Problem Set 5

Data Analysis and Interpretation (EE 223)

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1. Let $X_1, \dots, X_n$ be i.i.d. Poisson(λ). Find UMVUE for λ.
2. Let $X_1, \dots, X_n$ be i.i.d. Poisson(λ).
   (a) Find $E_\lambda[X_1^2]$.
   (b) Find $E_\lambda[X_1^2 | \sum_{i=1}^n X_i = y]$.
   (c) Find $\psi(\lambda)$ s.t. $E_\lambda[X_1^2 | \sum_{i=1}^n X_i]$ is UMVUE for $\psi(\lambda)$.
3. Let $X_1, \dots, X_n$ be i.i.d. Gaussian(μ, σ^2), where μ is known. Consider the following family of estimators for σ^2,
$$\delta_K(X_1, \dots, X_n) = \frac{1}{K} \sum_{i=1}^n (X_i - \bar{X})^2,$$
   where $\bar{X}$ is the sample mean.
   (a) Find MSE for $\delta_K(\cdot)$.
   (b) Find the optimal value of K for which MSE is the minimum.
4. Let $X_1, \dots, X_n$ be i.i.d. RVs with $f_\lambda(\cdot)$ s.t.
$$f_\lambda(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}, \quad \forall\, x \geq 0\,,\ \lambda \in (0, \infty)$$
   Let $\psi(\lambda) = \lambda^2$. Find UMVUE for $\psi(\cdot)$.
5. Let $X_1, \dots, X_n$ be i.i.d. RVs with $f_\theta(\cdot)$ where
$$\begin{aligned} f_\theta(x) &= 2x/\theta^2, \quad && 0 < x < 0, \\ &= 0, && \text{otherwise.} \end{aligned}$$
   Let $\delta_c(\bar{x}) = c \max\{x_1, \dots, x_n\}$.
   (a) Find MSE for $\delta_c(\bar{x})$.
   (b) Find c that minimizes MSE.
6. Let $X \sim N(\theta, \theta^2)$ and $\theta \in [0, \infty)$. Find MLE for θ^2.
7. Let $X_1, \dots, X_n$ be i.i.d. RVs. Find MLE for θ.
   (a) Bernoulli distribution with parameter θ
   (b) Geometric distribution
$$f_\theta(x) = (1 - \theta)^{x-1} \theta$$
   (c) Poisson distribution
$$f_\theta(x) = \frac{\theta^x e^{-\theta}}{x!}$$
   (d) Binomial distribution
$$f_\theta(x) = \frac{n!}{x!(n-x)!} \theta^x (1 - \theta)^{n-x}$$

(e) Negative Binomial distribution

$$f_\theta(x) = \binom{x-1}{r-1} \cdot \theta^r \cdot (1-\theta)^{x-r}$$

(f) Exponential distribution

$$f_\theta(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \qquad x \geq 0$$

(g) Gaussian distribution

$$f_\theta(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\theta)^2/2\sigma^2}$$

(h) Rayleigh distribution

$$f_\theta(x) = \frac{x}{\theta^2} e^{\left(\frac{-x^2}{2\theta^2}\right)}, \qquad x > 0$$

(i) Gamma distribution

$$f_\theta(x) = \frac{\theta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\theta x}, \qquad x > 0, \quad \theta, \alpha > 0.$$

(j) Pareto distribution

$$f_\theta(x) = \theta \frac{\beta^\theta}{x^{\theta+1}}, \qquad x \geq \beta, \quad \theta, \beta > 0.$$

8. Let $X_1, \ldots, X_n$ be i.i.d. Poisson(λ). Find MLE for $(1-\lambda)^2$.