MA 108 - Spring 2018
Tutorial Sheet 2

1. Solve the following ODE after finding an integrating factor.

   (a) $(27xy^2 + 8y^3)\ dx + (18x^2y + 12xy^2)\ dy = 0.$

   (b) $-y\ dx + (x^4 - x)\ dy = 0.$

   (c) $y \sin y\ dx + x(\sin y - y \cos y)\ dy = 0.$

   (d) $y(1 + 5 \ln |x|)\ dx + 4x \ln |x|\ dy = 0.$

   (e) $(3x^2y^3 - y^2 + y)\ dx + (-xy + 2x)\ dy = 0.$

   (f) $y\ dx + (2x - ye^y)\ dy = 0.$

   (g) $[a \cos(xy) - y \sin(xy)]\ \ dx + [b \cos(xy) - x \sin(xy)]\ \ dy = 0.$

2. Solve the following IVP.

   (a) $(4x^3y^2 - 6x^2y - 2x - 3)\ dx + (2x^4y - 2x^3)\ dy = 0, \quad y(1) = 3.$

   (b) $(y^3 - 1)e^x\ dx + 3y^2(e^x + 1)\ dy = 0, \ \ y(0) = 0.$

   (c) $(9x^2 + y - 1)\ dx - (4y - x)\ dy = 0, \ \ y(1) = 0.$

3. Based on the existence and uniqueness theorem, (i) find all the (x_0, y_0), for which theorem gives an interval on which the given IVP has a solution and (ii) an interval around x_0 for which it has a unique solution.

   (a) $y' = \dfrac{e^x + y}{x^2 + y^2}.$

   (b) $y' = (x^2 + y^2)y^{1/3}.$

   (c) $y' = \dfrac{1}{(\sin y)(x - 1)}$

4. Let $y' = 3x(y - 1)^{1/3}, \ \ y(x_0) = y_0.$

   (a) For what points (x_0, y_0) does IVP have a solution.

   (b) For what points (x_0, y_0) does IVP have a unique solution in an interval around x_0.

   (c) Observe that $y \equiv 1$ is a solution with initial value $y(0) = 1$.

   (d) Let $(x_0, y_0) = (0, 1)$. Find four solutions for the IVP which differ from each other for values of x in every open interval that contains $x_0 = 0$.

5. (a) From existence and uniqueness theorem, the IVP $y' = 3x(y - 1)^{1/3}, \quad y(3) = -7$ has a unique solution on some open interval that contains $x_0 = 3$. Determine the largest such open interval, and find the solution on this interval.

   (b) Find two solutions of the IVP defined on $(-\infty, \infty)$.

6. State on which rectangles the hypotheses of existence and uniqueness theorem for ODEs are satisfied.

   (a) $y' = \dfrac{\ln|xy|}{1 - x^2 + y^2}$.

   (b) $y' = \dfrac{1 + x^2}{3y - y^2}$.

7. Solve the IVP and determine how the interval in which the solution exists depends on the initial value y_0.

   (a) $y' + y^3 = 0, \quad y(0) = y_0$.

   (b) $y' = \dfrac{x^2}{y(1 + x^3)}, \quad y(0) = y_0$

8. Find ϕ_1, ϕ_2, ϕ_3, the first 3 Picard's iterations for following ODE's.

   (a) $y' = x + y^2, \ y(0) = 0$.

   (b) $y' = x^2 + y^2, \ y(0) = 0$.

   (c) $y' = x^2 + y, \ y(0) = 0$.