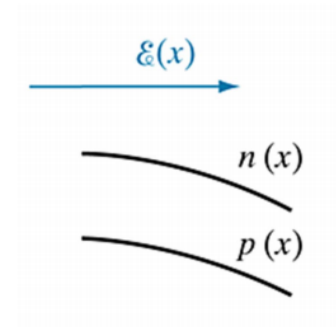
Tutorial-3

(1)



Flux Calculations (ϕ)

Drift:

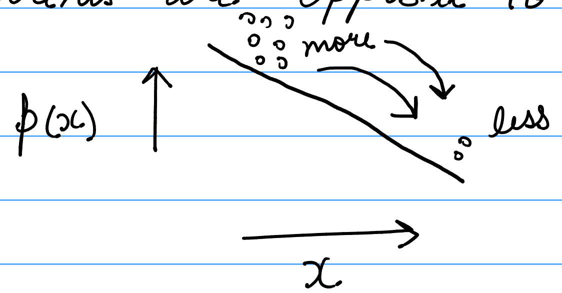
Hole : $\underbrace{p(x)\,\mu_p\,\mathcal{E}(x)}_{\text{Particles per unit area per unit time}}$

Electrons : $-n(x)\,\mu_n\,\mathcal{E}(x)$ [Electrons drift opposite to applied field]

Diffusion:

Hole : $-D_p \dfrac{\partial p}{\partial x}$; Electrons : $-D_n \dfrac{\partial n}{\partial x}$

Particle currents are opposite to concentration gradients



$\dfrac{dp}{dx} = -ve$

$\phi_p = +ve$

Current Calculations ($J \to$ Current Density)

$q = 1.6\,e{-19}\ C$

Drift:

Hole : $q\,p(x)\,\mu_p\,\mathcal{E}(x)$; Electrons : $q\,n(x)\,\mu_n\,\mathcal{E}(x)$

Diffusion:

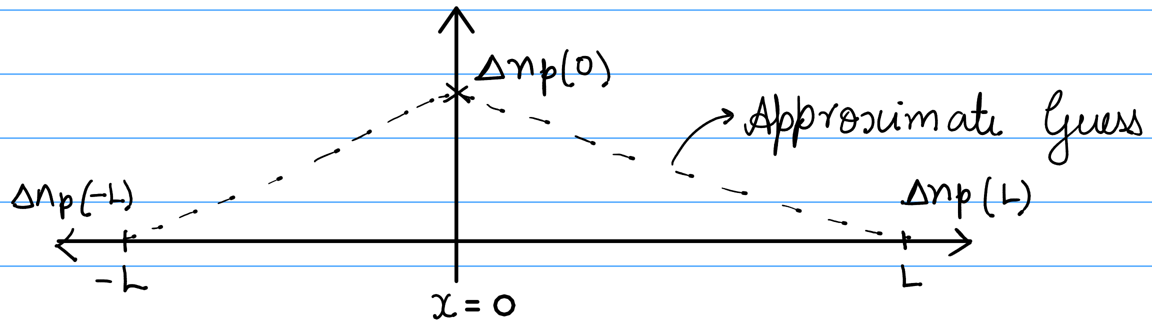
Hole : $-q D_p \dfrac{\partial p}{\partial x}$; Electrons : $q D_n \dfrac{\partial n}{\partial x}$

Important Takeaway: In an applied field, the holes and electrons drift in opposite directions but the currents are in the same direction (that of field). For similar concentration gradients, electrons and holes diffuse in the same direction but their currents are in opposite directions.

② (a) Dominant processes in the regions removed from $x = 0$ are diffusion and recombination.

* In the absence of $\mathcal{E}_l$ ($\mathcal{E}_l \cong 0$), there is no drift.
* In the presence of excess carriers and no laser beam for $x \neq 0$, there is recombination and no generation.
* Excess carriers at $x = 0$ develop a concentration gradient and diffusion sets in.

(b) Expected form of $\Delta n_p(x)$ in the bar



We can expect the excess carriers to recombine over the bar's length as we move away from $x = 0$, given the boundary conditions, we join them using a monotonic curve to form the expectation. Exact solution will require solving equations.

(c) $N_A = 10^{16}/cm^3$; $\Rightarrow p_{eq.} = 10^{16}/cm^3$; $n_{eq.} = 10^4/cm^3$ $\left(\frac{n_i^2}{p_{eq.}}\right)$

$\Delta n_p(0) = 10^{11}/cm^3$

Since $x=0$ is the region of generation, we expect

$$\Delta n_p(x) \leqslant \Delta n_p(0) \quad \forall x$$

The numbers imply:

$$n_{eq} << \Delta n_p(x) << p_{eq}$$

Or, in other words, the generation only disturbs the minority carrier concentration.

$$n(x) = n_{eq} + \Delta n_p(x) \sim \Delta n_p(x)$$

$$p(x) = p_{eq} + \Delta p(x) = p_{eq} + \Delta n_p(x) \sim p_{eq}$$

[Photogeneration results in equal no. of excess holes and electrons]

These conditions ($\Delta n_p \ll p_{eq}$) are representative of low-level injection.

(d) Starting from the continuity equation:

$$\frac{\partial n}{\partial t} = \frac{1}{-q}\nabla \cdot J_n - R \qquad \forall\ 0<x<L$$

$$\Rightarrow \frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p(x)}{\partial x^2} - \frac{\Delta n_p}{\tau_n}$$

In steady state, $\dfrac{\partial^2 \Delta n_p(x)}{\partial x^2} = \dfrac{\Delta n_p(x)}{D_n \tau_n} = \dfrac{\Delta n_p(x)}{L_n^2}$

(e) General Solution:

$$\Delta n_p(x) = A e^{-x/L_n} + B e^{x/L_n}$$

(f) Boundary Conditions : $\Delta n_p(0) = \Delta n_p = 10^{11}/cm^3$

$\Delta n_p(L) = 0$

(g) $\Rightarrow \quad A + B = \Delta n_p$

and $\quad Ae^{-L/L_n} + Be^{L/L_n} = 0$

$\Rightarrow \quad B = -Ae^{-2L/L_n}$

and $\quad A(1 - e^{-2L/L_n}) = \Delta n_p$

So, the complete solution is :

$$\Delta n_p(x) = \frac{\Delta n_p}{1 - e^{-2L/L_n}} e^{-x/L_n} - \frac{\Delta n_p e^{-2L/L_n}}{1 - e^{-2L/L_n}} e^{x/L_n}$$

$$= \frac{\Delta n_p}{1 - e^{-2L/L_n}} \left(e^{-x/L_n} - e^{(x-2L)/L_n} \right)$$

(h) In the limit $L \longrightarrow \infty$, $B = 0$

$$\Rightarrow \Delta n_p(x) = \Delta n_p e^{-x/L_n}$$

(i) In the limit $L << L_n$

$$\Delta n_p(x) = \frac{\Delta n_p}{1 - (1 - 2L/L_n)} \left(1 - \frac{x}{L_n} - \left(1 + \frac{x}{L_n}\right)\left(1 - \frac{2L}{L_n}\right) \right)$$

$$\Rightarrow \Delta n_p(x) = \frac{\Delta n_p}{2L} \times L_n \times \left(1 - \frac{x}{L_n} - 1 + \frac{2L}{L_n} - \frac{x}{L_n} + \cancel{\frac{2Lx}{L_n^2}}^{\,0} \right)$$

$$= \frac{\Delta n_p}{L} (L - x)$$

$$= \Delta n_p (1 - x/L)$$

Important takeaway : If $L_N >> L$, i.e. the diffusion lengths are much larger as compared to the device length, then the steady state excess carrier concentration falls off linearly as compared to exponentially if the diffusion lengths are much smaller.

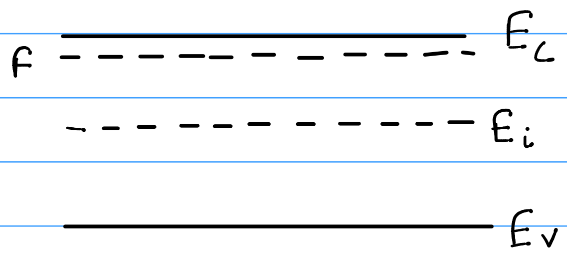
(3)

a) At $T = 300\,K$, in GaAs

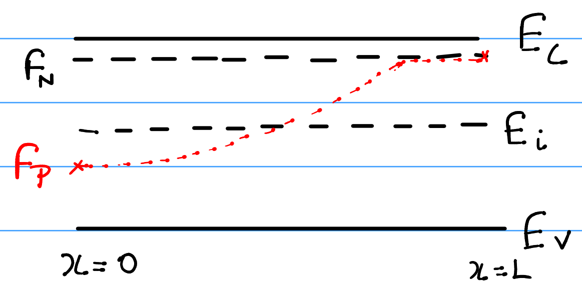
$$n_i = 2.1 \times 10^6 \,/cm^3$$
$$n_o = N_D - N_A = 5 \times 10^{15} \,/cm^3$$
$$p_o = \frac{n_i^2}{n_o} \cong 10^{-3} \,/cm^3$$

(b) Following the arguements as in previous question part (c), $\Delta p_{n_o} << n_{eq}$, this is a low-level injection state.

(c) In equilibrium,



After perturbation and $\mathcal{E}_f = 0$



Since $n_{per} = n_{eq}$, no significant change in F_N.
However; $\Delta p_n(x) = \Delta p_{no}(1 - x/L)$
At $x = L$, $\Delta p_n = 0$; $p_{per} = p_{eq}$ at $x = L$.
So F_p merges with F_N at $x = L$
For x away from L,

$$p_{per} \sim \Delta p_n(x) = \Delta p_{no}\left(1 - \frac{x}{L}\right) = n_i\, e^{(E_i - F_P)/kT}$$

This gives that F_P follows a $-\ln\left(1 - \frac{x}{L}\right)$ relationship.

(d) (i) There must be a hole diffusion current because there is a hole concentration gradient in steady state.

(ii) The hole diffusion current is dependent on a concentration gradient which is finite, on the other, in absence of $\mathcal{E}_f$, the drift current is negligible.

(iii) $$J_p = -qD_p \frac{\partial p}{\partial x} = -qD_p \times \frac{\Delta p_{n0}}{L}(-1)$$

$$= q\,\Delta p_n \frac{D_p}{L}$$

(e) In steady state and in the absence of all R-G processes we get from continuity,

$$\nabla \cdot J_p = 0$$

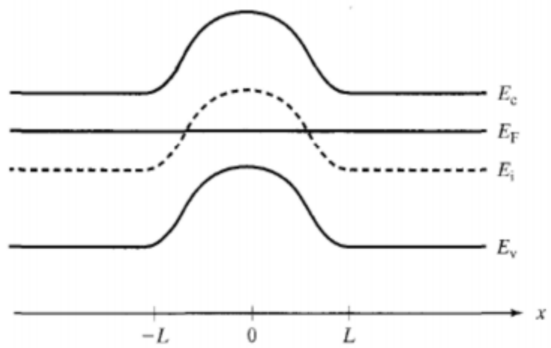
(Diffusion $\gg$ Drift) $\Rightarrow \dfrac{\partial^2 \Delta p}{\partial x^2} = 0$

$$\Rightarrow \Delta p_n(x) = a + bx$$

Using boundary conditions, $\Delta p_n(0) = \Delta p_{n0}$ & $\Delta p_n(L) = 0$

we get $\Delta p_n(x) = \Delta p_{n0}\left(1 - \dfrac{x}{L}\right)$

(4)

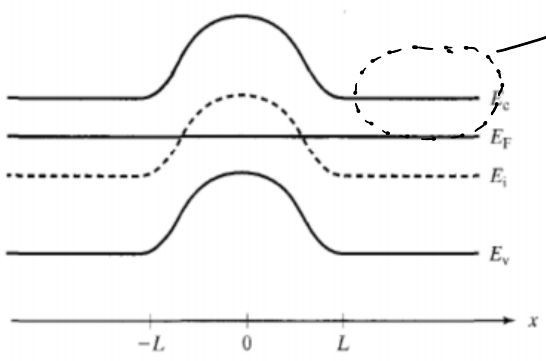


(a) The fermi-level throughout the semiconductor is flat. There is no applied bias or net current through the device. It is in equilibrium.

(b) Since there is no R-G (equilibrium, $np = n_i^2$, $R\text{-}G_{rate} = 0$ everywhere),

By continuity this implies, $\nabla \cdot J_n = 0$ and $\nabla \cdot J_p = 0$

J_n = constant and J_p = constant throughout the semiconductor.



In this region, there is no $\mathcal{E}_\ell$ $\Rightarrow$ $J_n(\text{drift}) = 0$

Also $E_c - E_F$ = constant

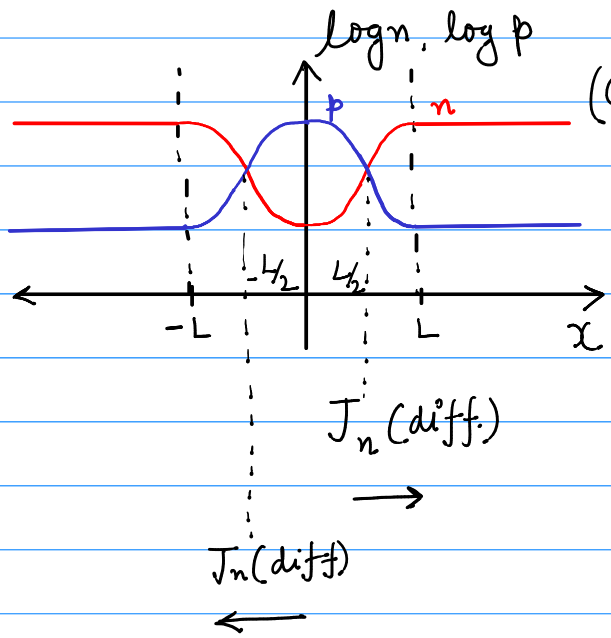
$\Rightarrow$ $n(x)$ = constant

$\Rightarrow$ $J_n(\text{diffusion}) = 0$

$J_n(\text{total}) = 0$

And since J_n = constant throughout the device, $J_n = 0$ at $x = \pm L/2$, Similarly $J_p(\pm L/2) = 0$

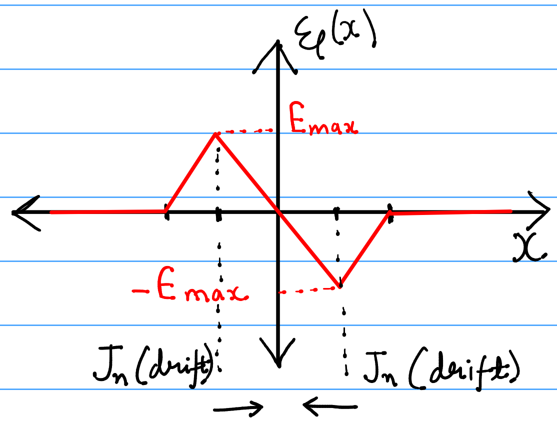
(c)



(d) At $x = \pm L/2$, $\dfrac{dn}{dx} \neq 0$

So, diffusion current is non-zero for electrons.

(e)

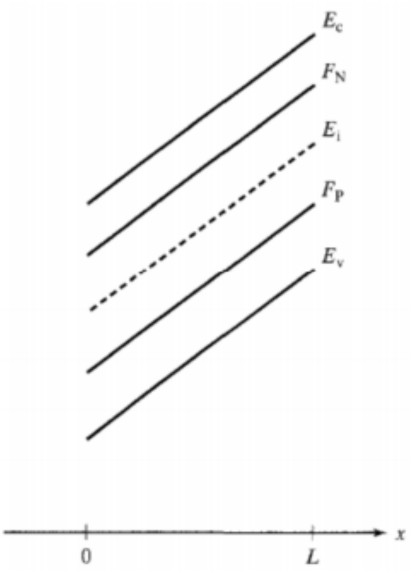


$$\mathcal{E}_\ell(x) = \frac{1}{q}\frac{dE_c(x)}{dx}$$

(f) At $x = \pm L/2$, $\mathcal{E}_\ell \neq 0$.

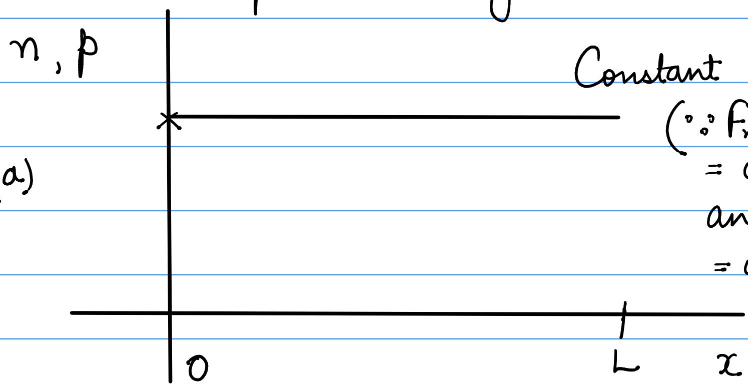
there is a drift current of electrons.

5



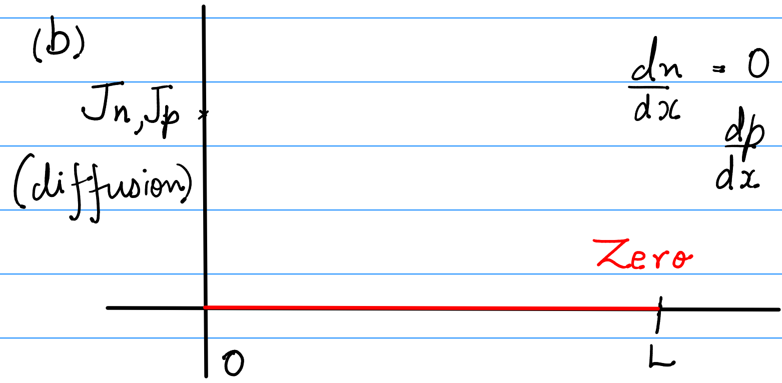
$n = p$ assuming $F_n - E_i = E_i - F_p$

(a)



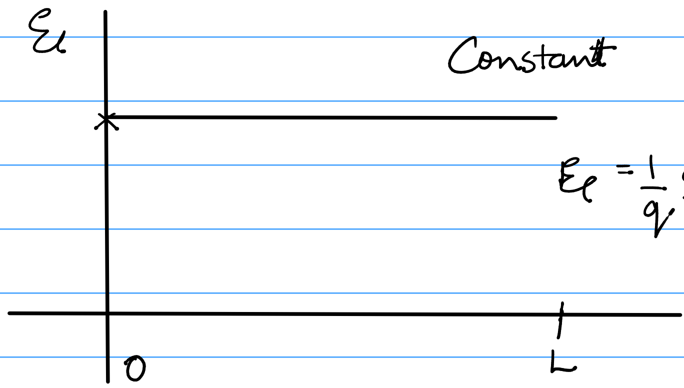
Constant ($\because F_n - F_i = \text{constant}$ and $F_p - F_i = \text{constant}$)

(b)



$\frac{dn}{dx} = 0$ $\quad \frac{dp}{dx} = 0$

(c)



Constant

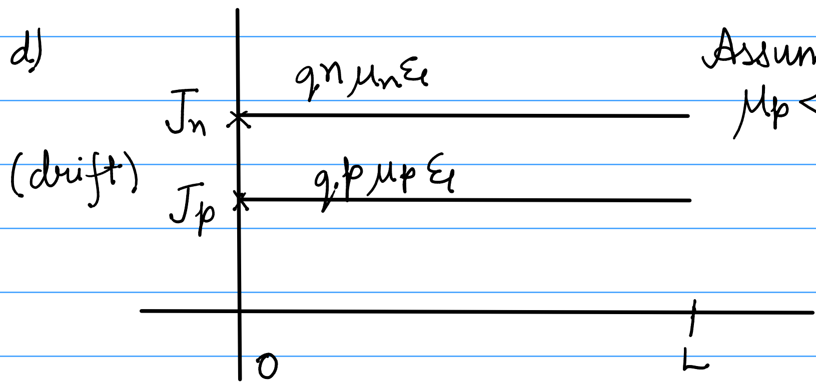
$\mathcal{E}_l = \frac{1}{q}\frac{dE_c}{dx}$

Drift:

$J_n = n q \mu_n \mathcal{E}_l$

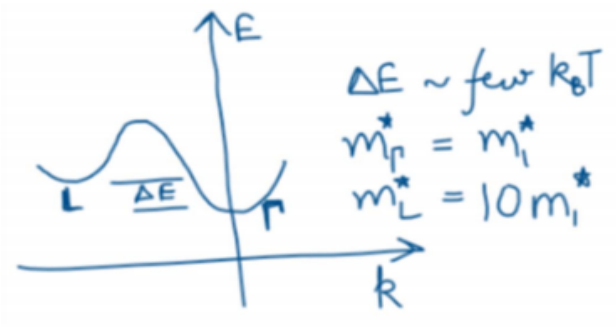
$J_p = p q \mu_p \mathcal{E}_l$

d)



Assume, $\mu_p < \mu_n$

(6)

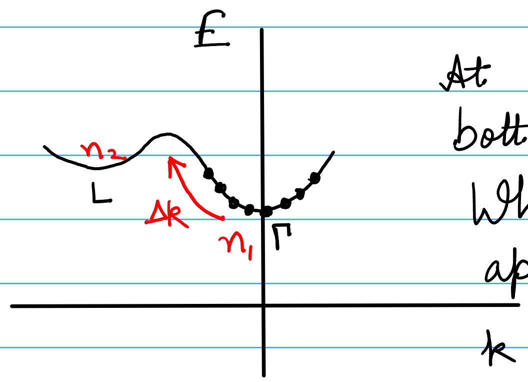


For a detailed discussion of electron transfer between valleys, please read the section on 'Intervalley Carrier Transfer' of Ch-6 'Carrier Transport' of the 'Advanced Semiconductor Fundamentals' book by Pierret.

This is the case of Drift-Velocity vs. Electric Field in GaAs. Using the expression for drift velocity as:

$$v_d = \frac{q \mathcal{E}_l \tau}{m^*}$$

$\tau \rightarrow$ Momentum relaxation time



At $\mathcal{E}_l = 0$, the electrons occupy the bottom of the Γ valley.
When field is applied, a $\Delta\vec{k}$ appears which is held steady by scattering. $\left(F = q\mathcal{E}_l \cong \hbar \frac{\Delta k}{\tau} \right)$

If these concepts are unfamiliar, it is sufficient to know that high enough electric fields allow some electrons to cross over the barrier between L and Γ valleys.
As a function of electric field, n_1 electrons remain in Γ valley and n_2 crossover over to L valley. (Assume $n_1 + n_2 = n$ = constant)
Now, the average drift velocity is,

$$V_d = \frac{n_2 v_{d_L} + n_1 v_{d_\Gamma}}{n}$$

$$= \frac{q \mathcal{E}_l \tau}{n} \left[\frac{n_2}{m_L^*} + \frac{n_1}{m_\Gamma^*} \right]$$

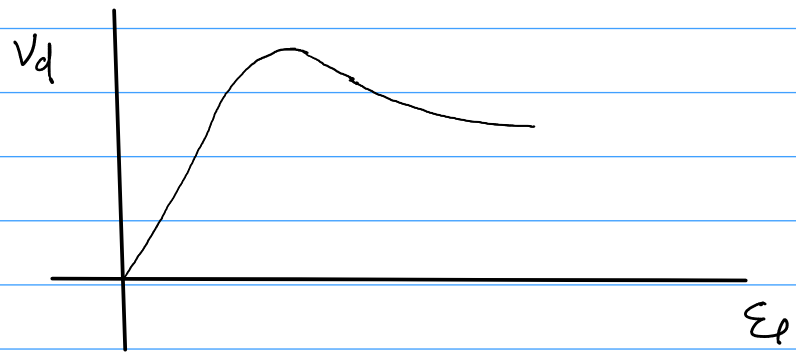
[Assuming same τ for both valleys]

$$V_d = \frac{q\,\mathcal{E}_\ell\,\tau}{n}\left[\frac{n_2}{10m_1^*} + \frac{n-n_2}{m_1^*}\right]$$

$$= \frac{q\,\mathcal{E}_\ell\,\tau}{m_1^*}\left[1 - \frac{9}{10}\frac{n_2}{n}\right]$$

As $\mathcal{E}_\ell$ increases, $\frac{n_2}{n}$ increases

This implies, there will be a maxima in V_d vs. $\mathcal{E}_\ell$ curve.



Only at high $\mathcal{E}_\ell$, the reduction in drift velocity due to high m_L^* becomes evident, for small fields, the linear relationship is followed. For even higher fields, the saturation kicks in as in other single valley cases.

For calculating, the peak position (i.e. $\frac{dV_d}{d\mathcal{E}_\ell}=0$) we need the $n_2(\mathcal{E}_\ell)$ dependence knowledge.

This would depend on the exact band diagram and the scattering mechanisms in the device. (Not relevant as of now)

7 (a) Continuity for holes:

$$\frac{\partial \Delta p}{\partial t} = -\frac{1}{q}\nabla \cdot J_p + G - R$$

We cannot write minority carrier diffusion equation for holes because holes are the majority carriers and diffusion is not the dominant current.

(b) Neglecting all R-G, the continuity simplifies to:

$$\frac{\partial \Delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x}$$

(c) Neglecting diffusion current,

$$\frac{\partial \Delta p}{\partial t} = -\frac{1}{q} \times \frac{\partial}{\partial x} q p \mu_p \mathcal{E}_e$$

$$\Rightarrow \frac{\partial \Delta p}{\partial t} = -N_A \mu_p \frac{\partial \mathcal{E}_e}{\partial x} \qquad [\because p \sim N_A]$$

(d) Poisson's equation: $\nabla^2 V = -\frac{\rho}{\varepsilon} \Rightarrow \frac{\partial \mathcal{E}_e}{\partial x} = \frac{\rho}{\varepsilon}$

$$\Rightarrow \frac{\partial \mathcal{E}_e}{\partial x} = \frac{q}{\varepsilon}[N_D + p - N_A - n]$$

$$\Rightarrow \frac{\partial \mathcal{E}_e}{\partial x} \sim \frac{q}{\varepsilon}[p - N_A] = \frac{q}{\varepsilon}\Delta p$$

(e)

$$\frac{\partial \Delta p}{\partial t} = -\frac{q N_A \mu_p}{\varepsilon}\Delta p \Rightarrow \Delta p(t) = \Delta p_0 e^{-t/\tau}$$

where $\tau = \frac{\varepsilon}{q N_A \mu_p} = \frac{K_S \varepsilon_0}{q N_A \mu_p}$

(f)

$$\tau = \frac{11.7 \times 8.85 \times 10^{-12}}{1.6 \times 10^{-19} \times 10^{21} \times 250 \times 10^{-4}} \text{ s (All in SI)} = 26 \text{ ps}$$