PH108 : Electricity & Magnetism : Tutorial 5

1. A conducting sphere (or a shell) of radius R has a charge Q.

   (a) Show that the force of repulsion between the two hemispheres is $Q^2/32\pi\epsilon_0 R^2$.

   (b) Now suppose one has a solid sphere of radius R with charge Q distributed uniformly over its volume. What will be the force of repulsion between the two hemispheres?

   (c) In which case is the force of repulsion larger?

2. A spherical soap bubble carries a charge Q. Its surface tension is T. The definition of surface tension is that it is an extra energy per unit area of the surface. So one needs to do some work to increase the surface area of a bubble. Identify all the sources of energy and then answer the following:

   (a) Work out what the excess pressure inside an *uncharged* soap bubble of radius R should be. The answer is $\Delta P = \dfrac{2T}{R}$. Make sure you understand why it is so. If a soap film has $T = 0.03\mathrm{Nm}^{-1}$ and a radius of 1cm, how much would be the excess pressure? (FYI : the surface tension of water at 20°C is $T = 0.072\mathrm{Nm}^{-1}$)

   (b) Now, if the bubble has Q charge in it - should ΔP increase or decrease? Give a qualitative argument first.

   (c) Now work out a quantitative expression for ΔP. How much charge is required to make the bubble burst?

3. Assume that an electron is a small sphere of radius R in which the charge $-|e|$ is distributed uniformly over its volume. Calculate the total energy of the system as a function of R. Now suppose you equate this energy to m_0c^2 where m_0 is the rest mass of the electron. What value of R do you get? How does it compare with the size of a hydrogen atom?

4. Consider an idealised vacuum diode consisting of two large parallel plates (like a capacitor). The distance between them is D, the plates are kept at $V(x = 0) = 0$ and $V(x = D) = V_0$. Here V_0 is positive. The plate at $V = 0$ is then heated, so that it starts emitting electrons which are attracted towards the other plate. You need to calculate the steady state current (I) - voltage (V_0) relation (called the Child-Langmuir law).

   (a) The space between the two plates will be filled with electrons so the charge density there is not zero. The velocity of the charged particles depend on the position x, however in the steady state the charge density remains time independent and the current is constant. Under these conditions, formulate the Poisson's equations such that $V(x)$ and x are the only variables.

   (b) Give a physical argument to show that the steady state would be reached when the electric field at the surface of the heated plate at $x = 0$ is zero.

   (c) Use this as your boundary condition to solve the differential equation fully. Establish the relation $I \propto V_0^{3/2}$