# EE225 Network Theory

TUTORIAL PROBLEMS - SET TWO,

**Q1.** Verify the following Laplace transform pairs and in each case specify the region of convergence

| S. No. | Function x(t) | Laplace Transform of x(t) |
|---|---|---|
| 1. | 1 | $1/s$ |
| 2. | t | $1/s^2$ |
| 3. | $t^{n-1}/(n-1)!$ ; n integer, n>1 | $1/s^n$ |
| 4. | $e^{at}$ | $1/(s-a)$ |
| 5. | $te^{at}$ | $1/(s-a)^2$ |
| 6. | $t^{n-1}e^{at}/(n-1)!$ ; n integer, n>1 | $1/(s-a)^n$ |
| 7. | $sin(\omega_0 t)$ | $\omega_0/(s^2+\omega_0^2)$ |
| 8. | $cos(\omega_0 t)$ | $s/(s^2+\omega_0^2)$ |
| 9. | $sinh(\omega_0 t)$ | $\omega_0/(s^2-\omega_0^2)$ |
| 10. | $cosh(\omega_0 t)$ | $s/(s^2-\omega_0^2)$ |
| 11. | $e^{-at}sin(\omega_0 t)$ | $\omega_0/((s+a)^2+\omega_0^2)$ |
| 12. | $(e^{at}-e^{bt})/(a-b)$ ; $a \neq b$ | $1/(s-a)(s-b))$ |

**Q2.** Obtain the inverse Laplace Transform of the following with a Region of convergence to the right of the rightmost pole. (corresponding right side function). In each case, specify thus the region of convergence

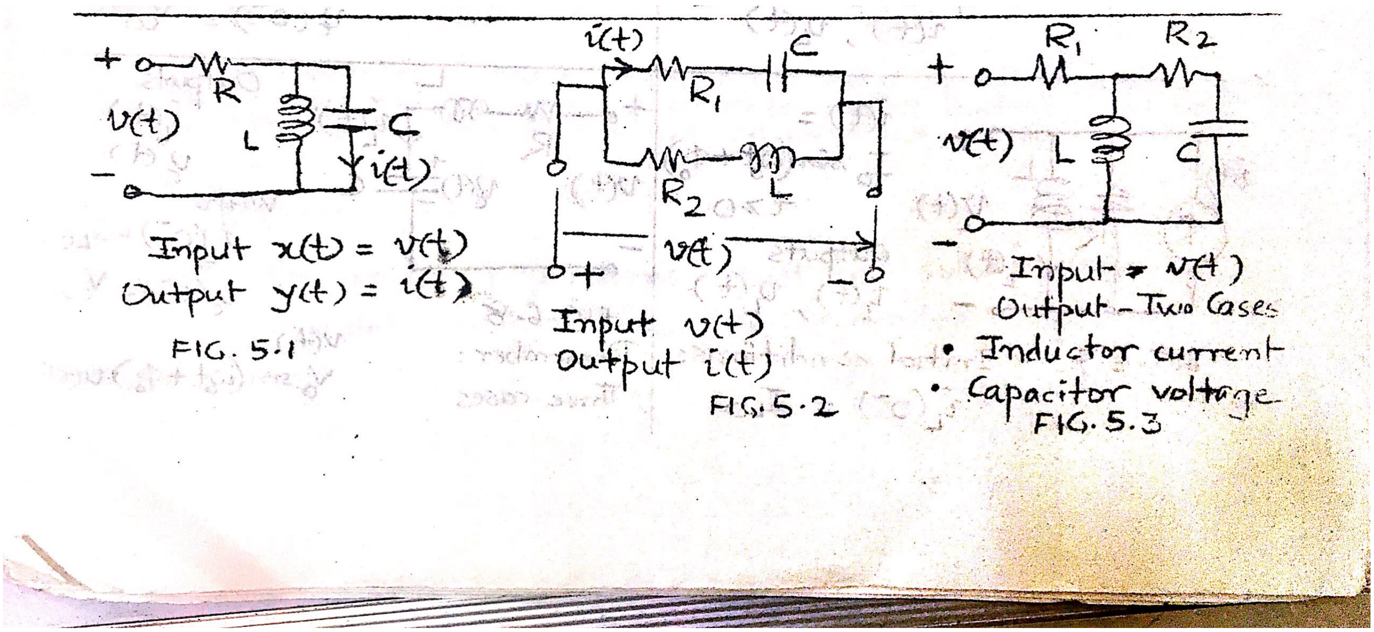
1. $3s/((s^2+1)(s^2+4))$
2. $(s+1)/(s^2+2s)$
3. $1/(s^3(s^2-1))$
4. $s^2/(s^2+1)^2$
5. $n!/(s(s+1)...(s+n))$ : Can you write it compactly ? (Hint : Binomial Theorem)

**Q3.** If the Laplace Transform of $x(t)$ be $X(s)$ with region of convergence R, show that the Laplace Transform of $x(t-t_0)$ is $e^{-st_0}X(s)$ with ROC R.

**Q4.** $g(t)$ is a periodic waveform with period $T$. Let $G_0(s)$ be the Laplace Transform of one period of $g(t)$ from 0 to $T$ ; i.e. that of $g_0(t) = \{g(t)\ for\ 0 < t < T$and zero otherwise$\}$. Show that the Laplace transform $G(s)$ of $g(t)u(t)$ is given by :
$G(s) = G_0(s)/(1-e^{-sT})$
Comment on the region of convergence

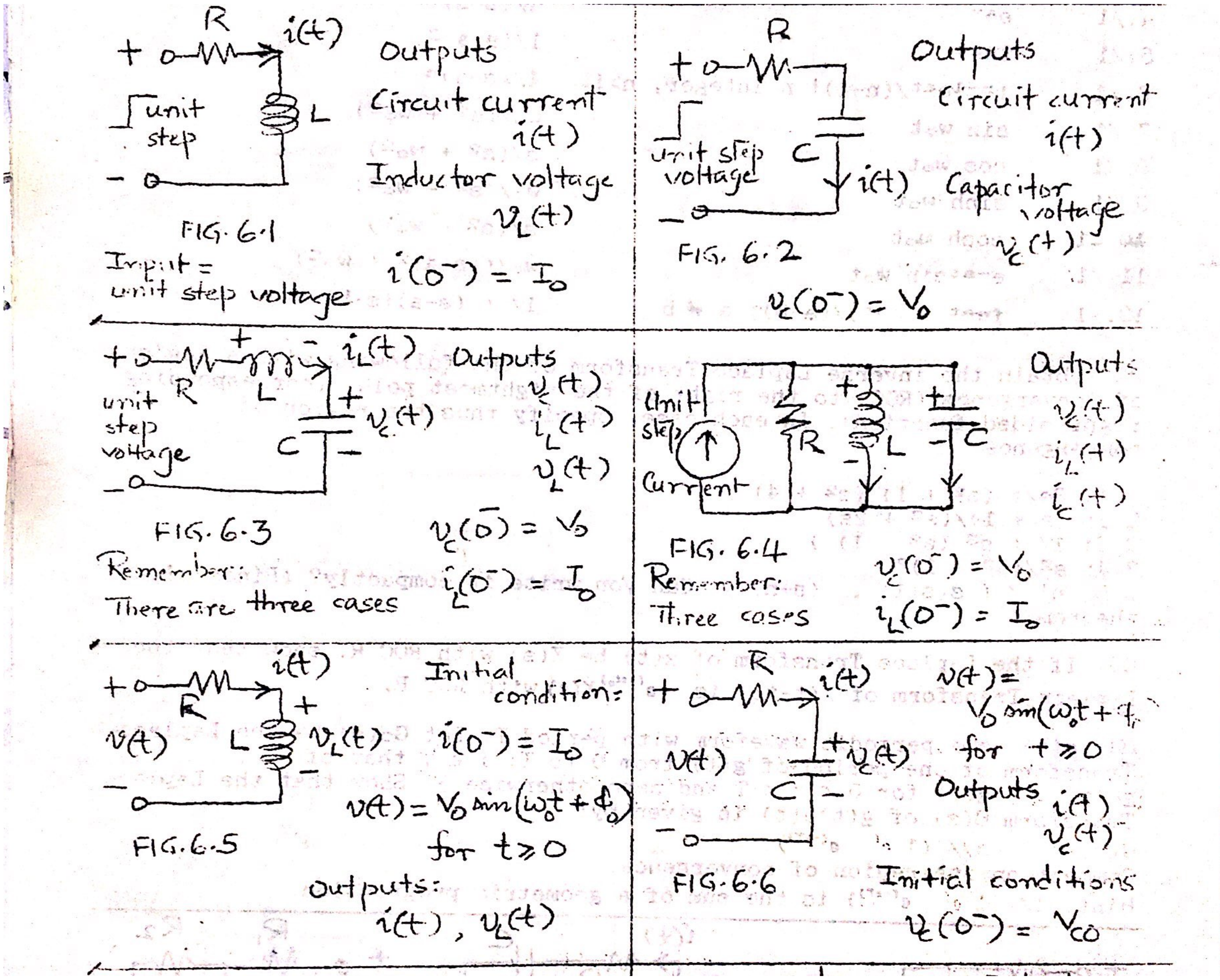
**Hint :** $1/(1 - e^{-sT})$ is the sum of a geometric progression.

**Q5.** In each of the circuits of Fig. 5 obtain :
(a) the driving point impedence and admittance
(b) the transfer function relating input to output as designated. You may consider the networks as initially unexcited for linearity.
(c) In Fig 5.2, is it possible for some cases to make the driving point impedance independent of s ? If so, under what condition(s) on the element values ? What is the implication of this ?




**Q6.** For each of the circuits of Fig 6., find the designated output waveforms as a function of time with the inputs and initial conditions as designated.

**Q7.** In each of the cases in Q5., find the poles and zeros of the driving point and transfer functions. In each of these functions, obtain the form of the output (numerator) quantity taking the input (numerator) quantity (a) as $u(t)$ : step response (b) as a unit impulse :impulse response. There may be more than one form of the output depending on parameters. Your solution to Q6 would guide you.


R
i(t)
unit step
L
Outputs
Circuit current i(t)
Inductor voltage v_L(t)
FIG. 6.1
Input = unit step voltage
i(0⁻) = I_0
R
Outputs
Circuit current i(t)
unit step voltage
C
i(t)
Capacitor voltage v_c(t)
FIG. 6.2
v_c(0⁻) = V_0
i_L(t)
Outputs
unit step voltage
R
L
C
v_c(t)
v_c(t)
i_L(t)
v_L(t)
FIG. 6.3
v_c(0⁻) = V_0
Remember:
There are three cases
i_L(0⁻) = I_0
Unit step Current
R
L
C
Outputs
v_c(t)
i_L(t)
i_c(t)
FIG. 6.4
Remember:
Three cases
v_c(0⁻) = V_0
i_L(0⁻) = I_0
i(t)
Initial condition:
R
v(t)
L
v_L(t)
i(0⁻) = I_0
v(t) = V_0 sin(ω_0 t + φ_0)
for t ≥ 0
FIG. 6.5
outputs:
i(t), v_L(t)
R
i(t)
v(t) = V_0 sin(ω_0 t + φ_0)
v(t)
v_c(t)
C
for t ≥ 0
Outputs i(t) v_c(t)
FIG. 6.6
Initial conditions
v_c(0⁻) = V_c0