EE 207: Assignment 3: Solutions

Solution 1:

Q1) a) given that $x_n > x_0$.

The equation, $\mathcal{E} = -\frac{dV}{dx}$

$$V = -\int_{-x_p}^{x_n} \mathcal{E}\, dx$$

$$= \int_{V(-x_p)}^{V(x_n)} dV = V(x_n) - V(-x_p) = V_{bi}$$

$$V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E}\, dx$$

$$= \frac{kT}{q} \int_{n(-x_p)}^{n(x_n)} \frac{dn}{n}$$

as
$$\mathcal{E} = -\frac{D_n}{\mu_n} \frac{dn/dx}{n}$$

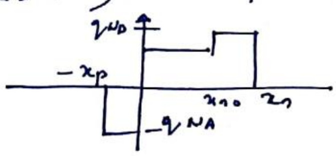
$$= -\frac{kT}{q} \frac{dn/dx}{n}$$

$n(-x_p) = \frac{n_i^2}{N_A}$ & $x_n > x_0$ $n(x_n) = N_D$ for the given profile

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

only the doping at the depletion region edges are relevant in determining V_{bi}.

b) $\rho = q(N_D - N_A)$ $\quad -x_p \le x \le x_n$



c)
$$\rho = \begin{cases} 0 & x < -x_p \\ -qN_A & -x_p < x < 0 \\ qN_D/2 & 0 \le x \le x_0 \\ qN_D & x_0 \le x < x_n \\ 0 & x > x_n \end{cases}$$

$$\frac{d\mathcal{E}}{dx} = \begin{cases} -qN_A/\epsilon_s\epsilon_0 & -x_p \le x \le 0 \\ qN_D/2\epsilon_s\epsilon_0 & 0 \le x < x_0 \\ qN_D/\epsilon_s\epsilon_0 & x_0 \le x \le x_n \end{cases}$$

$$\int_{\mathcal{E}(0)}^{\mathcal{E}(x)} d\mathcal{E} = \frac{qN_D}{2\epsilon_s\epsilon_0}\int_0^x dx$$

$$\mathcal{E}(x) = \mathcal{E}(0) + \frac{qN_D}{2\epsilon_s\epsilon_0}x \qquad (1) \qquad 0 \le x \le x_0$$

$$\int_{\mathcal{E}(-x_p)=0}^{\mathcal{E}(x)} d\mathcal{E} = -\frac{qN_A}{\epsilon_s\epsilon_0}\int_{-x_p}^{x} dx$$

$$\mathcal{E}(x) = -\frac{qN_A}{\epsilon_s\epsilon_0}(x_p) + c$$

~~at x (~~ at $x=-x_p$ $\mathcal{E}=0$

$$c = \frac{qN_A}{\epsilon_s\epsilon_0}(x_p)$$

$$\mathcal{E}(x) = -\frac{qN_A}{\epsilon_s\epsilon_0}(x_p+x)$$

$$\mathcal{E}(0) = -\frac{qN_A}{\epsilon_s\epsilon_0}(x_p) \qquad (2)$$

Put (2) in (1)

$$\mathcal{E}(x) = -\frac{qN_A}{\epsilon_s\epsilon_0}x_p + \frac{qN_D}{2\epsilon_s\epsilon_0}x \qquad 0 \le x \le x_0$$

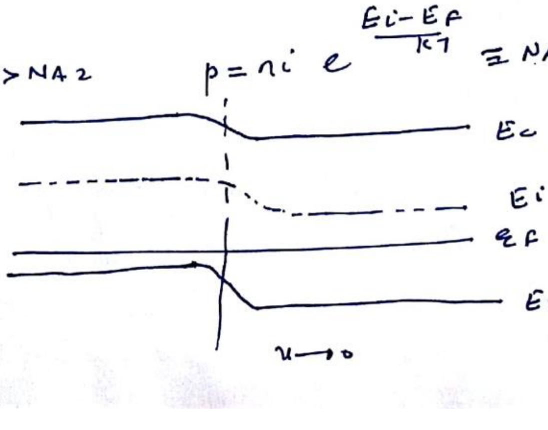
If integration is performed from $x=x_1$ backward into lighter doped region,

$$\mathcal{E}(x) = -\frac{qN_D}{\epsilon_s\epsilon_0}\left(x_1 - \frac{x_0}{2} - x/2\right) \qquad 0 \le x \le x_0$$

$$\mathcal{E}(x) = \begin{cases} -\frac{qN_A}{\epsilon_s\epsilon_0}(x_p+x) & \\ -\frac{q}{\epsilon_s\epsilon_0}\left(N_A x_p - \frac{N_D x}{2}\right) & 0 \le x \le x_0 \\ \text{or } -\frac{qN_D}{\epsilon_s\epsilon_0}\left(x_1 - \frac{x_0}{2} - x/2\right) & \\ -\frac{qN_D}{\epsilon_s\epsilon_0}(x_1 - x) & x_0 \le x \le x_1 \end{cases}$$

Solution 2:

Q2) $N_{A1} > N_{A2}$ $\quad p = n_i e^{\frac{E_i - E_F}{kT}} \cong N_A$ far from junction



b)

$$n(x_n) = \frac{n_i^2}{N_{A2}}$$

$$n(-x_p) = n_i^2/N_{A1}$$

$$V_{bi} = \frac{KT}{q}\int_{n(-x_p)}^{n(x_n)} \frac{dn}{n}$$

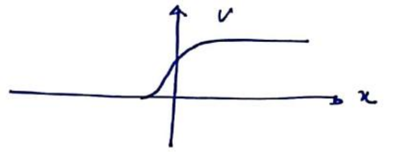
$$V_{bi} = \frac{KT}{q}\ln\left(\frac{N_{A1}}{N_{A2}}\right)$$

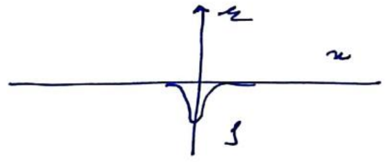
$$V_{bi} = \frac{1}{q}\left((E_i - E_F)_{p_1 side} - (E_i - E_F)_{p_2 side}\right)$$

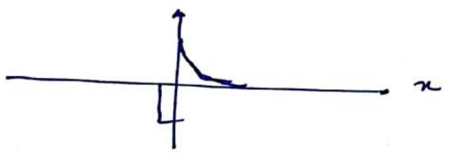
$$= \frac{1}{q}\left(KT\ln(N_{A1}/n_i) - KT\ln\left(\frac{N_{A2}}{n_i}\right)\right]$$

$$= \frac{KT}{q}\ln\frac{N_{A1}}{N_{A2}}$$

c)







exact functional dependencies can't be deduced employing a graphical approach.

Solution 3:

(a) Let us examine the minority carrier diffusion equation for hole. In general

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

For the steady state problem at hand $\partial\Delta p_n/\partial t = 0$. Also, $\partial^2\Delta p_n/\partial x^2 = 0$ if one goes far from the junction on the n-side where the carrier perturbation introduced by the junction has decayed to zero. Thus

$$0 = -\frac{\Delta p_n(x\rightarrow\infty)}{\tau_p} + G_L$$

or

$$\Delta p_n(x\rightarrow\infty) = G_L\tau_p \qquad \Leftarrow \text{boundary condition}$$

(b) One simply parallels the ideal diode derivation to obtain the desired I–V_A relationship. Given a p^+-n junction, however, we need consider only the lightly doped n-side of the junction. Specifically, under steady state conditions and with x' as defined in Fig. 6.5(a), we must solve

$$0 = D_P \frac{d^2\Delta p_n}{dx'^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

subject to the boundary conditions

$$\Delta p_n(x'=0) = (n_i^2/N_D)\left(e^{qV_A/kT} - 1\right)$$

$$\Delta p_n(x'\rightarrow\infty) = G_L\tau_p$$

The general solution is

$$\Delta p_n(x') = G_L\tau_p + A_1 e^{-x'/L_P} + A_2 e^{x'/L_P}$$

Because $\exp(x'/L_P) \rightarrow \infty$ as $x' \rightarrow \infty$, the only was the second boundary condition can be satisfied is for A_2 to be identically zero. With $A_2 = 0$, the application of the first boundary condition yields

$$\Delta p_n(x'=0) = G_L\tau_p + A_1 = (n_i^2/N_D)\left(e^{qV_A/kT} - 1\right)$$

or

$$A_1 = (n_i^2/N_D)\left(e^{qV_A/kT} - 1\right) - G_L\tau_p$$

and

$$\Delta p_n(x') = G_L\tau_p + \left[(n_i^2/N_D)\left(e^{qV_A/kT} - 1\right) - G_L\tau_p\right] e^{-x'/L_P}$$

The associated hole current density is then

$$J_P(x') = -qD_P \frac{d\Delta p_n}{dx'} = q\frac{D_P}{L_P}\left[(n_i^2/N_D)\left(e^{qV_A/kT}-1\right)-G_L\tau_p\right]e^{-x'/L_P}$$

and for a p^+-n diode

$$I = AJ = A[J_N(x{=}-x_p) + J_P(x{=}x_n)] \cong AJ_P(x'{=}0)$$

or

$$I = qA\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT}-1\right) - qA\frac{D_P\tau_p}{L_P}G_L$$

Finally noting $D_P\tau_p = L_P^2$, we conclude

$$\boxed{\begin{aligned} I &= I_0\left(e^{qV_A/kT}-1\right) + I_L \\ \text{where}\quad I_0 &= qA\frac{D_P}{L_P}\frac{n_i^2}{N_D} \\ I_L &= -qAL_PG_L \end{aligned}}$$

Solution 4:

Case (b)

→ Assuming $\tau_p \neq 0$ in 'second' N-region and $L_p << x_c - x_b$, We assume a finite non-zero excess charge carrier concentration at the boundary of two N-regions

→ According to continuity eqn

$$\frac{d\Delta p_n(x,t)}{dt} = D_p \frac{d^2\Delta p_n(x,t)}{dx^2} - \frac{\Delta p_n(x,t)}{\tau_p} \quad \textbf{(1)}$$

we have The following profiles of excess charge carrier in steady state

→ N-region with $\tau_p = \infty$

$$\Delta p_n(x) = Ax + B. \quad (2)$$

→ N-region with $\tau_p \to 0$, $\tau_p > 0$.

$$D_p \frac{d^2\Delta p_n(x)}{dx^2} = \frac{\Delta p_n(x)}{\tau_p}$$

$$\Rightarrow \frac{d^2\Delta p_n(x)}{dx^2} = \frac{\Delta p_n(x)}{L_p^2}$$

$$(\because D_p \tau_p = L_p^2)$$

which gives

$$\Delta p_n(x) = C \exp\left(\frac{x-x_b}{L_p}\right) + D \exp\left(-\frac{x-x_b}{L_p}\right) \quad \text{--- (3)}$$

Now, we have $L_p << x_b - x_c$

so we assume $\Delta p_n(\infty) = 0$

which gives $C = 0$ from eq^n (3).

Hence, we have the following profile for excess charge carriers in quasi-neutral N regions

$$\Delta p_n(x) = A\,x + B \quad ; \text{for } W_D < x \le x_b,$$

$$= D \exp\left(\frac{-(x-x_b)}{L_p}\right) ; \text{for } x_b \le x < x_c. \quad \text{--- (4)}$$

Now, we have following boundary conditions

i. $\Delta p_n(W_D) = p_{no} \exp\left(\frac{V_A}{V_T}\right)$

as shown in case (a)

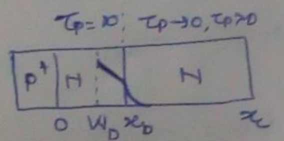
ii. $\Delta p_n(x_b^-) = \Delta p_n(x_b^+)$ and

iii. $\left.\frac{d}{dx}\Delta p_n(x)\right|_{x=x_b^-} = \left.\frac{d}{dx}\Delta p_n(x)\right|_{x=x_b^+}$

conditions (ii) and (iii) correspond to charge continuity and current continuity, respectively.

We can solve for three variables in eq^n 4 using these three boundary condition and get

$$\Delta p_n(x) = \frac{p_{no}\exp\left(\frac{V_A}{V_T}\right)}{W_D - x_b - L_p}(x - x_b - L_p) \quad \text{for } W_D < x \le x_b$$



(excess carrier profile on N-side)

$$= -\frac{L_p\, p_{no}\exp\left(\frac{V_A}{V_T}\right)}{W_D - x_b - L_p}\exp\left(\frac{-(x-x_b)}{L_p}\right) \quad \text{for } x_b \le x < x_c.$$

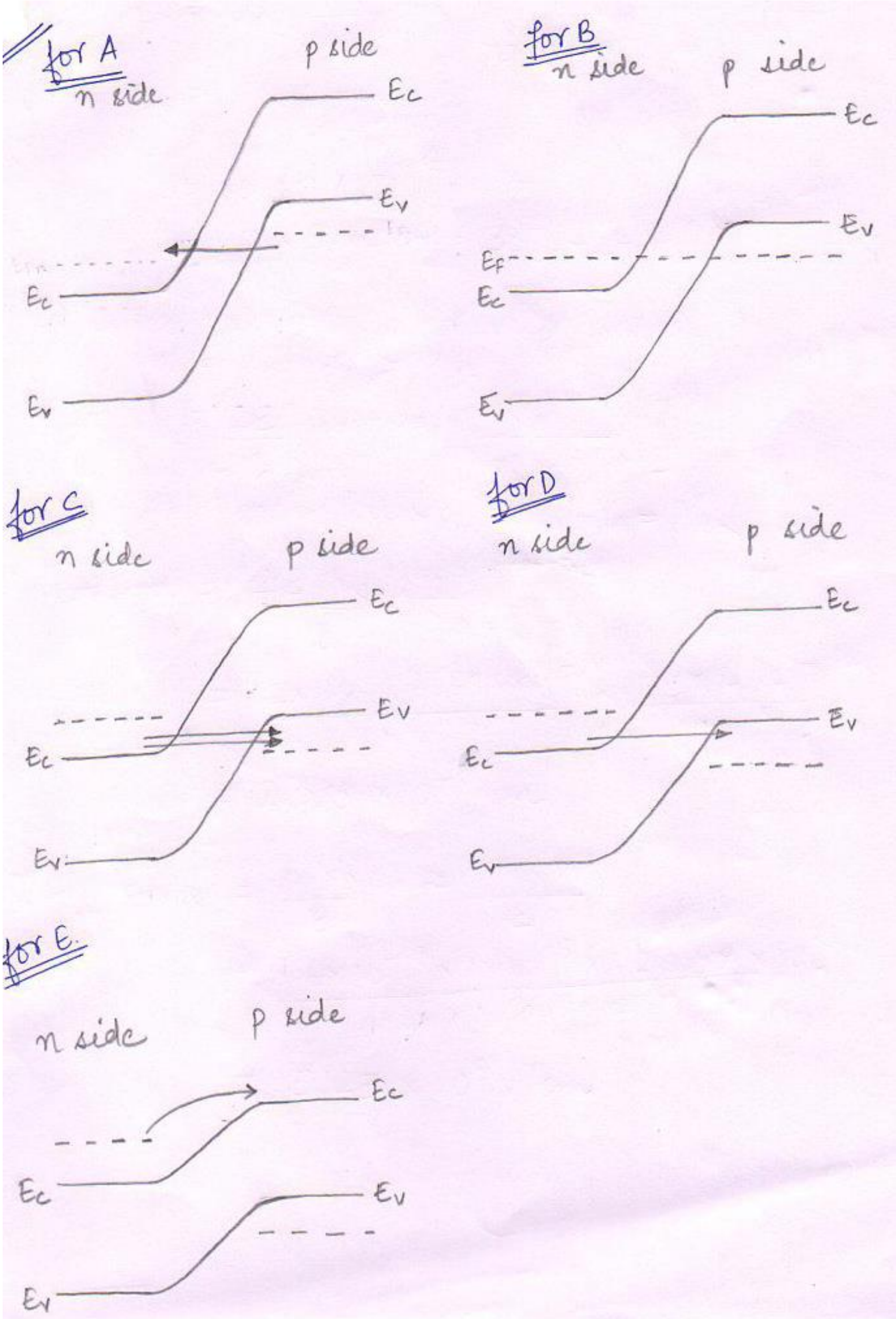
Using this we can compute current in N-region with $\tau_p = \infty$ as

$$J_p = -q D_p \left.\frac{d\,\Delta p_n(x)}{dx}\right|_{W_D < x \le x_b}$$

$$J_p = \frac{q D_p\, p_{no}\exp\left(\frac{V_A}{V_T}\right)}{x_b + L_p - W_D} \quad W_D < x \le x_b$$

again W_D is a function of V_A.

Solution 5:


for A
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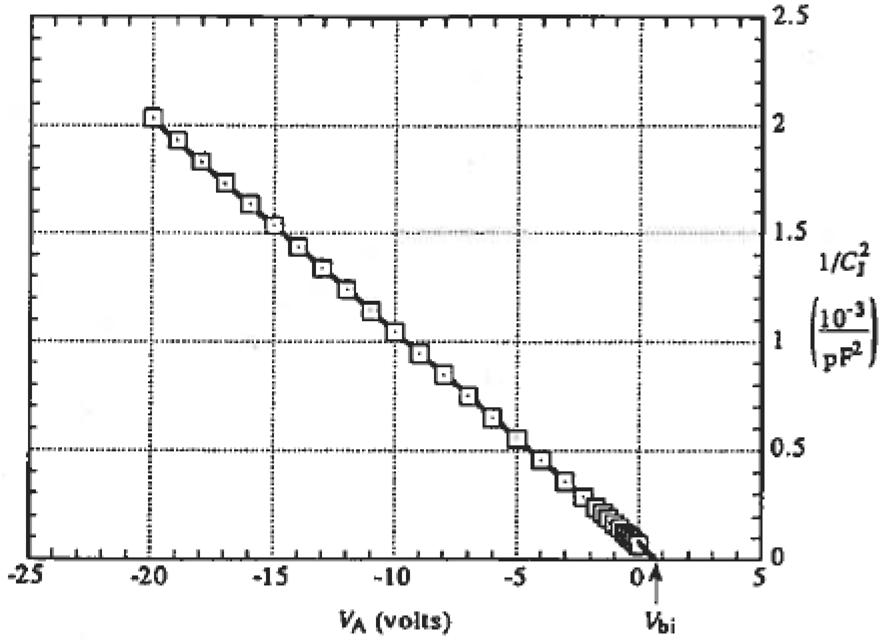
Solution 6:



$$N_A = \frac{2}{qK_S\varepsilon_0 A^2|\text{slope}|}$$

$$= \frac{2}{(1.6\times10^{-19})(11.8)(8.85\times10^{-14})(3.72\times10^{-3})^2(9.78\times10^{19})}$$

$$= 8.84\times10^{15}/\text{cm}^3$$

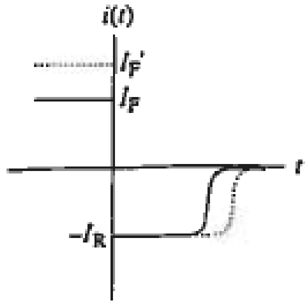
and

$$V_{bi} = V_A\big|_{1/C_J^2=0} = \frac{6.89\times10^{19}}{9.78\times10^{19}} = \mathbf{0.70\ V}$$

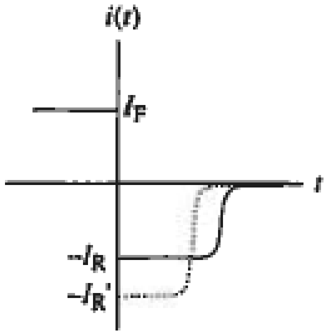
It should be noted that the deduced V_{bi} is lower than one would expect from the $N_A \cong 9\times10^{15}/\text{cm}^3$ p-side doping. The V_{bi} value deduced from the C–V data is subject to serious extrapolation errors and is sensitive to doping variations in the immediate vicinity of the metallurgical junction.

Solution 7:

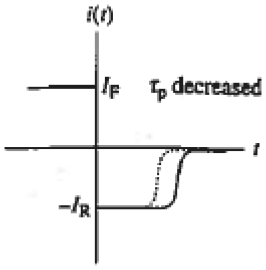
(i) Increased I_F: Increasing I_F increases the stored charge inside the diode. Since the stored charge is increased and the removal rate is unchanged, it will take longer to remove the stored charge, and hence storage delay time is expected to increase.



(ii) Increased I_R: Increasing I_R increases the rate at which the stored charge is removed by the reverse current flow. Hence, the storage delay time is reduced.



(iii) Reduced τ_p: A shorter minority carrier lifetime increases the carrier recombination rate and will therefore decrease the storage delay time.



Solution 8:

The first thing to do would be to calculate the charge transient using the Charge Control equation.

$$\frac{dQ_n}{dt} + \frac{Q_n}{\tau_n} = I_{diff}$$

The charge in the pre-switching steady-state is given by:

$$Q_n\left(0^-\right) = I_F \tau_n \Leftarrow \frac{Q_n\left(t<0\right)}{\tau_n} = I_{diff} = I_F$$

$\therefore Q_n\left(0^+\right) = Q_n\left(0^-\right) = I_F \tau_n$ is the initial condition.

For $t > 0$, we have $I_{diff} = 0$, since the diode is open-circuited.

$$\frac{dQ_n}{dt} + \frac{Q_n}{\tau_n} = 0 \Rightarrow Q_n\left(t\right) = Q_n\left(0^+\right) e^{-t/\tau_n}$$

$$Q_n\left(t\right) = I_F \tau_n e^{-t/\tau_n} \ldots[1]$$

For the voltage transient, we also need to use the quasi-static approximation. Under forward bias $v \gg k_B T$, this implies:

$$\frac{Q_n\left(t\right)}{\tau_n} = i\left(t\right) = I_0 e^{qv(t)/k_B T}$$

$$Q\left(t\right) = I_0 \tau_n e^{qv(t)/k_B T} \ldots[2]$$

From [1] and [2]:

$$I_F e^{-t/\tau_n} = I_0 e^{qv(t)/k_B T} \ldots[3]$$

Using $I_F = I_0 e^{qV_F/k_B T}$ in [3], we get:

$$e^{qV_F/k_B T} e^{-t/\tau_n} = e^{qv(t)/k_B T}$$

$$\Rightarrow v\left(t\right) = V_F - \frac{k_B T}{q} \frac{t}{\tau_n} \ldots[4]$$

This is the voltage transient.