BB101 Quiz2 : Solutions

① (i) Diffusion constant can be obtained from relation

$$\gamma D = K_B T \quad \text{or} \quad 6\pi\eta r D = K_B T$$

$$\Rightarrow \quad D = \frac{K_B T}{6\pi\eta r} = \frac{1.38\times10^{-23}\times300}{6\times3.14\times10^{-3}\times3\times10^{-9}}$$

$$= \frac{\cancel{3}\times1.38\times10^{-23+2}}{6\times\cancel{3}\times3.14\times10^{-9-3}}$$

$$= \frac{1.38\times10^{-21}}{18.84\times10^{-12}}$$

$$= \frac{138\times10^{-23}\times10^{12}}{18.84}$$

$$= 7.325\times10^{-11}\,\frac{m^2}{s}$$

$$= 73.25\times10^{-12}\,\frac{m^2}{s} \simeq 73.25\,\frac{\mu m^2}{s}$$

(ii) Drag coefficient is given by

$$\gamma = 6\pi\eta r = 6\times3.14\times10^{-3}\times10^{-9}\times3$$

$$= 3\times18.84\times10^{-12} = 18.84\times10^{-12}\,\frac{N.s}{m}$$

$$= 3\times18.84\,\frac{pN.s}{m} = 56.52\,\frac{pN.s}{m}$$

(iii) Distance travelled by means of diffusion is related to diffusion constant and time by following relation

$$x^2 = 2Dt$$

$$\text{or} \quad x = \sqrt{2Dt}$$

$$= \sqrt{2\times7.325\times10^{-11}\times6\times24\times60\times60}\ m$$

$$= \sqrt{7.595\times10^{-11}\times10^{6}}\ m$$

$$= \sqrt{7.595\times10^{-5}}\ m = \sqrt{75.95\times10^{-6}}\ m$$

$$= 8.715\times10^{-3}\,m \simeq 8.715\times10^{-3}\,m$$

$$\simeq 8.715\ mm$$

2 (i) Partition function is

$$Z = e^{-\frac{0}{K_BT}} + e^{-\frac{0}{K_BT}} + e^{-\frac{4.14}{K_BT}} + e^{-\frac{4.14}{K_BT}}$$

$\therefore$ At $T = 300K$

$$Z = 1 + 1 + e^{-1} + e^{-1}$$
$$= 2 + 2e^{-1}$$

(ii) In the limit $T \to 0$

$$Z = 1 + 1 + e^{-\infty} + e^{-\infty}$$
$$= 2$$

(iii) In the limit $T \to \infty$

$$Z = 1 + 1 + e^{-0} + e^{-0}$$
$$= 4$$

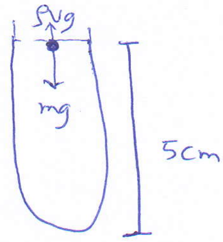
(iv) $P_A = \dfrac{e^{-\frac{0}{K_BT}}}{Z} = \dfrac{1}{2}$

(v) $P_D = \dfrac{e^{-\frac{4.14}{K_BT}}}{Z} = \dfrac{0}{2} = 0$

(vi) $P_A = \dfrac{e^{-\frac{0}{K_BT}}}{Z} = \dfrac{1}{4}$

(vi) $P_D = \dfrac{e^{-\frac{4.14}{K_BT}}}{Z} = \dfrac{e^{-0}}{4} = \dfrac{1}{4}$

③ The equation of motion is given by



$$mg - \rho V g = rv$$

$$19\rho V g - \rho V g = rv$$

$$18\rho V g = rv$$

$$\Rightarrow \quad v = \frac{18\rho V g}{r} = \frac{18\rho V g}{6\pi\eta r} = \frac{\cancel{18}\,\rho\, 4\cancel{\pi} \cancel{r^3}^{2} g}{\cancel{3}\times\cancel{6}\cancel{\pi}\eta\cancel{r}}$$

$$= \frac{4\rho r^2 g}{\eta}$$

∴ Therefore time taken is given by

$$t = \frac{x}{v} = \frac{\cancel{4}\times10^{-2}\times10^{-3}}{\cancel{4}\times1000\times10^{-9}\times10^{-9}\times10}$$

$$= \frac{10^{-5}}{10^{3}\times10^{-18}\times10}$$

$$= 10^{-5-3+18-1}$$

$$= 10^{9} \text{ seconds}$$

(4) The equation of motion for $t \leq \tau$ is given by

$$kx + rv = F$$

$$kx + r\frac{dx}{dt} = F$$

$$r\frac{dx}{dt} = F - kx$$

$$\Rightarrow \int \frac{dx}{F-kx} = \frac{1}{r}\int dt$$

at $t=0, x=0$

$$-\left[\frac{\ln(F-kx)}{k}\right]_0^x = \frac{1}{r}\left[t\right]_0^t$$

$$-\frac{\ln(F-kx) + \ln F}{k} = \frac{1}{r}t$$

$$+\ln\frac{F}{F-kx} = \frac{k}{r}t$$

$$\ln\frac{F}{F-kx} = \frac{k}{r}t$$

$$\Rightarrow \ln\frac{F-kx}{F} = -\frac{k}{r}t$$

$$\Rightarrow \frac{F-kx}{F} = e^{-\frac{k}{r}t}$$

$$\Rightarrow F-kx = Fe^{-\frac{k}{r}t}$$

$$\Rightarrow \boxed{x = \frac{F}{k}\left(1-e^{-\frac{k}{r}t}\right)}$$

$\Rightarrow$ At $t=\tau$ $\quad x(\tau) = F\left(1-e^{-\frac{k}{r}\tau}\right)$

Equation of motion for $t > \tau$ is given by

$$kx + rv = 0$$

$$\Rightarrow kx + r\frac{dx}{dt} = 0$$

$$\Rightarrow r\frac{dx}{dt} = -kx$$

$$\Rightarrow \quad \frac{dn}{n} = -\frac{k}{\gamma} dt$$

$$\Rightarrow \quad \int_{n(\tau)}^{n} \frac{dn}{n} = -\frac{k}{\gamma} \int_{0}^{t} dt$$

$$\left[\ln n\right]_{n(\tau)}^{n} = -\frac{k}{\gamma} \left[t\right]_{\tau}^{t}$$

$$\ln \frac{n}{n(\tau)} = -\frac{k}{\gamma}(t-\tau)$$

$$\Rightarrow \quad n = n(\tau)\, e^{-\frac{k}{\gamma}(t-\tau)}$$

$$\Rightarrow \quad \boxed{n = \frac{F}{k}\left(1 - e^{-\frac{k}{\gamma}\tau}\right) e^{-\frac{k}{\gamma}(t-\tau)}}$$

(5) Given that steady state concentration profile has been reached

$$\Rightarrow \quad \frac{\partial c}{\partial t} = 0$$

$$\Rightarrow \quad D\frac{\partial^2 c}{\partial x^2} = 0 \Rightarrow \frac{\partial c}{\partial x} = A_1$$

$$\Rightarrow \quad C(x) = A_1 x + A_2$$ (where A_1 & A_2 are arbitrary constants)

Given $C(100) = 10\mu M$ and $C(0) = 0$

$$\Rightarrow \quad 10 = A_1 100$$

and $A_2 = 0$

$$\Rightarrow \quad A_1 = \frac{10}{100} \frac{\mu M}{\mu m} = 0.1 \frac{\mu M}{\mu m}$$

$$\Rightarrow \quad C(x) = 0.1 x \ \mu M$$

$$\Rightarrow \quad J = -D\frac{\partial c}{\partial x} = -D \times 0.1$$

$$= -1000 \frac{\mu m^2}{s} \times \frac{\mu M}{\mu m}$$

$$= -1000 \frac{\mu M}{s} \mu m = -100 \frac{\mu M}{s} \mu m$$

-ve sign implies that direction of flux is from front to back.