Tutorial II

1. Show that $f(z) = e^x(\cos y + i \sin y)$ is holomorphic throughout $\mathbb{C}$.

2. Show that the CR equations take the form

$$u_r = \frac{1}{r} v_\theta \ \& \ v_r = -\frac{1}{r} u_\theta$$

in polar coordinates.

3. If u and v are harmonic conjugates of each other, show that they are constant functions.

4. Show that following functions are harmonic and find their harmonic conjugate.

   (i) $u(x, y) = xy + 3x^2y - y^3$;

   (ii) $u(x, y) = 3x^2 + 2x - 3y^2 - 1$.

5. Find the radius of convergence of the following power series :

   (i) $\sum_{k=1}^{\infty} kz^k$;

   (ii) $\sum_{p \text{ prime}} z^p$;

   (iii) $\sum_{k=1}^{\infty} \frac{k! z^k}{k^k}$.

6. Give an example of a series which can be shown to be convergent by root test but not by ratio test.