PH108 : Electricity & Magnetism

Weekly Quiz 1 - Use of Plane Polar Co-ordinates

24 January, 2018

Answer

$\int_0^{2\pi}\int_0^{\infty} e^{-r^2} dr\ r\, d\theta$ **[$\frac{1}{2}$ mark]**

I $= \sqrt{\pi}$ **[$\frac{1}{2}$ mark]**

Solution

I $= \int_{-\infty}^{\infty} e^{-x^2} dx = \int_{-\infty}^{\infty} e^{-y^2} dy \implies I^2 = (\int_{-\infty}^{\infty} e^{-x^2} dx) * (\int_{-\infty}^{\infty} e^{-y^2} dy)$

$\implies I^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2} e^{-y^2} dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2} dxdy$

$\implies I^2 = \int_0^{2\pi}\int_0^{\infty} e^{-r^2} dr\ r\, d\theta$

$\implies I^2 = 2\pi \int_0^{\infty} e^{-r^2} r\ dr$

Let t=r^2 $\implies$ dt = 2r dr

$\implies I^2 = \pi \int_0^{\infty} e^{-t} dt = \pi \implies I = \sqrt{\pi}$