by 7 without leaving a remainder, then it has to mean that when you subtract double of its last digit from the number formed by the remaining digits (or subtract the other way round to avoid negatives), you will get a number which is divisible by 7. And you can repeat the rule on each successive difference obtained, until you get one of the multiples of 7 in the multiplication table, or until you get a single digit which will be 0 or 7 if and only if the starting number is divisible by 7.

E.g. Out of the numbers 42, 58, 77, 161, 187 and 343; it is 42, 77, 161 and 343 that are divisible by 7.

1. Taking 42, 4 - (2 × 2) = 0; so 42 is divisible by 7 (could have avoided test by recognizing that 42 is a known multiple of 7)
2. Taking 58, (2 × 8) - 5 = 11; Then taking 11, (2 × 1) - 1 = 1, which is not 0 or 7; so 58 is not divisible by 7 (could have stopped test upon getting difference of 11, or could have avoided test altogether by recognizing that 58 is known not to be a multiple of 7)

3. Taking 77, (2 × 7) - 7 = 7; so 77 is divisible by 7 (could have avoided test by recognizing that 77 is a known multiple of 7)
4. Taking 161, 16 - (2 × 1) = 14; Then taking 14, (2 × 4) - 1 = 7; so 161 is divisible by 7 (could have stopped test upon getting difference of 14)
5. Taking 187, 18 - (2 × 7) = 4, which is not 0 or 7; so 187 is not divisible by 7
6. Taking 343, 34 - (2 × 3) = 28, which is a known multiple of 7; so 343 is divisible by 7 (could have avoided test by recognizing that 343 is the cube of 7)

Dividing with 9

A number is divisible by 9 if and only if the sum of its digits is divisible by 9. In other words, if a number can be divided by 9 without leaving a remainder, then it has to mean that when you add its digits together, you will get a number which is divisible by 9. And you can repeat the rule on each successive sum of digits obtained, until you get one of the multiples of 9 in the multiplication table, or until you get a single digit which will be 0 or 9 if and only if the starting number is divisible by 9.

E.g. Out of the numbers 43, 58, 75, 90, 1027 and 2934; it is 90 and 2934 that are divisible by 9.

1. Taking 43, 4 + 3 = 7, which is not 0 or 9; so 43 is not divisible by 9 (could have avoided test by recognizing that 43 is known not to be a multiple of 9)
2. Taking 58, 5 + 8 = 13. Then taking 13, 1 + 3 = 4, which is not 0 or 9; so 58 is not divisible by 9 (could have stopped test upon getting sum of 13, or could have avoided test altogether by recognizing that 58 is known not to be a multiple of 9)
3. Taking 75, 7 + 5 = 12. Then taking 12, 1 + 2 = 3, which is not 0 or 9; so 75 is not divisible by 9 (could have stopped test upon getting sum of 12, or could have avoided test altogether by recognizing that 75 is known not to be a multiple of 9)
4. Taking 90, 9 + 0 = 9; and so getting 9 means 90 is divisible by 9 (could have avoided test by recognizing that 90 is a known multiple of 9)
5. Taking 1027, 1 + 0 + 2 + 7 = 10, which is known not to be a multiple of 9; so 1027 is not divisible by 9
6. Taking 2934, 2 + 9 + 3 + 4 = 18, which is a known multiple of 9; so 2934 is divisible by 9

Dividing with 10

A number is divisible by 10 if and only if its last digit is 0.

E.g. Out of the numbers 43, 58, 75, 90, 1027 and 2934; it is 90 that is divisible by 10, because its last digit is 0.

NB: This rule can be generalized to any power of 10, ie a number which is 1 followed by zeros, e.g. 10, 100, 1000: A number is divisible by a power of 10 if and only if there are enough trailing zeros in the number to cancel out the zeros in the power of 10.

Dividing with 11

A number is divisible by 11 if and only if the difference between its last digit and the rest is divisible by 11. In other words, if a number can be divided by 11 without leaving a remainder, then it has to mean that when you subtract its last digit from the number formed by the remaining digits (or subtract the other way round to avoid negatives), you will get a number which is divisible by 11. And you can repeat the rule on each successive difference obtained, until you get one of the multiples of 11 in the multiplication table, or until you get a single digit which will be 0 if and only if the starting number is divisible by 11.

E.g. Out of the numbers 42, 58, 77, 161, 187 and 343; it is 77 and 187 that are divisible by 11.

1. Taking 42, 4 - 2 = 2, which is not 0; so 42 is not divisible by 11 (could have avoided test by recognizing that 42 is known not to be a multiple of 11)
2. Taking 58, 8 - 5 = 3, which is not 0; so 58 is not divisible by 11 (could have avoided test by recognizing that 58 is known not to be a multiple of 11)
3. Taking 77, 7 - 7 = 0; so 77 is divisible by 11 (could have avoided test by recognizing that 77 is a known multiple of 11)
4. Taking 161, 16 - 1 = 15; Then taking 15, 5 - 1 = 4, which is not 0; so 161 is not divisible by 11 (could have stopped test upon getting difference of 15)
5. Taking 187, 18 - 7 = 11, which is a known multiple of 11; so 187 is divisible by 11
6. Taking 343, 34 - 3 = 31, which is known not to be a multiple of 11; so 343 is not divisible by 11

Rule for Testing whether a Number in 1-150 is Prime or Composite

A *prime number* is a number greater than 1 that is not divisible by any number apart from itself and 1. The first 10 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. If a number is greater than 1 and is not a prime number, then it is a called a *composite number*. The number 1 is neither prime nor composite.

A sufficient test for determining whether a number in the range of 12-150 is prime or composite, is this: If a number between 12 and 150 is not divisible by each of the prime numbers less than 12 (i.e. 2, 3, 5, 7, 11), then the number is definitely a prime number. Else the number is definitely a composite number.

Tables for Factorisations of 1-100

NB: In the table below,

1. The numbers 1, 3, 5, 7 and 9 and any number whose last digit is 1, 3, 5, 7 or 9, have the additional description of odd numbers
2. The numbers 2, 4, 6 and 8 and any number whose last digit is 2, 4, 6, 8 or 0, have the additional description of even numbers
3. Multiples for numbers less than 13 are not listed because they can be found in the multiplication tables.

1	2	3
1 is • a perfect square: square root of 1 = 1 • a perfect cube: cube root of 1 = 1 • neither a prime number nor a composite number Some Factorisations: 1 = 1 × 1	2 is • a prime number Some Factorisations: 2 = 1 × 2	3 is • a prime number Some Factorisations: 3 = 1 × 3

4	5	6
4 is • a perfect square: square root of 4 = 2 • a composite number Some Factorisations: 4 = 2 × 2	5 is • a prime number Some Factorisations: 5 = 1 × 5	6 is • a composite number Some Factorisations: 6 = 2 × 3

7	8	9
7 is	8 is	9 is

<table>
<tr><th>7</th><th>8</th><th>9</th></tr>
<tr>
<td>a prime number
Some Factorisations:
7 = 1 × 7</td>
<td>a perfect cube: cube root of 8 = 2
a composite number
Some Factorisations:
8 = 2 × 4</td>
<td>a perfect square: square root of 9 = 3
a composite number
Some Factorisations:
9 = 3 × 3</td>
</tr>
</table>

<table>
<tr><th>10</th><th>11</th><th>12</th></tr>
<tr>
<td>10 is
a composite number
Some Factorisations:
10 = 2 × 5</td>
<td>11 is
a prime number
Some Factorisations:
11 = 1 × 11</td>
<td>12 is
a composite number
Some Factorisations:
12 = 2 × 6
12 = 3 × 4</td>
</tr>
</table>

<table>
<tr><th>13</th><th>14</th><th>15</th></tr>
<tr>
<td>13 is
a prime number
Some Factorisations:
13 = 1 × 13

Some Multiples:
13 × 2 = 26
13 × 3 = 39
13 × 4 = 52
13 × 5 = 65
13 × 6 = 78
13 × 7 = 91</td>
<td>14 is
a composite number
Some Factorisations:
14 = 2 × 7

Some Multiples:
14 × 2 = 28
14 × 3 = 42
14 × 4 = 56
14 × 5 = 70
14 × 6 = 84
14 × 7 = 98</td>
<td>15 is
a composite number
Some Factorisations:
15 = 3 × 5

Some Multiples:
15 × 2 = 30
15 × 3 = 45
15 × 4 = 60
15 × 5 = 75
15 × 6 = 90</td>
</tr>
</table>

<table>
<tr><th>16</th><th>17</th><th>18</th></tr>
<tr>
<td>16 is
a perfect square: square root of 16 = 4
a composite number</td>
<td>17 is
a prime number
Some Factorisations:
17 = 1 × 17</td>
<td>18 is
a composite number
Some Factorisations:
18 = 2 × 9
18 = 3 × 6</td>
</tr>
</table>

16	17	18
Some Factorisations: 16 = 2 × 8 16 = 4 × 4 Some Multiples: 16 × 2 = 32 16 × 3 = 48 16 × 4 = 64 16 × 5 = 80 16 × 6 = 96	Some Multiples: 17 × 2 = 34 17 × 3 = 51 17 × 4 = 68 17 × 5 = 85	Some Multiples: 18 × 2 = 36 18 × 3 = 54 18 × 4 = 72 18 × 5 = 90

19	20	21
19 is • a prime number Some Factorisations: 19 = 1 × 19 Some Multiples: 19 × 2 = 38 19 × 3 = 57 19 × 4 = 76 19 × 5 = 95	20 is • a composite number Some Factorisations: 20 = 2 × 10 20 = 4 × 5 Some Multiples: 20 × 2 = 40 20 × 3 = 60 20 × 4 = 80 20 × 5 = 100	21 is • a composite number Some Factorisations: 21 = 3 × 7 Some Multiples: 21 × 2 = 42 21 × 3 = 63 21 × 4 = 84

22	23	24
22 is • a composite number Some Factorisations: 22 = 2 × 11 Some Multiples: 22 × 2 = 44 22 × 3 = 66 22 × 4 = 88	23 is • a prime number Some Factorisations: 23 = 1 × 23 Some Multiples: 23 × 2 = 46 23 × 3 = 69 23 × 4 = 92	24 is • a composite number Some Factorisations: 24 = 2 × 12 24 = 3 × 8 24 = 4 × 6 Some Multiples: 24 × 2 = 48 24 × 3 = 72 24 × 4 = 96

<table>
<tr><th>25</th><th>26</th><th>27</th></tr>
<tr>
<td>25 is
a perfect square: square root of 25 = 5
a composite number
Some Factorisations:
25 = 5 × 5

Some Multiples:
25 × 2 = 50
25 × 3 = 75
25 × 4 = 100</td>
<td>26 is
a composite number
Some Factorisations:
26 = 2 × 13

Some Multiples:
26 × 2 = 52
26 × 3 = 78</td>
<td>27 is
a perfect cube: cube root of 27 = 3
a composite number
Some Factorisations:
27 = 3 × 9

Some Multiples:
27 × 2 = 54
27 × 3 = 81</td>
</tr>
</table>

<table>
<tr><th>28</th><th>29</th><th>30</th></tr>
<tr>
<td>28 is
a composite number
Some Factorisations:
28 = 4 × 7

Some Multiples:
28 × 2 = 56
28 × 3 = 84</td>
<td>29 is
a prime number
Some Factorisations:
29 = 1 × 29

Some Multiples:
29 × 2 = 58
29 × 3 = 87</td>
<td>30 is
a composite number
Some Factorisations:
30 = 3 × 10
30 = 5 × 6

Some Multiples:
30 × 2 = 60
30 × 3 = 90</td>
</tr>
</table>

<table>
<tr><th>31</th><th>32</th><th>33</th></tr>
<tr>
<td>31 is
a prime number
Some Factorisations:
31 = 1 × 31

Some Multiples:
31 × 2 = 62
31 × 3 = 93</td>
<td>32 is
a composite number
Some Factorisations:
32 = 4 × 8

Some Multiples:
32 × 2 = 64
32 × 3 = 96</td>
<td>33 is
a composite number
Some Factorisations:
33 = 3 × 11

Some Multiples:
33 × 2 = 66
33 × 3 = 99</td>
</tr>
</table>

34	35	36
34 is • a composite number Some Factorisations: 34 = 2 × 17 Some Multiples: 34 × 2 = 68	35 is • a composite number Some Factorisations: 35 = 5 × 7 Some Multiples: 35 × 2 = 70	36 is • a perfect square: square root of 36 = 6 • a composite number Some Factorisations: 36 = 3 × 12 36 = 4 × 9 36 = 6 × 6 Some Multiples: 36 × 2 = 72

37	38	39
37 is • a prime number Some Factorisations: 37 = 1 × 37 Some Multiples: 37 × 2 = 74	38 is • a composite number Some Factorisations: 38 = 2 × 19 Some Multiples: 38 × 2 = 76	39 is • a composite number Some Factorisations: 39 = 3 × 13 Some Multiples: 39 × 2 = 78

40	41	42
40 is • a composite number Some Factorisations: 40 = 4 × 10 40 = 5 × 8 Some Multiples: 40 × 2 = 80	41 is • a prime number Some Factorisations: 41 = 1 × 41 Some Multiples: 41 × 2 = 82	42 is • a composite number Some Factorisations: 42 = 6 × 7 Some Multiples: 42 × 2 = 84

43	44	45
43 is • a prime number	44 is • a composite number	45 is • a composite number

43	44	45
Some Factorisations: 43 = 1 × 43 Some Multiples: 43 × 2 = 86	Some Factorisations: 44 = 4 × 11 Some Multiples: 44 × 2 = 88	Some Factorisations: 45 = 5 × 9 Some Multiples: 45 × 2 = 90

46	47	48
46 is • a composite number Some Factorisations: 46 = 2 × 23 Some Multiples: 46 × 2 = 92	47 is • a prime number Some Factorisations: 47 = 1 × 47 Some Multiples: 47 × 2 = 94	48 is • a composite number Some Factorisations: 48 = 4 × 12 48 = 6 × 8 Some Multiples: 48 × 2 = 96

49	50	51
49 is • a perfect square: square root of 49 = 7 • a composite number Some Factorisations: 49 = 7 × 7 Some Multiples: 49 × 2 = 98	50 is • a composite number Some Factorisations: 50 = 5 × 10 Some Multiples: 50 × 2 = 100	51 is • a composite number Some Factorisations: 51 = 3 × 17

52	53	54
52 is • a composite number Some Factorisations: 52 = 4 × 13	53 is • a prime number Some Factorisations: 53 = 1 × 53	54 is • a composite number Some Factorisations: 54 = 6 × 9

55	56	57
55 is • a composite number Some Factorisations: 55 = 5 × 11	56 is • a composite number Some Factorisations: 56 = 7 × 8	57 is • a composite number Some Factorisations: 57 = 3 × 19

58	59	60
58 is • a composite number Some Factorisations: 58 = 2 × 29	59 is • a prime number Some Factorisations: 59 = 1 × 59	60 is • a composite number Some Factorisations: 60 = 5 × 12 60 = 6 × 10

61	62	63
61 is • a prime number Some Factorisations: 61 = 1 × 61	62 is • a composite number Some Factorisations: 62 = 2 × 31	63 is • a composite number Some Factorisations: 63 = 7 × 9

64	65	66
64 is • a perfect square: square root of 64 = 8 • a perfect cube: cube root of 64 = 4 • a composite number Some Factorisations: 64 = 8 × 8	65 is • a composite number Some Factorisations: 65 = 5 × 13	66 is • a composite number Some Factorisations: 66 = 6 × 11

67	68	69
67 is • a prime number	68 is • a composite number	69 is • a composite number

67	68	69
Some Factorisations: 67 = 1 × 67	Some Factorisations: 68 = 4 × 17	Some Factorisations: 69 = 3 × 23

70	71	72
70 is • a composite number Some Factorisations: 70 = 7 × 10	71 is • a prime number Some Factorisations: 71 = 1 × 71	72 is • a composite number Some Factorisations: 72 = 6 × 12 72 = 8 × 9

73	74	75
73 is • a prime number Some Factorisations: 73 = 1 × 73	74 is • a composite number Some Factorisations: 74 = 2 × 37	75 is • a composite number Some Factorisations: 75 = 5 × 15

76	77	78
76 is • a composite number Some Factorisations: 76 = 4 × 19	77 is • a composite number Some Factorisations: 77 = 7 × 11	78 is • a composite number Some Factorisations: 78 = 6 × 13

79	80	81
79 is • a prime number Some Factorisations: 79 = 1 × 79	80 is • a composite number Some Factorisations: 80 = 8 × 10	81 is • a perfect square: square root of 81 = 9 • a composite number Some Factorisations: 81 = 9 × 9

82	83	84
82 is	83 is	84 is

82	83	84
• a composite number Some Factorisations: 82 = 2 × 41	• a prime number Some Factorisations: 83 = 1 × 83	• a composite number Some Factorisations: 84 = 7 × 12

85	86	87
85 is • a composite number Some Factorisations: 85 = 5 × 17	86 is • a composite number Some Factorisations: 86 = 2 × 43	87 is • a composite number Some Factorisations: 87 = 3 × 29

88	89	90
88 is • a composite number Some Factorisations: 88 = 8 × 11	89 is • a prime number Some Factorisations: 89 = 1 × 89	90 is • a composite number Some Factorisations: 90 = 9 × 10

91	92	93
91 is • a composite number Some Factorisations: 91 = 7 × 13	92 is • a composite number Some Factorisations: 92 = 4 × 23	93 is • a composite number Some Factorisations: 93 = 3 × 31

94	95	96
94 is • a composite number Some Factorisations: 94 = 2 × 47	95 is • a composite number Some Factorisations: 95 = 5 × 19	96 is • a composite number Some Factorisations: 96 = 8 × 12

97	98	99
97 is • a prime number	98 is • a composite number	99 is • a composite number

97	98	99
Some Factorisations: 97 = 1 × 97	Some Factorisations: 98 = 7 × 14	Some Factorisations: 99 = 9 × 11

100
100 is • a perfect square: square root of 100 = 10 • a composite number Some Factorisations: 100 = 10 × 10

Procedure for Finding Factors of a Number

If possible, first find the prime factorization of the number for which factors are to be found, by making use of the table of factorizations. This involves starting with a factorization for the given number, and then replacing each factor in the factorization with one of the factor's own factorizations as well, until all numbers in the factorization are prime numbers. E.g.

- 5 = 5
- 12 = 3 × 4 = 3 × 2 × 2
- 19 = 19
- 20 = 2 × 10 = 2 × 2 × 5
- 60 = 4 × 15 = 2 × 2 × 5 × 3

To find factors of a number,

1. Start with 1 and the number itself.
2. *It is recommended to find prime factorization of the given number, in order to speed up subsequent steps in the procedure.*
3. *If the number itself can be determined to be prime, then the search is over.*
4. Else incrementally look for the next number between the numbers in the latest factorization found which divides the given number.
5. *Where a prime factorization was computed for the given number, the previous step is equivalent to finding the next number which can be expressed as a product of some of the prime factors of the given number.*
6. If such a number is not found, then the search is over.

7. Else determine the countepart number, which is the result of dividing the given number by the found number.
8. *Where a prime factorization was computed for the given number, the division can be replaced with a multiplication of the prime factors which were left over from expressing the found number as a product of prime factors.*
9. *If the counterpart number can be determined to be prime, then the search is over.*
10. Else search continues, but will now be restricted to the interval between the latest found number and its counterpart.

All of the numbers in the factorizations generated constitute the set of factors of the given number.

E.g. to find the factors of 12, and not make use of knowledge of primes

- *Found*: 1 × 12
- Searching between 1 and 12: Does 2 divide 12? Yes, and 12 ÷ 2 = 6
- *Found*: 2 × 6
- Searching between 2 and 6: Does 3 divide 12? Yes, and 12 ÷ 3 = 4
- *Found*: 3 × 4
- No integer exists between 3 and 4, so search ends.

So set of factors of 12 is: {1, 12, 2, 6, 3, 4}, or {1, 2, 3, 4, 6, 12}

E.g. to find the factors of 26, and not make use of knowledge of primes

- *Found*: 1 × 26
- Searching between 1 and 26: Does 2 divide 26? Yes, and 26 ÷ 2 = 13
- *Found*: 2 × 13
- Searching between 2 and 13: Does 3 divide 26? No. Does 4 divide 26? No. Does 5 divide 26? No. 6? No. 7? No. 8? No. 9? No. 10? No. 11? No. 12? No.
- No factor was found between 2 and 13, so search ends.

So set of factors of 26 is: {1, 26, 2, 13}, or {1, 2, 13, 26}

E.g. to find the factors of 12, and make use of knowledge of primes

- *Found*: 1 × 12
- Prime factorization of 12 = 6 × 2 = 2 × 3 × 2
- Search continues since 12 is not a prime number.
- Searching between 1 and 12: Can 2 be computed from the prime factors of 12? Yes, and the product of the leftover prime factors is 2 × 3 = 6.

- *Found*: 2 × 6
- Search continues since 6 is not a prime number.
- Searching between 2 and 6: Can 3 be computed from the prime factors of 12? Yes, and the product of the leftover prime factors is 2 × 2 = 4.
- *Found*: 3 × 4
- Search could have continued since 4 is not a prime number. However no integer exists between 3 and 4, so search ends.

So set of factors of 12 is once again: {1, 12, 2, 6, 3, 4}, or {1, 2, 3, 4, 6, 12}

E.g. to find the factors of 26, and make use of knowledge of primes

- *Found*: 1 × 26
- Prime factorization of 26 = 2 × 13
- Search continues since 26 is not a prime number.
- Searching between 1 and 26: Can 2 be computed from the prime factors of 26? Yes, and the leftover prime factor is 13.
- *Found*: 2 × 13
- Since 13 is a prime number, search ends.

So set of factors of 26 is once again: {1, 26, 2, 13}, or {1, 2, 13, 26}

Integer and Signed Number Procedures

Some Terminology

1. *Integer*: a signed whole number, i.e. zero, positive or negative whole number. E.g. 0, -1 and 7 are integers; 4/3 and 1/2 are not integers.
2. *Signed Number*: another name used in this document for real numbers. Real numbers are a set of numbers containing all integers and some non-integers. All numbers studied in junior high school are signed numbers.
3. *Magnitude of a signed number*: the part of a signed number without the sign. Equivalent to making a number positive. So magnitude of 0 is 0, magnitude of -1 is 1, magnitude of 3/4 is 3/4, magnitude of -1/2 is 1/2, magnitude of 4.5 is 4.5, magnitude of -0.65 is 0.65;

Addition and Subtraction

Given an addition or subtraction problem involving signed numbers (like 3 + -2, -2 − 5), first ensure that the second operand is not negative by using the following:

1. a + -b = a − b ; E.g. 3 + -2 = 3 - 2 ; -10 + -3 = -10 − 3
2. a − -b = a + b ; E.g. 3 − -2 = 3 + 2 ; -6 − -4 = -6 + 4

After that finish the calculation by employing the number line in the following way:

1. Start from zero, and locate the first operand on the number line to the left or right of zero depending on whether it is negative or positive respectively. Note this as an initial movement to the left or right.
2. Next, interpret an addition as a movement to the right and interpret a subtraction as a movement to the left.
3. Then if the initial movement and the movement suggested by the operation agree, add the magnitudes of the operands. The sign of the answer will be obvious from where the initial movement landed.
4. Else subtract the smaller magnitude from the larger magnitude. If the second movement is big enough to fully reverse the initial movement, then the sign of the answer will be the opposite of the destination of the initial movement. Else the sign of the answer will be where the initial movement landed.

Examples:

1. -2 − 5 = -7 ; -10 + -3 = -10 − 3 = -13 ; -3/4 − 1/4 = -1 ; In these cases first operand is negative, minus positive number. So the magnitude of the answer will be the sum of the magnitudes of the operands; and the answer will be negative.
2. 3 − -2 = 3 + 2 = 5 ; 3/4 + 1/4 = 1; Here the first operand is positive, plus positive number. So the magnitude of the answer will be the sum of the magnitudes of the operands; and the answer will be positive.
3. -3 + 2 = -1 ; -2 + 3 = 1 ; -6 − -4 = -6 + 4 = -2 ; -3/4 + 1/4 = -1/2; -1/4 + 3/4 = 1/2; In these cases the first operand is negative, plus positive number. So the magnitude of the answer will be the larger magnitude minus the smaller magnitude; and the sign of the answer will be the sign of the operand with the larger magnitude.
4. 3 − 2 = 1 ; 1 − 4 = -3 ; 3 + -2 = 3 − 2 = 1 ; 2 + -3 = 2 − 3 = -1 ; 3/4 − 1/4 = 1/2; 1/4 − 3/4 = -1/2; In these cases the first operand is positive, minus positive number. Therefore the magnitude of the answer will be the larger magnitude minus the smaller magnitude. If the first operand has the larger magnitude, then its sign will be the sign of the answer. Else the sign of the answer will be the opposite of the sign of the first operand.

Comparison

Procedure:

1. *Consider numbers as represent cash amounts*, in which a positive number means owning money, a negative number means owing money, zero means neither owing or owning money, and "larger than" means "more desirable than".
2. Positive numbers and zero are considered larger than negative numbers.
3. A negative number is considered larger than another negative number if it has a *smaller* magnitude than the other negative number. So even though $2 < 3$, $-2 > -3$, like how owing 2 is more desirable than owing 3.

Examples:

1. $10 > 5$, $5 < 10$, $4 > 0$, $0 < 4$
2. $5 > -10$, like how owning 5 is more desirable than owing 10 ; $-10 < 5$.
3. $0 > -4$, like how neither owing or owning money is more desirable than owing 4. ; $-4 < 0$.
4. $-5 > -10$, like how owing 5 is more desirable than owing 10.; $-10 < -5$.

Multiplication and Division

1. $-a \times -b = a \times b$; E.g. $-2 \times -3 = 6$
2. $-a \div -b = a \div b$; E.g. $-10 \div -2 = 5$
3. $-a \times b = -(a \times b)$; E.g. $-2 \times 3 = -6$
4. $-a \div b = -(a \div b)$; E.g. $-10 \div 2 = -5$
5. $a \times -b = -(a \times b)$; E.g. $2 \times -3 = -6$
6. $a \div -b = -(a \div b)$; E.g. $10 \div -2 = -5$

Negation

1. $-(a) = -1 \times a = -a$; E.g. $-(2) = -2$; $-(0) = 0$
2. $-(-a) = -1 \times -a = a$; E.g. $-(-2) = 2$; $---2 = -(-(-2)) = -2$

Powers and Roots

1. Square roots of negative numbers are undefined, ie are not real numbers. E.g. the square root of -4 is undefined; the square root of -9 is undefined.
2. Other than square roots of negative numbers, powers and roots of integers work as expected. E.g. the square of -2 is 4, not -4; the square of -3 is 9; the cube root of -8 is -2; the cube root of -27 is -3.

Fraction Procedures

Some Terminology

1. *Proper fraction*: a fraction in which the numerator is smaller than the denominator. E.g. 1/2, 1/3, 2/3
2. *Improper fraction*: a fraction in which the numerator is bigger than the denominator. E.g. 3/2, 4/3, 5/3
3. *Simple fraction*: a proper or improper fraction in which the numerator and denominator are whole numbers. E.g. 1/2 is a simple fraction, 1/(3/2) and (2/3)/5 are not simple fractions.
4. *Mixed fraction*: a fraction which is expressed in terms of a whole number part and a proper fraction part. E.g. 2 1/2, 5 3/4
5. *Ratio*: an expression involving positive numbers used to determine how to share a nonnegative number among two or more entities through repeated subtractions of whole numbers or parts of whole numbers.
    - A ratio for sharing between two entities is expressed as a ∶ b (read as "a is to b")
    - E.g. Kofi and Kwame share a collection of 100 pencils in the ratio of 2 ∶ 3 (read as "2 is to 3"), means that for every 2 pencils that Kofi takes out of the collection, Kwame gets to take 3 pencils.
    - Similarly a ratio for sharing among 3 entities is expressed as a ∶ b ∶ c (read as "a is to b is to c").
    - E.g. Ama, Yaa and Akos share a collection of 24 canned drinks in the ratio 2 ∶ 3 ∶ 1 (reas as "2 is to 3 is to 1"), means that for every 2 cans that Ama takes out of the collection, Yaa gets to take 3 cans and Akos gets to take 1 can.
6. *of* operator: a/b of c/d = a × (c/d ÷ b). E.g.
    - 3 of 4 = 4 + 4 + 4 = 12 ; 4 of 3 = 3 + 3 + 3 + 3 = 12
    - 4 of 1/2 = 1/2 + 1/2 + 1/2 + 1/2 = 2
    - 1/2 of 4 = 4/2 = 2 ; 1/3 of 5 = 5/3
    - 2/3 of 6 = 2 × (6/3) = 2 × 2 = 4 ; 3/5 of 7/10 = 3 × (7/10 ÷ 5) = 3 × 7/50 = 21/50
7. *Cross multiplication*: given two fractions a/b and c/d, cross multiplication refers to the action of multiplying a by d, and separately multiplying b by c to obtain two products (a × d) and (b × c). Cross multiplication is a heavily used step in fraction arithmetic.
8. *A fraction is treated in mathematics as being equivalent to division of the numerator by the denominator*, even if the numerator and denominator is not a whole number.
9. *A ratio of two numbers is treated in mathematics as being equivalent to a fraction*, even if one of the two numbers is not a whole number.
10. *The "of" operator is equivalent to multiplication of fractions in mathematics.*

Addition and Subtraction

Procedures:

1. a/b + c/d = (a × d + b × c)/(b × d)
2. a/b − c/d = (a × d − b × c)/(b × d)
3. If the denominators are the same, then the formulas simplify to become:
    i. a/e + c/e = (a + c)/e
    ii. a/e − c/e = (a − c)/e

Examples:

1. Same denominators:
    - 3/4 + 1/4 = (3 + 1)/4 = 4/4 = 1
    - -3/4 + 1/4 = (-3 + 1)/4 = -2/4 = -1/2
    - 3/4 - 1/4 = (3 - 1)/4 = 2/4 = 1/2
2. Different denominators: *Use cross multiply*.
    - 5/4 + 3/5 = (5 × 5 + 4 × 3)/(4 × 5) = (25 + 12)/20 = 37/20
    - 3/2 - 5/4 = (3 × 4 - 2 × 5)/(2 × 4) = (12 - 10)/8 = 2/8 = 1/4
    - -3/2 - 5/4 = (-3 × 4 - 2 × 5)/(2 × 4) = (-12 - 10)/8 = -22/8 = -11/4

Comparison

Procedure: To compare a/b and c/d to see which is larger, ensure denominators b and d have the same sign (by multiplying one fraction by -1/-1 if necessary), and then compare (a × d) and (b × c) to see which is larger

Examples: *Use cross multiply*.

- 3/4 vrs 2/3, which is larger? Equivalent to (3 × 3) vrs (4 × 2), which is larger? Equivalent to 9 vrs 8, which is larger? And so since 9 > 8, 3/4 > 2/3
- 2/4 vrs 1/2, which is larger? Equivalent to (2 × 2) vrs (4 × 1), which is larger? Equivalent to 4 vrs 4, which is larger? And so since 4 = 4, 2/4 = 1/2
- 1/4 vrs 2/7, which is larger? Equivalent to (1 × 7) vrs (4 × 2), which is larger? Equivalent to 7 vrs 8, which is larger? And so since 7 < 8, 1/4 < 2/7
- -11/4 vrs -20/7, which is larger? Equivalent to (-11 × 7) vrs (4 × -20), which is larger? Equivalent to -77 vrs -80, which is larger? And so since -77 > -80, -11/4 > -20/7
- 11/-4 vrs 20/7, which is larger? First ensure denominators are all positive by multiplying top and down of 11/-4 by -1 to get -11/4. So now it is -11/4 vrs 20/7. Equivalent to (-11 × 7) vrs (4 × 20), which is larger? Equivalent to -77 vrs 80, which is larger? And so since -77 < 80, 11/-4 < 20/7. Alternatively, comparison test could have

been made quicker by observing that 11/-4 is a negative number and 20/7 is a positive number. So since all negative numbers are considered less than positive numbers, 11/-4 < 20/7.

Multiplication

Procedure:

1. a/b × c/d = (a × c)/(b × d); NB: a/b of c/d is same as a/b × c/d

Examples:

- 3/5 of 7/5 = (3 × 7)/(5 × 5) = 21/25
- 5/4 of 3/15 = (5 × 3)/(4 × 15) = 1/4
- 5/4 × -3/5 = (5 × -3)/(4 × 5) = -15/20 = -3/4
- 5/4 × -3 = 5/4 × -3/1 = (5 × -3)/(4 × 1) = -15/4
- 7 × 2/3 = 7/1 × 2/3 = (7 × 2)/(1 × 3) = 14/3

Division

Procedure:

1. a/b ÷ c/d = (a × d)/(b × c)
2. If the denominators are the same, then the formula simplifies to become: a/e ÷ c/e = a/c

Examples:

1. Same denominators:
    - 4/5 ÷ 3/5 = 4/3
2. Different denominators: *Use cross multiply*.
    - 4/5 ÷ 3/15 = (4 × 15)/(5 × 3) = 60/15 = 4
    - -4/5 ÷ -3/4 = (-4 × 4)/(5 × -3) = -16/-15 = 16/15