Introduction to the Special Theory of Relativity

EP 207 Additional practice problems

9/9/2018

Question 1. A particle of rest mass M and momentum $\mathbf{P}$ and energy E decays into two particles of rest masses m_1 and m_2 and momenta $\mathbf{p_1}$ and $\mathbf{p_2}$. Take $c = 1$ in all cases (unit of velocity is rescaled).(J. D. Jackson)

a) Use the conservation of energy and momentum and the invariance of scalar products of the 4-vectors to show that the total energy of the first particle in the *frame of the decaying particle* is,
$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$$
and that the energy of the second particle, E_2 is obtained by interchanging m_1 and m_2

b) Show that the kinetic energy T_i of the i^{th} particle ($i = 1$ or 2) in the same frame is
$$T_i = \Delta M \left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right),$$
where $\Delta M = M - m_1 - m_2$. You may directly use the expression which is derived in part (a). [3]

c) The charged π-meson ($M = 130.6MeV$) decays into a μ-meson ($m_1 = 105.7MeV$) and a neutrino ($m_2 = 0$). Calculate the kinetic energies of the μ-meson and the neutrino in the π-meson's reference frame. You may directly use expressions which are derived in parts (a) and/or (b).

Question 2. A spaceship is moving on a straight line with a speed $v = 0.5c$ with respect to a stationary observer. The speed of the spaceship can be increased by igniting an internal booster engine. If each boosts increase the speed of the spaceship by $0.5c$ in its own frame, what is the final speed of the spaceship after *two* such boosts with respect to a stationary observer?

Question 3. Consider a uniformly charged rod has proper length ℓ and total charge $+Q$

(a) Construct the four vector current densities in frames (i) fixed on the centre of the rod and (ii) moving axially with a constant speed $-v$.

(b) Find the quantity which is invariant in both the frames (Lorentz invariant).

Question 4. The energy of a proton at rest is $10^9 eV$. If the energy of the proton is increased to $10^{12} eV$ in an accelerator, find out the speed of proton inside the accelerator.

Question 5. Consider three inertial frames S, S' and S'' which denotes the space-time coordinates of an event at (w, x), (w', x') and (w'', x'') respectively. S' moves along x direction with a speed v_1 with respect to S. S'' moves with a speed v_2 relative to S' and with v_3 relative to S.

(a) If the matrix $\mathbb{A}_1$ transforms $(w, x) \to (w', x')$ and $\mathbb{A}_2$ transforms $(w', x') \to (w'', x'')$ write down the matrix elements of $\mathbb{A}_1$ and $\mathbb{A}_2$.

(b) Express the transformation $(w, x) \to (w'', x'')$ in matrix form as two successive Lorentz transformation. Find the resultant matrix elements.

(c) Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity v_3.