PH-107 (2017) Tutorial Sheet 10

* Problems to be done in tutorial.

A. Statistical Mechanics

P83*: The national powerball lottery uses two sets of balls. The first set consists of 59 sequentially numbered balls; the second set consists of 35 sequentially numbered balls. Assume equal probability of choosing any ball, and that all the balls are differently numbered. Five balls are chosen without replacement from the set of 59. Then one ball is chosen from the set of 35. Calculate the number of ways these six balls can be chosen (and thus your probability of winning the grand powerball prize).

P84: Suppose we have 20 coins and we flip all of them together. Considering all the coins to be independent of each other, how many possible outcomes (no. of microstates) do you expect with such flipping?

How many ways are there for obtaining 12 heads and 8 tails? What is the probability of obtaining 12 heads and 8 tails regardless of the order? They are called macrostates.

- **P85*:** Three indistinguishable particles (say electrons) are to be arranged in three different energy levels of energy 0, E and 2E, with respective degeneracies (ignore spin degeneracy) 2, 10 and 20. The total energy available is 3E. What are the possible distributions and what are their probabilities?
- **P86.** Consider a particle confined to a 3D harmonic oscillator potential

$$V(x,y,z) = \frac{1}{2}m\omega^{2}(x^{2} + y^{2} + 4z^{2})$$

- (a) Calculate the ground state energy of the particle.
- (b) What is the degeneracy of the state with energy, $E = 7\hbar\omega$?
- **P87*:** In how many ways three electrons can occupy ten states (include spin degeneracy)? Is the number same as the way in which three persons can occupy ten chairs in a room? State the reason. In case the number is different, find the other number also.

P88*: The energy of a particle in a 3-D cubical box is given by:

$$E_{n_x,n_y,n_z} = \frac{\pi^2 \hbar^2}{2mL^2} (n_x^2 + n_y^2 + n_z^2)$$

If these levels are going to be occupied by electrons, write the energy values corresponding to the five lowest levels, taking into account the spin degeneracy. If three electrons occupy these states, find out the possible distributions which would yield a total energy of $\frac{18\pi^2\hbar^2}{2mL^2}$. Also find out the probability of each of the distributions.

P89*: A certain thermodynamic system has non-degenerate energy levels, with energies 0, E, 3E, 5E, 9E. Suppose that there are four particles, with total energy U = 9E. Identify the possible distribution of particles, and evaluate their microstates when (a) the particles are distinguishable. (b) the particles are identical bosons and (c) the particles are identical fermions.