PH-107 (2017)
Tutorial Sheet 8

* Problems to be done in tutorial.

A. Bound State Problem

P66*: A particle in a one-dimensional well (V=0 for 0<x<L, V=∞ elsewhere) has the wave function $\phi(x) = Ax(L-x)$ inside the box and $\phi(x) = 0$ elsewhere at $t=0$. Calculate the expectation value of energy. On making an energy measurement, what is the probability of finding the particle in the ground state?

Q67. You are given an arbitrary potential V(x) and the corresponding orthogonal and normalized bound-state solutions to the time-independent Schrodinger's equation, $\phi_n(x)$ with corresponding energy eigen values E_n. At time t = 0, the system is in the state,

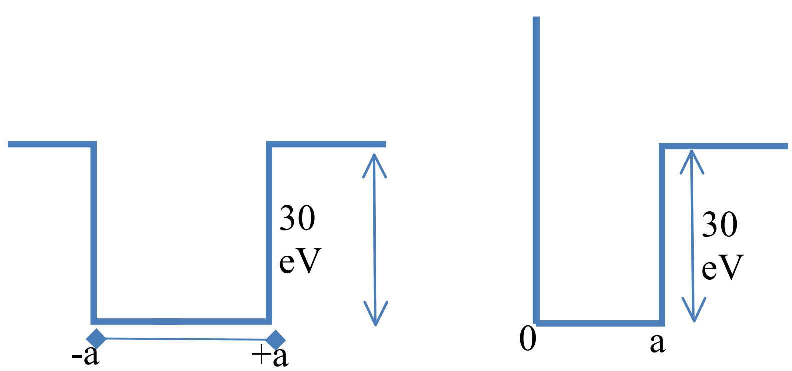
$$\psi(x,0) = A[\phi_1(x) + \phi_2(x) + \phi_4(x)].$$

(a) Find A for this wave function.

(b) What is the wave function at time t > 0.

(c) What is the expectation value of the energy at time t > 0.

Q68*. Suppose a finite square well (Figure shown below) has six bound states, 3, 7, 12, 17, 21, and 24 eV. If instead the potential is semi-infinite, with an infinite wall at x=0, how many bound states will exist in this semi-infinite well and what are the energies associated with it? Justify/explain in 1-2 sentences.



A. Scattering problems

P69: A beam of particles with energy E approaches from left hand side, a potential barrier defined by $V=0$ for $x<0$ and $V=V_o$ for $x>0$, where $V_o>E$.

- **(a)** Find the value of $x=x_o$ ($x_o>0$), for which the probability density is $1/e$ times the probability density at $x=0$.
- **(b)** Take maximum allowed uncertainty Δx for the particle to be localized in the classically forbidden region as x_o. Find the uncertainty this would cause in the energy of the particle. Can then one be sure that its energy E is less than V_o.

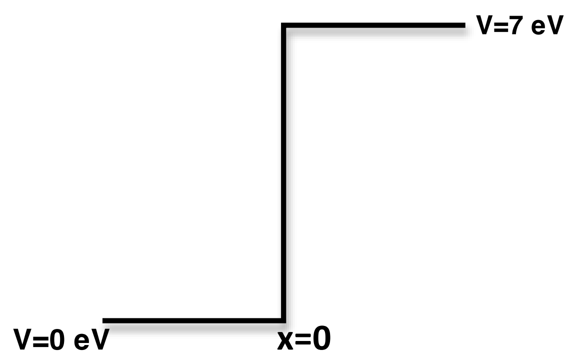
P70*: A beam of particles of energy E and de Broglie wavelength λ, traveling along the positive x-axis in potential free region, encounters a one-dimensional potential barrier of height $V=E$ and width L.

- **(a)** Obtain an expression for the transmission coefficient.
- **(b)** For what value of L (in terms of λ), will the reflection coefficient be half?

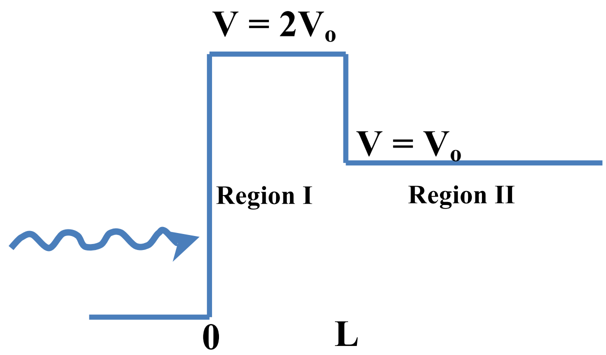
Q71. Particles of mass m and energy E impinge on a finite potential barrier of height V_0 and width L. Assume that $E < V_0$. P is the probability for the particle to tunnel through the barrier. Of the four statements below, which are true (T) and which are false (F)?

- **(a)** P is larger for larger value of E ______________
- **(b)** P is larger for larger value of m ______________
- **(c)** P is smaller for smaller values of V_0 ______________
- **(d)** P is smaller for smaller values of L. ______________

Q72*. Consider a potential V(x) as shown below. A beam of electrons, of energy 3 eV, collides with this potential. At what value of `x' will the probability of detecting the electron be half of the probability of detecting it at x = 0?



Q73. Consider a beam of particle of energy $E<V_o$ is incident from the left on a barrier of height $V = 2V_o$ as shown in the figure. It is claimed that for **region I** $(0<x<L)$ the solution is $\psi_I = Ae^{-k_1 x}$ where $k_1 = \sqrt{\frac{2m(2V_0 - E)}{\hbar^2}}$ and for **region II** $(x>L)$ the solution is of type $\psi_{II} = Be^{-k_2 x}$, where $k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$ Is this claim correct? Justify your answer in a few steps.

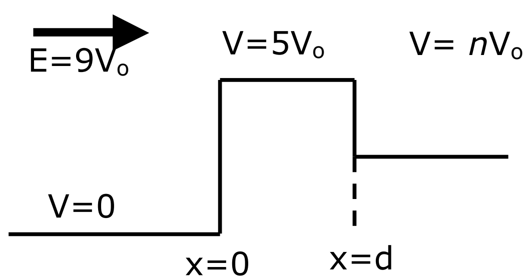


P74*: A beam of particles of mass 'm' and energy 9 V_o (V_o is a positive constant of energy dimension) is incident from left on a barrier given below.

$V=0$ for $x<0$

$V=5\,V_o$ for $x \le d$; where $d = \frac{\pi\ h}{\sqrt{8mV_o}}$

$V= nV_o$ for $x \ge d$; where n is a number, positive or negative.



It is found that the transmission coefficient from $x<0$ region to $x>d$ region is 0.75.

(a) Find 'n'. Is there more than one possible value for 'n'?

(b) Find the un-normalized wave function in all the regions in terms of the amplitude of the incident wave for each possible value of 'n'.

(c) Is there a phase change between the incident and the reflected beam at $x=0$? If yes determine it for each possible value of 'n'.

Give your answers by explaining all the steps and clearly writing the boundary conditions used