Tutorial III

1. Show that $\exp(z_1) = \exp(z_2)$ if and only if $z_1 - z_2 = 2\pi ni$ for some $n \in \mathbb{Z}$.

2. Prove that

   (i) $\sin z = \sin x \cosh y + i \cos x \sinh y$;

   (ii) $\cos z = \cos x \cosh y - i \sin x \sinh y$.

3. Let γ be the boundary of the triangle $\{0 \leq y \leq 1 - x; 0 \leq x \leq 1\}$ taken with the anticlockwise orientation. Evaluate:

   a) $\int_\gamma \mathrm{Re}(z)\ dz$;

   b) $\int_\gamma z^2\ dz$.

4. Show that $\sin; \cos : \mathbb{C} \to \mathbb{C}$ are surjective. How often does it attain a given value ?

5. Let γ be the circle with radius R around the origin with counter-clockwise orientation. Compute the following integrals :

   a) $\int_\gamma z^m\ dz, m \in \mathbb{Z}$;

   b) $\int_\gamma \bar{z}^m\ dz, m \in \mathbb{Z}$;

   c) $\int_\gamma |z|^m\ dz, m \in \mathbb{Z}$.

6. Show that if D is a bounded domain with C^1 boundary,

$$\int_{\partial D} \bar{z}\ dz = 2i\mathrm{Area}(D).$$