Tutorial 2 solutions

Q. 1) Consider a non- parabolic band structure satisfying the relation $E(1+\alpha E) = \frac{\hbar^2 k^2}{2m_o}$. Find the wave packet group velocity for the given band structure for large energies. (Assume α to be close to 1).

Sol) For large energies, E>>1/α or αE>>1 => $\alpha E^2 = \frac{\hbar^2 k^2}{2m_o}$ or E= $\frac{\hbar k}{\sqrt{2m_o\alpha}}$. The wave packet group velocity, $v_g = \frac{dE}{\hbar dk} = \frac{1}{\sqrt{2m_o\alpha}}$.

Q. 2) Find the density of states (DOS) expression for 2-dimensional (2D) material.

Sol) States between E+ΔE and E, $dn = 2\ (spin\ degeneracy) * \frac{\pi(k+dk)^2 - \pi k^2}{\frac{2\pi}{L} * \frac{2\pi}{W}} = \frac{Akdk}{\pi}$.

States per unit energy per unit area = $\frac{dn}{dE} = \frac{k}{\pi}\left(\frac{dk}{dE}\right)$ and $\frac{dk}{dE} = \frac{1}{\hbar}\sqrt{\frac{m^*}{2E}}$

because $E = \frac{\hbar^2 k^2}{2m^*}\ or\ k = \frac{\sqrt{2m^*E}}{\hbar}$. This gives DOS, $g_c = \frac{\sqrt{2m^*E}}{\pi\hbar} \times \frac{1}{\hbar}\sqrt{\frac{m^*}{2E}} = \frac{m^*}{\pi\hbar^2}$.

Q. 3) Calculate the position of Fermi level at room temperature (w.r.t. bottom of conduction band) in crystalline silicon doped with phosphorus atoms having doping concentrations:

a) 10^{16}/cm^3 b) 10^{17}/cm^3 c) 10^{18}/cm^3

Note: Use N_C = 2.81×10^{19} cm^{-3} for silicon.

Sol) For n-type semiconductor, n~N_D => $n = N_C\ e^{-\frac{(E_C - E_F)}{k_B T}} \cong N_D$.

At room temperature, k_BT = 0.026 eV, ∴ $E_C - E_F = 0.026 \ln\frac{2.81\times10^{19}}{N_D}$

a) $E_C - E_F = 0.026 \ln\frac{2.81\times10^{19}}{10^{16}}$ = 0.206 eV
b) $E_C - E_F = 0.026 \ln\frac{2.81\times10^{19}}{10^{17}}$ = 0.146 eV
c) $E_C - E_F = 0.026 \ln\frac{2.81\times10^{19}}{10^{18}}$ = 0.087 eV

Q. 4) Find the electron density in a 3D electron gas at T = 0 K in terms of Fermi energy, E_F.

Sol) States between E+ΔE and E, $dn = 2\ (spin\ degeneracy) * \frac{\frac{4}{3}\pi(k+dk)^3 - \frac{4}{3}\pi k^3}{\frac{2\pi}{L} * \frac{2\pi}{W} * \frac{2\pi}{H}} = \frac{Vk^2dk}{\pi^2}$.

=> $n = \int_0^{k_F} \frac{Vk^2 dk}{\pi^2} = \frac{Vk^3}{3\pi^2}$. We know that, $E_F = \frac{\hbar^2 k_F^2}{2m}$ which gives $n = \frac{V(2mE_F)^{\frac{3}{2}}}{3\pi^2\hbar^3}$.

Therefore, the electron density is: $n = \frac{(2mE_F)^{\frac{3}{2}}}{3\pi^2\hbar^3}$.

Q. 5) If both silicon and germanium are doped with $N_D = 5 \times 10^{14}$ cm^{-3} each, find their respective intrinsic carrier concentration at T = 400 K. Comment on the nature of the semiconductors at 400 K. Use $E_{g,Si}$ = 1.12 eV, $E_{g,Ge}$ = 0.66 eV, $n_{i,Si}^{300K} = 1.5 \times 10^{10}$ cm^{-3} and $n_{i,Ge}^{300K} = 2.5 \times 10^{13}$ cm^{-3}.

Sol) We know that, $n_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2k_BT}} = 2\left(\frac{2\pi k_B T}{\hbar^2}\right)^3 (m_n^* m_h^*)^{\frac{3}{2}} e^{-\frac{E_g}{2k_BT}}$. Hence by comparing n_i values at different temperatures,

For Si:

$$\frac{n_{i,Si}^{400K}}{n_{i,Si}^{300K}} = \left(\frac{400}{300}\right)^{\frac{3}{2}} e^{-\frac{0.66\times1.6\times10^{-19}}{2\times1.38\times10^{-23}}\left(\frac{1}{400}-\frac{1}{300}\right)} = 344.52$$

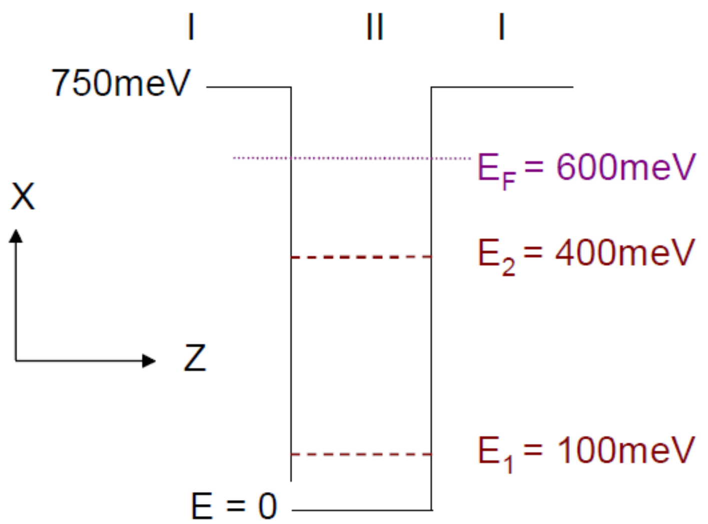
=> $n_{i,Si}^{400K} = 344.52\times1.5 \times 10^{10}$ cm^{-3} = 5.17×10^{12} cm^{-3} < N_D => n-type

For Ge:

$$\frac{n_{i,Ge}^{400K}}{n_{i,Ge}^{300K}} = \left(\frac{400}{300}\right)^{\frac{3}{2}} e^{-\frac{0.66\times1.6\times10^{-19}}{2\times1.38\times10^{-23}}\left(\frac{1}{400}-\frac{1}{300}\right)} = 24.25$$

=> $n_{i,Ge}^{400K} = 24.25\times2.5 \times 10^{13}$ cm^{-3} = 6.06×10^{14} cm^{-3} > N_D => intrinsic

Q. 6) Given that $g_{2D} = \frac{m_o}{\pi\hbar^2} \sim 4.2 \times 10^{16} cm^{-2} eV^{-1}$ and a quasi 2D system where a quantum well in the heterostructure formed by semiconductors I and II is shown as:



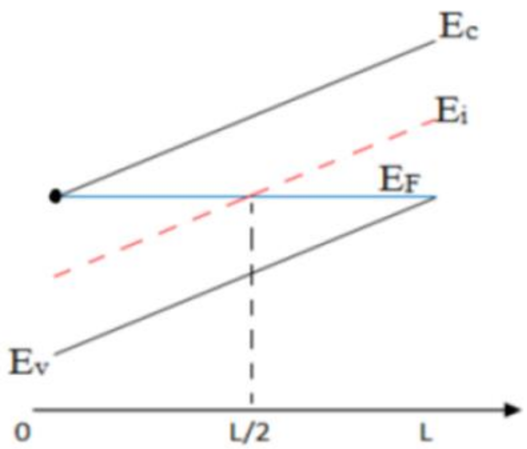
Find the 2D electron density at cryogenic temperature.

Sol) Cryogenic temperature means a very low temperature (not well defined but usually defined at 123 K)

$$n = \int g_c(E) f(E) dE = \int_0^{600\ meV} \frac{m_o}{\pi\hbar^2} dE = \frac{m_o}{\pi\hbar^2}\left((E_F - E_1) + (E_F - E_2)\right)$$

=> $n = 4.2\ \times 10^{16} \times (500 + 200) \times 10^{-3} cm^{-2} = 2.94 \times 10^{16}\ cm^{-2}$

Q. 7) For the band diagram given below, answer the following questions:

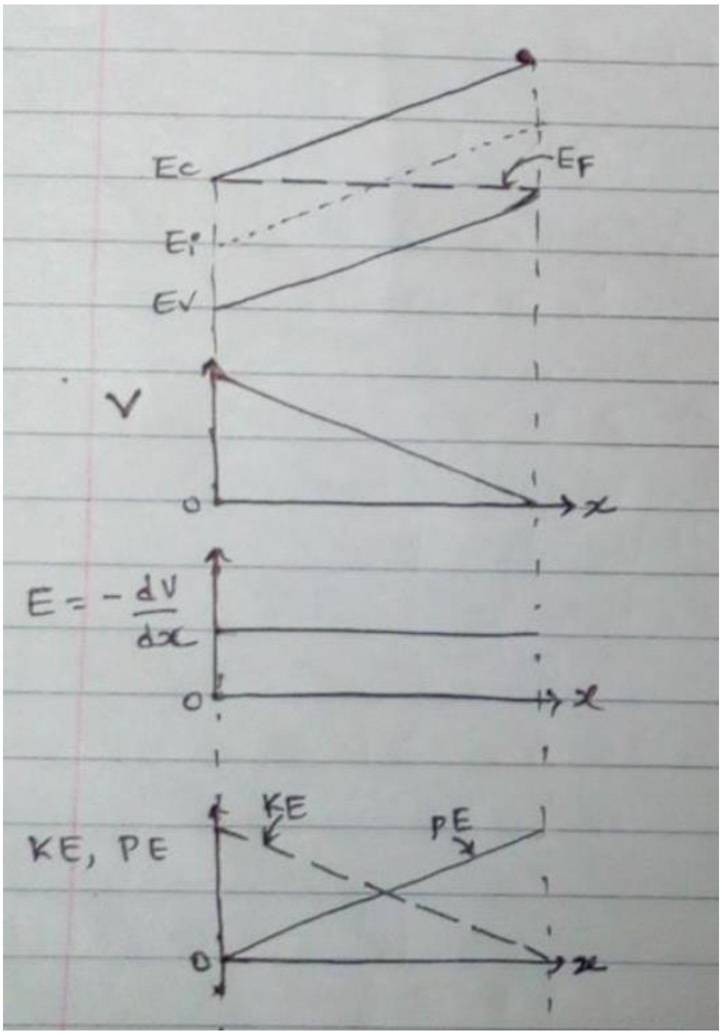


a) Is the semiconductor in equilibrium? Justify.
b) Sketch the electrostatic potential, electric field, potential and kinetic energies as a function of x inside the semiconductor.

Assume E_F as the reference level and particle shown in solid circle moves back and forth between x = 0 and x = L without changing the total energy.

Sol) a) Yes because Fermi level straight

b) minimum voltage is taken as zero. The offset can be taken as any value without affecting the shape and relative change in potential. E_F is taken as reference energy for calculating kinetic energy (KE) and potential energy (PE).



Ec
E_F
Ei
Ev
V
0
x
$E = -\frac{dV}{dx}$
0
x
KE
PE
KE, PE
0
x