EE225 Network Theory
Tutorial Problems- Set Five

August 23, 2017

Note: Unless otherwise stated, b denotes the number of branches in the graph in question; n, the number of nodes in the graph; A, a reduced incidence matrix for the graph.

Q1. Prove that the columns of ANY $(n-1) \times (n-1)$ non-singular sub-matrix of A for a connected graph correspond to the twigs of some tree. (Converse statement to the sub-matrix corresponding to a tree). Also show that the determinant of the non-singular matrix in question is always $+1$ or -1.

Q2. Show that if a set of branches of a graph contain a loop, the corresponding columns of A are linearly dependent.

Q3. For a graph, A may be partitioned as $[A_t \ g_1]$ where t corresponds to some tree; and the columns corresponding to the twigs are placed in A_t. What must be the structure for t so that A_t is an identity matrix? Prove and illustrate with an example.

Q4. A linear graph has five nodes and seven branches. For this matrix,

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & 0 \end{bmatrix}$$

(a) Verify that the branch numbers $1, 3, 4, 5$ form a tree without drawing the graph.

(b) For this tree, write down the f-loop matrix and f-cut-set matrix without drawing the graph.

(c) Now draw the graph and verify the results

(d) Enumerate the possible trees of the graph and write down this f-loop matrix and f-cut-set matrix for each tree.

(e) Verify that the number of trees possible is given by $\det(AA^T)$.

Q5. Prove that a graph contains atleast one loop if two or more branches are incident at each node.

Q6. For a connected planar graph, show that the loop sub-matrix corresponding to the meshes is of rank $(b - (n-1))$. Hence show that the mesh currents form an adequate basis for expressing all branch currents. Set up this loop sub-matrix for the meshes in Fig. 6

Q7. The f-cut-set determined by a twig x for some tree t in a linear graph contains a set of links l_1, l_2, $\cdots$ for that tree. Each of these Links defines an f-loop. Show that every one of these f-loops formed contains twig x.

Q8. Let $V_b(y)$ be a transformation on the voltage $v_b(t)$ of branch b, and $I_b(z)$, a transformation on the current $i_b(t)$ of branch b, such that both these transformations obey the following property: The transformation of an algebraic sum of voltages (resp. currents) is the corresponding algebraic sum of transforms of the voltages (resp. currents) in question. Further two matrices X and Y of appropriate sizes are said to be orthogonal if XY^T is the zero matrix of the appropriate size.

(a) Using the orthogonality of the incidence and loop matrices, show that the sum of products $V_b(y)I_b(z)$ over all branches b of the graph is always zero. This is more generalized version of Tellegen's theorem.

(b) Give some thought to the multifarious consequences of this form of the theorem.

(c) Show that among the three results: Tellegen's Theorem, KVL applied to $V_b(y)$, and KCL applied to $I_b(z)$; any two of them may be treated as fundamental, and the third a consequence of these two. Note that $V_b(y)$ could simply be $v_b(t)$; i.e. the transformation could simply be the identity transform.

Q9. Prove the statement: Every cut-set in a linear graph has an even number of branches in common with every loop. Hence or otherwise show that any cut-set sub-matrix is orthogonal to any loop sub-matrix.

Q10. Construct the duals of each of the networks shown in Fig. 10. In each case, verify that the role of current and voltage is interchanged between the network and its dual: by writing the relevant loop equations and nodal equations.

Q11. Is it always true that the incidence matrix is one particular f-cut-set matrix for some tree in a graph? If yes, prove. If not, demonstrate so through a counter example. Under what circumstances would this happen ?

Some Challenging Problems: (not mandatory in the course syllabus. These are meant to excite your imagination and inspire you to probe further)

QCH1 Is it possible to determine from the reduced incidence matrix of a linear graph whether the graph is planar or non-planar? (It must be, don't you think so?) How?

QCH2 Is it possible to come out with a dual for a non-planar network/graph? Or are there some classes of such non-planar networks/graphs for which it is possible? Or do we need to relax some condition(s)? How do we go about it? Illustrate with examples.

QCH3 Prove that the number of trees in a graph is given by $\det(AA^T)$.

QCH4 Given A for a graph, construct the COMPLETE loop matrix for the graph through a computer program/algorithm.

QCH5 Given A for a graph, construct the COMPLETE cut-set matrix for the graph through a computer program/algorithm.

QCH6 To what extent is the planar dual of a planar electrical network unique? Is it always possible to construct a planar dual for a planar electrical circuit with controlled sources? Explain.

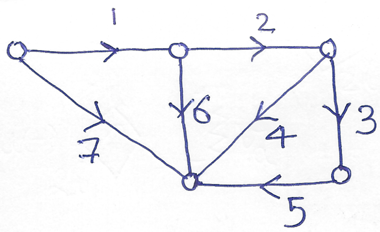


fig. 6-1

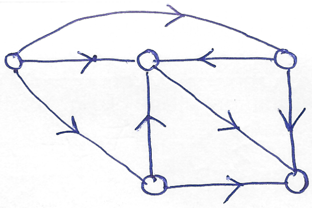


fig 6-2

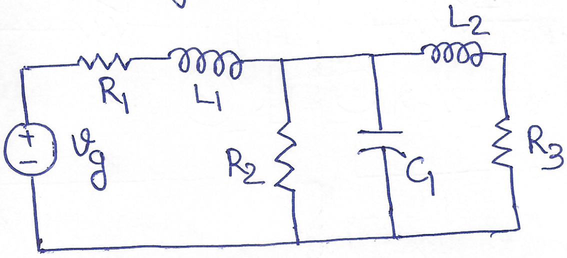
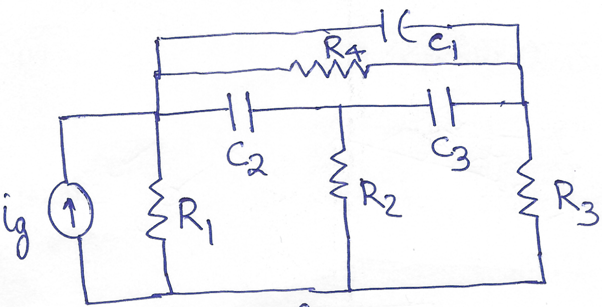


fig 10-1



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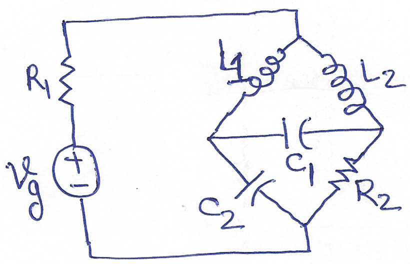

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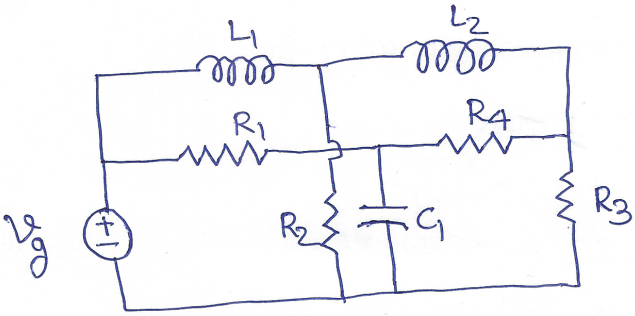

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