PH-107 (2017)
Tutorial Sheet 3

***** Problems to be done in tutorial.

A. de Broglie Wavelength:

P23. Calculate the wavelength of the matter waves associated with the following. Compare in each case the result with the respective dimension of the object. In which case will it be possible to observe the wave nature.

(i) A 2000 kg car moving with a speed of 100km/h.
(ii) A 0.28 kg cricket ball moving with a speed of 40 m/s.
(iii) An electron moving a speed of 10^7m/s.

[**Ans.:** 1.2×10^{-38}m, 5.9×10^{-35}m, 0.73Å]

P24. Show that the Bohr's condition of quantization of angular momentum leads to a condition of formation of standing wave of electron along the circumference in the Bohr model of hydrogen atom.

P25*. Buckminsterfullerene are soccer-like balls (called buck balls) made of 60 carbon atoms (C_{60}). Suppose a double slit experiment is performed using these buck balls travelling at a velocity of 100 m/sec, in which the slits are separated by a distance of 150 nm. The buck balls then strike an observation screen placed 1.25 m from the slits.
(a) Find the de Broglie wavelength of the buck balls.
(b) Find the distance between the maxima of the resultant interference pattern. Treat the buck balls as point like object.
(c) The size of the buck balls is approx. 1 nm (~10 Å). How does the size of the ball compare with the distance between the neighboring maxima of the interference patterns? Is the size of the C_{60} molecule likely to affect the visibility of the interference fringes? At what velocity for the C_{60} molecules would the interference fringes start to become difficult to detect?

P26*. A photon, an electron and a neutron all have energy 5 keV. (In the case of electron and neutron, the energy refers to non-relativistic kinetic energy). Calculate the de-Broglie wavelength for each of them. Express your answer in nanometers (nm).
(Given, electron mass = 500 keV/c^2, neutron mass = 1000 MeV/c^2)

[**Ans.:** λ_{Photon}=0.25 nm, $\lambda_{Electron}$=0.0175 nm, $\lambda_{Neutron}$=3.92×10^{-4} nm]

B. Phase and Group Velocity:

P27*. Two harmonic waves which travel simultaneously along a wire are $y_1 = 0.002 \cos(8.0\ x - 400\ t)$ and $y_2 = 0.002 \cos(7.6\ x - 380\ t)$, where x and y are in meters, and t in sec.

(a) What is the resultant wave and what are the phase and group velocities of the resultant wave?

(b) Calculate the range Δx between the zeros of the group wave and find the product of Δx and Δk ?

[**Ans.:** v_p=50 m/s, v_g=50 m/s, Δx=5π meter, Δx. Δk=2π]

P28. The dispersion relation for a lattice wave propagating in a one dimensional chain of atoms of mass m bound together by force constant β is given by the following equation.

$\omega = \omega_o \sin(ka/2)$, *where* $\omega_0 = \sqrt{4\beta/m}$

Here *a* is the distance between atoms.

(a) Show that in the long wavelength limits the medium is non-dispersive.

(b) Find the group and phase velocities at $k=\pi/a$.

[**Ans.:** 0, $\omega_o a/\pi$]

P29. Find the group and the phase velocity of the matter wave associated with a free particle under the assumption the frequency is defined using (i) the kinetic energy (ii) total relativistic energy.

P30*. The phase speed V_p of light in a certain wavelength range in a dispersive medium is given by the following expression:

$$V_p = \frac{C}{\left(A + \frac{4\pi^2 B}{\lambda^2}\right)}$$

Here λ is the wavelength, C is speed of light, and A and B is constants.

(a) Find an expression of the group speed in terms of wave vector k.

(b) Taking A = 1.7 and $B = \left(\frac{0.01}{4\pi^2}\right)(\mu m)^2$, calculate the group and phase speed of light for a wavelength of 400 nm. Which of the two speeds is physically important and why?

P31*. Consider an electromagnetic (EM) wave of the form $Ae^{i(kx-wt)}$. Its speed in free space is given by $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{w}{k}$; where ε_0 is the permittivity of free space, and μ_0 is the permeability of free space. The speed of EM waves in a material medium is defined by its electric permittivity ε and magnetic permeability μ.

(a) What is the expression for υ, the speed of EM waves in the medium, in terms of ε and μ ?

(b) Suppose the permittivity depends on the frequency according to the relation $\varepsilon = \varepsilon_0 (1 - \frac{w_P^2}{w^2})$. Assume $\mu=\mu_0$, and find a dispersion relation giving w(k) for the EM waves in a medium. w_p is a constant and is called the plasma frequency of the medium.

(c) Consider waves with $w=3w_p$. Find the phase velocity and group of the waves? What is the product of group and phase velocities?

(d) Based on the w(k) relation constructed, what happened to wave with $w<w_p$, that enters this medium.