Introduction to the Special Theory of Relativity

EP 207 Assignment

Submission deadline: 12/9/2018 11.00 am
Total marks 5

Question 1. Show that the homogeneous Maxwell's equation $\partial_i \mathscr{F}^{ij} = 0$ can be written as $\partial^i F^{jk} + \partial^j F^{ki} + \partial^k F^{ij} = 0$ where i, j, k takes any possible values $0, 1, 2, 3$, ∂^i is the *contravariant* 4-divergence, F^{ij} is the field tensor and $\mathscr{F}^{ij} = \epsilon^{ijkl} F_{kl}$.

Question 2. Show that the Lorentz force equation $\mathbf{F} = q(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B})$ and the energy rate equation $\frac{d\mathcal{E}}{dt} = \mathbf{J}.\mathbf{E}$ can be combined into a Lorentz invariant form

$$\frac{dp^i}{d\tau} = \frac{q}{c} F^{ij} v_j$$

Here $p^i = (\mathcal{E}/c, \gamma m_0 \mathbf{v})$, $d\tau$ is the differential *proper* time, $v^i = (\gamma c, \gamma \mathbf{v})$