Tutorial 10

P83. No. of ways = ${}^{59}C_5\,{}^{35}C_1$

Probability = $\dfrac{1}{{}^{39}C_5\,{}^{35}C_1}$ → (Depends on how you interpret the question)

P84. Total possible outcomes = 2^{20}

12 Heads, 8 Tails → ${}^{20}C_{12}$ ways

Probability = ${}^{20}C_{12}\left(\frac{1}{2}\right)^{12}\left(\frac{1}{2}\right)^{8}$

P85. Case I. $0, E, 2E$ ${}^{2}C_1\,{}^{10}C_1\,{}^{20}C_1 = 400$ microstates

Case II E, E, E ${}^{10}C_3$ $= 120$ microstates

$\text{Prob}(I) = \dfrac{400}{400+120} = \dfrac{10}{13}$, $\text{Prob}(II) = \dfrac{120}{400+120} = \dfrac{3}{13}$

P86. $V(x,y,z) = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\omega^2y^2 + \frac{1}{2}m(2\omega)^2z^2.$

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x,y,z) + V(x,y,z)\psi(x,y,z) = E\psi(x,y,z)$$

$$V(x,y,z) = V(x) + V(y) + V(z)$$
$$E = E_x + E_y + E_z.$$
$$\psi(x,y,z) = \psi(x)\psi(y)\psi(z)$$

→ ∴ can be separated into 3 equations of 1-D harmonic oscillator.

$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E_x\psi_x$. similarly, for Y & Z.

∴ $E_x = \left(n_x+\frac{1}{2}\right)\hbar\omega$, $E_Y = \left(n_y+\frac{1}{2}\right)\hbar\omega$, $E_z = \left(n_z+\frac{1}{2}\right)\hbar(2\omega)$

∴ $E = (n_x + n_y + 2n_z + 2)\hbar\omega$.

Ground state : $n_x = n_y = n_z = 0$. ∴ $E = 2\hbar\omega$.

$E = 7\hbar\omega$. $n_x + n_y + 2n_z = 5$.

$n_z = 0$ (0,5,0) (1,4,0) (2,3,0) (3,2,0) (4,1,0) (5,0,0)

$n_z = 1$ (0,3,1) (1,2,1) (2,1,1) (3,0,1)

$n_z = 2.$ (0,1,2) (1,0,2)

∴ 12 degenerate states

P87. If we include spin degeneracy,
$2 \times 10 = 20$ states.
Out of these, each state can be occupied by only 1 electron.
$\therefore {}^{20}C_3$ (Also, electrons → fermions → identical)

${}^{10}C_3\, 3!$ → No. of ways 3 persons can occupy 10 chairs.

P88. $E = \dfrac{\pi^2 \hbar^2}{2mL^2}$.

$\times 2$ → spin degeneracy

Total no. of degenerate states

$E_1 = 3E$	$1 \times 2 = 2$	(1, 1, 1)
$E_2 = 6E$	$3 \times 2 = 6$	(1, 1, 2) ↔ (2, 1, 1) ↔ (1, 2, 1)
$E_3 = 9E$	$3 \times 2 = 6$	(1, 2, 2) ↔ (2, 2, 1) ↔ (2, 1, 2)
$E_4 = 11E$	$3 \times 2 = 6$	(1, 1, 3) ↔ (3, 1, 1) ↔ (1, 3, 1)
$E_5 = 12E$.	$1 \times 2 = 2$.	(2, 2, 2).

Required energy state = 18E.
No. of electrons = 3.

(I) E_1, E_2, E_3 $\quad {}^2C_1\, {}^6C_1\, {}^6C_1 = 72$. $\quad P(I) = 72/94$

(II) E_2, E_2, E_2. $\quad {}^6C_3 = 20 \quad P(II) = 20/94$

(III) $E_1, E_1, E_5 \quad {}^2C_2\, {}^2C_1 = 2 \quad P(III) = 2/94$

P89. (a) Distinct (classical).

0	E	3E	5E	9E		
3	0	0	0	1	${}^4C_3 = 4$	
1	1	1	1	0	$4! = 24$	32.
1	0	3	0	0	${}^4C_3 = 4$	

(b) Bosons.

0	E	3E	5E	9E		
3	0	0	0	1	1	
1	1	1	1	0	1	3.
1	0	3	0	0	1	

(c) Fermions

0	E	3E	5E	9E		
1	1	1	1	0	1	1