Problem Set 6
Data Analysis and Interpretation (EE 223)
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1. Suppose $x_1, x_2, ..., x_n$ is a random sample from an exponential distribution with parameter θ. Is the hypothesis $H : \theta = 3$ a simple or a composite hypothesis? What about $H : \theta > 2$?

2. By using central limit theorem to approximate the distribution of $\sum_{i=1}^{n} X_i$, show that the smallest value for n required to make $\alpha = 0.05$ and $\beta \leq 0.1$ is approximately 213. Let α denote the Type I error probability and β denote the Type II error probability.

3. Suppose X is a single observation from a population with probability density given by $f(x) = \theta x^{\theta-1}$ for $0 < x < 1$. Find the test with best critical region. That is, find the most powerful test, with significance level $\alpha = 0.05$, for testing a single null hypothesis $H_0 : \theta = 3$ against the simple alternative hypothesis $H_A : \theta = 2$.

4. Suppose $X_1, X_2, ..X_n$ is a random sample from a normal population with mean μ and variance 16. Find the test with best critical region, with a sample size of $n = 16$ and a significance level $\alpha = 0.05$ to test the null hypothesis $H_0 : \mu = 10$ against the alternative hypothesis $H_A : \mu = 15$.

5. $X = (X_1, X_2, ..., X_n)$ is a sequence of Bernoulli trials with unknown success probability θ, the likelihood
$$L(\theta|x) = (1-\theta)^n (\frac{\theta}{1-\theta})^{x_1+x_2+...+x_n}$$
. For the test $H_0 : \theta = \theta_0$ vs $H_1 : \theta = \theta_1$, take $\theta_0 = 1/2, \theta > 1/2$ and $\alpha = 0.05$.

6. Suppose that $Y_1, Y_2, ..., Y_n$ are independent Poisson (λ) random variables and consider testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$, where $\lambda_1 > \lambda_0$. Significance level $= \alpha$. Determine the decision regions using Neyman-Pearson lemma.

7. Let X be a random variable whose pmf under H_0 and H_1 is given by

x	1	2	3	4	5	6	7
$f(x/H_0)$	0.01	0.01	0.01	0.01	0.01	0.01	0.94
$f(x/H_1)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

   Use Neyman-Pearson Lemma to find the most powerful test for H_0 versus H_1 with size $\alpha = 0.04$. Compute probability of Type II Error for this test.

8. The R.V X has the pdf $f(x) = e^{-x}, x > 0$. One observation is obtained on the R.V $Y = X^{\theta}$, and a test of $H_0 : \theta = 1$ versus $H_1 : \theta = 2$ needs to be constructed. Find the UMP level $\alpha = 0.1$ test and compute the Type II Error probability.

9. Show that for a random sample $X_1, X_2, ..., X_n$ from a $N(0, \sigma^2)$ population, the most powerful test of $H_0 : \sigma = \sigma_0$ versus $H_1 : \sigma = \sigma_1$, where $\sigma_0 < \sigma_1$ is given by

$$\phi(\Sigma x_i^2) = \begin{cases} 1, & \Sigma x_i^2 > c \\ 0, & x_i^2 \leq c \end{cases}$$

For a given value of α, the size of Type I error, show how the value of c is explicitly determined.