PH107 Quiz 1 2016

1. According to Planck, the spectral energy density u(λ) of a blackbody maintained at temperature *T* is given by

$$u(\lambda,T)=\frac{8\pi hc}{\lambda^5}\frac{1}{e^{(hc/\lambda k_BT)}-1}$$

where λ denotes the wavelength of radiation emitted by the blackbody.

(a) Find an expression for the wavelength $\lambda_{max}$ at which u(λ,T) attains its maximum value (at a fixed temperature T). $\lambda_{max}$ should be in terms of temperature T and fundamental constants h, c and $k_B$.
(b) Expressing $\lambda_{max}$ as $\alpha/T$, obtain an expression for $u_{max}(T)$ in terms of $\alpha$, T and the fundamental constants.

**Question-1(a)**:

$$\begin{aligned}
u(\lambda,T) &= \frac{8\pi hc}{\lambda^5}\frac{1}{[\exp(hc/\lambda k_BT)-1]}\\
\frac{du}{d\lambda} &= 8\pi hc\left[\frac{-5}{\lambda^6}\frac{1}{[\exp(hc/\lambda k_BT)-1]}+\frac{1}{\lambda^5}\frac{-1}{[\exp(hc/\lambda k_BT)-1]^2}\exp(hc/\lambda k_BT)\frac{hc}{k_BT}\frac{-1}{\lambda^2}\right]\\
&= \frac{8\pi hc}{\lambda^6}\frac{1}{[\exp(hc/\lambda k_BT)-1]}\left[-5+\frac{hc}{\lambda k_BT}\frac{\exp(hc/\lambda k_BT)}{[\exp(hc/\lambda k_BT)-1]}\right]=0 \qquad (1)
\end{aligned}$$

**1 mark** for getting $du/d\lambda$ correct.
Solving we get

$$5=\frac{hc}{\lambda k_BT}\frac{\exp(hc/\lambda k_BT)}{[\exp(hc/\lambda k_BT)-1]}.$$

We can make one of two approximations:

1. For $hc/\lambda k_BT \gg 1$, we have

$$\frac{\exp(hc/\lambda k_BT)}{[\exp(hc/\lambda k_BT)-1]}\approx 1,$$

leading to $\lambda_{\max}T=hc/5k_B$.

2. For $hc/\lambda k_BT \ll 1$, we do a Taylor expansion to get

$$5=\frac{hc}{\lambda k_BT}\frac{1+\frac{hc}{\lambda k_BT)}}{[1+\frac{hc}{\lambda k_BT}-1]}=1+\frac{hc}{\lambda k_BT},$$

which gives $\lambda_{max}T=hc/4k_B$.

**1 mark** for writing $\lambda_{max}T=hc/5k_B$ OR $\lambda_{max}T=hc/4k_B$.
**Question-1(b)**: Substituting $\lambda_{max}=\alpha/T$, we get

$$u(\lambda_{max},T)=\frac{8\pi hc}{(e^{hc/\alpha k_B}-1)}\frac{T^5}{\alpha^5}.$$

**1 mark**

2. A monochromatic light of intensity 1.0 μW/cm 2 falls on a metal
surface of area 1 cm 2 and work function φ = 4.5 eV. Assume that only
3% of the incident light is absorbed by the metal (The rest is reflected
back) and that the photoemission efficiency is 100 % (i.e. eachabsorbed photon produces one photo-electron). The measured
saturation current is 2.4 x 10 -9 Amp.
(a) Calculate the number of photons/second falling on the metal
surface. (b) What is the energy of the incident photon in eV ? (c) What
is the stopping potential ?

**Question-2(a)**

$N_e$ = Number of electrons emitted per second = I/q = $(2.4 \times 10^{-9})/(1.6 \times 10^{-19}) = 1.5 \times 10^{10}$. **1 mark**

$N_p$ = NUmber of photons falling on the metal per second = $N_e/(0.03) = 0.5 \times 10^{12}$ **1 mark**

**Question-2(b)**

Power on the metal = $N_p(h\nu)$.

So $h\nu = (10^{-6})/(0.5 \times 10^{12}) = 2 \times 10^{-18}$ Joule **1 mark**

$h\nu = (2 \times 10^{-18})/(1.6 \times 10^{-19}) = 12.5$ eV. **1 mark**

**Question-2(c)**

Therefore Maximum kinetic energy of photoelectrons is 8 eV.

Hence the stopping potential is 8 Volts. **1 mark**

3. Two Compton scattering experiments were performed where x-rays were scattered off free particles of mass **m**. In the first experiment, increase in wavelength, at an angle $\theta = 45^0$, is 7x10$^{-14}$ m.
(a) Calculate the Compton wavelength and the mass of the scatterer?

$E_1$ and $E_2$ are the energies of the incident x-rays in the first and the second experiments such that $E_2 = E_1/2$. In the second experiment, the wavelength of the scattered x-ray at an angle $\theta = 60^0$ is measured to be 9.9x10$^{-12}$ m.
(b) What are the wavelengths of the incident X-rays in the two experiments?

**Question-3(a)**

$\Delta\lambda = \lambda_1' - \lambda_1 = \lambda_C(1 - 1/\sqrt{2}) = 7 \times 10^{-14}$ meters.
$\lambda_c = (7/0.3) \times 10^{-14} = 23.3 \times 10^{-14}$ meters. **1 mark**
$\lambda_c = h/mc = hc/mc^2$. $mc^2 = hc/\lambda_c = (1240 \times 10^{-9})/(23.3 \times 10^{-14}) = 53.2 \times 10^5$ eV.
$m = 5.3$ MeV$/c^2 = 9 \times 10^{-30}$ kg. **1 mark**
Mark is to be given for writing $m$ either in MeV$/c^2$ or in kg.

**Question-3(b)**

$\lambda_2' - \lambda_2 = \lambda_c(1 - 1/2)$. Substituting the Compton wavelength of $23.3 \times 10^{-14}$ meters, in we get $\lambda_2 = (990 - 11.6) \times 10^{-14} = 978.4 \times 10^{-14}$ meters **1 mark**
$\lambda_1 = \lambda_2/2 = 489.2 \times 10^{-14}$ meters **1 mark**

4. A photon, an electron and a neutron all have energy 5 keV. (In the case of electron and neutron, the energy refers to non-relativistic kinetic energy). Calculate the de-Broglie wavelength for each of them. Express your answer in nanometers (nm).
(Given, electron mass = 500 keV/c$^2$, neutron mass = 1000 MeV/c$^2$ )

**Question-4(a)**

Photon: E = 5000 eV = pc. $\lambda_{photon} = h/p = hc/E = 1240/5000 = 0.25$ nm.
**1 mark**

**Question-4(b)**

Electron: $p = \sqrt{2mE}$. So $pc = \sqrt{2(mc^2)E} = \sqrt{2 \times 500 \times 5}$ keV $= 70.7$ keV.
$\lambda_{electron} = hc/pc = 1240/70700 = 17.5 \times 10^{-3}$ nm. **1 mark**

**Question-4(c)**

Neutron: $p = \sqrt{2mE}$. So $pc = \sqrt{2(mc^2)E} = \sqrt{2 \times 10^9 \times 5000}$ eV $= 3.16 \times 10^6$ eV.
$\lambda_{neutron} = hc/pc = (1240/3.16) \times 10^{-6} = 392 \times 10^{-6}$ nm. **1 mark**