PH108 : Electricity & Magnetism : Tutorial 9

1. Prove the following results involving magnetic field $\vec{B}$ in free space, where there is no current, *i.e.* $\vec{J} = 0$. Then use these results to prove the statement in part (c).

   (a) Each component of the magnetic field satisfies Laplaces's equation. This means:

   $$\nabla^2 B_x = 0 \qquad \nabla^2 B_y = 0 \qquad \nabla^2 B_z = 0$$

   (b) The magnetic field $B^2 = {B_x}^2 + {B_y}^2 + {B_z}^2$, satisfies the equation :

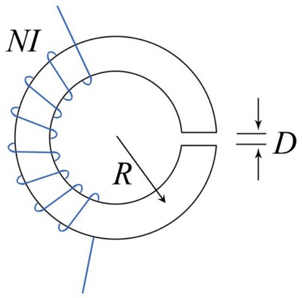
   $$\nabla^2 B^2 = 2\,|\nabla B_x|^2 + 2\,|\nabla B_y|^2 + 2\,|\nabla B_z|^2$$

   Notice that this result relates a second derivative to first derivatives - and is quite striking. You would need to use the result from the last problem to prove this one.

   (c) Now using these prove that in an inhomogeneous magnetic field an diamagnetic dipole can be in stable equilibrium but a paramagnetic dipole cannot be. *This effectively means that you can make a diamagnet, like a drop of water (!), float in a magnetic field, but you cannot do so with a paramagnetic material. Animal and plant bodies are often nearly 90% water - so they can be made to float too!*

2. The figure shows a toroidal ring of a soft magnetic material with a high (initial) permeability $\mu/\mu_0 >> 1$. The ring has a mean radius R and a small air gap D, such that $D << 2\pi R$. A coil of N turns with current I flowing through it is wrapped around it. Calculate the magnetic field B in the air gap of this simple electromagnet. In reality the exact shape of the iron core is not very important. You can also assume that the magnetic field has negligible "fringing" in the air gap. This is similar to what you assume for the electric field in a parallel plate capacitor, so do not worry too much about that ! Can you see an analogy with Ohm's law for this "magnetic circuit" ?

   Answer : $B \approx \dfrac{NI}{\frac{2\pi R}{\mu} + \frac{D}{\mu_0}}$

3. Consider a cylindrical bar magnet of cross sectional area A that is magnetised along its length, and has a magnetic field B just outside the pole. A flat object with a high permeability is stuck on it. How much force would be required to pull them apart? Now apply what you have found to estimate the mass of the largest object a magnet can lift (against gravity), for a magnet that has a surface field $B \approx 0.5$ Tesla and a cross section $A = 1$ cm^2. The numbers given are reasonable approximations for rare earth magnets that are quite popular nowadays.