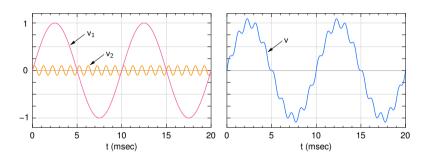
## Op-Amp Circuits: Part 3



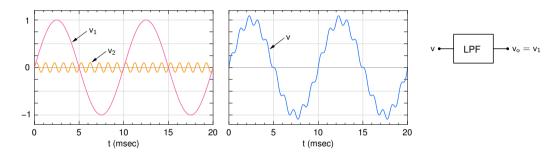
M. B. Patil
mbpatil@ee.iitb.ac.in
www.ee.iitb.ac.in/~sequel

Department of Electrical Engineering Indian Institute of Technology Bombay

Consider 
$$v(t) = v_1(t) + v_2(t) = V_{m1} \sin \omega_1 t + V_{m2} \sin \omega_2 t$$
.

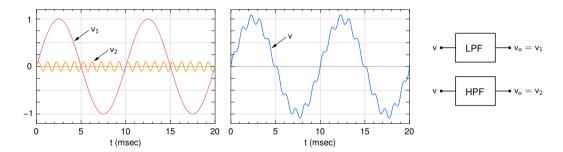


Consider  $v(t) = v_1(t) + v_2(t) = V_{m1} \sin \omega_1 t + V_{m2} \sin \omega_2 t$ .



A low-pass filter with a cut-off frequency  $\omega_1 < \omega_c < \omega_2$  will pass the low-frequency component  $v_1(t)$  and remove the high-frequency component  $v_2(t)$ .

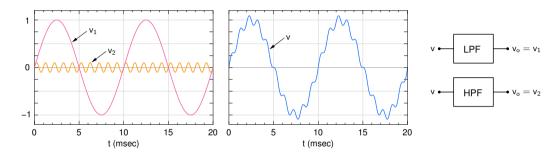
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A high-pass filter with a cut-off frequency  $\omega_1 < \omega_c < \omega_2$  will pass the high-frequency component  $v_2(t)$  and remove the low-frequency component  $v_1(t)$ .

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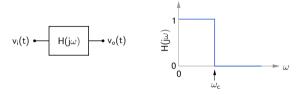


A low-pass filter with a cut-off frequency  $\omega_1 < \omega_c < \omega_2$  will pass the low-frequency component  $v_1(t)$  and remove the high-frequency component  $v_2(t)$ .

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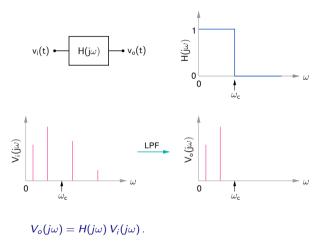
There are some other types of filters, as we will see.

# Ideal low-pass filter

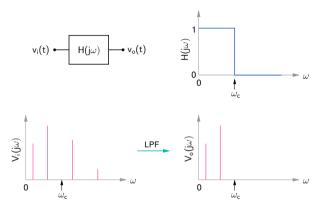


$$V_o(j\omega) = H(j\omega) V_i(j\omega)$$
.

# Ideal low-pass filter



## Ideal low-pass filter



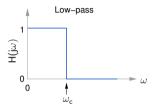
$$V_o(j\omega) = H(j\omega) V_i(j\omega)$$
.

All components with  $\omega < \omega_c$  appear at the output without attenuation.

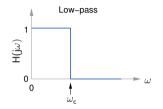
All components with  $\omega > \omega_c$  get eliminated.

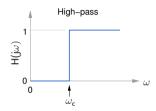
(Note that the ideal low-pass filter has  $\angle H(j\omega)=1$ , i.e.,  $H(j\omega)=1+j0$ .)

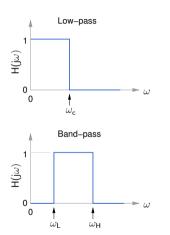
# Ideal filters

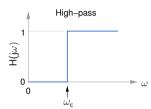


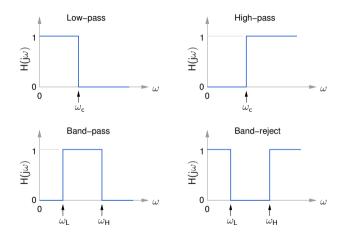
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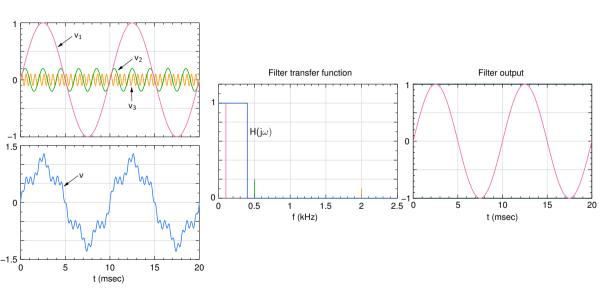




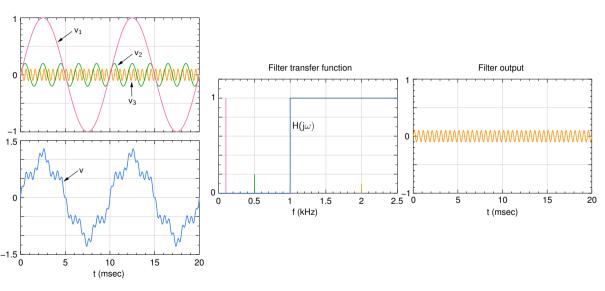




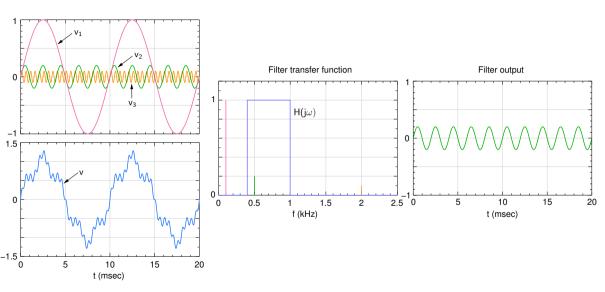


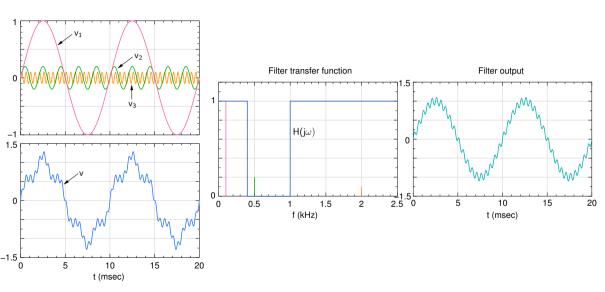


## Ideal high-pass filter: example



## Ideal band-pass filter: example





#### Practical filter circuits

\* In practical filter circuits, the ideal filter response is approximated with a suitable  $H(j\omega)$  that can be obtained with circuit elements. For example,

$$H(s) = \frac{1}{a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}$$

represents a 5<sup>th</sup>-order low-pass filter.

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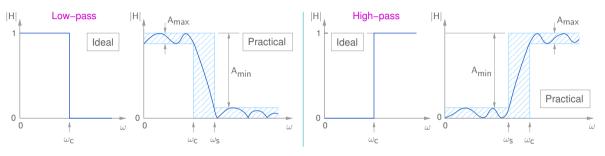
\* Some commonly used approximations (polynomials) are the Butterworth, Chebyshev, Bessel, and elliptic functions.

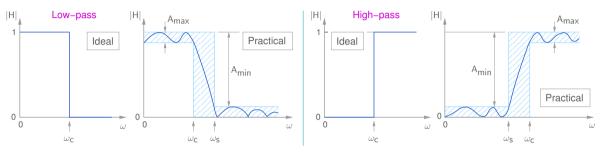
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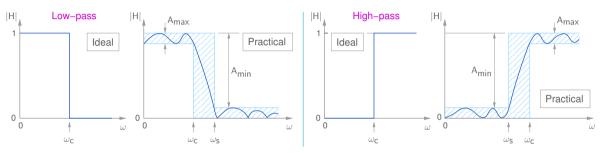
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- \* Coefficients for these filters are listed in filter handbooks. Also, programs for filter design are available on the internet.

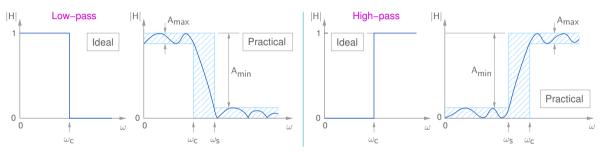




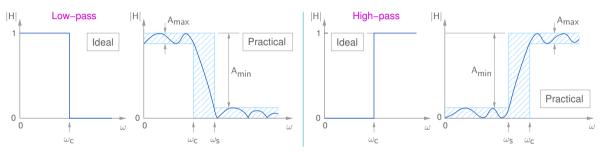
\* A practical filter may exhibit a ripple.  $A_{\max}$  is called the maximum passband ripple, e.g.,  $A_{\max}=1$  dB.



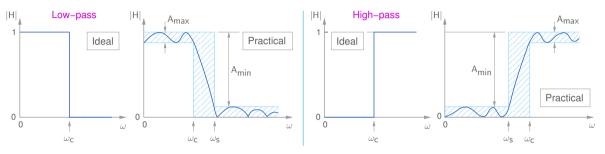
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- \*  $\omega_c < \omega < \omega_s$ : transition band.

For a low-pass filter, 
$$H(s) = rac{1}{\displaystyle\sum_{i=0}^{n} a_i (s/\omega_c)^i}$$
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Coefficients  $(a_i)$  for various types of filters are tabulated in handbooks. We now look at  $|H(j\omega)|$  for two commonly used filters.

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Butterworth filters:

$$|H(j\omega)| = rac{1}{\sqrt{1+\epsilon^2(\omega/\omega_c)^{2n}}}$$
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Chebyshev filters:

$$|H(j\omega)| = rac{1}{\sqrt{1+\epsilon^2 C_n^2(\omega/\omega_c)}}$$
 where

$$C_n(x) = \cos\left[n\cos^{-1}(x)\right]$$
 for  $x \le 1$ ,

$$C_n(x) = \cosh \left[ n \cosh^{-1}(x) \right]$$
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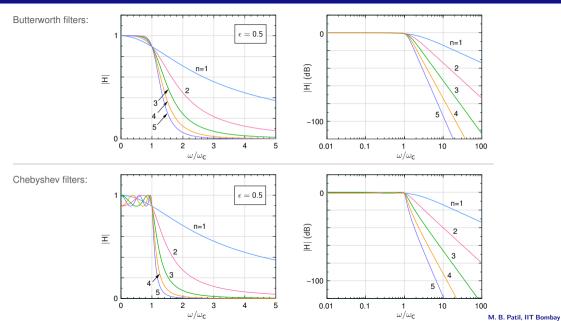
$$|H(j\omega)| = rac{1}{\sqrt{1+\epsilon^2 C_n^2(\omega/\omega_c)}}$$
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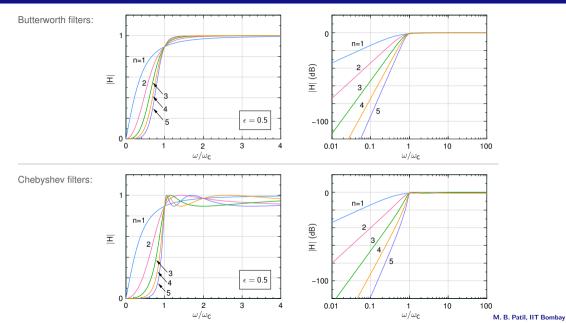
$$C_n(x) = \cosh \left[ n \cosh^{-1}(x) \right] \text{ for } x \ge 1,$$

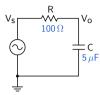
H(s) for a high-pass filter can be obtained from H(s) of the corresponding low-pass filter by  $(s/\omega_c) o (\omega_c/s)$ .

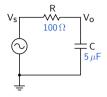
## Practical filters (low-pass)



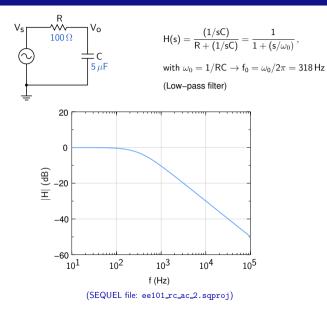
## Practical filters (high-pass)

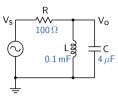


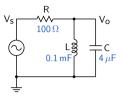




$$\begin{split} &H(s)=\frac{(1/sC)}{R+(1/sC)}=\frac{1}{1+(s/\omega_0)}\,,\\ &\text{with }\omega_0=1/RC\to f_0=\omega_0/2\pi=318\,\text{Hz}\\ &\text{(Low-pass filter)} \end{split}$$

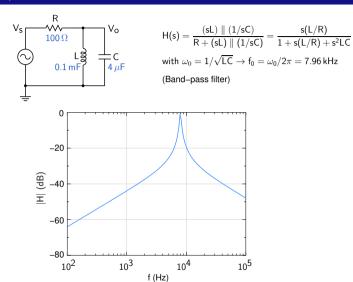






$$\begin{split} H(s) &= \frac{(sL) \parallel (1/sC)}{R + (sL) \parallel (1/sC)} = \frac{s(L/R)}{1 + s(L/R) + s^2L} \\ \text{with } \omega_0 &= 1/\sqrt{LC} \rightarrow f_0 = \omega_0/2\pi = 7.96 \, \text{kHz} \\ \text{(Band–pass filter)} \end{split}$$

#### Passive filter example



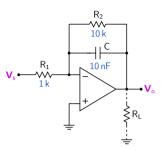
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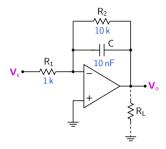
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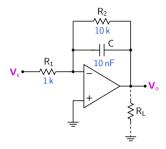
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- \* With op-amps, a filter circuit can be designed with a pass-band gain.
- \* Op-amp filters can be easily incorporated in an integrated circuit.
- \* However, there are situations in which passive filters are still used.
  - high frequencies at which op-amps do not have sufficient gain
  - high power which op-amps cannot handle





Op-amp filters are designed for op-amp operation in the linear region  $\rightarrow$  Our analysis of the inverting amplifier applies, and we get,

$$\mathbf{V_o} = -rac{R_2 \parallel (1/sC)}{R_1} \, \mathbf{V_s} \; \; (\mathbf{V_s} \; ext{and} \; \mathbf{V_o} \; ext{are phasors})$$
  $H(s) = -rac{R_2}{R_1} rac{1}{1+sR_2C}$ 

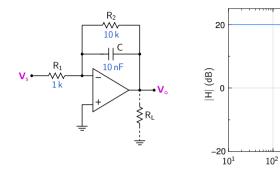


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This is a low-pass filter, with  $\omega_0=1/R_2 \, {\it C}$  (i.e.,  $f_0=\omega_0/2\pi=1.59\,{\rm kHz}$ ).



Op-amp filters are designed for op-amp operation in the linear region  $\rightarrow$  Our analysis of the inverting amplifier applies, and we get,

 $10^{3}$ 

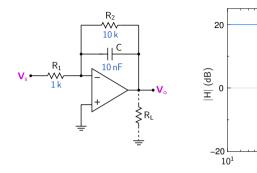
f (Hz)

 $10^{4}$ 

 $10^{5}$ 

$$egin{aligned} \mathbf{V_o} &= -rac{R_2 \parallel (1/sC)}{R_1}\,\mathbf{V_s} \;\; (\mathbf{V_s} \;\; ext{and} \;\; \mathbf{V_o} \;\; ext{are phasors}) \ H(s) &= -rac{R_2}{R_1}\,rac{1}{1+sR_2C} \end{aligned}$$

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Op-amp filters are designed for op-amp operation in the linear region  $\rightarrow$  Our analysis of the inverting amplifier applies, and we get,

 $10^{2}$ 

 $10^{3}$ 

f (Hz)

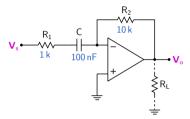
 $10^{4}$ 

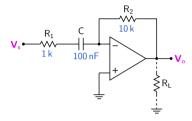
 $10^{5}$ 

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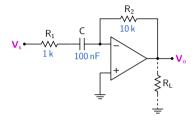
This is a low-pass filter, with  $\omega_0=1/R_2C$  (i.e.,  $f_0=\omega_0/2\pi=1.59\,\mathrm{kHz}$ ).

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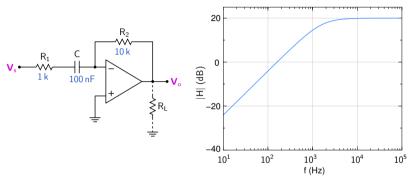


$$H(s) = -\frac{R_2}{R_1 + (1/sC)} = -\frac{sR_2C}{1 + sR_1C}$$
.



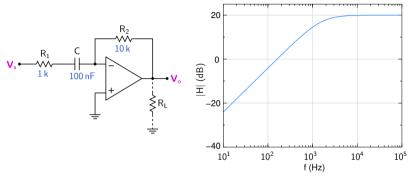
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This is a high-pass filter, with  $\omega_0=1/R_1C$  (i.e.,  $\mathit{f}_0=\omega_0/2\pi=1.59\,\mathrm{kHz}$ ).



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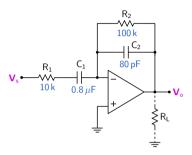
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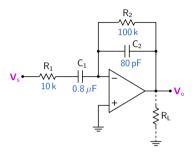


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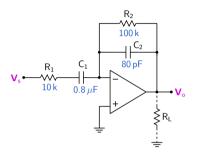
This is a high-pass filter, with  $\omega_0=1/R_1C$  (i.e.,  $\mathit{f}_0=\omega_0/2\pi=1.59\,\mathrm{kHz}$ ).

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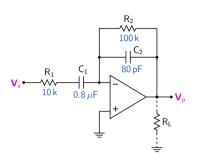


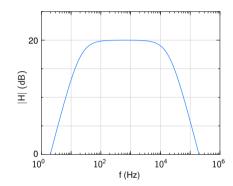
$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1} \frac{sR_1C_1}{(1 + sR_1C_1)(1 + sR_2C_2)}.$$



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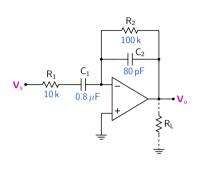
This is a band-pass filter, with  $\omega_L=1/R_1\,C_1$  and  $\omega_H=1/R_2\,C_2$  .  $\to$   $f_L=20$  Hz,  $f_H=20$  kHz.

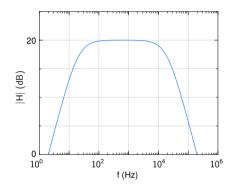




$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1} \frac{sR_1C_1}{(1 + sR_1C_1)(1 + sR_2C_2)}.$$

This is a band-pass filter, with  $\omega_L=1/R_1\,C_1$  and  $\omega_H=1/R_2\,C_2$  .  $\to f_L=20\,{\rm Hz},\, f_H=20\,{\rm kHz}.$ 



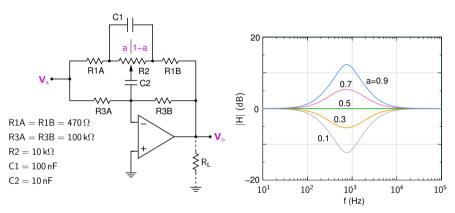


$$H(s) = -\frac{R_2 \parallel (1/sC_2)}{R_1 + (1/sC_1)} = -\frac{R_2}{R_1} \frac{sR_1C_1}{(1 + sR_1C_1)(1 + sR_2C_2)}.$$

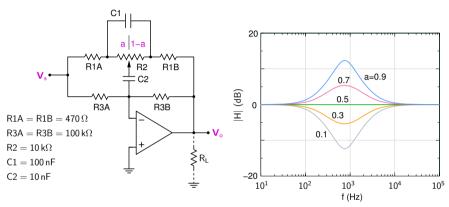
This is a band-pass filter, with  $\omega_L=1/R_1\,C_1$  and  $\omega_H=1/R_2\,C_2$  .

$$\rightarrow f_L = 20 \text{ Hz}, f_H = 20 \text{ kHz}.$$

(SEQUEL file: ee101\_op\_filter\_3.sqproj)

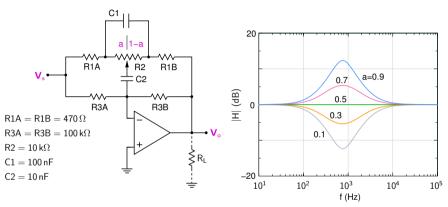


(Ref.: S. Franco, "Design with Op Amps and analog ICs")



(Ref.: S. Franco, "Design with Op Amps and analog ICs")

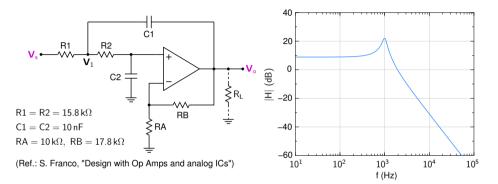
\* Equalizers are implemented as arrays of narrow-band filters, each with an adjustable gain (attenuation) around a centre frequency.

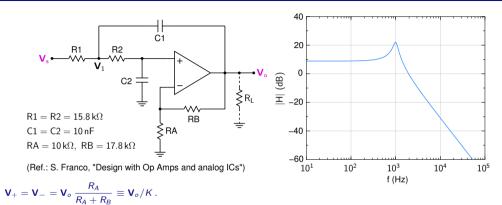


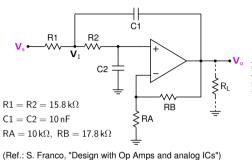
(Ref.: S. Franco, "Design with Op Amps and analog ICs")

- \* Equalizers are implemented as arrays of narrow-band filters, each with an adjustable gain (attenuation) around a centre frequency.
- \* The circuit shown above represents one of the equalizer sections.
  (SEQUEL file: ee101\_op\_filter\_4.sqproj)





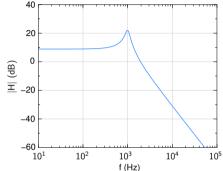


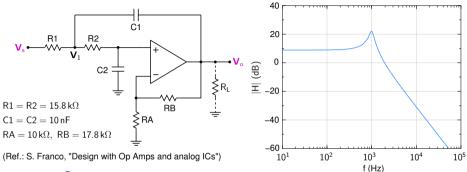


(Ref.: S. Franco, "Design with Op Amps and analog ICs")

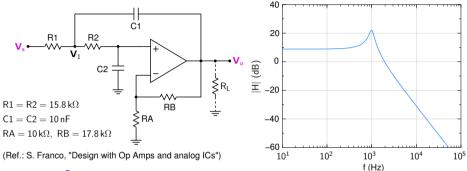
$$\mathbf{V}_{+} = \mathbf{V}_{-} = \mathbf{V}_{o} \frac{R_{A}}{R_{A} + R_{B}} \equiv \mathbf{V}_{o} / K$$
.

Also,  $\mathbf{V}_{+} = \frac{(1/sC_{2})}{R_{2} + (1/sC_{2})} \mathbf{V}_{1} = \frac{1}{1 + sR_{2}C_{2}} \mathbf{V}_{1}$ .





$$\begin{split} \mathbf{V}_{+} &= \mathbf{V}_{-} = \mathbf{V}_{o} \, \frac{R_{A}}{R_{A} + R_{B}} \equiv \mathbf{V}_{o} / \mathcal{K} \, . \\ \text{Also, } \mathbf{V}_{+} &= \frac{(1/sC_{2})}{R_{2} + (1/sC_{2})} \, \mathbf{V}_{1} = \frac{1}{1 + sR_{2}C_{2}} \, \mathbf{V}_{1} \, . \\ \text{KCL at } \mathbf{V}_{1} &\to \frac{1}{R_{1}} (\mathbf{V}_{s} - \mathbf{V}_{1}) + sC_{1} (\mathbf{V}_{o} - \mathbf{V}_{1}) + \frac{1}{R_{2}} (\mathbf{V}_{+} - \mathbf{V}_{1}) = 0 \, . \end{split}$$



$$\mathbf{V}_+ = \mathbf{V}_- = \mathbf{V}_o \, rac{R_A}{R_A + R_B} \equiv \mathbf{V}_o / K \, .$$

Also, 
$$\mathbf{V}_{+}=rac{(1/s\mathcal{C}_{2})}{R_{2}+(1/s\mathcal{C}_{2})}\,\mathbf{V}_{1}=rac{1}{1+sR_{2}\mathcal{C}_{2}}\,\mathbf{V}_{1}\,.$$

$$\mathsf{KCL} \; \mathsf{at} \; \boldsymbol{\mathsf{V}}_1 \to \frac{1}{R_1} (\boldsymbol{\mathsf{V}}_s - \boldsymbol{\mathsf{V}}_1) + s \mathcal{C}_1 (\boldsymbol{\mathsf{V}}_o - \boldsymbol{\mathsf{V}}_1) + \frac{1}{R_2} (\boldsymbol{\mathsf{V}}_+ - \boldsymbol{\mathsf{V}}_1) = 0 \; .$$

Combining the above equations, 
$$H(s) = \frac{K}{1 + s[(R_1 + R_2)C_2 + (1 - K)R_1C_1] + s^2R_1C_1R_2C_2}$$

(SEQUEL file: ee101\_op\_filter\_5.sqproj)

#### Sixth-order Chebyshev low-pass filter (cascade design)

