BB 101: Physical Biology

TUTORIAL 1: Solutions

1. Work done=Force × Distance

Given Force= 5 pN

Therefore, Distance =Work done/Force $=10\times100\times10^{-21}/5\times10^{-12} = 200\times10^{-9}$ m = 200 nm

2. We know $F=\gamma v$

This implies, $v=F/\gamma$

Now $\gamma=6\pi\eta r$

Therefore, $v=F/6\pi\eta r$

Given $F = 5$ pN $= 5\times10^{-12}$ N

$\eta = 1000\times10^{-3}$ Pa.s

$r= 3/\pi$ μm $= 3/\pi\times10^{-6}$ m

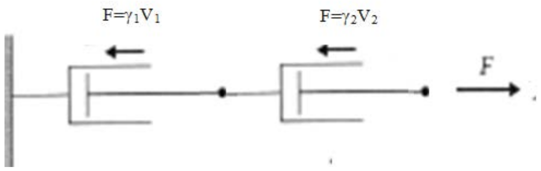
Therefore, $v=5\times10^{-12}$ N $/(6\pi\times1000\times10^{-3}$ Pa.s$\times3/\pi\times10^{-6})$

$= (5/18)\times10^{-6}$ m/s

$= 5/18$ μm/s

$= 0.27$ μm/s

3. When dashpots are in series as show below then both of them would experience the same applied force. However, both of them would move with different velocities such that net velocity V is sum of the velocities of the individual dashpots



$V = V_1 + V_2$

Or, $F/\gamma_{eff} = F/\gamma_1 + F/\gamma_2$

Or, $1/\gamma_{eff} = 1/\gamma_1 + 1/\gamma_2$

4. When an abrupt force F is applied at $t = 0$, let's assume that it leads to a net displacement of x_0. This net displacement would be the sum of the extension x_s of spring and displacement x_s of dashpot

i.e.$x_0 = x_s + x_d$

Further, both spring and dashpot would feel the same applied force and hence

$F = kx_s$ and $F = \gamma v = \gamma \frac{dx_d}{dt}$

Therefore, $kx_s = \gamma \frac{dx_d}{dt}$

Or, $k(x_0 - x_d) = \gamma \frac{dx_d}{dt}$

Or, $\int_0^{x_d} \frac{dx_d}{(x_0 - x_d)} = \int_0^t \frac{k}{\gamma} dt$

Or, $\ln \frac{(x_0 - x_d)}{x_0} = -\frac{k}{\gamma} t$

Or, $\frac{(x_0 - x_d)}{x_0} = e^{-\frac{k}{\gamma} t}$

Or, $1 - \frac{x_d}{x_0} = e^{-\frac{k}{\gamma}t}$

Or, $x_d = x_0(1 - e^{-\frac{k}{\gamma}t})$

Therefore, $x_d = x_0(1 - e^{-\frac{k}{\gamma}t})$, and $x_s = x_0 e^{-\frac{k}{\gamma}t}$

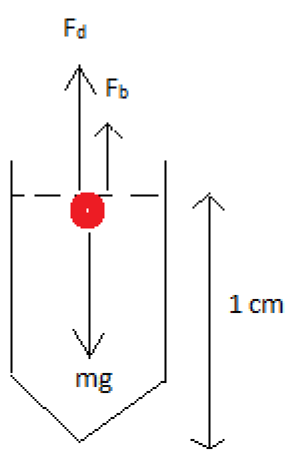
Let's look at the limits $t = 0$ and $t = \infty$

At $t = 0$, $x_s = x_0$ and $x_d = 0$

At $t = \infty$, $x_s = 0$ and $x_d = x_0$

This mean when force is applied at $t = 0$, the spring instantaneously stretches to x_0. However, as time progress extension of the spring starts to decrease and applied load starts to elongate dashpot. In the limit $t = \infty$, extension of the spring would become equal to zero.

5.



When protein sediments with contant velocity then

$$\gamma v = F_{net} = F_g - F_b$$

$$= mg - \rho V g$$

$$= mg - \rho . \frac{4}{3} \pi r^3 g$$

Given m=100 KDa, $r = 3$nm, g=10 m/s^2

Therefore,

$$\gamma v = 1.6 \times 10^{-27} \times 100 \times 1000 \times 10 - 1000 \times (4/3) \times 3.14 \times 27 \times 10^{-27} \times 10$$

$$= 1.6 \times 10^{-21} - (339.12/3) \times 10^{-23}$$

$$= 1.6 \times 10^{-21} - 1.13 \times 10^{2} \times 10^{-23}$$

$$= 1.6 \times 10^{-21} - 1.13 \times 10^{-21} \text{ N}$$

$$= 4.7 \times 10^{-22} \text{ N}$$

Therefore,

$$v = \frac{4.7 \times 10^{-22}}{6\times 3.14 \times 10^{-3} \times 3 \times 10^{-9}} = \frac{4.7 \times 10^{-22}}{56.52 \times 10^{-12}} = \frac{470 \times 10^{-24}}{56.52 \times 10^{-12}}$$

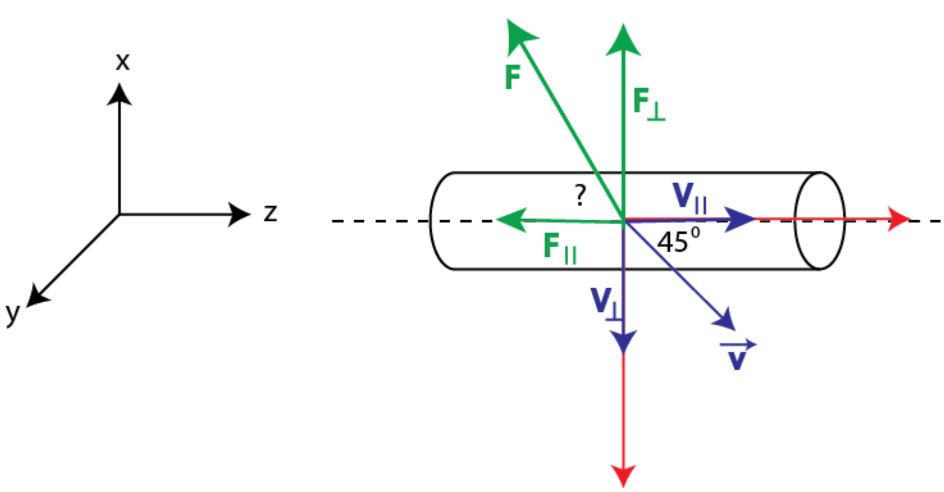
$$\approx 8.3 \times 10^{-12} \text{ m/s}$$

Therefore, Time required for sedimentation = $x/v = \dfrac{1 \times 10^{-2}}{8.3 \times 10^{-12}} = \dfrac{10 \times 10^{-3}}{8.3 \times 10^{-12}}$

$$\approx 1.2 \times 10^{9}\text{s}$$

$$\approx 38.05 \text{ Years}$$

6.



$v_{\perp} = v_{\parallel}$ (since angle is 45^0)

$F^2 = F_{\parallel}^2 + F_{\perp}^2 = \gamma_{\parallel}^2 v_{\parallel}^2 + \gamma_{\perp}^2 v_{\perp}^2 = \gamma_{\parallel}^2 v_{\parallel}^2 + \gamma_{\parallel}^2 v_{\perp}^2 = 4\gamma_{\parallel}^2 v_{\parallel}^2 = 4F_{\parallel}^2$ since ($\gamma_{\perp} = \sqrt{3}\gamma_{\parallel}$)

Therefore, $F = 2F_{\parallel}$

$$\cos\theta = \frac{F_{\parallel}}{F}$$

Therefore, $\theta = \cos^{-1}\left(\frac{F_{\parallel}}{F}\right) = \cos^{-1}\left(\frac{F_{\parallel}}{2F_{\parallel}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^0$