PH-107 (2017)
Tutorial Sheet 1

***** Problems to be done in tutorial.

A. Introduction:

P1*: Show that if the nucleus in the Bohr atom is assumed to be of finite mass, the angular momentum of the system, the allowed radii and energies are all given by identical expressions except for replacement of m by the reduced mass μ.

P2: Two similar particles of mass *m* are connected to each other by a spring of negligible natural length and mass and spring constant *k*. The particles are made to rotate in a circle about their common centre of mass, such that the distance between them is *R*. Assume that the only force between the particles is the one provided by the spring. Apply Bohr's quantization rule to this system and find the allowed value of *r* and the energies in terms of *m*, *k* and fundamental constants. (This problem illustrates that the Bohr's quantization condition leads to Planck's quantization condition.)

P3: Assume that the wavelengths λ of the hydrogen atom spectra are given by the following expression instead of the usual one.

$$\frac{1}{\lambda} = R\left(\frac{1}{m^3} - \frac{1}{n^3}\right)$$

Here *R* is a constant and *m* and *n* are integers with $n>m$. If we write the angular momentum (L) quantization condition as $L=a\hbar$, what values *"a"* should take so as to explain the above spectra. Construct a theory similar to Bohr's using this quantization condition and find an expression of the energy and Bohr's radius.

P4*: A muon is an elementary particle of charge $-e$ and mass that is 207 times the mass of the electron. A muonic atom consists of a nucleus of charge Ze with muon circulating about it.

(a) Calculate the radius of the first Bohr orbit of a muonic atom with $Z=1$.

(b) Calculate the binding energy of a muonic atom with Z=1.

(c) Find the wavelength of the first line in the Lymann series for such an atom.

[**Ans.:** 2.85×10^{-3} Å, 2.53 keV, 6.55 Å]

P5*: One of the lines in the Hydrogen atom has a wavelength 4861.320 Å. It was later discovered this line has a faint companion located at 4859.975 Å. The explanation for this line was the presence of a small amount of heavier isotope deuterium in hydrogen. Use this data to compute the deuterium mass to the proton mass.

P6: A positronium atom (consisting of an electron and a positron revolving about common centre of mass; positron being a particle with mass equal to the mass of electron but charge plus *e*) is excited from a state with n=1 to n=4. Apply Bohr's theory with suitable modifications. (a) Calculate the energy that would have been absorbed by the atom. (b) Calculate minimum possible wavelength emitted when such an electron de-excites. (c) Calculate the recoil speed and recoil energy of the positronium atom, assumed initially at rest, after the excitation takes place.

P7: (a) Calculate the recoil speed of hydrogen atom assumed initially at rest, when it makes a transition from n=4 to n=1.
(b) What is the kinetic energy given to the hydrogen atom?
(c) At what temperature would the hydrogen atoms have this as the average speed assuming the gas to be classical?
(d) What would you expect if the hydrogen atoms were in motion approaching the photon?
(e) Can you think of a simple experiment that can be used to cool a gas?

[**Ans.:** 4.1 m/s, 8.65×10^{-8} eV, 0.7 mK]

P8*: (a) A diatomic molecule consists of two point masses, each of mass *m*, separated by a distance D (called bond length). Consider a gas of such di-atomic molecules. The molecular specific heat at constant volume C_v of this gas changes from *1.5R* to *2.5R* at temperature T_1. The quantum of rotational energy of the molecule is given by $\hbar^2/2I$, where $\hbar = h/2\pi$ and I is the moment of Inertia of the molecule. Obtain an expression for D in terms of m and T_1.

(b) C_v of the same diatomic gas, changes from 2.5R to 3.5R at temperature T_2. The quantum of energy of vibration is given by $\hbar\omega$, where ω is the angular frequency of vibrations. Obtain an expression for the bond strength (or the spring constant of the bond) in terms of m and T_2.

[**Ans.:** $D = \hbar\sqrt{2/mk_BT_1}$, $k_{sp} = (k_B^2T_2^2m)/(2\hbar^2)$]

P9: Graphite is a layered structured solid with carbon atoms arranged in xy-plane. Each atom in the structure can, in principle, perform simple harmonic motion in 3 mutually orthogonal directions. The restoring forces in direction parallel to a layer are very large; hence the natural frequency of oscillations in the x and y direction lying within the plane of a layer are both equal to a value w_1 which is so large that $\hbar w_1 >> 300k_B$, the thermal energy k_BT at room temperature. On the other hand, the restoring forces perpendicular to a layer are quite small; hence the frequency of oscillation w_2 of an atom in z-direction, perpendicular to the layer is so small that $\hbar w_2 << 300 k_B$. On the basis of this information, what is the molar specific heat (at constant volume) of graphite at 300 K. You may assume that equi-partition theorem is valid in this case.

[**Ans:** $C_V = R$]

P10*: Show that the Einstein's expression of specific heat gives a value 3R for large temperature (T) and zero as $T \rightarrow 0$.