Question 1. (3 Marks)

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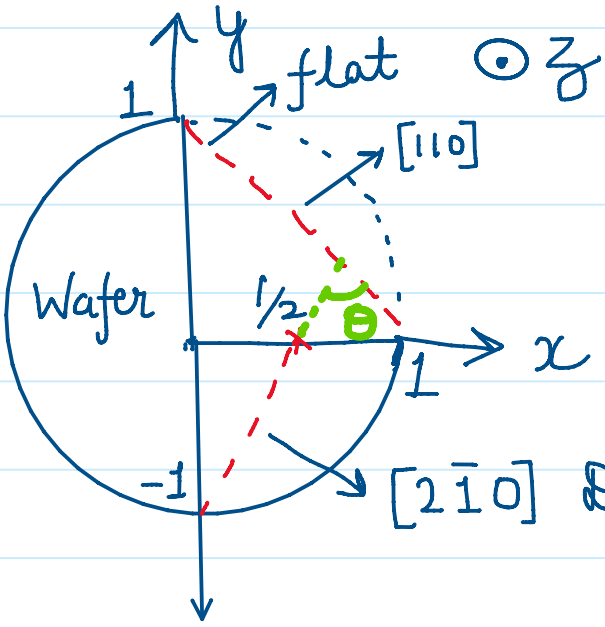

z - direction is perpendicular to the wafer surface.

(1 Mark for pictorial)

Desired direction.

Need to identify θ.

$$\cos\theta = \frac{[2\hat{i} - \hat{j}] \cdot [1\hat{i} + \hat{j}]}{\sqrt{5}\ \sqrt{2}}$$

$$= \frac{1}{\sqrt{10}}$$

(1 Mark for calculations)

Devices should be aligned Lar to the shown direction ([2 1̄ 0]).

(a) For part (a), you need to show that [2 1̄ 0] and the wafer normal [0 0 1] are perpendicular, [2 1̄ 0].[0 0 1] = 0 This implies the devices can be fabricated in the plane of the wafer.

(1 mark for reason)

Question 2. (6 Marks)

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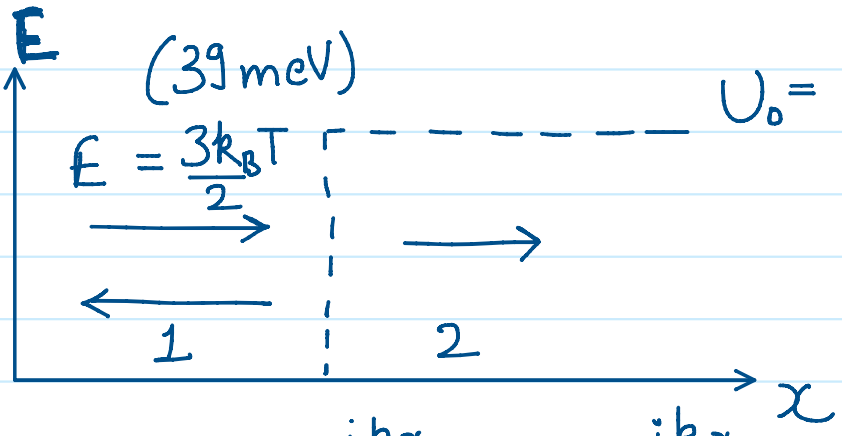

$$\psi_1(x) = 1e^{ikx} + re^{-ikx}$$

$$\psi_2(x) = Ce^{-\alpha x}$$

a) $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U_0\psi = E\psi \quad \text{(in region 2)}$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E - U_0)\psi$$

$$\Rightarrow \quad = \alpha^2\psi$$

$$\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar}} \quad \text{(1 mark)}$$

b) Interpreting $\psi_2(x) = Ce^{-x/(1/\alpha)}$ (1 mark for interpretation)

$\Rightarrow$ Decay length, $L_D = 1/\alpha$

$$L_D = \frac{\hbar}{\sqrt{2m_0(U_0 - E)}} = \frac{6.626 \times 10^{-34}/2\pi}{\sqrt{2 \times 9.1 \times 10^{-31} \times (3.38 - 39 \times 10^{-3})e}} \text{ m}$$

$\sim$ 1 Å (1 mark for answer)

L_D or $2L_D$ or $L_D/2$ are fine.

c) Continuity of ψ at $x = 0$,

$$1 + r = C \qquad (1 \text{ mark})$$

Continuity of ψ' at $x = 0$,

$$ik(1 - r) = -\alpha C \qquad (1 \text{ mark})$$

$$\Rightarrow ik(1 - r) = -\alpha(1 + r)$$

$$\Rightarrow r = -\frac{(\alpha + ik)}{(\alpha - ik)}$$

$$|r|^2 = \frac{\sqrt{|\alpha|^2 + |k|^2}}{\sqrt{|\alpha|^2 + |k|^2}} = 1 \qquad (1 \text{ mark})$$

Question 3 (2 marks)

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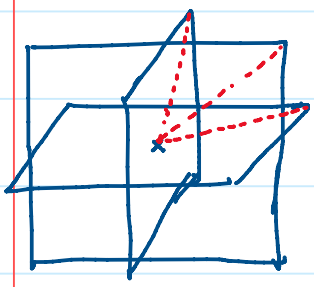
For the [110] directions, there will be 12 equivalent band minima



And the minima occur at halfway on the edges

$\Rightarrow$ All 12 contribute to m_{DOS}. (1 mark)

(In contrast, Ge has 8 equivalent band minima right at the edge, so only 4 effectively contribute to m_{DOS})

$$m_{DOS} = (M_c)^{2/3} (m_l\, m_t\, m_t)^{1/3}$$

M_c: 12 $\quad$ m_l: $1.12 m_0$ $\quad$ m_t: $0.22 m_0$

$$= 1.98\, m_0 \quad (1 \text{ mark})$$

Question 4 (2 marks)

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$$p = N_V e^{(E_V - E_F)/k_B T} \quad ; \quad N_V \propto (m^*)^{3/2}$$

(1 mark)

Assuming E_V is the same for both the bands (hh & lh)

$$\frac{p_{hh}}{p_{hh} + p_{lh}} = \frac{m_{hh}^{3/2}}{m_{hh}^{3/2} + m_{lh}^{3/2}} = \frac{(0.54)^{3/2}}{(0.54)^{3/2} + (0.15)^{3/2}}$$

$$= 0.87$$ (1 mark)