PH108 : Electricity & Magnetism

Weekly Quiz 3 - Finding the Levi-Civita Symbol

31 January, 2018

Answer

$$M_{ij} = \begin{matrix} 0 & \text{-}a_z & a_y \\ a_z & 0 & \text{-}a_x \\ \text{-}a_y & a_x & 0 \end{matrix}$$ $[\frac{1}{2}$ **mark**$]$

$|a_\times| = 0$ $[\frac{1}{2}$ **mark**$]$

Solution

$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y)\hat{\imath} - (a_x b_z - a_z b_x)\hat{\jmath} + (a_x b_y - a_y b_x)\hat{k}$

$= [0(b_x) + (-a_z)(b_y) + (a_y)(b_z)]\,\hat{\imath}$
$+ [(a_z)(b_x) + (0)(b_y) + (-a_x)(b_z)]\,\hat{\jmath}$
$+ [(-a_y)(b_x) + (a_x)(b_y) + 0(b_z)]\,\hat{k}$

Skew symmetric square matrix of odd order $\implies$ determinant is zero!