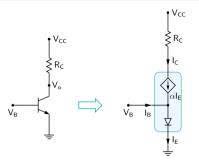
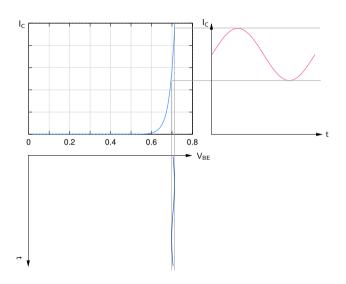
BJT Amplifiers: Part 1

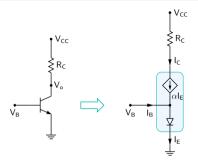


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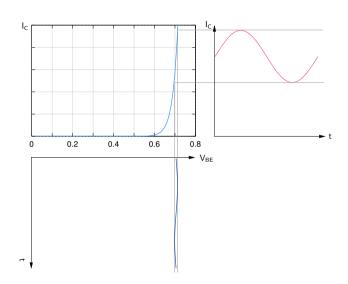
Department of Electrical Engineering Indian Institute of Technology Bombay

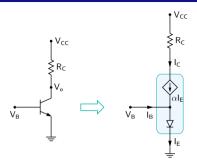




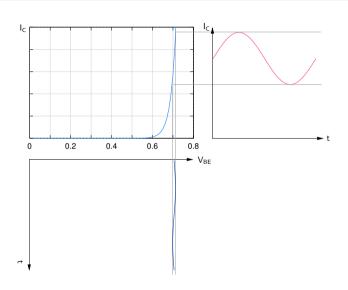


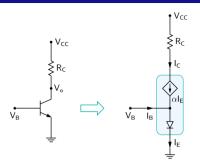
* In the active mode, I_C changes exponentially with V_{BE} : $I_C = \alpha_F I_{ES} \left[\exp(V_{BE}/V_T) - 1 \right]$



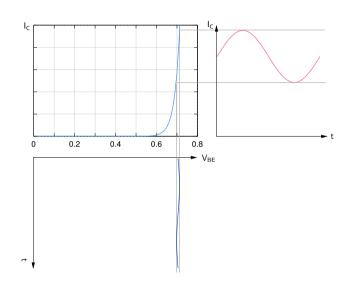


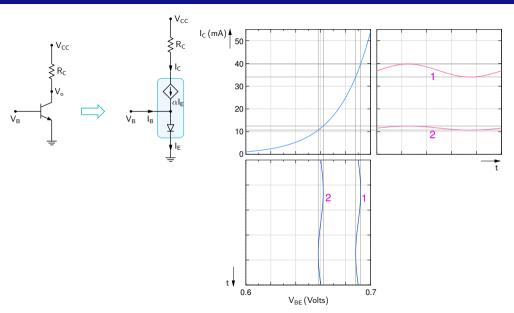
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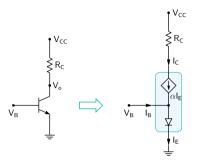




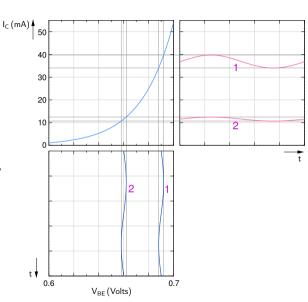
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- * Note that both the input (V_{BE}) and output (V_o) voltages have DC ("bias") components.

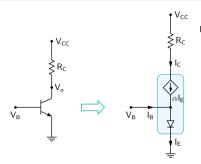




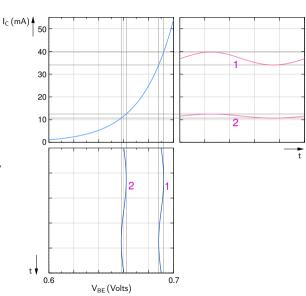


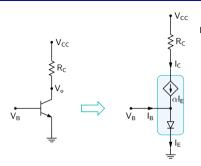
* The gain depends on the DC (bias) value of V_{BE} , the input voltage in this circuit.



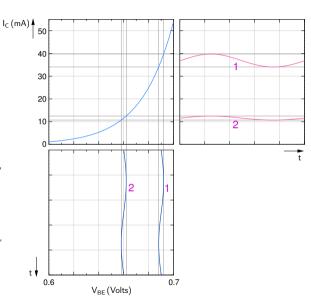


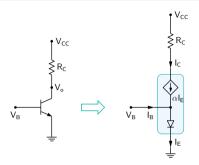
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- In practice, it is not possible to set the bias value of the input voltage to the desired value (e.g., 0.673 V).





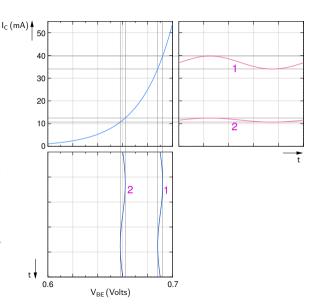
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- Even if we could set the input bias as desired, device-to-device variation, change in temperature, etc. would cause the gain to change.
 - → need a better biasing method.

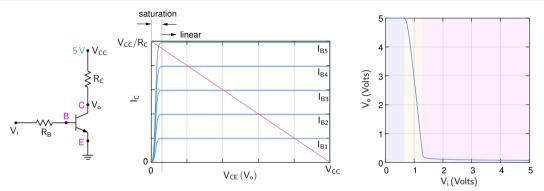




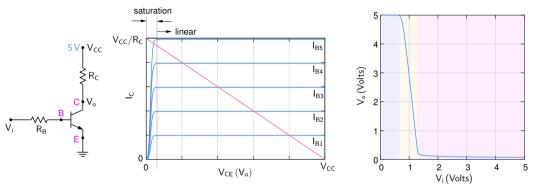
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 → need a better biasing method.
- * Biasing the transistor at a specific V_{BE} is equivalent to biasing it at a specific I_C .



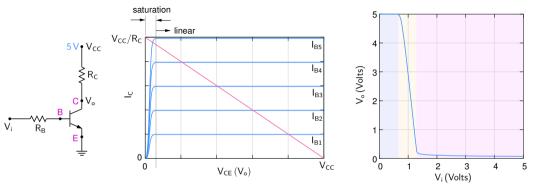


Consider a more realistic BJT amplifier circuit, with R_B added to limit the base current (and thus protect the transistor).



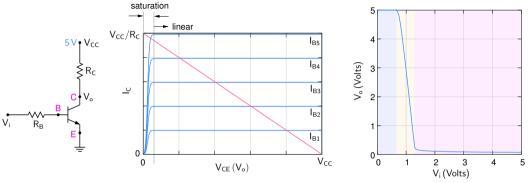
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* When $V_i < 0.7 \,\text{V}$, the B-E junction is not sufficiently forward biased, and the BJT is in the cut-off mode $(V_{BE} = V_i, \ V_{BC} = V_i - V_{CC})$



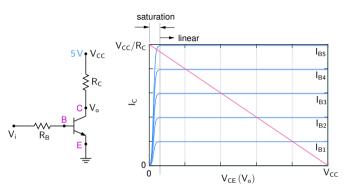
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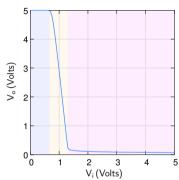
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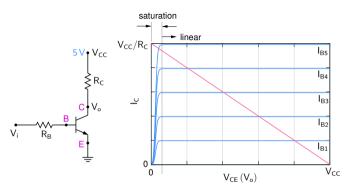


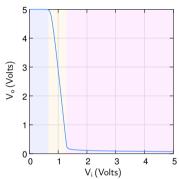
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- * As V_i is increased further, V_o reaches $V_{CE}^{\rm sat}$ (about 0.2 V), and the BJT enters the saturation region (both B-E and B-C junctions are forward biased).

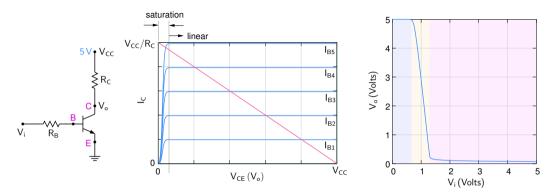




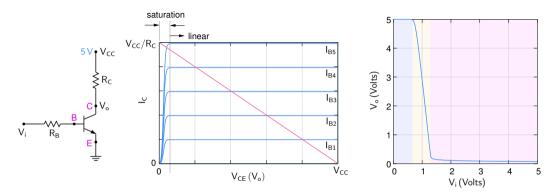




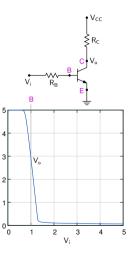
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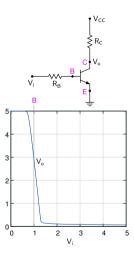


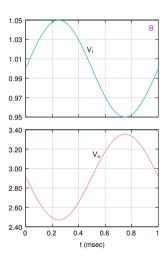
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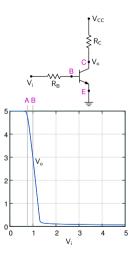


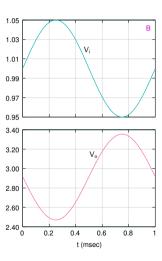
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- * Further, to get a large swing in V_o without distortion, the DC bias of V_i should be at the centre of the amplifying region, i.e., $V_i \approx 1 \ V$.

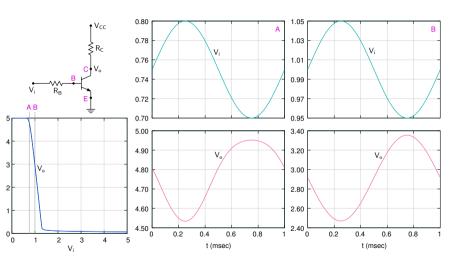


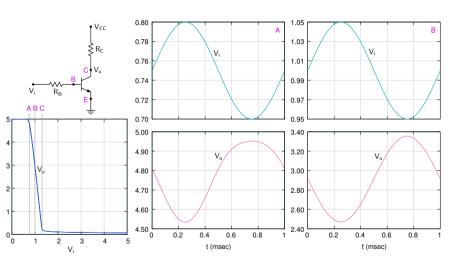


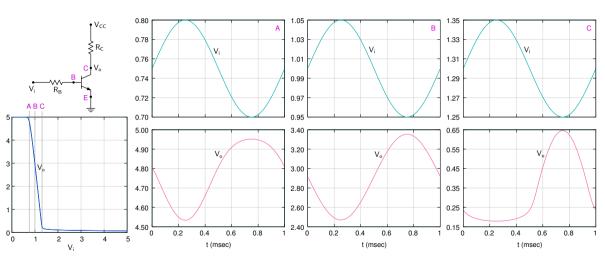


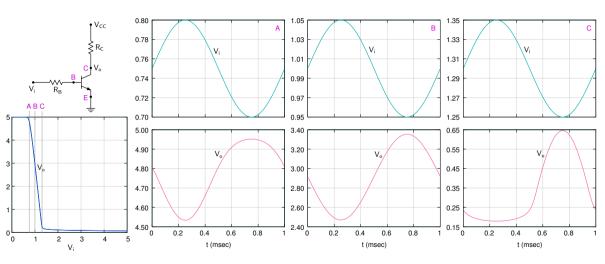




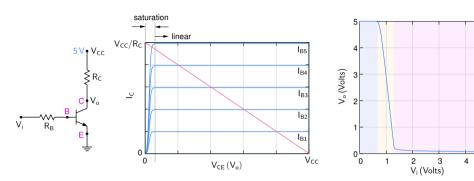






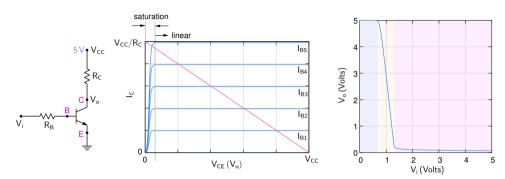


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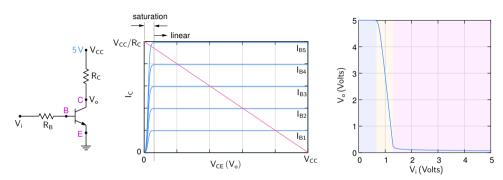


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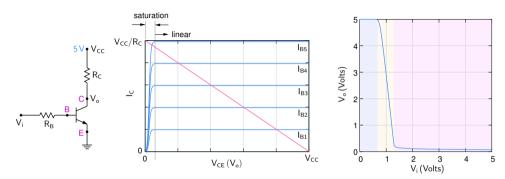
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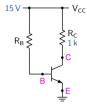
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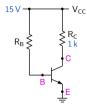
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- * The key challenges in realizing this amplifier in practice are
 - adjusting the input DC bias to ensure that the BJT remains in the linear (active) region with a certain bias value of V_{BE} (or I_C).
 - mixing the input DC bias with the signal voltage.
- * The first issue is addressed by using a suitable biasing scheme, and the second by using "coupling" capacitors.

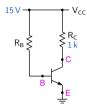


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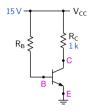
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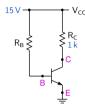


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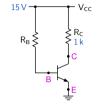
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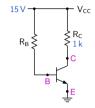
$$\rightarrow R_B = \frac{14.3 \, V}{33 \,\mu\text{A}} = 430 \,\text{k}\Omega \,.$$

BJT amplifier: a simple biasing scheme (continued)



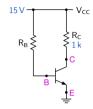
With $R_B=430\,\mathrm{k}$, we expect $I_C=3.3\,\mathrm{m}A$, assuming $\beta=100$.

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However, in practice, there is a substantial variation in the β value (even for the same transistor type). The manufacturer may specify the nominal value of β as 100, but the actual value may be 150, for example.



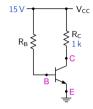
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With $\beta = 150$, the actual I_C is,

$$I_C = \beta imes rac{V_{CC} - V_{BE}}{R_B} = 150 imes rac{(15 - 0.7) \ V}{430 \ \mathrm{k}} = 5 \ \mathrm{mA} \, ,$$

which is significantly different than the intended value, viz., 3.3 mA.



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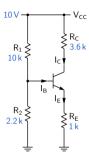
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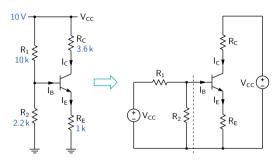
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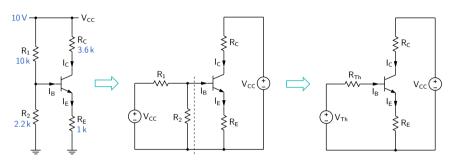
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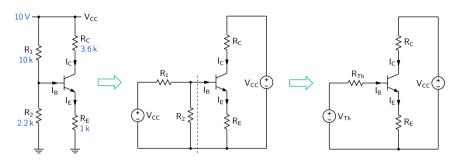
which is significantly different than the intended value, viz., 3.3 mA.

 \rightarrow need a biasing scheme which is not so sensitive to β .

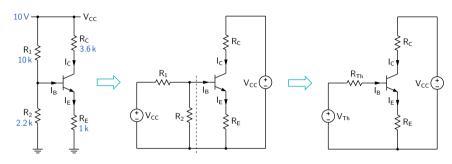








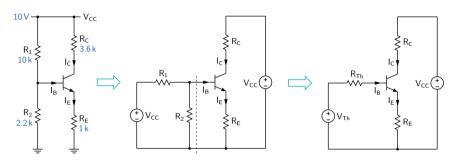
$$V_{Th} = rac{R_2}{R_1 + R_2} \; V_{CC} = rac{2.2 \, \mathrm{k}}{10 \, \mathrm{k} + 2.2 \, \mathrm{k}} imes 10 \; V = 1.8 \, V, \quad R_{Th} = R_1 \parallel R_2 = 1.8 \, \mathrm{k}$$



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Assuming the BJT to be in the active mode,

$$\mathsf{KVL} \colon V_{\mathit{Th}} = R_{\mathit{Th}} \, I_{\mathit{B}} + V_{\mathit{BE}} + R_{\mathit{E}} \, I_{\mathit{E}} = R_{\mathit{Th}} \, I_{\mathit{B}} + V_{\mathit{BE}} + (\beta + 1) \, I_{\mathit{B}} \, R_{\mathit{E}}$$

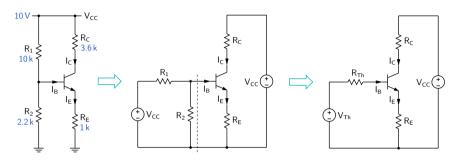


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$$\rightarrow I_{B} = \frac{V_{Th} - V_{BE}}{R_{Th} + \left(\beta + 1\right)R_{E}}, \quad I_{C} = \beta I_{B} = \frac{\beta \left(V_{Th} - V_{BE}\right)}{R_{Th} + \left(\beta + 1\right)R_{E}}.$$



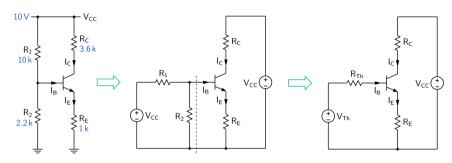
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$$\rightarrow \textit{I}_{\textit{B}} = \frac{\textit{V}_{\textit{Th}} - \textit{V}_{\textit{BE}}}{\textit{R}_{\textit{Th}} + (\beta + 1) \textit{R}_{\textit{E}}}, \quad \textit{I}_{\textit{C}} = \beta \textit{I}_{\textit{B}} = \frac{\beta \left(\textit{V}_{\textit{Th}} - \textit{V}_{\textit{BE}}\right)}{\textit{R}_{\textit{Th}} + (\beta + 1) \textit{R}_{\textit{E}}}.$$

For $\beta = 100$, $I_C = 1.07 \text{ m}A$.



$$V_{Th} = \frac{R_2}{R_1 + R_2} \ V_{CC} = \frac{2.2 \, \text{k}}{10 \, \text{k} + 2.2 \, \text{k}} \times 10 \ V = 1.8 \, V, \quad R_{Th} = R_1 \parallel R_2 = 1.8 \, \text{k}$$

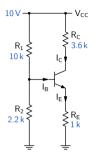
Assuming the BJT to be in the active mode,

KVL:
$$V_{Th} = R_{Th} I_B + V_{BE} + R_E I_E = R_{Th} I_B + V_{BE} + (\beta + 1) I_B R_E$$

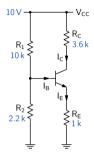
$$\rightarrow \textit{I}_{\textit{B}} = \frac{\textit{V}_{\textit{Th}} - \textit{V}_{\textit{BE}}}{\textit{R}_{\textit{Th}} + (\beta + 1) \textit{R}_{\textit{E}}}, \quad \textit{I}_{\textit{C}} = \beta \textit{I}_{\textit{B}} = \frac{\beta \left(\textit{V}_{\textit{Th}} - \textit{V}_{\textit{BE}}\right)}{\textit{R}_{\textit{Th}} + (\beta + 1) \textit{R}_{\textit{E}}}.$$

For
$$\beta = 100$$
, $I_C = 1.07 \,\text{m}A$.

For
$$\beta = 200$$
, $I_C = 1.085$ mA.

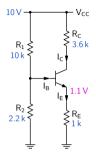


With $I_{C}=1.1\,\mathrm{m}\textit{A}$, the various DC ("bias") voltages are



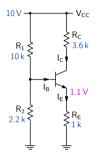
With $I_{C}=1.1\,\mathrm{m}\text{A}$, the various DC ("bias") voltages are

$$\label{eq:VE} \textit{V}_{\textit{E}} = \textit{I}_{\textit{E}} \, \textit{R}_{\textit{E}} \approx 1.1 \, \text{mA} \times 1 \, \text{k} = 1.1 \, \textit{V},$$



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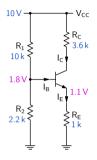
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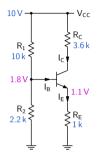
$$V_B = V_E + V_{BE} \approx 1.1 V + 0.7 V = 1.8 V$$
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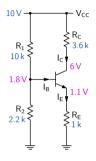


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,

$$V_C = V_{CC} - I_C R_C = 10 V - 1.1 \text{ mA} \times 3.6 \text{ k} \approx 6 V$$

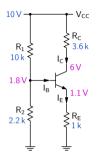


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$$\label{eq:Vc} \textit{V}_{\textit{C}} = \textit{V}_{\textit{CC}} - \textit{I}_{\textit{C}} \, \textit{R}_{\textit{C}} = 10 \; \textit{V} - 1.1 \, \textrm{mA} \times 3.6 \, \textrm{k} \approx 6 \; \textit{V},$$



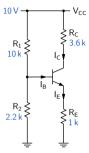
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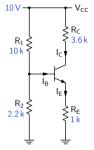
$$V_E = I_E R_E \approx 1.1 \,\mathrm{m}A \times 1 \,\mathrm{k} = 1.1 \,V,$$

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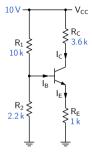
$$V_C = V_{CC} - I_C R_C = 10 V - 1.1 \,\text{mA} \times 3.6 \,\text{k} \approx 6 \,V,$$

$$V_{CE} = V_C - V_E = 6 - 1.1 = 4.9 V.$$



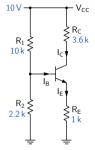


$$V_B = \frac{R_2}{R_1 + R_2} V_{CC} = \frac{2.2 \text{ k}}{10 \text{ k} + 2.2 \text{ k}} \times 10 V = 1.8 V.$$



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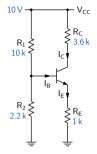
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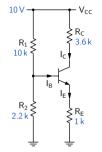


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$$I_C = \alpha I_E \approx I_E = 1.1 \, \text{mA}.$$



P- 2.2 k

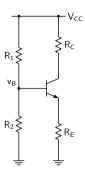
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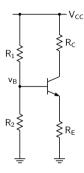
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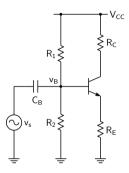
$$I_C = \alpha I_E \approx I_E = 1.1 \,\mathrm{m}A.$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 10 V - (3.6 \text{ k} \times 1.1 \text{ mA}) - (1 \text{ k} \times 1.1 \text{ mA}) \approx 5 V.$$

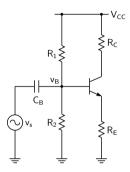




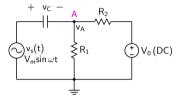
* As we have seen earlier, the input signal $v_s(t) = \widehat{V} \sin \omega t$ (for example) needs to be mixed with the desired bias value V_B so that the net voltage at the base is $v_B(t) = V_B + \widehat{V} \sin \omega t$.



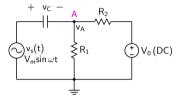
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- * This can be achieved by using a coupling capacitor C_B .
- * Let us consider a simple circuit to illustrate how a coupling capacitor works.

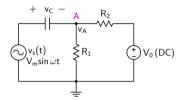


We are interested in the solution (currents and voltages) in the "sinusoidal steady state" when the exponential transients have vanished and each quantity x(t) is of the form X_0 (constant) + $X_m \sin(\omega t + \alpha)$.



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There are two ways to obtain the solution:

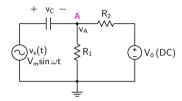


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(1) Solve the circuit equations directly:

$$\frac{v_A(t)}{R_1} + \frac{v_A(t) - V_0}{R_2} = C \frac{d}{dt} (v_s(t) - v_A(t)).$$



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(2) Use the DC circuit + AC circuit approach.

$$+ v_R(t) - \overline{\downarrow}_{R(t)} R$$

$$+ v_R(t) - \overline{i_R(t)} R$$

Let
$$v_R(t) = V_R + v_r(t)$$
 where $V_R = \text{constant}$, $v_r(t) = \widehat{V}_R \sin{(\omega t + \alpha)}$, $i_R(t) = I_R + i_r(t)$ where $I_R = \text{constant}$, $i_r(t) = \widehat{I}_R \sin{(\omega t + \alpha)}$.

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Since $v_R(t) = R \times i_R(t)$, we get $[V_R + v_r(t)] = R \times [I_R + i_r(t)]$.

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This relationship can be split into two:

$$V_R = R \times I_R$$
, and $v_r(t) = R \times i_r(t)$.

Resistor in sinusoidal steady state

$$+ v_R(t) - \overline{i_R(t)} R$$

Let
$$v_R(t) = V_R + v_r(t)$$
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$$V_R = R \times I_R$$
, and $v_r(t) = R \times i_r(t)$.

In other words, a resistor can be described by

$$\begin{array}{c|c} + & \mathsf{V}_{\mathsf{R}} & - \\ \hline \bullet & & \\ \hline \mathsf{I}_{\mathsf{R}} & \mathsf{R} \\ \hline \mathsf{DC} \\ \end{array} \qquad \begin{array}{c|c} + & \mathsf{v}_{\mathsf{r}}(\mathsf{t}) & - \\ \hline \bullet & & \\ \hline \mathsf{i}_{\mathsf{r}}(\mathsf{t}) & \mathsf{R} \\ \hline \mathsf{AC} \\ \end{array}$$



$$+ v_C(t) - \overline{\downarrow}_{i_C(t)} C$$

Let
$$v_C(t) = V_C + v_c(t)$$
 where $V_C = \text{constant}$, $v_c(t) = \widehat{V}_C \sin(\omega t + \alpha)$, $i_C(t) = I_C + i_c(t)$ where $I_C = \text{constant}$, $i_C(t) = \widehat{I}_C \sin(\omega t + \beta)$.

$$+ v_C(t) - \overline{\downarrow}_{i_C(t)} C$$

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$$+ v_C(t) - \overline{\downarrow}$$
 $i_C(t) C$

Let
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This relationship can be split into two:

$$I_C = C \frac{dV_C}{dt} = 0$$
, and $i_c(t) = C \frac{dv_c}{dt}$.

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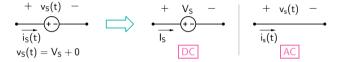
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In other words, a capacitor can be described by

Voltage sources in sinusoidal steady state

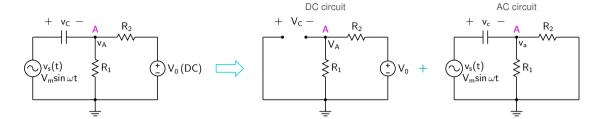
DC voltage source:

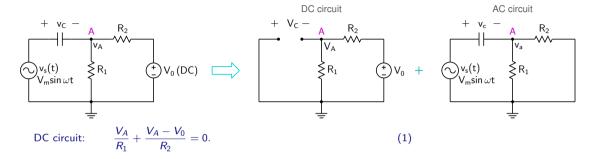


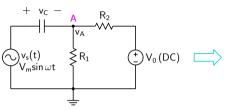
Voltage sources in sinusoidal steady state

DC voltage source:

AC voltage source:

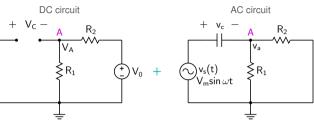


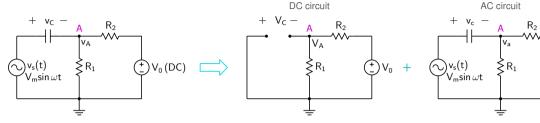






AC circuit:
$$\frac{v_a}{R_1} + \frac{v_a}{R_2} = C \frac{d}{dt} (v_s - v_a). \tag{2}$$

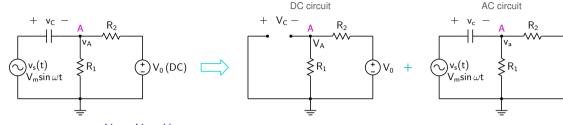




DC circuit:
$$\frac{V_A}{R_1} + \frac{V_A - V_0}{R_2} = 0.$$
 (1)

AC circuit:
$$\frac{v_a}{R_1} + \frac{v_a}{R_2} = C \frac{d}{dt} (v_s - v_a). \tag{2}$$

Adding (1) and (2), we get
$$\frac{V_A + v_a}{R_1} + \frac{V_A + v_a - V_0}{R_2} = C \frac{d}{dt} (v_s - v_a)$$
. (3)



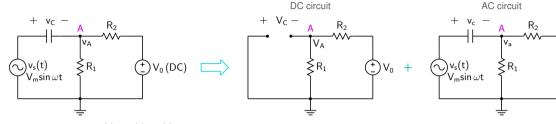
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Compare with the equation obtained directly from the original circuit:

$$\frac{v_A}{R_1} + \frac{v_A - V_0}{R_2} = C \frac{d}{dt} (v_s - v_A). \tag{4}$$



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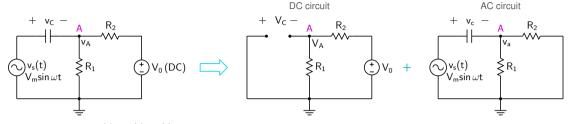
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Eqs. (3) and (4) are identical since $v_A = V_A + v_a$.



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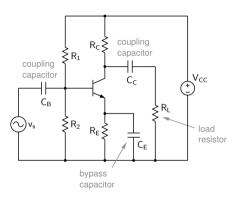
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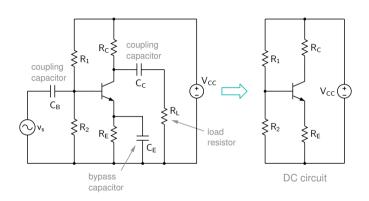
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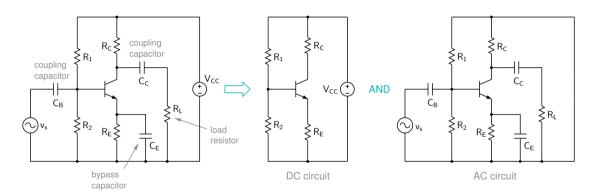
$$\frac{v_A}{R_1} + \frac{v_A - V_0}{R_2} = C \frac{d}{dt} (v_s - v_A). \tag{4}$$

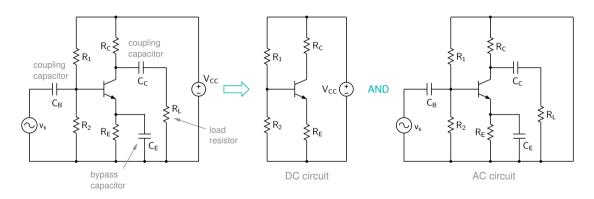
Eqs. (3) and (4) are identical since $v_A = V_A + v_a$.

 \rightarrow Instead of computing $v_A(t)$ directly, we can compute V_A and $v_a(t)$ separately, and then use $v_A(t) = V_A + v_a(t)$.

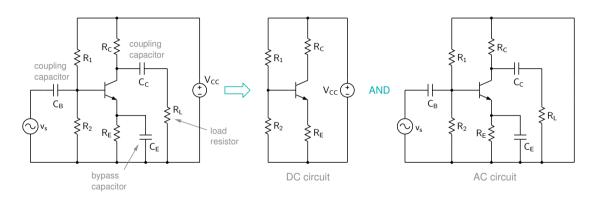




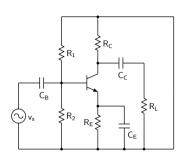


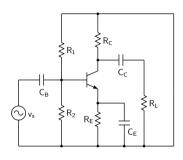


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- * This enables us to bias the amplifier without worrying about what load it is going to drive.





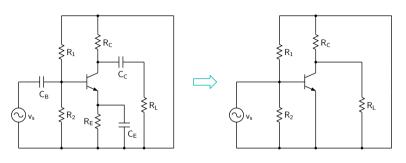
* The coupling and bypass capacitors are "large" (typically, a few μF), and at frequencies of interest, their impedance is small.

For example, for $C=10\,\mu\text{F}$, $f=1\,\text{kHz}$,

$$Z_C = \frac{1}{2\pi \times 10^3 \times 10 \times 10^{-6}} = 16 \,\Omega$$

which is much smaller than typical values of R_1 , R_2 , R_C , R_E (a few $k\Omega$).

 \Rightarrow C_B , C_C , C_E can be replaced by short circuits at the frequencies of interest.



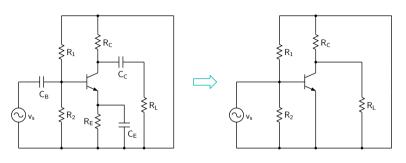
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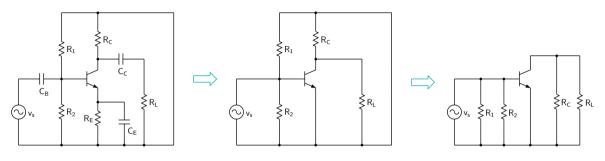
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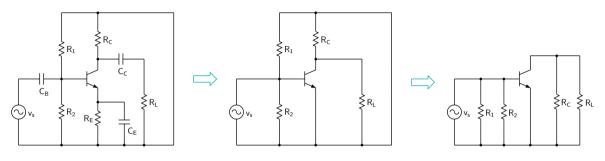
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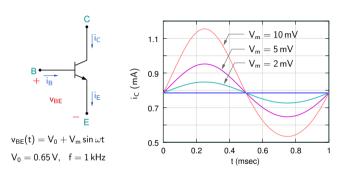
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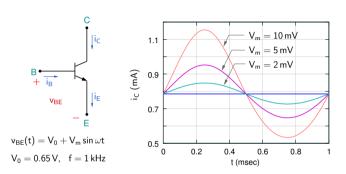
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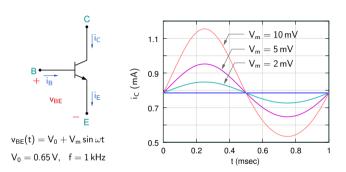
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- * The circuit can be re-drawn in a more friendly format.
- * We now need to figure out the AC description of a BJT.

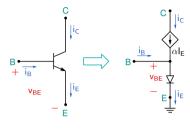




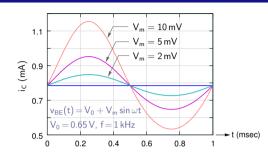
* As the v_{BE} amplitude increases, the shape of $i_C(t)$ deviates from a sinusoid ightarrow distortion.

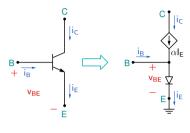


- * As the v_{BE} amplitude increases, the shape of $i_C(t)$ deviates from a sinusoid \rightarrow distortion.
- * If $v_{be}(t)$, i.e., the time-varying part of v_{BE} , is kept small, i_C varies linearly with v_{BE} . How small? Let us look at this in more detail.



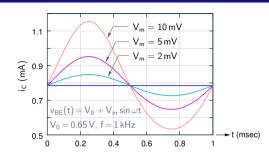
Let $v_{BE}(t) = V_{BE} + v_{be}(t)$ (bias+signal), and $i_C(t) = I_C + i_c(t)$.

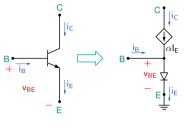




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Assuming active mode,
$$i_C(t) = \alpha i_E(t) = \alpha I_{ES} \left[\exp \left(\frac{v_{BE}(t)}{V_T} \right) - 1 \right].$$



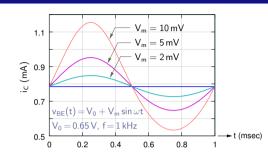


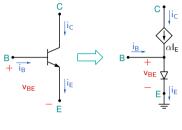
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Since the B-E junction is forward-biased, $\exp\left(\frac{v_{BE}(t)}{V_T}\right)\gg 1$, and we get

$$i_{\mathcal{C}}(t) = \alpha \, \mathit{I}_{\mathit{ES}} \, \exp\left(\frac{\mathit{v}_{\mathit{BE}}(t)}{\mathit{V}_{\mathit{T}}}\right) = \alpha \, \mathit{I}_{\mathit{ES}} \, \exp\left(\frac{\mathit{V}_{\mathit{BE}} + \mathit{v}_{\mathit{be}}(t)}{\mathit{V}_{\mathit{T}}}\right) = \alpha \, \mathit{I}_{\mathit{ES}} \, \exp\left(\frac{\mathit{v}_{\mathit{BE}}}{\mathit{V}_{\mathit{T}}}\right) \times \exp\left(\frac{\mathit{v}_{\mathit{be}}(t)}{\mathit{V}_{\mathit{T}}}\right).$$





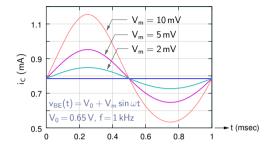
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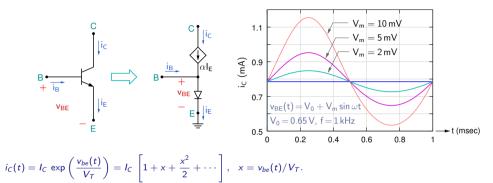
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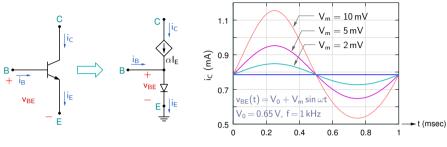
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If
$$v_{be}(t) = 0$$
, $i_C(t) = I_C$ (the bias value of i_C), i.e., $I_C = \alpha I_{ES} \exp\left(\frac{V_{BE}}{V_T}\right)$
 $\Rightarrow i_C(t) = I_C \exp\left(\frac{v_{be}(t)}{V_C}\right)$.



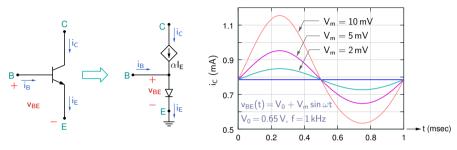




$$i_C(t) = I_C \exp\left(\frac{v_{be}(t)}{V_T}\right) = I_C \left[1 + x + \frac{x^2}{2} + \cdots\right], \quad x = v_{be}(t)/V_T.$$

If x is small, i.e., if the amplitude of $v_{be}(t)$ is small compared to the thermal voltage V_T , we get

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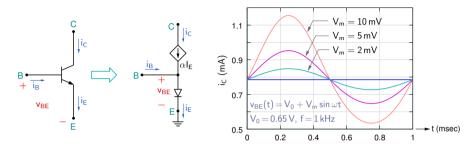


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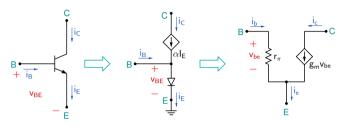
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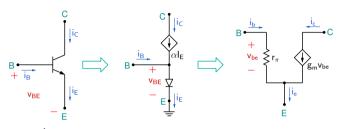
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The relationship, $i_c(t)=\frac{I_C}{V_T}\,v_{be}(t)$ can be represented by a VCCS, $i_c(t)=g_m\,v_{be}(t)$, where $g_m=I_C/V_T$ is the "transconductance."



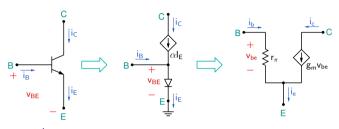
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For the base current, we have,

$$i_B(t) = I_B + i_b(t) = \frac{1}{\beta} [I_C + i_c(t)]$$

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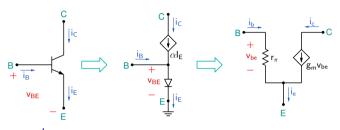
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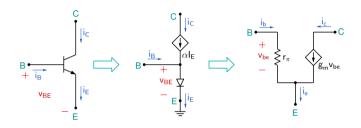
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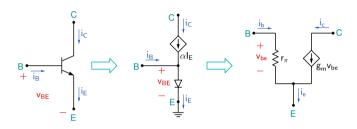
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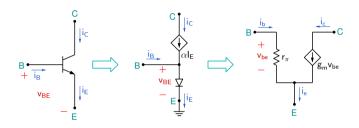
The resulting model is called the π -model for small-signal description of a BJT.



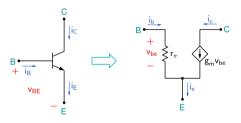
* The transconductance g_m depends on the biasing of the BJT, since $g_m = I_C/V_T$. For $I_C = 1\,\text{mA}$, $V_T \approx 25\,\text{mV}$ (room temperature), $g_m = 1\,\text{mA}/25\,\text{mV} = 40\,\text{m}$ (milli-mho or milli-siemens).



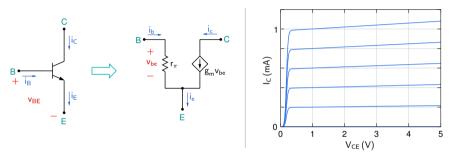
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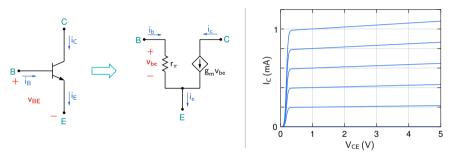
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- * Note that the small-signal model is valid only for small v_{be} (small compared to V_T).



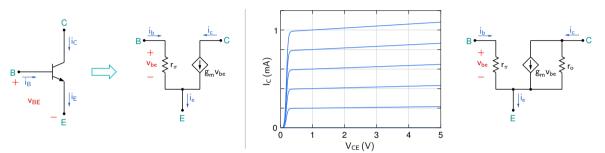
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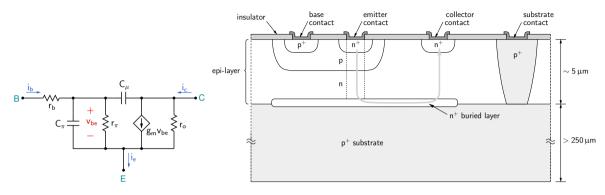
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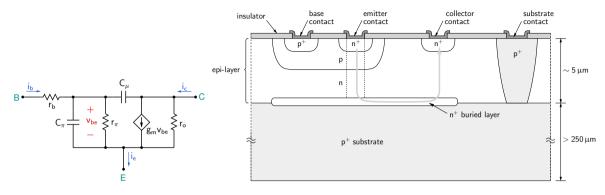


* A few other components are required to make the small-signal model complete:

 r_b : base spreading resistance

 C_{π} : base charging capacitance + B-E junction capacitance

 C_{μ} : B-C junction capacitance



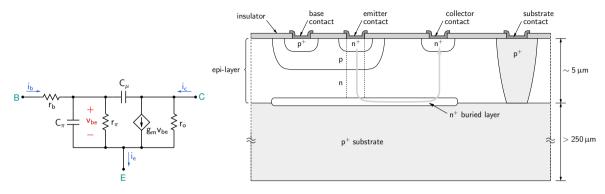
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- * The capacitances are typically in the pF range. At low frequencies, $1/\omega C$ is large, and the capacitances can be replaced by open circuits.
- * Note that the small-signal models we have described are valid in the active region only.