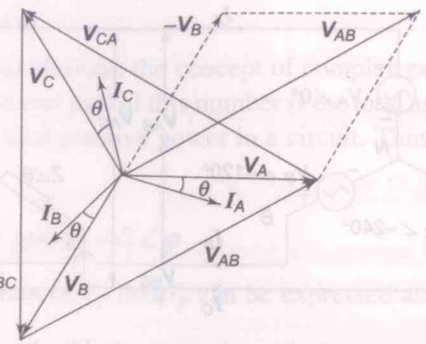

Fig. 5.15 Phasor diagram of balanced star-connected three-phase load connected to three-phase star-connected supply

For a three-phase four-wire star-connected supply system, the three phase voltages and the neutral conductor are available at the supply terminals. The neutral conductor connects the neutral point of the source with the neutral point of the star-connected load. This connection is generally employed for low-voltage distribution systems. If the three-phase load is unbalanced, the neutral conductor carries current I_N given by

$$I_N = I_A + I_B + I_C$$

For a balanced load, I_N is zero.

Example 5.1 In a three-phase four-wire system the line voltage is 400 V. Non-inductive loads of 12 kW, 10 kW, and 8 kW are connected between the three line conductors and the neutral point as shown in Fig. 5.16. Calculate (a) the current in each line, and (b) the current in the neutral conductor.

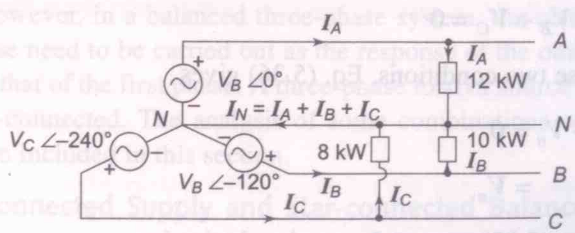

Fig. 5.16 Circuit diagram for Example 5.1

Solution

The phase voltages are given by

$$V_A = \frac{400}{\sqrt{3}} \angle 0° = 230.94 \angle 0°$$

$$V_B = 230.94 \angle -120°$$

$$V_C = 230.94 \angle -240°$$

(a) The line currents are given by

$$I_A = \frac{12 \times 10^3}{230.94 \angle 0°} = 51.96\angle 0° \text{ A}$$

$$I_B = \frac{10 \times 10^3}{230.94 \angle -120°} = 43.3 \angle 120° \text{ A}$$

$$I_C = \frac{8 \times 10^3}{230.94 \angle -240°} = 34.64 \angle 240° \text{ A}$$

(b) The current in neutral

$$I_N = I_A + I_B + I_C$$

$$= 51.96\angle 0° + 43.3 \angle 120° + 34.64 \angle 240°$$

$$= 51.6(1 + j0) + 43.3(-0.5 + j0.866) + 34.64(-0.5 - j0.866)$$

$$= 51.96 + (-21.65 + j37.5) + (-17.32 - j30.0)$$

$$= 12.99 + j7.5 = 15 \angle 30° \text{ A}$$

5.4.2 Star-connected Supply and Delta-connected Balanced Load

Figure 5.17 shows a delta-connected balanced load, with an impedance $Z = Z\angle\theta$ inserted in each pair of lines, fed from a three-phase star-connected supply with balanced phase voltages $V_A = V_P\angle 0°$, $V_B = V_P \angle -120°$, and $V_C = V_P\angle -240°$ and balanced line voltages, each having magnitude $V_L = \sqrt{3}V_P$. The line voltage phasors can then be expressed as $V_{AB} = \sqrt{3}V_P \angle 30°$, $V_{BC} = \sqrt{3}V_P \angle -90°$, and $V_{CA} = \sqrt{3}V_P \angle -210°$. For this connection, the phase currents from the source and the line currents drawn by the delta-connected loads are the same.

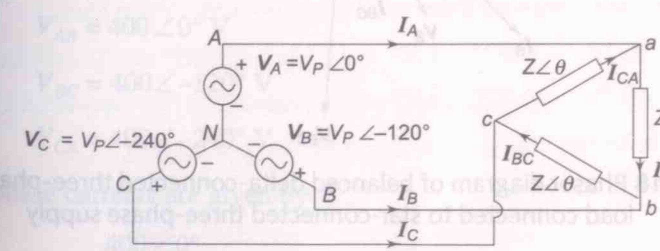

Fig. 5.17 Delta-connected load across star-connected supply

The load currents may be obtained as

$$I_{AB} = \frac{V_{AB}}{Z} = \frac{\sqrt{3}V_P \angle 30°}{Z\angle\theta} = \frac{\sqrt{3}V_P}{Z} \angle (30 - \theta)° \quad (5.39)$$

$$I_{BC} = \frac{V_{BC}}{Z} = \frac{\sqrt{3}\, V_P \angle -90°}{Z\angle\theta} = \frac{\sqrt{3}\, V_P}{Z} \angle (-90 - \theta)° \quad (5.40)$$

$$I_{CA} = \frac{400\angle -240^\circ}{30 + j70} = \frac{400\angle -240^\circ}{76.16\angle 66.8} = 5.252\angle -173.2$$

$$= (-5.215 - j0.622) \text{ A}$$

(b) The line currents are given by

$$I_A = I_{AB} - I_{CA} = 2.667 - (-5.215 - j0.622)$$

$$= 7.882 + j0.662 = 7.91\angle 4.8^\circ \text{ A}$$

$$I_B = I_{BC} - I_{AB} = (48.95 - j23.26) - 2.667$$

$$= 46.283 - j23.26 = 51.8\angle 25.68^\circ \text{ A}$$

$$I_C = I_{CA} - I_{BC} = -5.215 - j0.622 - 48.95 - j23.26$$

$$= -54.165 - j23.882 = 59.2\angle -156.2^\circ \text{ A}$$

Example 5.3 A balanced star-connected load of $(4 + j\,3)$ Ω per phase is connected to a 400-V, 3-phase, 50-Hz supply. Find the (a) line current, (b) power factor, (c) power, (d) reactive volt-ampere, and (e) total volt-ampere.

Solution

(a) Load impedance, $Z_L = 4 + j3 = 5\angle 36.87^\circ$

Assuming the phase voltage $V_A = \angle 0^\circ$ as the reference phasor and using Eq. (5.19), the line current I_A may be calculated as

$$I_A = \frac{V_A}{Z_L} = \frac{(400/\sqrt{3})\angle 0^\circ}{5\angle 36.87^\circ} = 46.19\angle -36.87^\circ \text{ A}$$

As the load is balanced, the phase currents are also balanced. They have equal magnitude but are displaced from each other by 120°. Therefore,

$$I_B = 46.19\angle -36.87^\circ - 120^\circ = 46.19\angle -156.87^\circ \text{ A}$$

$$I_C = 46.19\angle -36.87^\circ - 240^\circ = 46.19\angle -276.87^\circ = 46.19\angle 83.13^\circ \text{ A}$$

(b) The power factor, $\cos\varphi = \cos(-36.87^\circ) = 0.8$ lagging

(c) The three-phase power, $P = \sqrt{3}\,V_L I_L \cos\varphi = \sqrt{3} \times 400 \times 46.19 \times 0.8$
$= 25.6$ kW

(d) The reactive volt-ampere, $Q = \sqrt{3}\,V_L I_L \sin\varphi = \sqrt{3} \times 400 \times 46.19 \times 0.6$
$= 19.2$ kvar

(e) The total volt-ampere, $\sqrt{3}\,V_L I_L = \sqrt{3} \times 400 \times 46.19 = 32$ kVA

The total power is given by

$$P_1 + P_2 = V_{AC}I_A\cos(30° - \varphi) + V_{BC}I_B\cos(30° + \varphi) \qquad (5.51)$$

Since the load is balanced, substituting $V_{BC} = V_{AC} = V$ and $I_A = I_B = I$ in Eq. (5.51),

$$P_1 + P_2 = VI\{(\cos 30° - \varphi) + (\cos 30° + \varphi)\}$$

$$= \sqrt{3}\ VI\cos\varphi \qquad (5.52)$$

Subtracting Eq. (5.50) from Eq. (5.49) and simplifying

$$P_1 - P_2 = VI\{(\cos 30° - \varphi) - (\cos 30° + \varphi)\}$$

$$= VI\sin\varphi \qquad (5.53)$$

Dividing Eq. (5.53) by Eq. (5.52) gives

$$\tan\varphi = \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2} \qquad (5.54)$$

$$\varphi = \tan^{-1}\sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2} \qquad (5.55)$$

Equation (5.55) provides an expression for the determination of the power factor angle φ of the load by the two-wattmeter method.

It is important to note that, in general, the two wattmeters work under quite different phase angle conditions. Thus, the phase angle of one is $(30 - \varphi)$ and that of the other is $(30 + \varphi)$. Hence, only in the special case of $\varphi = 0$, that is, unity power factor, can the two readings be equal. For all other power factors, even under balanced conditions, the two readings are not equal. It must also be noted that if one of the phase angles becomes greater than 90°, that is $\varphi > 60°$, the wattmeter will give a negative reading, which must be corrected by reversing the connections to the terminals of its pressure coil. Under these circumstances its reading is reckoned as negative, and the total power is then the difference of the readings. Thus, in general, the total power is given by the algebraic sum of the two readings.

Example 5.6 A 3-phase, 415-V, mesh-connected system shown in Fig. 5.25 has the following loads: 25 kW at power factor 1.0 for branch AB, 40 kVA at power factor 0.85 lagging for branch BC, 30 kVA at power factor 0.6 leading for branch CA. Find the line currents and the readings on wattmeters whose current coils are in phases A and C. Also, sketch the phasor diagram.

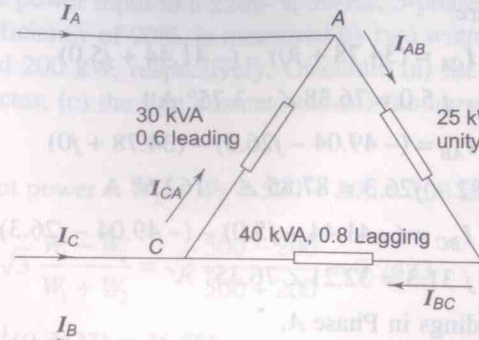

Fig. 5.25 Circuit diagram for a three-phase delta connected loads

Solution

Let the line voltage V_{AB} be the reference phasor. Then the line voltages are

$$V_{AB} = 415\angle 0°$$

$$V_{BC} = 415\angle -120°$$

$$V_{CA} = 415\angle -240°$$

The phase currents are

$$I_{AB} = \frac{\text{kW}\times 10^3}{\sqrt{3}\times V_{AB}\times \text{pf}} = \frac{25\times 10^3}{\sqrt{3}\times 415\times 1}$$

$= 34.78$ A in phase with respect to V_{AB}

or

$$I_{AB} = 34.78\angle 0° = 34.78 + j0$$

$$I_{BC} = \frac{\text{kVA}\times 10^3}{\sqrt{3}\times V_{BC}} = \frac{40\times 10^3}{\sqrt{3}\times 415}$$

$= 55.65$ A at pf 0.85 lagging with respect to V_{BC}

$$\cos^{-1}0.85 = \angle 31.8°$$

Then

$$I_{BC} = 55.65\angle(-120 - 31.8)° = 55.65\angle -151.8°$$

$$= -49.04 - j26.3 \text{ A}$$

$$I_{CA} = \frac{\text{kVA}\times 10^3}{\sqrt{3}\times V_{AB}} = \frac{30\times 10^3}{\sqrt{3}\times 415}$$

$= 41.74$ A at pf 0.6 leading with respect to V_{CA}

Also, $\cos^{-1}0.6 = \angle 53.13°$. Then

$$I_{CA} = 41.74\angle(-240 + 53.13)° = 41.74\angle -186.87°$$

$$= -41.44 + j5.0 \text{ A}$$

The line currents are

$$I_A = I_{AB} - I_{CA} = (34.78 + j0) - (-41.44 + j5.0)$$
$$= 76.22 - j\,5.0 = 76.38\angle -3.75^\circ \text{ A}$$
$$I_B = I_{BC} - I_{AB} = (-49.04 - j26.3) - (34.78 + j0)$$
$$= -83.82 - j26.3 = 87.85\angle -162.6^\circ \text{ A}$$
$$I_C = I_{CA} - I_{BC} = (-41.44 + j5.0) - (-49.04 - j26.3)$$
$$= 7.6 + j\,31.3 = 32.21\angle 76.35^\circ \text{ A}$$

The wattmeter readings in Phase A,

$$W_1 = V_{AB} \times I_A \times \cos\varphi_A$$

where φ_A is the phase angle between the phasors V_{AB} and I_A. Then

$$W_1 = 415 \times 76.38 \times \cos(-3.75^\circ) = 31.63 \text{ kW}$$

The wattmeter readings in Phase C,

$$W_2 = V_{CB} \times I_C \times \cos\varphi_C$$

where φ_C is the phase angle between the phasors V_{CB} and I_C

$$V_{CB} = -V_{BC} = -415\angle -120^\circ = 415\angle 60^\circ$$

$$\varphi_C = 76.35 - 60 = 16.35^\circ.$$

Then, $W_2 = 415 \times 32.21 \times \cos(16.35^\circ) = 12.827$ kW

The sketch of the phasors is shown in Fig. 5.26.

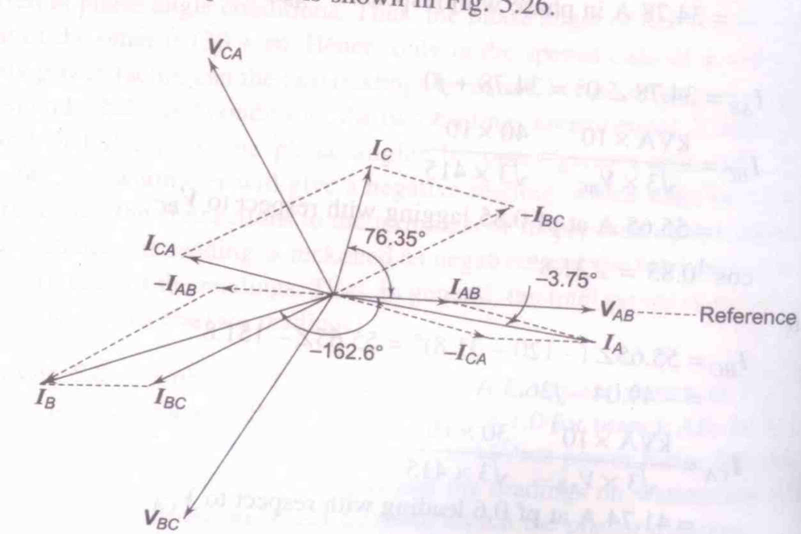

Fig. 5.26 Phasor diagram for Example 5.6

Example 5.7 The power input to a 2200-V, 50-Hz, 3-phase motor, running on full load at an efficiency of 90%, is measured by two wattmeters, which indicate 500 kW and 200 kW, respectively. Calculate (a) the total input power, (b) the power factor, (c) the line current, and (d) the horse power output.

Solution

(a) The total input power = $W_1 + W_2 = 500 + 200 = 700$ kW

(b) $$\tan\varphi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{500 - 200}{500 + 200} = 0.7423$$

$$\varphi = \tan^{-1}(0.7423) = 36.58^\circ$$

The power factor, $\cos\varphi = \cos 36.58^\circ = 0.803$.

(c) The line current can be determined from

$$P = \sqrt{3}\,V_L I_L \cos\varphi$$
$$= 3 \times 2200 \times I_L \times 0.803$$

$$\therefore \quad I_L = \frac{700 \times 10^3}{\sqrt{3} \times 2200 \times 0.803} = 228.77 \text{ A}$$

(d) Efficiency η = output/input

Then,

output = $\eta \times$ input = $0.9 \times 700 = 630$ kW

Now, 1 hp = 746 W

$$\therefore \quad \text{Output} = \frac{630 \times 10^3}{746} = 844.5 \text{ hp}$$

Recapitulation

For a star-connected system: $V_L = \sqrt{3}\,V_P$ and $I_L = I_P$

For a delta-connected system: $V_L = V_P$ and $I_L = \sqrt{3} I_P$

Active power in a three-phase network, $P = 3\,V_P I_P \cos\varphi = \sqrt{3}\,V_L I_L \cos\varphi$

Reactive power in a three-phase network, $Q = 3\,V_P I_P \sin\varphi = \sqrt{3}\,V_L I_L \sin\varphi$

Volt-amperes in a three-phase network, $VA = 3\,V_P I_P = \sqrt{3}\,V_L I_L$