Tutorial I

1. Show that a complex polynomial of degree n has exactly n roots. (Assuming the fundamental theorem of algebra)

2. Show that a real polynomial that is irreducible has degree at most two i.e., if

$$f(x) = a_0 + a_1 x + \cdots + a_n x^n, \quad a_i \in \mathbb{R},$$

then there are non-constant real polynomials g and h such that $f(x) = g(x)h(x)$ if $n \geq 3$.

3. Check for differentiability and holomorphicity:

   (i) $f(z) = c$, $c \in \mathbb{C}$;

   (ii) $f(z) = z$;

   (iii) $f(z) = z^n$, $n \in \mathbb{N}$;

   (iv) $f(z) = \mathrm{Re}(z)$;

   (v) $f(z) = |z|^2$;

   (vi) $f(z) = \bar{z}$;

   (vii) $f(z) = \begin{cases} \frac{z}{\bar{z}} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0. \end{cases}$

4. If $f(z)$ is a real valued function in a domain $\Omega \subseteq \mathbb{C}$, then show that for any $z \in \Omega$ either $f'(z) = 0$ or $f'(z)$ does not exist. Hence, conclude that a real valued function f defined on a domain is holomorphic if and only if it is constant.