PH-107 (2017)
Tutorial Sheet 7

* Problems to be done in tutorial.

A. Particle in a box, finite potential well problems

Q61. Consider a particle of mass m in an infinite potential well extending from x=0 to x=L. Wave function of the particle is given by

$$\psi(x) = A\left[\sin\left(\frac{\pi x}{L}\right)+\sin\left(\frac{2\pi x}{L}\right)\right],$$ Where A is the normalization constant

(a) Calculate A

(b) Calculate the expectation values of x and x^2 and hence the uncertainty Δx.

(c) Calculate the expectation values of p and p^2 and hence the uncertainty Δp.

(d) What is the probability of finding the particle in the first excited state, if an energy measurement is made?

$$\left[Given, \quad \int_0^L x\cos\left(\frac{n\pi x}{L}\right)\ dx = 0, \ \int_0^L x^2\cos\left(\frac{n\pi x}{L}\right)\ dx = 0, \ \ for \ \ all \ \ n\right]$$

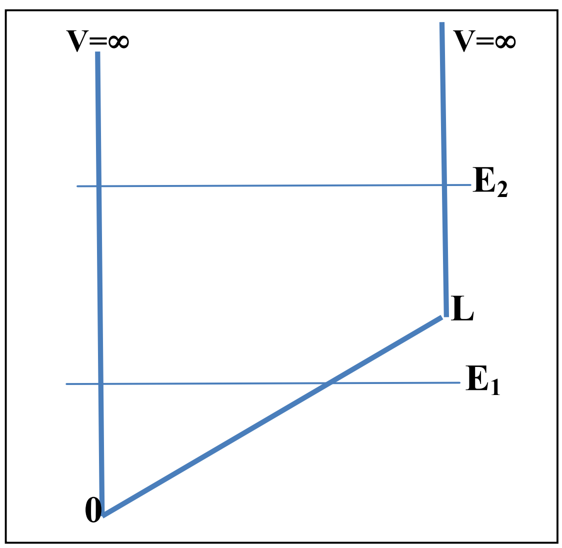
Q62*. An electron is bound in an infinite potential well extending from $x = 0$ to $x = L$. At time t=0, its normalized wave function is

$$\psi(x,0) = \frac{2}{\sqrt{L}}\sin\left(\frac{3\pi x}{2L}\right)\cos\left(\frac{\pi x}{2L}\right)$$

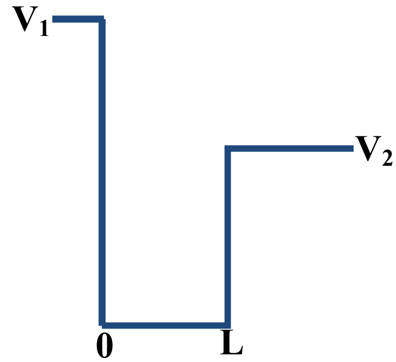
(a) Calculate $\psi(x,t)$ at a later time t.

(b) Calculate the probability of finding the electron between $x = L/4$ and $x = L/2$ at time t.

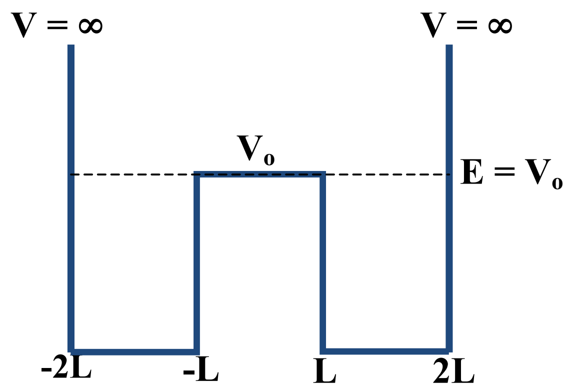
Q63. Consider a particle bound inside an infinite well whose *floor* is sloping (variation is small) as shown in the figure. Without solving the Schrodinger equation, sketch a plausible wave function when the energy is E_1, assuming it has no nodes. Sketch the wave function with 5 nodes when energy is E_2. Provide proper justification for your answers.



Q64*. Consider the asymmetric finite potential well of width L, with a barrier V_1 on one side and a barrier V_2 on the other side. Obtain the energy quantization condition for the bound states in such a well. From this condition derive the energy quantization conditions for a semi-infinite potential well (when $V_1 \rightarrow \infty$ and V_2 is finite)



Q65*. A particle of mass m is bound in a double well potential shown in the figure. Its energy eigen state $\psi(x)$ has energy eigenvalue $E = V_o$ (where V_o is the energy of the plateau in the middle of the potential well). It is known that $\psi(x) = C$ (C is a constant) in the plateau region.



(a) Obtain $\psi(x)$ in the regions $-2L<x<-L$ and $L<x<2L$ and the relation between the wave number `k' and L.

(b) Determine `C' in terms of L.

(c) Assume the bound particle to be an electron and L = 1 Å. Calculate the 2 lowest values of V_o (in eV) for which such a solution exists.

(d) For the smallest allowed k, calculate the expectation values for x, x^2, p and p^2 and show that Heisenberg's Uncertainty Relation is obeyed.