Problem Set 1
Data Analysis and Interpretation (EE 223)

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1 Warming Up

1. Let X be a discrete random variable with probability mass function as given below:

$$\mathbb{P}(X=x)=\begin{cases} kx & \text{for } x=1,2,4 \\ k(x-1) & \text{for } x=3,5,6 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find k.
   (b) Find $\mathbb{E}[X]$ and $\mathbb{E}[X^2]$.

2. Let X be a Gaussian random variable with density function given as below:

$$f_X(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}.$$

   Find mean and variable of X.

3. Let X be a Poisson random variable with probability mass function given as below:

$$\mathbb{P}(X=k)=\frac{\lambda^k}{k!}e^{-\lambda},$$

   for $k=0,1,2,\ldots$ and $\lambda>0$. Find mean and variable of X.

4. Let X be a continuous random variable with distribution F_X (denoted as $X\sim F_X$). Let $Y=\max\{X,c\}$ for some constant c. Find F_Y.

5. Let $X\sim F_X$ and $Y=I_{\{X>0\}}$. Find F_Y.

2 Getting more into it

1. Let X be a uniform random variable on $[0,1]$. Define $Y=-\log X$.
   (a) Show that Y is a random variable.
   (b) Find F_Y.

2. Let X be a continuous random variable with distribution F_X. Define $Y=aF_X(X)+b$.
   (a) Show that Y is a random variable.
   (b) Find F_Y.

3. Show that if $X\geqslant 0$ with $\mathbb{E}[X]=\eta$, then $\mathbb{P}(X\geqslant\sqrt{\eta})\leqslant\sqrt{\eta}$.

4. X is Gaussian random variable with $\mu=0$ and $\sigma^2=4$ and $Y=3X^2$. Find mean and variance of Y.

5. Let X and Y be two continuous random variables. Show that

$$\mathbb{E}[\log f_X(X)]\geqslant\mathbb{E}[\log f_Y(X)].$$

3 All in

1. Let X and Y be random variables such that $f_{XY}(x, y) = e^{-(x+y)}$ whenever x and y are positive. Find the distribution for $X+Y$, $X-Y$, XY, X/Y, $\min\{X, Y\}$ and $\max\{X, Y\}$.
2. Let $f_{XY}(x, y) = 2(1-x)$ whenever $(x, y) \in [0, 1]^2$ and zero otherwise. Find f_Z for $Z = XY$.
3. Let $g : \Re \to \Re$ be a monotone increasing function and let $Y = g(X)$. Find F_{XY}.
4. Let $f_{XY}(x, y) = e^{-x}$ if $0 \leqslant y \leqslant x < \infty$ and zero otherwise. Find $\mathbb{E}[Y|X]$.
5. Show that

$$\mathbb{E}[Y|X \leqslant 0] = \frac{1}{F_X(0)} \int_{-\infty}^{0} \mathbb{E}[Y|X] f_X(x) dx.$$

4 If you like that kinda thing...

1. Let X and Y be two random variables defined on $(\Omega, \mathcal{F}, P)$. Show that $X + c$, $\max\{X, Y\}$, $\min\{X, Y\}$, $X+Y$, XY and X/Y are random variables $(\Omega, \mathcal{F}, P)$.