P61 please check. Most probably I might've made a mistake which is carried to the other path.

$$\int_0^L x\cos\left(\frac{n\pi x}{L}\right)dx = \left[\frac{x\sin\left(\frac{n\pi x}{L}\right)}{\frac{n\pi}{L}}\right]_0^L \to 0 - \int_0^L \frac{\sin\frac{n\pi x}{L}}{\frac{n\pi}{L}}dx$$

$$\left[\frac{\cos\frac{n\pi x}{L}}{\left(\frac{n\pi}{L}\right)^2}\right]_0^L = \frac{\cos n\pi - 1}{\left(\frac{n\pi}{L}\right)^2}$$

$\to 0$ if n is even.

$\to \frac{-2}{\left(\frac{n\pi}{L}\right)^2}$ if n is odd.

$$\int_0^L x^2\cos\left(\frac{n\pi x}{L}\right)dx = \left[\frac{x^2\sin\left(\frac{n\pi x}{L}\right)}{\frac{n\pi}{L}}\right]_0^L - \int_0^L \frac{2x\sin\left(\frac{n\pi x}{L}\right)}{\frac{n\pi}{L}}dx$$

$$= -2\left[\left[\frac{-x\cos\frac{n\pi x}{L}}{\left(\frac{n\pi}{L}\right)^2}\right]_0^L + \int_0^L\left[\frac{\cos\frac{n\pi x}{L}}{\left(\frac{n\pi}{L}\right)^2}\right]dx\right]$$

$$= \frac{2L\cos n\pi}{\left(\frac{n\pi}{L}\right)^2}$$

$\to \frac{2L}{\left(\frac{n\pi}{L}\right)^2}$ if n is even

$\to \frac{-2L}{\left(\frac{n\pi}{L}\right)^2}$ if n is odd.

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(a) Normalize:

$$\frac{1}{A^2} = \int_0^L \left[\sin^2\frac{\pi x}{L} + \sin^2\frac{2\pi x}{L} + 2\sin\frac{\pi x}{L}\sin\frac{2\pi x}{L}\right]dx$$

$$RHS = \int_0^L \left(\frac{1-\cos\frac{2\pi x}{L}}{2} + \frac{1-\cos\frac{4\pi x}{L}}{2} + \cos\frac{\pi x}{L} - \cos\frac{3\pi x}{L}\right)dx$$

$$L = \frac{1}{A^2} \quad \therefore \quad A = \pm\sqrt{\frac{1}{L}}$$

(b) $\langle \psi | x | \psi \rangle$ α

$$= A^2 \int_0^L \left[x - \cancel{\frac{x\cos\frac{2\pi x}{L}}{2}}^{0} - \cancel{\frac{x\cos\frac{4\pi x}{L}}{2}}^{0} + x\cos\frac{\pi x}{L} - x\cos\frac{3\pi x}{L}\right]dx$$

$$\langle x \rangle = A^2\left[\frac{L^2}{2} - \frac{2}{(\pi/L)^2} + \frac{2}{(3\pi/L)^2}\right]$$

(c) $\langle \psi | x^2 | \psi \rangle$

$$= A^2 \int_0^L \left[x^2 - \frac{x^2\cos\frac{2\pi x}{L}}{2} - \frac{x^2\cos\frac{4\pi x}{L}}{2} + x^2\cos\frac{\pi x}{L} - x^2\cos\frac{3\pi x}{L}\right]dx$$

$$\langle x^2 \rangle = A^2\left[\frac{L^3}{3} - \right.$$

Accordingly solve.

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(c) $\langle \psi | -i\hbar \frac{\partial}{\partial x} | \psi \rangle$

$$= -i\hbar A^2 \int_0^L \left(\sin\frac{\pi x}{L} + \sin\frac{2\pi x}{L}\right) \times \frac{\pi}{L}\left(\cos\frac{\pi x}{L} + 2\cos\frac{2\pi x}{L}\right) dx.$$

solve ahead.

$\langle \psi | -i\hbar \frac{\partial}{\partial x} \left(-i\hbar \frac{\partial}{\partial x} \right) | \psi \rangle .$

$$= +\frac{\pi^2 \hbar^2 A^2}{L^2} \int_0^L \left(\sin\frac{\pi x}{L} + \sin\frac{2\pi x}{L}\right) \times \left(\sin\frac{\pi x}{L} + 4\sin\frac{2\pi x}{L}\right) dx$$

dx

solve ahead.

(d) $\phi_2 = \sqrt{\frac{2}{L}} \sin\frac{2\pi x}{L}$.

prob of finding in 1st excited state

$$= \langle \phi_2 | \psi \rangle^2 = \left[\int_0^L \left(\sqrt{\frac{2}{L}} \sin\frac{2\pi x}{L} \right) \left(\frac{1}{\sqrt{2}} \sin\frac{\pi x}{L} + \frac{1}{\sqrt{2}} \sin\frac{2\pi x}{L} \right) dx \right]^2$$

Solve to get.

$\langle \phi_2 | \psi \rangle^2 = 1/2$.

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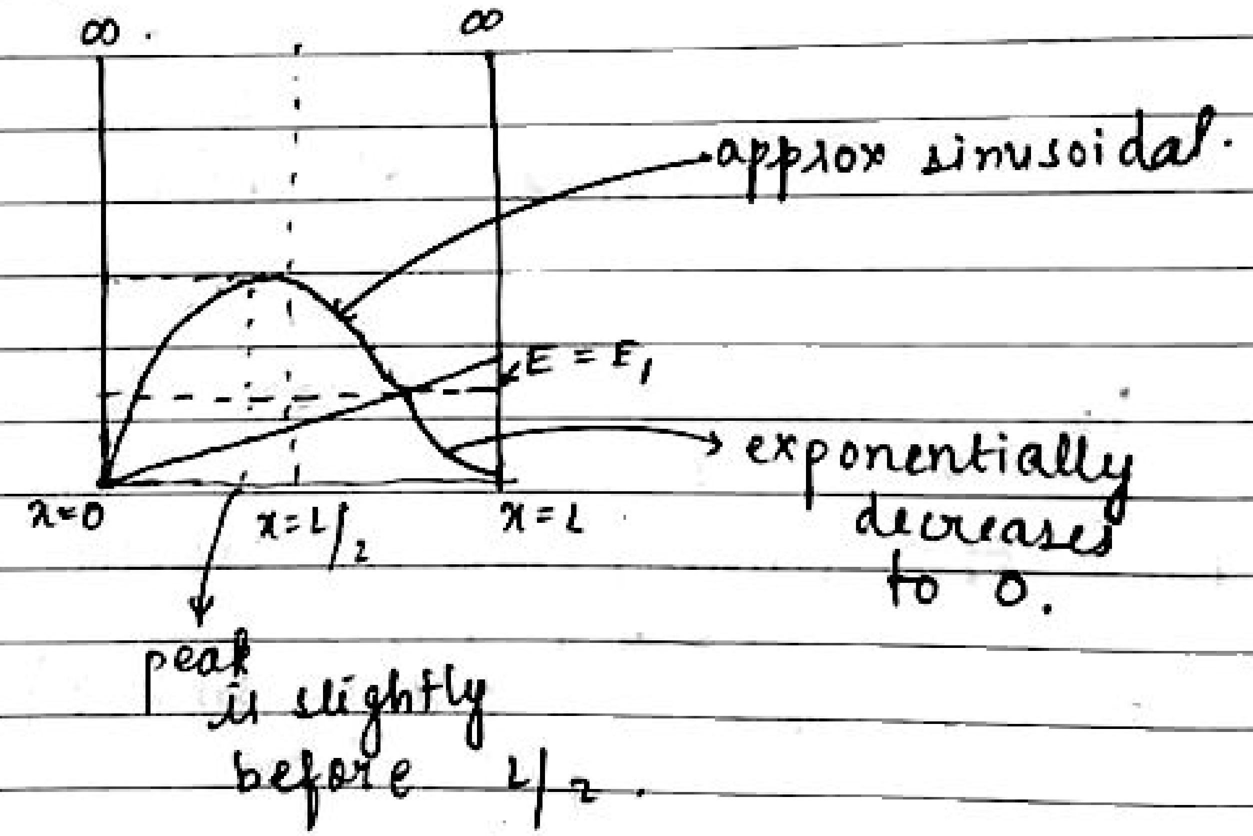
62.(a) $$\psi(x,t) = \frac{1}{\sqrt{L}} \sin\left(\frac{2\pi x}{L}\right) e^{-iE_2t/\hbar} + \frac{1}{\sqrt{L}} \sin\left(\frac{\pi x}{L}\right) e^{-iE_1t/\hbar}$$

$$E_2 = \frac{4h^2}{8mL^2} \qquad E_1 = \frac{h^2}{8mL^2}.$$

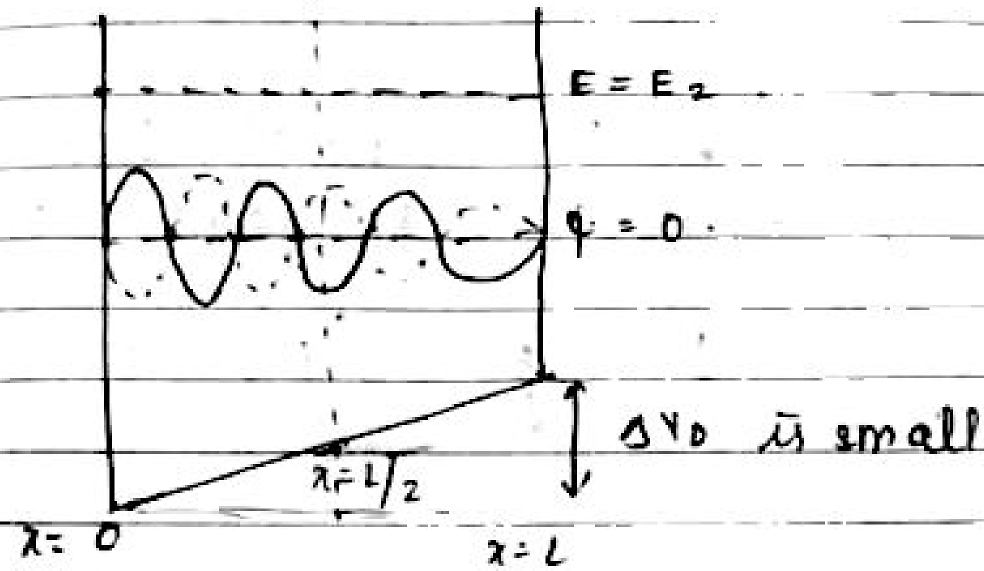
~~(b)~~

(b) $$\int_{L/4}^{L/2} \psi^* \psi \, dx$$ solve.

63.(a) $E = E_1$.

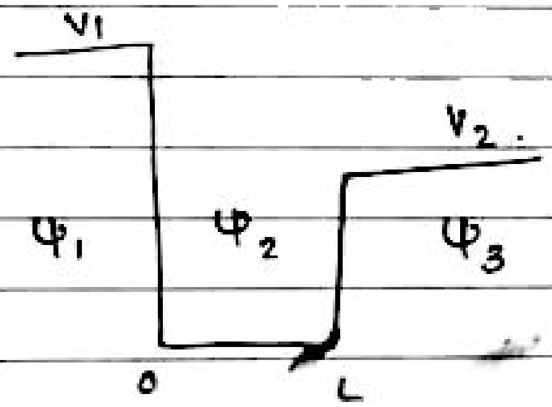


(b) $E = E_2$.



→ 3^{rd} node is slightly before $L/2$
→ λ is increasing as k is decreasing slightly
→ Amplitude is decreasing slightly
→ Approx sinusoidal

E4.



Assuming $E < V_2$

$x<0$ $\quad \psi_1 = \cancel{A}e^{-\alpha x} + Be^{\alpha x}$ $\quad \alpha = \sqrt{\frac{2m(V_1-E)}{\hbar^2}}$ ($A \to 0$)

$0<x<L$ $\quad \psi_2 = C\sin kx + D\cos kx$. $\quad k = \sqrt{\frac{2mE}{\hbar^2}}$

$L<x$ $\quad \psi_3 = Fe^{-\beta x} + \cancel{E}e^{\beta x}$ $\quad \beta = \sqrt{\frac{2m(V_2-E)}{\hbar^2}}$ ($E \to 0$)

(A & E are 0 as ψ is a well-behaved wave function).

Boundary conditions:

$x=0$ $\quad \psi_1 = \psi_2 \qquad B = D$

$\psi_1' = \psi_2' \qquad B\alpha = CK. \qquad \therefore C = B\alpha/K.$

$x=L$ $\quad \psi_2 = \psi_3 \qquad Fe^{-\beta L} = C\sin KL + D\cos KL$

$\psi_2' = \psi_3' \qquad -\beta F e^{-\beta L} = K(C\cos KL - D\sin KL).$

$$\left(\frac{\cancel{B}\alpha}{K}\frac{\sin KL}{\cos KL} + \cancel{B}\right)(-\beta) = \left(\cancel{B}\alpha - \frac{\sin KL}{\cos KL}\cdot \cancel{B}K\right)$$

$$\therefore \frac{\alpha+\beta}{K - \frac{\beta\alpha}{K}} = \tan KL$$

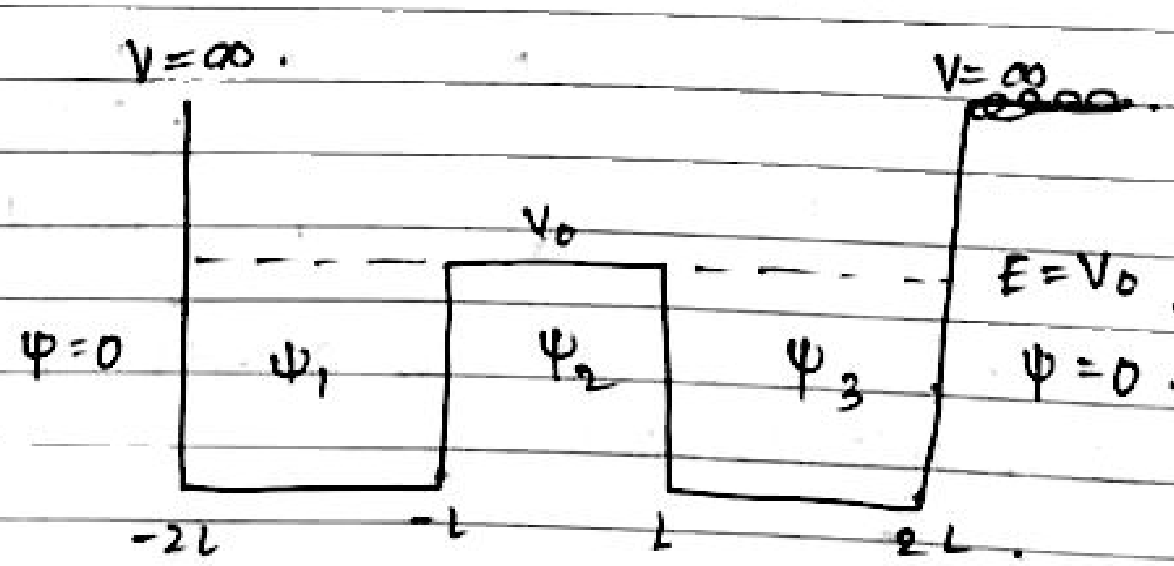
$$\boxed{\therefore \tan KL = \frac{K(\alpha+\beta)}{K^2 - \alpha\beta}.}$$

as $\alpha \to \infty$ $\quad \left(V_1 \to \infty \quad \alpha = \sqrt{\frac{2m(V_1 - E)}{\hbar^2}}. \quad \therefore \alpha \to \infty.\right)$

$$\therefore \tan KL = \lim_{\alpha\to\infty} \frac{\alpha K + \beta K}{K^2 - \alpha\beta}.$$

$$\therefore \boxed{\tan KL = \frac{-K}{\beta}.}$$

P65.



$\psi_1 = A\sin kx + B\cos kx$ $\quad k = \sqrt{\dfrac{2mE}{\hbar^2}}$ $\quad E = V_0$

$\psi_2 = C$

$\psi_3 = -A\sin kx + B\cos kx$

(ψ_3 we get from ψ_1 due to symmetry).

Boundary conditions:

$-A\sin 2kL + B\cos 2kL = 0$ $\quad x = -2L$

$Ak\cos kL + kB\sin kL = 0.$ $\quad x = -L.$

$C = -A\sin kL \Leftarrow -A\sin kL + B\cos kL = C$ $\quad x = -L.$

$A\sin 2kL = B\cos 2kL.$

$B\sin kL = -A\cos kL.$

$\therefore \dfrac{\sin 2kL}{\cos 2kL} = \dfrac{-\cos kL}{\sin kL}.$

$\sin 2kL \sin kL + \cos kL \cos 2kL = 0.$

$\cos kL = 0.$

substitute

$$\boxed{\therefore kL = \left(\frac{2n+1}{2}\right)\pi}$$

$\therefore \underline{B = 0}$

$\therefore$ (a) Normalize.

$\psi_1 = A\sin kx$

$\psi_2 = -A\sin kL.$

$\psi_3 = -A\sin kx.$

$$\int_{-2L}^{2L} \psi^* \psi \, dx = 1.$$

$$= \int_{-2L}^{-L} A^2 \sin^2 kx + \int_{-L}^{L} C^2 + \int_{L}^{2L} A^2 \sin^2 kx$$

$$A = \frac{1}{\sqrt{3L}}.$$

(b) $c = -\sqrt{\frac{1}{3l}} \sin\left(\frac{(2n+1)\pi}{2}\right)$.

(c) $\because$ $k = \sqrt{\frac{2mV_0}{\hbar^2}}$

$\therefore$ $\sqrt{\frac{2mV_0}{\hbar^2}} = \frac{(2n+1)\pi}{2L}$.

The two lowest values are:

$\sqrt{\frac{2mV_0}{\hbar^2}} = \frac{\pi}{2L}$ $\qquad$ $\sqrt{\frac{2mV_0}{\hbar^2}} = \frac{3\pi}{2L}$.

$\downarrow$ $\qquad$ $\downarrow$

$V_0 = 9.43\, eV$ $\qquad$ $V_0 = 84.9\, eV$

$\downarrow$

(d) Calculate K.

Accordingly calculate expectation values.