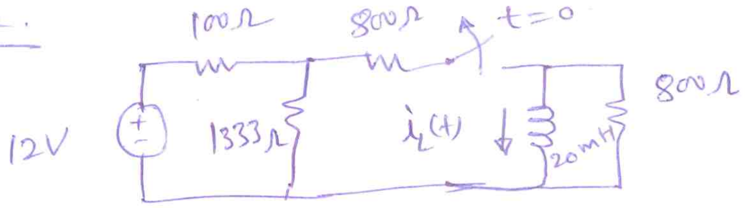
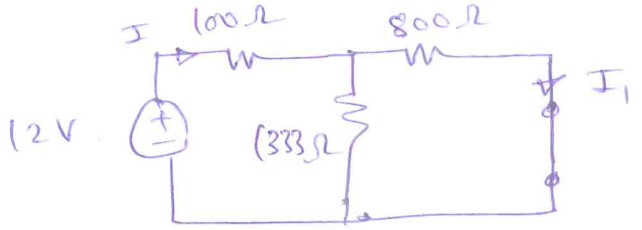
Assignment - 2

Ques 1.



∵ 12-V source has been applied for a long time before the switch open at $t=0$. So the inductor will behave as short circuited prior to $t=0$.
And above circuit will look as -



Resistance seen by the 12-V source

$$= 100 + \frac{1333 \times 800}{1333 + 800} = 600\ \Omega$$

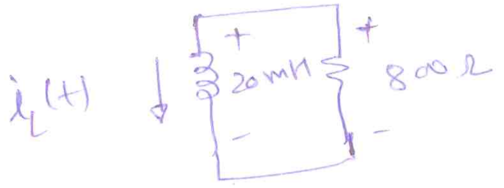
$$I = \frac{12}{600} = 0.02\,A.$$

Current through inductor

$$I_1 = 0.02 \times \frac{1333}{1333+800} = 0.01249 = 12.5\,mA$$

So $\boxed{I_L(0^+) = 12.5\,mA}$

when switch is opend at $t=0$. the ckt to find inductor will be as -



writing KVL

$$v_L(t) + R \times i_L(t) = 0$$

$$v_L(t) = -R\, i_L(t)$$

$$L \frac{di_L(t)}{dt} = -R\, i_L(t)$$

$$\frac{di_L(t)}{i_L(t)} = -\frac{R}{L} dt$$

Integrating both the sides.

$$\log i_L(t) = -\frac{R}{L} t + K$$

$$i_L(t) = e^{(-R/L\, t + K)}$$

$$i_L(t) = e^{-R/L\, t}\, e^{K} \qquad C = e^{K} = \text{constant.}$$

$$i_L(t) = e^{-R/L\, t}\, C$$

which we ~~find~~ by initial condition.

$$12.5\,mA = e^{-R/L \times 0}\, C$$

$$\boxed{C = 12.5\,mA} \quad \& \quad R = 800\,\Omega \;\&\; L = 20\,mH$$

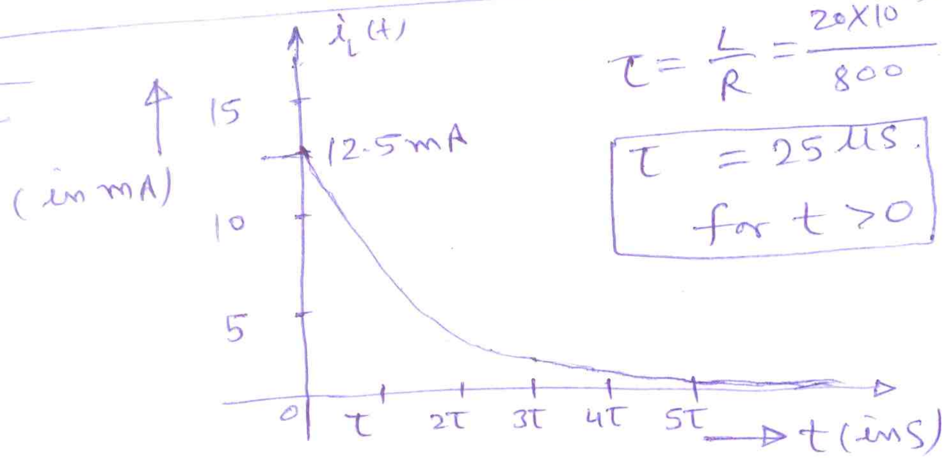
So $i_L(t) = 12.5\, e^{-800/20\times10^{-3}\, t}$

$$\boxed{i_L(t) = 12.5\, e^{-40000t}\ mA} \ \boxed{\text{for } t > 0}$$

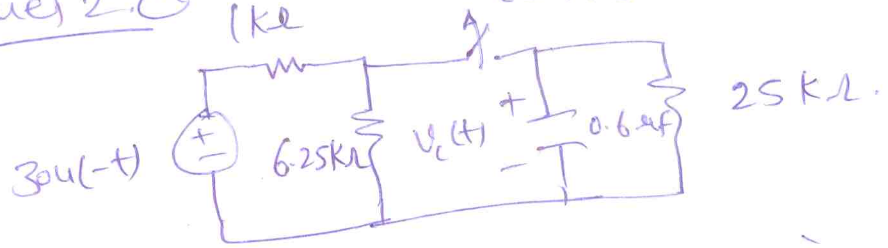
$$\tau = \frac{L}{R} = \frac{20\times10^{-3}}{800}$$

$$\boxed{\tau = 25\,\mu s. \text{ for } t > 0}$$

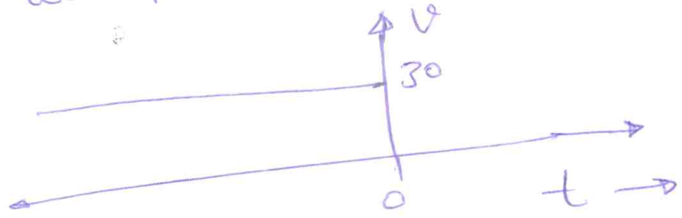
Sketch of $i_L(t)$ —



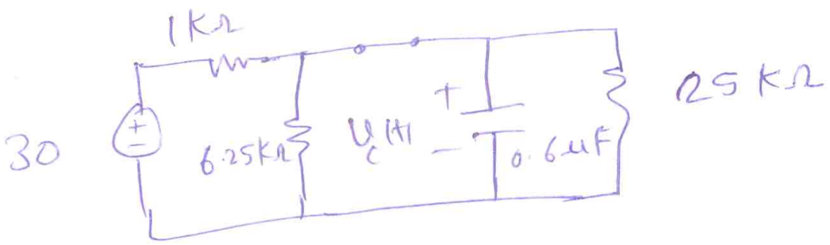
Ques 2 (a)



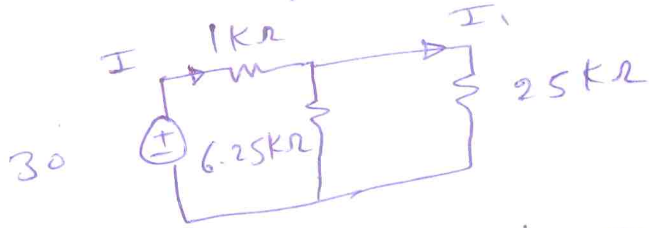
The waveform of $30u(-t)$ is as



So Above circuit prior to $t=0^+$ will be as.



The capacitor will act as open circuited so voltage across it will be the same which will be across 25 kΩ resistance at $t=0^-$.



Resistance seen by 30 V is

$$R = 1\,k\Omega + \frac{6.25 \times 25}{6.25+25}\,k\Omega$$

$$= 6\,k\Omega$$

$$I = \frac{30}{6\,k\Omega} = 5\,mA$$

$$I_1 = 5 \times \frac{6.25}{6.25+25} = 1\,mA$$

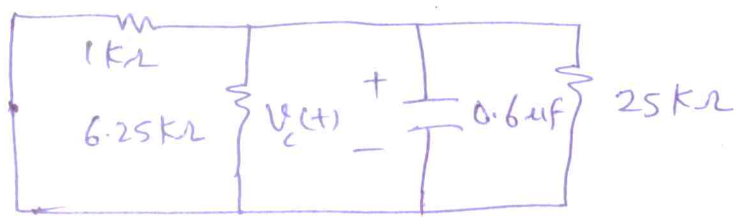
Voltage across 25 KΩ resistor

$= 25\,K\Omega \times 1\,mA = 25\,V.$

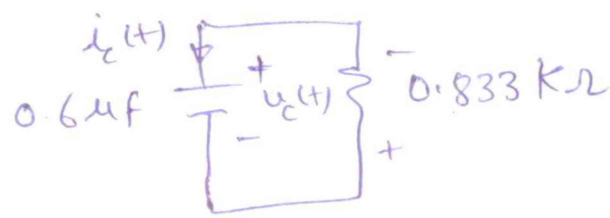
So it is the same across cap at $t=0^+$

$$\boxed{V_c(t=0^+) = 25\,V}$$

At $t=0$ the voltage source vanishes & followin circuit remains. till $t = 1\,ms$.



∵ All the resistance are in parallel to capacitor.
So following eq. circuit results



writing KVL

$$V_c(t) + R\,i_c(t) = 0$$

$$V_c(t) + RC\frac{dV_c(t)}{dt} = 0$$

$$\frac{dV_c(t)}{dt} = -\frac{1}{RC}V_c(t)$$

$$\frac{dV_c(t)}{V_c(t)} = -\frac{1}{RC}dt.$$

Integrating on both the sides.

$$\log v_c(t) = -\frac{1}{RC}t + k$$

$$v_c(t) = e^{-t/RC}\, e^{k}$$

Again the constant is find out by initial condition.

$$25 = e^{-0/RC} \times e^{k}$$

$$e^{k} = 25$$

So

$$v_c(t) = 25\, e^{-t/(0.833 \times 0.6 \times 10^{-3})}$$

$$\boxed{v_c(t) = 25\, e^{-2000t}\ V \quad 0 < t < 1\ ms}$$

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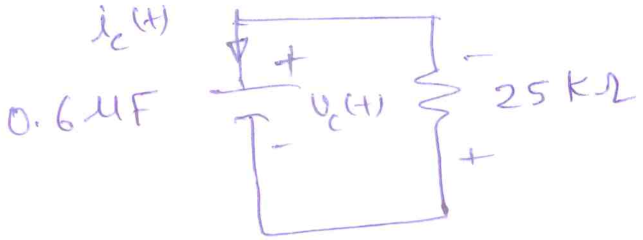
$$v_c(1^{+}ms) = v_c(1^{-}ms)$$

$$= 25\, e^{-2000 \times 1 \times 10^{-3}}$$

$$\boxed{V_c(1^{+}ms) = 3.383\ V}$$

Now for $1 \leq t$ ms the circuit will be as -



$$v_c(t) + R\, i_c(t) = 0.$$

$$v_c(t) + RC\frac{dv_c(t)}{dt} = 0.$$

$$\frac{dv_c(t)}{dt} = -\frac{1}{RC}v_c(t).$$

$$\int_{V(1^+ ms) = 3.383}^{v_c(t)} \frac{dv_c(t)}{v_c(t)} = \int_{t=1ms}^{t} -\frac{1}{RC}\,dt$$

$$\left[\log v_c(t)\right]_{3.383}^{v_c(t)} = -\frac{1}{RC}(t-0.001)$$

$$\log\left[v_c(t) - 3.383\right] = -\frac{1}{RC}(t-0.001)$$

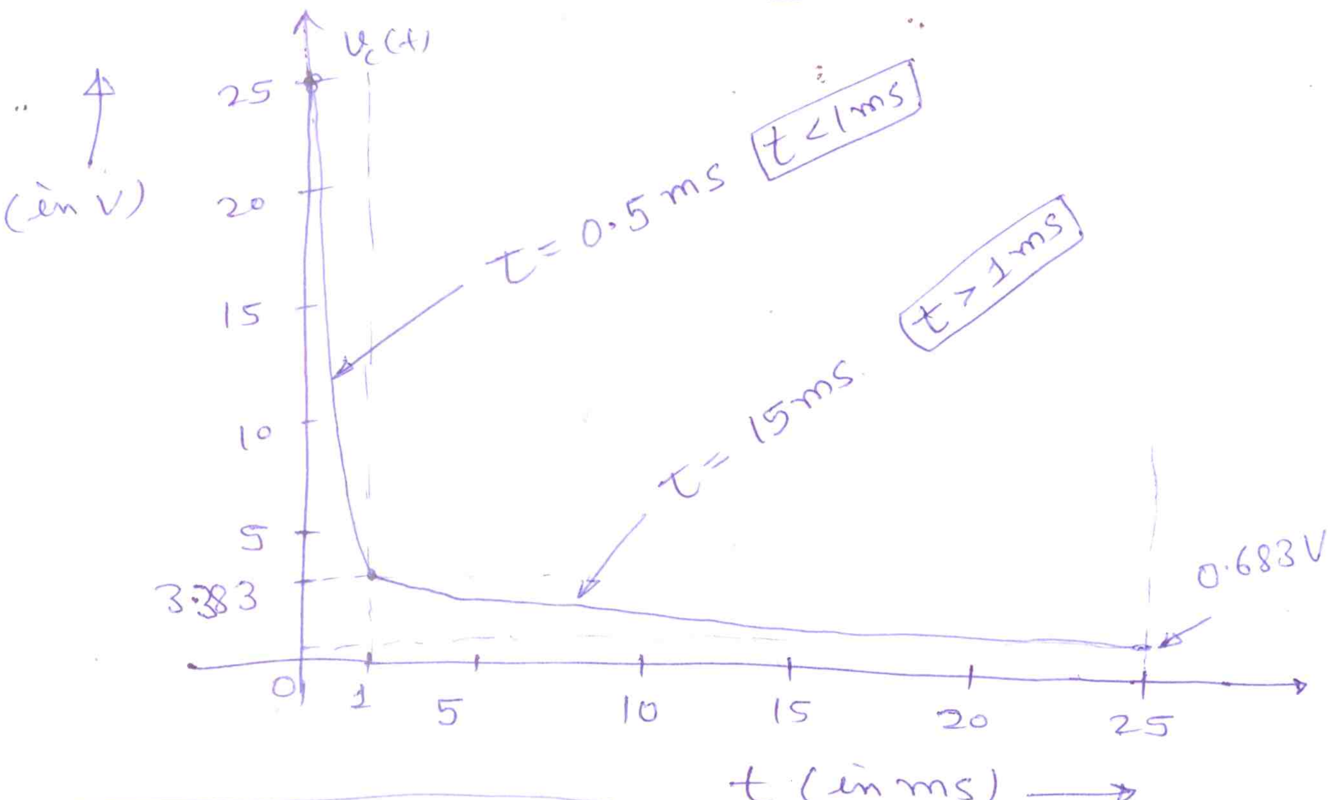
$$v_c(t) = 3.383\, e^{-(t-0.001)/RC}$$

where $R = 25\,k\Omega$, $C = 0.6\,\mu F$.

$$\tau = RC = 25 \times 0.6 \times 10^{-3} = 0.015\ s$$

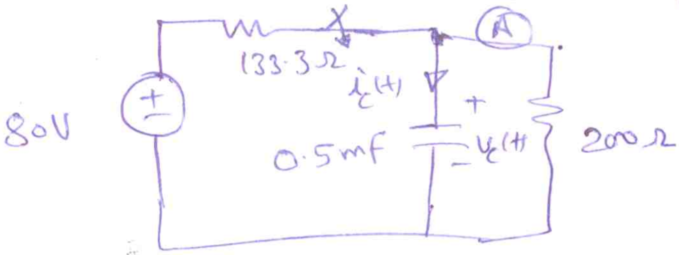
$$\boxed{v_c(t) = 3.383\, e^{-\frac{(t-0.001)}{0.015}}\ V \cdot\ 1 \le t\,ms}$$

(c) Sketch $v_c(t)$ for $0 \le t \le 25\,ms$



$$\boxed{v_c(25ms) = 0.683\ V}$$

Que 3 (a) when $t=0$ the switch s is closed



for $0 \le t \le 0.06$ s writing KCL at node A

$$\frac{V_c(t)-80}{133.3} + i_c(t) + \frac{v_c(t)}{200} = 0$$

$$\frac{v_c(t)-80}{133.3} + c\frac{dv_c(t)}{dt} + \frac{v_c(t)}{200} = 0$$

$$0.04\frac{dv_c(t)}{dt} + v_c(t) = 48$$

$$V_c(t) = 48\left(1-e^{-t/0.04}\right)$$

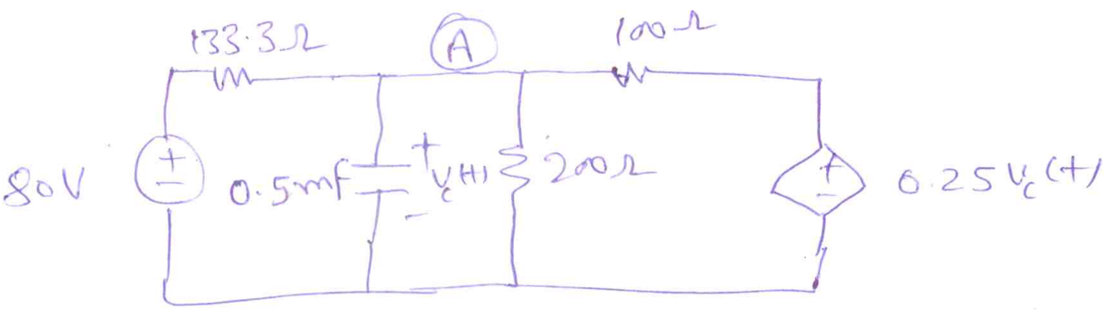
$$\boxed{v_c(t) = 48\left(1-e^{-25t}\right)} \quad \boxed{0 \le t \le 60\,ms}$$

$$v_c(t) = v_c(0.06)$$

$$= 48\left(1-e^{-25\times 0.06}\right)$$

$$\boxed{V_c(t) = 37.29\ V}$$

Circuit for $t \geq 60$ ms.



writing again KCL at node A

$$\frac{v_c(t) - 80}{133.3} + C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{200} + \frac{v_c(t) - 0.25\,v_c(t)}{100} = 0$$

$$\frac{v_c(t) - 80}{133.3} + C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{200} + \frac{0.75\,v_c(t)}{100} = 0$$

$$\frac{v_c(t) - 80}{133.3} + C\frac{dv_c(t)}{dt} + \frac{v_c(t) + 1.5\,v_c(t)}{200} = 0$$

$$\frac{v_c(t) - 80}{133.3} + C\frac{dv_c(t)}{dt} + \frac{2.5\,v_c(t)}{200} = 0.$$

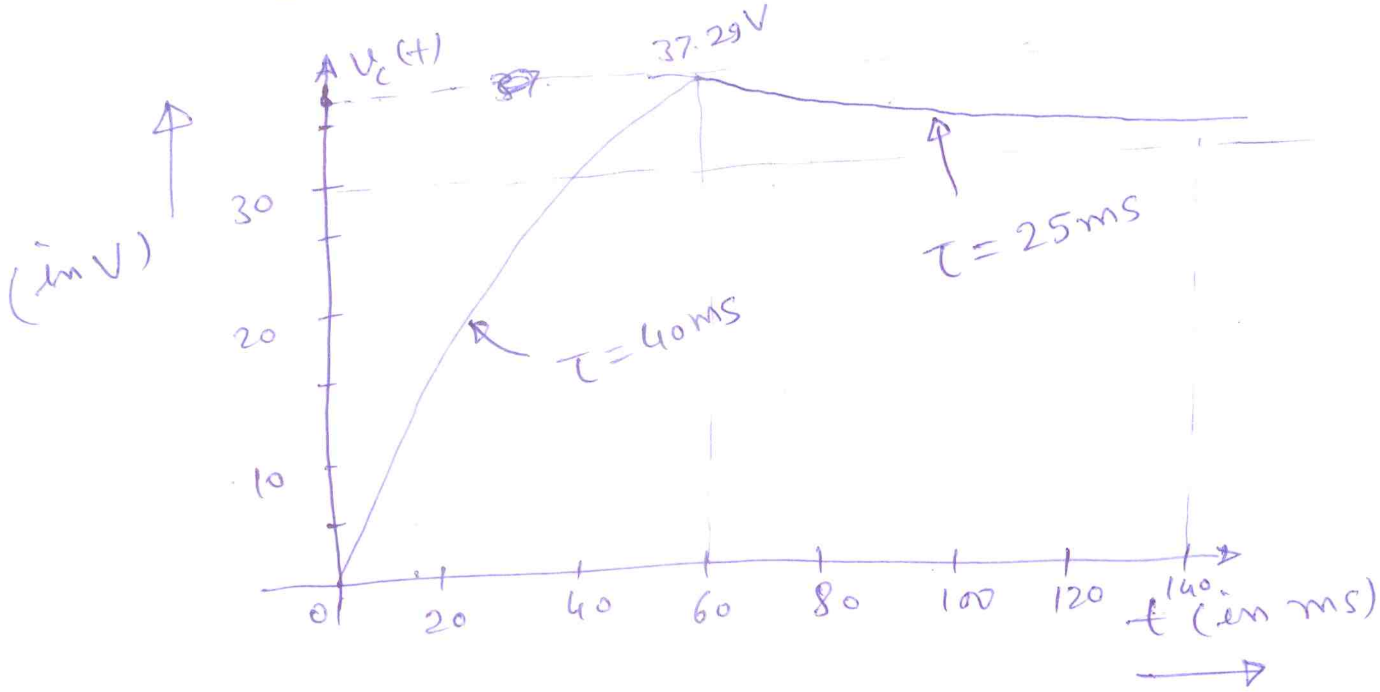
$$0.025\frac{dv_c(t)}{dt} + v_c(t) = 30.$$

$$v_c(t) = 30 + 7.29\,e^{-t'/0.025}$$

$$v_c(t) = 30 + 7.29\,e^{-40t'}$$

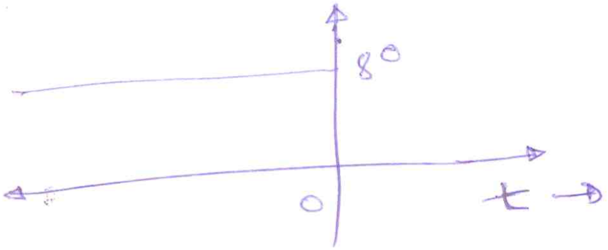
where $t' = t - 0.06$

$$\boxed{v_c(t) = 30 + 7.29\,e^{-40(t-0.06)}} \quad \boxed{\text{for } t \geq 60\text{ ms}}$$

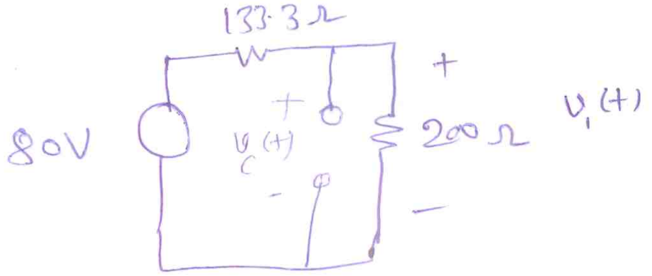


(b)

The 80u(−t) waveform is as.



At t=0 the capacitor will behave as open circuited due to connected to 80 V source a long time prior to t=0. Hence the circuit will be as follows at t=0⁻.

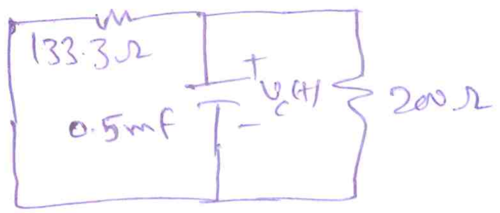


Voltage across capacitor will be equal to voltage across 200Ω resistance. So.

$$v_1(t) = 80 \times \frac{200}{200+133.3} = 48\ V.$$

Hence $v_c(0^-) = v_c(0^+) = 48\ V$.

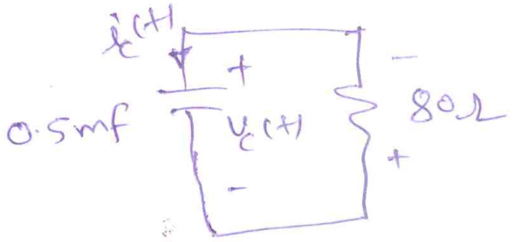
from t=0⁺ to t=0.06 s the circuit will be as-



The resistance seen by the capacitor

$$R = \frac{200 \times 133.3}{200+133.3} = 80\ \Omega$$

Hence the eq. circuit will be as.



writing KVL in loop.

$$v_c(t) + 80\, i_c(t) = 0.$$

$$v_c(t) + 80\, C \frac{d v_c(t)}{dt} = 0$$

$$\int \frac{d v_c(t)}{v_c(t)} = \int -\frac{1}{80 \times 0.5 \times 10^{-3}}\, dt$$

$$\log v_c(t) = -\frac{1}{0.04}\, t + K.$$

$$v_c(t) = K e^{-25t}$$

Again from Initial condition.

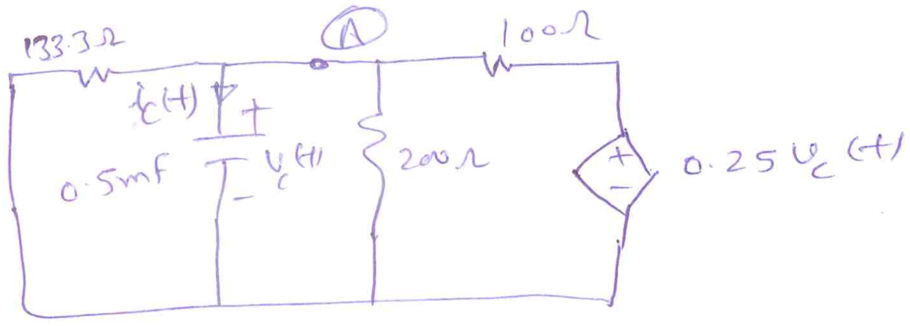
$$K = v_c(0^+) = 48V.$$

$$\boxed{v_c(t) = 48\, e^{-\frac{t}{0.04}}\ V, \quad 0 \le t \le 0.06\, S}$$

Now $\quad v_c(t) = v_c(0.06) = 48 \cdot e^{-\left(\frac{0.06}{0.04 \times}\right)}$

$$= 10.71\ V$$

Now $t = 60ms$ the switch is closed then the circuit topology will be.



writing KCL at node A.

$$\frac{v_c(t)}{133.3} + i_c(t) + \frac{v_c(t)}{200} + \frac{0.75\, v_c(t)}{100} = 0.$$

$$0.02\, v_c(t) + c\,\frac{d v_c(t)}{dt} = 0.$$

$$\int^{v_c(t)} \frac{d v_c(t)}{v_c(t)} = \int_{t=0.06}^{t} -\frac{0.02}{c}\, dt$$

$$\left[\log v_c(t)\right]_{10.71}^{v_c(t)} = \frac{-0.02}{0.5\times 10^{-3}}\left[t-0.06\right]$$

$$\log\left[v_c(t)\right] - \log\left[10.71\right] = -40\left[t-0.06\right]$$

$$\boxed{v_c(t) = 10.71\, e^{-40(t-0.06)} \qquad t \geq 0.06\,s}$$