PH108: Electricity & Magnetism

Weekly Quiz 2 - Continuity equation & Gauss's law

31 January, 2018

Answer

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\sigma}{\epsilon_0} \rho &= 0 \\ \rho(t) &= \rho(0) e^{-\frac{\sigma}{\epsilon_0} t} \\ \textbf{Solution} \end{aligned} \qquad \begin{bmatrix} \frac{1}{2} \text{ mark} \end{bmatrix}$$

$$\frac{\partial \rho}{\partial t} + \nabla . \mathbf{J} = 0$$

$$\mathbf{J} = \sigma \mathbf{E} \Longrightarrow \nabla . \mathbf{J} = \sigma \nabla . \mathbf{E} = \frac{\sigma}{\epsilon_0} \rho$$

$$\Longrightarrow \frac{\partial \rho}{\partial t} + \frac{\sigma}{\epsilon_0} \rho = 0$$

$$\frac{d\rho}{\rho} = -\frac{\sigma}{\epsilon_0} dt$$

$$\Longrightarrow \rho(t) = \rho(0) e^{-\frac{\sigma}{\epsilon_0} t}$$