

CH 107 Tutorial 1

Please solve these problems BEFORE the tutorial session

1. Consider the eigenvalue equation $C^2\Psi = \Psi$ where C is a quantum mechanical operator, and Ψ is an eigenfunction. What are the possible eigenvalues of the operator C ?
2. The eigenvalue equation is given as $\hat{A}\Psi = a\Psi$. Suggest eigenfunctions for the following operators
(i) $-i\hbar \frac{\partial}{\partial q}$ (ii) $\frac{d^2}{dx^2}$ (iii) $\int dx$ (iv) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ (v) $\frac{1}{\sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d}{d\theta} \right)$
3. Plot the following functions and hence, explain which of these CAN NOT be a valid wavefunction.
(i) $x \sin x$ (ii) $\frac{1}{x} \sin x$ (iii) $Ae^{-\alpha x^2}$
4. Under what conditions will a linear combination of two or more eigenfunctions also be an eigenfunction of a quantum mechanical operator \hat{A} ?
5. Following the postulates of Quantum Mechanics, how would one calculate the expectation value of a physically observable quantity? How would one calculate the most probable value?
6. When does one use (a) eigenvalues, (b) expectation values? When will the two be the same?
7. Suppose that the wavefunction for a system can be written as

$$\psi(x) = \frac{1}{2}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{3 + \sqrt{2}i}{2}\phi_3(x)$$

where, $\phi_1(x)$, $\phi_2(x)$, $\phi_3(x)$ are orthogonal to each other and are normalized eigenfunctions of the kinetic energy operator, with eigenvalues E_1 , $3E_1$ and $7E_1$ respectively.

- a) Is $\psi(x)$ normalized?
- b) What are the possible values that you could obtain in measuring the kinetic energy on the system described by $\psi(x)$?
- c) What is the (i) average value and (ii) most probable value of kinetic energy that will be obtained for a large number of measurements?

CH 107 Tutorial 2

Please solve these problems BEFORE the tutorial session

1. Calculate the wavelength of light absorbed in the transition from $n = 1$ to $n = 2$ for an electron in a one dimensional box of length of 1.0 nm.
2. For the particle in a box given in the above question, what is the probability of finding the electron between (i) $x = 0.49$ and 0.51 , (ii) $x = 0$ and 0.020 and (iii) $x=0.24$ and 0.26 (x in nm) for both $n=1$ and $n=2$. Rationalize your answers.
3. Consider a particle in a 2-D box with $L_x=L_y$. How many *distinct* transitions can be possible (*i.e. may be observed*) in the system if you only consider levels with $n_i=1,2,3$ (for $i=x,y$)?
4. The wavefunctions of a particle in a 1D box are orthonormal to each other, i.e. $\langle \psi_i | \psi_j \rangle = \delta_{ij}$ (Kroneker delta) Verify this for $i = 2, j = 1, 2$. Given, $\sin \theta \sin \varphi = \frac{\cos(\theta - \varphi) - \cos(\theta + \varphi)}{2}$.
5. Draw a sketch in which the two wavefunctions in question 4 are overlapped. Using this sketch, verify the orthogonality of wavefunctions.
6. Let $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_N)$ be an eigenfunction of the Hamiltonian operator ($\hat{H}(\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_N)$). Often, the relationship $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_N) = \prod_{i=1}^N \Psi(\vec{r}_i)$ is used. When is this relationship exact? In such condition, what is the expression for the total energy?
7. Why are spherical polar coordinates used in preference over Cartesian co-ordinates for the hydrogen atom problem?
8. Derive the formula for the volume element in spherical polar coordinates.
9. Assuming the ground state wave function for hydrogen atom to be $\Psi(r, \theta, \phi) = N \exp\left(\frac{-r}{a_0}\right)$ find the normalization constant N . Use $\int x^n \cdot e^{-ax} dx = n! / a^{n+1}$