MA 108 - Spring 2018
Tutorial Sheet 4

1. Given one solution y_1 of homogeneous part, find a particular solution of the ODE by putting $y = vy_1$ into the ODE and solving for v.

   (a) $y'' + 4xy' + (4x^2 + 2)y = 8e^{x(x+2)}; \quad y_1 = e^{-x^2}$.

   (b) $4x^2y'' - 4x(x + 1)y' + (2x + 3)y = 4x^{5/2}e^{2x}, \quad y_1 = x^{1/2}$.

   (c) $xy'' - y' + 4x^3y = 0, \quad x > 0; \quad y_1(x) = \sin x^2$.

2. Find the general solution of the ODE and then solve the IVP. Here y_1 is a solution of homogeneous part.

   (a) $x^2y'' - 3xy' + 4y = 4x^4, \quad y(-1) = 7,\ y'(-1) = 8;\ y_1 = x^2$.

   (b) $(3x - 1)y'' - (3x + 2)y' - (6x - 8)y = 0, \quad y(0) = 2,\ y'(0) = 3;\ y_1 = e^{2x}$.

3. Find a general solution to the following ODE and IVP where mentioned.

   (a) $y''' - y = 0$.

   (b) $y^{(4)} + 64y = 0$.

   (c) $y^{(5)} + y^{(4)} + y''' + y'' + y' + y = 0$.

   (d) $y^{(4)} + 2y'' + y = 0$.

   (e) $y''' - 2y'' + 4y' - 8y = 0, \quad y(0) = 0,\ y'(0) = -2,\ y''(0) = 0$

   (f) $y''' - 6y'' + 12y' - 8y = 0, \quad y(0) = 1,\ y'(0) = -1,\ y''(0) = -4$

   (g) $y^{(4)} + 2y''' - 2y'' - 8y' - 8y = 0, \quad y(0) = 5,\ y'(0) = -2,\ y''(0) = 6,\ y'''(0) = 8$.

4. Find the fundamental set of solutions for the following equations.

   (a) $(D^2 + 9)^3D^2y = 0$.

   (b) $D^3(D - 2)^2(D^2 + 4)^2y = 0$.

   (c) $[(D - 1)^4 - 16]y = 0$

5. Using the annihilator method, find the general solution (i.e. $y = y_p + c_1y_1 + c_2y_2$, where y_p is a particular solution of ODE and y_1, y_2 is a basis of solutions of homogeneous part).

   (a) $y'' - 2y' - 3y = e^x(-8 + 3x)$.

   (b) $y'' + 5y' + 6 = \cos x + \sin x$.

   (c) $y''' - y'' - y' + y = 2e^{-x} + 3$

6. Find the form of the particular solution (without explicitly finding the particular solution) of ODE's.

   (a) $y'' + y = e^{-x}(2 - 4x + 2x^2) + e^{3x}(8 - 12x - 10x^2)$.

(b) $y'' + 6y' + 13y = e^{-2x}[(4 + 20x)\cos 3x + (26 - 32x)\sin 3x]$.

(c) $y'' + 2y' + y = 8x^2 \cos x - 4x \sin x$.

(d) $y^{(4)} - 4y'' = 3x + \cos x$.

(e) $y''' - y'' - y' + y = e^x(7 + 6x)$.

(f) $4y^{(4)} - 11y'' - 9y' - 2y = -e^x(1 - 6x)$.

(g) $y''' + 3y'' + 4y' + 12y = 8\cos 2x - 16\sin 2x$.

(h) $y^{(4)} + 3y''' + 2y'' - 2y' - 4y = -e^{-x}(\cos x - \sin x)$

7. Consider the ODE $Ly = f$. Find the annihilator A for f. Then write down a basis for the solutions of the equation $ALy = 0$.

(a) $y''' - 2y'' + y' = x^3 + 2e^x$

(b) $y^{(4)} - y''' + y'' + y' = x^2 + 4 + x\sin x$.

(c) $y^{(4)} + 4y'' = \sin 2x + xe^x + 4$.

(d) $y''' - 2y'' + y' - 2y = -e^x[(9 - 5x + 4x^2)\cos 2x - (6 - 5x - 3x^2)\sin 2x]$

(e) $y^{(4)} - 7y''' + 18y'' - 20y' + 8y = e^{2x}(3 - 8x - 5x^2)$.

(f) $y^{(4)} + 5y''' + 9y'' + 7y' + 2y = e^{-x}(30 + 24x) - e^{-2x}$.