Tutorial 9

P75 $E_x = n_x^2 \alpha$ $\quad \left(\alpha = \dfrac{h^2}{8mL^2}\right)$.

$E_y = 4n_y^2 \alpha$

$E_z = 4n_z^2 \alpha$.

$$E_T = \frac{h^2}{8mL^2}\left(n_x^2 + 4n_y^2 + 4n_z^2\right).$$

Wave function

$$\psi = \left(\sqrt{\frac{2}{L}} \sin \frac{n_x \pi x}{L}\right)\left(\sqrt{\frac{4}{L}} \sin \frac{n_y \pi y}{L/2}\right)\left(\sqrt{\frac{4}{L}} \sin \frac{n_z \pi z}{L/2}\right)$$

Lowest energy level that exhibits degeneracy

$(1,1,2)$ $(1,2,1)$ $\rightarrow \dfrac{21h^2}{8mL^2}$.

P76. $$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi + (-V_0)\psi = E\psi$$

$\psi = \psi_x \psi_y \psi_z \rightarrow$ variable separable

$$E_T = \frac{h^2}{8mL^2}\left(n_x^2 + 4n_y^2 + 4n_z^2\right) - V_0.$$

Lowest energy level that exhibits degeneracy

$(1,1,2) \rightarrow (1,2,1) \Rightarrow \dfrac{21h^2}{8mL^2} - V_0$

Get unnormalized wave function

Max value of V_0 for -ve state.

$$V_0 = \frac{9h^2}{8mL^2}.$$

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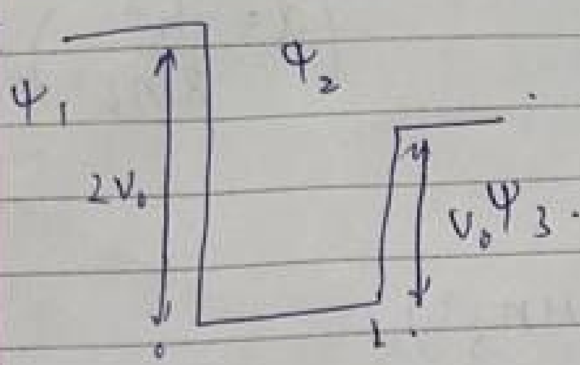

Assume $E < V_0$.

$\psi_1 = Ae^{\alpha x}$, $\psi_3 = De^{-\beta x}$

$$\left(\alpha = \sqrt{\frac{2m(2V_0-E)}{\hbar^2}}, \quad \beta = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}\right).$$

($\psi_1, \psi_3 \rightarrow$ well-behaved).

$$\psi_2 = B\sin kx + C\cos kx \qquad k = \sqrt{\frac{2mE}{\hbar^2}}.$$

Boundary conditions.

$x=0$ $\psi_1 = \psi_2 \Rightarrow A = C$ $\quad \psi_1' = \psi_2' \Rightarrow A\alpha = Bk.$

$x=L$ $\psi_2 = \psi_3 \Rightarrow \frac{A\alpha}{k}\sin kL + A\cos kL = De^{-\beta L}.$

$$A\alpha\cos kL - Ak\sin kL = -\beta De^{-\beta L}.$$

$$\tan kL = \frac{k(\alpha+\beta)}{k^2 - \alpha\beta}.$$

$$\tan kL = \frac{\sqrt{E}\left(\sqrt{2V_0-E} - \sqrt{V_0-E}\right)}{E - \sqrt{2V_0-E}\sqrt{V_0-E}} = p.$$

$$\therefore kL = \tan^{-1} p + n\pi$$

Now, for exactly one bounded state, $E_1 < V_0$ & $E_2 > V_0$.

$\therefore$ In limiting conditions $E_1 = V_0$ & $E_2 = V_0$.

$$\therefore \quad \frac{\pi}{4} < \underset{E=V_0}{kL} < \pi + \frac{\pi}{4}$$

$$\frac{\pi\hbar}{4\sqrt{2mV_0}} < L < \frac{5\pi\hbar}{4\sqrt{2mV_0}}$$

(for details PTO).

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Explanation:
for exactly one bound state,
$E_1 < V_0 \quad E_2 > V_0.$
$\therefore$ As a limiting condition,
~~i.e. tan~~, we take $E_1 = E_2 = V_0$,
as it's an increasing function
$\left(\frac{dL}{dE} > 0\right) \longrightarrow$ (Ask me in class if you don't get it)

$\therefore \quad n = 0$:

$$\tan\left(\sqrt{\frac{2mE_1}{\hbar^2}}\,L\right) = \frac{\sqrt{E_1}\left(\sqrt{2V_0 - E_1} - \sqrt{V_0 - E_1}\right)}{E_1 - \sqrt{2V_0 - E_1}\sqrt{V_0 - E_1}}$$

$$E_1 = V_0.$$

$$\tan\left(\sqrt{\frac{2mV_0}{\hbar^2}}\,L\right) = 1.$$

$$\therefore \quad L > \frac{\pi}{4}\frac{\hbar}{\sqrt{2mV_0}}$$

Similarly $\quad E_2 = V_0.$

$$L < \frac{5\pi}{4}\frac{\hbar}{\sqrt{2mV_0}}$$

I mistakenly told in class,
$n\pi + \tan^{-1}p < kL < (n\pi + \pi) + \tan^{-1}p.$
This is the condition for exactly 'n' states.

$$\therefore \quad \frac{5\pi}{4}\frac{\hbar}{\sqrt{2mV_0}} > L > \frac{\pi}{4}\frac{\hbar}{\sqrt{2mV_0}}$$

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P78. $\dfrac{h^2}{8mL^2} = \alpha$

(n_x, n_y, n_z).

state			Energy.
(1, 1, 4)	(2, 1, 2)	(1, 2, 2)	$24/4\,\alpha$
(2, 1, 1)	(1, 2, 1)		$21/4\,\alpha$
(1, 1, 3)			$17/4\,\alpha$
(1, 1, 2)			$12/4\,\alpha$
(1, 1, 1)			$9/4\,\alpha$

Q9. $\psi(x,y,z) = \frac{1}{2\pi^{3/2}} \sin 3x \, e^{i5y} e^{iz}$.

$$= \frac{1}{2\pi^{3/2}} \left(\frac{e^{i3x} - e^{-i3x}}{2i} \right) e^{i5y} e^{iz}.$$

$$= \frac{1}{4\pi^{3/2} i} \left(\underbrace{e^{i(3x+5y+z)}}_{\psi_I} + \underbrace{\left(-e^{+i(-3x+5y+z)}\right)}_{\psi_{II}} \right).$$

(a) $-\frac{\hbar^2}{2m} \nabla^2 (\psi(x,y,z)) = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2}\psi + \frac{\partial^2}{\partial y^2}\psi + \frac{\partial^2}{\partial z^2}\psi \right)$.

(TISE:

$-\frac{\hbar^2}{2m} \nabla^2 \psi(x,y,z) + \underbrace{V}_{0} \psi(x,y,z) = E \psi(x,y,z)$.

(Eigen values : energy of operator $-\frac{\hbar^2}{2m}\nabla^2$)).

$\psi_I = \psi_{II} = (3^2 + 5^2 + 1^2) \frac{\hbar^2}{2m} = \frac{35\hbar^2}{2m}$.

$\therefore$ They will observe the value of energy $= \frac{35\hbar^2}{2m}$ with equal probability of collapsing into ~~each~~ both eigen states.

(b) momentum operator : $-i\hbar \nabla(\psi)$.

$\therefore \psi_I \Rightarrow \hbar(3\hat{i} + 5\hat{j} + \hat{k})$
$\psi_{II} \Rightarrow \hbar(-3\hat{i} + 5\hat{j} + \hat{k})$ } possible values of (p_x, p_y, p_z).

$\Downarrow$

$\hbar(3, 5, 1)$

$\hbar(-3, 5, 1)$

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(c) For $\psi(r,t)$ multiply by $e^{-iEt/\hbar}$.

$$\therefore \psi(r,t) = \psi_I e^{-iEt/\hbar} + \psi_{II} e^{-iEt/\hbar}$$

where $E = \frac{35\hbar}{2m}$,

(d) If ψ collapses to a state (here ψ_I),

$$\psi = \psi_I .$$

$$\therefore \psi(x,y,z) = \frac{1}{2\pi^{3/2}} \times \frac{1}{2i} \times e^{i(3x+5y+z)}$$

steps in (a), (b), (c)

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V

P80. $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + \frac{1}{2}m\omega^2 x^2\psi = E\psi.$

$\psi = Axe^{-\left(\frac{m\omega}{2\hbar}\right)x^2}.$

$LHS = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + \frac{1}{2}m\omega^2 x^2\psi.$

substitute.
You'll get LHS = RHS.

P81. $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + \frac{1}{2}m\omega^2 x^2\psi = E\psi.$

$\psi = Axe^{-\alpha x^2}.$

substitute ψ in LHS.
In the expression you get,
you'll have to make coefficient
of x^2 0 for it to be an eigen equation

$\alpha = \frac{m\omega}{2\hbar}$

P82. Assuming $E_0 \xrightarrow{\text{+ve X-axis}}$ (Assuming reference origin)

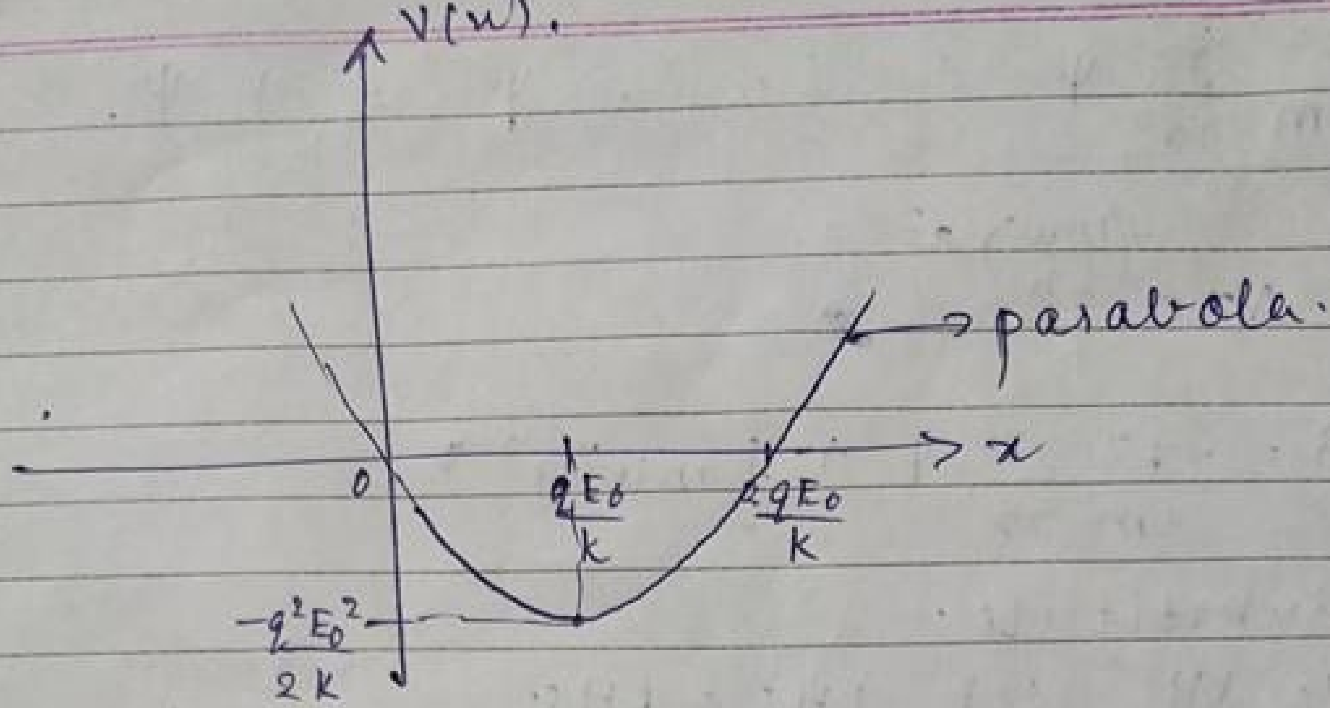
(a) $\therefore V(x) = \frac{1}{2}m\omega^2 x^2 - qE_0 x$

(b) $m\omega^2 = K.$

$\frac{m\omega^2}{2}\left(x - \frac{qE_0}{m\omega^2}\right)^2 - \frac{q^2E_0^2}{2m\omega^2}.$

$V(x) = \frac{K}{2}\left(x - \frac{qE_0}{K}\right)^2 - \frac{q^2E_0^2}{2K}.$

(c)



(d) Ground state $\rightarrow Ae^{-\alpha x^2}$.

~~you'll get something like~~ $Ae^{-\alpha(x-\frac{qE_0}{k})^2}$

$$\psi = Ae^{-\alpha(x-\frac{qE_0}{k})^2}.$$

substitute in schrodinger's equation with $V(x)$ as in part(c).
Solve to get solution

(According to my guess,
$\frac{\hbar\omega}{2} - \frac{q^2E_0^2}{2k}$ (I'm not at all sure of answer).