

# **Projection of Solids**

## **Lecture - 5**

Slides by Prof. Salil S. Kulkarni

# Motivation

- Most of the objects of practical interest are three-dimensional solids

## Typical Problem

- To find the projections of a solid given its orientation
- To graphically determine the relative positions of objects

# Types of Solids

- **Polyhedra**
- **Solids of revolution**

## □ **Polyhedra**

A solid bounded by planes called faces.

- **Tetrahedron (4), Cube (6), Octahedron(8), Dodecahedron(12), Icosahedron(20)**

When all the faces are equal in shape and size, the polyhedra is said to be regular

- **Prism** – A polyhedron having two equal faces called bases or ends, parallel to each other and joined by faces which are parallelograms.

A right and regular prism has its axis perpendicular to the bases and the bases are regular polygons. All faces are rectangles and are perpendicular to the bases

- **Pyramid** – A polyhedron having a plane as its base and a number of triangular faces meeting at a point called the vertex or apex.

A right and regular pyramid has its axis perpendicular to the base and the base is a regular polygon. All faces are isosceles triangles

Prisms and pyramids are named according to the shape of their bases.

# Types of Solids

## ❑ Solids of revolutions

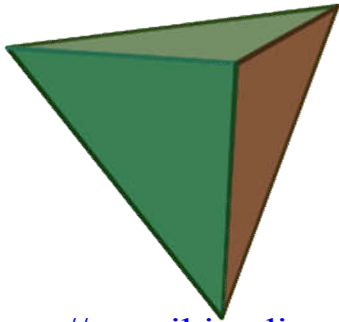
A solid generated by revolution of a plane about an axis

- **Cylinder** – A right circular cylinder is a solid generated by the revolution of a rectangle about one of its sides
- **Cone** – A right circular cone is a solid generated by the revolution of a right-angled triangle about one of its perpendicular sides
- **Sphere** – A solid generated by the revolution of a semi-circle about its diameter
- **Frustum** – A solid obtained by cutting a pyramid or a cone by a plane parallel to its base
- **Truncated** – A solid obtained by cutting a pyramid or a cone by a plane inclined to its base

# Types of Regular Polyhedra

## **Tetrahedron**

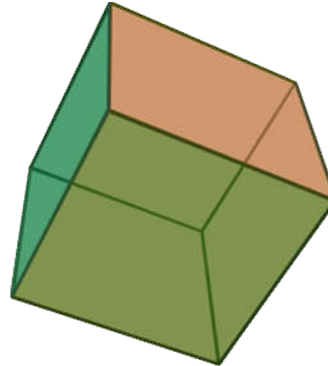
Four equal faces, each an equilateral triangle



<http://en.wikipedia.org/wiki/File:Tetrahedron.svg>

## **Cube**

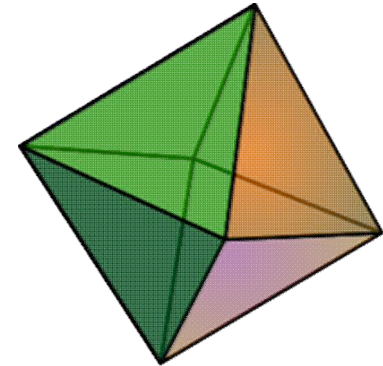
Six equal faces, each a square



<http://en.wikipedia.org/wiki/Cube>

## **Octahedron**

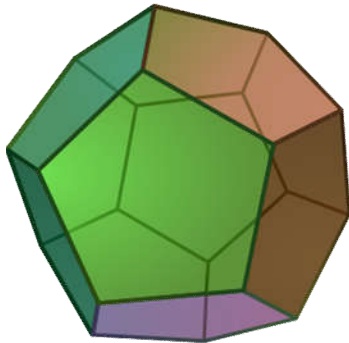
Eight equal faces, each an equilateral triangle



<http://en.wikipedia.org/wiki/Octahedron>

## **Dodecahedron**

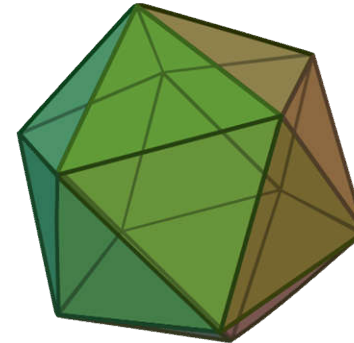
Twelve equal faces, each a regular pentagon



<http://en.wikipedia.org/wiki/File:POV-Ray-Dodecahedron.svg>

## **Icosahedron**

Twenty equal faces, each an equilateral triangle

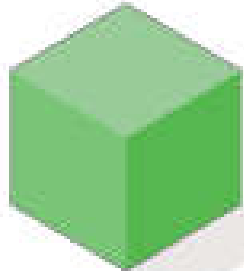


<http://en.wikipedia.org/wiki/File:Icosahedron.svg>

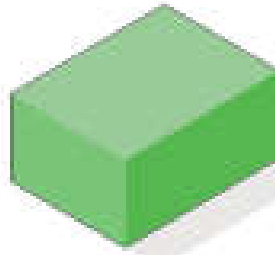
# Types of Solids



Sphere



Cube



Cuboid



Cylinder



Hexagonal Prism



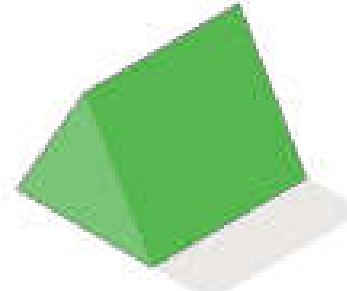
Cone



Square-based  
pyramid



Triangular-based  
pyramid



Triangular  
prism

[http://www.bbc.co.uk/schools/ks3bitesize/maths/shape\\_space/3d\\_shapes/revise2.shtml](http://www.bbc.co.uk/schools/ks3bitesize/maths/shape_space/3d_shapes/revise2.shtml)

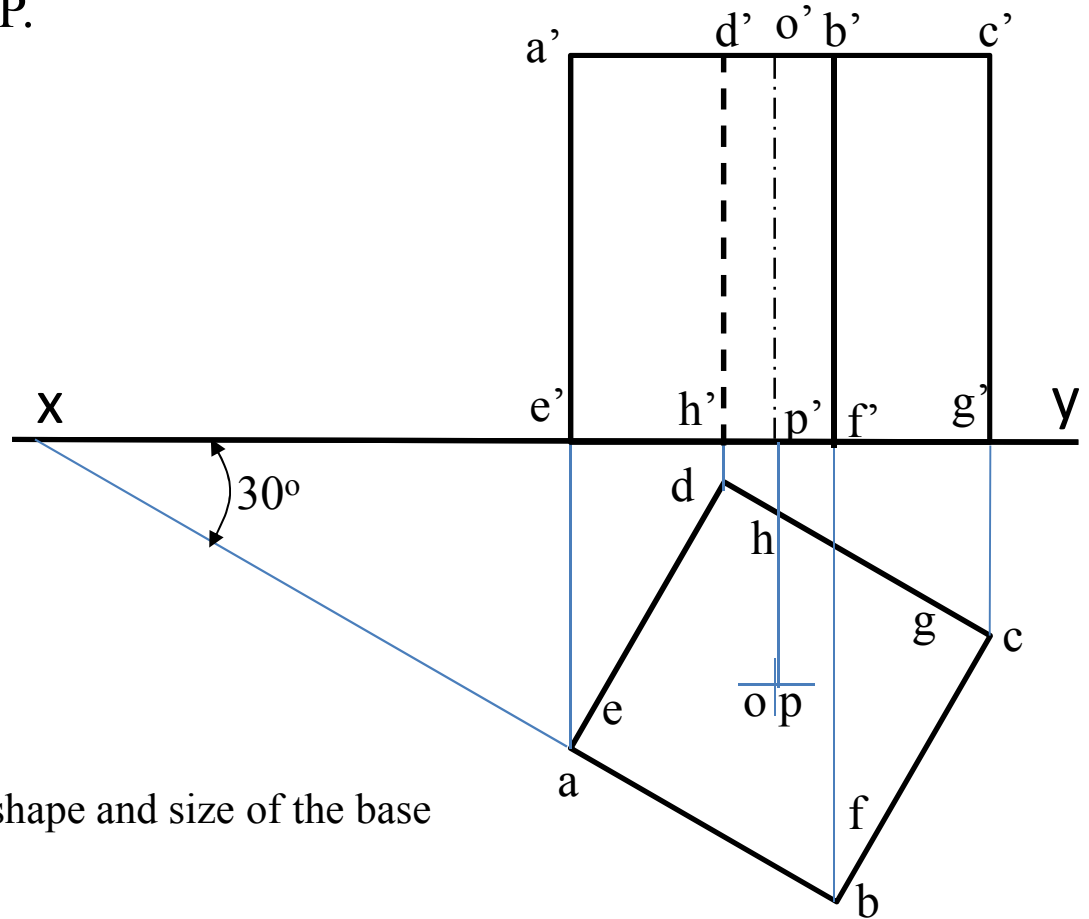
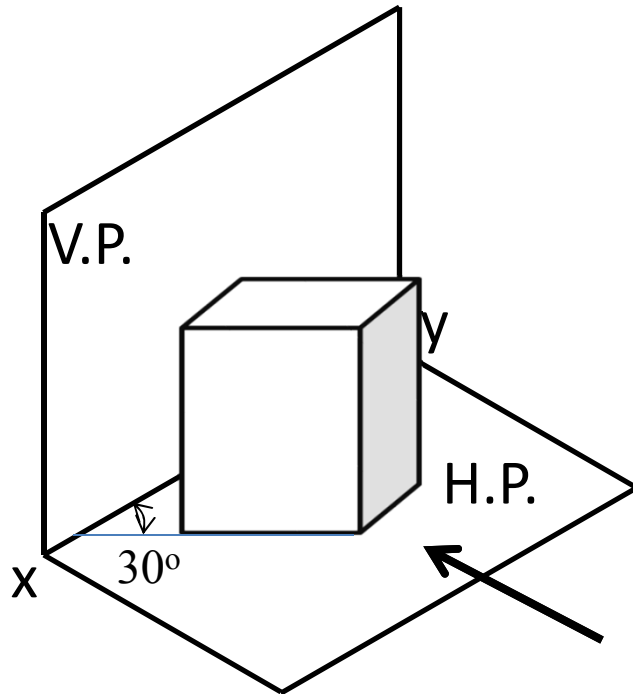
# Types of Projections of Solids

Depending on the orientation of the axes of the solids with the reference planes, the projections of the solids can be classified as

- **Projections of solids in simple positions**
  - Axis perpendicular to the H.P.
  - Axis perpendicular to the V.P.
  - Axis parallel to both the H.P. and the V.P.
- **Projections of solids with axes inclined to one of the reference planes and parallel to the other**
  - Axis inclined to the V.P. and parallel to the H.P.
  - Axis inclined to the H.P. and parallel to the V.P.
- **Projections of solids with axes inclined to both the reference planes**

# Projection of a Solid with Axis Perpendicular to the H.P.

Draw projections of a square prism resting on the H.P. and one of the faces making an angle of  $30^\circ$  with the V.P.



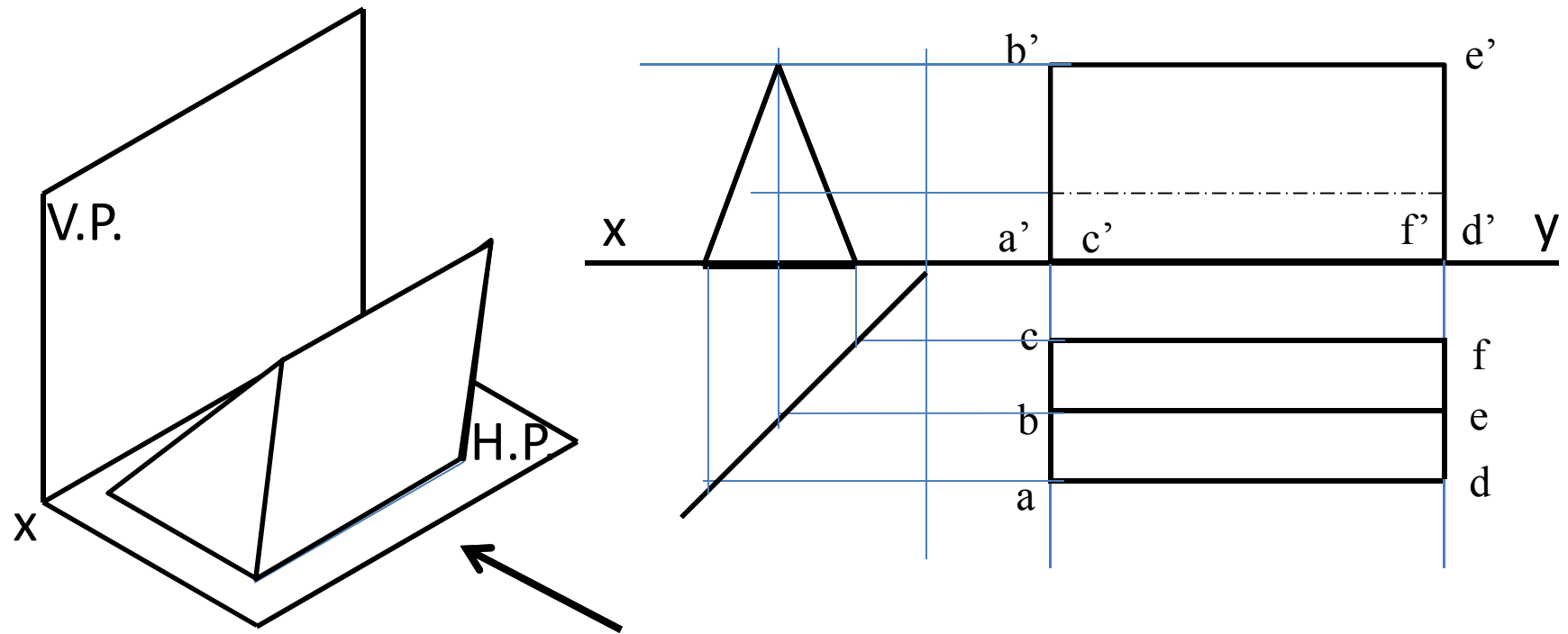
- Draw the top view. It will show the true shape and size of the base
- Project the front view from the top view

Projection of a solid on the plane to which its axis is perpendicular, will show the true size and shape of the base



# Projection of a Solid with Axis Parallel to the H.P. and the V.P.

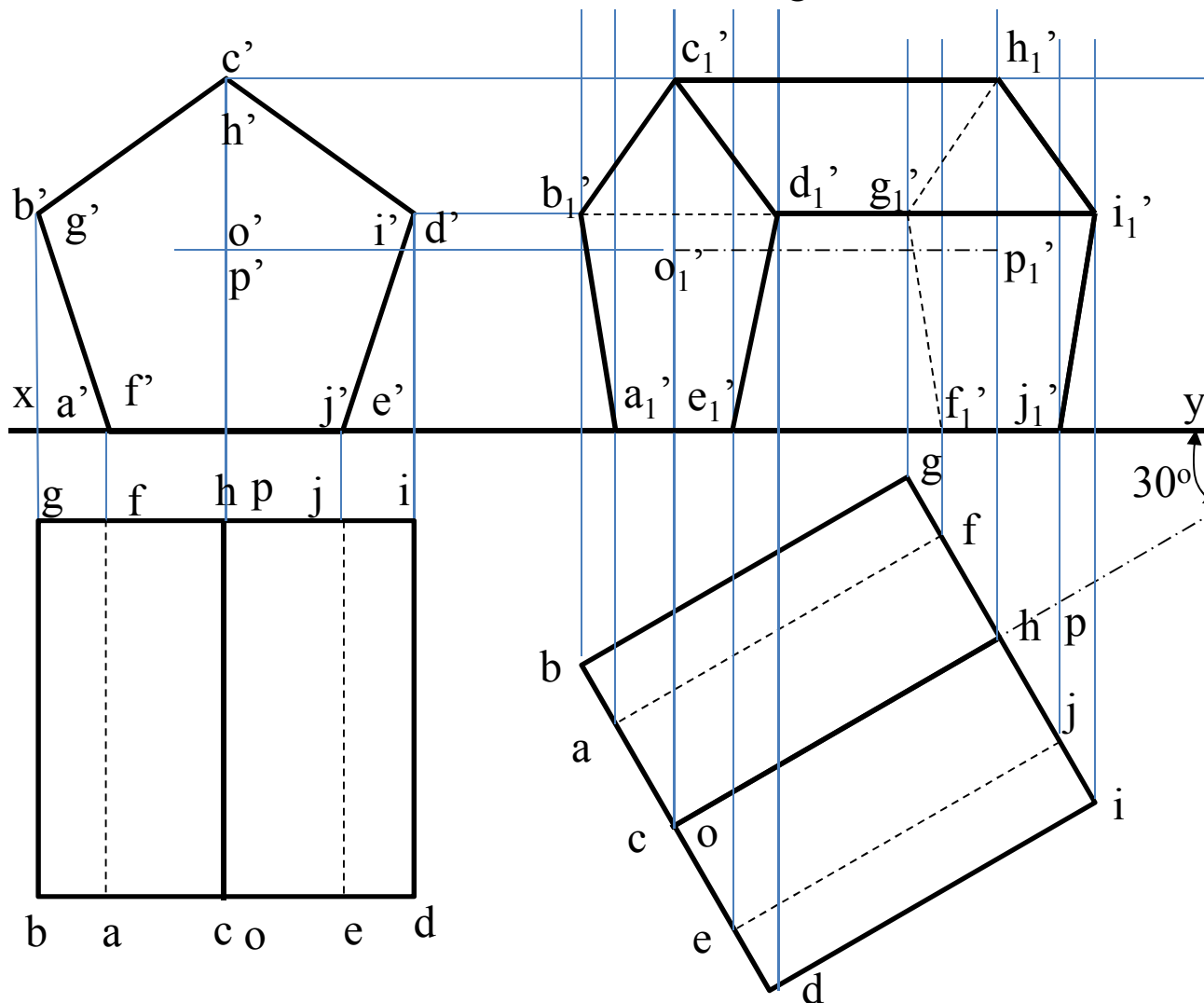
Draw projections of a triangular prism resting on the H.P. with its axis parallel to both the H.P. and the V.P. The end faces of the prism are isosceles triangles and the face of the prism in contact with the H.P. includes the bases of the isosceles triangles.



- The axis of the prism is perpendicular to the profile plane. Hence the side view will show the true shape and size of the base. Therefore draw the side view first
- Project the front view from the side view
- Project the top view from the front view and the side view

# Projection of a Solid with Axis Parallel to the H.P. and Inclined to the V.P.

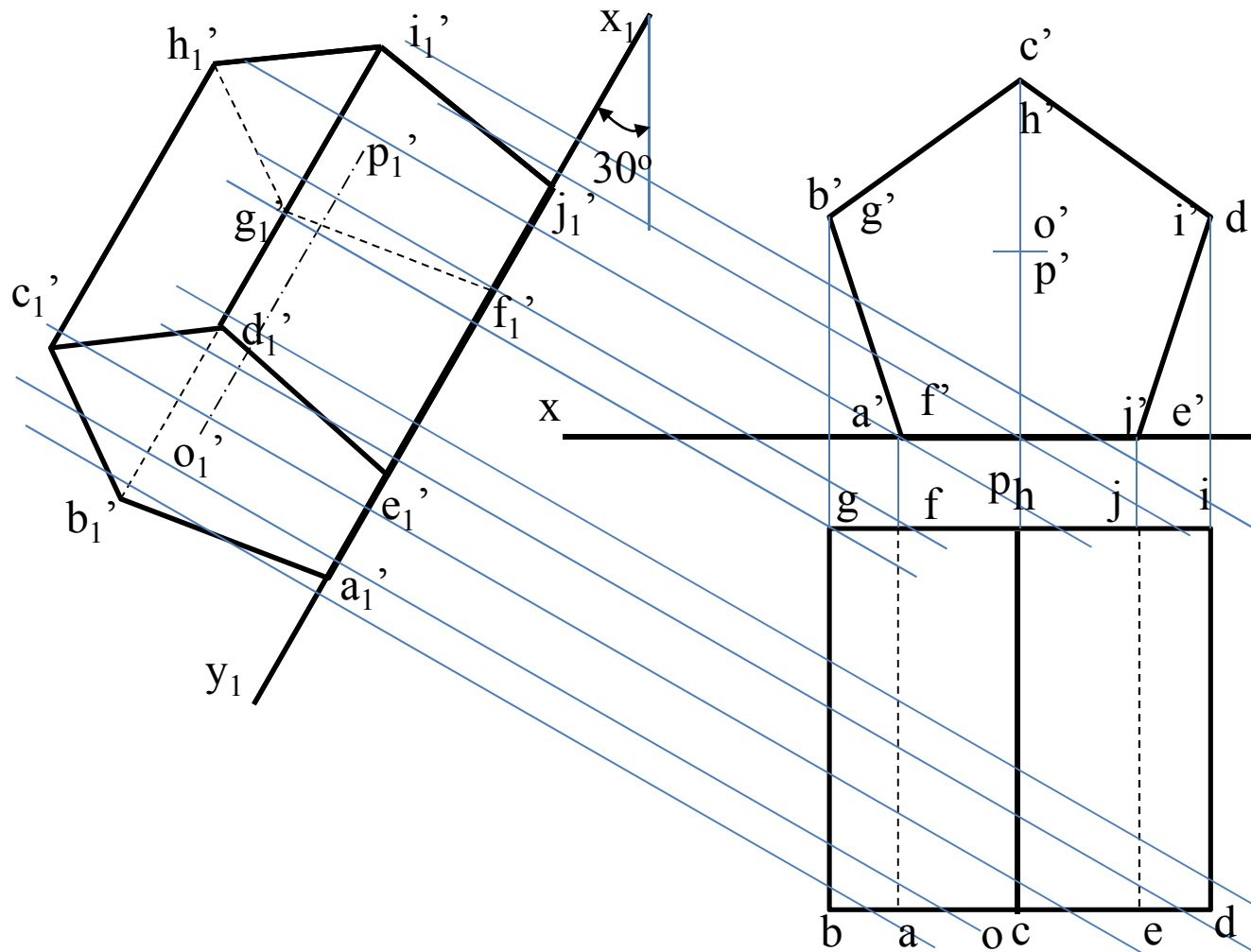
Draw the projections of a pentagonal prism, base 25 mm and axis 50 mm long, resting on one of the faces on the H.P. and the axis making  $30^\circ$  with the V.P.



- Draw the projection with one of the faces in the H.P. and the axis perpendicular to the V.P. The front view shows the true shape of the base.
- Redraw the top view with the axis inclined at  $30^\circ$  with the xy line. The size and shape of the top view does not change.
- Project the front view from the redrawn top view and the original front view. The distance (height) of the corner points from the xy line does not change.
- Connect the points to get the edges. Take into account the hidden edges

## Projection of a Solid with Axis Parallel to the H.P. and Inclined to the V.P.

Draw the projections of a pentagonal prism, base 25 mm and axis 50 mm long, resting on one of the faces on the H.P. and the axis making  $30^\circ$  with the V.P.



-Draw the projection with one of the faces in the H.P. and the axis perpendicular to the V.P. The front view shows the true shape of the base.

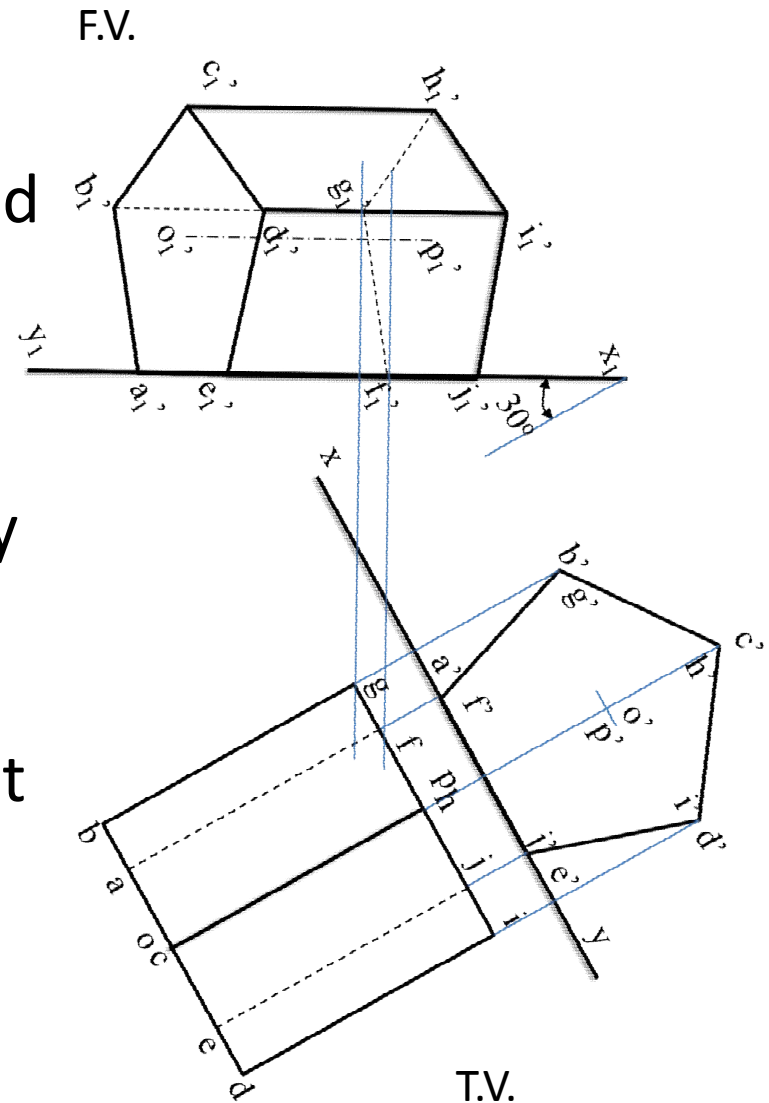
-Draw an auxiliary vertical plane such that the angle between  $x_1y_1$  and the top view of the axis is  $30^\circ$

- Project the new front view. The distance (height) of the corner points from the xy line does not change.

- Connect the points to get the edges. Take into account the hidden edges

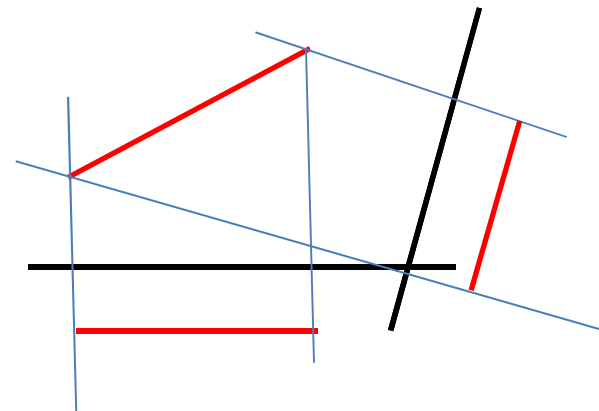
# How to Choose Auxiliary Planes

- Say the F.V. and T.V. projections are given around  $x_1-y_1$ : Find true shape of cross section
- We can find true shape of cross section by drawing  $x-y$  perpendicular to axis in T.V. (po is true view of axis)
- The previous problem is just the reverse of this one !



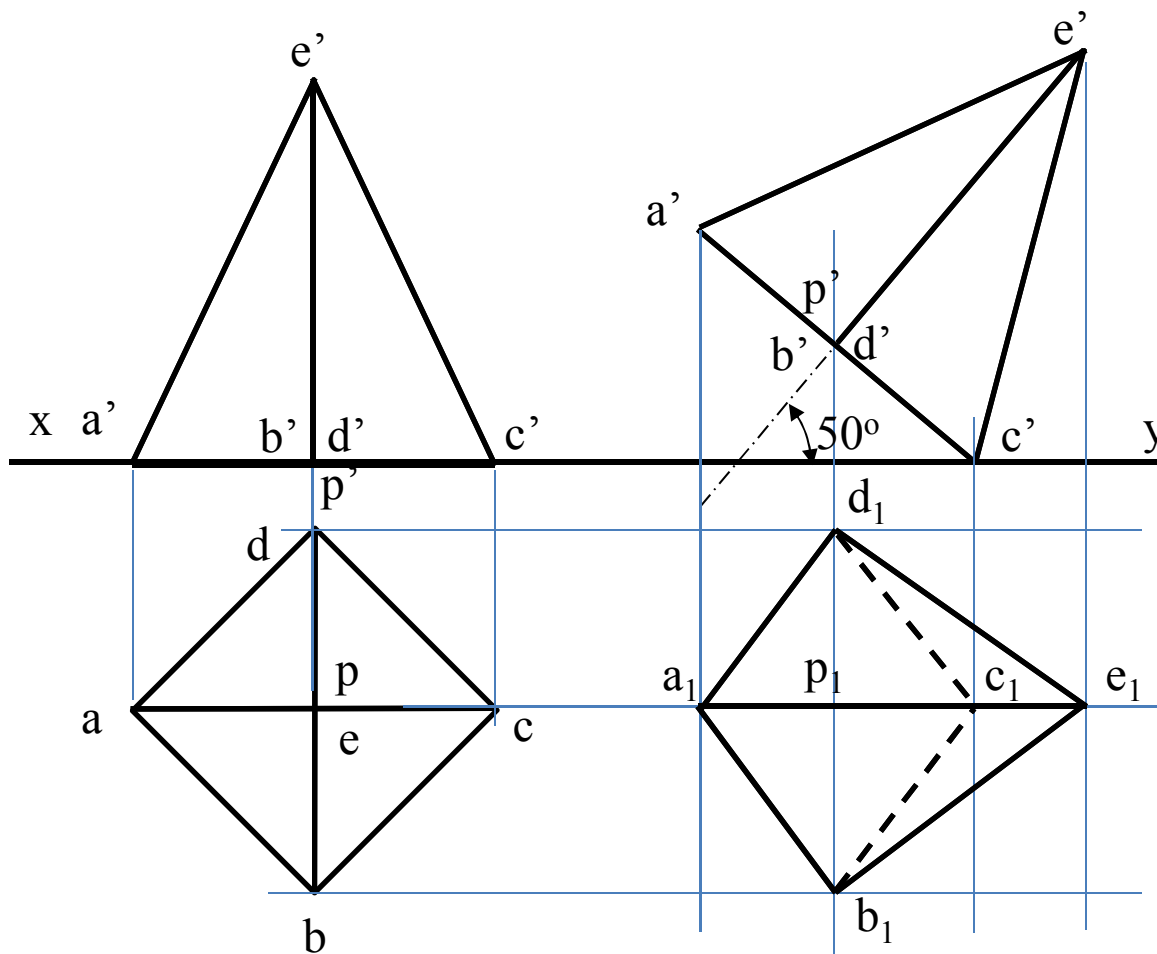
# Auxiliary Planes: Basic Rule of Thumb

- If a true view of a line in the solid/plane needs to be at angle  $\theta$  to H.P. then the Auxiliary **Horizontal** Plane needs to be drawn at angle  $\theta$  to the line in **F.V.** and new Auxiliary view needs to be projected from **F.V.**



# Projection of a Solid with Axis Parallel to the V.P. and Inclined to the H.P.

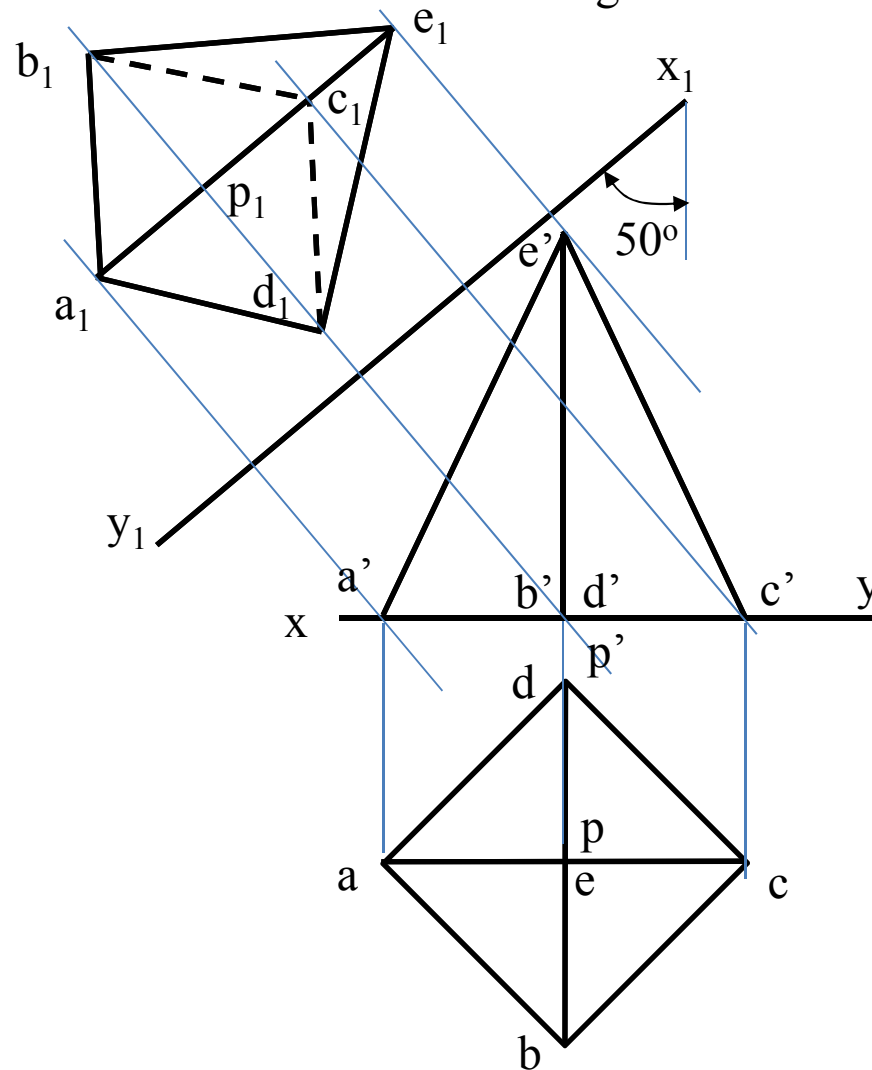
Draw the projections of a square pyramid, base 25 mm and axis 50 mm long, resting on one of the corners in the H.P. and its axis making  $50^\circ$  with the H.P. and parallel to the V.P.



- Draw the projections with the axis perpendicular to the H.P. The top view shows the true shape of the base. The object will be rotated @ point C. Make sure that the line joining C to the center of the base is parallel to the xy line
- Redraw the front view with the axis inclined at  $50^\circ$  with the xy line. The size and shape of the front view does not change.
- Project the top view from the redrawn front view and the original top view. The distance (depth dimension) of the corner points from the xy line does not change.
- Connect the points to get the edges. Take into account the hidden edges

# Projection of a Solid with Axis Parallel to the V.P. and Inclined to the H.P.

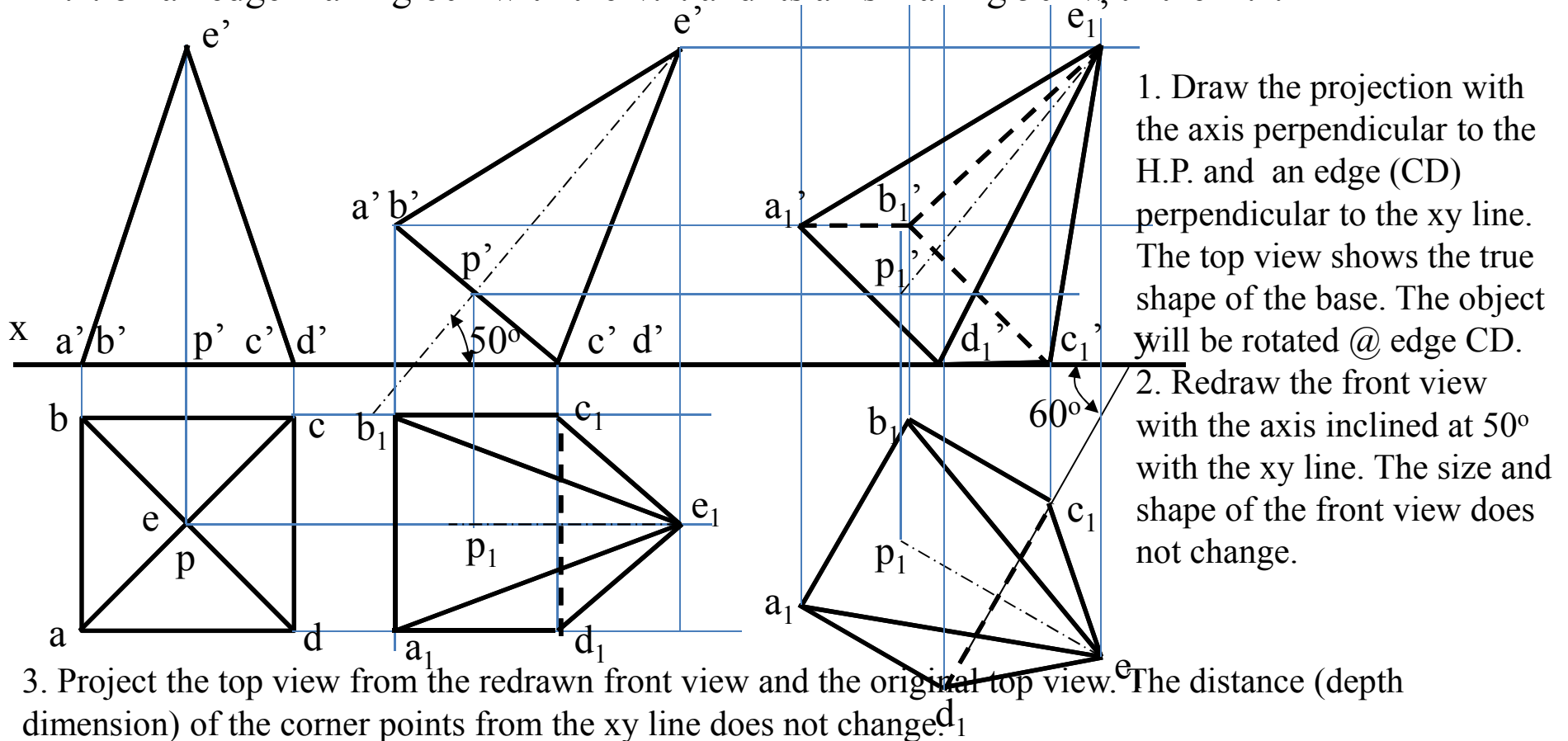
Draw the projections of a square pyramid, base 25 mm and axis 50 mm long, resting on one of the corners in the H.P. and its axis making  $50^\circ$  with the H.P. and parallel to the V.P.



- Draw the projections with the axis perpendicular to the H.P. The top view shows the true shape of the base. The object will be rotated @ point C. Make sure that the line joining C to the center of the base is parallel to the xy line
- Draw an auxiliary plane such that the angle between  $x_1y_1$  and the front view of the axis is  $50^\circ$
- Project the new top view. The distance (depth dimension) of the corner points from the xy line does not change.
- Connect the points to get the edges. Take into account the hidden edges

# Projection of a Solid with Axis Inclined to Both the H.P. and the V.P.

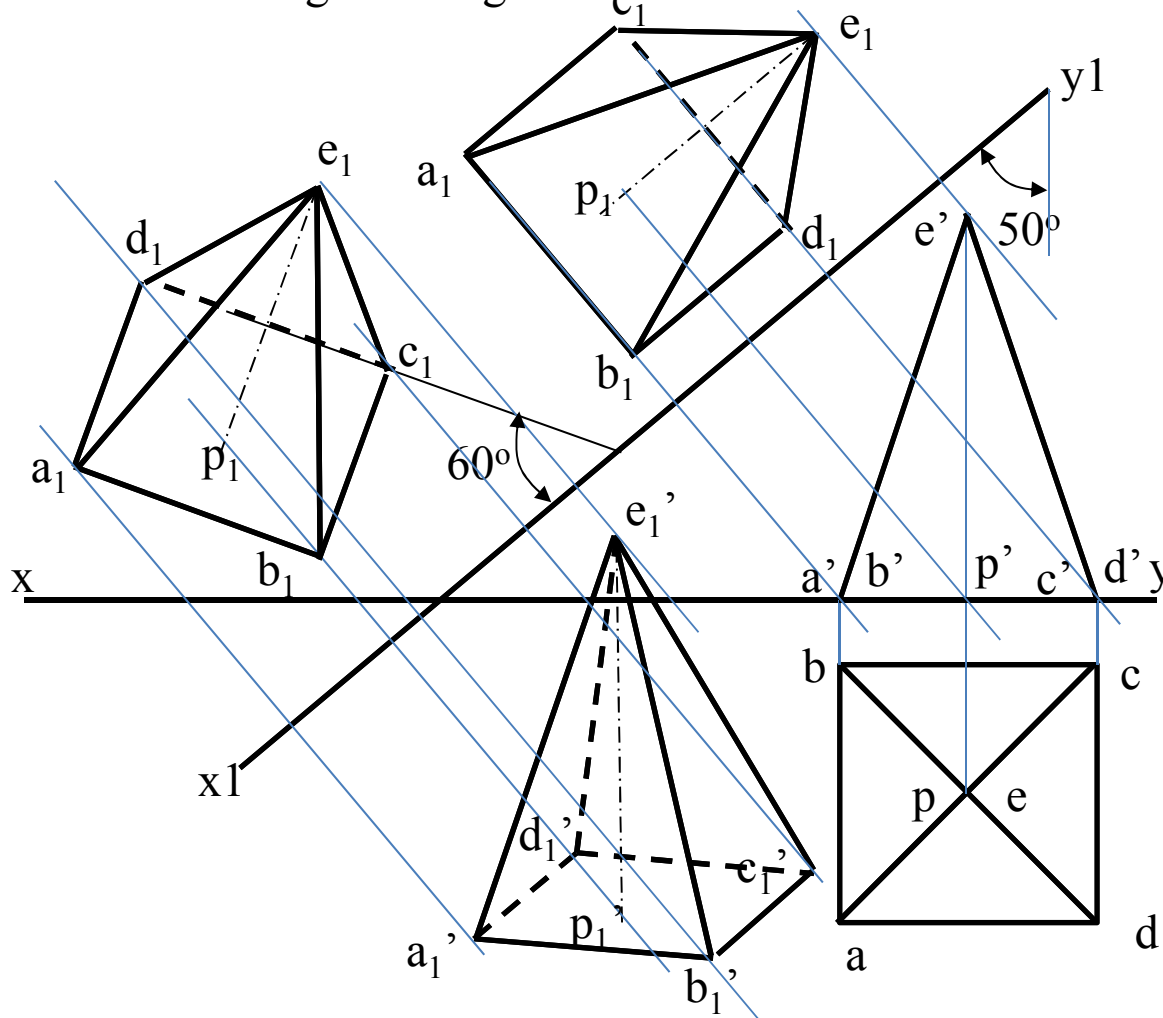
Draw the projections of a square pyramid, base 25 mm and axis 50 mm long, resting in the H.P. on an edge making  $60^\circ$  with the V.P. and its axis making  $50^\circ$  with the H.P.





# Projection of a Solid with Axis Inclined to Both the H.P. and the V.P.

Draw the projections of a square pyramid, base 25 mm and axis 50 mm long, resting in the H.P. on an edge making  $60^\circ$  with the V.P. and its axis making  $50^\circ$  with the H.P.



-Draw the projection with the axis perpendicular to the H.P. and an edge (CD) perpendicular to the xy line.

The top view shows the true shape of the base. The object will be rotated @ edge CD.

-Draw an auxiliary inclined plane inclined at  $50^\circ$  with the front view of the axis as shown.

-Project the new top view.

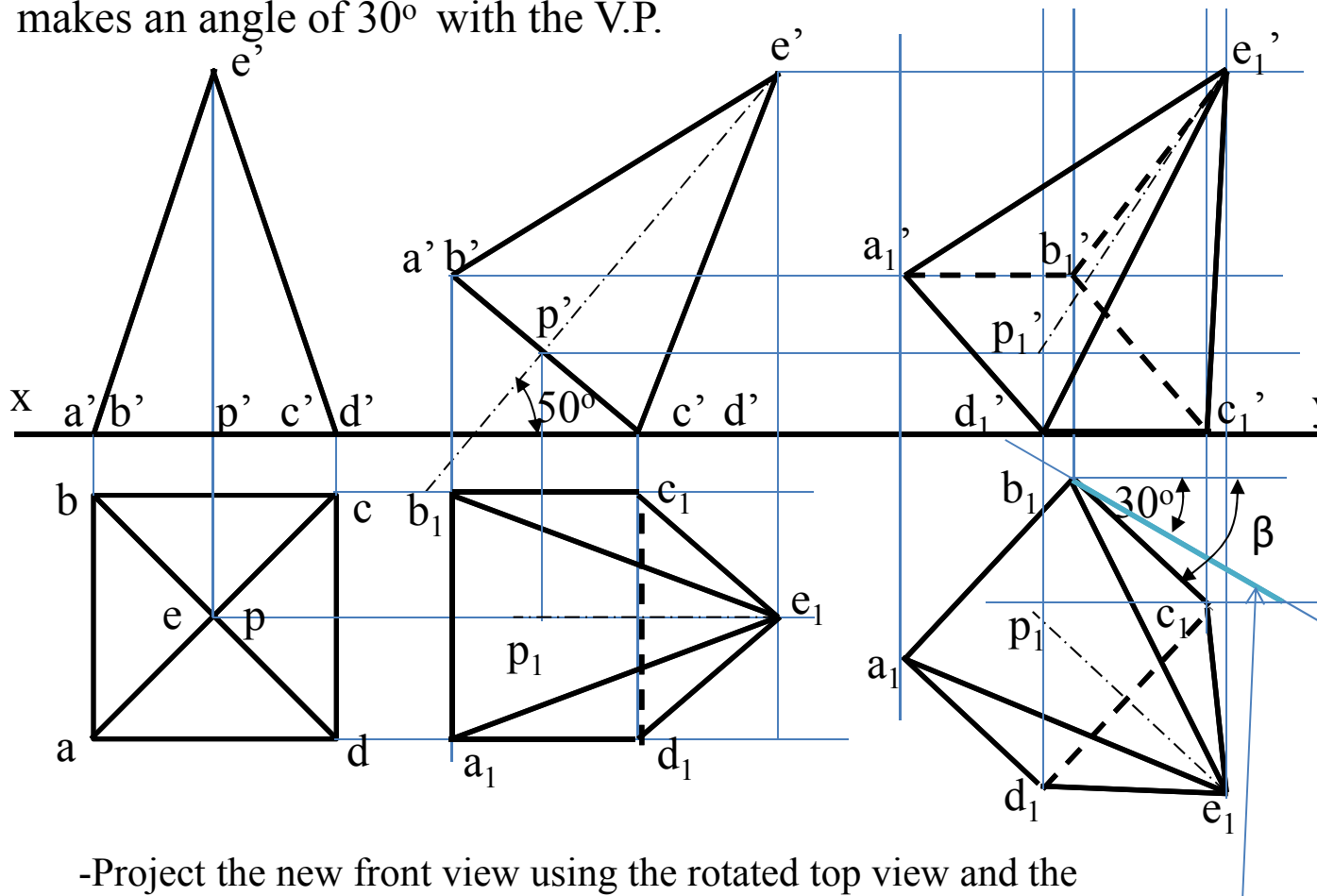
The distance (depth dimension) of the corner points from the xy line does not change.

-Make the edge CD inclined to  $60^\circ$  with the auxiliary plane. Redraw the new top view. The size and shape of the top view does not change.

-Project the new front view using the rotated top view and the front view (the height dimension is preserved).

# Projection of a Solid with Axis Inclined to Both the H.P. and the V.P.

Draw the projections of a square pyramid, base 25 mm and axis 50 mm long, resting in the H.P. on an edge and its axis making  $50^\circ$  with the H.P. The edge adjacent to the edge in the H.P. makes an angle of  $30^\circ$  with the V.P.



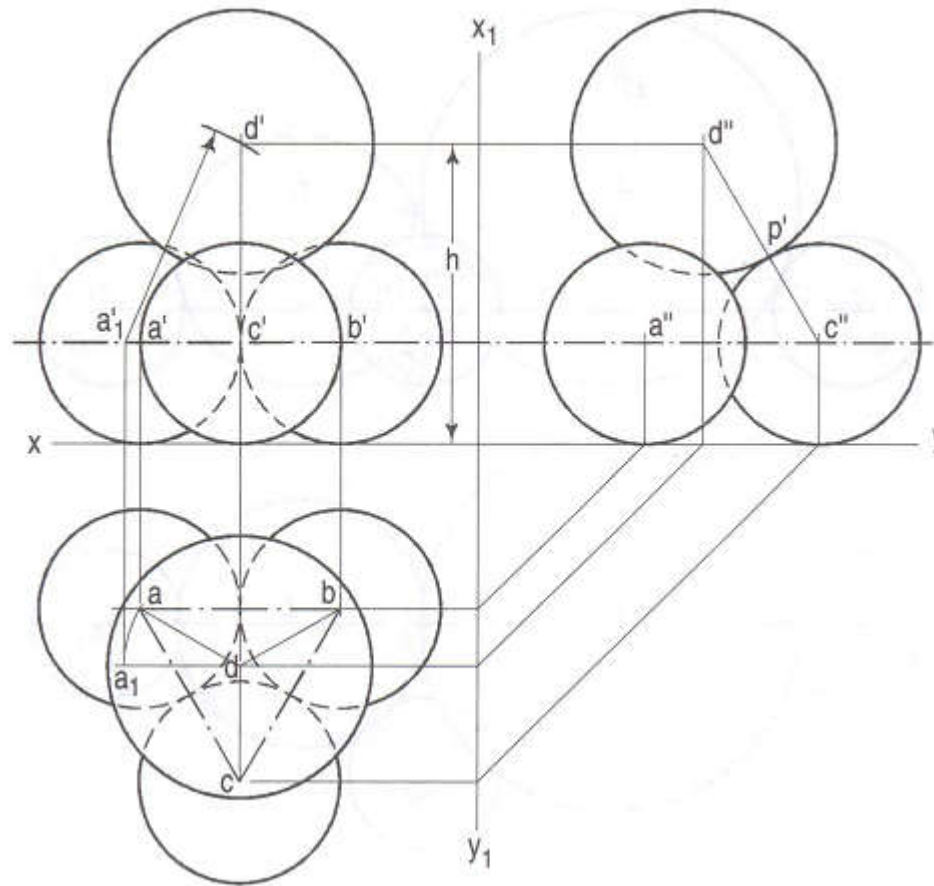
-Project the new front view using the rotated top view and the inclined front view (the height dimension is preserved).

True length of BC

-Follow steps 1-3 given on slide 14.  
 -Rotate the top view such that the true angle made by the edge BC with the V.P. is  $30^\circ$ . Note that since the true length of the edge BC is not seen in the V.P., the apparent angle BC makes with the V.P. ( $\beta$ ) needs to be obtained by taking into account its true length and true angle of inclination.  
 -Make the edge BC inclined to the V.P. at the apparent angle  $\beta$ . Redraw the new top view. The size and shape of the top view does not change.

# Projection of a Spheres

Three equal spheres of 38 mm diameter are resting on the ground so that each touches the other two and the line joining the centers of two of them are parallel to the V.P. A fourth sphere of 50 mm diameter is placed on the top of the three spheres. Draw the three views of the arrangement and find the distance of the center of the fourth sphere from the ground.



## **Summary: Projection of a solid with axis inclined to one reference plane and parallel to the other reference plane**

1. Assume the solid perpendicular to that reference plane to which it is to be made inclined. The projection on that plane shows the true shape and size of the base. (If the solid is to be titled about an edge in the reference plane, that edge should be drawn perpendicular to the xy line in the initial projection. If the solid is to be titled about a corner in the reference plane, the line joining that corner to the center of the base should parallel to the xy line in the initial projection).
2. Draw the appropriate projection such that the axis makes the required angle with the given plane.
3. Project the remaining view so that the distances of the corners from the xy axis are preserved.

## **Summary: Projection of a solid with axis inclined to both the reference planes**

1. Assume the solid perpendicular to that reference plane to which it is to be made inclined. The projection on that plane shows the true shape and size of the base. (If the solid is to be tilted about an edge in the reference plane, that edge should be drawn perpendicular to the xy line in the initial projection. If the solid is to be tilted about a corner in the reference plane, the line joining that corner to the center of the base should be parallel to the xy line in the initial projection).
2. Draw the appropriate projection such that the axis makes the required angle with the given plane
3. Project the remaining view.
4. Rotate the appropriate view so that the given edge makes the required angle with the reference axis. Make sure that the angle of rotation is drawn taking into account the true length of the edge. The size and shape of this view does not change during the rotation. Project the other view so that the distances of the corners from the xy axis are preserved.

## Points to Remember

1. If the axis of the solid is perpendicular to a reference plane, then the true shape and size of the base is seen in the projected view on that plane.
2. If the front (top) view is not the true shape of the base of the solid, then the solid is inclined to the V.P. (H.P.)
3. If a plane is in the V.P. (H.P.), its true shape and size is seen in the front (top) view.
4. If a solid does not change its relation with the reference plane (e.g. the angle of inclination of its axis), the projection on that reference plane does not change in size and shape.
5. If two lines representing the edges cross each other, one of them must be hidden and should therefore be drawn as a dashed line

**END**