Quizes: Week 2

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D1/D2 Quiz 2

Let $f(x)$ be defined as follows.

$$f(x) = \begin{cases} a & \text{if} \quad x \leq 0 \\ bx & \text{otherwise.} \end{cases}$$

Using the $\epsilon - \delta$ definition of the limit, determine if $f(x)$ is continuous at 0 for the following value of a and b.

A) $a = 2,\ b = 3$ B) $a = 1,\ b = 2$

C) $a = 2,\ b = 1$ D) $a = 3,\ b = 2$.

Marking scheme

Again, I'll give the marking scheme for A)
We will show that $f(x)$ is discontinous at 0. We need to find an $\epsilon > 0$ such that for every $\delta > 0$ there exists at least one x with $0 < |x - 0| < \delta$ and $|f(x) - 2| \geq \epsilon$.

(1 mark)

Above, you can give 1 mark if the student writes $f(0)$ instead of 2. You can also give 1 mark if the students negates the definition of sequential continuity.

Let $\epsilon = 1$ (anything less than or equal to 2 will work, but the interval in which we take x below will change)

(1 mark)

For all $x \in (0, 1)$, $f(x) < 1$. Hence $|f(x) - 2| \geq 1$. It follows that in every interval $(-\delta, \delta)$ there is a point $x \neq 0$ such that $|f(x) - 2| \geq 1$. This shows that f is discontinuous at 0.

(1 mark)

Alternate solution

Some of you have apparently shown that the left-hand and right-hand limits are different. If you have correctly used the $\epsilon - \delta$ definition of the left- and right-hand limits to do this, you will get credit for this. Those of you who have just asserted that the limits are different (i.e., you have guessed the values of the limits but have not shown that these values are actually the limits usng the $\epsilon - \delta$ definition) will not be given credit, since the question asked you explicitly to use the $\epsilon - \delta$ definition.