PH-107 (2017)
Tutorial Sheet 6

***** Problems to be done in tutorial.

A. Wave function, Operator

P49*: If $\phi_n(x)$ are the solutions of time independent Schrödinger equation, with energies E_n, show that $\psi(x,t) = \sum_n c_n \phi_n(x) e^{\frac{-iE_n t}{\hbar}}$, where C_n are constants, is a solution of time dependent Schrödinger equation. However, show that $\psi(x,0)$ is not a solution of the time independent Schrödinger equation.

P50. If $\psi_1(x,t)$ and $\psi_2(x,t)$ are solutions of time dependent Schrödinger equation, show that $a\psi_1(x,t) + b\psi_2(x,t)$, where a and b are constants, is also a solution of the same.

P51: Consider two operators $\hat{O}_1$, $\hat{O}_2$ as defined below:

$$\hat{O}_1 \equiv \left(-i\frac{\partial}{\partial x}\right) \quad , \quad \hat{O}_2 \equiv \left(\hat{x} - i\frac{\partial}{\partial x}\right)$$

Calculate $O_1\Psi(x)$ and $O_2\Psi(x)$, where $\Psi(x)$ is a function of x. Is $\Psi(x)$ an eigen function of the two operators if it has the form $\Psi(x)=\exp(ikx)$.

P52*: Find the eigen function $\varphi(x)$ of the following operator

$$\hat{G} = -i\hbar\frac{d}{dx} + Ax$$

Here A is a constant. If this eigen function is subjected to a boundary condition $\varphi(a)= \varphi(-a)$, find out the eigen values.

P53*: Consider a large number (N) of identical experimental set-ups. In each of these set ups, a single particle is described by the wave function $\phi(x) = A\exp(-x^2/a^2)$ at $t=0$, where A is the normalization constant, and a is a constant of the dimension length. If a measurement of position of the particle is carried out at time $t=0$ in all these set-ups, it is found that in 100 of these the particle is found within infinitesimal interval of $x=2a$ and $2a+dx$. Find out in how many of the measurements the particle would have been found in the infinitesimal interval of $x=a$ and $a+dx$.

P54*: $\psi_1(x)$ and $\psi_2(x)$ are the normalized eigen functions of an operator $\hat{P}$, with eigen values P_1 and P_2 respectively. If the wave function of the particle is $0.25\psi_1(x)+0.75\psi_2(x)$ at $t=0$; find the probability of observing P_1?

P55: An observable 'A' is represented by an operator $\hat{A}$. Two of its normalized eigen functions are given as $\phi_1(x)$ and $\phi_2(x)$, corresponding to distinct eigen values a_1 and a_2 respectively. Another observable 'B' is represented by an operator $\hat{B}$. Two normalized eigen functions of this operator are given as u_1 and u_2 with distinct eigen values b_1 and b_2 respectively. The eigen functions $\phi_1(x)$ and $\phi_2(x)$ can be written in terms of u_1 and u_2 in the following way.

$$\phi_1 = D(3u_1 + 4u_2);\ \ \phi_2 = F(4u_1 - Pu_2)$$

At time t=0, a particle is in a state given as $(2/3)\phi_1 + (1/3)\phi_2$.

(a) Find the values of 'D', 'F' and 'P'.

(b) If a measurement of 'A' is carried out at t=0, what are the possible results and what are their probabilities?

(c) Assume that the measurement of 'A' mentioned above yielded a value a_1. If a measurement of 'B' is carried out immediately after this, what would be the possible outcomes and what would be their probabilities?

(d) If instead of following the above path, a measurement of 'B' was carried out initially at t=0, what would be the possible outcomes and what would be their probabilities?

Assume that after performing the measurements described in (c), the outcome was b_2.What would be the possible outcomes, if 'A' were measured immediately after this and what would be the probabilities

A. Particle in a Box

P56*: For a particle in one-dimensional box of side L, show that the probability of finding the particle between $x=B$ and $x=B+b$ approaches the classical value b/L, if the energy of the particle is very high.

P57: Consider a particle confined to a one-dimensional box. Find the probability that the particle in its ground state will be in the central one-third region of the box.

P58*: Consider a one dimensional infinite square well potential of length L. A particle is in $n=3$ state of this potential well. Find the probability that this particle will be observed between $x = 0$ and $x = (L/6)$. Can you guess the answer without solving the integral? Explain how.

P59: Suppose we have 10,000 rigid boxes of same length L from $x = 0$ to $x = L$. Each box contains one particle of the same mass. All these particles are in the ground state. If a measurement of position of the particle is made in all of these boxes at the same time, in how many of them, the particle is expected to be found between $x = 0$ and $L/4$. In a particular box, the particle was found to be between $x = 0$ and $L/4$. Another measurement of position of the particle is carried out in this box immediately after the first measurement. What is the probability that the particle is again found between $x = 0$ and $L/4$.

P60*: Solve the time independent Schrödinger equation for a particle in one dimensional box taking the origin at its mid-point and the ends at $\pm(L/2)$, where L is the length of the box.