Roll No:

Q1. Two identical particles each of mass m are connected to each other by a spring of spring constant k. Assume that the natural length and mass of the spring are negligible. The particles are made to rotate in a circle about their common center of mass, such that the distance between them is r. Assume that the only force between the particles is the one provided by the spring. Apply Bohr's quantization rule to this system and derive expressions for

(a) r (distance between particles)

(b) energy "E" of the system in terms of m, k and h.

[3+1 Marks]

We do the problem using CM cordinate $\vec{R} = \frac{\vec{r}_1 + \vec{r}_2}{2}$ and relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$.

Equations of motion are $(2m)\ddot{\vec{R}} = 0$, $\mu\ddot{\vec{r}} = -k\vec{r}$

where $\mu = \frac{m}{2}$ is the reduced mass.

a) Bohr quantization condition $\mu v r = n\hbar$

where v is the relative velocity (modulus).

$\mu\ddot{\vec{r}} = -k\vec{r}$ for circular motion becomes

$$\mu\frac{v^2}{r} = kr \Rightarrow \mu v^2 = kr^2$$

$$\mu\cdot\left(\frac{n\hbar}{\mu r}\right)^2 = kr^2 \Rightarrow \frac{n^2\hbar^2}{\mu r^2} = kr^2$$

$$r^4 = \frac{n^2\hbar^2}{k\mu} = \frac{2n^2\hbar^2}{km} \Rightarrow r = \left[\frac{2n^2\hbar^2}{km}\right]^{1/4}$$

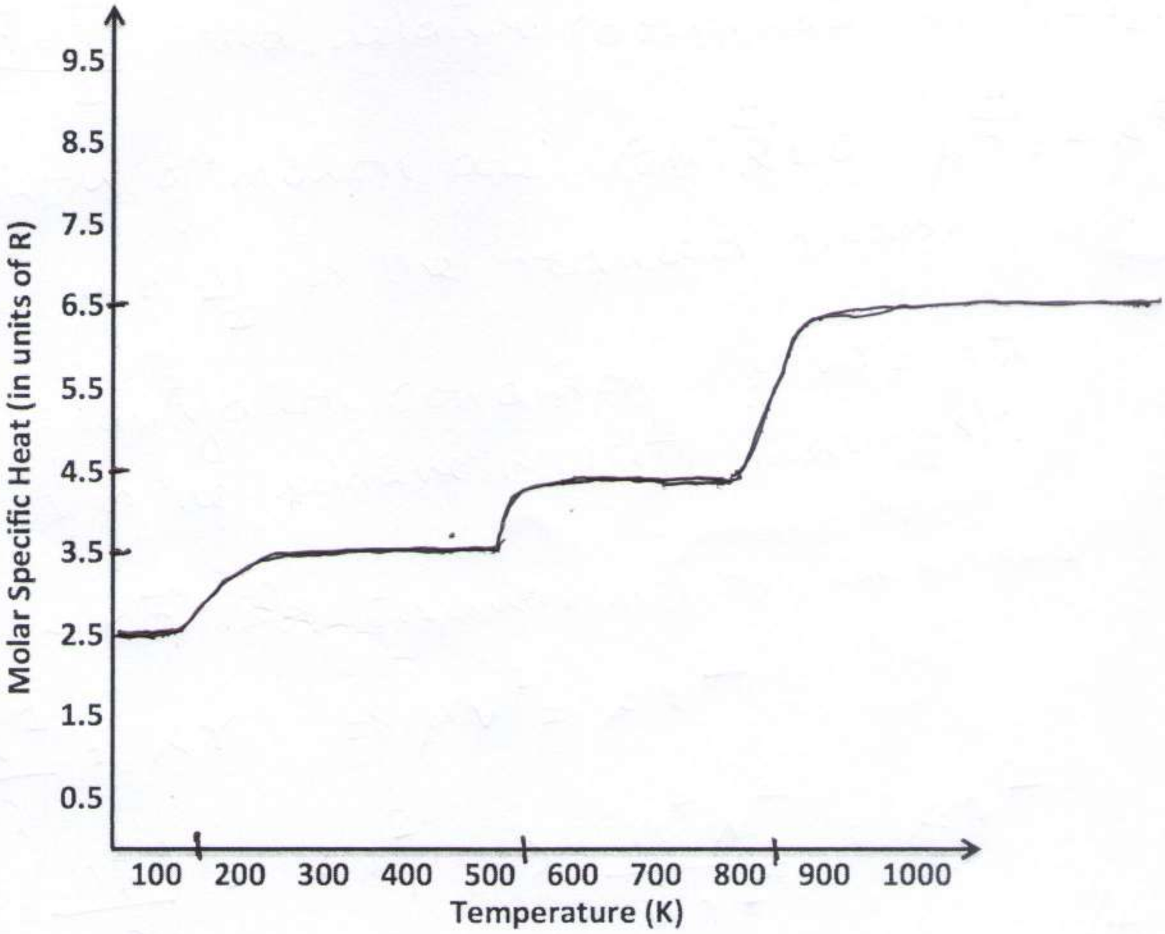
b) $$E = \frac{1}{2}\mu v^2 + \frac{1}{2}kr^2 = kr^2 = k\frac{\sqrt{2}\,n\hbar}{\sqrt{km}}$$

$$= \sqrt{\frac{2k}{m}}\,n\hbar.$$

Roll No:

Q2. Carbon dioxide is an example of a linear tri-atomic molecule, with the carbon atom in the middle and the two oxygen atoms connected on either side as shown in the figure below.

This molecule has two rotational and four vibrational degrees of freedom. Assume that the excitation energy of the rotational degrees of freedom is very small. The quantum of energy required to excite the lowest vibrational mode is E, and that for the next mode is 4E. The next two modes have equal excitation energy of 6E. Given that E = 12×10^{-3} eV, sketch the molar specific heat of CO_2 from T = 100 K to T = 1000 K in the graph below.



[4 Marks]

$K T_1 = 12\times10^{-3}\,eV \Rightarrow T_1 = \frac{12}{8.6}\times10^{2} = 140\,K$

$K T_2 = 4 K T_1 \Rightarrow T_2 = 4\times140 = 560\,K$

$T_3 = 6T_1 = 840\,K.$

Until T_1, $C_V = \frac{3}{2}R + R = \frac{5}{2}R$ (3 translations + 2 rotations)

At T_1, C_V increases by R, At T_2, C_V increases by R

At T_3, C_V increases by 2R [Two equal vibrational states].

Roll No:

Q3. **(a)** Given Planck's formula for the energy density $U(\nu,T)=\frac{8\pi h\nu^3}{c^3}\frac{1}{e^{(h\nu/k_BT)}-1}$, obtain an expression for the Rayleigh Jeans formula for $U(\nu,T)$.

(b) For a black body at temperature T, $U(\nu,T)$ was measured at $\nu=\nu_0$. This value is found to be one tenth of the value estimated using Rayleigh Jeans formula. Obtain an implicit equation in terms of $(h\nu/k_BT)$

(c) Solve the above equation to obtain the value of $(h\nu/k_BT)$, up to the first decimal place.

[1+1+2 Marks]

a) We get Raleigh-Jeans formula from Planck's formula in the limit $kT \gg h\nu$

$$e^{h\nu/kT}-1 = 1+\frac{h\nu}{kT}-1 = \frac{h\nu}{kT}$$

$$U_{RJ}(\nu,T) = \frac{8\pi\nu^2}{c^3}.kT$$

b) $U(\nu_0,T) = 0.1\ U_{RJ}(\nu_0,T)$

$$\frac{8\pi h\nu_0^3}{c^3}\frac{1}{e^{h\nu_0/kT}-1} = 0.1\frac{8\pi\nu_0^2 kT}{c^3}$$

$$\Rightarrow 10\frac{h\nu_0}{kT} = e^{(h\nu_0/kT)}-1$$

c) Set $x = \frac{h\nu_0}{kT}$. $10x = e^x-1$

x	LHS	RHS
2	20	6.4
3	30	19.1
4	40	53.7
3.5	35	32.2
3.6	36	35.68

Therefore

$$x = \frac{h\nu_0}{kT} \simeq 3.6$$

Roll No:

Q4. A photon strikes an electron at rest and scatters from it elastically. The photon ends up with a wavelength equal to three times the Compton wavelength and loses half its energy in the collision. At what angle relative to its initial propagation direction is the photon scattered? Calculate the momentum of the scattered electron.

[2+1 Marks]

$$\lambda' = 3\lambda_c \quad \text{and} \quad E' = \frac{E}{2} \Rightarrow \lambda' = 2\lambda$$

$$\lambda = \frac{\lambda'}{2} = \frac{3}{2}\lambda_c .$$

$$\lambda' - \lambda = \lambda_c(1-\cos\theta) \Rightarrow 3\lambda_c - \frac{3}{2}\lambda_c = \lambda_c(1-\cos\theta)$$

$$1-\cos\theta = \frac{3}{2} \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ .$$

~~$E_{el} = m_e c^2 + E'$ by energy conservation~~

$$m_e c^2 + E = E_{el,sc} + E'$$

$$E_{el,sc} = m_e c^2 + (E-E') = m_e c^2 + \frac{E}{2}$$

$$= m_e c^2 + \frac{hc}{2\lambda} = m_e c^2 + \frac{hc}{3\lambda_c}$$

$$\lambda_c = \frac{h}{mc} \Rightarrow \frac{hc}{3\lambda_c} = \frac{1}{3} m_e c^2$$

$$E_{el,sc} = \frac{4}{3} m_e c^2 = \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

$$p_e^2 c^2 = \left(\frac{16}{9} - 1\right)(m_e c^2)^2 = \frac{7}{9} m_e^2 c^4$$

$$p_e = \sqrt{(7/9)}\, m_e c .$$

Partial Marking Scheme for Quiz-1

Problem-1:

- One mark for writing the Bohr Quantization condition correctly. In terms of reduced mass and relative coordinates, it is $\mu vr = n\hbar$. In terms of individual particles, it is $2mv_1r_1 = n\hbar$, where v_1 is the speed of each particle and r_1 is the radius of the circle.
- One mark for writing the equation of motion correctly. In terms of reduced mass and relative coordinates, it is $\mu v^2/r = kr$. In terms of individual particles it is $mv_1^2/r_1 = 2kr_1$.
- One mark for obtaining the expression for r (or $r_1 = r/2$).
- One mark for obtaining the expression for the energy.

Problem-2:

- The specific heat changes at three temperatures. One mark for getting these temperatures correct. (We assume that the student gets all three correct or all three wrong. In the event, the student gets one or two temperatures correct, give **half** mark).
- Below 140K, C_V is (5/2)R. (3/2)R coming from translational dof and R coming from rotational dof. **Half** mark each.
- Above 140K, C_V jumps by R. **Half** mark.
- Above 560K, C_V jumps by R. **Half** mark.
- Above 840K, C_V jumps by 2R. One mark.

Problem-3:

- One mark for deriving Raleigh-Jeans formula.
- One mark for obtaining $10x = e^x - 1$.
- If answer for x is between 3 and 4, give one mark. If it is between 3.5 to 3.7, give **two** marks.

Problem-4:

- **Half** mark for obtaining λ.
- **Half** mark for writing the Compton Scattering formula correctly.
- One mark for $\theta = 120$ (No mark for $\theta = 60$).
- If the student writes the momentum as a fraction of $m_e c$, give the full mark.