PH108 : Electricity & Magnetism : Tutorial 11

1. (a) The amount of solar energy the upper atmosphere receives is approximately 1300 $\mathrm{W.m^{-2}}$. Calculate the rms value of the electric field due to this radiation assuming the entire radiation is concentrated in one wavelength.

   (b) All India Radio has a 100 kW transmitter roughly 10 km away from the IITB campus. Calculate the rms value of the electric field of the radio wave incident on a receiver here.

   For these estimates you can ignore any non uniformities of the atmosphere or the surrounding terrain. The values you get from the simple estimates are quite useful.

2. It has been proposed that a spacecraft may be propelled by harnessing the pressure of sunlight. Assume that a spacecraft is sufficiently away from earth and is under the sun's gravitational field alone. A very large and fully reflecting sail is oriented at right angles to the sun's rays and attached to the craft. How large must the sail be so that the craft can start sailing away from the sun?
   The sun radiates 10^{26} Watt and has a mass of 10^{30} kg, the total mass of the spacecraft and sail is 1500 kg.

3. The Maxwell's equations in "matter" are usually written as

$$\begin{aligned} \nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H} &= \frac{\partial\vec{D}}{\partial t} \end{aligned}$$

   where $\vec{B} = \mu\vec{H}$ and $\vec{D} = \epsilon\vec{E}$ in a linear medium.

   It is usually taken for granted that ϵ and μ are both positive. Suppose the linear relations hold but $\epsilon < 0$ and $\mu < 0$ (Don't worry why and how). An electromagnetic plane wave with wavevector $\vec{k}$ propagates in the medium. Find a relation connecting $\vec{E}$, $\vec{H}$, $\vec{k}$. Interpret the physical significance of what you have got.

4. The wave equation does not commit you to any specific set of co-ordinates just like the Laplace's equation. So it can be written out is cartesian/ spherical/ cylindrical co-ordinates using very similar procedures. Consider a scalar wave equation (e.g satisfied by the scalar potential or each component of the vector potential in free space in Lorenz gauge).

$$\nabla^2 V = \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2}$$

   Set up the problem in spherical polar co-ordinates. Assume that there is no θ or ϕ dependence. What is the equation satisfied by $V(r)$? Show that the most general form of the solution is

$$V(r) = const.\frac{1}{r}\left[F(r-ct) + G(r+ct)\right]$$

   Here F, G are some scalar functions.

5. Consider a propagating wave in free space given by

$$\vec{E} = E_0\frac{\sin\theta}{r}\left[\cos(kr-\omega t) - \frac{\sin(kr-\omega t)}{kr}\right]\hat{\phi}$$

(a) Calculate the magnetic field $\vec{B}$ and the Poynting vector $\vec{S}$. You would need to use the expansions of $\nabla\times$ in spherical co-ordinates.

(b) What is the total average power radiated by the source?

6. Consider a plane electromagnetic wave of wavelength λ propagating in vacuum. The electric field is given by

$$\vec{E} \quad = \quad E_0 \hat{x} \cos \frac{2\pi}{\lambda} \left(\frac{\sqrt{3}}{2} y + \frac{1}{2} z - ct \right)$$

(a) Find an expression for the instantaneous $\vec{H}$.

(b) If the amplitude of this field is H_0 , find the numerical value of the ratio $\dfrac{E_0}{H_0}$ in SI units. Your answer must be a pure number.

(c) Which commonly used quantity has the same dimension (unit) as this number ?