Roll No:

1. In a photoelectric experiment, a photocathode is illuminated separately by two light sources of wavelengths 480 nm and 613 nm and of identical intensity. The resulting photocurrent (nano amps) is measured as a function of potential difference (V) between the cathode and anode. Observed photocurrent for three values of V is given below:

V	Current in nano amperes	
	480 nm	**613 nm**
-0.1	76.31	64.70
-0.2	67.62	44.41
-0.3	58.93	24.11

(a) Using this data obtain (i) the Work function of the photocathode (iii) the value of Planck's constant and (ii) the threshold wavelength.

(b) What is the maximum kinetic energy (in eV) of electron for 480 nm light source? What should be the wavelength of light to emit electrons half this kinetic energy?

(c) When the photocathode material is changed, the threshold frequency becomes 1.2 times the threshold frequency of old material. Calculate the work function of the new material?

[3+1+1 Marks]

(a) For 480 nm

$I_1 = m_1 V_1 + c_1$. From data we find

$m_1 = 86.9$ and $c_1 = 85$

$I_1 = 0$ when $V_1 = -\frac{c_1}{m_1} = \frac{85}{86.9} = -0.98\,V$

For 613 nm

$I_2 = m_2 V_2 + C_2$. From data we find

$m_2 = 203$ and $C_2 = 84.3$

$I_2 = 0$ when $V_2 = \frac{-C_2}{m_2} = -0.42\,V.$

For $\nu_1 = c/480\text{-nm}$, $K_{max\,1} = 0.98\,eV$

For $\nu_2 = c/613\text{-nm}$, $K_{max\,2} = 0.42\,eV$.

$$\left.\begin{aligned} h\nu_1 &= \phi + 0.98 \\ h\nu_2 &= \phi + 0.42 \end{aligned}\right\} \text{where } \phi \text{ is the work function.}$$

$$\nu_1 = 6.25 \times 10^{14}\,s^{-1}, \quad \nu_2 = 4.89 \times 10^{14}\,s^{-1}.$$

$$\cancel{h = \nu_1 - \nu_2} \qquad h(\nu_1 - \nu_2) = 0.98 - 0.42$$

$$h = \frac{0.56\ eV}{1.36 \times 10^{14}\ s^{-1}} = 4.1 \times 10^{-15}\ eV\,s$$

$$= 4.1 \times 10^{-15} \times 1.6 \times 10^{-19} = 6.6 \times 10^{-34}\ J\text{-}s$$ — 1

$$\phi = 4.1 \times 10^{-15} \times 6.25 \times 10^{4} - 0.98 = 1.58\,eV$$ — 1

$$\nu_{th} = \frac{1.58\ eV}{4.1 \times 10^{-15}\ eV\text{-}s} = 3.86 \times 10^{14}\,s^{-1}$$

$$\lambda_{th} = \frac{3 \times 10^{8}}{3.86 \times 10^{14}} = 0.777 \times 10^{-6}\ m = 777\ nm.$$ — 1

(b) K_{max} for $\lambda = 480\,nm$ is $0.98\,eV$.

Half of this $= 0.49\ eV$.

Corresponding photon energy $= \phi + K_{max}$

$= 1.58 + 0.49$

$= 2.07\ eV.$

$$\lambda = \frac{hc}{E} = \frac{1240\,eV\text{-}nm}{2.07\,eV} = 594\ nm$$ — 1

(c) $\phi_2 = 1.2\,\phi_1 = 1.2 \times 1.58 \simeq 1.9\ eV.$ — 1

Roll No:

2. (a) In a Compton scattering, show that the energy of the scattered photon will never exceed $2m_ec^2$, irrespective of the incident photon, if the angle of scattering exceeds a limiting value α. Find the angle α.

(b) A Compton scattering experiment was done with a spectrometer whose angular resolution (minimum angle it can measure) is 10 milliradians. What is the smallest wavelength shift this instrument can measure?

[2+2 Marks]

(a) $\lambda' - \lambda = \lambda_c (1 - \cos\theta)$

$$\frac{hc}{E'} - \frac{hc}{E} = \frac{hc}{m_e c^2}(1-\cos\theta)$$

$$\frac{1}{E'} = \frac{1}{E} + \frac{1-\cos\theta}{m_e c^2}$$

$$E' = \frac{m_e c^2 \cdot E}{m_e c^2 + E(1-\cos\theta)} = \frac{m_e c^2}{1-\cos\theta + \frac{m_e c^2}{E}} \text{ — } 1$$

E' is always less than $\frac{m_e c^2}{1-\cos\theta}$.

$$\frac{m_e c^2}{1-\cos\theta} < \frac{m_e c^2}{0.5} \left(= 2 m_e c^2\right)$$

if $1-\cos\theta > 0.5$ or if $\cos\theta < 0.5$

$\Rightarrow \theta > 60°$. — 1

(b) $\lambda' - \lambda = \Delta\lambda = \lambda_c(1-\cos\theta) = \frac{\lambda_c \theta^2}{2}$ for $\theta \ll 1$ — 1

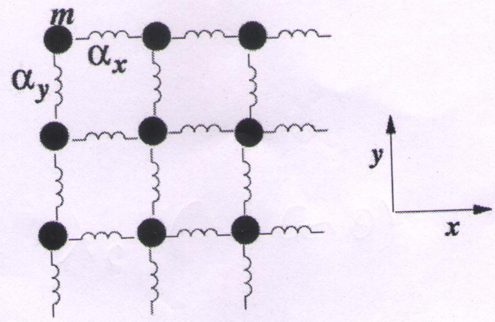
Smallest wavelength shift $= \frac{hc}{m_e c^2} \times \frac{10^{-4}}{2}$

$$= \frac{1240 \times 10^{-4}\ \text{eV-nm}}{10^6\ \text{eV}} = 1.24 \times 10^{-7}\ \text{nm.}$$

— 1

Roll No:

3. Consider a two dimensional system with small balls (each of mass m) connected by springs (see the figure below).



The spring constants along x- and y- directions are α_x and α_y respectively. The dispersion relation for this system is given by

$$-\omega^2 m + 2\alpha_x(1-\cos k_x b_x) + 2\alpha_y(1-\cos k_y b_y) = 0$$

where the wave-vector $\mathbf{k}=k_x\mathbf{i}+k_y\mathbf{j}$. b_x and b_y are the distance between two successive balls along x- and y-directions respectively. Find the group velocity $\vec{V}_g(k_x,k_y)$ and the angle that it makes with the x-axis

[3+1 Marks]

$$\omega^2 = \frac{2}{m}\left[\alpha_x(1-\cos k_x b_x) + \alpha_y(1-\cos k_y b_y)\right]$$

$$\vec{V_g} = \frac{\partial \omega}{\partial k_x}\hat{i} + \frac{\partial \omega}{\partial k_y}\hat{j} \quad — 1$$

$$2\omega \frac{d\omega}{dk_x} = \frac{2\alpha_x}{m}(-1)(-\sin k_x b_x) \cdot b_x$$

$$\frac{d\omega}{dk_x} = \frac{1}{\omega m}\left[\alpha_x b_x \sin k_x b_x\right] \quad — 1$$

$$\frac{d\omega}{dk_y} = \frac{1}{\omega m}\left[\alpha_y b_y \sin k_y b_y\right] \quad — 1$$

$$\vec{V_g} = \frac{1}{\omega m}\left[\alpha_x b_x \sin k_x b_x \hat{i} + \alpha_y b_y \sin k_y b_y \hat{j}\right]$$

$$\tan\phi = \frac{V_{gy}}{V_{gx}} = \frac{\alpha_y b_y \sin k_y b_y}{\alpha_x b_x \sin k_x b_x} \quad — 1$$

Roll No:

4. A wave packet is of the form

$$f(x) = \exp(-\alpha|x|).\exp(ik_0 x) \quad (\text{for } -\infty \le x \le \infty)$$ where α, k_0 are positive constants.

(a) Plot $|f(x)|$ versus x.

(b) At what values of x does $|f(x)|$ attain half of its maximum value? Consider the full width at half maxima (FWHM) as a measure of the spread (uncertainty) in x, find Δx

(c) Calculate the Fourier transform of $f(x)$, i.e. $g(k) = \int_{-\infty}^{+\infty} f(x) e^{ikx} dx$

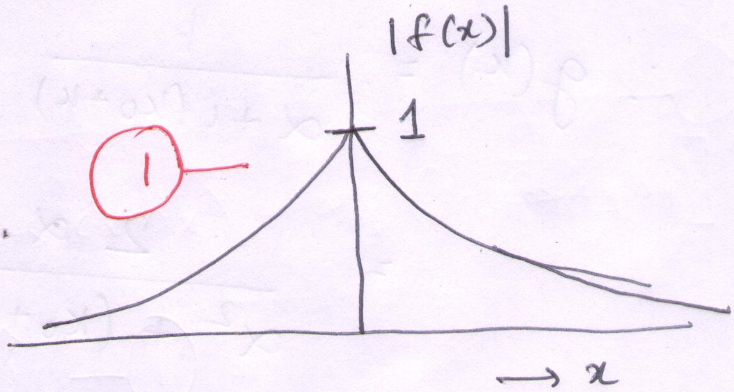
(d) Plot $g(k)$ versus k.

(e) Find the values of k at which $g(k)$ attains half its maximum value? Using the same concept of FWHM as in part (b), calculate Δk? Hence calculate the product $\Delta x.\Delta k$

$$\left[Given: \int_0^{\infty} e^{-(\alpha - ik)x} dx = \frac{1}{\alpha - ik} \right]$$

[1+1+2+1+1 Marks]

(a) $|f(x)| = e^{-\alpha|x|}$



Deduct half mark if $|f(x)|=1$ at $x=0$ is not shown.

(b) $|f(x)|_{max} = 1$ for $x = 0$.

$e^{-\alpha|x|} = \frac{1}{2}$ occurs when $\alpha|x| = 0.693$

$\therefore \Delta x = \frac{2 \times 0.693}{\alpha} \simeq \frac{1.4}{\alpha}$ — 1

(c) $g(k) = \int_{-\infty}^{\infty} e^{-\alpha|x| + ik_0x + ikx} dx$

$$g(k) = \int_{-\infty}^{0} e^{\alpha x + ik_0 x + ikx} dx + \int_0^{\infty} e^{-\alpha x + ik_0 x + ikx} dx$$

To do the first integral, substitute

$y = -x \Rightarrow dy = -dx$

$$\text{First integral} = \int_{\infty}^{0} e^{-(\alpha + ik_0 + ik)y}\,(-dy)$$

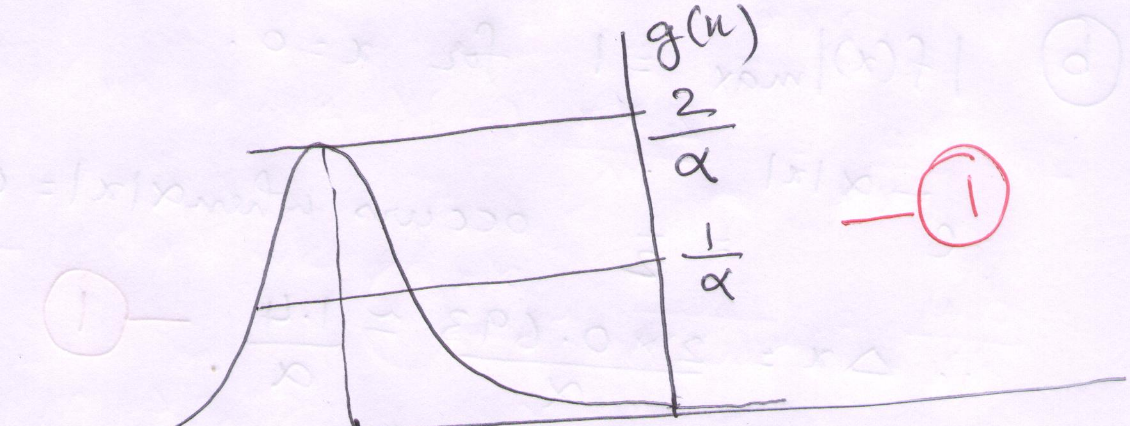
$$= \int_{0}^{\infty} e^{-(\alpha + ik_0 + ik)y}\,dy = \frac{1}{\alpha + i(k_0 + k)}$$ — 1

$$\text{Second Integral} = \frac{1}{\alpha - i(k_0 + k)}$$

$$\therefore\ g(k) = \frac{1}{\alpha + i(k_0+k)} + \frac{1}{\alpha - i(k_0+k)}$$

$$= \frac{2\alpha}{\alpha^2 + (k_0+k)^2}$$ — 1

(d)



— 1

(e) $g(k) = \frac{1}{\alpha}$ when $\alpha^2 = (k_0+k)^2$

or $k = -k_0 \pm \alpha \Rightarrow \Delta k = 2\alpha$

$$(\Delta x)(\Delta k) = \frac{1.4}{\alpha} \times 2\alpha = 2.8$$ — 1

Roll No:

5. For a non-relativistic electron, using the uncertainty relation $\Delta x \Delta p_x = \hbar/2$

(a) Derive the expression for the minimum kinetic energy of the electron localized in a region of size 'a'.

(b) If the uncertainty in the location of a particle is equal to its de Broglie wavelength, show that the uncertainty in the measurement of its velocity is same as the particle velocity.

(c) Using the expression in (b), calculate the uncertainty in the velocity of an electron having energy 0.2 keV

(d) An electron of energy 0.2 keV is passed through a circular hole of radius 10^{-6} m. What is the uncertainty introduced in the angle of emergence in radians? (Given $\tan\theta \cong \theta$)

[1+2+1+2 Marks]

(a) $E_{min} = \dfrac{P_{min}^2}{2m}$. $P_{min} = \Delta P_x = \dfrac{\hbar}{2\Delta x} = \dfrac{\hbar}{2a}$

$E_{min} = \dfrac{\hbar^2}{8ma^2}$ — 1

(b) $\lambda = \dfrac{h}{P}$ $\quad P = \sqrt{2mE}$

$= \dfrac{h}{\sqrt{2mE}}$ — 1

Given $\lambda = \Delta x \Rightarrow \Delta x = \dfrac{h}{\sqrt{2mE}}$

$\Delta p = m\Delta V$

$\therefore \Delta x \Delta p = \dfrac{\hbar}{2}$

$\dfrac{h}{\sqrt{2mE}} \cdot m \Delta V = \dfrac{h}{4\pi}$

$\Rightarrow \Delta V = \dfrac{1}{4\pi} \cdot \dfrac{\sqrt{2mE}}{m} = \cancel{\dfrac{1}{2\pi}\sqrt{\dfrac{E}{2m}}}$

$= \dfrac{V}{4\pi}$ — 1

(c) $v = \frac{\sqrt{2mE}}{m} = \sqrt{\frac{2E}{m}}$

$\frac{v}{c} = \sqrt{\frac{2E}{mc^2}} = \sqrt{\frac{2\times 200}{0.5\times 10^6}} = \sqrt{\frac{800}{10^6}}$

$= 2.8\times 10^{-2}$

$\frac{\Delta v}{c} = \frac{1}{4\pi}\frac{v}{c} \approx \frac{2.8\times 10^{-2}}{12}$

$= 2.3\times 10^{-3}$

$\Delta v = 7\times 10^{-3}\times 10^{8} = 7\times 10^{5}$ m/s — 1

(d) $p = \sqrt{2mE}$

$\Delta p = \frac{\hbar}{2\Delta x} = \frac{\hbar}{4r}$ — 1

Uncertainty in angle of emergence

$\Delta\theta = \frac{\Delta p}{p} = \frac{\hbar}{4r}\cdot\frac{1}{\sqrt{2mE}}$

$= \frac{\hbar c}{4r\sqrt{2mc^2E}} = \frac{0.2\ \text{eV}-\mu\text{m}}{4\times 1\,\mu\text{m}\times\sqrt{10^6\times 200}\ \text{eV}}$

$= \frac{0.2}{4\times\sqrt{2}\times 10^4} = 0.035\times 10^{-4}$ radians

$= 3.5\times 10^{-6}$ radians. — 1

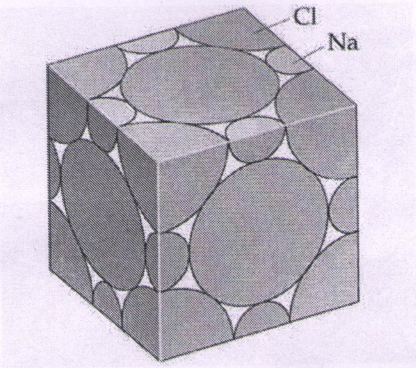
Roll No:

6. Sodium Chloride (NaCl) crystal is made up of cubes of edge length 'd', as shown in the figure. Each cube contains a full sodium ion at its body center, which is not shown in the figure. In a Davisson-Germer experiment performed using electrons of kinetic energy 40 eV, the NaCl crystal gives a first order (n = 1) scattering peak at 20.11 degrees.

(a) Compute 'd'

(b) Compute the number of NaCl molecules in the given cube.

(c) Given density and molecular weight of NaCl to be 2.17 gm/cm^3 and 58.44 gm/mol respectively, compute Avogadro's number.



[2+1+2 Marks]

(a) $E_e = 40$ eV.

$$p = \sqrt{2 m_e E_e} \Rightarrow pc = \sqrt{2 m_e c^2 E_e}$$

$$= \sqrt{10^6 \times 40}$$

$$= 6.32 \times 10^3 \text{ eV}$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240 \text{ eV-nm}}{6.32 \times 10^3 \text{ eV}} = 196 \times 10^{-3} \text{ nm} \text{ — } 1$$

$$= 1.96 \times 10^{-10} \text{ m}$$

$$\lambda = d \sin\phi = d \times 0.344$$

$$\therefore d = \frac{1.96 \times 10^{-10}}{0.344} = 5.7 \times 10^{-10} \text{ m} \text{ — } 1$$

(b) $\frac{1}{2}$ Cl atom per face $+ \frac{1}{8}$ Cl atom per corner

$\Rightarrow \frac{1}{2} \times 6 + \frac{1}{8} \times 8 = 4$ Cl atoms.

$4 \times \frac{1}{4}$ Na atom in top layer

$+ 4 \times \frac{1}{4}$ Na atom in middle layer +1 at center

$+ 4 \times \frac{1}{4}$ Na atom in botthom layer = 4 Na atoms.

$\Rightarrow$ Given cell contains 4 NaCl molecules. — 1

(c) $V_{cell} = (5.7 \times 10^{-10})^3 \, m^3$

$= 185 \times 10^{-30} \, m^3 = 1.85 \times 10^{-28} \, m^3$

$= 1.85 \times 10^{-22} \, cm^3$

Mass of NaCl molecule $= \dfrac{V_{cell} \times \text{density}}{\text{No. of molecules}}$

$M_m = \dfrac{1.85 \times 10^{-22} \times 2.17}{4}$ gm

$= 1 \times 10^{-22}$ gm — 1

Molar weight of NaCl $= A \times M_m = 58.44$ gm/Mol

$\Rightarrow A = \dfrac{58.44}{10^{-22}} = 5.844 \times 10^{23}$

$\simeq 6 \times 10^{23}$ — 1