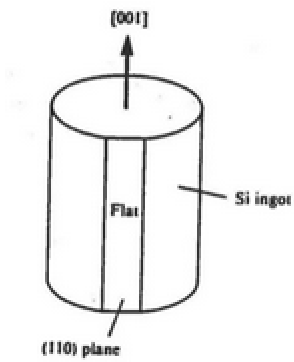
EE 207: Assignment 1

Submission deadline: 11:59 PM on 6th August, 2018 (Monday)

Submission mode: Online Moodle interface. Link will be shared soon.

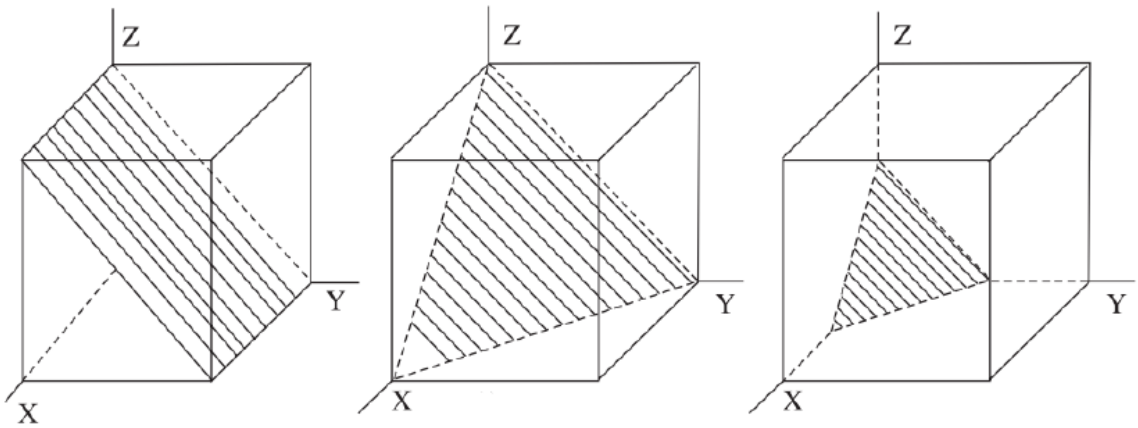
Crystal Structure

1. A cylindrical Si ingot is produced whose axis is oriented in the [001] direction. A flat is subsequently machined along the side of the cylinder forming a (110) plane as shown in the following figure. A research program requires wafers whose surfaces are (112) planes. Indicate how the ingot must be sawed to achieve the desired wafers. **(2 Marks)**



2. (a) Find the ratio of intercepts (x:y:z) on the crystal axes by plane (231) in a simple cubic lattice. **(1 Mark)**

   (b) Find the Miller indices of the following planes in a simple cubic lattice. **(3 Marks)**



   (Note: In the third figure the plane cuts the unit cell at the centre of the edges on all axes)

3. Show that reciprocal lattice of a reciprocal lattice gives a direct lattice; and hence show that BCC and FCC are reciprocal lattice pairs. **(4 Marks)**

4. Consider the plane (hkl) in direct lattice of a crystal structure and $\boldsymbol{G} = h\boldsymbol{b_1} + k\boldsymbol{b_2} + l\boldsymbol{b_3}$ is the reciprocal lattice vector. Prove that the distance between two adjacent parallel planes of the lattice is $d(hkl) = \frac{2\pi}{|\boldsymbol{G}|}$. **(4 Marks)**

5. (a) Show that the ideal c/a ratio for a hexagonal closed pack structure (hcp) is $\sqrt{\frac{8}{3}}$. Assume that the atoms touch each other. (Hint: A Hexagonal closed pack structure consists of two interpenetrating simple Hexagonal Bravais Lattices, displaced from one another by $\frac{a_1}{3} + \frac{a_2}{3} + \frac{a_3}{3}$) **(4 Marks)**

(b) Sodium transforms from bcc to hcp at about 23K. Assuming that density remains fixed throughout the transition, find the lattice constant a_h of the hexagonal phase in terms of the lattice constant a_b of the cubic phase. (Use the $\frac{c}{a}$ ratio from the previous question). **(2 Marks)**

Quantum Mechanics

6. Consider the wavefunctions $\psi(k) = \frac{1}{\sqrt{L}} e^{ikx}$ defined over the interval $x \in \left(-\frac{L}{2}, \frac{L}{2}\right)$. Show that
   (a) These wavefunctions are normalized $\forall\, L$. **(2 Marks)**
   (b) These wavefunctions are orthogonal for $L \rightarrow \infty$. **(3 Marks)**

7. A particle of mass m that moves freely inside an infinite well of length a has the following wave function:

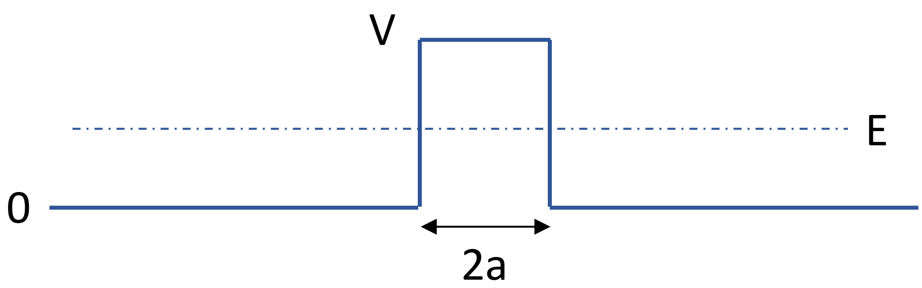
$$\psi(x,0) = \frac{A}{\sqrt{a}}\sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}}\sin\left(\frac{3\pi x}{a}\right) + \sqrt{\frac{1}{5a}}\sin\left(\frac{5\pi x}{a}\right)$$

   where a is a real constant.
   (a) Find a so that $\psi(x,0)$ is normalized. **(2 Marks)**
   (b) If measurements are carried out for energy, what energies are observed, and what are the corresponding probabilities? Calculate the average energy. **(2 Marks)**
   (c) Find the wave function $\psi(x,t)$ at any later time t. **(1 Mark)**

8. Prove that the transmission coefficient for the following single potential barrier is: **(3 Marks)**

$$\frac{1}{T} = 1 + \frac{1}{4}\frac{V^2}{E(V-E)}\sinh^2(2\kappa a)$$

$$where\ \frac{\hbar^2\kappa^2}{2m} = V - E$$



(a) Also plot $T\ versus\ 2a$, where $V = 2eV$, $m = m_0$ and $2a$ varies from 1 nm to 10 nm for various $\frac{E}{V} = 0.5, 1, 1.05, 1.5\ and\ 3$ **(5 Marks)**
(b) Repeat the above for $m = 0.5m_0$ **(2 Marks)**
(c) What are your conclusions from the plots in (a) and (b)? **(5 Marks)**
Note: Use MATLAB for plotting. Please submit the codes as well.