Department of Electrical Engineering

Indian Institute of Technology Bombay, Powai

EE111 : Introduction to Electrical Systems

Solution to Assignment 2

1. $X_{L1} = \omega L_1 = 8\Omega \qquad L_1 = 0.0255\text{H}$
   $X_C = \frac{1}{\omega C} = 8\Omega \qquad C = 318\mu\text{F}$
   Z_{eq} of parallel branch $= \frac{(5+j8)(8-j10)}{(5+j8)+(8-j10)} = 9.15\angle 15.4°\Omega$
   $X_{L2} = \omega L_2 = 12\Omega \qquad L_2 = 0.038\text{H}$
   $Z_{tot} = 9.15\angle 15.4°\Omega + (7 + j12) = 21.4\angle 42°\Omega$
   $\theta = 42° \qquad$ Inductive
   $V = IZ = 214\text{V}$

2. $\theta = 116.6° - 45° = 71.6° \qquad$ (lag)
   $Z = \frac{V}{I} = 10 + j30\Omega$
   $L = \frac{30}{2000} = 0.015\text{H}$
   With second source, $\theta = 30°$
   $Z = \frac{R}{\cos\theta} = 11.5\Omega$
   $X_L = Z\sin\theta = 5.75\Omega$
   $\omega = \frac{X_L}{L} = 385$ rad/s
   Case II:
   $|I| = 6\text{A}$ and $|Z| = 150/6 = 25\Omega$
   $X_L = \sqrt{Z^2 - R^2} = 22.9\Omega$
   $\omega = \frac{22.9}{0.015} = 1528$ rad/s
   Change in $\omega = \frac{2000-1528}{2000} = 23.6\%$
   I is maximum when Z is minimum. Z is minimum when $X_L = 0$, $\omega = 0$

$I_{max} = 150/10 = 15\text{A}$

3. Current lags voltage by $10° + 53.4° = 63.4°$.
   $\tan\theta = X_L/R$, so $X_L = 2R$
   $\frac{V_m}{I_m} = |Z| = \sqrt{R^2 + X_L^2}$, so $R = 5\Omega$
   $\omega = 500$ rad/s, therefore $L = 0.02\text{H}$

4. $\theta = 60°$ and $\tan\theta = X_C/R$
   $R = 5\Omega$ therefore $X_C = 8.66\Omega$
   $C = \frac{1}{\omega X_C} = 57.7\mu\text{F}$

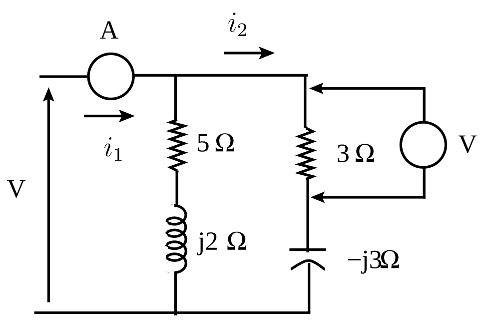

Figure 1: For Solution 5

5. $|i_2| = V/3 = 15\text{A}$
   Taking i_2 as reference, $I_2 = 15\angle 0°$
   $V = 15\angle 0°(3 - j3) = 63.6\angle{-45°}\text{V}$
   $I_1 = 11.8\angle 66.8°$ A
   $I_{tot} = I_1 + I_2 = 22.4\angle{-29°}$ A

6. $Z_{//} = 5.52\angle 88.45°\Omega$
   $Z_{eq} = 8.5\angle 30° + 5.52\angle 88.45° = 12.3\angle 52.4°\Omega$
   Voltage across parallel branch $= IZ_p = 50\text{V}$
   $\frac{V}{Z_{eq}} = \frac{V_1}{Z_p}$

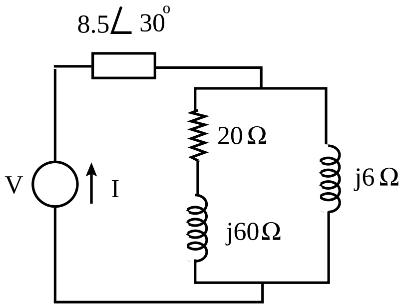

Figure 2: For Solution 6

$|V| = 50\frac{12.3}{5.52} = 111.5\text{V}$

7. $Z_{eq} = \frac{1}{\frac{1}{j5}+\frac{1}{5+j8.66}+\frac{1}{15}+\frac{1}{-j10}} = 4.55\angle 58°\Omega$
   $I = \frac{V}{Z_{eq}} = 33\angle{-13}°\text{A}$

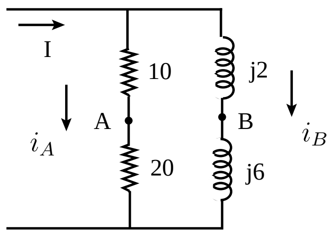

Figure 3: For Solution 8

8. $I_A = I(\frac{j8}{30+j8}) = 4.66\angle 120°\text{A}$
   $I_B = I(\frac{30}{30+j8}) = 17.5\angle 30°\text{A}$
   $V_{20\Omega} = I_A(20) = 93.2\angle 120°\text{V}$
   $V_{j6\Omega} = I_B(j6) = 105\angle 120°\text{V}$
   $V_{AB} = 11.8\angle{-60}°\text{V}$

9. $I_1 = \frac{20\angle 0°}{13-j4} = 1.47\angle 17.1°\text{A}$
   $V_{10} = I_1(10) = 14.7\angle 17.1°\text{V}$
   $V_{AB} = 20\angle 0° - 10\angle 45° - 14.7\angle 17.1° = 11.39\angle 264.4°\text{V}$
   $Z' = 5 + \frac{10(3-j4)}{13-j4} = 7.97 - j2.16\Omega$

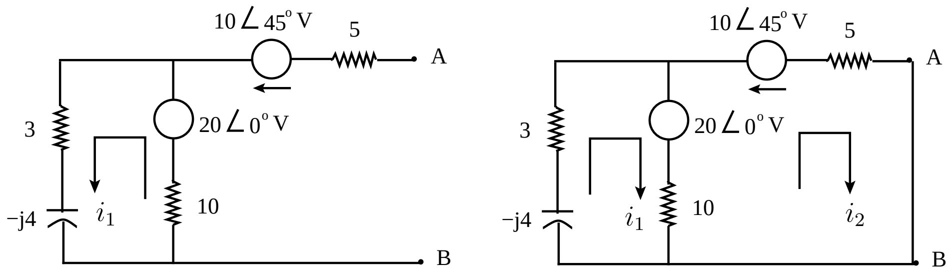

Figure 4: For Solution 9

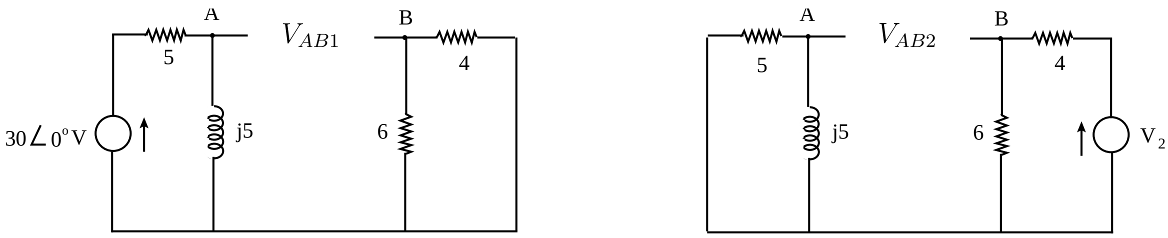

Figure 5: For Solution 10

Norton's circuit:

$(13 - j4)I_1 - 10I_2 = -20\angle 0^\circ$

$-10I_1 + 15I_2 = 20\angle 0^\circ - 10\angle 45^\circ$

$I_2 = 1.39\angle 279^\circ \text{A}$

10. $V_{AB1} = (\frac{30\angle 0^\circ}{5+j5})j5 = 15 + j15\text{V}$
    $V_{AB2} = (\frac{V_2}{4+6})6 = 0.6V_2$ V
    $V_{AB} = 15 + 0.6V_2 + j15$ V
    For zero current to flow in $2 + j3\Omega$ resistance, $V_{AB} = 0$
    $V_2 = -25 - j25$ V

11. For given loads, we have

$$\begin{array}{cc} P & Q \\ 250 & 0 \\ 1500 & 726.5 \\ 1000 & 750 \\ 700 & -339 \end{array} \tag{1}$$

So Total P=3450 kW and Total Q= 1137.5 kVAr

Using $Q = P\tan\theta$ we have $\cos\theta = \cos(\tan^1(\frac{Q}{P})) = 0.95$ lag and $S = \sqrt{P^2 + Q^2} =$ 3633 kVA.

Hence for the same current, load that the cable can carry at UPF is 3633 kW.

12. Active Power $= 440 \times 40 \times 0.7 = 12320$ W and

    Reactive Power $= 440 \times 40 \times \sin(\cos^{-1}(0.7)) = 12569$ VAr.

    At 0.9 lag, P remains same and

    Q supplied by the capacitor $= 12569 - 12320\tan(\cos^{-1}(0.9)) = 6602$ VAr

    Capacitive reactance $X_c = \frac{V^2}{6602}$ so $C = \frac{6602}{440^2 \times 314} = 108\mu$F

13. Voltage across the lamp at UPF $= \frac{500}{4} = 125$ V

    Hence voltage across the Choke $= \sqrt{250^2 - 125^2} = 216.5$ V

    Therefore, $2\pi f L = \frac{216.5}{4}$ and so $L = 0.17234$ H

    $S = 250 \times 4 = 1000$ VA and $Q = \sqrt{1000^2 - 500^2} = 866$ VAr

    Hence Q supplied by C = 866 VAr so $866 = \frac{250^2}{X_c}$ and so $C = 44\mu$F