8.7 BOUNDARY CONDITIONS (SURFACES OF DISCONTINUITY)

The boundary conditions are similar to those discussed for the electrostatics and the steady currents. The conditions are now the relations for the vectors **B** and **H** at the two adjacent points on the two sides of an interface between the two different magnetic surfaces (Figure 8.7).

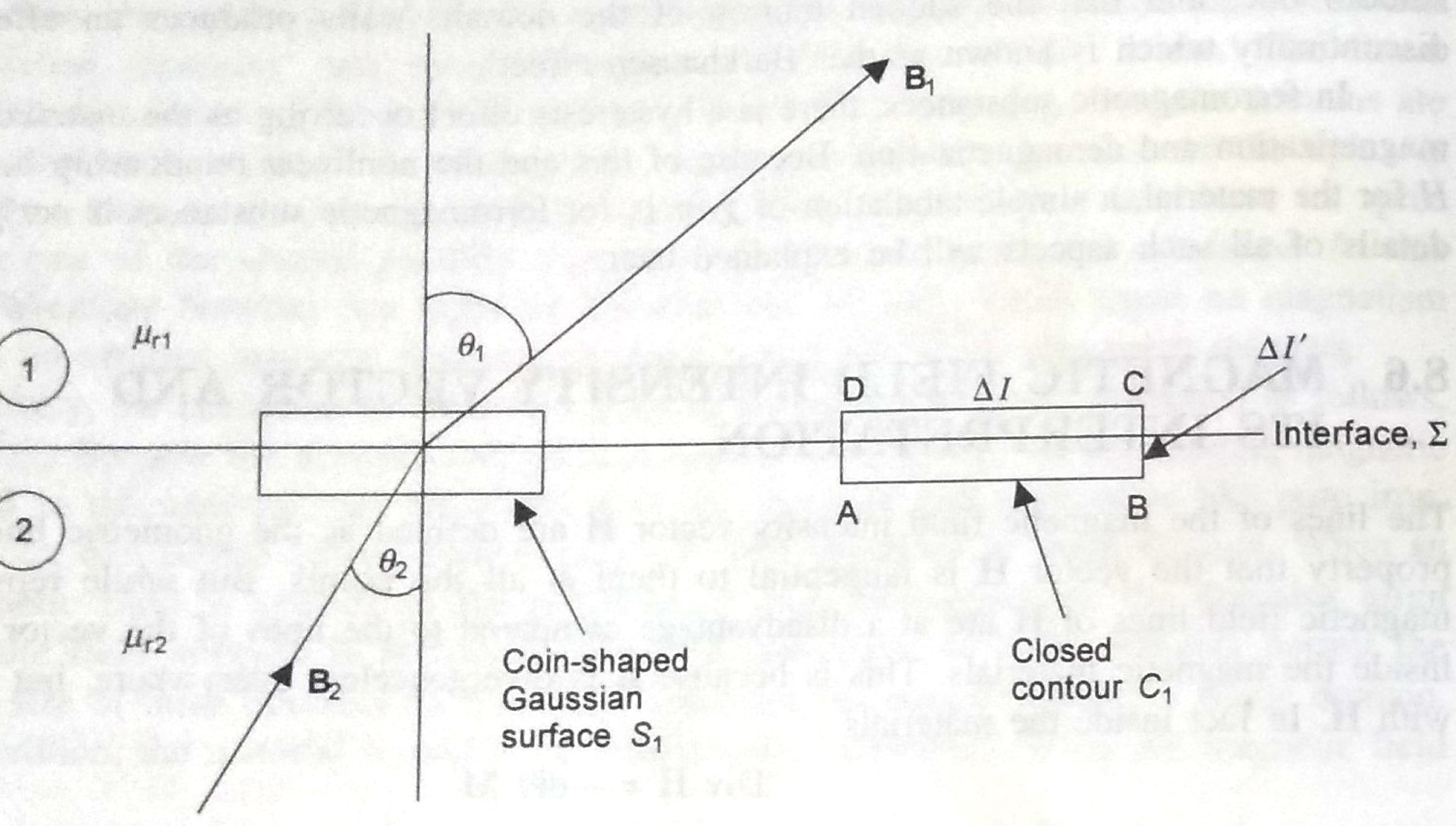

Figure 8.7 Interface between two different magnetic media.

We consider the interface, Σ, between the two different magnetic media (1) and (2) of relative permeabilities μ_{r1} and μ_{r2}, respectively. On this interface, we take a coin-shaped Gaussian surface of cross-sectional area δS_1 and find the outward flux of **B** from S_1. This flux must be zero because of the solenoidal property of **B**, i.e. div **B** = 0. (This is an alternative way of applying the 'principle of conservation of magnetic flux'.) So,

$$(B_1 \cos \theta_1 - B_2 \cos \theta_2)\delta S_1 = 0$$

as the flux out of the peripheral edge of the closed surface becomes zero in the limit.

$$\therefore \qquad B_1 \cos \theta_1 = B_2 \cos \theta_2 \quad \text{or} \quad B_{n1} = B_{n2} \tag{8.41}$$

i.e. the normal component of **B** is continuous across such a surface of discontinuity.

We next consider a closed contour C_1, i.e. ABCDA on the interface Σ. If there is no surface current on the interface, then

$$\oint_{\text{ABCDA}} \mathbf{H} \cdot d\mathbf{l} = 0$$

or

$$(H_1 \sin \theta_1 - H_2 \sin \theta_2)\Delta l + \text{contribution due to } \Delta l' = 0$$

The second term in the above equation → 0 in the limit.

$$\therefore \qquad H_1 \sin \theta_1 = H_2 \sin \theta_2 \quad \text{or} \quad H_{t1} = H_{t2} \tag{8.42}$$

This can be written as

$$\frac{B_{t1}}{\mu_{r1}} = \frac{B_{t2}}{\mu_{r2}} \tag{8.43}$$

The above equation implies that if there is no surface current on the surface of discontinuity, then the tangential component of the vector **H** is continuous. Combining the above three equations, we get

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_{r1}}{\mu_{r2}} \tag{8.44}$$

This is the law of refraction of the magnetic field lines.

Let us consider an important practical case, when the medium (1) is air, so that $\mu_{r1} = 1$ and $\mu_1 = \mu_0$, and the medium (2) is some ferromagnetic substance, such that $\mu_{r2} >> \mu_{r1}$. In that case,

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_{r1}}{\mu_{r2}} \simeq 0 \tag{8.45}$$

in which case $\theta_1 = 0$, i.e. the magnetic flux density vector **B** in air is practically normal to the surfaces of ferromagnetic materials.

Next, we consider again the interface of discontinuity, but now with a surface current I_S per unit width (Figure 8.8).

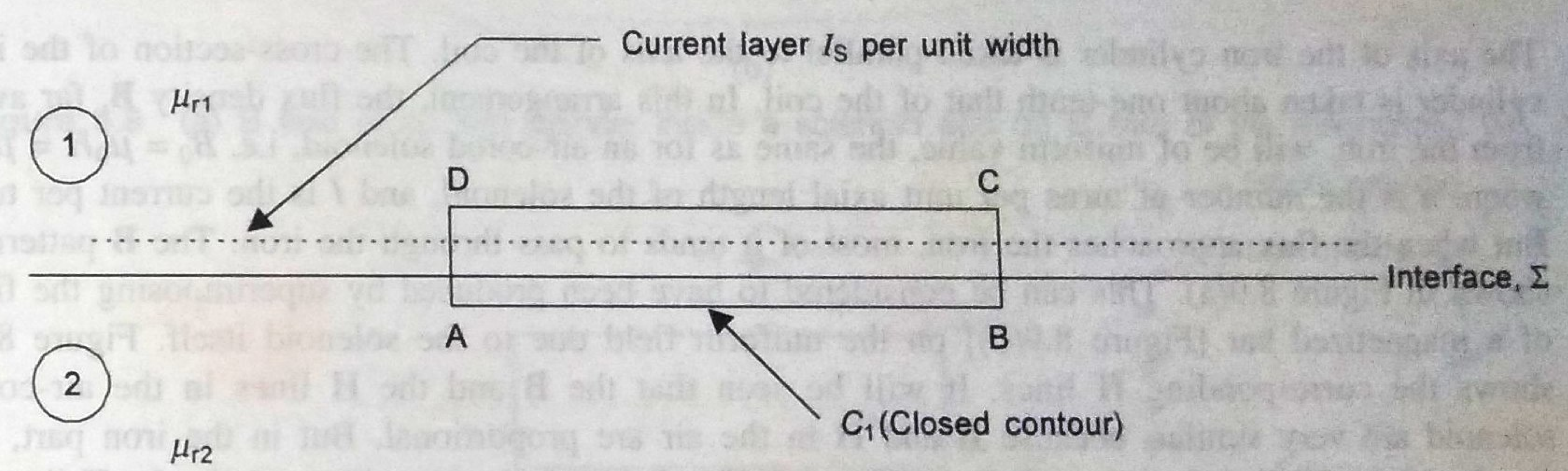

Figure 8.8 Surface of discontinuity with surface current on it.

In this case as well, the normal component of **B** would be continuous, for the same reason as before. But when we consider the closed contour ABCDA,

$$(H_{t1} - H_{t2})\Delta l = I_S \Delta l \quad \text{or} \quad H_{t1} - H_{t2} = I_S \tag{8.46}$$

i.e. the tangential component of **H** is discontinuous now. Furthermore if the magnitude of I_S is large enough, the direction of H_t might get reversed as one passes across the interface plane Σ.

8.8 THE MAGNETIC CHARACTERISTICS OF IRON (FERROMAGNETIC MATERIALS)

As we have seen earlier, the magnetic materials are divided into three main groups. The first two groups, i.e. the diamagnetic and the paramagnetic substances, have their relative permeabilities