ARPIT Course on

Electronic Systems for Sensor Applications

Lecture 2: Measurements, and how to use them

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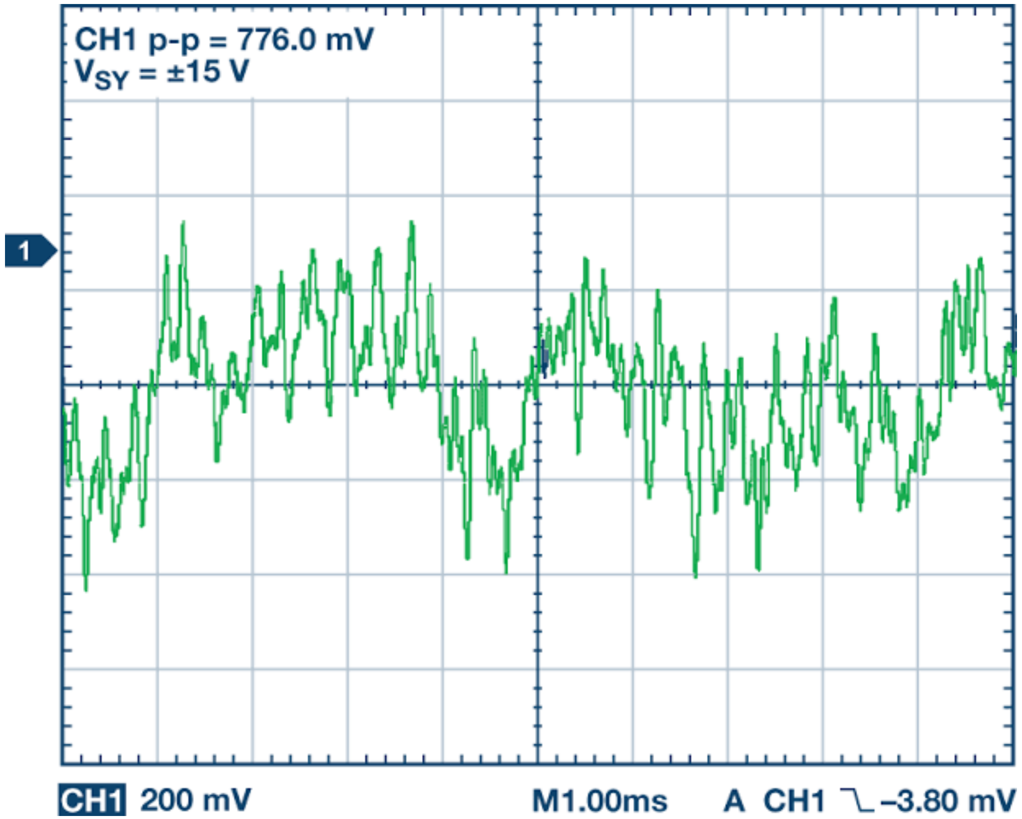
Learning objectives

- How to correctly use datasheet specifications to design a solution for a target application
- Case study for understanding how to address design trade-offs in sensor system design

Measuring offset and noise



- Shift in offset (e.g. offset drift) is very hard to distinguish from change in the measurand (physical quantity to be measured). Noise variations are easier to manage (e.g. filtering, averaging etc.)
- Typically require additional information (auxiliary sensors) to debug.
- Impact of errors due to offset are often underestimated. Variations in offset and sensitivity are much harder to calibrate/compensate as compared to high noise.

Measuring offset and noise

- What do you think offset and noise measurements (variation) could be impacted by?
  - Inherent (stimulus + sensor + circuit) variation
  - Variation due to different operators (reproducibility)
  - Variation of measurement due to equipment error (repeatability)
- <u>Repeatability</u> refers to the measurement variation obtained when one person repeatedly measures the same item with the same gage.
- <u>Reproducibility</u> refers to the variation due to different operators using the same gage measuring the same item.

$$\sigma_m^2 = \sigma_p^2 + \sigma_o^2 + \sigma_e^2$$

Variance of actual measurement

Variance of true product characteristic

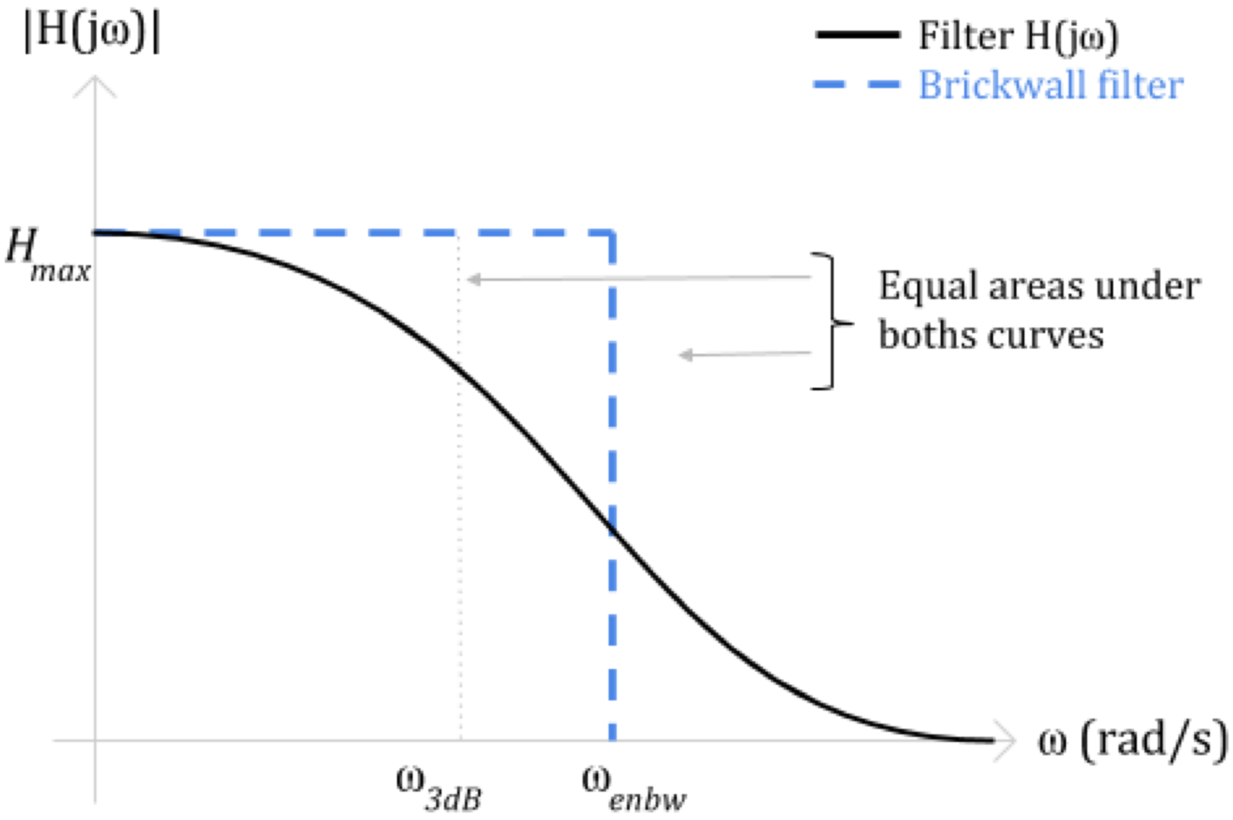
Variance due to operator

Variance due to equipment error

Units of noise

- Consider a sensor that produces a voltage output
- RMS noise is specified in units of [V-rms]
- Peak-to-peak noise ~ 5.5 x RMS noise
- Noise density is specified in units of [V^2/Hz] (power) or [$V/\sqrt{Hz}$] (voltage)
- RMS noise = Voltage noise density x $\sqrt{(\text{bandwidth})}$

Equivalent noise bandwidth



ENBW of First Order Lowpass filter

Transfer function of I-order lowpass filter is

$$H(j\omega) = \frac{H_{max}}{1 + j\frac{\omega}{\omega_c}} \tag{2}$$

where ω_c is the corner frequency of the filter. It is same as -3dB frequency for first order filter.

The equivalent noise bandwidth of I-order lowpass filter, substituting Eq-(2) in Eq-(1), is given by

$$\omega_{enbw} = \int_0^\infty \left|\frac{H(j\omega)}{H_{max}}\right|^2 d\omega = \int_0^\infty \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^2} d\omega \tag{3}$$

$$= \omega_c \tan^{-1} \frac{\omega}{\omega_c}\Big|_0^\infty = \frac{\pi}{2}\omega_c$$

The 3-dB bandwidth (ω_{3dB}) is found by evaluating the value of (ω) where, $20\log|H(j\omega)| - 20\log|H_{max}| = -3dB$. Hence

$$20\log\left|\frac{1}{1 + j\frac{\omega}{\omega_c}}\right| = -3dB$$

$$\boxed{\omega_c = \omega_{3dB}} \tag{4}$$

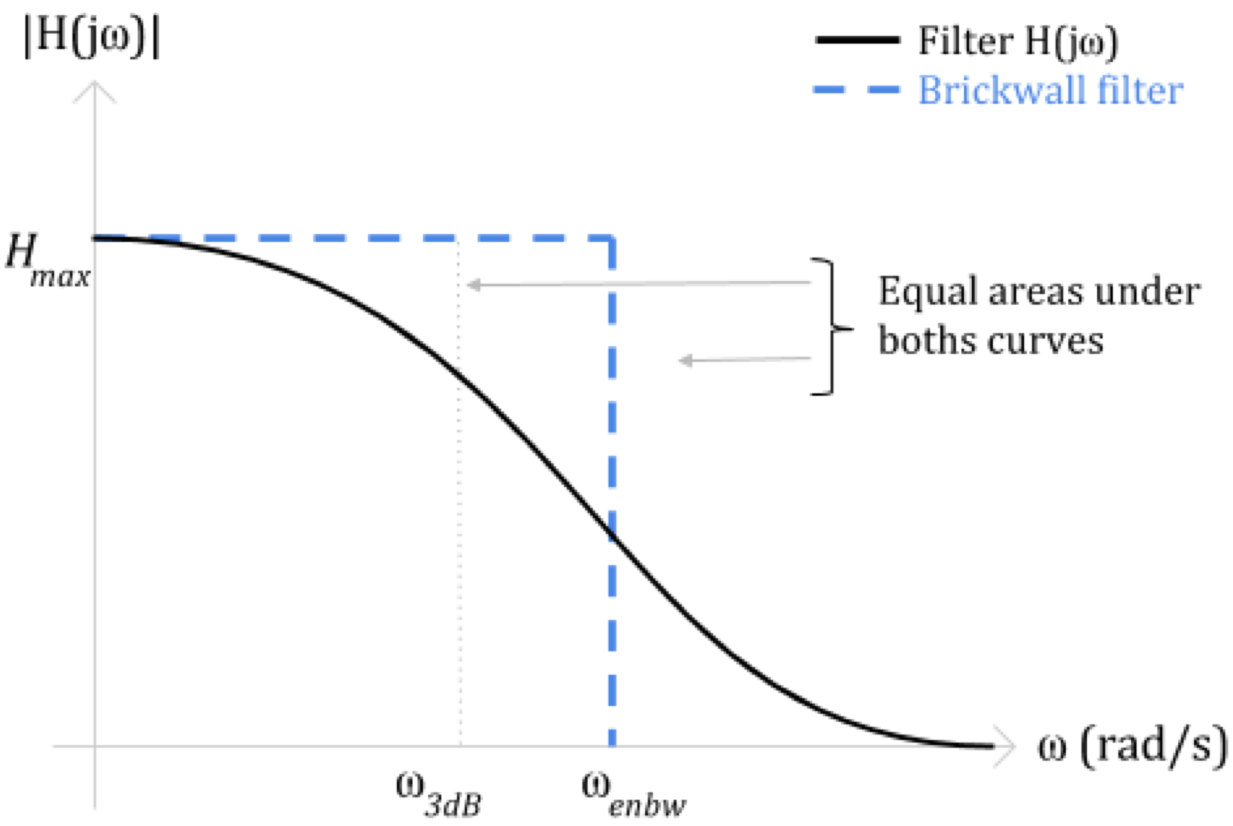
Therefore, for first order lowpass filter the corner frequency(ω_c) is same as 3dB frequency (ω_{3dB}).

From Eq-(4) and Eq-(3), the I-order passive low-pass filter ENBW is related to 3dB frequency as

$$\boxed{\omega_{enbw} = \frac{\pi}{2}\omega_{3dB} = 1.57\ \omega_{3dB}} \tag{5}$$

https://analog.intgckts.com/equivalent-noise-bandwidth/

Equivalent noise bandwidth

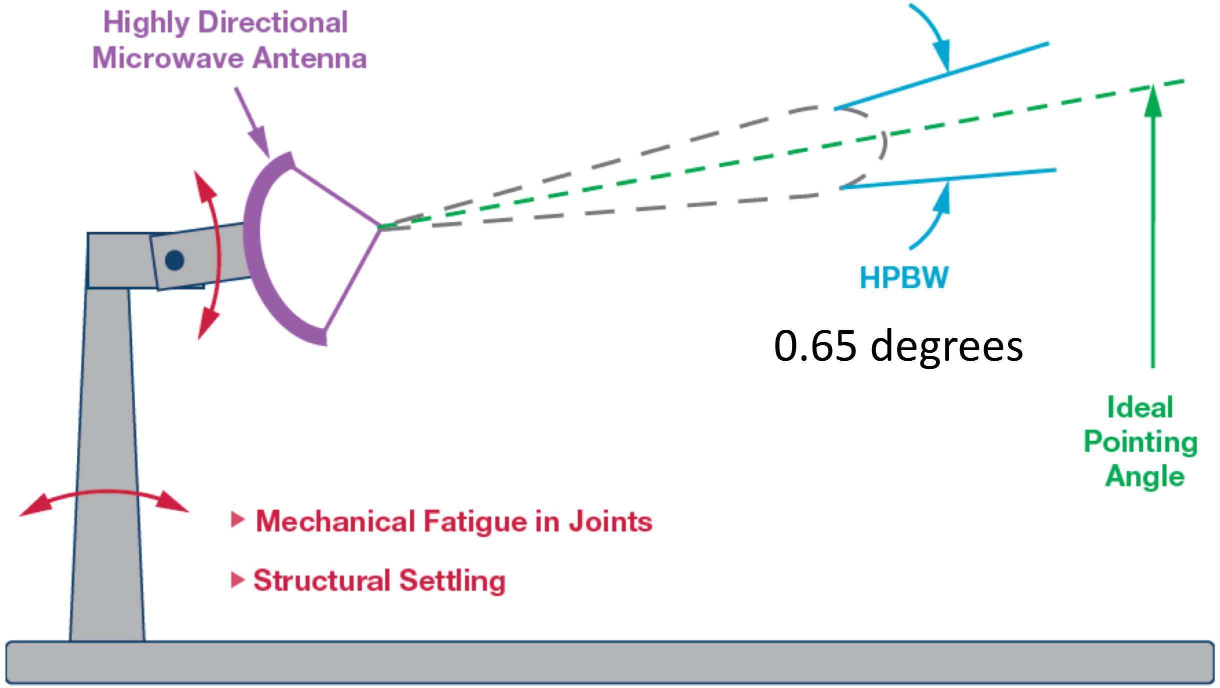


Filter Order	$\omega_{enbw} / \omega_{3dB}$
1	1.57
2	1.22
3	1.15
4	1.13
5	1.11

Table 1. Filter order vs Multiplication factor ($\omega_{enbw} / \omega_{3dB}$)

https://analog.intgckts.com/equivalent-noise-bandwidth/

Case study: Low noise vs low power tradeoff



Device	Noise (µg/√Hz)	Power (µW)
XL_1	20	340
XL_2	178	26

The reliability of the data link depends on the accuracy of the pointing angle. Maintaining the pointing angle can require manual adjustment, especially after earthquakes and other disturbances to the platforms that these antennas rest on. You must choose a MEMS accelerometer to implement a tilt sensor.

The installation will be in a remote location, so low power is important so as to not replace batteries often. However, accuracy is important for data link reliability, so low noise is also desirable. Suggest a methodology to choose a sensor from the table above.

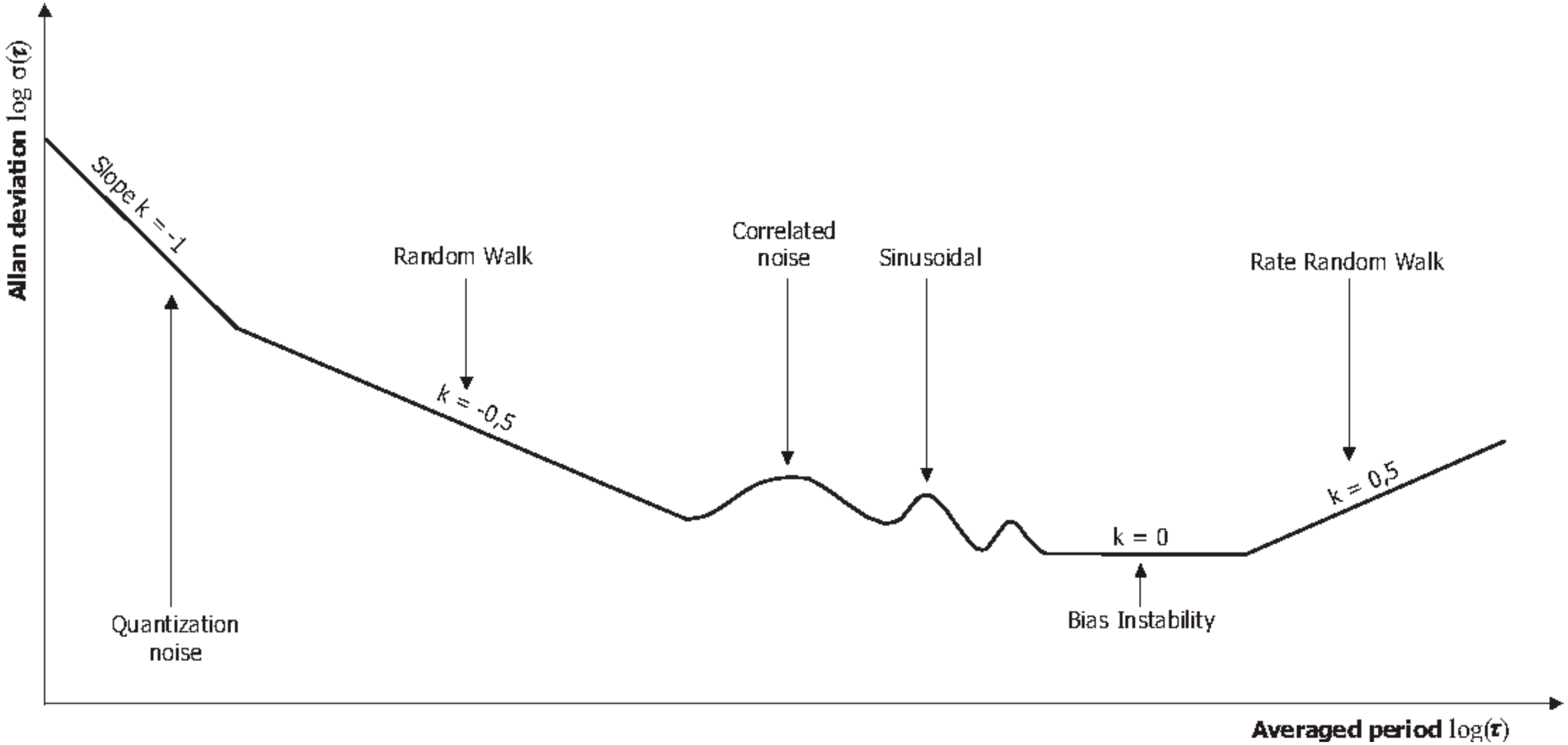
Methodology (Heuristics)

1. Identify a trigger value of tilt to alert e.g. schedule maintenance visit. Base this off the HPBW of the antenna. E.g. 25% of HPBW
2. Identify a reasonable measurement resolution. Say 10% accuracy around the trigger value is sufficient. Be wary of rms to peak power conversion: Noise angle < (0.25*HPBW)/(10*5.5) ~ 3mdeg
3. Convert to acceleration noise: Acceleration noise < 1g*sin(noise angle) ~ 50µg
4. Find out **how long must you average** the sensor output to achieve this resolution (How do you decide how long to average?)
5. You can duty cycle the power to the sensors, as they need not be ON the whole time. Calculate energy per measurement = power*averaging time.

Allan variance

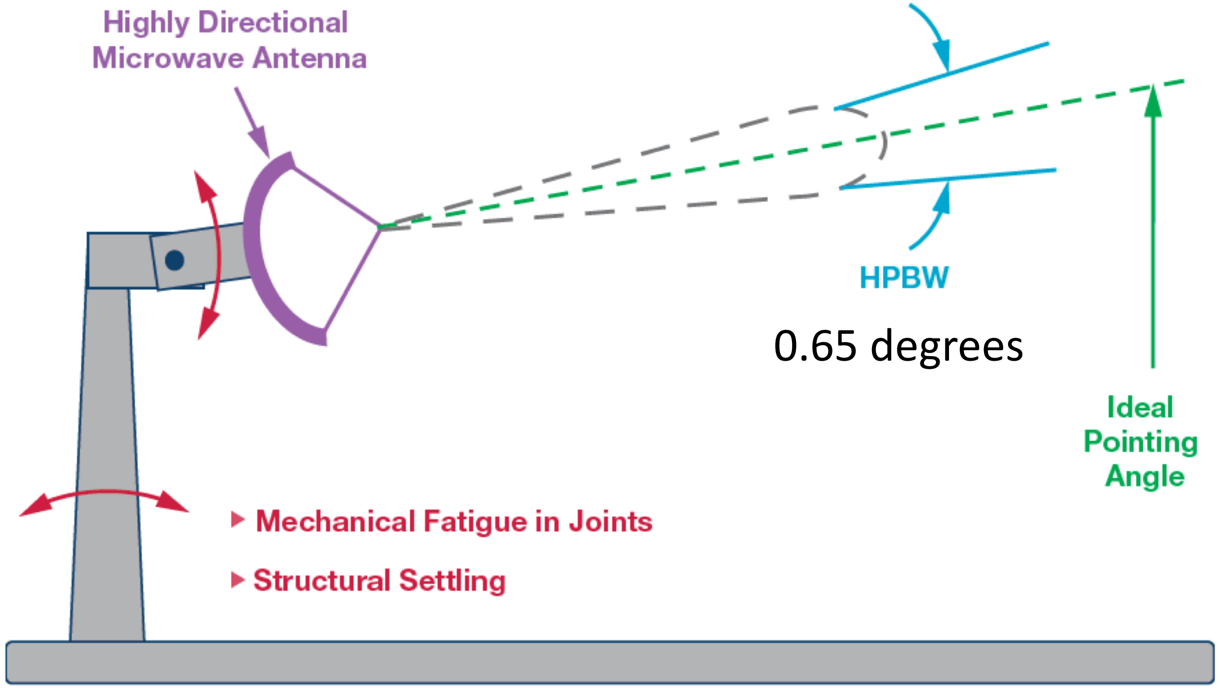
$$\sigma_y^2(\tau) = \frac{1}{2}\langle(\overline{y_{n+1}} - \overline{y_n})^2\rangle$$

Allan deviation: $\sqrt{\sigma_y^2(\tau)}$



Additional reading: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.210.2092&rep=rep1&type=pdf

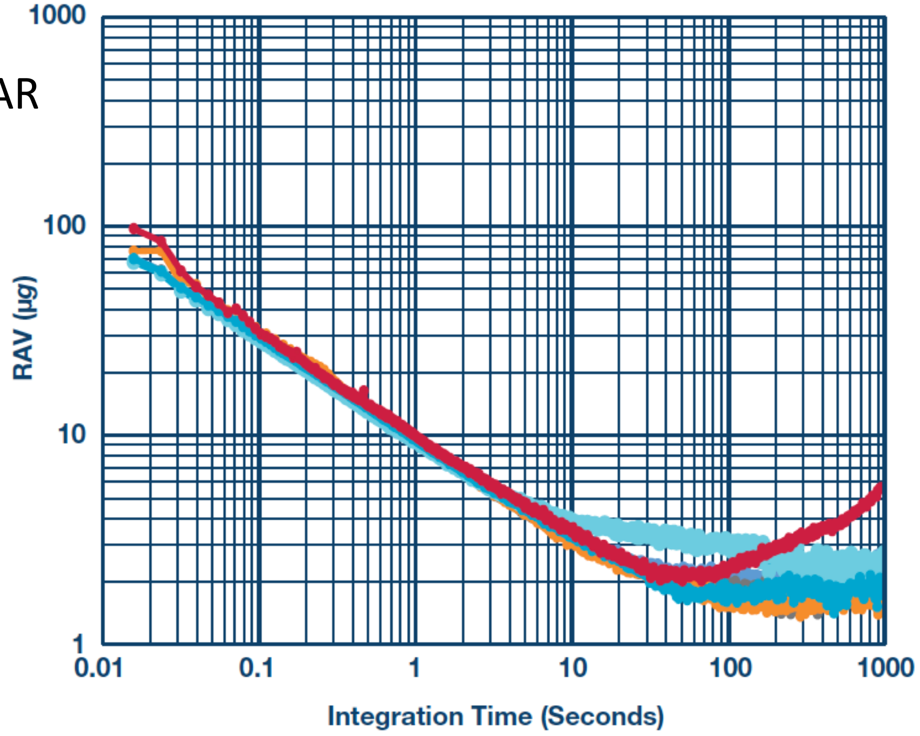
Case study: Low noise vs low power tradeoff



0.65 degrees

Device	Noise (µg/√Hz)	Power (µW)
XL_1	20	340
XL_2	178	26

XL_1 AVAR



Methodology (Heuristics)

1. Identify a trigger value of tilt to schedule maintenance visit. Base this off the HPBW of the antenna. E.g. 25% of HPBW
2. Identify a reasonable measurement resolution. 10% accuracy around the trigger value is sufficient. Be wary of rms to peak power conversion: Noise angle < (0.25*HPBW)/(10*5.5) ~ 3mdeg
3. Convert to acceleration noise: Acceleration noise < 1g*sin(noise angle) ~ 50µg
4. Find out how long must you average the sensor output to achieve this resolution (from Allan variance curve). Typically you will find this information in datasheet of low noise sensors but not low power sensors. For XL_1, this corresponds to roughly 0.05s.
5. XL_2 has 9X worse noise, so assume 81X longer averaging time (slope = -0.5). This corresponds to roughly 4.05s.
6. You can duty cycle the power to the sensors, as they need not be ON the whole time. Calculate energy per measurement = power*averaging time. XL_1: 0.05*340 = 17µJ, XL_2: 4.05*26 = 105.3µJ
7. Thus, there is no tradeoff between low power and low noise in this application. Due to the nature of the application, where we can power cycle the sensor, a low noise sensor needs to be on for shorter time, requiring less power consumption.

Summary

- Careful study of datasheet necessary for calculating application level requirements – eludicated through case study
- Beware of units when comparing quantities!