Q1.

Graphite is a layered structured solid with carbon atoms arranged in xy-plane. Each atom in the structure can, in principle, perform simple harmonic motion in 3 mutually orthogonal directions. The restoring forces in direction parallel to a layer are very large; hence the natural frequency of oscillations in the x and y direction lying within the plane of a layer are both equal to a value $\omega_{\parallel}$ which is so large that $\hbar\omega_{\parallel} >> 300k_B$, the thermal energy k_BT at room temperature. On the other hand, the restoring forces perpendicular to a layer are quite small; hence the frequency of oscillation $\omega_{\perp}$ of an atom in z-direction, perpendicular to the layer is so small that $\hbar\omega_{\perp} << 300k_B$. On the basis of this information, what is the molar specific heat (at constant volume) of graphite at 300K. You may assume that equipartition theorem is valid in this case. 2

[3 Marks]

At room temp, ie $\hbar\omega_{\parallel} >> 300k_B$ $\therefore$ thermal fluctuations are not enough to excite degree of freedom in the plane.

$\therefore$ Degree of freedom $\omega_{\parallel}$ is FROZEN.

$\hbar\omega_{\perp} << 300k_B$ or $300k_BT >> \hbar\omega_{\perp}$ $\therefore$ atoms can move only in $\perp^{er}$ direction ($\perp^{er}$ to plane).

— 1 Mark —

$\therefore$ P.E. + K.E in $\perp^{er}$ direction will be $\underbrace{k_BT \ (\tfrac{1}{2}k_BT \text{ each})}$ → – 1.5 Marks –

$$\therefore \frac{dU}{dT} = C_V = N_A k_B = R$$

→ – ½ Mark —

Comments: 1 Mark for stating
either $\omega_{\parallel}$ are not excited
or only $\omega_{\perp}$ are excited
or atoms can oscillate ONLY in z-dir.

1½ Marks for saying $U = \frac{1}{2}k_BT + \frac{1}{2}k_BT = k_BT$

½ Marks for $C_V = R$

[Students will get ½ Mark even if expression for U is wrong but writes $C_V = dU/dT$]

3

Q2.

A beam of x-ray with wavelength of 0.24 nm is directed towards a sample. The x-ray gets scattered from the electrons within the sample, imparting momentum to the electrons which are initially at rest. After scattering, the x-rays are detected at various angles relative to the direction of the incoming beam, using a detector that can resolve their wavelengths.

(a) What is the longest wavelength measured by the detector?

(b) For this wavelength what is the K.E. of the recoiling electrons?

(c) If the detector measures a wavelength for the scattered x-rays of 0.2412 nm, what is the x-ray scattering angle?

(d) What is the direction of the travel of the recoil electrons in case (c)? **[4 Marks]**

→ (a) The longest shift in the wavelength occurs for $\cos\theta = -1$

$$\therefore \lambda' = \lambda + 2h/m_0c = 0.24\,nm + 2\cdot\frac{1240\ eV\,nm}{511\times10^3\ eV} = 0.2449\,nm$$ } ½ Mark

→ (b) $E + m_0c^2 = E' + \gamma m_0c^2$

$\therefore E - E' = (\gamma - 1)m_0c^2$

$E = \frac{hc}{\lambda} = \frac{1240}{0.24} = 5167\ eV$, $E' = \frac{hc}{\lambda'} = \frac{1240}{0.2449} = 5063\ eV$

$\therefore E - E' = 104\ eV$ } 1 Mark

→ (c) $\lambda' = 0.2412\ nm$, $\theta = ?$

$$\cos\theta = (\lambda - \lambda')\frac{m_0c^2}{hc} + 1 = -0.0012\,\frac{511\times10^3}{1240} + 1 = 0.5$$ } ½ Mark

$\therefore \theta = \pi/3$

→ (d) $E' = \frac{hc}{\lambda'} = \frac{1240}{0.2412} = 5141\,eV$, $p' = E'/c = 5141\,eV/c$

$\theta = 60^\circ$, $\therefore p'_x = 5141\cos(\pi/3) = 2570.5\ eV/c$

$p'_y = 5141\sin(\pi/3) = 4452.3\ eV/c$ } ½ Mark

Conserve both x- and y- components of momentum } → 1 Mark

x-dir^n $p_{i,tot} = p_{f,tot}$ $5167\ eV/c = 2570.5\ eV/c + p'_{e,x}$

y-dir^n ——→ $0 = 4452.3\ eV/c + p'_{y,e} \Rightarrow p'_{y,e} = -4452.3\ eV/c$

$$\therefore \tan\phi = \frac{p'_{e,y}}{p'_{e,x}} = \frac{-4452.3}{2596.5} \Rightarrow \phi = 59.75^\circ$$ } ½ Mark

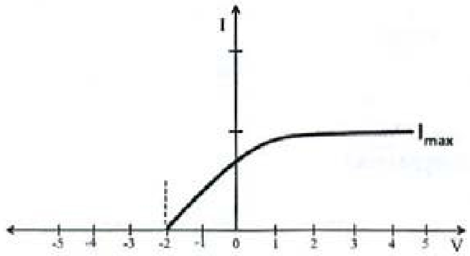
→ Comments: (a) & (b) are related. If λ' from (a) is wrong but all manipulations in (b) with wrong λ' from (a) are done correctly, students get 1 mark. Sly (c) & (d) are related, if θ from (c) is wrong but all manipulations in (d) with wrong θ from (c) are done correctly, students get (2) mark.

$KE_e + W_f = h\nu$
$KE_e = h\nu - W_f$

Q3.

4

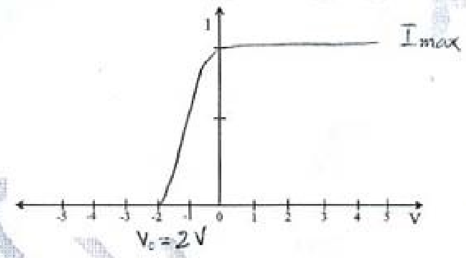
In a photoelectric setup following current-voltage curvewas obtained for a light of frequency f_1 and intensity 100 W/m^2. The metal's work-function is 4 eV.



Sketch the IV-curves (as below), which you would expect for a light of a) $f=f_1$, and intensity 200 W/m^2 b) $f=2f_1$ and intensity 100 W/m^2 and c) $f = f_1/2$ and intensity 100 W/m^2. Label your V_o.[4 Marks]

(a)

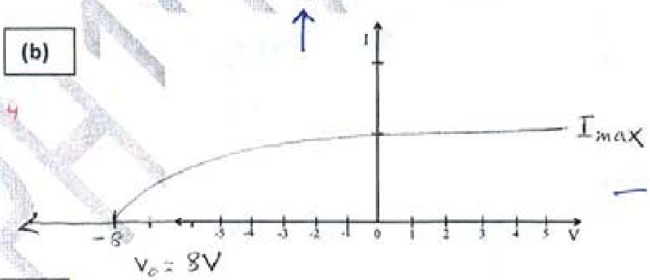
6 - 4



– (1 Mark) –

(b)

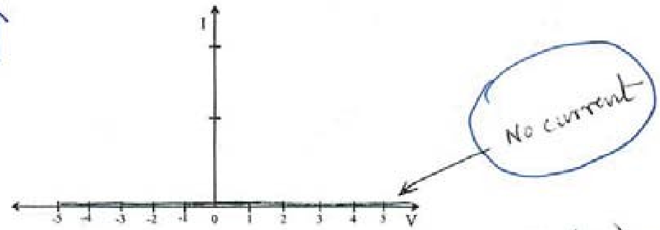
12 - 4



– (1 Mark) –

(c)

N.P.
3-4 < 0



– (2 Marks) –

Q4. Consider two one-dimensional harmonic waves travelingalong +x-direction are $y_1 = 0.002 \cos(8.0x - 400t)$ and $y_2 = 0.002 \cos(7.6x - 380t)$, where x and y are in meters, and t in sec. (a) Whatis the resultant wave form? Determine the phase and group velocities of the resultantwave? (b) Calculate the range Δx between the zeros of the group wave and find the product of Δx and Δk. **[2 + 2 Marks]** 5

(a) Use $\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$, where $A = 8.0x - 400t$, $B = 7.6x - 380t$

$\therefore y = y_1 + y_2 = \cancel{0.002[\cos}\ 0.002(2)\cos\left(\frac{15.6x - 780t}{2}\right)\cos\left(\frac{0.4x - 20t}{2}\right)$

$\therefore y(x,t) = \underbrace{0.004\ \cos(0.2x - 10t)}\ \cos(7.8x - 390t)$ —1 Mark

From above resultant value, $\Delta k = 0.4\,m^{-1}$, $\Delta\omega = 20\,s^{-1}$, $\bar{k} = 7.8\,m^{-1}$, $\bar{\omega} = 390\,s^{-1}$

(b) Phase velocity $v_p = \bar{\omega}/\bar{k} = \frac{390\,s^{-1}}{7.8\,m^{-1}} = 50\ m/s$

(c) Group velocity $v_g = \Delta\omega/\Delta k = \frac{20\,s^{-1}}{0.4\,m^{-1}} = 50\ m/s$ —1 Mark

$\because v_g = v_p$ $\therefore$ Medium is non-dispersive

(d) The group envelop shape is governed by the factor $\cos(0.2x - 10t)$. Assume x_1 and x_2 as two neighboring points at which envelop goes to zero at a given instant 't', $\therefore$ we know the phase difference betn these pts. is π.

i.e $(0.2x_2 - 10t) - (0.2x_1 - 10t) = 0.2(x_2 - x_1) = 0.2\Delta x = \pi$

$\therefore \Delta x = \pi/0.2 = 5\pi$ meters

$= 15.7\ m$

Since $\Delta k = 0.4\,m^{-1}$, $\Delta x \cdot \Delta k = 0.4 \times 5\pi$ or $\Delta x = \frac{\pi}{(\frac{1}{2}\Delta k)}$

$= 2\pi$

— 2 Marks

Comments: Wrong value of Δx shall fetch a partial credit.