

PH108 : Electricity & Magnetism

Weekly Quiz 3 - Using the Dirac Delta Function

31 January, 2018

Answer

$$g(\theta) = \delta(\theta - \frac{\pi}{2})$$

$[\frac{1}{2}]$ **mark**

$$f(r) = \frac{1}{r}$$

$[\frac{1}{2}]$ **mark**

Solution

X-Y plane $\implies \theta = \frac{\pi}{2}$. ρ only non zero for this value of $\theta \implies$ Dirac Delta function.

$$Q = \sigma \pi R^2 = \int_0^{2\pi} d\phi \int_0^{\pi} \sin(\theta) d\theta \int_0^R r^2 dr \rho(\mathbf{r}) = 2\pi \sigma \int_0^{\pi} \sin(\theta) d\theta \delta(\theta - \frac{\pi}{2}) \int_0^R r^2 dr f(r)$$

$$\implies \int_0^R r^2 dr f(r) = R^2/2 \implies f(r) = \frac{1}{r}$$

OR,

$$[\text{L.H.S.}] = [\rho] = [\text{charge}] [\text{length}]^{-3} \text{ where } [] \equiv \text{dimension.}$$

$$[\text{R.H.S.}] = [\sigma] [g(\theta)] [f(r)] = [\text{charge}] [\text{length}]^{-2} [f(r)] \implies [f(r)] = [\text{length}]^{-1}$$

(θ is dimensionless.)