# Inertial Navigation System

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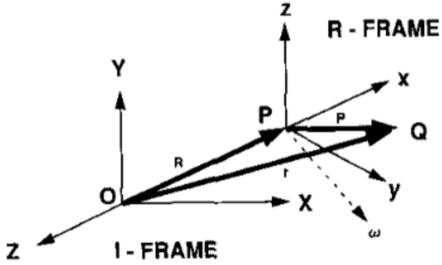
- **Navigation**: Science of directing a vehicle to the destination by determining its position from observation of landmarks, celestial bodies, or radio beams.
- **Inertial Navigation Systems**:
  - Self-contained determination of the instantaneous position and other parameters of motion of a vehicle
  - Using measuring specific force, angular velocity, and time in a selected coordinate system.
  - Velocity and position are determined through real-time integration of the governing differential equations, with measured specific force as an input.
- Errors in INS
  - Initial condition errors
  - Gravitational mass attraction compensation errors
  - Coordinate frame transformation errors
  - Sensor errors such as accelerometers, gyroscopes, and external navigation aids
- Complicated error equations due to the different coordinate frames involved and the many error sources inherent in the instruments.




- Consider a rigid body fixed at point $O$ of fixed Cartesian coordinate system $XYZ$, called as $I$-frame.
- Assume a point in body with position $\boldsymbol{r}$ and velocity $\dot{\boldsymbol{r}}$ relative to origin $O$.

$$\boldsymbol{r} = X\boldsymbol{I} + Y\boldsymbol{J} + Z\boldsymbol{K}$$
$$\dot{\boldsymbol{r}} = \frac{d\boldsymbol{r}}{dt} = \dot{X}\boldsymbol{I} + \dot{Y}\boldsymbol{J} + \dot{Z}\boldsymbol{K}$$

- For a rigid body with fixed point,

$$\dot{\boldsymbol{r}} = \boldsymbol{\omega} \times \boldsymbol{r}$$

where, angular velocity $\boldsymbol{\omega}$ is given by

$$\boldsymbol{\omega} = \Omega_X\boldsymbol{I} + \Omega_Y\boldsymbol{J} + \Omega_Z\boldsymbol{K}$$

- Components of velocity vector in fixed frame

$$\boldsymbol{\omega} \times \boldsymbol{r} = \begin{vmatrix} \boldsymbol{I} & \boldsymbol{J} & \boldsymbol{K} \\ \Omega_X & \Omega_Y & \Omega_Z \\ X & Y & Z \end{vmatrix}$$
$$= \underbrace{(\Omega_Y Z - \Omega_Z Y)}_{v_X} \boldsymbol{I} + \underbrace{(\Omega_Z X - \Omega_X Z)}_{v_Y} \boldsymbol{J} + \underbrace{(\Omega_X Y - \Omega_Y X)}_{v_Z} \boldsymbol{K}$$

- Assume a second coordinate system $R$-frame.
- Consider $P$ at any point in the body and $O$ the origin of space axes.
- Velocity of any point $Q$ in the body w.r.t. space axes at $O$ is given by

$$\boldsymbol{v}_Q = \boldsymbol{v}_P + \boldsymbol{\omega} \times \boldsymbol{r}_{Q/P}$$

where, $\boldsymbol{r}_{Q/P}$ is the relative distance of $Q$ w.r.t. $P$.
- Relative velocity of $Q$ w.r.t. $P$ is defined as

$$\boldsymbol{v}_{Q/P} = \boldsymbol{v}_Q - \boldsymbol{v}_P = \boldsymbol{\omega} \times \boldsymbol{r}_{Q/P}$$

- Vector function $\boldsymbol{A}(t)$ can be expressed in two coordinate frames as

$$\boldsymbol{A}(t) = A_X \boldsymbol{I} + A_Y \boldsymbol{J} + A_Z \boldsymbol{K} = A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}$$

- Its derivatives are given by

$$\begin{aligned}\dot{\boldsymbol{A}}(t) = \frac{d\boldsymbol{A}(t)}{dt} &= \dot{A}_X \boldsymbol{I} + \dot{A}_Y \boldsymbol{J} + \dot{A}_Z \boldsymbol{K} \\ &= \underbrace{\dot{A}_x \boldsymbol{i} + \dot{A}_y \boldsymbol{j} + \dot{A}_z \boldsymbol{k}} + A_x \dot{\boldsymbol{i}} + A_y \dot{\boldsymbol{j}} + A_z \dot{\boldsymbol{k}} \\ &= \frac{\delta \boldsymbol{A}}{\delta t} + A_x \dot{\boldsymbol{i}} + A_y \dot{\boldsymbol{j}} + A_z \dot{\boldsymbol{k}}\end{aligned}$$

- Derivative of unit vector in fixed frame

$$\boxed{\frac{d\boldsymbol{i}}{dt} = \boldsymbol{\omega} \times \boldsymbol{i}, \quad \frac{d\boldsymbol{j}}{dt} = \boldsymbol{\omega} \times \boldsymbol{j}, \quad \frac{d\boldsymbol{k}}{dt} = \boldsymbol{\omega} \times \boldsymbol{k}}$$

where, $\boldsymbol{\omega}$ is the angular velocity of coordinate axes.

- Derivative of a vector in two frames $XYZ$ and $xyz$ are related as

$$\left[\frac{d\boldsymbol{A}(t)}{dt}\right]_{XYZ} = \frac{d\boldsymbol{A}(t)}{dt} = \frac{\delta \boldsymbol{A}}{\delta t} + \boldsymbol{\omega} \times \boldsymbol{A}$$

$$\boxed{\left[\frac{d\boldsymbol{A}(t)}{dt}\right]_{XYZ} = \left[\frac{d\boldsymbol{A}(t)}{dt}\right]_{xyz} + \boldsymbol{\omega} \times \boldsymbol{A}}$$

where, $\boldsymbol{\omega}$ is the angular velocity of $xyz$ w.r.t. $XYZ$.

- Alternatively,

$$\frac{\delta \boldsymbol{A}}{\delta t} = \frac{d\boldsymbol{A}(t)}{dt} - \boldsymbol{\omega} \times \boldsymbol{A} = \frac{d\boldsymbol{A}(t)}{dt} + (-\boldsymbol{\omega}) \times \boldsymbol{A}$$

where, $(-\boldsymbol{\omega})$ is the angular velocity of $XYZ$ w.r.t. $xyz$.

- From a kinematic point of view, it makes no difference which system is considered as fixed and which one as rotating.

- Consider two points $P$ and $Q$ with position vectors denoted by $\boldsymbol{R}$ and $\boldsymbol{r}$, respectively, w.r.t. the point $O$.
- Relative position of $Q$ w.r.t. $P$ is denoted by $\boldsymbol{p}$.
- Relative equation of motion

$$\boxed{\boldsymbol{r} = \boldsymbol{R} + \boldsymbol{p}, \quad \dot{\boldsymbol{r}} = \dot{\boldsymbol{R}} + \dot{\boldsymbol{p}}, \quad \ddot{\boldsymbol{r}} = \ddot{\boldsymbol{R}} + \ddot{\boldsymbol{p}}}$$

- Let $XYZ$ with origin at $O$ be fixed and $xyz$ with origin at $P$ is moving with the angular velocity $\boldsymbol{\omega}$.
- Derivative of relative position vector $\boldsymbol{p}$

$$\dot{\boldsymbol{p}} = \frac{d\boldsymbol{p}}{dt} = \frac{\delta \boldsymbol{p}}{\delta t} + \boldsymbol{\omega} \times \boldsymbol{p}$$

$$\begin{aligned}\ddot{\boldsymbol{p}} =& \frac{d\dot{\boldsymbol{p}}}{dt} = \frac{\delta \dot{\boldsymbol{p}}}{\delta t} + \boldsymbol{\omega} \times \dot{\boldsymbol{p}} = \frac{\delta^2 \boldsymbol{p}}{\delta t^2} + \frac{\delta(\boldsymbol{\omega} \times \boldsymbol{p})}{\delta t} + \boldsymbol{\omega} \times \left(\frac{\delta \boldsymbol{p}}{\delta t} + \boldsymbol{\omega} \times \boldsymbol{p}\right) \\ =& \frac{\delta^2 \boldsymbol{p}}{\delta t^2} + \frac{\delta \boldsymbol{\omega}}{\delta t} \times \boldsymbol{p} + 2\boldsymbol{\omega} \times \frac{\delta \boldsymbol{p}}{\delta t} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{p})\end{aligned}$$

- As $\dfrac{d\boldsymbol{\omega}}{dt} = \dfrac{\delta\boldsymbol{\omega}}{\delta t} = \dot{\boldsymbol{\omega}}$, we have

$$\ddot{\boldsymbol{p}} = \frac{\delta^2 \boldsymbol{p}}{\delta t^2} + \dot{\boldsymbol{\omega}} \times \boldsymbol{p} + 2\boldsymbol{\omega} \times \frac{\delta \boldsymbol{p}}{\delta t} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{p})$$

- Complete relative equation of motion

$$\begin{aligned}
\boldsymbol{r} &= \boldsymbol{R} + \boldsymbol{p} \\
\frac{d\boldsymbol{r}}{dt} &= \frac{d\boldsymbol{R}}{dt} + \frac{\delta \boldsymbol{p}}{\delta t} + \boldsymbol{\omega} \times \boldsymbol{p} \\
\frac{d^2\boldsymbol{r}}{dt^2} &= \underbrace{\frac{d^2\boldsymbol{R}}{dt^2} + \frac{\delta^2 \boldsymbol{p}}{\delta t^2}}_{\text{Linear acceleration terms}} + \underbrace{\dot{\boldsymbol{\omega}} \times \boldsymbol{p}}_{\text{Tangential component due to } \dot{\boldsymbol{\omega}}} + \underbrace{2\boldsymbol{\omega} \times \frac{\delta \boldsymbol{p}}{\delta t}}_{\text{Coriolis acceleration}} \\
&\quad + \underbrace{\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{p})}_{\text{centripetal acceleration}}
\end{aligned}$$

- Differential equation of motion of inertial navigation of vehicle relative to inertial frame

$$\dot{\boldsymbol{R}} = \boldsymbol{\mathcal{V}}$$
$$\left.\frac{d\boldsymbol{\mathcal{V}}}{dt}\right|_I = \boldsymbol{A} + \boldsymbol{g}_m(\boldsymbol{R})$$

where,

$\boldsymbol{R}$ = Geocentric position vector

$\boldsymbol{\mathcal{V}}$ = Velocity of the vehicle relative to the inertial frame

$\boldsymbol{A}$ = Non-gravitational specific force

$\boldsymbol{g}_m(\boldsymbol{R})$ = Gravitational acceleration due to mass attraction, considered positive toward the center of the Earth

- Gravity effect of Moon, Sun, and other stars are neglected.

- We can rewrite previous equation as

$$\boxed{\boldsymbol{A} = \left.\frac{d^2\boldsymbol{R}}{dt^2}\right|_I - \boldsymbol{g}_m(\boldsymbol{R})}$$

- Specific force (accelerometer's output) $\boldsymbol{A}$ is proportional to the inertial acceleration of the system due to all forces, except gravity.
- Since the Earth is rotating and moving w.r.t. inertial space, a transformation is necessary to relate measurements taken in inertial space to observations of position, velocity, and acceleration in a moving vehicle.
- An ideal accelerometer measures the specific force, that is, the difference between the inertial acceleration and gravitational acceleration.
- For Earth-centered inertial (ECI) system,

$$\boldsymbol{A}^P = \boldsymbol{C}_I^P \left[\ddot{\boldsymbol{R}}^I - \boldsymbol{g}_m^I(\boldsymbol{R})\right]$$

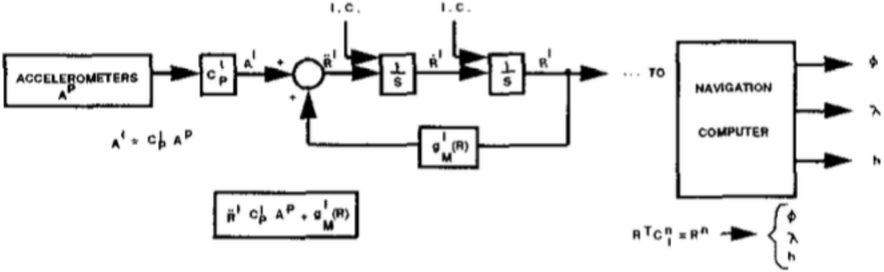
where $\boldsymbol{C}_I^P$ is the transformation matrix from inertial to platform coordinates.

- Earth-centered inertial (ECI) acceleration in terms of specific force and gravity can be written as

$$\boxed{\ddot{\boldsymbol{R}}^I = \boldsymbol{C}_P^I \boldsymbol{A}^P + \boldsymbol{g}_m^I(\boldsymbol{R})}$$

where $\boldsymbol{C}_P^I$ is the transformation matrix from platform to inertial coordinates.

- Block diagram representation



- For navigation at or near the surface of the earth, the position and velocity of vehicle should be referred in an ECEF coordinate system.
- From the Law of Coriolis, the expression relating ECI and ECEF velocities,

$$\left[\frac{d\boldsymbol{R}}{dt}\right]_I = \left[\frac{d\boldsymbol{R}}{dt}\right]_E + \boldsymbol{\Omega} \times \boldsymbol{R} = \boldsymbol{V} + \boldsymbol{\Omega} \times \boldsymbol{R}$$

where, $\boldsymbol{\Omega}$ is the angular rate of Earth relative to the inertial frame, and $\boldsymbol{V}$ is true velocity of vehicle w.r.t. the Earth.

- As angular rate of earth is constant, we have $d\boldsymbol{\Omega}/dt = 0$.
- Differentiating w.r.t. inertial coordinates,

$$\left[\frac{d^2\boldsymbol{R}}{dt^2}\right]_I = \left[\frac{d\boldsymbol{V}}{dt}\right]_I + \boldsymbol{\Omega} \times \left[\frac{d\boldsymbol{R}}{dt}\right]_I$$

- On substituting for $[d\boldsymbol{R}/dt]_I$,

$$\left[\frac{d^2\boldsymbol{R}}{dt^2}\right]_I = \left[\frac{d\boldsymbol{V}}{dt}\right]_I + \boldsymbol{\Omega} \times \boldsymbol{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{R})$$

- As output of accelerometer gives measurements in platform frame, differentiation and integration need to be carried out in same frame.
- Relation between derivatives of $\boldsymbol{V}$ w.r.t. platform and inertial space is

$$\left[\frac{d\boldsymbol{V}}{dt}\right]_I = \left[\frac{d\boldsymbol{V}}{dt}\right]_P + \boldsymbol{\omega} \times \boldsymbol{V}$$

where $\boldsymbol{\omega}$ is the angular rate of platform w.r.t. inertial space (*spatial rate*).

$$\left[\frac{d^2\boldsymbol{R}}{dt^2}\right]_I = \left[\frac{d\boldsymbol{V}}{dt}\right]_P + (\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{R})$$

- Finally, we have

$$\boldsymbol{A} = \left[\frac{d\boldsymbol{V}}{dt}\right]_P + (\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{R}) - \boldsymbol{g}_m(\boldsymbol{R})$$

- As the centripetal acceleration of Earth is a function of position of Earth only, it can be combined with gravity term.

$$\boldsymbol{g}(\boldsymbol{R}) = \boldsymbol{g}_m(\boldsymbol{R}) - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{R}) = \omega_s^2 \boldsymbol{R}$$

where $\omega_s = \sqrt{g(R)/R}$ is the Schuler angular frequency.
- $\boldsymbol{g}(\boldsymbol{R})$ is dominant feedback term for principal mode of behavior of INS.
- Generalized mechanization equation

$$\boldsymbol{A} = \left[\frac{d\boldsymbol{V}}{dt}\right]_P + (\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{V} - \boldsymbol{g}(\boldsymbol{R})$$

- It does not refer to any particular type of system coordinate frame.

- **Locally level platform coordinate frame**: spatial rate being equal to sum of Earth rate and vehicle (or platform) angular rate $\boldsymbol{\rho}$ w.r.t. Earth-fixed frame.
- Term $\boldsymbol{\rho}$ is called as transport rate and mathematically, $\boldsymbol{\omega} = \boldsymbol{\rho} + \boldsymbol{\Omega}$.
- On rearranging, we get

$$\left[\frac{d\boldsymbol{V}}{dt}\right]_P = \boldsymbol{A} - (\boldsymbol{\rho} + 2\boldsymbol{\Omega}) \times \boldsymbol{V} + \boldsymbol{g}(\boldsymbol{R})$$

- Generalized navigation equation of a vehicle, expressed in the platform or computational frame, which is referenced to the Earth.
- On expanding this equation,

$$\begin{aligned}
\dot{V}_x &= A_x - (\rho_y + 2\Omega_y)V_z + (\rho_z + 2\Omega_z)V_y + g_x \\
\dot{V}_y &= A_y - (\rho_z + 2\Omega_z)V_x + (\rho_x + 2\Omega_x)V_z + g_y \\
\dot{V}_z &= A_z - (\rho_x + 2\Omega_x)V_y + (\rho_y + 2\Omega_y)V_x + g_z
\end{aligned}$$

- Gravitational model is based on a spherical harmonic expansion of the gravitational potential.
- Two commonly used expansions of the gravitational potential
  - Spherical or zonal harmonics: depend on the geocentric latitude only.
  - Tesseral and sectoral harmonics: depend on both latitude and longitude.
- Tesseral and sectoral harmonics
  - Indicate deviations from rotational symmetry
  - Can be neglected without compromising system performance or accuracy
- Derivation of the gravitational potential is based on the reference ellipsoid.
- Assumptions
  - Earth's mass distribution is symmetric about the polar axis.
  - Gravitational potential $U(R, \phi)$ in ECEF is at distance $R$ from Earth's center, independent of longitude.

- Gravitational potential in ECEF frame in terms of spherical harmonics

$$U(R,\phi) = -\frac{\mu}{R}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{a}{R}\right)^n P_n(\sin\phi)\right]$$

$\mu$ = Earth's gravitational constant

$a$ = Mean equatorial radius of the Earth (or semimajor axis)

$R$ = Magnitude of the geocentric position vector

$\phi$ = Geocentric latitude

$J_n$ = Coefficients of zonal harmonics of the Earth potential function

$P_n(\sin\phi)$ = Associated Legendre polynomials of the first kind as functions of $\phi$ and degree $n$

- $\dfrac{\mu}{R}$ denotes mean value and is simplified gravitational potential of the Earth.
- It is due to spherically mass symmetric body.
- Remaining terms account for asymmetricity of the Earth.
- **Second harmonics** $J_2$: Earth flattening, the meridional cross-section being an ellipse rather than a circle
- **Third harmonics** $J_3$: tendency toward a triangular shape
- **Fourth harmonics** $J_4$: tendency toward a square shape
- If the symmetry w.r.t. equator is assumed then

$$J_1 = J_3 = J_5 \cdots = 0$$

- As $R$ is very large, all the terms within the are small as compared with unity.

- Gravitation vector is given as the gradient of gravitational potential as

$$\boldsymbol{g}(\boldsymbol{R}) = [g_x \quad g_y \quad g_z]^T, \quad g_x = \frac{\partial U}{\partial x} \quad g_y = \frac{\partial U}{\partial y} \quad g_z = \frac{\partial U}{\partial z}$$

- Assumptions: Direction of the gravity vector $\boldsymbol{g} \perp$ the reference ellipsoid, positive in the downward direction.

$$\boldsymbol{g}(\boldsymbol{R}) = -\, g_z \mathbf{1}_z$$

- For a spherical Earth model, $\boldsymbol{g}(\boldsymbol{R}) = -\mu \boldsymbol{R}/R^3$ and $\boldsymbol{R} = [x \ y \ z]^T$.
- Components of the apparent gravity vector along the platform $x, \ y$ axes can be neglected because their magnitude is less than $10^{-5} \ g$.

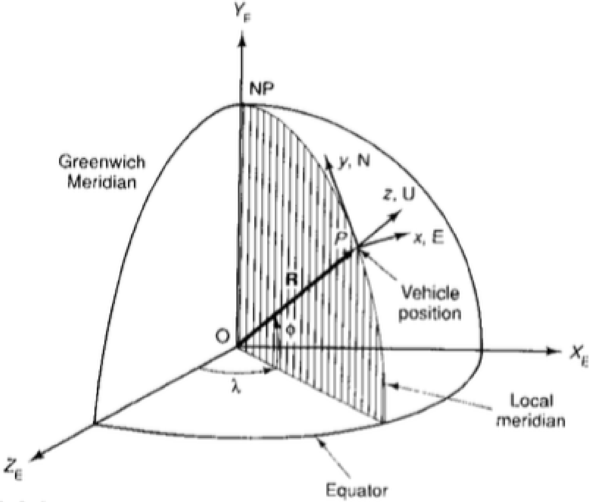
$$\boldsymbol{g}(\boldsymbol{R}) = \begin{bmatrix} 0 \\ 0 \\ g_z \end{bmatrix}, \ \boldsymbol{R} = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}$$

- Navigation in ECI coordinates involves integration of a simple set of differential equations driven by the measured specific force $\boldsymbol{A}$.
- For most terrestrial applications, it is more convenient to refer the position and velocity of the vehicle to ECEF, which rotates with the earth.
- Equations of motion must account for the rotation of the coordinate frame.
- In north-east-up (NEU) coordinate system,

$$\boldsymbol{\Omega} = \underbrace{0}_{\Omega_x, \Omega_E} \boldsymbol{i} + \underbrace{\Omega \cos\phi}_{\Omega_y, \Omega_N} \boldsymbol{j} + \underbrace{\Omega \sin\phi}_{\Omega_z, \Omega_U} \boldsymbol{k}$$

- By using gravity and angular rate components,

$$\begin{aligned}
\dot{V}_x &= A_x - (\rho_y + 2\Omega_y)V_z + (\rho_z + 2\Omega_z)V_y \\
\dot{V}_y &= A_y - (\rho_z + 2\Omega_z)V_x + \rho_x V_z \\
\dot{V}_z &= A_z - \rho_x V_y + (\rho_y + 2\Omega_y)V_x - g_z
\end{aligned}$$



$Y_E$
NP
Greenwich Meridian
y, N
z, U
P
x, E
R
Vehicle position
O
φ
$X_E$
λ
Local meridian
$Z_E$
Equator

- Gyroscope torquing rate w.r.t inertial space

$$\begin{aligned}\omega_x &= \omega_E = \ \rho_E \\ \omega_y &= \omega_N = \ \rho_N + \Omega \cos\phi \\ \omega_z &= \omega_z = \ \rho_z + \Omega \sin\phi\end{aligned}$$

- $\omega_x, \omega_y$: level angular rates of platform required to maintain platform level
- $\omega_z$: platform azimuth rate to maintain platform orientation to north
- To maintain platform level, gimbal axes must have

$$\dot{\phi} = -\rho_E, \quad \dot{\lambda}\cos\phi = \rho_N$$

- Generalized mechanization equation

$$\begin{aligned}\boldsymbol{A} &= \left[\frac{d\boldsymbol{V}}{dt}\right]_P + (\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{V} - \boldsymbol{g}(\boldsymbol{R}) \\ \Rightarrow \left[\frac{d\boldsymbol{V}}{dt}\right]_P &= \boldsymbol{A} - (\boldsymbol{\Omega} + \boldsymbol{\omega}) \times \boldsymbol{V} + \boldsymbol{g}(\boldsymbol{R})\end{aligned}$$

- Level and vertical velocity equations

$$
\begin{aligned}
\dot{V}_E &= A_E - (\omega_N + \Omega\cos\phi)V_z + (\omega_z + \Omega\sin\phi)V_N \\
\dot{V}_N &= A_N - (\omega_z + \Omega\sin\phi)V_E + \omega_E V_z \\
\dot{V}_z &= A_z - \omega_E V_N + (\omega_N + \Omega\sin\phi)V_E - g_z + K_2(h_B - h)
\end{aligned}
$$

where, $\dot{h} = V_z + K_1(h_B - h)$ and $h_B$ is barometric altitude.

- For a spherical Earth model,

$$
\begin{aligned}
\omega_x &= \omega_E = -\dot{\phi} \\
\omega_y &= \omega_N = \dot{\lambda}\cos\phi + \Omega\cos\phi = \frac{V_x}{R} + \Omega\cos\phi \\
\omega_z &= \omega_z = \dot{\lambda}\sin\phi + \Omega\sin\phi = \frac{V_x}{R}\tan\phi + \Omega\sin\phi
\end{aligned}
$$

- To maintain platform level, longitude and latitude gimbal axes rates

$$
\dot{\phi} = \frac{V_y}{R} = \frac{V_N}{R}, \quad \dot{\lambda} = \frac{V_x}{R\cos\phi} = \frac{V_E}{R}\sec\phi
$$

- Longitude and latitude computations

$$\phi = \phi(0) + \frac{V_y}{R}t, \quad \lambda = \lambda(0) + \int_0^t \dot{\lambda} dt$$

- Now torquing rate becomes

$$\begin{aligned}
\omega_x &= -\frac{V_y}{R} \\
\omega_y &= \frac{V_x}{R} + \Omega \cos\left(\phi(0) + \frac{V_y}{R}t\right) \\
\omega_z &= \frac{V_x}{R} \tan\left(\phi(0) + \frac{V_y}{R}t\right) + \Omega \sin\left(\phi(0) + \frac{V_y}{R}t\right)
\end{aligned}$$

- Platform rotation rate relative to Earth, $\boldsymbol{\rho} = \boldsymbol{\omega} - \boldsymbol{\Omega}$

$$\rho_x = -\frac{V_y}{R} \quad \rho_y = \frac{V_x}{R}, \quad \rho_z = \frac{V_x}{R} \tan\left(\phi(0) + \frac{V_y}{R}t\right)$$

## Reference

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2. D. H. Titterton and J. L. Weston, *Strapdown Inertial Navigation Technology*, Progress in Astronautics and Aeronautics, Vol. 207, ed. 2, ch. 4.

Thank you for your attention !!!