PH-107 (2017)
Tutorial Sheet 11

* Problems to be done in tutorial.

Statistical Mechanics (Density of States and Fermi Energy)

P90. A system has 1 state with energy 0, 4 states with energy 2E and 8 states with energy 3E. Six electrons (spin-half particles obeying Pauli's exclusion principle) are to be distributed among these states such that their total energy is 12E. Consider a configuration (j,m,n) in which j electrons are in state of energy 0, m electrons are in states of energy 2E and n electrons are in states of energy 3E.
(a) Calculate the total number of microstates for the configuration (1,3,2).
(b) What is the ratio of probability of occurrence of a configuration (2,0,4) to that of a configuration (1,3,2)?

P91*: The spin independent energy levels of a carbon nanotube are described by those of a 1-dimensional infinite potential well. In a carbon nanotube of length 1 μm, electrons occupy all the energy levels, up to 0.1 eV.
(a) Calculate the number of electrons in the carbon nanotube.
(b) Calculate the density of states g(E) at the Fermi energy (E_F) in units of $[(\text{eV})^{-1} (\mu\text{m})^{-1}]$.

P92: Consider a non-interacting Fermi gas of N particles in two dimension, confined in a square area $A=L^2$ (a) Derive a formula for the density of states, g(E) (b) Find the Fermi energy E_F (in terms of N and A), and show that the average energy per particle E/N at T = 0 is $E_F/2$.

P93. Consider a particle confined to a potential

$$V(x,y,z)=\frac{1}{2}m\omega^2(x^2+y^2+4z^2)$$

(a) Calculate the ground state energy of the particle.
(b) What is the degeneracy of the state with energy, $E=7\hbar\omega$?
(c) For $E=n\hbar\omega\ (n>>1)$, calculate the density of states $g(n)$.

Q94*. Consider a gas of N identical bosons confined by an isotropic three-dimensional harmonic potential. The energy levels in this potential are $\varepsilon = nhf$, where n is non-negative integer and f is classical oscillation frequency. The degeneracy of the level is (n+1)(n+2)/2. Find the density of states, for atoms confined by this potential. You may assume n>>1.

P95. Mass of the Sun is given by M_{sun}=2x10^{30} Kg. Ignore electron spin in this problem.
(a) Estimate the number of electrons (N_{sun}) in the Sun.
(b) In a white dwarf star, this number of electrons (N_{sun}) is contained in a sphere of radius 2x10^{7} m. Assuming these electrons to be non-relativistic, estimate the Fermi energy (in eV) of this electron system.

(c) Consider another star of volume V. All its electrons, N in number, are extremely relativistic such that the electron rest mass energy m_ec^2<<pc, where p is the momentum of the electron. Obtain an expression for the Fermi energy of this star.

P96: Use Bose-Einstein Statistics and the density of state expression, with suitable modifications, if any, to derive Planck's formula of black body radiation.

P97*: Show that the kinetic energy of a three dimensional gas of *N* free electrons at *0 K* is $(3/5)N\varepsilon_F$.

P98*: (a) Using the Fermi Dirac (FD) Statistics, find the probability that a state is occupied if its energy is higher than ε_F by $0.1k_BT$, $1.0k_BT$, $2.0k_BT$ and $10.0k_BT$, where ε_F is the Fermi Energy. How good is the approximation of neglecting *1* in the denominator for an energy equal to $10k_BT$.(b) In the Fermi Dirac distribution substitute $\varepsilon = \varepsilon_F+\delta$. Compute δ for the probability of occupancy equal to *0.25* and *0.75*. (c) Show that for a distribution system governed by F.D. distribution, the probability of occupation of a state with energy higher than ε_F by an amount ΔE is equal to the probability that a state with energy lower than ε_F by ΔE is unoccupied.

P99: The Fermi energy of Cu is 7.04 eV. Calculate the velocity and de Broglie wavelength of electrons at the Fermi energy of Cu. Can these electrons be diffracted by a crystal?

P100*: Show that the fraction of electrons within k_BT of the Fermi energy is $1.5k_BT/\varepsilon_F$, under the assumption that the temperature is so low that the probability of occupancy of levels is not altered from the one at 0^oK. Calculate numerically the value of this fraction for copper (ε_F= *7.04 eV*) at 300^oK and 1360^oK (approximate melting point of Cu). This fraction is of interest because it is a rough measure of the percentage of electrons excited to higher energy states at a temperature *T*. Find roughly the electronic contribution to specific heat of Cu using this expression.