PH108 : Electricity & Magnetism

Weekly Quiz 2 - Use of Variable Unit Vectors

31 January, 2018

Answer

$$\mathbf{r} = (R - vt)\hat{r}$$

$$\mathbf{a} (+ \omega^2 \mathbf{r}) = -2 \omega \mathbf{v} \hat{\theta}$$

Solution

$$\mathbf{r} = (\mathbf{R} - \mathbf{v}t)\hat{r} = (\mathbf{R} - \mathbf{v}t)(\cos(\omega t) \hat{\imath} + \sin(\omega t) \hat{\jmath})$$

$$\implies \mathbf{v} = -v(\cos(\omega t)\hat{\imath} + \sin(\omega t)\hat{\jmath}) + (R - vt)(R - vt)(-\sin(\omega t)\hat{\imath} + \cos(\omega t)\hat{\jmath}) = -v\hat{r} + (R - vt)\hat{\theta}$$

$$\implies \mathbf{a} = -v\omega(-\sin(\omega t)\hat{\imath} + \cos(\omega t)\hat{\jmath}) - v\omega(-\sin(\omega t)\hat{\imath} + \cos(\omega t)\hat{\jmath}) + (R - vt)\omega^{2}(-\cos(\omega t)\hat{\imath} - \sin(\omega t)\hat{\jmath})$$

$$\implies \mathbf{a} + \omega^{2}\mathbf{r} = -2v\omega\hat{\theta}$$