Roll No:

Q1. A 1.0 microgm speck of dust is trapped to roll inside a tube of length L = 1.0 micrometer. The tube is capped at both ends and the motion of speck is considered along the length.

(a) Modeling this as a one-dimensional infinite square well, determine the value of the quantum number 'n' if the speck has an energy of 1.0 microJoule.

(b) What is the probability of finding this speck within 0.1 micrometer (0.45<x<0.55) of the center of the tube.

(c) How much energy is needed to excite this speck to an energy level next to 1 microJoule ? Compare this excitation energy with the thermal energy at room temperature (T=300 K).

[2+2+2 Marks]

a) $$\frac{n^2\pi^2\hbar^2}{2mL^2} = 10^{-6} \text{ Joule} \longrightarrow \frac{1}{2} \text{ marks}$$

$$\frac{n^2 \times (3.14)^2 \times (1.05\times10^{-34})^2}{2\times 10^{-9}\times(10^{-6})^2} = 10^{-6}$$

$$\boxed{n \simeq 1.356\times10^{+20}} \longrightarrow 1\frac{1}{2} \text{ marks}$$

(Give full marks if $n \sim 10^{19} - 10^{21}$

(b) Because n is too large, this actually can be considered as a classical analogue of the Quantum problem.

And the probability of finding the spee with 0.1 μm will be 0.1. → 2 marks

** Few students will start to calculate the Integral from x = 0.45 to x = 0.55 and arrive at ~0.1. Give full marks.

(c) Excitation Energy $\Delta E = E_{n+1} - E_n = \frac{(2n+1)\pi^2\hbar^2}{2mL^2}$

or $\Delta E \simeq \frac{n\,\pi^2\hbar^2}{mL^2}$ [because n is $\gg 1$]

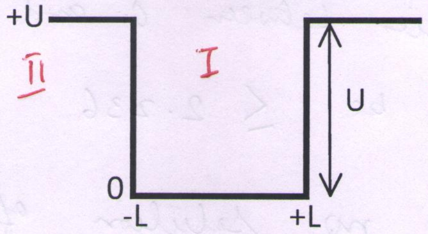
$\Delta E \sim 14.74\times10^{-27}$ Joule

$\boxed{\Delta E \sim 9.21\times10^{-8}\ eV} \longrightarrow$ 1 marks

Thermal Energy $= k_B T = 2.6\times10^{-2}$ eV. $\longrightarrow$ 1 Marks

Roll No:

Q2. A particle with energy E is bound in a finite square well potential with height U and width 2L (as shown in the figure below)



(a) Consider the case E<U, obtain the energy quantization condition for the symmetric wave functions in terms of K and α, where $K=\sqrt{2mE/\hbar^2}$ and $\alpha=\sqrt{2m(U-E)/\hbar^2}$

(b) Apply this result to an electron trapped at a defect site in a crystal. Modeling the defect as a finite square well potential with height 5 eV and width 200 pm, calculate the ground state energy ?

(c) Calculate the total number of bound states with symmetric wavefunction ?

[2+2+2 Marks]

(a) Inside the well $(-L \le x \le L)$, the particle is free

The wavefunction symmetric about $x=0$ is

$$\psi_I(x) = A\cos Kx \quad , \text{ where } K=\sqrt{\frac{2mE}{\hbar^2}}$$

Outside the well,

$$\psi_{II}(x) = B e^{\alpha x} \quad ; \quad \psi_{III}(x) = C e^{-\alpha x} ,$$

$$\alpha = \sqrt{\frac{2m(U-E)}{\hbar^2}}$$

1 marks for correctly writing the wavefunction in three Region

Boundary Conditions at $x=L$ implies

$$A\cos KL = Ce^{-\alpha L} \quad ; \quad AK\sin KL = C\alpha e^{-\alpha L}$$

1 marks for proper boundary condition either at $x=-L$ or $+L$

$$\Rightarrow \boxed{K\tan KL = \alpha} \quad \text{——— (1)}$$

(b)

From Eqn (1), $\sec^2(KL) = \dfrac{U}{E}$

or, $\sec^2\left(\dfrac{\sqrt{2mE}}{\hbar} L\right) = \dfrac{U}{E}$

$\sec^2(\sqrt{E}) = \dfrac{5}{E}$ $\quad$ $\left[\begin{array}{l} U = 5eV, \; L = 200\times10^{-12} m \\ E \text{ is in } eV \end{array}\right]$

or $\cos(\sqrt{E}) = \dfrac{\sqrt{E}}{2.236}$ ——— (2)

Gr. State Energy → $\boxed{E_1 = 1.1475}$

Any value between ~~[illegible]~~ 1.1 to 1.2 deserves full marks, Else cut 1 mark.

(c) Eq^n (2) Can also be written as

$$\cos(\theta) = \frac{\theta}{2.236} \qquad (\theta = \sqrt{E})$$

Because $\cos\theta$ lies between 0 and 1 (for all θ)

θ should be ≤ 2.236 → 2 marks

But there exists no solution of the above transendental equation for $\theta \leq 2.236$ except the ground state. Hence there exists only one bound state for $U = 5eV$.

Roll No:

Q3. A scanning tunneling microscope (STM) can be approximated as an electron tunneling into a step potential [$V(x)=0$ for $x\leq 0$, $V(x)=V_0$ for $x>0$]. The tunneling current (or probability) in a STM reduces exponentially as a function of the distance from the sample. Considering only single electron-electron interaction, an applied voltage of 5 V and sample work function of 7 eV, calculate the amplification in the tunneling current if the separation is reduced from 2 atoms to 1 atom thickness.
(Take approximate size of an atom to be 3 Angstrom)

[3 Marks]

Tunnelling Current (or probababil:ity)

$$I \sim e^{-2\alpha x} \quad , \quad \alpha = \frac{\sqrt{2m|(U-E)|}}{\hbar}$$ (1) marks for Correct expression

Now, an applied votate of 5V for a Single electron will generate ~~area~~ a potential $V = 5eV$

Also the work function $\Phi = E = 7eV$.

$\therefore$ $$\boxed{\alpha \simeq 7.27 \times 10^{9}\ m^{-1}}$$ (Momentum or wave vector of $\bar{e}$)

(1) Marks for Correct value of 'α'

If Seperation is reduced from 2-atoms to 1-atom thickness,

Amplification in Tunnellig Current $= \dfrac{e^{-2\alpha x_1}}{e^{-2\alpha x_2}}$ $\left[\begin{array}{l} x_1 = 3\mathring{A} \\ x_2 = 6\mathring{A} \end{array}\right]$

$$= e^{+2 \cdot \times 7.27 \times 10^{9} \times 3 \times 10^{-10}}$$

$$= e^{4.36}$$

$$\simeq 78$$

(1) Marks