Example 3.1.3 [Bestsekas]

$$\min \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$$ [spheres

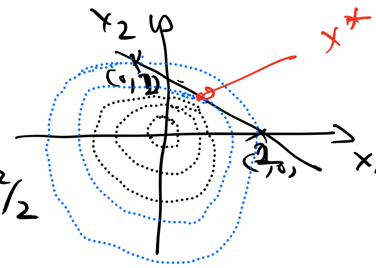
subject to $x_1 + x_2 + x_3 = 3$

$x_1^2 + x_2^2 + x_3^2 = r^2$ $\rightarrow$ sphere of radius r

$x_1 + x_2 + x_3 = 3$ $\rightarrow$ represents a plane

$$\min \frac{1}{2}(x_1^2 + x_2^2)$$

s.t. $x_1 + x_2 = 2$



$x_1^2 + x_2^2 = r^2$ obj function $r^2/2$

$$\min_x f(x)$$
$$Ax = b$$

$A = [1 \quad 1 \quad 1]$ $b = [1]$

$$L(x, \lambda) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) + \lambda(x_1 + x_2 + x_3 - 3)$$

Look at its stationary point.

$\frac{\partial L}{\partial x_1} = 0 \Rightarrow x_1^* + \lambda^* = 0$

$\frac{\partial L}{\partial x_2} = 0 \Rightarrow x_2^* + \lambda^* = 0$

$\frac{\partial L}{\partial x_3} = 0 \Rightarrow x_3^* + \lambda^* = 0$

$x_1^* = x_2^* = x_3^* = 1$; $\lambda^* = -1$;

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow x_1 + x_2 + x_3 - 3 = 0$ [feasibility]

$$\nabla f(x^*) + A^T \lambda^* = 0$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}(-1) = [0]$$

Necessary condition for optimality is satisfied

$Z^T H Z$

$$\frac{\partial^2 f}{\partial x_i^2} = 1$$

$$H = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial x_1 \partial x_2} = 0$$

trivial p.d. matrix

$Z^T H Z$ will be s.p.d. if H is s.p.d.

$$x^T (Z^T H Z) x = \underbrace{(Zx)^T}_{y^T} H \underbrace{Zx}_{y} \geq 0$$

$y^T H y \geq 0$

$\to$ H is p.d.

$Z \Rightarrow$ null space basis

$x \neq 0 \quad Zx \neq 0$

$y^T H y > 0$

$y = Zx$

$Z^T H Z$ projected Hessian is p.d. $\quad x \neq 0, \ y \neq 0$

stationary pt. of Lagrangian

$Z^T H Z$ is s.p.d.

(1) $(1,1,1) \to$ is strict local <u>min</u>.