Properties of Solids
(Drude Model)

S Uma Sankar

Department of Physics
Indian Institute of Technology Bombay
Mumbai, India

A Short History of Metals

- Metals have played an important role in the developement of civilization.
- They were originally prized for their tensile strength and their malleability.
- Advent of electricity led to the studies of electrical conductivity of the metals.
- Two important quantitative laws obtained in the 19th century are
  1. Ohm's Law: Described the ability of a metal to conduct electricity.
  2. Hall Effect: Attempted to describe the nature of charge conduction in electrical currents.
- In addition, an attempt was made to relate the good heat conduction properties of the metals to their good electrical conduction properties.

Ohm's Law

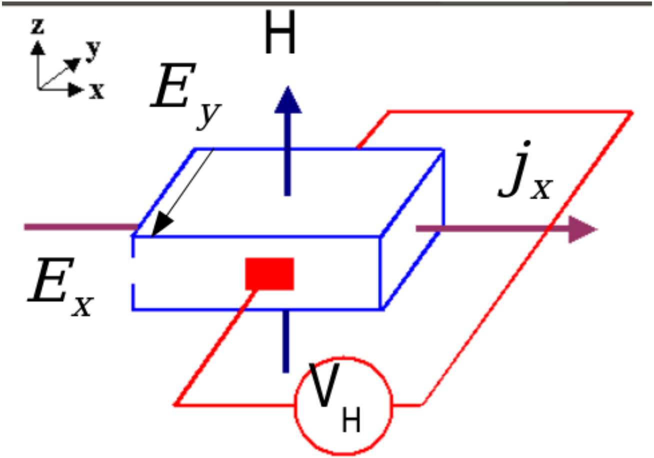
- It is usualy written as $I = (1/R)V$, current is proportional to the voltage difference applied. $(1/R)$ is called the conductance. Conductance depends on the geometry of the metallic piece involved.
- We want to write Ohm's law in a form which depends only the property of the material involved. It is written as

$$\vec{J} = \sigma \vec{E},$$

  where $\vec{J}$ is the current density (current per unit area of the wire), $\vec{E}$ is the electric field inside the wire (in the direction of the current) and σ is called the conductivity of the material.
- We can convert $\vec{J}$ into current by multiplying it by the cross sectional area A and convert $\vec{E}$ into voltage across by multiplying it by the length of the wire l.
- This enables us to identify $(1/R) = (\sigma A/l)$ or $R = \rho l/A$, where $\rho = 1/\sigma$ is called the resistivity of the material. ρ increases linearly with temperature.

Hall Effect

- Hall argued that there are tiny carriers of electric charge in the metals, which move under the influence of the applied electric field.
- To study them, he constructed an ingenious experiment.
- He established a current in $+x$ direction by applying electric field E_x. And then added a magnetic field in $+z$ direction.
- Lorentz force law $\vec{F} = q\vec{v} \times \vec{B}$. Since the current is $+x$ direction, $q\vec{v}$ is along $\hat{i}$. The charge carriers, whether positive or negative, experience a force in $-y$ direction.
- They pile up on the side of the metal and create an induced electric field E_y. We define the Hall Coefficient $R_H = E_y/j_x H$ is positive (negative) if the charge carriers carry positive (negative) charge.
- Expriment showed that R_H is positive for some metals and negative for others.


z
y
x
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E_y
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E_x
V_H

Free Electron (Drude) Model of Metals

- After the discovery of the electron, the mystery of the charge carrier is solved. Drude constructed the Free Electron Model of a metal in 1900 (before Quantum Mechanics).
- He imagined that the electrons in a metal move freely about in the metal, just as gas molecules move about in a container of a gas.
- The metal is assumed to contain lattice points where the ions are held fixed. In the course of their random motions, the electrons periodically undergo collisions with the ions.
- The average time between two collisions of an electron is taken to be τ and the average distance travelled between the collisions is called the mean free path.
- Note that, in Drude model, the electrons are assumed to be classical particles. The Coulomb interactions beween electrons and ions and also those between two electrons are ignored.

Conductivity in Drude Model

- Now apply a potential difference across the metal. This creates an electric field $\vec{E}$ inside the metal which acts on the electrons.
- In studying electrostatics, you learned that there can't be any electric field inside a metal. However, to sustain a current in the metal, one needs a small electric field.
- Moreover, there is a potential difference across the metal. And the law $V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{r}$ must be satisfied.
- If there are n electrons per unit volume in the metal and each of them carry charge $(-e)$, then the current density can be written as

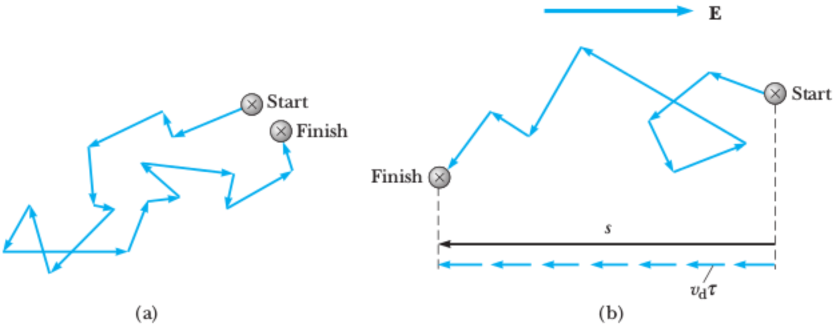
$$\vec{j} = n(-e)\langle\vec{v}\rangle$$

- Electrons in the metal experience an acceleration $\vec{a} = (-e)\vec{E}/m$ and their velocity is given by

$$\vec{v}_i(t) = \vec{v}_{i0} + \frac{(-e)\vec{E}}{m} t,$$

where $\vec{v}_{i0}$ is the velocity of the electron immediately after a collision.

Conductivity in Drude Model



Conductivity in Drude Model

- When we take average over all the electrons, $\langle \vec{v}_{i0} \rangle = 0$. Therefore we have
$$\langle \vec{v}_i \rangle = \frac{(-e)\vec{E}\tau}{m}.$$
- Substituting in the expression for current density, we obtain an expression for the conductivity
$$\sigma = \frac{ne^2\tau}{m}.$$
- We assume one free electron per atom. From the mass number of the metal and its density, we can calculate n.
- We need to know the value of τ to calculate the conductivity.
- We make the simple assumption that the electron collides with each ion in its way.
- This gives $\tau = L/\langle v \rangle$, where L is the inter-ion spacing, which we take to be a few Angstroms.

Conductivity in Drude Model

- What do we take for $\langle v \rangle$? One possibility is to take it as the *rms* speed of the electrons in the metal, which we assume to be $\sqrt{3kT/m}$ from Maxwell-Boltzmann distribution.
- Substituting it in the expression for σ we get

$$\sigma = \frac{ne^2 L}{\sqrt{3kTm}}.$$

- Substituing the values relevant for copper for various terms in the above expression, we get its conductivity to be $5.6 \times 10^6\ (\Omega\ m)^{-1}$,
- This is about a factor 10 smaller than the experimental value of $59 \times 10^6\ (\Omega\ m)^{-1}$.
- Another important difference between the Drude model and the experimental observations is the temperature dependence.
- Drude model predicts $\sigma \propto T^{-1/2}$ whereas experimentally $\sigma \propto T^{-1}$.

Problems with Drude Model

- Predicts the conductivity of metals to be an order of magnitude smaller than the measured value.
- Predicts the electronic contribution to the specific heat to be $3R$, whereas the experimental measurement is less than 1% of that.
- Predicts that conductivity varies with temperature as $T^{-1/2}$ whereas the experiment shows the variation to be T^{-1}.
- Predicts negative Hall coefficient for all metals, whereas experiment shows positive Hall coefficient for some metals.

Despite all the problems, we believe that Drude model must be right at some level. Why?
The model made a lot of simplifying assumptions. A lot of details were ignored. In spite of all these shortcomings, it predicts a value of conductivity which is within a factor of 10 of the correct answer.
That can't be a coincidence. There must be something right about the model.