MA 108 - Spring 2018
Tutorial Sheet 3

1. (a) Verify that $y_1 = 1/(x-1)$ and $y_2 = 1/(x+1)$ are solutions of $(x^2-1)y''+4xy'+2y = 0$ on $\mathbb{R} - \{\pm 1\}$. Find the general solution.
   (b) Find the solution with initial conditions $y(0) = -5,\ y'(0) = 1$.
   (c) What is the interval of validity of this solution?

2. Compute the Wronskians of the given set of functions.
   (a) $\{e^x, e^x \sin x\}$.
   (b) $\{x^{1/2}, x^{-1/3}\}$.
   (c) $\{x \ln|x|, x^2 \ln|x|\}$.

3. Let y_1 and y_2 be two solutions of ODE. Find the Wronskian $W(y_1, y_2; x)$ of following ODE's.
   (a) $y'' + 3(x^2+1)y' - 2y = 0$ given that $W(\pi) = -1$.
   (b) $(1-x^2)y'' - 2xy' + a(a+1)y = 0$ given that $W(0) = 1$.
   (c) $x^2y'' + xy' + (x^2 - \nu^2)y = 0$ given that $W(1) = 1$.

4. Suppose p_1, p_2, q_1, q_2 are continuous on (a, b) and the equations $y'' + p_1(x)y' + q_1(x)y = 0$ and $y'' + p_2(x)y' + q_2(x)y = 0$ have the same solutions on (a, b). Show that $p_1 = p_2$ and $q_1 = q_2$ on (a, b). [Hint. Use Abel's formula.]

5. Solve the following IVPs.
   (a) $y'' - 2y' + 2y = 0, \quad y(0) = 3,\ y'(0) = -2$.
   (b) $y'' + 14y' + 50y = 0,\ \ y(0) = 2,\ y'(0) = -17$.
   (c) $6y'' - y' - y = 0,\ \ y(0) = 10,\ y'(0) = 0$.
   (d) $4y'' - 4y' - 3y = 0,\ \ y(0) = \frac{13}{12},\ y'(0) = \frac{23}{24}$.
   (e) $4y'' - 12y' + 9y = 0,\ \ y(0) = 3,\ y'(0) = \frac{5}{2}$.

6. Find the general solution of following ODE.

   [Hint: Find two linearly independent solution of homogeneous part and then use variation of parameter method to find a particular solution.]
   (a) $x^2y'' + xy' - 4y = 2x^4$.
   (b) $x^2y'' - 3xy' + 3y = x$.
   (c) $y'' - 3y' + 2y = 1/(1+e^{-x})$.
   (d) $x^2y'' + xy' - 4y = -6x - 4$.

(e) $x^2y'' - 2xy' + 2y = x^{9/2}$.

(f) $y'' - 2y' + y = 14x^{3/2}e^x$.

(g) $y'' + 4y = \sin 2x \sec^2 2x$.

7. (Principle of Superposition) Assume y_1 is a solution of $a(x)y'' + b(x)y' + c(x)y = f_1(x)$ and y_2 is a solution of $a(x)y'' + b(x)y' + c(x)y = f_2(x)$. Show that $y_1 + y_2$ is a solution of $a(x)y'' + b(x)y' + c(x)y = f_1(x) + f_2(x)$.

8. Find a particular solution of $y'' + 4xy' + (4x^2 + 2)y = 4e^{-x(x+2)}$, given that $y_1 = e^{-x^2}$, $y_2 = xe^{-x^2}$ are solutions of homogeneous part.

9. Find a particular solution using variation of parameters method.

(a) $y'' - 2y' + y = 14x^{3/2}e^x$.

(b) $y'' - y = \dfrac{4e^{-x}}{1 - e^{-2x}}$.

(c) $y'' + y = \sec x \tan x$.

(d) $y'' - 3y' + 2y = \sin e^{-x}$.

(e) $x^2y'' - x(x+2)y' + (x+2)y = 2x^3, \quad x > 0,$
the fundamental set of solutions of homogeneous part is $\{x, xe^x\}$

(f) $(1 - x)y'' + xy' - y = 2(x-1)^2e^{-x}, \quad 0 < x < 1;$
the fundamental set of solutions of homogeneous part is $\{e^x, x\}$.