EP 207: Introduction to Special Theory of Relativity
Tutorial 1 - Kinematics

Basic Postulates

Q1: The special theory of relativity rests on two principles - that there is no preferred inertial frame (which is common sensical once we realize that all motion is relative motion) and that the speed of light has the same value when measured in any inertial frame (which is implied by Maxwell's equations and can be verified experimentally).

Come up with some thought experiment to argue that if the speed of light is invariant then it must also be the maximum speed. Is there some general physical principle that would be violated if that was not so? (Hint: think causality)

Q2: The ancients seemed to implicitly believe that light travelled at infinite speed. Galileo was the first to try to experimentally measure the speed of light - though he failed. Try to think of some experimental device to do it. Google when you have given it enough thought.

Time-intervals between events are relative

Q3: Consider the special Michelson-Morley-like clock in the figure. It has a period of $\frac{2L}{c}$ (for an observer stationary to it). Check that its period is $\frac{2L}{\sqrt{1-\beta^2}c}$ for the moving observer. (Hint: Use the Pythagorean Theorem)

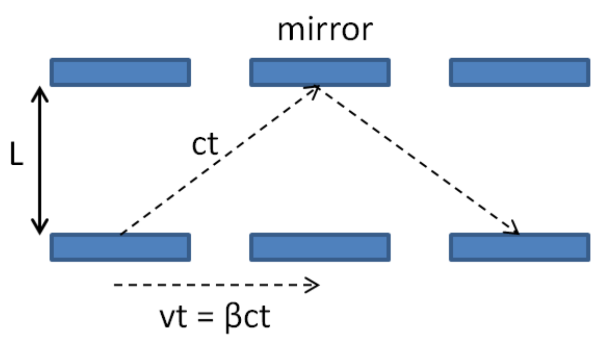

Figure 1: Q3

Q4: Suppose there are several other kinds of clocks kept (at rest) near the special clock. If the time-periods of all these clocks do not change in the same manner (for the moving observer) which principle of relativity is violated?

Length-intervals between events are relative

Q5: A rod of proper length L is kept at rest in frame I. A clock moves from one end (event A) to the other (event B). The clock is rest w.r.t the II.

a) According to frame I, what time does the clock show at event B?

b) The clock is stationary w.r.t the frame II but the time it shows at event B is the same as calculated in (a). How does observer II explain this?

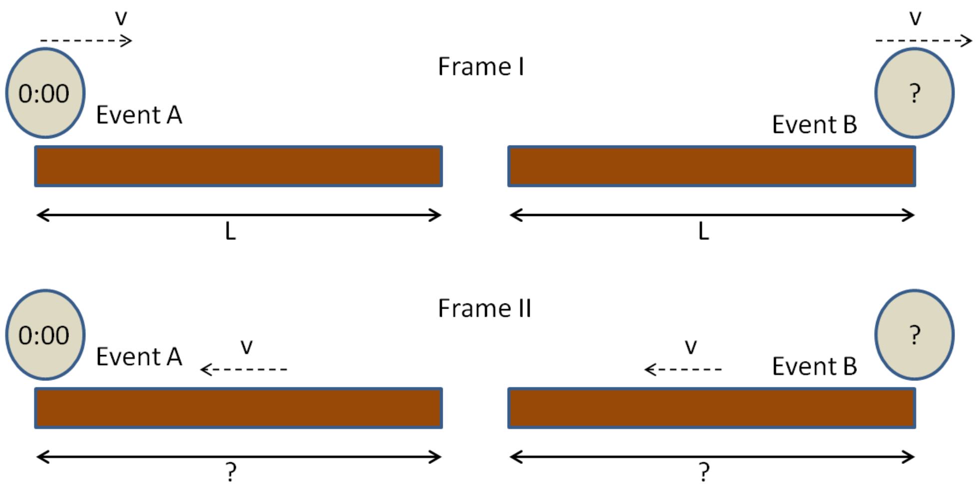

Figure 2: Q5

Simultaneity of events is relative

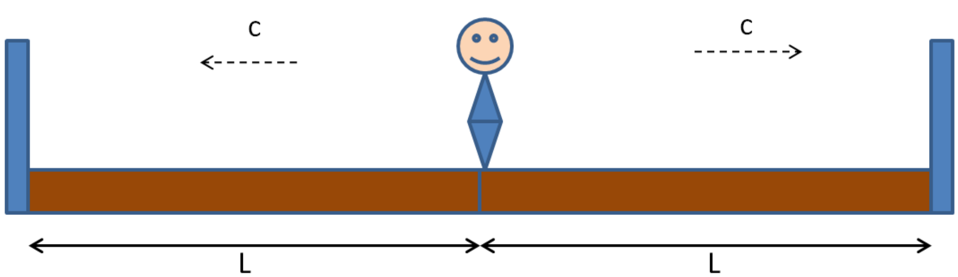

Figure 3: Q6

Q6: Consider a person standing in the center of a train compartment. He points two beams of light towards either end of the compartment and records the time at which the beams hit the walls(events A and B). The events are simultaneous (to the person).

a) Are the events still simultaneous to an observer travelling with a speed v? Which of events A or B happen earlier? How is that related to the direction of motion?

b) Calculate the time-difference between the two events for the observer. (Hint: Be careful about length-contraction).

c) If instead of light, the person had thrown a massive projectile towards the walls would the events still be simultaneous? (Hint: Be careful about relative speed arguments).

d) As the simultaneity of events is relative, clocks that are synchronized in one frame may not be synchronized in the other. Keeping that in mind, draw little clocks at the walls of the compartment and at the center during the occurring of events A and B for the two observers.

Putting it all together

Q7: A train passes a platform with velocity v. Two clocks are placed on the edge of the platform separated by a distance L and synchronized relative to the platform inertial system. Clock 1 reads $4:00$ when it coincides with the front of the train, and clock 2 reads $4:00$ when it coincides with the train rear of the train. Answer questions relative to an observer on the train.
a) What is the length of the train?
b) What is the reading of the clock 2 when clock 1 coincides with the front of the train?
c) What is the time interval between the two end events?
⟨taken from Basic Relativity by Richard Mould ⟩

Q8: Consider the transformations:

$$x' = \gamma(x - \beta\tau) \qquad\qquad x = \gamma(x' + \beta\tau') \tag{1}$$
$$\tau' = \gamma(\tau - \beta x) \qquad\qquad \tau = \gamma(\tau' + \beta x') \tag{2}$$

Here, $\tau = ct$. This is convenient as this will make the Lorentz equations symmetric.
a) Which one, S or S' is moving with positive relative velocity βc?
b) Redo Q6 and Q5 using only Lorentz transformations.

Q9 (relativistic velocity addition): Consider a compartment of proper length L with a pair of synchronized (w.r.t. the compartment) clocks at either end. A ball is released from one end (event A) to the other (event B) with speed u' as shown in the first figure.
If the compartment is moving with velocity v relative to the ground, then

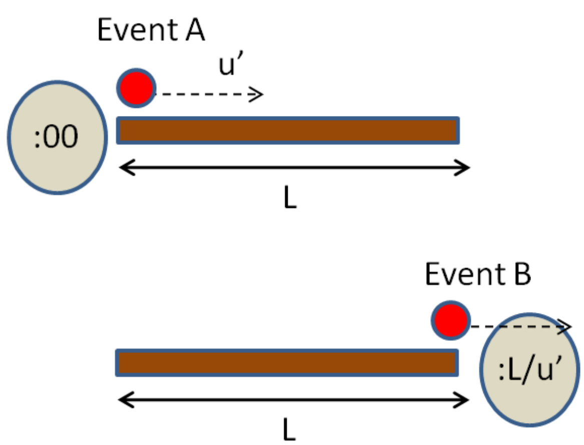

Figure 4: Q10 Frame of the compartment

a) Convince yourself that the measured length of the compartment relative to the ground will be

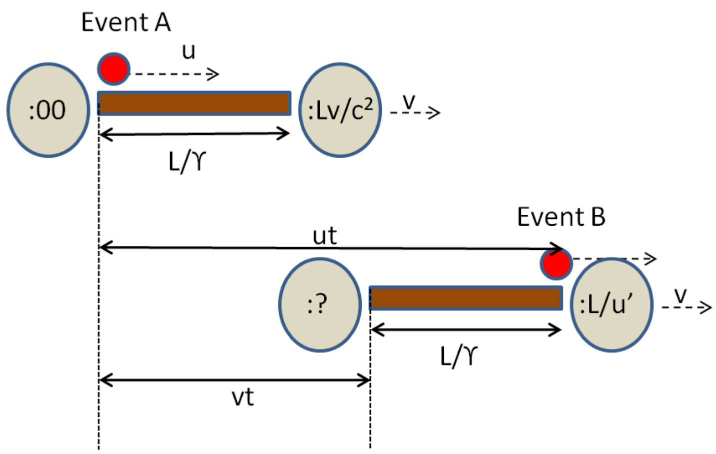

Figure 5: Q10 Ground frame

L/γ_v where $\gamma_v = 1/\sqrt{1 - v^2/c^2}$
b) Convince yourself that the clocks relative to the ground will be observed to be running slower by the factor $1/\gamma_v$.
c) Convince yourself that the clock at the front end will be lagging behind by $\frac{Lv}{c^2}$ as the clocks are not synchronized w.r.t. the ground. (The diagram misses out the sign).
d) Convince yourself that

$$\frac{L}{u'} = -\frac{Lv}{c^2} + \frac{t}{\gamma}$$

e) Find another equation for t, use the two equations to prove that

$$u = \frac{u' + v}{1 + u'v/c^2}$$

f) It is frequently said that an object's maximum speed is the velocity of light because the mass keeps increasing with increase the speed. However, one can argue that the velocity barrier is because of the form of the addition of velocities itself.
g) Redo the problem formally using Lorentz transformations.

"Relativity does not relativize everything; it only goes deeper to find constancy" - Richard Mould

Q10: Consider (x, τ) and (x', τ') related to each other by (1) and (2).
a) Construct a quantity $f(x', \tau')$ which is *invariant* (remains constant irrespective of v).
b) Verify that $s^2 = x'^2 - \tau'^2$ is such an invariant. s^2 is called the invariant interval between events.
c) The invariant-interval can be defined between any two space-time events by:

$$s^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - (\tau_1 - \tau_2)^2$$

d) For events that happen simultaneously w.r.t. any given frame of reference check that s^2 is positive. The order of occurrence of such events may change depending on the frame of reference.
e) For events that can in principle be causally connected s^2 will be negative.