Navigation & Guidance

(Course Code: AE 410/641)

Department of Aerospace Engineering
Indian Institute of Technology Bombay

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Date: **26-30 Nov 2020** **End-Semester Examination** Total Points: 100

General Instructions

- It is highly recommended (not mandatory though) that students submit neatly typeset document containing answers to each of the questions.
- The answer sheet needs to be submitted online in .pdf format on Moodle.
- Marks for each portion of question are given separately.
- **Please show your steps clearly while solving problems. Reasonably assume any data or information you feel is missing. However, the same must be mentioned clearly.**
- Do not indulge in academic dishonesty. In cases where the answers of two students are found to be copied, both of them will be awarded zero marks for that particular question. Furthermore, those students may face disciplinary actions.
- A viva may also be conducted in case of suspicion.
- Submission is due on **30th November 2020, 6 pm**.
- Late submissions will not be entertained.

1. Consider a planar engagement between a missile and a target. If the missile is guided using deviated pursuit from the beginning of the engagement, then

   (a) show analytically that the time-to-go of missile-target engagement is zero if and only if the relative distance between the missile and target is zero.

   (b) derive the condition for which the missile intercepts the target in minimum time.

[6+6]

2. Consider a three-dimensional engagement between a missile and a target wherein the missile uses parallel navigation from the beginning of the engagement.

   (a) Prove that the engagement is instantaneously planar.

   (b) Consider a scenario where the missile is initially at the origin and the target starts at $(12, 3, 4)$ km. The target is moving in $-x$ (negative x) direction with a speed of 250 m/s, while the missile moves with twice the target's speed. Determine the velocity components of the missile, and the missile's flight time.

[6+6]

3. Consider a planar engagement between a missile and a constant velocity target wherein the vehicles are separated by a radial distance of 10 km at the time of their launch. The target has a speed of 200 m/s and is launched with a flight path angle of 90°. Initially, the line-of-sight between the vehicles is 0°. The missile has a speed of 400 m/s and is launched with a heading angle error of +30°. *Note that heading angle error is measured positive in the anti-clockwise directions from the collision course.*

   (a) What is the flight path angle, γ_{M}, of the missile at the time of launch if $\gamma_{\mathrm{M}} \in (-\pi/2, \pi/2)$?

   (b) Suppose after three seconds of engagement, the missile is alerted that it is launched with a flight path angle inconsistent with interception. As a result, the missile is now commanded by a pure proportional-navigation (PPN) guidance law after three seconds of engagement. What would be the lateral acceleration of the missile at the instant it starts executing PPN guidance law with an optimal value of N?

   (c) Plot the trajectories of missile and target in inertial space, and plot the time evolution of lateral acceleration of the missile until interception. Please label your plots clearly and provide a brief explanation of the same.

[2+8+6]

4. Consider a planar engagement between a missile and a non-maneuvering target whose speeds are denoted by V_{M} and V_{T}, respectively. Let ν be the ratio of speeds of the missile and that of the target. Assume that $\gamma_{\mathrm{T}} \triangleq 0$ and $\nu > 1$. If the missile is guided using proportional-navigation guidance such that $\dot{\gamma}_{\mathrm{M}} = 2\dot{\theta}$, then prove the following.

   (a) Near interception (i.e. $r \to 0$), the lateral acceleration of the missile, a_{M}, satisfies

$$a_{\mathrm{M}} \to \begin{cases} 0 & \text{if } \nu \cos \alpha_0 > -1 \\ \infty & \text{if } \nu \cos \alpha_0 < -1 \\ \text{a finite constant} & \text{if } \nu \cos \alpha_0 = -1 \end{cases}$$

   where $\alpha_0 = 2\theta_0 - \gamma_{\mathrm{M}_0}$, while θ_0 and γ_{M_0} respectively denote the initial LOS and flight path angles of the missile.

   (b) If $0 < \theta_0,\ \gamma_{\mathrm{M}_0} - \theta_0 < \pi$, then the lead angle of the missile is strictly decreasing until interception. (*Hint: To show a variable x strictly decreases, it is sufficient to show* $\dot{x} < 0\ \forall\ t\ \in \mathbb{R}_+$).

[10+5]

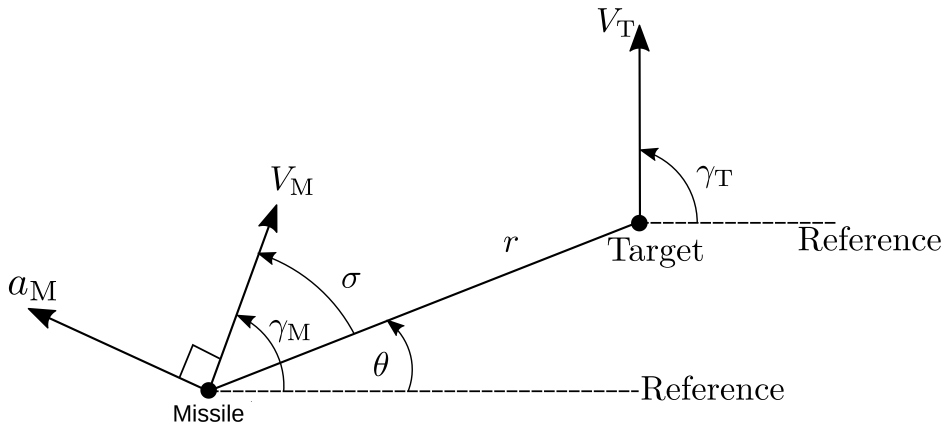

Figure 1: Engagement geometry between a missile and a constant velocity target.

5. Consider a planar engagement between a constant velocity target and an unguided missile as shown in Figure 1. Assuming that the missile and the target are on collision course,

   (a) derive the relationship between line-of-sight angle, θ, of the collision course and η, where $\eta = \gamma_{\mathrm{T}} - \gamma_{\mathrm{M}}$.

(b) compute the time of interception if the initial relative distance is 10 km, $\eta = 30°$, $\gamma_\mathrm{T} = 60°$, and $V_\mathrm{M} = 2V_\mathrm{T} = 200$ m/s.

[5+5]

6. Prove that in parallel navigation, if $\dfrac{V_\mathrm{M}}{V_\mathrm{T}} > 1$, then the ratio $\left|\dfrac{a_\mathrm{M}}{a_\mathrm{T}}\right| \leq 1$, where a_T is the target's acceleration applied perpendicular to its velocity vector. [5]

7. Consider a rotation of the basis vector $\mathbf{e} = (1, 0, 0)$ about an axis defined by a unit vector $q = (q_1, q_2, q_3)$ through an angle θ. The components q_1, q_2, q_3 are a Pythagorean triplet of an isosceles right-angled triangle, as shown in Figure 2. Compute the effect of this rotation on the basis vector $\mathbf{e}$. [6]

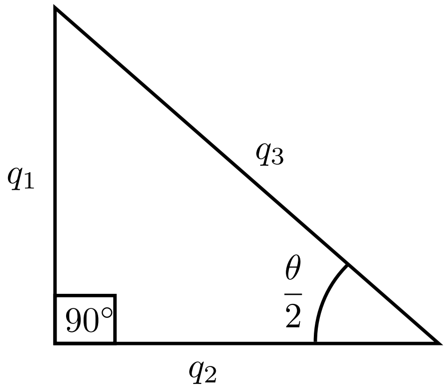

Figure 2: An **isosceles** right-angled triangle whose sides are components of a unit vector.

8. (a) Consider a simple vertical accelerometer with its input axis along Z axis. Calculate the altitude divergence (Δh) after 1 second if the error in vertical acceleration is 5×10^{-3} m/s^2. Assume that the altitude is measured from the surface of earth, whose radius is 6400 km and $\Delta h(0) = \Delta \dot{h}(0)$.

   (b) Consider two stations situated at T_1 and T_2 transmitting synchronized signals of a frequency 100 MHz. What is the relative distance between T_1 and T_2 if Omega Navigation is used and the phase difference between the signals is 10 KHz?

[2+2]

9. (a) Consider a LORAN (hyperbolic navigation) equipped aircraft located at a distance of 500 km from the master and 200 km from the slave ground stations. If the aircraft measures a time delay of 0.25 ms, then compute the the distance between the master and the slave stations. Assume that the slave station transmits its own signal after a time delay of 0.3 ms.

(b) Assume the master and the slave stations are separated by a distance as computed in part (a), and the slave station transmits its own signal after a delay of 0.1 ms. What is the time delay measured by the aircraft in the following situations?

(i) The aircraft is equidistant from the master and the slave ground stations.

(ii) The aircraft and the ground stations are collinear such that the slave station lies between the master station and the aircraft.

(iii) The aircraft and the ground stations are collinear such that the master station lies between the slave station and the aircraft.

[4+6]

10. Consider five satellites denoted by S_1, S_2, S_3, S_4 and S_5. Table 1 represents the line-of-sight orientation of these satellites as observed by a receiver.

Satellite	S_1	S_2	S_3	S_4	S_5
Elevation (in °)	10	10	10	10	90
Azimuth (in °)	0	100	150	200	0

Table 1: Satellite orientations as observed by the receiver.

(a) Calculate visibility matrices for all possible sets of 4-satellite configuration.

(b) Determine the values of *geometric dilution of precision* (GDOP) and *position dilution of precision* (PDOP) for all these possible sets of 4-satellite configuration.

(c) Depending on PDOP values calculated in part (b), select the configuration of satellites for the most accurate calculation of 3D position.

[4+4+2]