EE 207 — Midsem Solutions — Autumn 2018

1(a) The conductivity ~~of~~ can be defined as

$$\sigma = nq\,\frac{q\langle\langle \tau_m \rangle\rangle}{m^*}$$, where $\langle\langle \tau_m \rangle\rangle$ is the average momentum relaxation time

Assuming electric field in x-direction, electrons are expected to respond to electric field with the effective mass in the direction of electric field

Neare equilibrium, one sixth of electrons are in each of the six ellipsoid in silicon.

For ellipsoids along x-direction,

$$\sigma_l = \sigma_{x_1, x_2} = \frac{n}{6} q \frac{q\langle\langle \tau_m \rangle\rangle}{m_l^*}$$, where m_l^* = longitudinal ~~effective~~ mass

1

For ellipsoids along y and z-direction,

$$\sigma_t = \sigma_{y1, y2, z1, z2} = \frac{n}{6} q \frac{q\langle\langle \tau_m \rangle\rangle}{m_t^*}$$, where m_t^* = transverse mass

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∴ The total conductivity can be given as

$$\sigma = 2\sigma_L + 4\sigma_t = nq\left[\frac{1}{3m_l^*} + \frac{2}{3m_t^*}\right] q\langle\langle \tau_m \rangle\rangle$$

or $$\cancel{n}q\frac{\cancel{q\langle\langle \tau_m \rangle\rangle}}{m_{ce}^*} = \cancel{nq}\left[\frac{1}{3m_l^*} + \frac{2}{3m_t^*}\right]\cancel{q\langle\langle \tau_m \rangle\rangle}$$ 1

or $$\boxed{\frac{1}{m_{ce}^*} = \frac{1}{3}\left[\frac{1}{m_l^*} + \frac{2}{m_t^*}\right]}$$ 1

b) As explained in part (a), for electric field in y and z direction also, there would be one component of m_l and two components of m_t. This indicates that the movement of electrons in the conduction band would be isotiopic that would lead to current flow in silicon

1 → isotropic

1 → explanation

Q2 (a) for 2D

Number of states in a given k-space volume

$$dN = \frac{\Delta k}{\delta k} \times \underbrace{2}_{\text{due to spin}}$$

$$= \frac{\Delta k}{(2\pi/L)^2} \times 2$$

$$dN = \frac{L^2}{4\pi^2} \times 2 \times 2\pi k\, dk$$

$$= \frac{L^2}{\pi} k\, dk$$

$$dn = \frac{dN}{L^2} = \frac{k}{\pi} dk$$ 1

Now $$DOS = \frac{dn}{dE} = \frac{dn}{dk} \cdot \frac{dk}{dE}$$

$E = \hbar v |k|$

$\frac{dE}{dk} = \pm \hbar v$

$$= \frac{k}{\pi} \cdot \left(\pm \frac{1}{\hbar v} \right)$$

$$= \frac{E}{\pi(\pm \hbar v)} \left[\pm \frac{1}{\hbar v} \right]$$

$$\boxed{DOS = \frac{E}{\pi \hbar^2 v^2}}$$ 1

for 1D

Number of states in a given k-space

$$dN = \frac{\Delta k}{\delta k} \times 2$$

$$= \frac{\Delta k}{(2\pi/L)} \times 2$$

$$dN = \frac{L}{2\pi} \times 2 \times 2\, dk$$

$$dN = \frac{2L}{\pi}\, dk$$

$$dn = \frac{dN}{L} = \frac{2\, dk}{\pi} \qquad (1)$$

$$DOS = \frac{dn}{dE} = \frac{dn}{dk} \cdot \frac{dk}{dE}$$

$$E = \hbar v |k|$$

$$\frac{dE}{dk} = \cancel{\frac{1}{\pm\hbar v}} \pm \hbar v$$

$$= \frac{2}{\pi}\left(\pm \frac{1}{\hbar v}\right)$$

$$\boxed{DOS = \pm \frac{2}{\pi \hbar v}} \approx \frac{2}{\pi \hbar v} \qquad (1)$$

(b)

$$n = \int_0^{\infty} DOS\, f(E)\, dE$$

at 0K, $f(E) = \begin{cases} 1 & E \le E_F \\ 0 & E > E_F \end{cases}$ (1)

$$n = \int_0^{E_F} DOS\, dE$$

Single bound state is at 3eV

$$n = \int_{3eV}^{E_F} DOS\, dE$$

$$= \int_{3eV}^{5eV} DOS\, dE \qquad (1)$$

$$= \frac{2}{\pi \hbar v}(5-3)\,eV$$

$$= \frac{2 \times 2\pi}{\pi \times (6.634e-34) \times 10^5} \times 2 \times 1.6 \times 10^{-19}$$

$$\boxed{\begin{aligned} n &= 1.929 \times 10^{10} /m \\ n &= 1.929 \times 10^{8} /cm \end{aligned}} \qquad (1)$$

$N = n \times L = 1.929 \times 10^{10} \times 10^{-2}$

$= 1.929 \times 10^{+6}$

Q3. (a)

$$\rho = \frac{1}{q\mu_n n + q\mu_p p}$$

(i) Now $n \approx N_D - N_A$

$\approx 10^{16}/cm^3$

$$p = \frac{n_i^2}{n} = 10^4 /cm^3$$

$L = \sqrt{D\tau}$ and $\mu = \frac{D}{(kT/q)}$

$$\Rightarrow \mu = \frac{L^2}{\tau (kT/q)}$$

$$\mu_n = \frac{L_n^2}{\tau_n (kT/q)}$$

$$= \frac{(10 \times 10^{-6})^2}{10^{-6} \times 0.026}$$

$= 38.46\ cm^2/V sec$ 1

$$\mu_p = \frac{(100 \times 10^{-6})^2}{10^{-6} \times 0.026} = 3846\ cm^2/Vsec$$

$$\rho = \frac{1}{q[38.46 \times 10^{16} + 3846 \times 10^4]}$$

$$= \frac{1}{1.6 \times 10^{-19} \times 38.46 \times 10^{16}}\ \Omega\ cm$$

$\boxed{\rho = 0.1624\ \Omega m}$ $\boxed{\rho = 16.24\ \Omega\ cm}$ 1

(b) $\Delta n = \Delta p = 10^{16}/cm^3$

$n = 2 \times 10^{16}/cm^3$

$p \approx 10^{16}/cm^3$

Yes, resistivity will change due to perturbation 1

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$$\rho_{new} = \frac{1}{q\mu_n n + q\mu_p P}$$

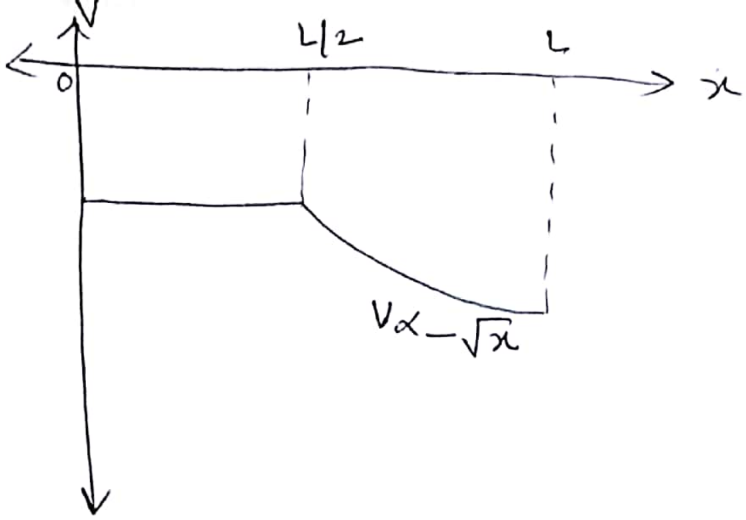
$$= \frac{1}{1.6\times10^{-19}\left[38.46\times 2\times10^{16} + 3846 \times 10^{16}\right]}$$

$$= \frac{10^3}{1.6\left[76.92 + 3846\right]}$$

$$= 10^3 \times (0.000159)\ \Omega\ cm$$

$$\boxed{\rho_{new} = 0.16\ \Omega\ cm}$$ 1

Q 4 a)



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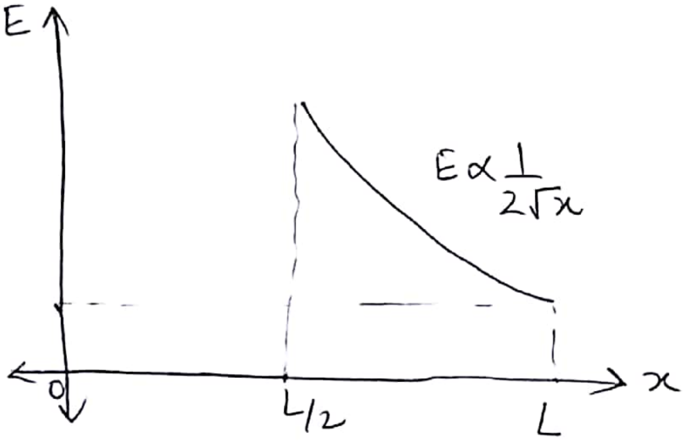
b)



$E = -\frac{dV}{dx}$ 1

for $x = 0$ to $L/2$

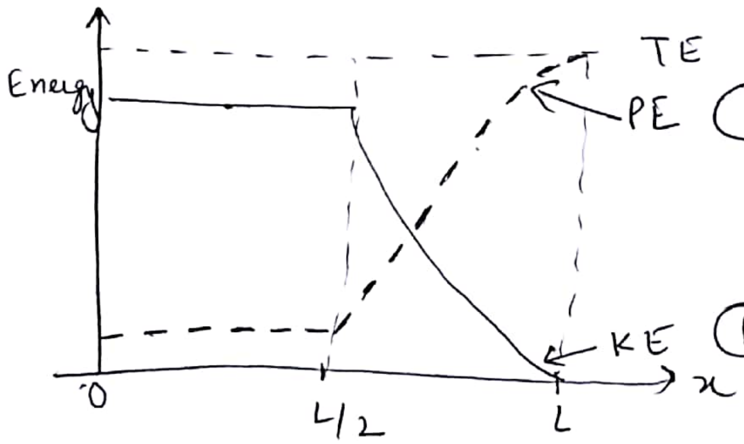
$V =$ constant $\Rightarrow E = 0$

for $x = L/2$ to L

$V \propto -\sqrt{x}$

$E \propto \frac{1}{2\sqrt{x}}$

c)



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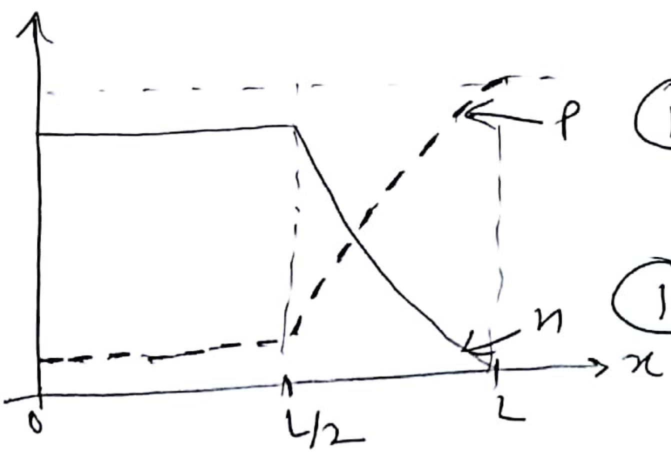
d)



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(Q5 (a) Using continuity equation.

$$-\frac{1}{q}\nabla J_p + G_p - R_p = \frac{dp}{dt}$$

Under steady state (dark)

$$-\frac{1}{q}\nabla \cdot J_p - R_p = 0 \qquad (1)$$

$$-\frac{1}{q}\frac{d}{dx}\left[q\mu_p p \mathcal{E} - qD_p\frac{dp}{dx}\right] - \frac{\Delta p}{\tau_p} = 0$$

$$-\mu_p \frac{d}{dx}(p_0 + \Delta p)\mathcal{E} + D_p\frac{d^2\Delta p}{dx^2} - \frac{\Delta p}{\tau_p} = 0$$

$$-\mu_p \mathcal{E}\frac{d\Delta p}{dx} + D_p\frac{d^2\Delta p}{dx^2} - \frac{\Delta p}{\tau_p} = 0$$

$$D_p\frac{d^2\Delta p}{dx^2} - \mu_p\mathcal{E}\frac{d}{dx}\Delta p - \frac{\Delta p}{\tau_p} = 0 \qquad (1)$$

$$\Rightarrow \Delta p = C_1 e^{r_1 x} + C_2 e^{r_2 x}$$

r_1, r_2 will be obtained by solving

$$D_p r^2 - \mu_p \mathcal{E} r - \frac{1}{\tau_p} = 0$$

$$\Rightarrow r = \frac{\mu\mathcal{E} \pm \sqrt{\mu^2\mathcal{E}^2 + \frac{4D_p}{\tau_p}}}{2D_p}$$

$$= \frac{\mu}{2D_p}\mathcal{E} \pm \sqrt{\frac{\mu^2\mathcal{E}^2}{4D_p^2} + \frac{1}{D_p\tau_p}}$$

$$r_1, r_2 = \frac{\mathcal{E}}{2\phi_t} \pm \sqrt{\frac{\mathcal{E}^2}{4\phi_t^2} + \frac{1}{L_p^2}} \qquad \phi_t = \frac{kT}{q} = \frac{D_p}{\mu_p}$$

Now, we need to find C_1, C_2

at $x=0$ $\Delta p(x=0) = C_1 + C_2$ (0.5) at $x = L$

$$\Delta p(x=L) = 0 = C_1 e^{r_1 L} + C_2 e^{r_2 L}$$

0.5

$$C_1 = -C_2 e^{(r_2 - r_1)L}$$

$$\Delta p(x=0) = C_2 \left(1 - e^{(r_2 - r_1)L}\right)$$

$$C_2 = \frac{\Delta p(x=0)}{1 - e^{(r_2 - r_1)L}}$$

$$\Rightarrow \quad C_1 = -\frac{\Delta p(x=0)}{1 - e^{(r_2 - r_1)L}} \times e^{(r_2 - r_1)L}$$

$$= \frac{\Delta p(x=0)}{1 - e^{(r_1 - r_2)L}}$$

$$\boxed{\Delta p(x) = \frac{\Delta p(x=0)}{1 - e^{(r_1 - r_2)L}} e^{r_1 x} + \frac{\Delta p(x=0)}{1 - e^{(r_2 - r_1)L}} e^{r_2 x}}$$

1

where

$$r_1 = \frac{\varepsilon}{2\phi_t} + \sqrt{\frac{\varepsilon^2}{4\phi_t^2} + \frac{1}{L_p^2}}$$

$$r_2 = \frac{\varepsilon}{2\phi_t} - \sqrt{\frac{\varepsilon^2}{4\phi_t^2} + \frac{1}{L_p^2}}$$

(b) : $r_1 = \frac{10}{2\times0.026} + \sqrt{\left(\frac{10}{2\times 0.026}\right)^2 + \left(\frac{1}{10^{-3}}\right)^2}$

$= 1210/\text{cm}$

$r_2 = -826\ /\text{cm}$

1

$J_{drift} = q\mu_p E\, \Delta p(x)$

$= 1.6\times10^{-19} \times 400 \times 10 \times 10^{15} \left[\frac{e^{1210x}}{1-e^{2}} + \frac{e^{-826x}}{1-e^{-2}}\right]$

$= \frac{0.64}{1-e^{2}}\left[e^{1210x} - e^{(2-826x)}\right]$

1

at $x=0 \Rightarrow J_{drift} = 0.64\ A/cm^2$

$J_{diff} = -qD_p \frac{d}{dx}\Delta p$

$= -1.6\times10^{-19} \times 400 \times 0.026 \times 10^{15} \left[\frac{1210\, e^{1210x}}{1-e^{2}} - \frac{826\, e^{-826x}}{1-e^{-2}}\right]$

$= \frac{-1.664}{1-e^{2}}\left[1.21\, e^{1210x} + 0.826\, e^{(2-826x)}\right]$

1

at $x=0 \Rightarrow J_{diff} = 1.9\ A/cm^2$