1. The number of roots of $P_{101}(x)$ lying in the open interval $(0,1)$ equals

   (A) 49 (B) 50 (C) 51 (D) 52

2. For $x>0$, the equation $x^2y''-x(1+x)y'+y=0$ has a solution $xe^x\log x+\sum_{n=1}^{\infty} b_nH_nx^{n+1}$ with b_n equal to

   (A) $\frac{-1}{(n-1)!}$ (B) $\frac{-1}{n!}$ (C) $\frac{2^n}{n!}$ (D) $\frac{1}{(n-1)!}$

3. The domain of analyticity of a real-valued function on $\mathbb{R}$ can be

   (A) $\{0\}$ (B) $\bigcup_{n=1}^{\infty}\{1/n\}$ (C) $[0,1]$ (D) $(-1,1)\setminus\{0\}$

4. A pair (a,b) of real numbers is said to be good if there exists a real number p such that $aJ_p(x)+bJ_{-p}(x)=0$ for all $x>0$. The set of all good pairs is defined by

   (A) $a^2-b^2=0$ (B) $a=b=0$ (C) $a-b=0$ (D) $a+b=0$

5. If $x^{50}+x^{49}=\sum_{n=0}^{50}c_nP_n(x)$, then the sum of even coefficients $c_0+c_2+c_4+c_6+\cdots+c_{50}$ equals

   (A) 0 (B) 1 (C) 50/99 (D) 51/101

6. The equation $x(e^x-1)y''+(\sin x)y'+y=0$ has a

   (A) irregular singular point at $x=0$ (B) irregular singular point at $x=1$
   (C) regular singular point at $x=0$ (D) regular singular point at $x=1$

7. In the interval $(-1,217)$, the equation $(1+x)y'=-y/2$ with $y(0)=1$ has a power series solution $\sum_{n\geq 0} a_n(x-108)^n$ with the value of $a_{207}(109)^{207}$ equal to

   (A) $a_0P_{414}(0)$ (B) $a_0P_{414}(108)$ (C) $a_0P_{207}(0)$ (D) $a_0J_{207}(108)$

8. The value of ${J_0}^2(2)-{J_2}^2(2)$ equals

   (A) 0 (B) $J_0(2)J_2'(2)$ (C) $J_1(2)J_1'(2)$ (D) $2J_1(2)J_1'(2)$

9. The radius of convergence of the power series $\sum_{n=0}^{\infty}\frac{(2n)!}{3^{2n}(n!)^2}x^{2n}$ equals

   (A) 3 (B) 9 (C) 3/2 (D) 9/4

10. Let $g(x)$ be the quadratic polynomial with roots $\pm\sqrt{\frac{1}{3}}$ with $g(1)=2/3$. Let $f(x)$ be the polynomial solution of the equation $((1-x^2)y')'+6y=0$ with $f(1)=1$. The value of $\int_{-1}^{1} f(x)g(x)dx$ equals

    (A) 0 (B) 2/3 (C) 2/5 (D) 4/15

11. The recursion obtained while solving $y''-xy'+y=0$ by the power series method is

    (A) $(n+2)(n+1)a_{n+2}=(n-1)a_n$ (B) $(n+2)(n+1)a_{n+2}=na_n$
    (C) $(n+2)(n+1)a_{n+2}=(n-1)a_{n-1}$ (D) $(n+2)(n+1)a_{n+2}=(n+1)a_{n+1}-a_n$

12. Let a and b be the number of solutions of $J_0(x) = P_0(x)$ and $J_1(x) = P_1(x)$ respectively in the interval $[0, 1]$. Then (a, b) is

(A) $(0, 1)$ (B) $(0, 2)$ (C) $(1, 1)$ (D) $(1, 2)$

13. An inner product on $\mathbb{R}^2$ can be defined by setting $\langle (a_1, a_2), (b_1, b_2) \rangle$ equal to

(A) $a_1b_1 - a_2b_2$ (B) $a_1^2b_1^2 + a_2^2b_2^2$ (C) $(a_1 + a_2)(b_1 + b_2)$ (D) $2a_1b_1 - a_1b_2 - a_2b_1 + 5a_2b_2$

14. The set of all points where the Taylor series of the function $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{x^2}{(1 + x^2)^n}$ around the point $x = e$ converges to $f(x)$ is

(A) $\emptyset$ (B) $(0, 2e)$ (C) $\mathbb{R} \setminus \{0\}$ (D) $\mathbb{R}$

15. The value of $\displaystyle\lim_{x \to 1^+} \frac{J_p(x^2 - 1)}{(x - 1)^p}$ at $p = 4$ equals

(A) 0 (B) 1/24 (C) 1/120 (D) ∞