Roll Number:

PH107(August 27, 2014) QUIZ 1

a. **Please write your Name, Roll Number, Division and Tutorial Batch on answer sheets and Roll Number on the Question Paper.**
b. All steps must be shown. All explanations must be clearly given for getting credit. Just the correct final answer does not guarantee the full credit.
c. Possession of mobile phone and exchange of calculators during the examination are strictly prohibited.

Weightage: 15% **Time: 50 minutes**

1.
(a) Using equipartition law, find the molar specific heat of a gas (at constant volume), which contains tri-atomic molecules with the atoms bonded (non rigidly) to each other in a triangular form.

[2 marks]

Triatomic molecule has 3 atoms and hence total no. of degrees of freedom=3x3=9
Out of these, 3 are translational, 3 are rotational and the remaining 3 are vibrational.
According to the equipartition theorem, the average energy per degree of freedom for translation and rotation is $\frac{1}{2}k_BT$ each and that for vibrational freedom is k_BT.

..................(1 mark)

Therefore, the total energy per mole is $\left[(3+3)\frac{1}{2}k_BT + 3k_BT\right]N_A = 6N_Ak_BT$

And $C_V = \frac{dE}{dT} = 6N_Ak_B = 6R$ *..................(1 mark)*

(b) In a dispersive medium, the frequency- wavelength relationship of certain type of waves is such that phase velocity is v_o for a wavelength λ_o, but is $v_o/2$ for the wavelength $2\lambda_o$. Find the group velocity of the waves at the wavelengths λ_o and $2\lambda_o$, in terms of v_o. Also find the angular frequency ω of the wave for these two wavelengths, in terms of v_o and λ_o.

[4 marks]

$$v_p = \frac{\omega}{k} \propto \frac{1}{\lambda} \quad or \quad \frac{\omega}{k} \propto k$$

$$i.e., \omega = Ak^2 = v_p k \; (A \text{ is a constant})$$

..(1 mark)

$$v_g = \frac{d\omega}{dk} = 2Ak = 2v_p$$

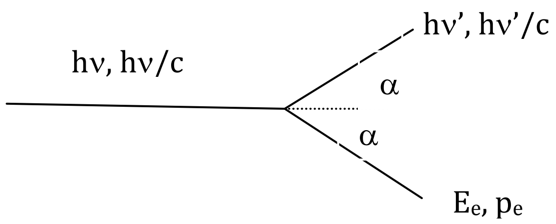
$$\therefore v_g(\lambda = \lambda_0) = 2v_0 \text{ and}$$

$$v_g(\lambda = 2\lambda_0) = 2\frac{v_0}{2} = v_0$$ (1 mark)

$$\omega(\lambda = \lambda_0) = v_0 k = v_0 \times \frac{2\pi}{\lambda_0} = \frac{2\pi v_0}{\lambda_0}$$

$$\omega(\lambda = 2\lambda_0) = v_0 k = \frac{v_0}{2} \times \frac{2\pi}{2\lambda_0} = \frac{2\pi v_0}{4\lambda_0}$$ (2 marks)

2. A photon with energy equal to $3m_o c^2$ is scattered by a particle of rest mass m_o, initially at rest. After the scattering, both the photon and the particle move in different directions making equal angles α with the direction of the incident photon. Find (a) the value of α (b) the energy of the scattered photon in terms of $m_o c^2$ and (c) de Broglie wave length of the scattered particle. [5 marks]



$$p_e \sin\alpha = \frac{h\nu'}{c}\sin\alpha ....(1)$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c} + p_e \cos\alpha ....(2)$$ (1 mark)

$$\therefore p_e = \frac{h\nu'}{c}$$

Putting this in eqn.(2),

$$\frac{h\nu}{c} = 2p_e \cos\alpha = 2\frac{h\nu'}{c}\cos\alpha$$

$$\therefore \frac{\nu}{\nu'} = 2\cos\alpha.$$

$$or \ \frac{\lambda'}{\lambda} = 2\cos\alpha$$ (1 mark)

Using Compton effect equation, $\lambda' \text{-} \lambda = \frac{h}{m_0 c}(1-\cos\alpha)$

$$i.e., \lambda(2\cos\alpha - 1) = \frac{h}{m_0 c}(1-\cos\alpha)..............(3)$$

Also, $\frac{hc}{\lambda} = 3m_0 c^2 \Rightarrow \lambda = \frac{h}{3m_0 c}$

putting in equation 3, $\alpha = \cos^{-1}(4/5)$ (1 mark)

$$\frac{h\ \nu}{h\ \nu'} = \frac{8}{5},$$

$$\therefore h\nu' = \frac{15}{8} m_0 c^2$$ (1 mark)

$$p_e = \frac{h\nu'}{c} = \frac{15}{8} m_0 c$$

$$\lambda_{dB} = \frac{h}{p_e} = \frac{8}{15}\left(\frac{h}{m_0 c}\right) = 1.3\times 10^{-3}\ nm$$ (1 mark)

3. A particle of mass m is confined to a region where a potential $V = V_0 \frac{|x|}{a}$ $(-\infty < x < \infty)$ exists. It is given that V_0 has the dimensions of potential and a has the dimension of length. Both V_0 and a are constants. Using the uncertainty principle with $\Delta x \Delta p_x = h$, find out the ground state energy of the particle. [4 marks]

$$V = V_0 \frac{|x|}{a}$$

$$\Delta x = 2x$$

$$\Delta p_x = \frac{h}{2x}$$

since $\langle p_x \rangle = 0, \langle p^2 \rangle = (\Delta p_x)^2 = \left(\frac{h^2}{4x^2}\right)$ (1 mark)

$$E = \frac{\langle p^2 \rangle}{2m} + V(x) = \left(\frac{h^2}{8mx^2}\right) + V_0 \frac{x}{a}$$ (1 mark)

$$\frac{dE}{dx} = -\left(\frac{h^2}{4mx^3}\right) + \frac{V_0}{a} = 0$$

$$\left(\frac{h^2}{4mx^3}\right) = \frac{V_0}{a} \Rightarrow x_0 = \left(\frac{h^2 a}{4mV_0}\right)^{1/3}$$ (1 mark)

$$E_0 = \left(\frac{h^2}{8mx_0^{\ 2}}\right) + V_0 \frac{x_0}{a} \Rightarrow \left(\frac{h^2}{8m}\right)\left(\frac{h^2 a}{4mV_0}\right)^{-2/3} + \frac{V_0}{a}\left(\frac{h^2 a}{4mV_0}\right)^{1/3}$$

$$\Rightarrow \left(\frac{h^2 a V_0^{\ 2}}{4m}\right)^{1/3}\left[\left(\frac{h^2}{8m}\right)\left(\frac{4m}{h^2 a}\right) + \frac{1}{a}\right]$$

$$\Rightarrow \left(\frac{27h^2 V_0^{\ 2}}{32ma^2}\right)^{1/3}$$ (1 mark)

Even if a student has taken $\Delta x = x$, full credit should be give, provided that all other steps are correct.

Useful data

$$E^2 = p^2c^2 + m_o{}^2c^4$$

$$\lambda' - \lambda = \frac{h}{m_oc}\left(1 - \cos\theta\right)$$

$$\frac{h}{m_oc} = 2.43 \times 10^{-3} \text{ nm}$$