PH-107 (2017)
Tutorial Sheet 5

***** Problems to be done in tutorial.

A. Wave packet and Fourier Theory (continue):

P39. A wave packet is of the form

$$f(x) = x \text{ (for } -1 \le x \le 1) \text{ and } f(x) = 0 \text{ elsewhere}$$

(a) Plot $f(x)$ versus x.

(b) Calculate the Fourier transform of $f(x)$, i.e. $g(k) = \int_{-\infty}^{+\infty} f(x) e^{ikx} dx$?

(c) Plot $|g(k)|$ versus k.

(d) At what value of k, $|g(k)|$ attains a minimum value?

P40*. Following is the form of a triangular pulse,

$$f(x) = 1 - |x| \text{ (for } -1 \le x \le 1) \text{ and } f(x) = 0 \text{ elsewhere}$$

(a) Plot $f(x)$ versus x.

(b) Calculate the Fourier transform of $f(x)$, i.e. $g(k) = \int_{-\infty}^{+\infty} f(x) e^{ikx} dx$?

(c) Plot $g(k)$ versus k.

(d) At what value of k, $|g(k)|$ attains its maximum value?

(e) Find the values of k, at which $g(k)$ attains half of this maximum value

(f) From the values obtained in part (e), find the value of the spread Δk? Hence calculate the spread Δx using the uncertainty relation $\Delta x . \Delta k = 1/2$

B. Uncertainty Principle

P41. The position and the momentum of a *1keV* electron are determined simultaneously. Its position is known to an accuracy of only *1 Å* along x-axis. Using uncertainty principle, what is the minimum permissible percentage uncertainty in its momentum along the x-axis? From the above data can you determine the uncertainty along y-axis?

P42. An electron falls from a height of *10 m* and passes through a hole of radius *1 cm*. To study the motion of the electron afterwards, should we apply the wave aspect or the particle aspect?

P43*. A wave packet is constructed by superposing waves, their wavelengths varying continuously in the following way

$$y(x,t) = \int A(k)\cos(kx - wt)\,dk$$

Where $A(k) = A$ for $(k_o - \Delta k/2) \le k_o \le (k_o + \Delta k/2)$ and $= 0$ otherwise. Sketch approximately $y(x,t)$ and estimate Δx by taking the difference between two values of x for which the central maximum and nearest minimum is observed in the envelope. Verify uncertainty principle from this.

P44*. A beam of electron of energy $0.025\ eV$ moving along x-direction, passes through a slit of variable width w placed along y-axis. Estimate the value of the width of the slit for which the spot size on a screen kept at a distance of $0.5\ m$ from slit would be minimum.

P45*. Assume that in case the average value of momentum p_x is zero, the uncertainty in the x-component of momentum Δp_x is related to the average of square of x component of momentum $\langle px^2 \rangle$ by the following relation.

$$\Delta p_x = \sqrt{\langle p_x{}^2 \rangle}$$

Using the above relation estimate the following. [Use the following uncertainty principle for this problem $\Delta x\, \Delta p_x \ge \hbar/2$]

(a) The minimum kinetic energy that proton and electron would have if they were confined to a nucleus of approximate diameter $10^{-14}\ m$. This is an argument used against the existence of electron in nuclei.
(b) The ground state energy of a particle of mass m bound by a potential $V=(1/2)kx^2$
(c) The radial distance r for which the sum of kinetic and potential energies is minimum in hydrogen atom.

P46. Estimate the minimum possible energy consistent with uncertainty principle (i.e. ground state energy) for a particle of mass 'm' bound in a potential $V=kx^4$. Use the following uncertainty principle for this problem $\Delta x\, \Delta p_x \ge \hbar/2$.

P47*. A photon of energy E is emitted as a result of a particular transition. What would be the value of the recoil energy, assuming that the atom recoils with non-relativistic speed. Let the lifetime of the state be of the order of 10^{-8} s. What would be the order of natural line width of the emitted line. For what value of E, would the recoil energy be of the same order of magnitude as the natural line width? For order of magnitude calculation take the mass number of the atom as 100. What conclusions would you draw from this regarding resonant absorption?

[**Ans.:** $E^2/(2mc^2)$, 6.6×10^{-8} eV, 111 eV]

P48. A container contains monatomic hydrogen gas in thermal equilibrium at a temperature T, for which $k_BT = 0.025$ eV. Let E_1 is the difference between the ground state and the first excited state energy of the atom when at rest. Let E_2 be the energy of photon (in the frame of container) required to make this transition in an atom, which is traveling towards the photon (in an antiparallel direction) with the average energy at the above specified temperature.
(i) Find E_1-E_2.
(ii) After the absorption of photon what would be the final velocity of the hydrogen atom.
(iii) If the lifetime of the first excited state is 10^{-8} s, will the photon with energy E_2 be able to cause a transition, had the atom been at rest. Discuss quantitatively.
You are free to make any assumption, provided you justify it.