PH108 : Electricity & Magnetism : Tutorial 7

1. A cylinder of radius R and height L is positioned such that the origin is at the center and the z-axis is along the axis of the cylinder. The cylinder carries a frozen polarisation $\vec{P} = P_o\hat{z}$. Calculate $\vec{E}$ and $\vec{D}$ at all points on the z-axis. Are the quantities $\vec{D}$ and $\vec{E}$ proportional to each other inside the material?

2. Consider a dielectric with displacement vector given by $\vec{D_0} = \varepsilon_0\vec{E_0} + \vec{P}$, where all the three quantities are uniform.

   (a) A small spherical cavity is hollowed out of the material. Find the electric field $\vec{E}$ and $\vec{D}$ at the centre of the cavity.
   [Ans. $\vec{E} = \vec{E_0} + \frac{\vec{P}}{3\varepsilon_0}, \vec{D} = \vec{D_0} - \frac{2}{3}\vec{P}$]

   (b) Find the electric field $\vec{E}$ and $\vec{D}$ inside a needle-shaped cavity running parallel to the direction of polarization.[Ans. $\vec{E} = \vec{E_0}, \vec{D} = \vec{D_0} - \vec{P}$]

   (c) Find the electric field $\vec{E}$ and $\vec{D}$ for a thin wafer shaped cavity with its plane perpendicular to the direction of polarization. [Ans. $\vec{E} = \vec{E_0} + \frac{\vec{P}}{3\varepsilon_0}, \vec{D} = \vec{D_0}$]

3. Write down the boundary conditions that the field vectors must satisfy at the interface between two dielectrics. If the electric field vector $\vec{E_1}$ in a semi-infinite medium of permittivity ε_1 makes an angle θ with the interface with a second infinite dielectric medium of permittivity ε_2 , find the angle ϕ that the field vector $\vec{E_2}$ in the second medium makes with the interface.[Ans. $\tan\phi = (\varepsilon_2/\varepsilon_1)\tan\theta$]

4. A parallel plate capacitor of plate area A and separation d , as shown in the figure is completely filled with a dielectric slab of dielectric constant $\kappa = \dfrac{\varepsilon}{\varepsilon_0}$. It is connected to a constant voltage source (battery) V, through a switch S. The slab is then *slowly* pulled out. *Slowly* or *quasi-statically* means that the kinetic energy of the slab need not be considered in the process.

   (a) What is the work done in the process when the switch is kept closed during the process?

   (b) Now suppose the capacitor is charged to voltage V as before but then the switch is *opened*. The slab is pulled out slowly as before. Calculate the work done in this process. Why are the two results different, depending on the state of the switch?