PH108 : Electricity & Magnetism : Tutorial 1

1. For a circle given by the equation, $r = 2R\cos\theta$, where R is the radius of the circle and (r, θ) are the polar coordinates. Calculate the following.

   (a) area of the circle.

   (b) centroid of the semi-circular area in the first quadrant.

   (c) length and centroid of the semi-circular arc in the first quadrant.

   (d) area common to the given circle and the circle given by $r = R$

2. The area bounded by the curve $r = 2R\cos\theta$ has a surface charge density $\sigma(r, \theta) = \sigma_0 \frac{r}{R} \sin^4\theta$. What is the total amount of charge?

3. Consider a sphere of *unit* radius centered at the origin. Show that the area of the surface enclosed between $\theta = 0$ and $\theta = \alpha$ is given by $2\pi(1 - \cos\alpha)$

4. Consider the section of a cone described by the equation $z^2 = x^2 + y^2$, between the planes $z = 1$ and $z = 2$. Determine the volume of this part using

   (a) spherical polar co-ordinates,

   (b) cylindrical co-ordinates.

5. Compute the divergence of the vector field given by:

$$\vec{v} \;=\; r\cos\theta\hat{r} \;+\; r\sin\theta\hat{\theta} \;+\; r\sin\theta\cos\phi\hat{\phi}$$

   Check the divergence theorem for this using the volume of an inverted hemisphere of radius R, resting on the xy plane and centered at the origin.

6. A vector field is given by

$$\vec{v} \;=\; xy\hat{i} \;+\; 2yz\hat{j} \;+\; 3z\hat{k}$$

   Verify the validity of Stoke's theorem, using a triangular area with vertices $(0, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 2)$

7. A vector field is given by

$$\vec{v} \;=\; ay\hat{i} + bx\hat{j}$$

   where a, b are constants.

   (a) Find the line integral of this field over a circular path of radius R, lying in the xy plane and centered at the origin using (i) plane polar and then (ii) Cartesian system.

   (b) Imagine a right circular cylinder of length L with its axis parallel to the z axis standing on the circle. Use cylindrical co-ordinate system to show that the theorem is valid over its surface.

8. A ship sails from the southernmost point of India (6.75^{o}N, 93.84^{o}E) to the southernmost point of Africa (34.5^{o}S, 20.00^{o}E) following the shortest possible path.

   (a) Given that the radius of the earth is 6400 km, what is the distance it has covered?

   (b) If instead of sailing, one had travelled in an aeroplane - by what percentage would the shortest possible distance change?