Tutorial 12

P101 $I = neAv_d$

Using this data:

Atomic weight of Cu = 63.5, density = 8.94 g/cc.

8940 kg occupies $1\,m^3$, each contributes 1 electron.

$$n = \frac{8940}{63.5} \times \frac{N_A \times 10^{23}}{1\,m^3} = 8.475 \times 10^{28}\ m^{-3}$$

$$\frac{I}{neA} = v_d = \frac{10}{8.475 \times 10^{28} \times 1.6 \times 10^{-19} \times 10^{-6}} = 7.375 \times 10^{-4}\ m/s$$

P102 Relaxation time $\tau = \frac{\sigma m}{ne^2} = \frac{m}{\rho n e^2}$

$$= \frac{9.1 \times 10^{-31}}{1.5 \times 10^{-8} \times 8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2} = 2.78 \times 10^{-14}\ \text{sec.}$$

Resistivity $\rho = \frac{5.8 \times 10^{-3}}{1} \times 3.3 \times 10^{-6} = 1.914 \times 10^{-8}$ ohm m

$\therefore$ conductivity $\sigma = 5.22 \times 10^{7}$ $(\text{ohm-m})^{-1}$

Current density $J = \frac{25}{3.3 \times 10^{-6}} = 7.58 \times 10^{6}$ A/m^2

Now $J = neV_D$ we have to find n to find drift velocity V_D.

63.5 kg of Cu has $10^3 N_A$ atoms ($N_A \rightarrow$ Avagadro's number)

8940 kg of Cu will have $\frac{8940(N_A \times 10^3)}{63.5} = 8.475 \times 10^{28}$

As 8940 kg occupies 1m^3 volume, assuming each atom contributes one free electron, we have

$n = 8.475 \times 10^{28}$ m^{-3}

Thus, $V_D = \frac{7.58 \times 10^{6}}{8.475 \times 10^{28} \times 1.6 \times 10^{-19}} = 5.6 \times 10^{-4}$ m/s

Note: drift velocity is very small in comparison to even classical rms speed. To calculate mean free path in Drude model, one has to calculate rms speed

$\frac{1}{2} m u_{rms}^2 = \frac{3}{2} kT$ $\quad$ kT ≈ .025 eV at room temperature

$\therefore$ $u_{rms} = \sqrt{\frac{3 \times 0.025 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} \approx 1.15 \times 10^{5}$ m/s

Relaxation time $\tau = \frac{\sigma m}{ne^2} = \frac{5.22 \times 10^{7} \times 9.1 \times 10^{-31}}{8.475 \times 10^{28} \times (1.6 \times 10^{-19})^2}$

$\approx 2.19 \times 10^{-14}$ s

Mean free path in Drude model

$l = u_{rms} \times \tau = 1.15 \times 10^5 \times 2.19 \times 10^{-14} \approx 2.52\,nm$

Mean free path in Sommerfield model

$l = v_F \times \tau$.

E_F for $Cu = 7.06\,eV \Rightarrow v_F = 1.58 \times 10^6 \quad (E_F = \frac{1}{2} m v_F^2)$.

$\therefore\ l = 34.7\,nm$

P104 $\quad \sigma = \dfrac{ne^2\tau}{m^*}$, where m^* is the effective mass

for Na

$$n = \frac{6.02 \times 10^{26} \times 970}{23} \approx 2.54 \times 10^{28}.$$

$$\tau = \frac{\sigma m^*}{ne^2} = \frac{2.17 \times 10^7 \times 1.2 \times 9.1 \times 10^{-31}}{2.54 \times 10^{28} \times (1.6 \times 10^{-19})^2} \approx 3.64 \times 10^{-14}\,s.$$

$l = 1.15 \times 10^5 \times 3.64 \times 10^{-14} \approx 4.19\,nm$ (in Drude model)

(for sommerfield, $E_F = \frac{1}{2} m^* v_F^2$ & calculate l using v_F)

$$v_D = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{2.17 \times 10^7 \times 100}{2.54 \times 10^{28} \times 1.6 \times 10^{-19}} \approx 0.53\,m/s.$$

P105

Let E, v vary as $e^{-i\omega t}$

$$m\left(\frac{dv}{dt} + \frac{v}{\tau}\right) = -eE.$$

$\therefore v(t) = ve^{-i\omega t}$

$\therefore E(t) = Ee^{-i\omega t}$

$\therefore \frac{dv(t)}{dt} = -i\omega v e^{-i\omega t}$

$$m\left(-i\omega v e^{-i\omega t} + \frac{ve^{-i\omega t}}{\tau}\right) = -eEe^{-i\omega t}$$

$$m\left(-i\omega v + \frac{v}{\tau}\right) = -eE. \Rightarrow v = \frac{-eE/m}{1/\tau - i\omega} = \frac{-eE\tau}{m}\,\frac{1+i\omega\tau}{1+(\omega\tau)^2}$$

Electric current density $= J = n(-e)v = \sigma E$

$$\frac{ne^2E\tau}{m}\left[\frac{1+i\omega\tau}{1+(\omega\tau)^2}\right] = \sigma E. \Rightarrow \sigma = \frac{ne^2\tau}{m}\left[\frac{1+i\omega\tau}{1+(\omega\tau)^2}\right]$$

P106.

Fermi energy for any element:

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{2/3} \approx 5.84 \times 10^{-38} \left(\frac{N}{V}\right)^{2/3} \text{ Joules}$$

$$\frac{N}{V} = \frac{N_A \times \text{density in g/cc} \times 10^6}{\text{Atomic weight in g}}$$

$$\approx 3.646 \times 10^{-19} \left(\frac{N}{V}\right)^{2/3} \text{eV}$$

Example for Na.

$$\frac{N}{V} = \frac{6.02 \times 10^{26} \times 0.971 \times 10^6}{22.99} = 2.543 \times 10^{28}.$$ Substituting

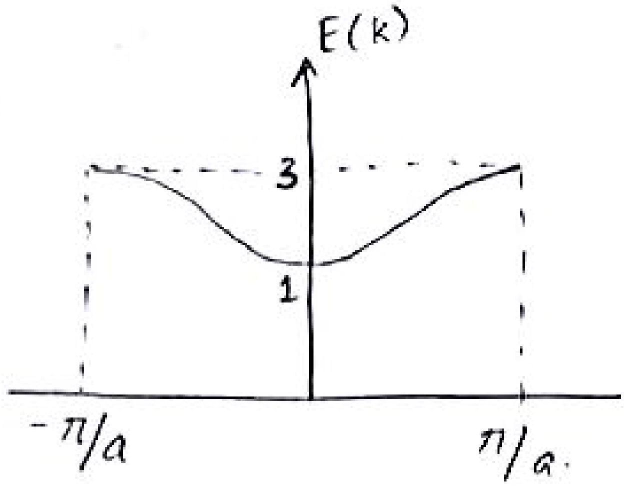
$$E_F \approx 3.16 \text{ eV}$$

	Li	Na	K	Rb	Cs.
E_F (in eV)	4.72	3.16	2.14	1.82	1.53

(approx).

P108.

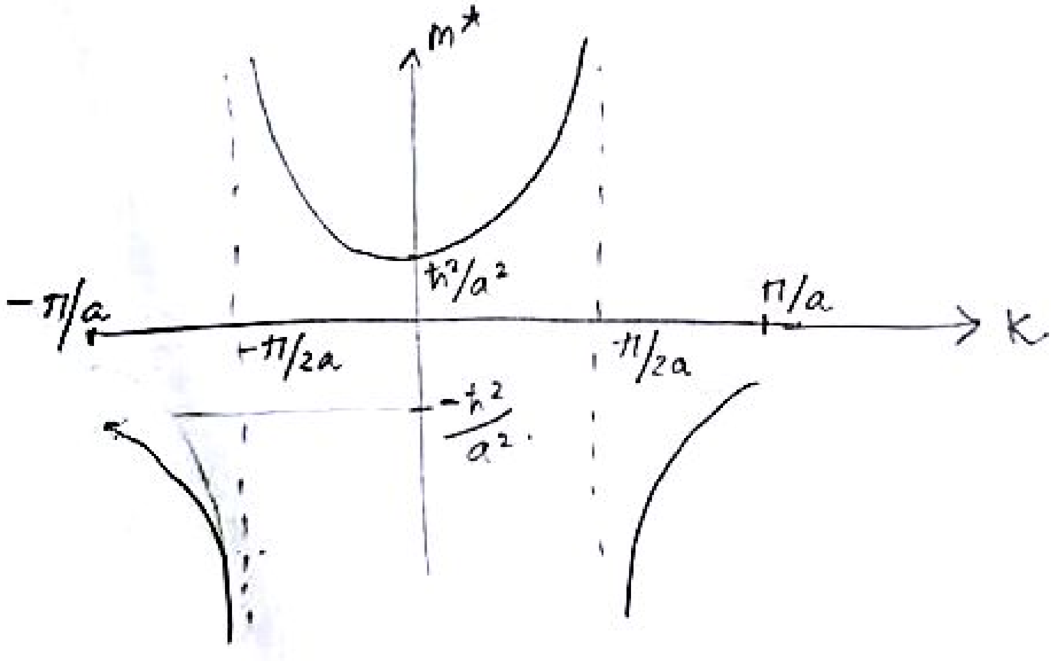
(a)



(b) $v = \frac{p}{m} = \frac{\hbar k}{m} = \frac{1}{\hbar}\frac{dE}{dk} = \frac{1}{\hbar} a \sin ka$

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}} = \frac{\hbar^2}{a^2 \cos(ka)}$$

(c)



(P107 → same as P99, Tutorial 11).