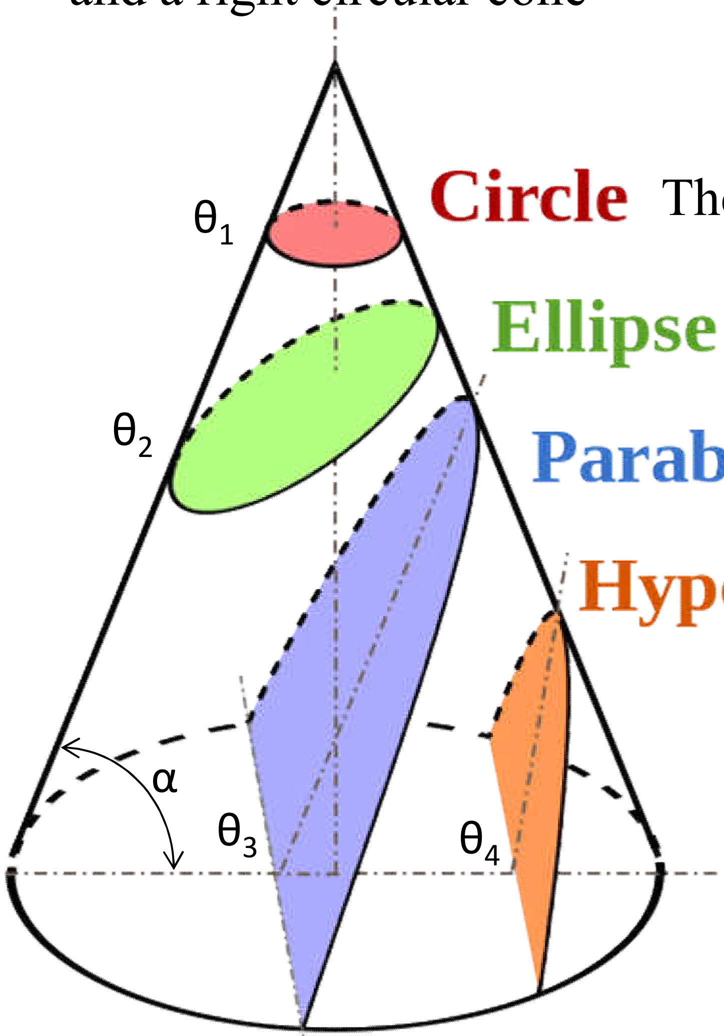
Engineering Curves
Lecture – 2

Adapted by Prof. Amitabh Bhattacharya
from slides by Prof. Salil S. Kulkarni

Conic Sections (Conics)

Conic section: curves formed by the intersection of a plane (cutting plane, CP) and a right circular cone



Circle The CP is parallel to the base of the cone ($\theta_1 = 0$)

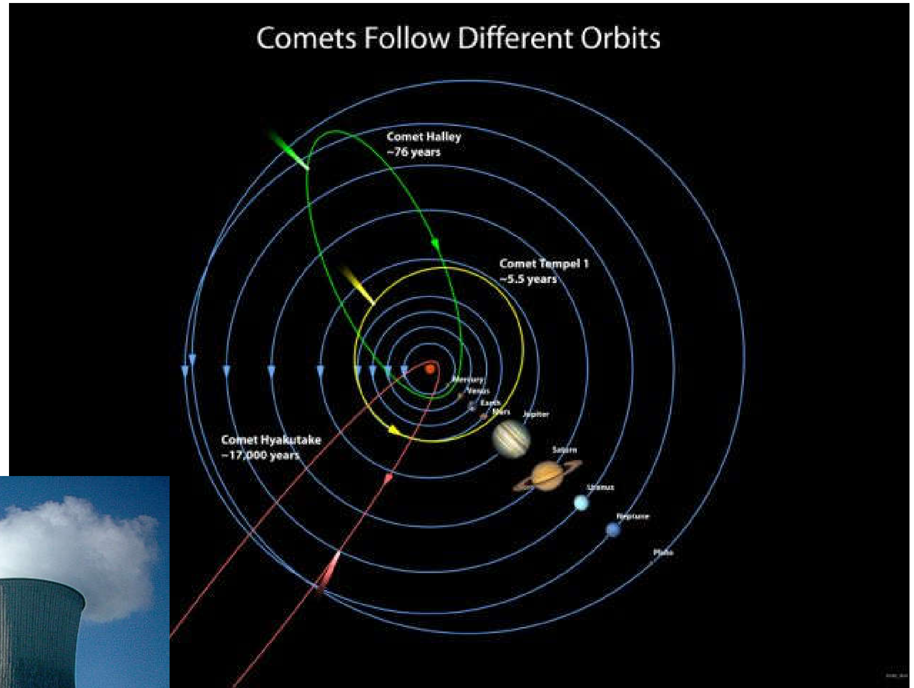
Ellipse The CP is inclined at an angle smaller than that made by the generator with the base ($\theta_2 < \alpha$)

Parabola The CP is inclined at an angle equal to that made by the generator with the base ($\theta_3 = \alpha$)

Hyperbola The CP is inclined at an angle greater than by that made the generator with the base ($\theta_4 > \alpha$)

https://en.wikipedia.org/wiki/Conic_section

Conic Sections in Real World Applications



http://deepimpact.umd.edu/gallery/jpg/D1381_001d.jpg

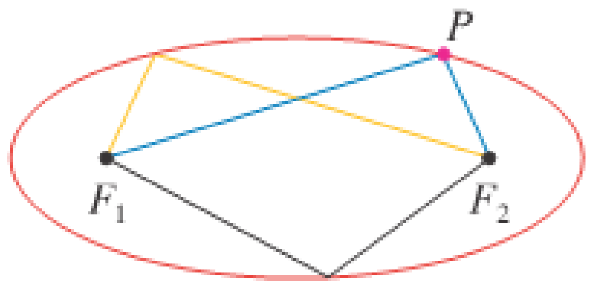
http://en.wikipedia.org/wiki/List_of_Hyperboloid_structures

http://britton.disted.camosun.bc.ca/jbconics.htm

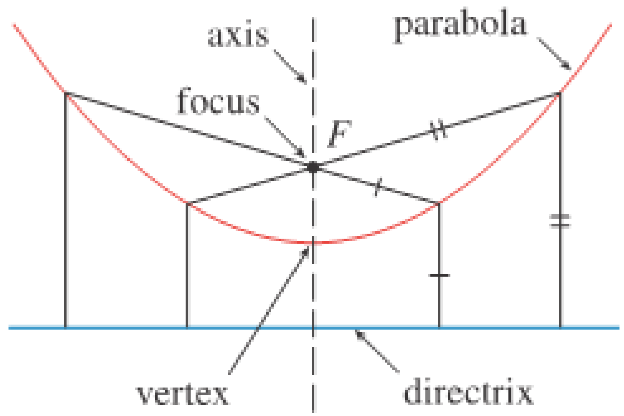
http://en.wikipedia.org/wiki/Parabola#Parabolae_in_the_physical_world

Conics – Geometric properties

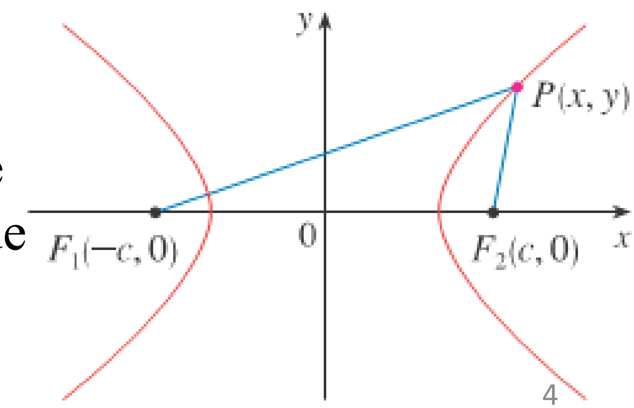
Ellipse: A locus of all points for which the sum of distances from two given points F_1, F_2 has a fixed value



Parabola: A locus of all points having equal distances from a given point F (focus) and a given line (directrix

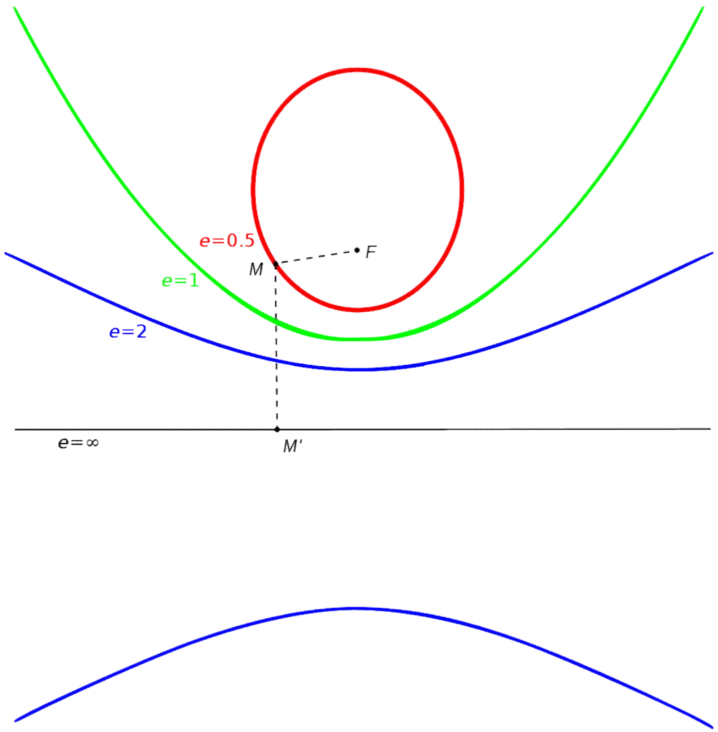


Hyperbola: A locus of all points for which the difference of distances from two given points F_1, F_2 has a fixed value

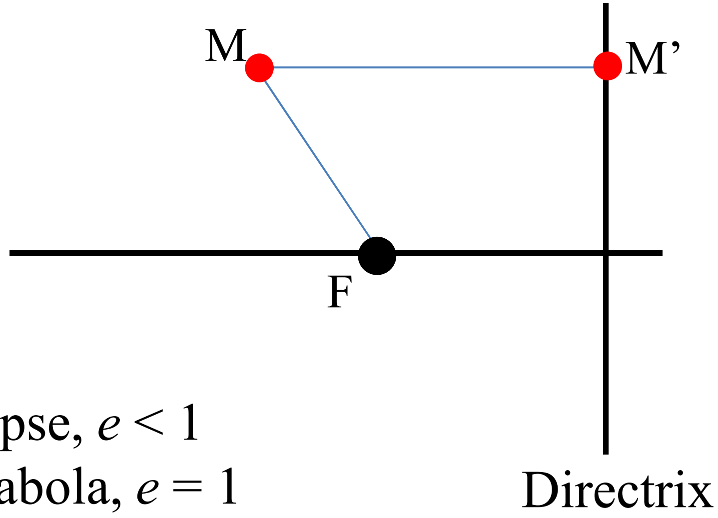


Conic Sections (Conic) – Alternate definition

- A locus of a point (M) which moves so that its distance from a fixed point is in a constant ratio to its perpendicular distance from a fixed straight line.
- The fixed point is called the **Focus** (F)
- The fixed straight line is called the **Directrix**
- The constant ratio is called the **Eccentricity** and is denoted by e



$$e = \frac{l(FM)}{l(MM')}$$



- Ellipse, $e < 1$
- Parabola, $e = 1$
- Hyperbola $e > 1$
- Circle, $e = 0$ (limiting case, directrix at infinity)

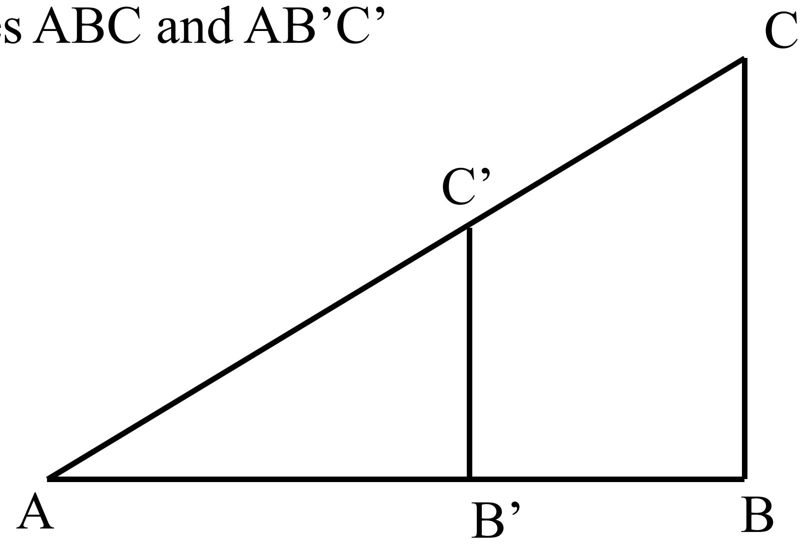
https://en.wikipedia.org/wiki/Conic_section

Method to Construct the Conic Section given the distance of the focus from the directrix and its eccentricity

Locus of points: Based on the fact that the eccentricity is constant. For an assumed distance from the focus, the distance from the directix can be obtained using the eccentricity.

Use is made of the fact that for similar triangles ABC and AB'C'

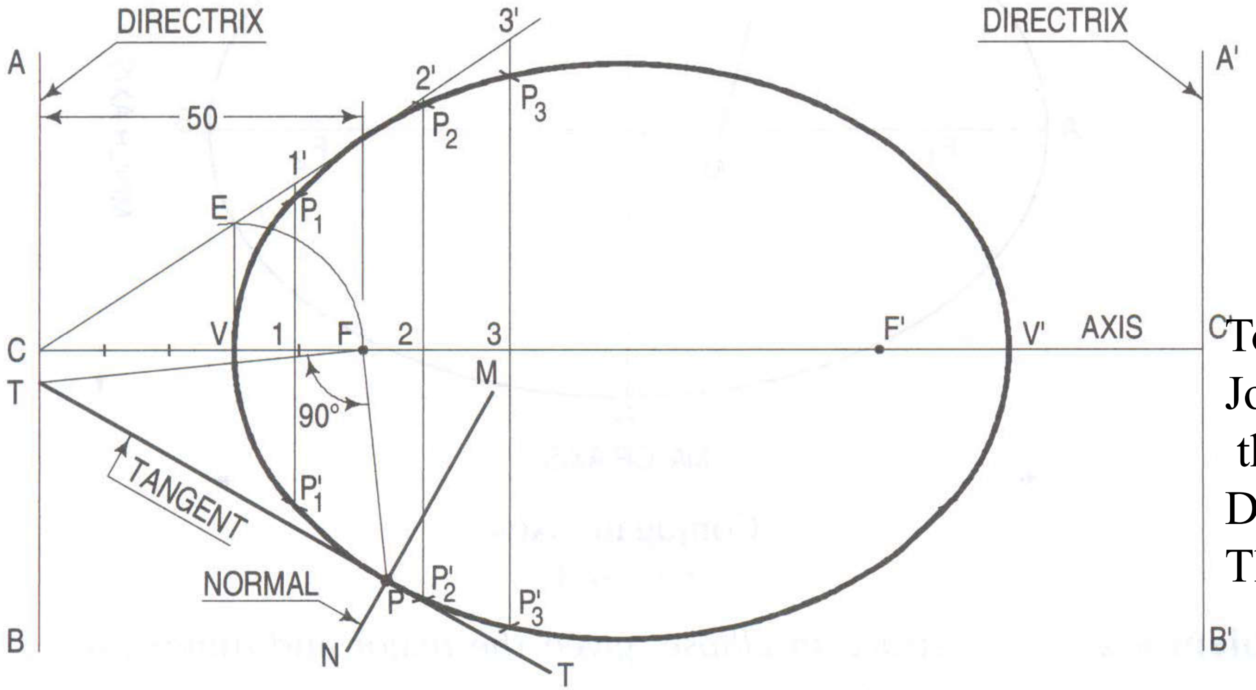
$$\frac{l(AB')}{l(AB)} = \frac{l(B'C')}{l(BC)}$$



Tangent: To draw the tangent at a given point on a conic section, use is made of the fact that the line joining the given point to the focus is perpendicular to the line joining the focus to the point on the directrix which is the intersection of the tangent and the directrix

Construction of an ellipse

Construct an ellipse when the distance of the focus from the directrix is equal to 50mm and its eccentricity is 2/3.



To draw the tangent at P:
Join P to F. Draw FT such that FT is perpendicular to FP.
Draw a line through T and P.
This is the required tangent

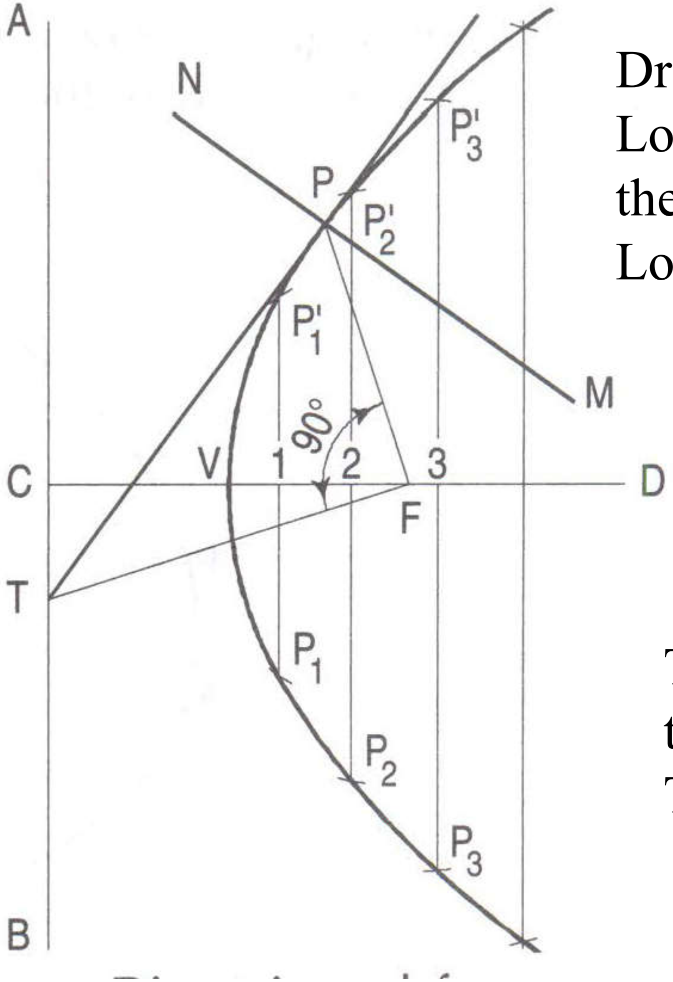
Draw the directrix AB and the axis CD. Locate the focus F. Locate the vertex V taking into account the eccentricity. Locate the subsequent points on the ellipse using

$$e = \frac{l(FV)}{l(CV)} = \frac{l(EV)}{l(CV)} = \frac{l(11')}{l(C1)} = \frac{l(FP_1)}{l(C1)} = \frac{l(22')}{l(C2)} = \frac{l(FP_2)}{l(C2)}$$

Ref: Engineering Drawing by N. D. Bhatt et. al

Parabola – Geometric properties

Construct a parabola when the distance of the focus from the directrix is equal to 50mm.



Draw the directrix AB and the axis CD
Locate the focus F. Locate the vertex V taking into account the eccentricity
Locate the subsequent points on the parabola using

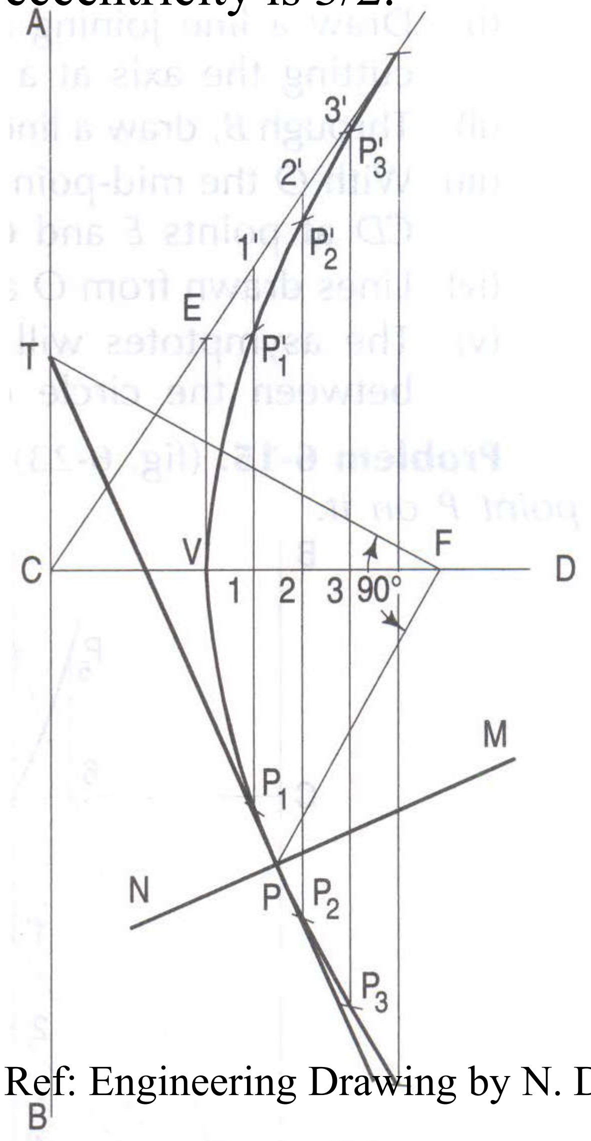
$$e = \frac{l(FV)}{l(CV)} = \frac{l(FP_1)}{l(C1)} = \frac{l(FP_2)}{l(C2)} = 1$$

To draw the tangent at P: Join P to F. Draw FT such that FT is perpendicular to FP. Draw a line through T and P. This is the required tangent

Ref: Engineering Drawing by N. D. Bhatt et. al

Hyperbola – Geometric properties

Construct a hyperbola when the distance of the focus from the directrix is 65mm and eccentricity is 3/2.



Draw the directrix AB and the axis CD

Locate the focus F. Locate the vertex V taking into account the eccentricity

Locate the subsequent points on the hyperbola using

$$e = \frac{l(FV)}{l(CV)} = \frac{l(EV)}{l(CV)} = \frac{l(11')}{l(C1)} = \frac{l(FP_1)}{l(C1)} = \frac{l(22')}{l(C2)} = \frac{l(FP_2)}{l(C2)}$$

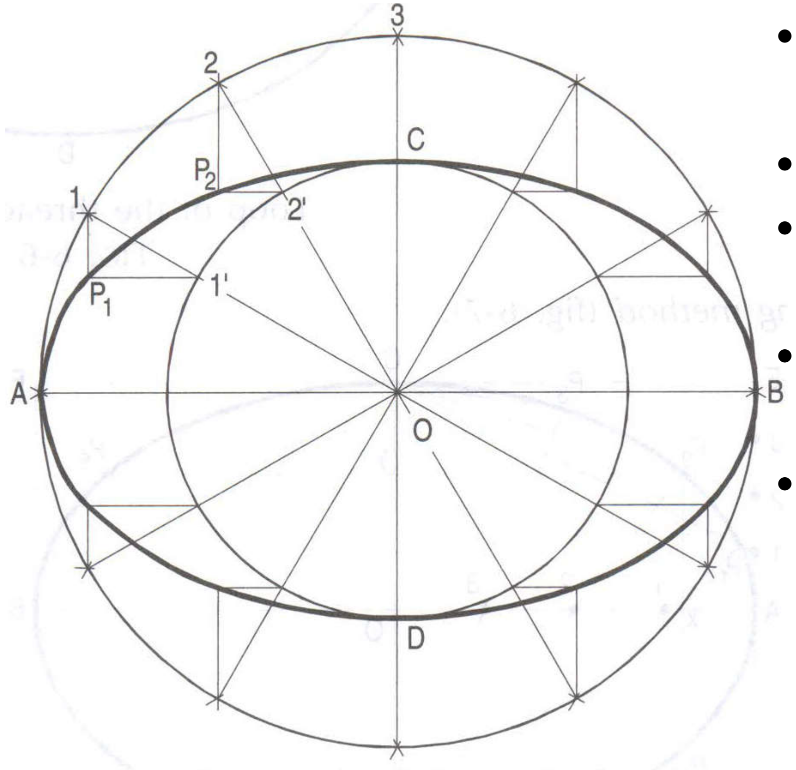
Ref: Engineering Drawing by N. D. Bhatt et. al

Methods to Construct an Ellipse given its Major and the Minor Axes

- **Exact methods**
  - Arc of circle method
  - Concentric circle method
  - Rectangle method
- **Approximate methods**
  - Four center method – the ellipse is approximated as a union of arcs of four circles – will study this in the later part of the course

Ellipse – Concentric circle method

Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using concentric of circles method.



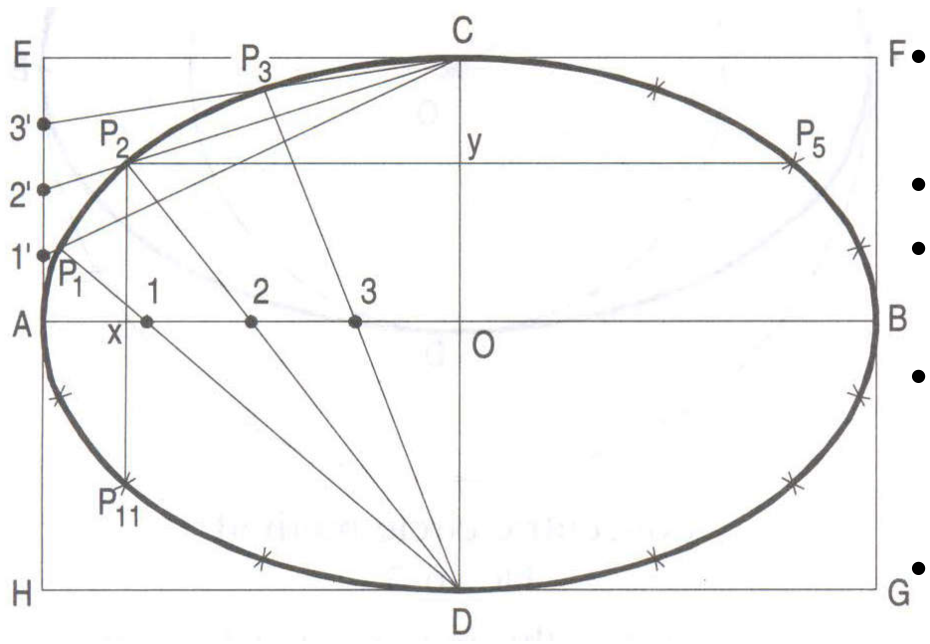
- Draw two concentric circles of radii 60mm and 40 mm
- Divide the circles into 12 equal parts
- Through the intersection points of the radial lines with the outer circle draw vertical lines
- Through the intersection points of the radial lines with the inner circle draw horizontal lines
- The intersection of a vertical line with the horizontal line gives a point on the ellipse. eg: the intersection of the vertical line through point 2 and the horizontal line through point 2’ gives point P_2

The most accurate of the methods to draw an ellipse given the semi-major and the semi-minor axis

Ref: Engineering Drawing by N. D. Bhatt et. al

Ellipse – Rectangle method

Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using the rectangle method.

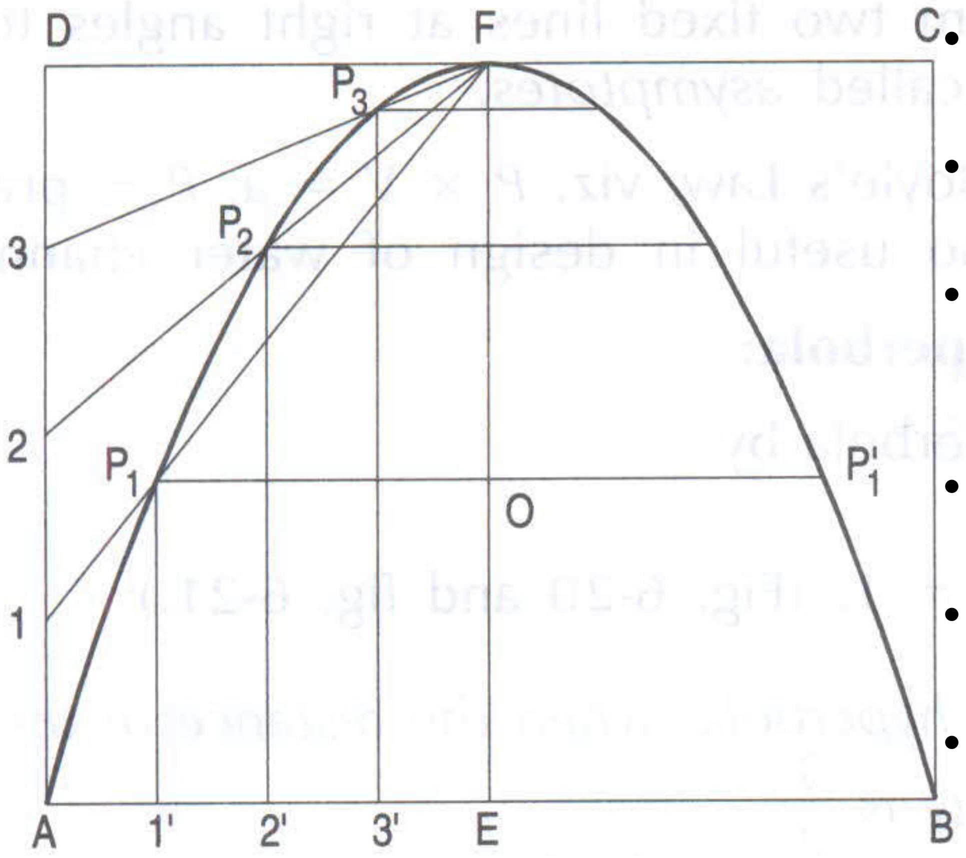


- Draw the bounding rectangle EFGH with EF = 120mm and FG = 80mm
- Draw the lines AB and CD so as to divide the rectangle into four equal parts
- The point of intersection of AB and CD is C
- Divide AO into n equal parts – in the present case n = 4 and mark points 1, 2, 3
- Divide AE into n equal parts – in the present case n = 4 and mark points 1', 2' 3'
- Draw lines through D passing through points 1, 2, 3
- Draw lines through C passing through points 1', 2', 3'
- The intersection of the line D1 with line C1' gives point P_1, a point on the ellipse
- Similarly locate points P_2, P_3, etc.
- Draw a smooth curve passing through points A, P_1, P_2, etc

Ref: Engineering Drawing by N. D. Bhatt et. al

Parabola – Rectangle method

Construct a parabola inscribed in a rectangle of 120mm x 90mm using the rectangle method

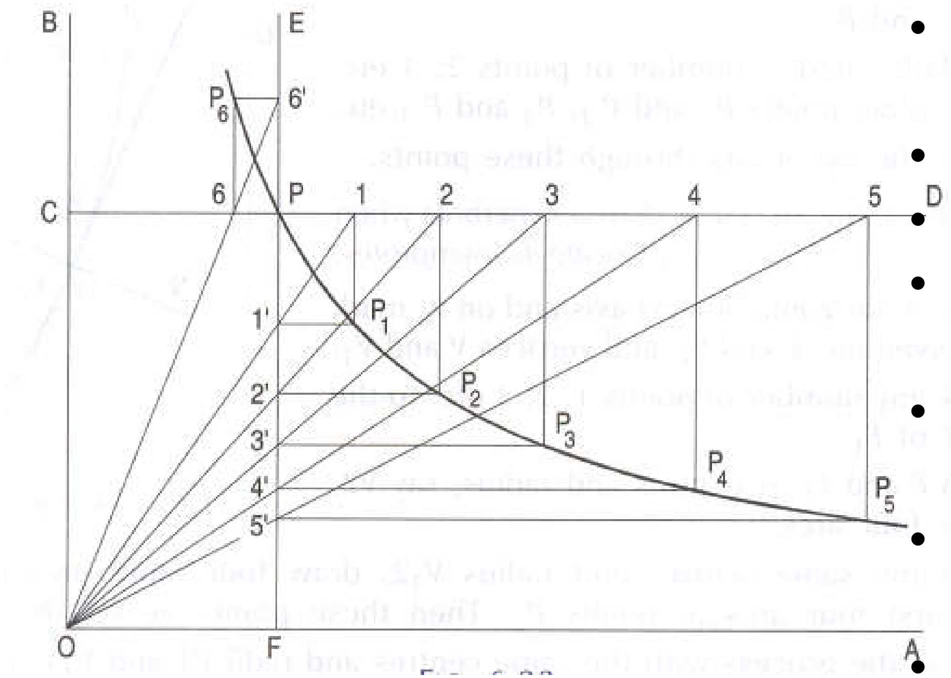


Ref: Engineering Drawing by N. D. Bhatt et. al

- Draw the bounding rectangle ABCD with AB = 120mm and BC = 90mm
- Draw line EF so as to divide the rectangle into two equal parts
- Divide AD into n equal parts – in the present case n = 4 and mark points 1, 2, 3
- Divide AE into n equal parts – in the present case n = 4 and mark points 1', 2' 3'
- Draw lines through F passing through points 1, 2, 3
- Draw vertical lines passing through points 1', 2', 3'
- The intersection of the line F1 with the vertical line through 1' gives point P_1, a point on the parabola
- Similarly locate points P_2 , P_3 , etc.
- Draw a smooth curve passing through points A, P_1, P_2 , etc

Rectangular Hyperbola: *xy = c*

Construct a rectangular hyperbola given a point P on it



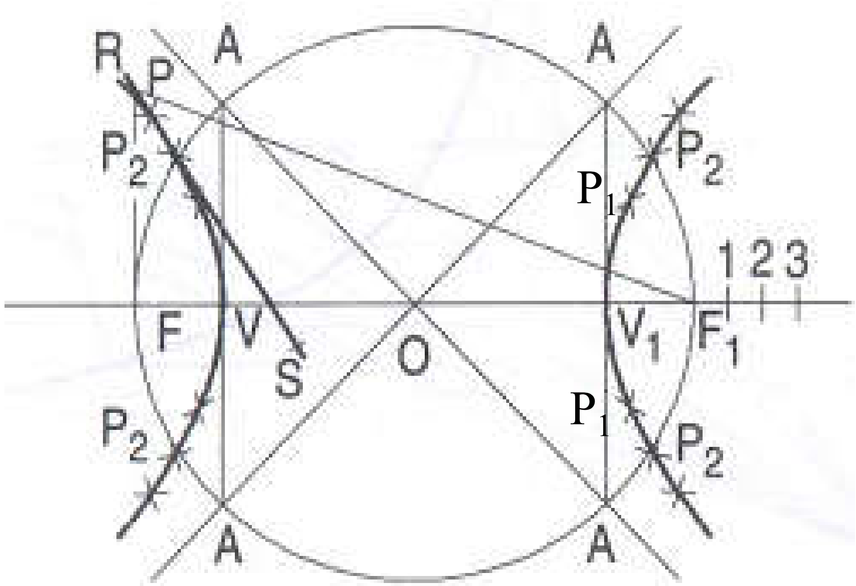
- Draw OA and OB perpendicular to each other and locate point P.
- Through P draw a line EF parallel to OB
- Through P draw a line CD parallel to OA
- Mark points 1, 2, 3, …on CD – need not be equispaced
- Draw lines through O passing through points 1, 2, etc
- The intersection of these lines with EF gives points 1', 2', etc
- Through points 1, 2, etc draw vertical lines
- Through points 1', 2', etc draw horizontal lines
- The intersection of the vertical line, say through 1, and the horizontal line through 1' gives point P_1, a point on the rectangular hyperbola
- Similarly locate points P_2, P_3, etc

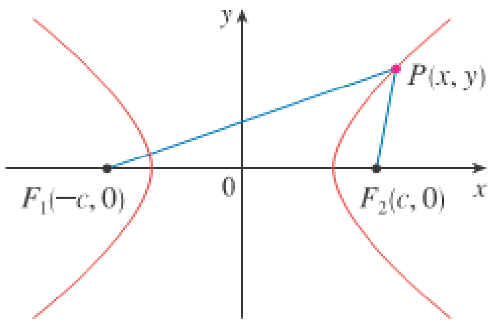
Rectangular hyperbola: The asymptotes are right angles to each other

Ref: Engineering Drawing by N. D. Bhatt et. al

Hyperbola

Construct a hyperbola given its vertices V and V_1 and foci F and F_1





- Draw a horizontal line
- Locate points V and V_1 and foci F and F_1
- Know that $FP-F_1P=FV_1 – F_1V_1 = VV_1$ is a constant
- Mark points 1, 2, 3 on the horizontal line
- Measure $V_1 1$ and draw arc with center F_1
- Measure V1 and draw an arc with center F
- The point of intersection of the two arcs gives point P_1
- Because of about the symmetry we get two locations of P_1
- Similarly one can locate other points

Ref: Engineering Drawing by N. D. Bhatt et. al

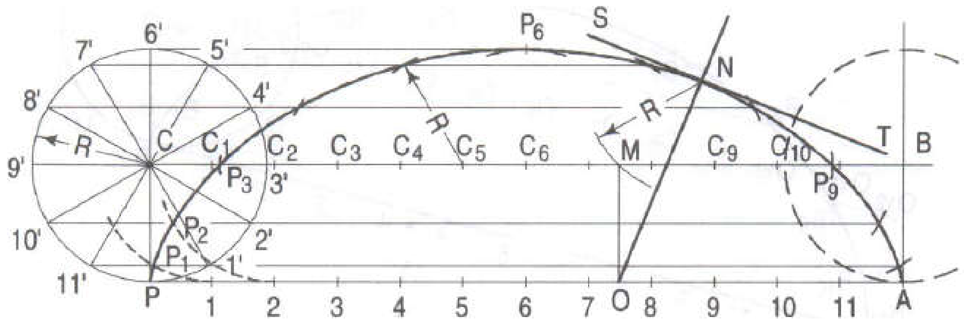
Cycloidal Curves

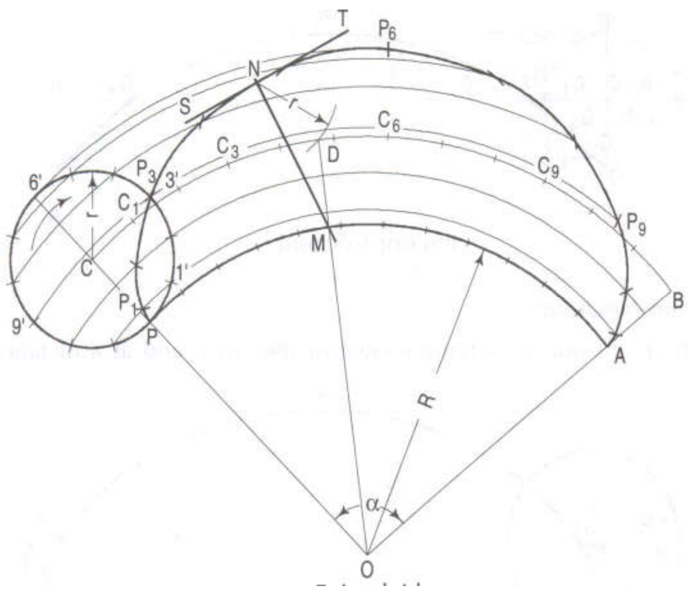
Cycloidal Curves: Curves generated by a fixed point on the circumference of a circle which rolls without slipping along a fixed straight line or a circle.

Terminology:

Generating circle: Rolling circle is called as the generating circle

Directing line or directing circle: The fixed line or the fixed circle is called as the directing line or directing circle



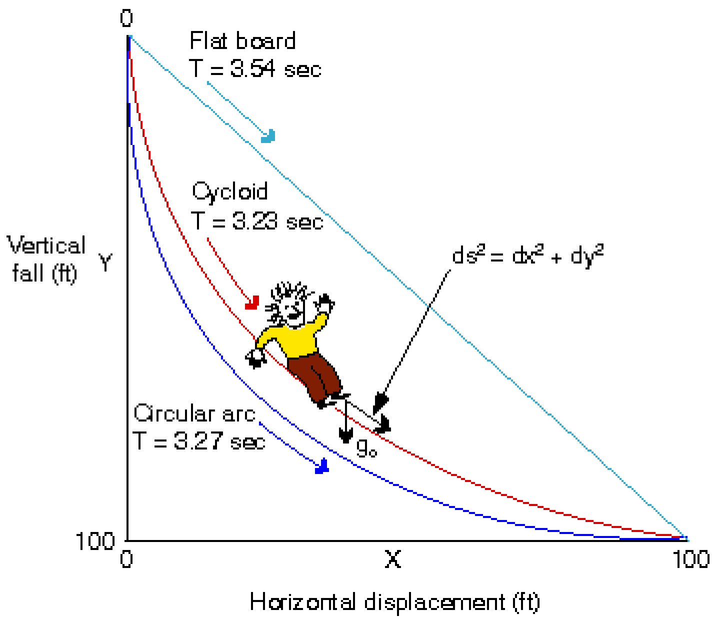


Ref: Engineering Drawing by N. D. Bhatt et. al

Interesting Properties of a Cycloid

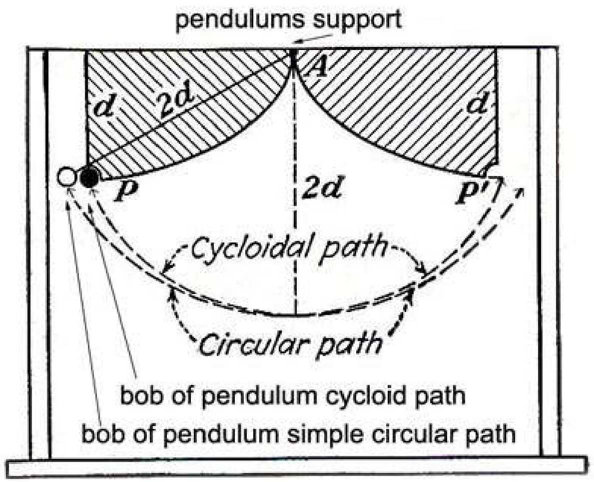
Brachistochronous

(curve of quickest descent)



Tautochronous

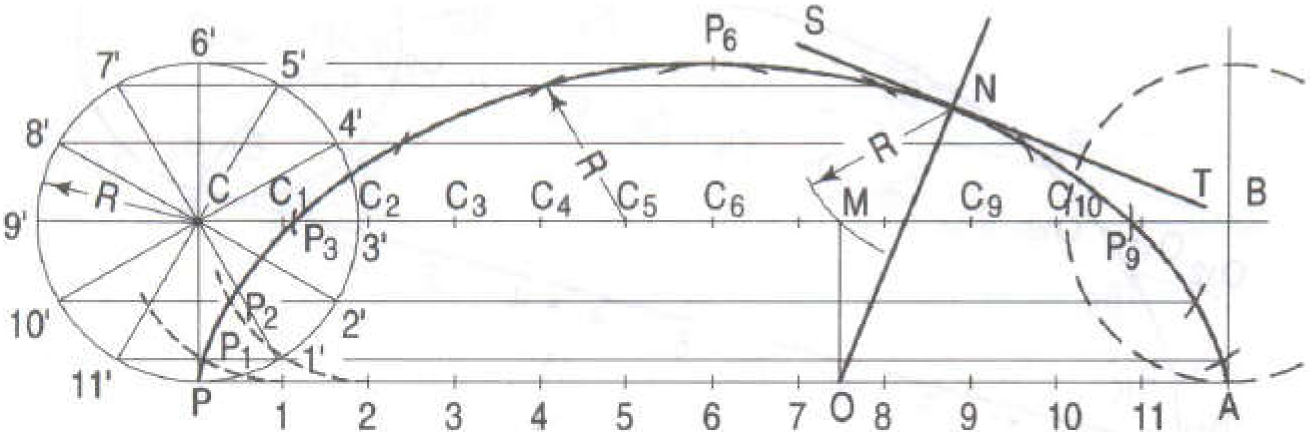
(period of oscillation is independent of the amplitude of oscillation)



Used in construction of pendulum clocks where the period of oscillation is Independent of the amplitude

http://www.scitechantiques.com/cycloidhtml/

Cycloid Curves



Draw the generating circle

Draw the directing line with length equal to the circumference of the generating circle

Divide the generating circle into 12 equal parts – label the points 1’,2’,…,12’

Divide the directing line into 12 equal parts – label the points 1,2,…,12

From the points 1’,2’,…, draw lines parallel to the directing line

From the points 1,2,… draw lines perpendicular to the directing line

Label the intersection of these lines with the line through the center of the generating circle as C_1, C_2,…,C_{12}

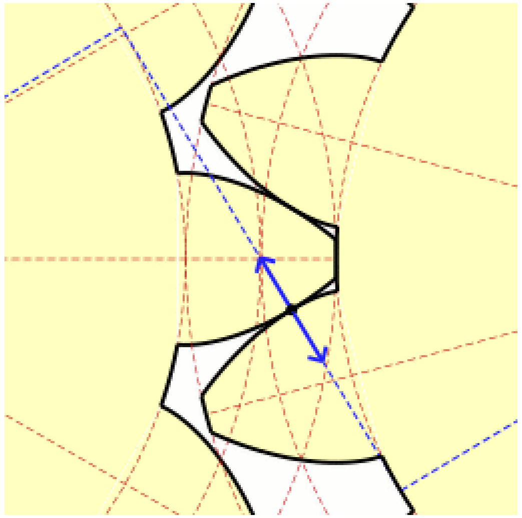
With C_1, C_2, …. as centers and radius equal to the radius of the generating circles, mark points P_1, P_2, on lines passing through 1,2,….

Join points P_1, P_2, … This is the required cycloid

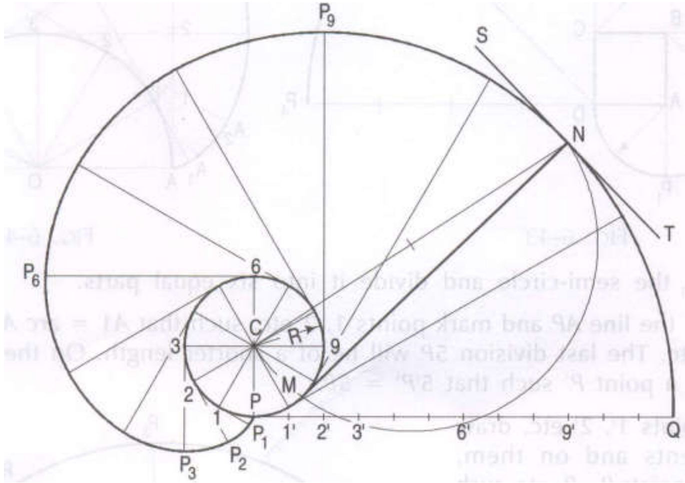
Ref: Engineering Drawing by N. D. Bhatt et. al

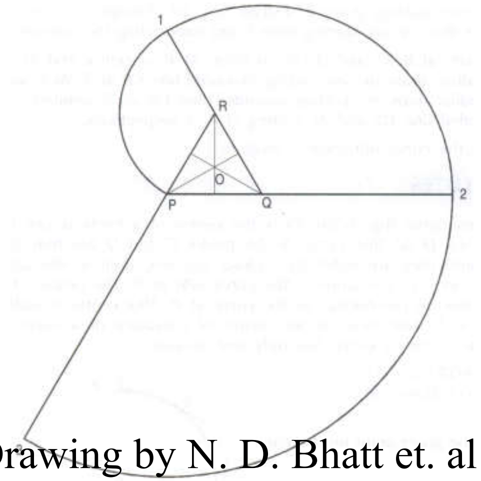
Involute

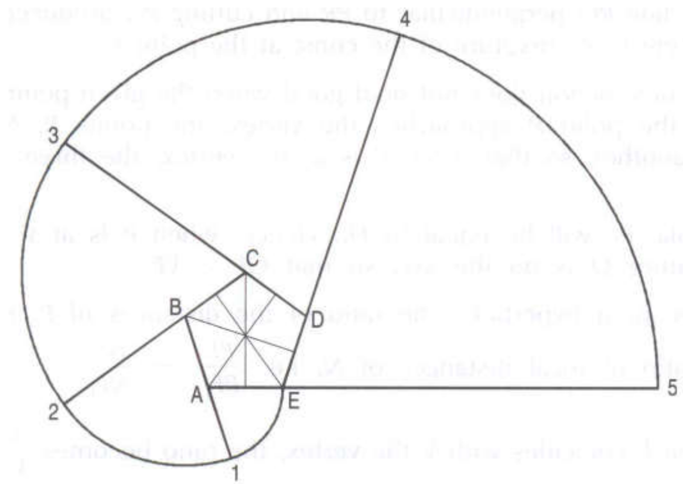
Involute: Involute is a curve traced out by an end of a piece of a thread unwound from a circle or a polygon.



https://en.wikipedia.org/wiki/Involute_gear

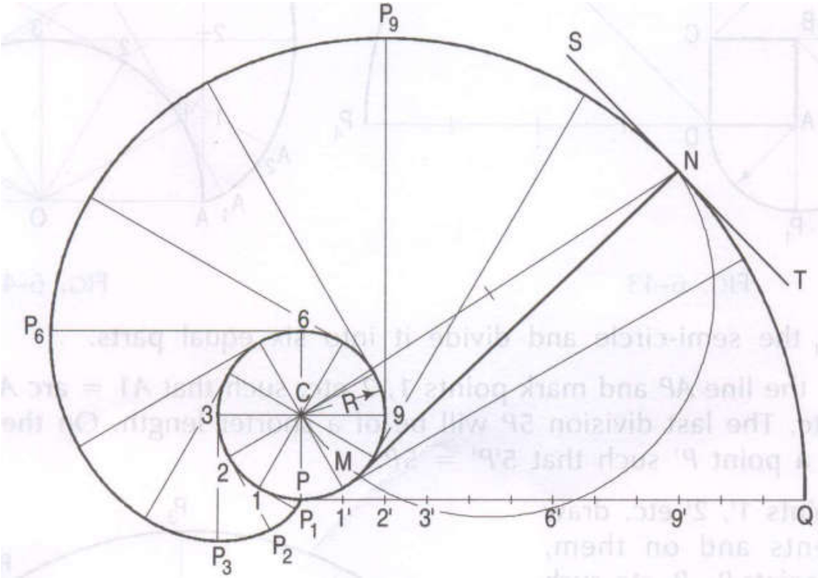






Ref: Engineering Drawing by N. D. Bhatt et. al

Involute to a Circle



With center C draw the given circle of radius R

Draw a line PQ, tangent to the circle and with length equal to the circumference of the circle

Divide the circle into 12 equal parts – label the points 1,2,…,12

Divide the line into 12 equal parts – label the points 1’,2’,…,12’

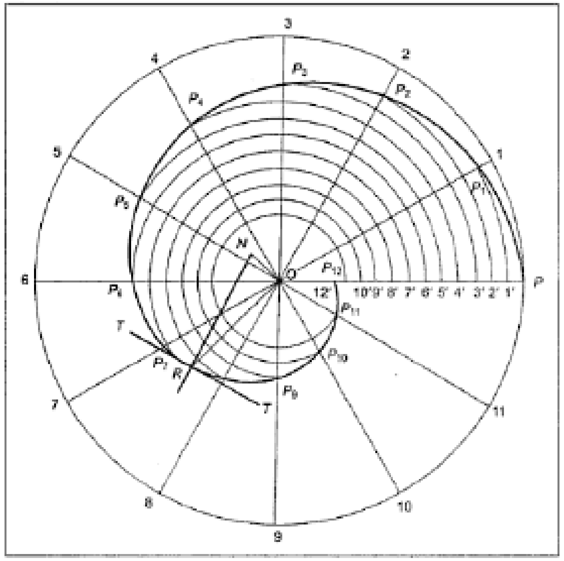
Draw tangents at points 1, 2, 3, and mark on them points P1, P2, P3, etc, such that $1P_1$=P1’, $2P_2$ = P2’, etc

Draw the involute through points P_1, P_2, P_3, etc

Spiral

Spiral: If a line rotates in a plane about one of its ends and if the at the same time, a point moves along the line continuously in one direction, the curve traced out by the moving point is called the spiral

Terminology:
Pole: The point about which the line rotates
Radius Vector: The line joining any point on the curve with the pole
Vectorial Angle: The angle between the radius vector and the line in the initial position
Convolution: Curve generated during one complete revolution of the straight line



Archimedian Spiral

https://en.wikipedia.org/wiki/Logarithmic_spiral

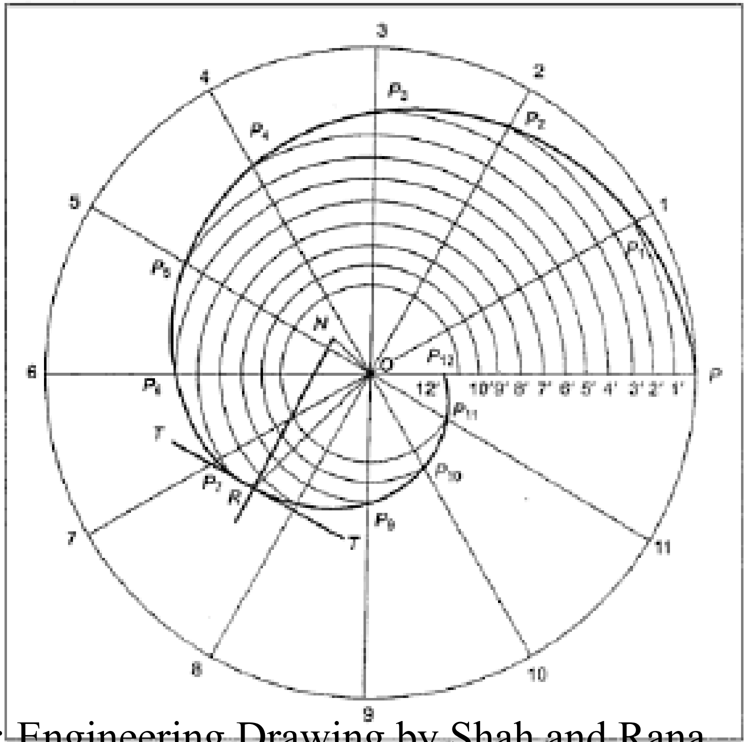
Logarithmic Spiral

Archimedian Spiral

Curve traced out by a pint moving in such a way that its movement towards or away from the pole is uniform with the increase of the vectorial angle from the starting line.

$$r = r_0 + K\theta$$

To draw an Archimedian spiral of one convolution with known shortest and longest radius vectors



Draw OP with length equal to the longest radius vector

Draw OP_{12} with length equal to the shortest radius vector

Draw a circle with center O and radius OP

Divide the circle into 12 equal parts – label the points 1, 2, 3, etc

Divide the line PP_{12} into 12 equal parts and label the points 1', 2', 3' etc

With center O and radius O1' mark point P_1 on the line O1

Similarly with center O and radius O2' mark point P_2 on the line O2

Points P_1, P_2, etc from the required spiral

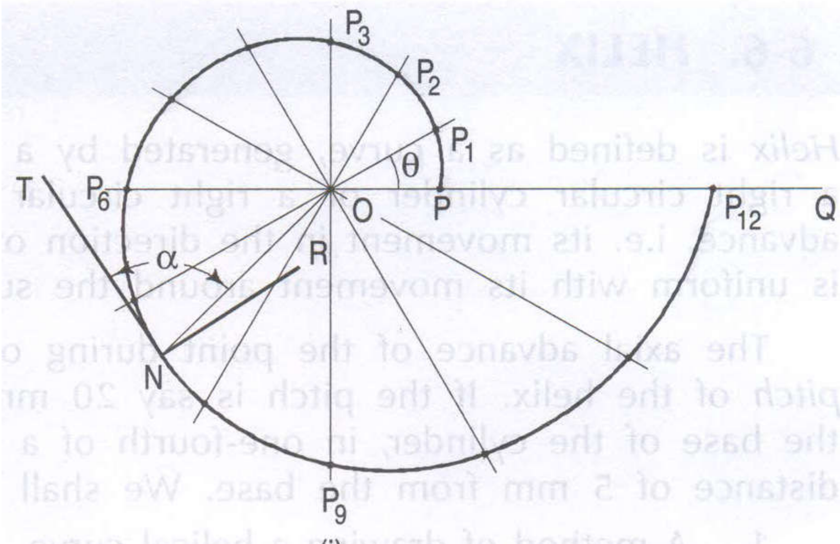
Ref: Engineering Drawing by Shah and Rana

Logarithmic Spiral

Growth spiral, equiangular spiral

If the lengths of successive radius vectors enclosing equal angles at the pole are in geometric progression, i.e., the ratio of the successive radius vectors is constant, a logarithmic spiral is generated.

$$r = ae^{b\theta}$$



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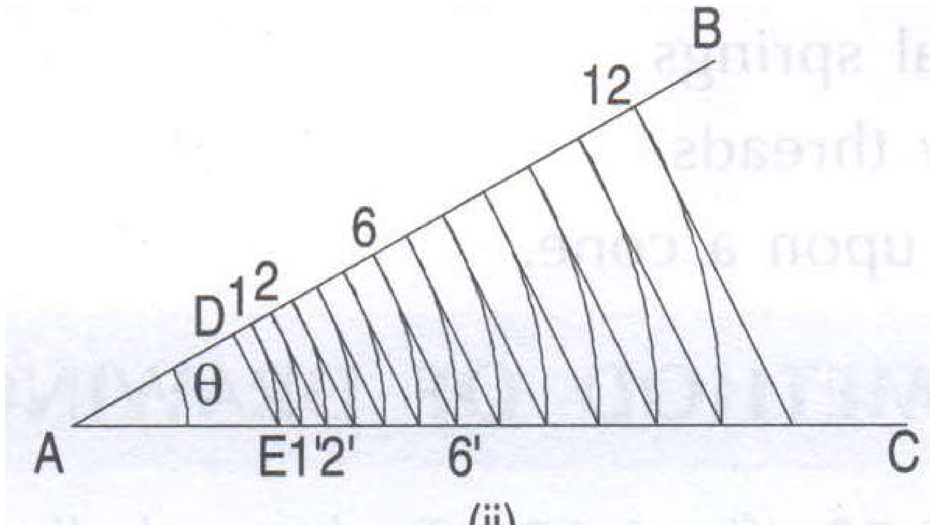

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Ref: Engineering Drawing by N. D. Bhatt et. al

Helix

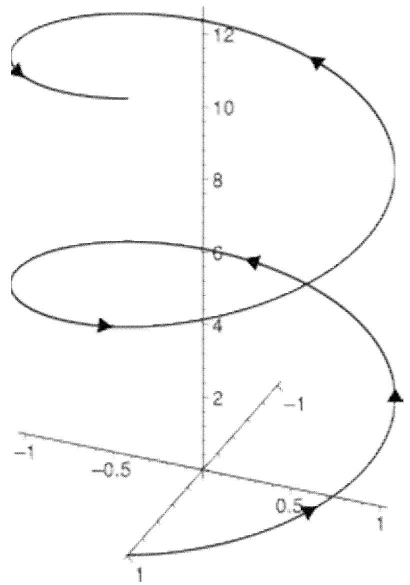
Curve generated by a point moving around the surface of a right circular cylinder or a right circular cone in such a way that its movement in the direction of the axis of the cylinder or the cone is uniform with the movement around the surface of the cylinder or cone. <u>Helix is a space curve</u>

Parametric equation of helix generated using a cylinder

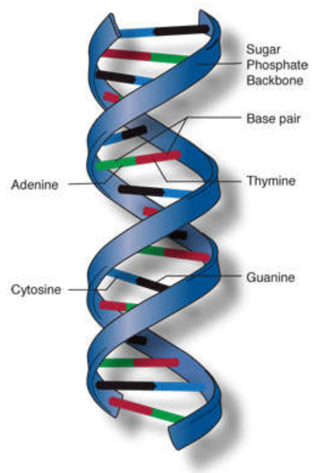
$$x = r\cos t, y = r\sin t\,, z = ct$$

<u>Terminology</u>

<u>Pitch</u>: The axial advance of the point during one complete revolution



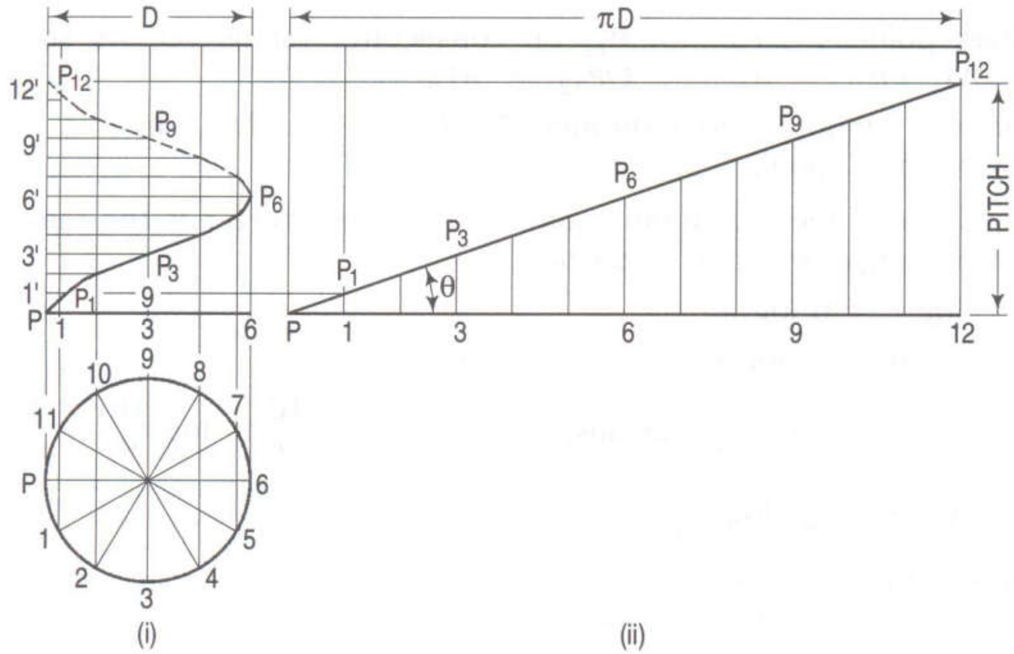
https://en.wikipedia.org/wiki/Helix



https://geneed.nlm.nih.gov/topic_subtopic.php?tid=15&sid=16

Helix

To draw a helix of one convolution given the pitch and the diameter of the cylinder



Draw the front view and the top view of the cylinder – The front view is a rectangle and the top view is a circle.
Divide the circle into 12 equal parts – label the points 1, 2, 3, etc
Draw the projections from points 1,2, etc in the front view.
Draw P12 equal to the circumference of the cylinder
Mark a length P12' equal to the pitch along the vertical side of the cylinder.
Joint P to P_{12}
Divide P12 into 12 equal parts and label the points 1,2, etc.
Draw vertical projections from 1,2, etc
Label the intersection of the projections with PP_{12} as P_1, P_2, etc

Draw horizontal projections from P_1, P_2, etc and find the intersection point with the Corresponding vertical projections on the front view. Label these points P_1, P_2, etc. Join P_1, P_2, etc to obtain the required helix

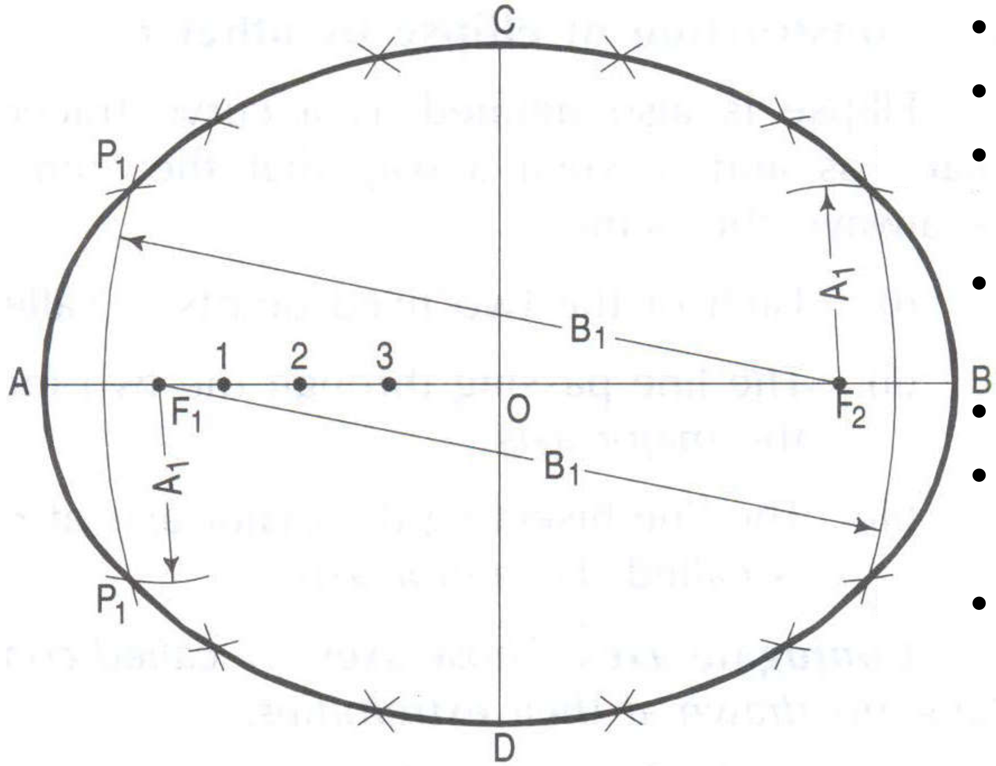
Ref: Engineering Drawing by N. D. Bhatt et. al

END

Refer to some additional slides given next

Ellipse – Arc of circle method

Construct an ellipse whose semi-major axis is 60mm and semi-minor axis 40mm using arcs of circles method.



- Draw AB of length 120mm
- Draw its perpendicular bisector
- Mark points C and D such that OC = OD = 40 mm.
- With centers C and D and radius 60mm locate the foci F_1 and F_2 , respectively
- On AB mark points 1, 2, 3, etc.
- With radius A1 and center F_1 draw arcs on either side of AB
- With radius B1 and center F_2 draw arcs intersecting the previously drawn arcs at P_1 and P_1'. These points lie on the ellipse as A1+B1 = $2a$

$CF_1 = CF_2 = a$

The sum of distance of a point P from the foci F_1 and F_2 has a fixed value $2a$, the length of the major axis

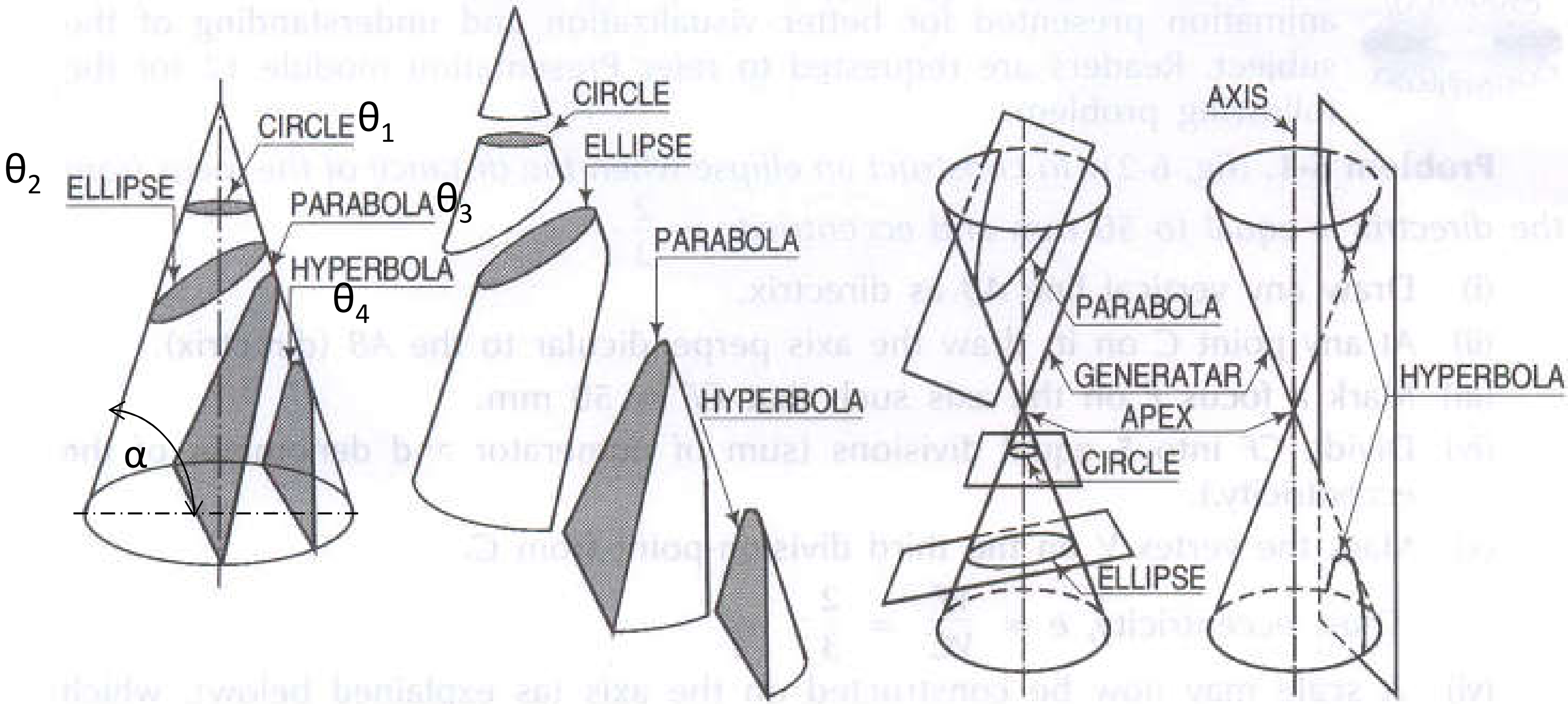
Ref: Engineering Drawing by N. D. Bhatt et. al

Parabola – Tangent Method

Read N. D. Bhat – Problem 6-11, page 111

Conic Sections (Conics)

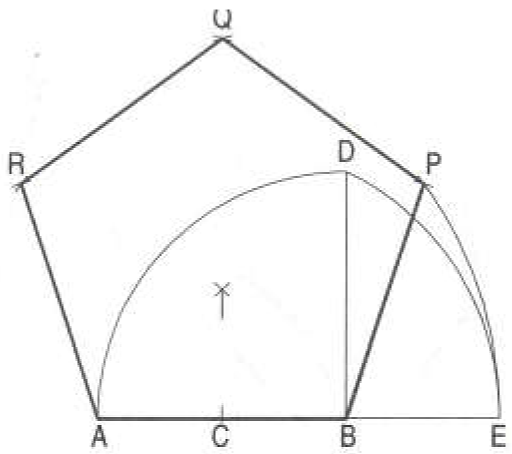
Conic section: curves formed by the intersection of a plane (cutting plane, CP) and a right circular cone



Ref: Engineering Drawing by N. D. Bhatt et. al

Important Geometric Constructions

Draw a pentagon given the length of the side



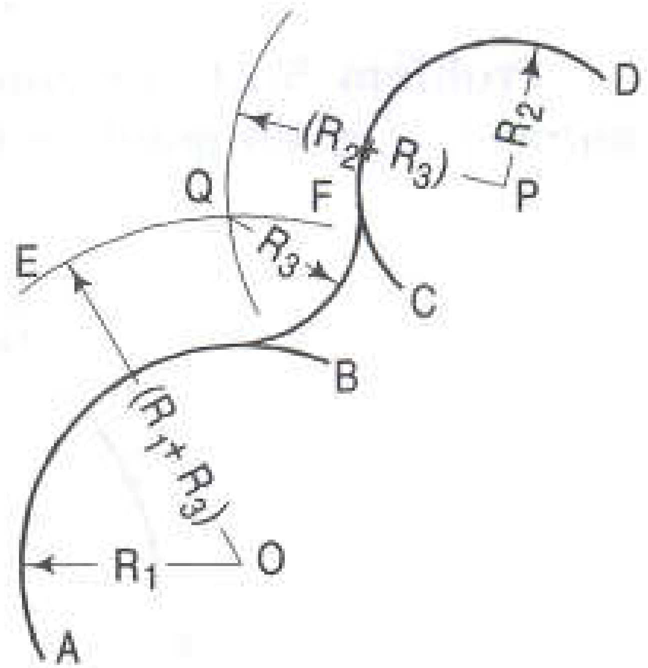
- Draw line AB of required length
- At B, draw a line perpendicular to AB and mark point D such that BD = AB
- Find C, the midpoint of AB
- With center C and radius CD draw an arc intersecting the extended line AB at E
- With center A and radius AE and center B and radius AB draw arcs intersecting at P. Join BP
- With center B and radius AE and center P and radius AB draw arcs intersecting at Q. Join PQ
- With center P and radius AE and center Q and radius AB draw arcs intersecting at R. Join QR and RA

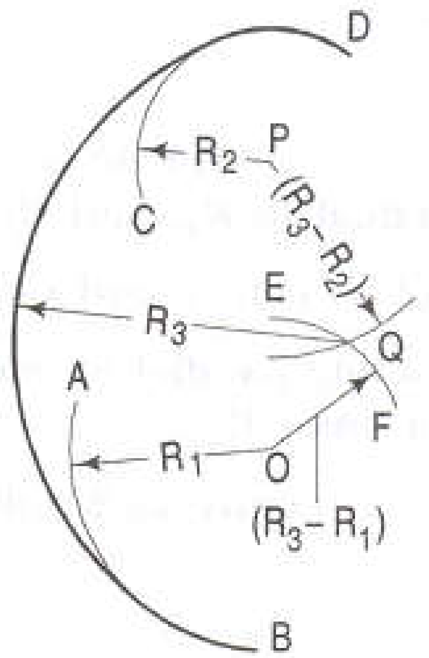
Based on the fact that ratio of the length of the diagonal of the pentagon (AP, AQ, BQ, BR and PR) to the length of the side is $(1+\sqrt{5})/2$. This number is referred to the golden ratio

Ref: Engineering Drawing by N. D. Bhatt et. al

Important Geometric Constructions

To draw an arc of given radius R_3 touching arc AB (center O and radius R_1) and arc CD (center P and radius R_2)





Aim: To locate a point Q which is equidistant (distance R_3) from both the arcs – AB and AC. The distances are measured along the radial lines from O and C.

Ref: Engineering Drawing by N. D. Bhatt et. al