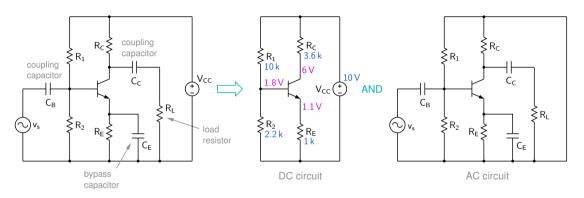
BJT Amplifiers: Part 2

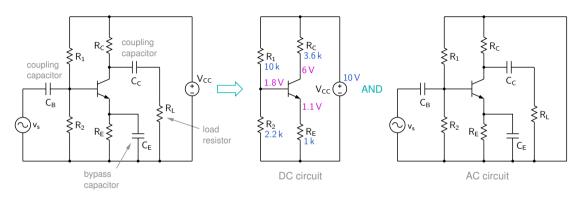


M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

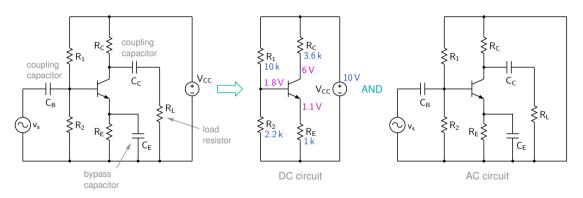
Department of Electrical Engineering Indian Institute of Technology Bombay



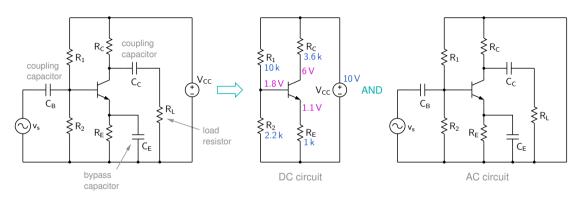
* We have already analysed the DC (bias) circuit of this amplifier and found that $V_B=1.8~V,~V_E=1.1~V,~V_C=6~V,~{\rm and}~I_C=1.1~{\rm m}A.$



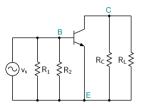
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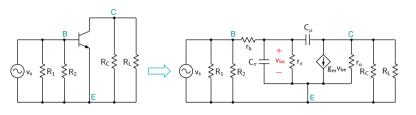


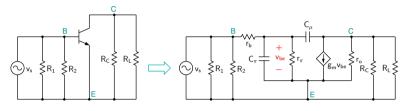
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- * We will then get the complete solution by simply adding the DC and AC results, e.g., $i_C(t) = I_C + i_C(t)$.
- * We will assume that C_B , C_C , C_E are large enough so that, at the signal frequency (say, 1 kHz), they can be replaced by short circuits.



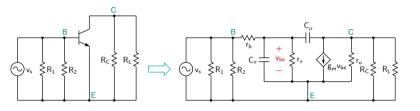




* The parasitic capacitances C_{π} and C_{μ} are in the pF range. At a signal frequency of 1 kHz, the impedance corresponding to these capacitances is

$$\mathbf{Z} \sim rac{-j}{\omega C} = rac{-j}{2\pi imes 10^3 imes 10^{-12}} \sim -j\,100\,\mathrm{M}\Omega$$

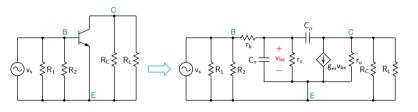
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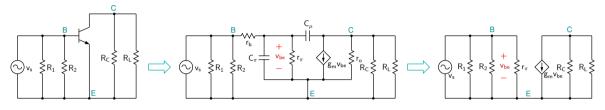
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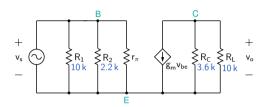
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- * The above considerations significantly simplify the AC circuit.



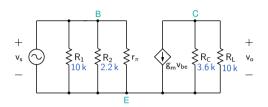
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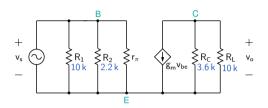
$$v_o = -(g_m \, v_{be}) imes (R_C \parallel R_L) = -(g_m \, v_s) imes (R_C \parallel R_L)$$



$$v_o = -(g_m v_{be}) \times (R_C \parallel R_L) = -(g_m v_s) \times (R_C \parallel R_L)$$

 $\rightarrow A_V^L = \text{voltage gain} = \frac{v_o}{v_s} = -g_m (R_C \parallel R_L)$

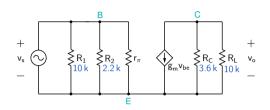
(superscript L is used because the gain includes the effect of \mathcal{R}_L .)



$$\begin{aligned} & v_o = -(g_m \, v_{be}) \times (R_C \parallel R_L) = -(g_m \, v_s) \times (R_C \parallel R_L) \\ & \rightarrow A_V^L = \text{voltage gain} = \frac{v_o}{V} = -g_m \, (R_C \parallel R_L) \end{aligned}$$

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Since I_C (bias current) = 1.1 mA, $g_m = I_C/V_T = 1.1 \, \mathrm{mA/25.9 \, mV} = 42.5 \, \mathrm{m} \odot$.



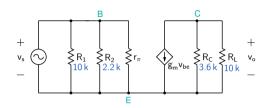
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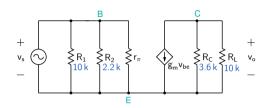
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For $v_s(t)=(2\,\mathrm{m} V)\,\sin\omega t$, the AC output voltage is,



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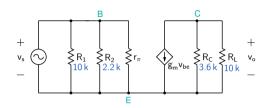
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For $v_s(t) = (2 \text{ mV}) \sin \omega t$, the AC output voltage is,

$$v_o = A_V^L v_s = -(112.5)(2 \,\mathrm{m} V) \sin \omega t = -(225 \,\mathrm{m} V) \sin \omega t$$



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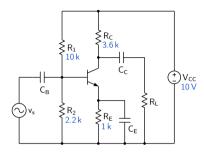
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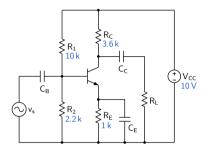
$$i_c = g_m v_{be} = g_m v_s = 42.5 \,\mathrm{m} \mho \times (2 \,\mathrm{m} V) \sin \omega t = 85 \,\sin \omega t \,\mu A.$$



For $v_s(t) = (2 \,\mathrm{m} \, V) \sin \omega t$, we can now obtain expressions for the instantaneous currents and voltages:

$$v_C(t) = V_C + v_c(t) = V_C + v_o(t) = 6 V - (225 \,\mathrm{m}V) \sin \omega t$$
.

$$i_{C}(t) = I_{C} + i_{c}(t) = 1.1 \, \mathrm{mA} + 0.085 \, \sin \omega t \, \, \mathrm{mA} \, .$$

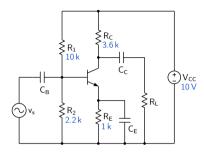


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Note that the above procedure (DC + AC analysis) can be used *only if* the small-signal approximation (i.e., $|v_{be}| \ll V_T$) is valid. In the above example, the amplitude of v_{be} is 2 mV, which is much smaller than V_T (about 25 mV).



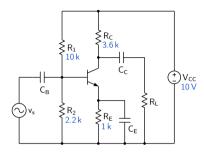
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For $v_s(t) = (20 \,\mathrm{mV}) \sin \omega t$, for example, the small-signal approximation will not hold, and a numerical simulation will be required to obtain the currents and voltages of interest.



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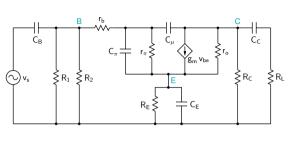
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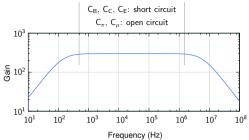
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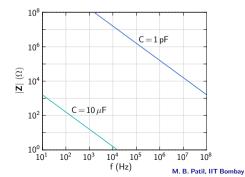
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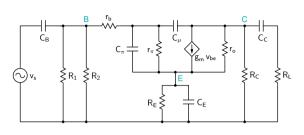
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In practice, such a situation is anyway not prevalent (because it gives rise to distortion in the output voltage) except in special types of amplifiers.

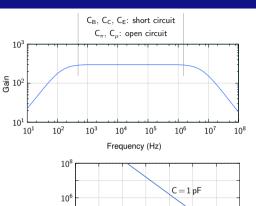


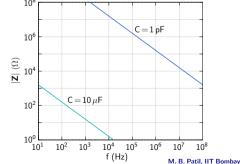


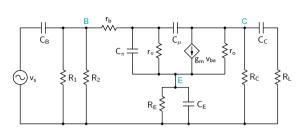




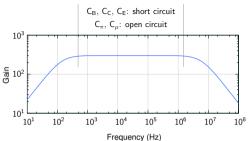
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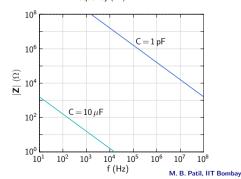


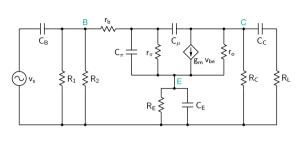


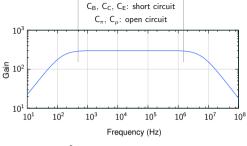


- * C_B , C_E , C_C are large capacitances $\rightarrow 1/\omega C$ is negligibly small (short circuit) except at low frequencies.
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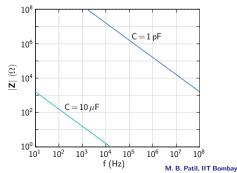


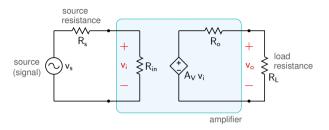




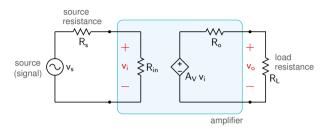


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- * C_{π} , C_{μ} are small capacitances $\rightarrow 1/\omega C$ is very large (open circuit) except at high frequencies.
- * In the "mid-band" range (which we have considered so far), the large capacitances behave like short circuits, and the small capacitances like open circuits. In this range, the gain is independent of frequency.

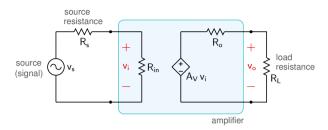




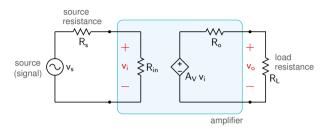
* An amplifier is represented by a voltage gain, an input resistance $R_{\rm in}$, and an output resistance $R_{\rm o}$. For a voltage-to-voltage amplifier, a large $R_{\rm in}$ and a small $R_{\rm o}$ are desirable.



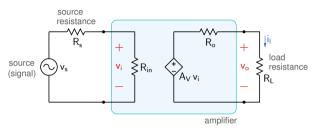
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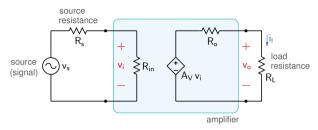
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- * The DC bias of the circuit can affect parameter values in the AC equivalent circuit $(A_V, R_{\rm in}, R_{\rm o})$. For example, for the common-emitter amplifier, $A_V \propto g_m = I_C/V_T$, I_C being the DC (bias) value of the collector current.



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- * Suppose we are given an amplifier as a "black box" and asked to find A_V , R_{in} , and R_o . What experiments would give us this information?



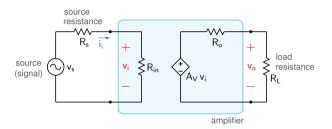
If $R_L \to \infty$, $i_l \to 0$, and $v_o \to A_V v_i$.



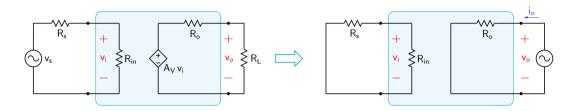
If $R_L \to \infty$, $i_l \to 0$, and $v_o \to A_V v_i$.

We can remove R_L (i.e., replace it with an open circuit), measure v_i and v_o , then use $A_V = v_o/v_i$.

Input resistance R_{in}



Measurement of v_i and i_i yields $R_{in} = v_i/i_i$.



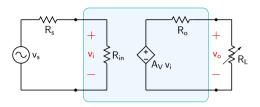
Method 1:

If
$$v_s \to 0$$
, $A_V v_i \to 0$.

Now, connect a test source v_o , and measure $i_o \rightarrow R_o = v_o/i_o$.

(This method works fine on paper, but it is difficult to use experimentally.)

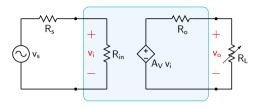
Output resistance R_o



Method 2:

$$v_o = \frac{R_L}{R_L + R_o} \, A_V \, v_i$$

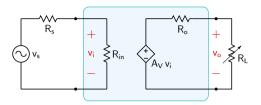
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$$v_o = \frac{R_L}{R_L + R_o} A_V v_i.$$
If $R_L \to \infty$, $v_{o1} = A_V v_i$.

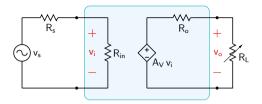
Output resistance Ro



Method 2:

$$\begin{split} v_o &= \frac{R_L}{R_L + R_o} \ A_V \ v_i. \\ \text{If } R_L &\to \infty, \ v_{o1} = A_V \ v_i. \\ \text{If } R_L &= R_o, \ v_{o2} = \frac{1}{2} \ A_V \ v_i = \frac{1}{2} \ v_{o1}. \end{split}$$

Output resistance Ro



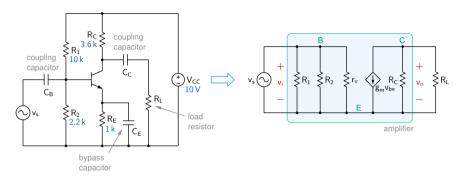
Method 2:

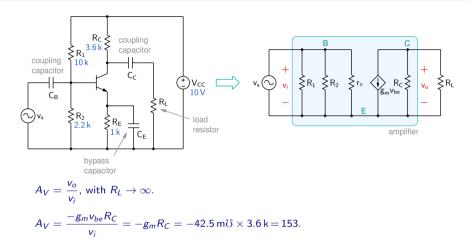
$$v_o = \frac{R_L}{R_L + R_o} A_V v_i.$$
If $R_L \to \infty$, $v_{o1} = A_V v_i$.

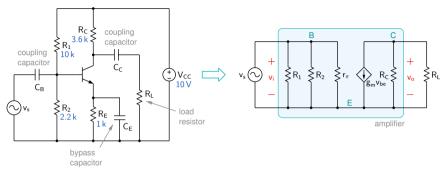
If
$$R_L = R_o$$
, $v_{o2} = \frac{1}{2} A_V v_i = \frac{1}{2} v_{o1}$.

Procedure:

- Measure v_{o1} with $R_L \to \infty$ (i.e., R_L removed).
- Vary R_L and observe v_o .
- When v_o is equal to $v_{o1}/2$, measure R_L (after removing it).
- R_o is the same as the measured resistance.





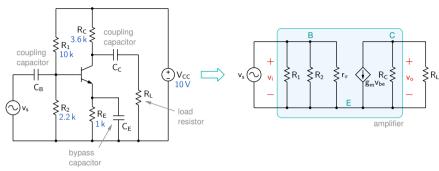


$$A_V=rac{v_o}{v_i}$$
, with $R_L o\infty$.

$$A_V = \frac{-g_m v_{be} R_C}{v_i} = -g_m R_C = -42.5 \,\mathrm{m} \mho \times 3.6 \,\mathrm{k} = 153.$$

The input resistance of the amplifier is, by inspection, $R_{\text{in}} = (R_1 \parallel R_2) \parallel r_{\pi}$.

$$r_{\pi} = \beta/g_{m} = 100/42.5 \,\mathrm{m} \odot = 2.35 \,\mathrm{k} \to R_{\mathrm{in}} = 1 \,\mathrm{k}.$$



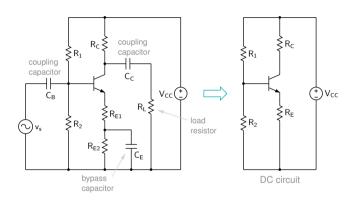
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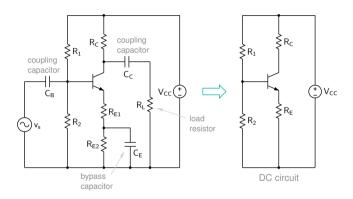
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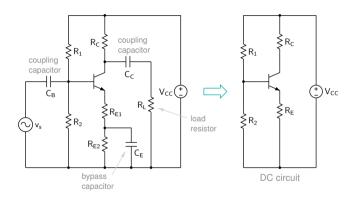
$$r_{\pi} = \beta/g_{m} = 100/42.5 \,\mathrm{m} \odot = 2.35 \,\mathrm{k} \to R_{\mathrm{in}} = 1 \,\mathrm{k}.$$

The output resistance is $R_{\mathcal{C}}$ (by "Method 1" seen previously).

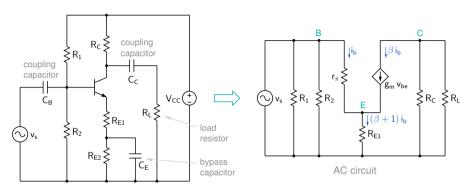




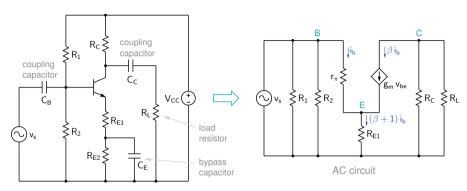
* For DC computation, C_E is open, and the DC analysis is therefore identical to our earlier amplifier, with $R_E \leftarrow R_{E1} + R_{E2}$.



- * For DC computation, C_E is open, and the DC analysis is therefore identical to our earlier amplifier, with $R_E \leftarrow R_{E1} + R_{E2}$.
- * Bypassing a part of R_E (as opposed to all of it) does have an impact on the voltage gain (see next slide).

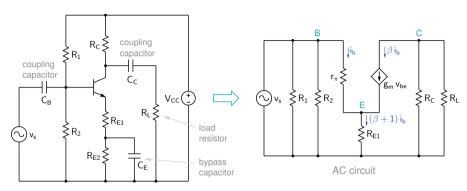


Again, assume that, at the frequency of operation, C_B , C_C , C_E can be replaced by short circuits, and the BJT parasitic capacitances by open circuits.



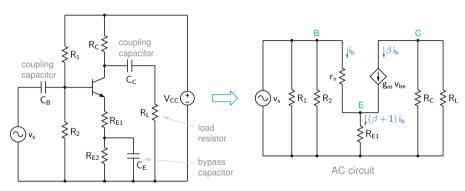
Again, assume that, at the frequency of operation, C_B , C_C , C_E can be replaced by short circuits, and the BJT parasitic capacitances by open circuits.

$$v_s = i_b r_\pi + (\beta + 1) i_b R_{E1} \rightarrow i_b = \frac{v_s}{r_\pi + (\beta + 1) R_{E1}}.$$



Again, assume that, at the frequency of operation, C_B , C_C , C_E can be replaced by short circuits, and the BJT parasitic capacitances by open circuits.

$$\begin{aligned} v_s &= i_b \, r_\pi + (\beta + 1) \, i_b \, R_{E1} \to i_b = \frac{v_s}{r_\pi + (\beta + 1) \, R_{E1}}. \\ v_o &= -\beta \, i_b \times (R_C \parallel R_L) \to \frac{v_o}{v_s} = -\frac{\beta \, (R_C \parallel R_L)}{r_\pi + (\beta + 1) \, R_{E1}} \approx -\frac{(R_C \parallel R_L)}{R_{E1}} \text{ if } r_\pi \ll (\beta + 1) \, R_{E1}. \end{aligned}$$

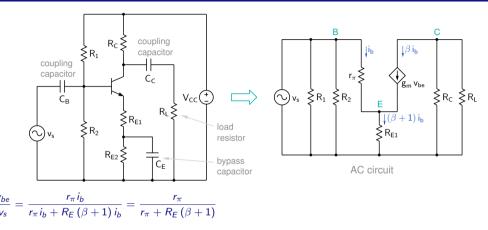


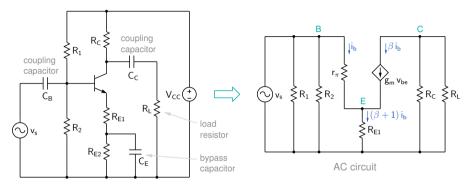
Again, assume that, at the frequency of operation, C_B , C_C , C_E can be replaced by short circuits, and the BJT parasitic capacitances by open circuits.

$$v_{s} = i_{b} r_{\pi} + (\beta + 1) i_{b} R_{E1} \rightarrow i_{b} = \frac{v_{s}}{r_{\pi} + (\beta + 1) R_{E1}}.$$

$$v_{o} = -\beta i_{b} \times (R_{C} \parallel R_{L}) \rightarrow \frac{v_{o}}{v_{s}} = -\frac{\beta (R_{C} \parallel R_{L})}{r_{\pi} + (\beta + 1) R_{E1}} \approx -\frac{(R_{C} \parallel R_{L})}{R_{E1}} \text{ if } r_{\pi} \ll (\beta + 1) R_{E1}.$$

Note: R_{E1} gets multiplied by $(\beta + 1)$.

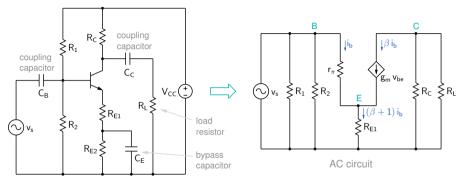




$$\frac{v_{be}}{v_s} = \frac{r_{\pi}i_b}{r_{\pi}i_b + R_E(\beta + 1)i_b} = \frac{r_{\pi}}{r_{\pi} + R_E(\beta + 1)}$$

The small-signal condition, viz., $|v_{be}(t)| \ll V_T$ now implies

$$|v_s| \, rac{r_\pi}{r_\pi + R_E \, (eta + 1)} \ll V_T \;\; ext{ or } \; |v_s| \ll V_T imes rac{r_\pi + R_E \, (eta + 1)}{r_\pi}, ext{ which is much larger than } V_T.$$



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 \to Although the gain is reduced, partial emitter bypass allows larger input voltages to be applied without causing distortion in $v_o(t)$. (For comparison, we required $|v_s| \ll V_T$ for the CE amplifier.)