PH 107 Mid-Sem Review

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What you should know...

- Wien's Displacement Law

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- **Rayleigh-Jeans Law** - The energy per unit volume in the frequency interval ν to $\nu + d\nu$ of the blackbody spectrum of a cavity at temperature T is

$$u(\nu, T)d\nu = \frac{8\pi\nu^2 \mathbf{kT}}{c^3} d\nu$$

$\langle E \rangle = kT$; The Ultraviolet Catastrophe

What you should know...

- **Plank's Theory** $\rightarrow E = nh\nu$ The Birth of Quantum Mechanics!

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} nh\nu \exp\left(nh\nu/kT\right)}{\sum_{n=0}^{\infty} \exp\left(nh\nu/kT\right)} = \frac{h\nu}{\exp\left(nh\nu/kT\right) - 1}$$

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- Specific Heat of Solids

$$U = 3N \frac{h\nu}{e^{nh\nu/kT} - 1}$$

$$C = \frac{\partial U}{\partial T} = 3R \left(\frac{h\nu}{kT}\right)^2 e^{-h\nu/kT}$$

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- **Bohr's Quantisation**

$$mvr = n\hbar$$

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- Relativistic Quantites
  - Rest Mass Energy

$$E_{rest} = mc^2 = 0.511 MeV \text{ for } e$$

  - Momentum

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \gamma mv$$

  - Total Relativistic Energy

$$E_{total} = \sqrt{p^2c^2 + m^2c^4} = \gamma mc^2$$

  - Kinetic Energy

$$T = E_{total} - E_{rest} = \sqrt{p^2c^2 + m^2c^4} - mc^2 = (\gamma - 1)\, mc^2$$

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- Matter Wave as a Wave Packet with finite extension

$$v_p = \frac{\omega}{k}$$

$$v_g = \frac{d\omega}{dk} = v_p + k\frac{dv_p}{dk}$$

When $\hbar\omega = E_{total}$

$$v_g = \frac{c^2}{v_p}$$

What you should know...

- **Davisson & Germer Experiment**

$$n\lambda = 2d \sin\theta$$

Where θ is the angle with the crystal plane. or,

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- **Young's Double-Slit Experiment** Position of n^{th} bright fringe from central-maxima

$$x_n = \frac{n\lambda D}{d} = n\beta$$

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- **Wave-Packet**

$$\psi(x) = \int_{-\infty}^{\infty} A(k) \sin\left(kx + \phi(k)\right) dk$$

A, ϕ are obtained from the 'Fourier Transform' of $\psi(x)$

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- Uncertainty Relations

$$\Delta x \Delta p_x \geq \hbar/2 \ , \ \Delta E \Delta t \geq \hbar/2$$

Some Questions for Practise

Question from Quiz-1 2014

Using equipartition law, find the molar specific heat of a gas (at constant volume), which contains tri-atomic molecules with the atoms bonded (non rigidly) to each other in a triangular form.

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$C_V = 6R$

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In a dispersive medium, the frequency- wavelength relationship of certain type of waves is such that phase velocity is v_0 for a wavelength λ_0, but is $v_0/2$ for the wavelength $2\lambda_0$. Find the group velocity of the waves at the wavelengths λ_0 and $2\lambda_0$ in terms of v_0. Also find the angular frequency ω of the wave for these two wavelengths, in terms of v_0 and λ_0.

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$v_g(\lambda_0) = 2v_0$
$v_g(2\lambda_0) = v_0$
$\omega(\lambda_0) = \frac{2v_0}{\lambda_0}$
$\omega(2\lambda_0) = \frac{2v_0}{4\lambda_0}$

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Question from Quiz-1 2014

A photon with energy equal to $3m_0c^2$ is scattered by a particle of rest mass m_0, initially at rest. After the scattering, both the photon and the particle move in different directions making equal angles α with the direction of the incident photon. Find

- the value of α
- the energy of the scattered photon in terms of m_0c^2 and
- de-Broglie wave length of the scattered particle.

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$\alpha = \cos^{-1}(4/5)$
$h\nu' = \frac{15}{8} m_0 c^2$
$\lambda_{de-Broglie} = \frac{8}{15}\frac{h}{m_0 c}$

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A particle of mass m is confined to a region where a potential $V(x) = V_0 \frac{|x|}{a}$, $(-\infty < x < \infty)$ exists. It is given that V_0 has the dimensions of potential and a has the dimension of length. Both V_0 and a are constants. Using the uncertainty principle with $\Delta x \Delta p_x = h$, find out the ground state energy of the particle.

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$E_0 = \left(\frac{27h^2 V_0^2}{32ma^2}\right)^{1/3}$

Some Questions for Practise

Question from Serway's Book

A woman on a ladder drops small pellets toward a spot on the floor.

- Show that, according to the uncertainty principle, the miss distance must be at least
$$\Delta x = \left(\frac{\hbar}{2m}\right)^{1/2}\left(\frac{2H}{g}\right)^{1/4}$$
where H is the initial height of each pellet above the floor and m is the mass of each pellet.
- If $H = 2.0m$ and $m = 0.50g$, what is x?

Some Questions for Practise

Question from Mid-Sem 2014

The absolute temperature of a perfectly black body is changed so that the peak wavelength in its spectrum increases by 10%.

- What will be the ratio of the total initial and final power radiated by the black body?

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- Assume a one-dimensional chain of Avogadro number N_A of atoms. If all motions are confined only in one dimension, use Einstein's model to find the specific heat at an absolute temperature T for such a chain. Also find the lower and higher temperature limits of the specific heat.

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$$C_V = \frac{(h\nu)^2}{(kT)^2\left(e^{\frac{h\nu}{kT}}-1\right)^2} R$$

$$C_V = \frac{(h\nu)^2}{(kT)^2} e^{-\frac{2h\nu}{kT}} R \text{ (low T)}$$

$$C_V = R \text{ (high T)}$$

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- The dispersion relation for electrons in a medium is given by $\omega = A\sin\left(\frac{ka}{2}\right)$. Here A is a constant, a is a constant with the dimension of length. ω and k have their usual meaning. Plot the variation of the group velocity of the electron as a function of k in the range $0 \leq k \leq \frac{\pi}{a}$

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- Assume that in a hydrogen atom, the life time of an excited state is $\approx 10^{-8} s$. Taking the uncertainty product as $\hbar$, find the width of the energy levels. For which value of quantum number 'n', the separation between two consecutive levels will be of the same order as the width of the level.

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$n \approx 746$

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Question from Mid-Sem 2014

A photon is incident on a hydrogen atom at rest. The hydrogen atom is initially in its ground state. The photon ionizes the atom and as a result an electron is released with certain kinetic energy. Neglect the recoil energy of the hydrogen atom nucleus. Immediately after the release, the electron encounters a positron (a particle with same mass as that of electron but with positive charge), which is at rest. The electron and positron form a positronium atom in the first excited state. (A positronium atom is one in which electron and positron revolve around their common centre of mass). As a result of formation of this atom, a photon is released, which moves in the same direction as that of the incident electron. It is found that this photon has a wavelength which is four times the incident (original) photon. (Apply Bohr's model to this problem and assume that all energy and speeds are non-relativistic). Find **(a)** the energy of the original photon and **(b)** the speed with which the positronium atom would be moving.

Some Questions for Practise

Question from Tut-3 2014

The band structure of a solid in the low wave vector limit is approximately given by the following equation

$$\hbar\omega = Ak^2 - Bk^4$$

where A and B are constants.

- Find the angular frequency ω at which the group & phase velocities are the same.
- Show that if the second term in the dispersion relation is neglected, the group velocity of electrons would be twice that phase velocity.

Some Questions for Practise

Question from End-Sem 2014

The atomic number and mass number of triply ionized beryllium (Be^{3+}) atom are 4 and 9 respectively. Assume this as a hydrogen-like atom, with the nuclear mass much larger than the electron mass. Initially an isolated Be^{3+} ion is in an excited state of quantum number $2n$. By emitting a photon of energy $40.8eV$, it de-excites to the state with quantum number n. Find the values of n and the ground state energy (in eV). Also find the energy (in eV) of the photon with maximum energy that this ion can emit when it de-excites from the level with quantum number $2n$.

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$n = 2$

$E_{gs} = -217.6eV$

$\Delta E_{max} = 204eV$

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The width is less than the separation between the emission and absorption lines, hence resonance absorption will not take place

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$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{2}{3}}$