Blackbody Radiation and Quantum Hypothesis

S Uma Sankar

Department of Physics
Indian Institute of Technology Bombay
Mumbai, India

Problems with Classical Radiation Theories

1. Specific heat of diatomic gases not properly explained.
   (Theory prediction: $C_V = 2.5R$ for all temperatures.
   Experimental Observation for Hydrogen: $C_V = 1.5R$ until about 100 K, $2.5R$ around room temperature (300 K) and rises to $3.5R$ for 1000 K.)
2. Incorrect prediction for specific heat of solids at low temperatures
   (Theory: $C = 3R$ for all temperatures.
   Experiment: Theory is correct for room temperatures but for low temperatures $C \to 0$).
3. No consistent explanation for blackbody radiation.
   Rayleigh-Jeans law could explain low frequency part of the blackbody radiation.
   Wien's law could explain the high frenquency part of the radiation.
4. No explanation for the discrete spectral lines in the radiation from atoms.

Classical Radiation Theory

Sometime around 1855, Maxwell constructed a unified picture of electricity and magnetism. He did this by modifying Ampere's law such that it applies to time varying currents also.

With this unified picture, he predicted that there exist electromagnetic waves which travel with speed of light. He also claimed that light is electromagnetic wave.

Using his theory, he derived the laws of reflection and refraction.

He also showed that an oscillating charge or an oscillating current emits electromagnetic radiation of the same frequency as the oscillation.

Heinrich Hertz verified a number of these predictions.

Blackbody Radiation

(Sections 3.2 and 3.3 of Serway, Moses and Moyer)

Any heated solid emits radiation in a continuous spectrum. Some empirical observations:

- The hotter the body the higher the frequency of radiation. (First the body becomes red hot and then becomes white hot)
- The frequency of radiation is independent of the object being heated. It depends only on the temperature.

Kirchhoff studied the emission of radiation from heated bodies and proved the following theorem based on thermodynamic arguments. If $e(\nu)$ is the emissivity (power emitted per unit area per unit frequency) and $A(\nu)$ is the fraction of incident power absorbed per unit area per unit frequency, then

$$e(\nu) = J(\nu, T)A(\nu),$$

where $J(\nu, T)$ is a **universal** function.

Blackbody is a body for which $A(\nu) = 1$, that is it absorbs all the power incident on it (and radiates some of it out).

Stefan's Law

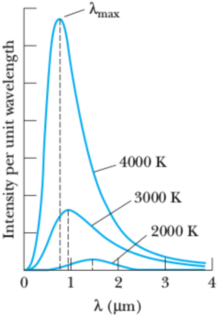
Based on an experimental study of different blackbodies, Stefan established that the total power, per unit area, emitted by a blackbody overall the frequencies, is given by

$$e_{\rm total} = \int_0^\infty e(\nu) d\nu = \sigma T^4.$$

The above equation is called Stefan's law and σ is a universal constant with value $\sigma = 5.67 \times 10^{-8}$ W-m^{-2}-K^{-4}. It is called Stefan-Boltzmann constant.

Wien's displacement Law

A blackbody emits radiation over a wide range of frequencies. However, there is a single frequency where the emitted power is maximum. The corresponding wavelength is denoted λ_{max}.



Wilhelm Wien, using thermodynamical arguments, showed that

$$\lambda_{max} T = \text{constant} \approx 3 \times 10^{-3}\ \mathrm{m-K}.$$

Wien's Exponential Law

A universal function $J(\nu, T)$ was introduced to describe the power emitted by a blackbody.

This can be related to $u(\nu, T)$, the energy per unit volume per unit frequency, carried by the blackbody as

$$J(\nu, T) = \frac{c}{4} u(\nu, T).$$

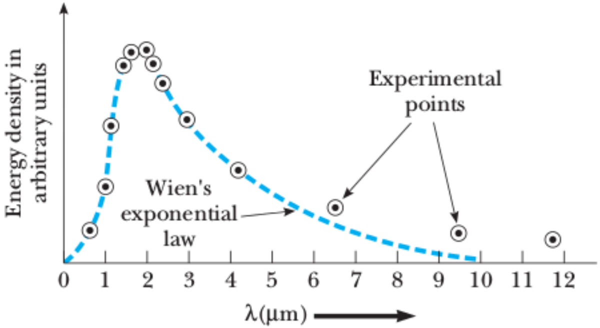
Since $J(\nu, T)$ is a universal function, so is $u(\nu, T)$. Different people made different attempts to calculate this universal function by various means.

Wien, based on some thermodynamical considerations, guessed that

$$u(\nu, T) = A\nu^3 \exp(-B\nu/T),$$

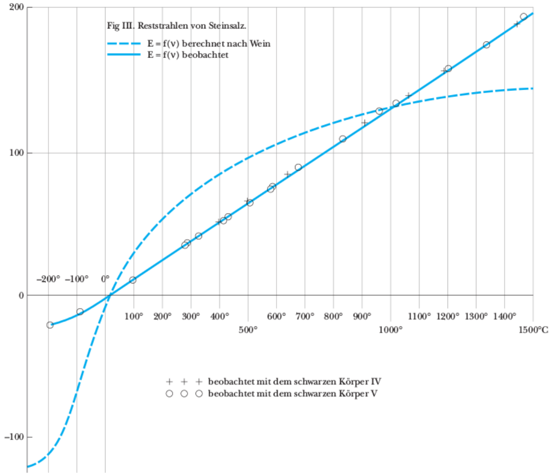
where A and B are universal constants. This is called Wien's exponential law.

Wien's Law



Notice that the data begins to disagree with Wien's law at higher wavelengths (or lower frequencies).

Wien's Law



The temperature dependence of blackbody radiation, measured by Rubens and Kurlbaum at $\lambda = 51.2\mu\mathrm{m}$, is very different from that predicted by Wien.

Rayleigh-Jeans Law

Rayleigh and Jeans derived another expression for $u(\nu, T)$ based on the following assumptions:

1. The walls of the blackbody consist of a large number (Avagadro number) of oscillators.
2. Different oscillators oscillate with different frequencies and they emit and absorb radiation.
3. The oscillators are in thermal equilibrium with the radiation surrounding them.
4. By equipartition theorem, each oscillator has average energy kT. (half from kinetic energy and half from potential energy).

The probability for an oscillator at temperature T to have energy E is given by $P(E) \propto \exp(-E/kT)$. Hence the average energy is

$$\langle E \rangle = \frac{\int_0^\infty EP(E)dE}{\int_0^\infty P(E)dE} = kT.$$

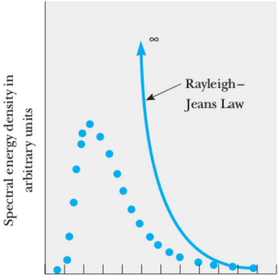
Rayleigh-Jeans Law

From the assumption of the equilibrium, Rayleigh-Jeans calculated

$$n(\nu)d\nu = \frac{8\pi\nu^2}{c^3}d\nu,$$

where $n(\nu)d\nu$ is the number of oscillators per unit volume, oscillating with frequency between ν and $\nu + d\nu$. Then

$$u(\nu, T)d\nu = kTn(\nu)d\nu = kT\frac{8\pi\nu^2}{c^3}d\nu.$$



Planck's Hypothesis

Rayleigh-Jean's law provides a good description for $u(\nu, T)$ at low frequencies but is a disaster at high frequencies. It predicts increasingly high energy densities for high frequencies.

This is called Ultraviolet Catastrophe.

Max Planck, inspired by the experiments of Lummer and Prigsheim and also of Rubens and Kurlbaum on blackbody radiation, made a very daring and radical proposal.

He assumed that the walls of the blackbody emits radiation only in certain QUANTA.

He said an oscillator in the wall of the blackbody, oscillating at frequency ν will emit energy in units of $h\nu$ (0 hν, 1 hν, 2 hν, ...).

Now, the probability of emission of energy is $\propto e^{-E/kT} = e^{-nh\nu/kT}$.

Planck's Derivation of Blackbody Radiation

Using Planck's hyothesis, let us calculate the average energy for a mode of oscillation of frequency ν.

$$\begin{aligned} \langle E \rangle &= \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}} \\ &= \frac{h\nu}{e^{h\nu/kT} - 1} \end{aligned}$$

Substituting this formula, in stead of kT, in the expression for $u(\nu)$, we get

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}.$$

In the limit $h\nu/kT \ll 1$, we recover Rayleigh-Jeans law.
We can **derive** Stefan's law by integrating $u(\nu)$

$$U = \frac{c}{4} \int_0^{\infty} u(\nu) d\nu = \frac{2\pi^5 k^4}{15 h^3 c^2} T^4. = \sigma T^4$$

We get an expression for Stefan's constant σ in terms fundamental constants k, c and h.