Roll No:

Q1(a). A triangular pulse (wave packet) is of the form

$$\Psi(x)=\begin{cases}\left(1-\dfrac{|x|}{b}\right) & -b<x<b\\[2ex] 0 & elsewhere\end{cases}$$

(i) Calculate the Fourier transform of $\Psi(x)$, i.e. $A(k)=\int_{-\infty}^{\infty}\Psi(x)e^{-ikx}\,dx$

(ii) Plot $\Psi(x)$ versus x and $A(k)$ versus k for b=1

(ii) Calculate the product $\Delta x.\Delta k$ using the width from the first minima of both the functions.

[2+2+1 Marks]

1(b) A particle of mass m moves in a 1-dimensional potential V(x)=A|x|, where A is a positive constant. Use the Heisenberg uncertainty principle [$\Delta x.\Delta p_x \approx h/(4\pi)$] to estimate the minimum total energy (kinetic+potential) of the particle as a function of m, A and Planck's constant.

[3 Marks]

Solution 1(a)

(i) $$A(K) = \int_{-\infty}^{\infty} \Psi(x)\, e^{-iKx}\, dx = \int_{-b}^{+b}\left[1-\frac{|x|}{b}\right]e^{-iKx}dx$$

$$A(K) = \int_{-b}^{+b} e^{-iKx}dx - \frac{1}{b}\left[\int_{-b}^{0} -x e^{-iKx} + \int_{0}^{b} x\, e^{-iKx}dx\right]$$

$$= \frac{1}{iK}\left(2i \sin Kb\right) - \frac{1}{b}\left[\int_{b}^{0} x\, e^{iKx}(-dx) + \int_{0}^{b} x e^{-iKx}dx\right]$$

$$= \frac{2}{K}\sin Kb - \frac{1}{b}\left[\int_{0}^{b} x\cdot 2\cos Kx\, dx\right]$$

$$= \frac{2}{K}\sin Kb - \frac{2}{b}\left[\frac{b}{K}\sin Kb + \frac{\cos Kb}{K^2} - \frac{1}{K^2}\right]$$

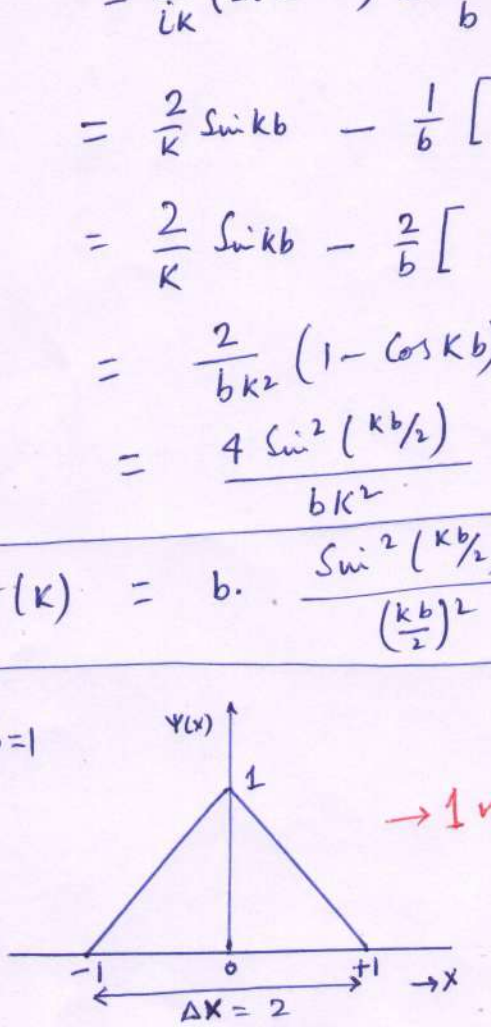
$$= \frac{2}{bK^2}\left(1-\cos Kb\right)$$

$$= \frac{4\sin^2(Kb/2)}{bK^2}$$

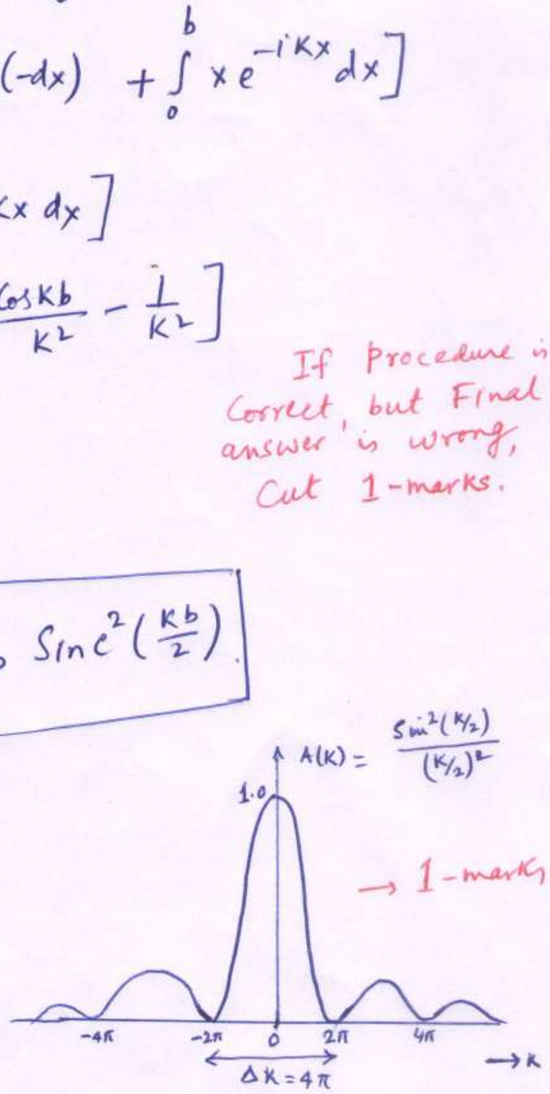
If Procedure is Correct, but Final answer is wrong, Cut 1-marks.

$$\boxed{A(K) = b\cdot\frac{\sin^2(Kb/2)}{\left(\frac{Kb}{2}\right)^2} = b\,\mathrm{Sinc}^2\left(\frac{Kb}{2}\right)}$$

(ii) for b=1



→ 1 marks



→ 1-marks

(iii) $\Delta x = 2$, $\Delta K = 4\pi$, $\Delta x.\Delta K = 8\pi$ [$\frac{1}{2}$ for Δx. $\frac{1}{2}$ for ΔK].

1(b)

$$E_{tot} = \frac{p^2}{2m} + A|x|$$

Heisenberg's Uncertainty reln: $\Delta x . \Delta p \geq \hbar/2$

At minimum Energy ; $xp \simeq \frac{\hbar}{2} \Rightarrow \boxed{p \simeq \frac{\hbar}{2x}}$ 1-marks.

$$\therefore E_{tot}^{(x)} = \frac{\hbar^2}{8mx^2} + A|x|$$

Now, because $E_{tot}(x)$ is an even function of x, $E_{tot}(x) = E_{tot}(-x)$

Let us choose "$+x$",

$$E_{tot} = \frac{\hbar^2}{8mx^2} + Ax$$

Minimize w.r. to x, $\frac{\partial E_{tot}}{\partial x} = 0$

$$\Rightarrow -\frac{\hbar^2}{4mx^3} + A = 0 \Rightarrow \boxed{x = \left(\frac{\hbar^2}{4mA}\right)^{1/3}}$$ 1 marks

$\therefore$ Minimum Total Energy; $E_{tot}^{min} = \frac{\hbar^2}{8m}\left(\frac{4mA}{\hbar^2}\right)^{2/3} + A\cdot\left(\frac{\hbar^2}{4mA}\right)^{1/3}$

$$\text{or } E_{tot}^{min} = \left(\frac{\hbar^2}{m}\right)^{1/3} \cdot \frac{A^{2/3}}{2^{5/3}} + \left(\frac{\hbar^2}{m}\right)^{1/3} \cdot A^{2/3} \cdot \frac{1}{2^{2/3}}$$

$$= \left[\frac{1}{2^{5/3}} + \frac{1}{2^{2/3}}\right]\left(\frac{\hbar^2 A^2}{m}\right)^{1/3}$$

$$= \frac{3}{2^{5/3}} \cdot \left(\frac{\hbar^2 A^2}{m}\right)^{1/3}$$

$$\boxed{E_{tot}^{min} = \frac{3}{2}\left(\frac{\hbar^2 A^2}{4m}\right)^{1/3}}$$

1 marks

P.S.

Few people will do this problem by expressing E_{tot} in terms of 'p' instead of x and then minimize w.r.to p.

Follow same marking scheme as above.

2a:

(i) Since the particle is in region R_1, its wave function $\psi(x)$ must be $\psi_1(x)$ (because $\psi_2(x) = 0$ in R_1). — $\frac{1}{2}$

$$\therefore \psi(x,t) = e^{-iE_1 t} \psi_1(x).$$

$$|\psi(x,t)|^2 = |\psi_1(x)|^2$$

and $\int_{R_1} |\psi(x,t)|^2 dx = \int_{R_1} |\psi_1(x)|^2 dx = 1.$ — $\frac{1}{2}$

Particle stays in R_1 for ever.

(ii) $\psi(x,0) = \frac{1}{\sqrt{2}}\left[\psi_1(x) + \psi_2(x)\right]$

$$\psi(x,t) = \frac{1}{\sqrt{2}}\left[e^{-iE_1 t/\hbar} \psi_1(x) + e^{-iE_2 t/\hbar} \psi_2(x)\right]$$ — $\frac{1}{2}$

$$|\psi(x,t)|^2 = \frac{1}{2}\left[|\psi_1(x)|^2 + |\psi_2(x)|^2 + e^{i(E_1-E_2)/\hbar} \psi_1^*(x)\psi_2(x) + e^{i(E_2-E_1)/\hbar} \psi_2^*(x)\psi_1(x)\right]$$ — $\frac{1}{2}$

If there is no overlap of R_1 and R_2,

$\psi_1^*(x)\psi_2(x) = 0 = \psi_2^*(x)\psi_1(x)$ everywhere

P.T.O.

$$\therefore\ |\psi(x,t)|^2 = |\psi_1(x)|^2 + |\psi_2(x)|^2 \quad 1$$

is independent of time.

iii) If R_1 and R_2 overlap, then $\psi_1^*(x)\psi_2(x) \neq 0$ and so is $\psi_2^*(x)\psi_1(x)$.

We get

$\cancel{\psi_1^*(x) e^{i(E_1-E_2}}$

$$e^{i(E_1-E_2)t/\hbar}\,\psi_1^*(x)\psi_2(x) + e^{i(E_2-E_1)t/\hbar}\,\psi_2^*(x)\psi_1(x)$$

$$= 2\,\mathrm{Re}\left[e^{i(E_2-E_1)t/\hbar}\,\psi_2^*(x)\psi_1(x)\right] \quad 1$$

Here

$$= 2\left[\cos\left\{\frac{(E_2-E_1)t}{\hbar}\right\}\mathrm{Re}\left(\psi_2^*(x)\psi_1(x)\right) - \sin\left\{\frac{(E_2-E_1)t}{\hbar}\right\}\mathrm{Im}\left(\psi_2^*(x)\psi_1(x)\right)\right]$$

$$\therefore\ |\psi(x,t)|^2 = |\psi_1(x)|^2 + |\psi_2(x)|^2 + 2\left[\cos\left\{\frac{(E_2-E_1)t}{\hbar}\right\}\mathrm{Re}\left(\psi_2^*(x)\psi_1(x)\right) - \sin\left\{\frac{(E_2-E_1)t}{\hbar}\right\}\mathrm{Im}\left(\psi_2^*(x)\psi_1(x)\right)\right] \quad 1$$

Here.

or

Q2(a) A potential V(x) is defined over a region R, which consists of two sub regions R_1 and R_2 ($R=R_1 \cup R_2$). This potential has two normalized energy eigenfunctions $\Psi_1(x)$ and $\Psi_2(x)$ with energy eigenvalues E_1 and E_2 ($E_1 \neq E_2$) respectively. $\Psi_1(x) = 0$ outside the region R_1, and $\Psi_2(x) = 0$ outside the region R_2.

(i) Suppose regions R_1 and R_2 do not overlap. Show that if the particle is in the region R_1 it will stay there forever.

(ii) If the initial state is $\Psi(x,0) = [\Psi_1(x) + \Psi_2(x)]/\sqrt{2}$, show that the probability density $|\Psi(x,t)|^2$ is independent of time.

(iii) Show that if regions R_1 and R_2 overlap, the probability density $|\Psi(x,t)|^2$ oscillate in time for the initial state given in (ii).

[1+1+2 Marks]

2(b) An electron is bound in an infinite potential well of width one nano-meter. A measurement of its position found it in the region $4.5 \leq x \leq 5.5$ Angstrom. There is a uniform probability of finding the electron in this region.

(i) Sketch the wavefunction after the position measurement.

(ii) If an energy measurement is made later, calculate the probability of finding the electron in the ground state and in the first excited state.

(iii) Find the value of quantum number 'n' such that the probability of electron being in the n-th energy eigenstate ψ_n is < 0.01, but nonzero.

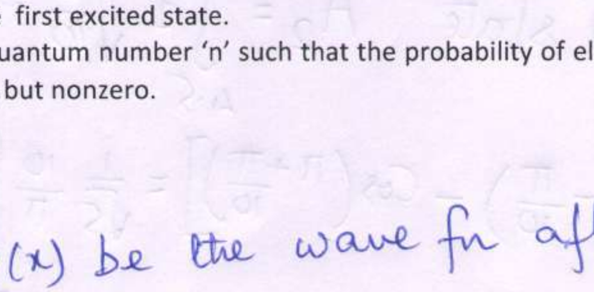
[1+1+2 Marks]

2b

(i) Let $\psi(x)$ be the wave fn after position measurement.

$\psi(x) = c$ for $4.5 \leq x \leq 5.5$
$ = 0$ for rest of the well.

$\int_{4.5}^{5.5} |\psi(x)|^2 = 1 \Rightarrow c^2 \cdot 1 = 1 \Rightarrow c = 1$.



$\frac{1}{2}$ mark for shape

$\frac{1}{2}$ mark for height.

(ii) Amplitude for $\psi(x)$ to be in ground state $A_1 = \int_{4.5}^{5.5} 1 \cdot \sqrt{\frac{2}{10}} \sin\left(\frac{\pi x}{10}\right) dx$

$= \frac{1}{\sqrt{5}} \cdot \frac{10}{\pi} \left[\cos\left(\frac{4.5\pi}{10}\right) - \cos\left(\frac{5.5\pi}{10}\right) \right]$

P.T.O.

$$= \frac{1}{\sqrt{5}} \frac{10}{\pi} \left[\cos\left(\frac{\pi}{2} - \frac{\pi}{20}\right) - \cos\left(\frac{\pi}{2} + \frac{\pi}{20}\right) \right] = \frac{1}{\sqrt{5}} \frac{10}{\pi} \cancel{\left[\sin\left(\frac{\pi}{20}\right) + S \right.}$$

$$= \frac{1}{\sqrt{5}} \frac{10}{\pi} \left[\sin\left(\frac{\pi}{20}\right) + \sin\left(\frac{\pi}{20}\right) \right] = \frac{1}{\sqrt{5}} \cdot \frac{20}{\pi} \cdot \sin\left(\frac{\pi}{20}\right).$$

$\frac{\pi}{20}$ is small and we can approximate $\sin\left(\frac{\pi}{20}\right) \sim \frac{\pi}{20}$.

$$A_1 = \frac{1}{\sqrt{5}} \cdot \frac{20}{\pi} \cdot \frac{\pi}{20} = \frac{1}{\sqrt{5}}.$$

Probability for ground state $= \frac{1}{5}$. — $\frac{1}{2}$

For first excited state $A_2 = \int_{4.5}^{5.5} 1 \sqrt{\frac{2}{10}} \sin\left(\frac{2\pi x}{10}\right) dx$

$$= \frac{1}{\sqrt{5}} \frac{10}{2\pi} \left[\cos\left(\pi - \frac{\pi}{10}\right) - \cos\left(\pi + \frac{\pi}{10}\right) \right] = \frac{1}{\sqrt{5}} \frac{10}{\pi} \left[-1 \cdot \cos\left(\frac{\pi}{10}\right) - (-1) \cos\left(\frac{\pi}{10}\right) \right] = 0.$$

$\therefore$ Probability for first excited state $= 0$. — $\frac{1}{2}$

iii) ~~Ro~~ For n^{th} state $A_n = \int_{4.5}^{5.5} 1 \sqrt{\frac{2}{10}} \sin\left(\frac{n\pi x}{10}\right) dx$ — $\frac{1}{2}$

$$= \frac{1}{\sqrt{5}} \frac{10}{n\pi} \left[\cos\left(\frac{n\pi}{2} - \frac{n\pi}{20}\right) - \cos\left(\frac{n\pi}{2} + \frac{n\pi}{20}\right) \right]$$

$$= \frac{1}{\sqrt{5}} \frac{20}{n\pi} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{20}\right)$$ — $\frac{1}{2}$

$$\text{Prob} = |A_n|^2 = \frac{1}{5} \left[\frac{\sin\left(\frac{n\pi}{20}\right)}{\left(\frac{n\pi}{20}\right)} \right]^2 < 0.01$$

$$\Rightarrow \frac{\sin\left(\frac{n\pi}{20}\right)}{\left(\frac{n\pi}{20}\right)} < 0.1 * \sqrt{5} = 0.224$$ — $\frac{1}{2}$

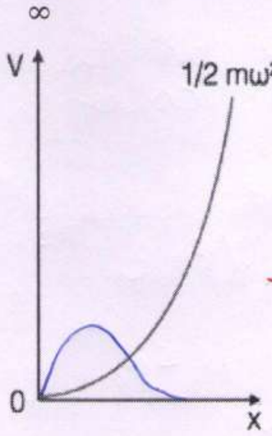
Numerical calculation shows $\frac{n\pi}{20} > 2.5 \Rightarrow n > \frac{50}{\pi} \simeq 16$.

L $\frac{1}{2}$

Roll No:

Q3(a) Consider a half-harmonic oscillator potential as shown in the figure below.

(i) Sketch the ground state wavefunction in the figure itself.

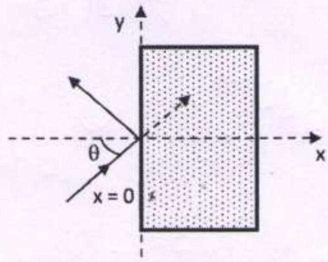


— 1

(ii) The ground state energy in the above case is $(1+\frac{1}{2})\hbar\omega = \frac{3}{2}\hbar\omega$ — 1

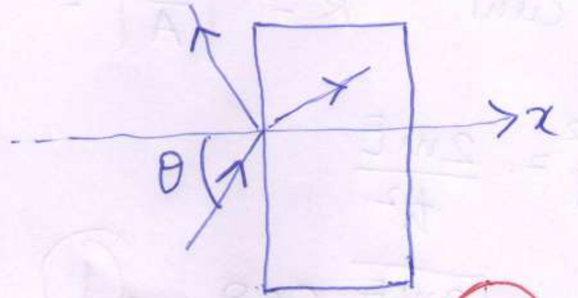
[1+1 Marks]

3(b) A monoenergetic parallel beam of non-relativistic neutrons of energy E is incident on an infinite metal surface. Within the metal, the neutrons experience a *uniform* negative potential V. The incident beam makes an angle θ with respect to the surface normal (as shown in the figure below). What fraction of the incident beam is reflected?



[6 Marks]

3b:



$\psi_{in} = A e^{i(k_x x + k_y y)}$ — $\frac{1}{2}$

$\psi_{ref} = B e^{i(-k_x x + k_{Ry} y)}$ — $\frac{1}{2}$

$\psi_{tr} = C e^{i(k'_x x + k_{ty} y)}$ — $\frac{1}{2}$

k_{Ry} is the wave number of reflected wave in y-dir

k_{ty} is the wave number of transmitted wave in y-dir.

There is no change of potential in y-dir.

Therefore $k_{Ry} = k_{ty} = k_y$. — 1

We can drop the y dependence of the wave fns.

Continuity of wave fn at $x=0$ gives

$$A + B = C. \quad \frac{1}{2}$$

Continuity of $\frac{d\psi}{dx}$ at $x=0$ gives

$$i k_x (A - B) = i k'_x C. \quad \frac{1}{2}$$

Adding we get $2A = \left(1 + \frac{k'_x}{k_x}\right) C$

or $\frac{C}{A} = \frac{2}{1 + \frac{k'_x}{k_x}} = \frac{2k_x}{k_x + k'_x}$

$\Rightarrow \frac{B}{A} = \frac{C}{A} - 1 = \frac{k_x - k'_x}{k_x + k'_x}$ — $\frac{1}{2}$

P.T.O.

Reflection coefficient $R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_x - k_x'}{k_x + k_x'}\right|^2$

Here $k_x^2 + k_y^2 = \frac{2mE}{\hbar^2}$

$$\Rightarrow \quad k_x = \sqrt{\frac{2mE}{\hbar^2}} \cos\theta \quad — 1$$

$$k_x'^2 + k_y^2 = \frac{2m(E + V_0)}{\hbar^2} \quad \text{where } V = -V_0.$$

$$(k_x')^2 = \frac{2mE}{\hbar^2} + \frac{2mV_0}{\hbar^2} - \frac{2mE}{\hbar^2}\sin^2\theta$$

$$k_x' = \sqrt{\frac{2m(E\cos^2\theta + V_0)}{\hbar^2}} \quad — 1$$

Q4(a). Consider nitrogen molecules in earth's atmosphere. Calculate

(i) the fraction of molecules with velocities between 199 m/s and 201 m/s at T=300 K

(ii) the mean energy, root-mean square (rms) energy and the uncertainty in energy per molecule (spread-out-ness of energy) at temperature T.

(iii) the average value of inverse of velocity, i.e., $\langle v^{-1} \rangle$.

[1+2+1 Marks]

(i) Mass of nitrogen molecule = 28 amu. 1 amu = 1.67×10^{-27} kg

$k_B=1.38\times10^{-23}$ JK^{-1}, T=300 K

v= (199+201)/2 = 200 m/s, dv=201-199 = 2 m/s

$$fraction = \frac{N(v)dv}{N} = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right)dv$$ → ½ marks for this expression

$$Fraction = 4\pi\left(\frac{28\times1.67\times10^{-27}}{2\pi\times1.38\times10^{-23}\times300}\right)^{3/2}(200)^2 \exp\left(-\frac{28\times1.67\times10^{-27}\times200^2}{2\times1.38\times10^{-23}\times300}\right)(2)$$

$$\therefore Fraction = 4\pi\left(1.7985\times10^{-6}\right)^{3/2}(4\times10^4)\exp\left(-22.589\times10^{-2}\right)(2)$$

$$\boxed{\therefore Fraction = 1.93\times10^{-3}}$$ → ½ marks if order of magnitude is correct.

Note: (1) Do we need to give M=28 amu, 1 amu=1.67×10^{-27} kg ✓

(2) Hope the students can get k_B= 1.38×10^{-23} JK-1 from k_B=8.6×10^{-5} eVK^{-1}, ✓

(II) $$f(E)dE = \frac{2\pi}{(\pi k_B T)^{3/2}} E^{1/2} e^{-E/k_B T} dE$$

$$\boxed{\text{Mean energy}=\langle E\rangle = \frac{2\pi}{(\pi k_B T)^{3/2}}\int_0^\infty E^{3/2} e^{-E/k_B T} dE = \frac{3}{2}k_B T}$$ → ½ marks

$$\langle E^2\rangle = \frac{2\pi}{(\pi k_B T)^{3/2}}\int_0^\infty E^{5/2} e^{-E/k_B T} dE = \frac{15}{4}(k_B T)^2$$

$$\boxed{\text{RMS energy} = \sqrt{\frac{15}{4}}k_B T}$$ → ½ marks.

Uncertainty in energy = ΔE

$$\Delta E^2 = \langle E^2\rangle - \langle E\rangle^2 = \left(\frac{15}{4}-\frac{9}{4}\right)(k_B T)^2 = \frac{3}{2}(k_B T)^2$$; ½ mark, for writing correct expression $(\Delta E)^2 = \langle E^2\rangle - \langle E\rangle^2$

$$\boxed{\therefore \Delta E = \sqrt{\frac{3}{2}}k_B T}$$; ½ marks for final result for ΔE.

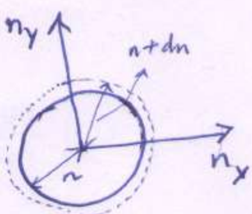
(III)

$\frac{1}{2}$ marks

$$\left\langle \frac{1}{v} \right\rangle = \int_0^\infty \frac{1}{v} f(v)dv = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_0^\infty \frac{1}{v} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right) dv$$

$$= 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} \frac{k_B T}{m} \frac{k_B T}{m}\int_0^\infty e^{-x}dx = \boxed{\left(\frac{2m}{\pi k_B T}\right)^{1/2}} \longrightarrow \frac{1}{2} \text{ marks.}$$

4(b) (i) $E = \frac{\pi^2\hbar^2}{2mL^2}(n_x^2 + n_y^2) = \frac{\pi^2\hbar^2}{2mL^2} n^2$



Density of state in n-space

$g(n)\,dn = \frac{1}{4}\left[\pi(n+dn)^2 - \pi n^2\right]$

$= \frac{1}{2}\pi n\,dn$

Including spin degeneracy, $g(n)dn = \pi n\,dn$

In terms of Energy $g(E)dE = \pi \cdot \left(\frac{mL^2}{\pi^2\hbar^2} dE\right)$

Cut $\frac{1}{2}$ marks if spin degeneracy or $\frac{1}{4}$-factor is missing $\longrightarrow$ $\boxed{g(E)dE = \frac{mL^2}{\pi\hbar^2} dE}$

(ii)

Total # of particles; $N = \int_0^\infty g(E)f(E)dE \longrightarrow \frac{1}{2}$-marks

or, $N = \int_0^{E_F} \frac{mL^2}{\pi\hbar^2} dE$ $\left[f(E) = \begin{cases} 1 & \text{for } E \le E_F \\ 0 & \text{for } E > E_F \end{cases} \right]$ at $T=0$

$N = \frac{mL^2}{\pi\hbar^2} E_F$

$\boxed{E_F = \frac{\pi\hbar^2}{mL^2} N} \longrightarrow \frac{1}{2}$-marks.

(iii) $\langle E \rangle = \frac{1}{N} \cdot \int_0^\infty E\, g(E) f(E) dE \longrightarrow 1$-marks

$= \frac{1}{N} \cdot \frac{mL^2}{\pi\hbar^2} \int_0^{E_F} E\,dE$

$= \frac{mL^2}{\pi\hbar^2} \cdot \frac{(E_F^2/2)}{N}$

$= \frac{\cancel{mL^2}}{2\cancel{\pi\hbar^2}} \cdot \frac{E_F^2}{\left(\frac{\cancel{mL^2}}{\cancel{\pi\hbar^2}}\right) E_F}$

$\boxed{\langle E \rangle = \frac{1}{2} E_F} \longrightarrow 1$-marks.

Roll No:

Q5(a). Assuming that Silver is a monovalent metal obeying Sommerfeld model, calculate the following quantities :

(i) Radius (k_F) of Fermi sphere

(ii) Average energy of free electrons at 0 K.

(iii) the temperature at which the average molecular energy in the ideal gas will have the same value as the average energy of free electrons at 0 K.

(iv) the speed of electron with this energy.

[*Given, density of Ag= 10.5 g cc ; Atomic wt. of Ag= 107.87; Resistivity of Ag at 295K = 1.61 x 10^{-6} ohm cm and at 20K= 3.8 x 10^{-9} ohm cm.*]

[1+1+1+1 Marks]

5(b). Consider a system of five particles trapped in a 1-dimensional harmonic oscillator potential.

(i) What are the microstates of the ground state of this system for classical particles, identical Bosons and identical spin half Fermions.

(ii) Suppose that the system is excited and has one unit of energy ($\hbar\omega$) above the corresponding ground state energy in each of the three cases. Calculate the number of microstates of the system for each of the three cases.

(iii) Suppose that the temperature of this system is low, so that the total energy is low (but above the ground state). Describe in a couple of sentences, the difference in the behavior of the system of identical bosons from that of the system of classical particles?

[1.5+1.5+1 Marks]

5 (a) (i) $K_F = (3\pi^2 n)^{1/3}$

Given, Density of Ag $(\rho_{Ag}) = 10.5$ g/cc ; At. wt. of Ag $(W_{Ag}) = 107.87$ g/mol

$\therefore$ Electron Density $(n) = \dfrac{\rho_{Ag} \times \text{Avagadros No.}}{W_{Ag}}$

$= \dfrac{10.5 \times 6.023 \times 10^{23}}{107.87}$ cm^{-3}

$\boxed{n = 5.86 \times 10^{28} \text{ m}^{-3}}$ —— $\frac{1}{2}$ marks

$K_F = (3 \times (3.14)^2 \times 5.86 \times 10^{28})^{1/3}$,

$\boxed{K_F = 1.2 \times 10^{10}}$ —— $\frac{1}{2}$ marks

(ii) Averag Energ $(\bar{E}) = \frac{3}{5} E_F = \frac{3}{5} \frac{\hbar^2}{2m} (K_F)^2$ —— $\frac{1}{2}$ marks

$\bar{E} = \dfrac{3 \times (1.05 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}} \times (1.2 \times 10^{10})^2$ Joule

$\boxed{\bar{E} = \dfrac{8.836 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 3.31 \text{ eV}}$ —— $\frac{1}{2}$ marks

(iii) $\frac{3}{2} k_B T = \bar{E}_F \Rightarrow T = \dfrac{2 \times 3.31 \times 1.6 \times 10^{-19}}{3 \times 1.38 \times 10^{-23}}$; $\boxed{T \simeq 25584 \text{ K}}$

—— 1 mark

(iv) $\frac{1}{2} m v^2 = \bar{E} \Rightarrow V = \sqrt{\dfrac{2\bar{E}}{m}}$

$\frac{1}{2}$ mark

$\boxed{V = 1.08 \times 10^7 \text{ m/sec}}$ —— $\frac{1}{2}$ mark

5(b) (i) Ground State

Type of particles	Energy States S_1	S_2	S_3	S_4	S_5	S_6	Degeneracy
$\frac{1}{2}$ marks — classical	5	0	0	0	0	0	1
$\frac{1}{2}$ marks — Bosons	5	0	0	0	0	0	1
$\frac{1}{2}$ marks — spin $\frac{1}{2}$ Fermions	↑↓	↑↓	↑				2
	↑↓	↑↓	↓				

(ii) Excited State with one Unit of energy ($\hbar\omega$) above gr. state

Type of Particle	S_1	S_2	S_3	S_4	S_5	S_6	Degeneracy
$\frac{1}{2}$ marks — Classical	4	1	0	0	0	0	5
$\frac{1}{2}$ marks — Boson	4	1	0	0	0	0	1
spin $1/2$ Fermion	↑↓ ↑↓	↑↓ ↑↓		↑ ↓	0	0	2
$\frac{1}{2}$ mark	↑↓ ↑↓	↑ ↓	↑↓ ↑↓	0	0	0	2

(iii)

Since the degeneracy of distinguishable (classical) particle states is much large than the bosonic particle states, small energy unit leads to excite the particle easily at the distinguishable particles. So we can find the particles in the ground state for bosonic case easily than the classical case.