PH108 : Electricity & Magnetism : Tutorial 3

1. A charge distribution produces an electric field

$$\vec{E} = c\left(1 - e^{-r/r_0}\right)\frac{\hat{r}}{r^2}$$

where c and r_0 are constants. Find the net charge within a sphere of radius $r = r_0$

2. A sphere has a uniform volume charge density everywhere except inside an off-centre spherical cavity within. Show that the field inside the cavity is uniform.

3. Imagine a sphere of radius R filled with negative charge of uniform density, the total charge being equivalent to that of two electrons. Embed in this jelly two protons so that the total charge of the system is zero. Assume that the charge density is not altered because of the presence of the protons. Where must the protons be located so that the force on each on them is zero? This is a primitive model for the hydrogen molecule.

4. A charge Q is uniformly distributed in a spherical volume of radius R. Find the potential everywhere. Suppose the same charge Q is uniformly distributed on a very thin *shell* of radius R. How does the situation change?

5. An infinitely long cylinder with its axis along the z-axis has a volume charge density given by

$$\begin{aligned} \rho(r, \phi, z) &= \rho_0\left(1 - \frac{r}{a}\right) \qquad \text{for} \quad r < a \\ &= 0 \qquad\qquad\qquad \text{for} \quad r > a \end{aligned}$$

where ρ_0 is a constant.

   (a) Calculate electric field for $r < a$ and $r > a$

   (b) the potential difference between $r = a$ and $r = 0$, and between $r = 2a$ and $r = a$

6. Consider a point charge q moving with speed v along the z-axis ($i.e.$ $\theta = 0$ direction. The electric field due to such a moving charge is given by (do not worry how it is derived, you do not need to know that to solve this)

$$\vec{E}(r, \theta) = \frac{q}{4\pi\epsilon_0}\left(1 - \beta^2\right)\frac{1}{\left(1 - \beta^2\sin^2\theta\right)^{3/2}}\frac{\hat{r}}{r^2}$$

$\beta = \dfrac{v}{c}$, where c is the speed of light in vacuum. Notice that the field remains radial at all points.

   (a) Compared to the usual Coulomb field, at which location does it become stronger/weaker?

   (b) Calculate the flux of the electric field through the curved surface of the top hemisphere between ($\theta = 0$ to $\theta = \pi/2$) and the bottom hemisphere ($\theta = \pi/2$ to $\theta = \pi$). Which is larger? What is the sum total of the flux through the two hemispheres?

   (c) Calculate the flux through the forward cone between ($\theta = 0$ to $\theta = \pi/4$) and compare it with the value for $v = 0$. Does this increase or decrease?

Note: You would need to work out a substitution to evaluate the integral.