Introduction to Navigation & Guidance

(Course Code: AE 410/641)

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Assignment - 2

General Instructions

- It is highly recommended that students submit neatly typeset document containing answers to questions in Assignment-2.
- Marks for each portion of question are given separately.
- This assignment includes computer simulation questions. You may choose any programming language to simulate the results.
- Your assignment needs to be submitted online in .pdf format along with a .zip file containing the codes for the submitted results on Moodle.
- Please mention your assumptions (if any) and show your steps clearly while solving problems.
- Do not indulge in academic dishonesty. In cases where the answers of two students are found to be copied, both of them will be awarded zero marks for that particular question. Furthermore, those students may face disciplinary actions.
- Assignment is due on **16^{th} November 2020**.
- Late assignment submission: **20% reduction** in total weightage per late day.

1. Consider an engagement where a missile aims to intercept a constant velocity target using deviated pursuit guidance law. Assume that the speeds of missile and target are constant, and are denoted by V_M and V_T, respectively, while their flight path angles are denoted by γ_M and γ_T, respectively. The speed ratio of the participants is denoted by $K = V_M/V_T$. The lead angle of the target is represented as θ_T, whereas the constant deviation angle of deviated pursuit is denoted by δ. The engagement geometry for deviated pursuit guidance is shown in Figure 1.

   (a) Derive the expression for relative distance between missile and target as a function of target's lead angle, and show that it can be written as

$$r = C\frac{\sin^{\mu-1}\left(\frac{\theta_T-\beta}{2}\right)}{\cos^{\mu+1}\left(\frac{\theta_T+\beta}{2}\right)},\ \beta = \sin^{-1}(K\sin\delta),\ \mu = K\frac{\cos\delta}{\cos\beta},$$

   where C depends on the initial conditions. Provide all the major steps.

(b) Show that the lateral acceleration requirement on missile, under deviated pursuit strategy, remains bounded if the speed ratio K satisfies

$$1 < K \leq \frac{2}{\sqrt{1 + 3\sin^2\delta}}.$$

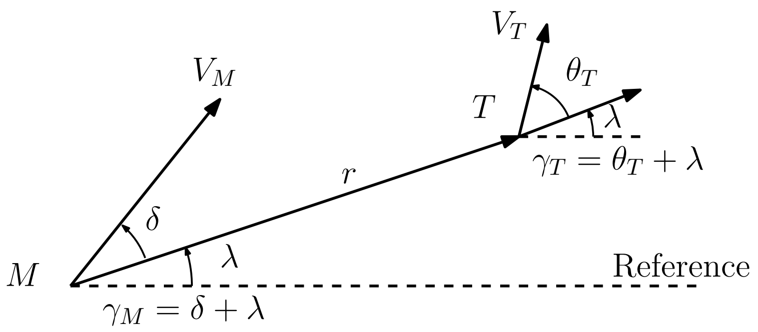

Figure 1: Engagement geometry for deviated pursuit guidance.

[10+5]

2. Consider an engagement scenario where missile and target are launched from ground with a separation of 5 km and an initial LOS of 0°. Assume that the speed of missile and target are 500 m/s and 300 m/s, respectively. The target is moving at an angle of 90° from the ground. If the missile misses the interception by 5 m, what would have been the heading angle of missile? Find out the desired missile heading angle to achieve a successful interception. [5]

3. Consider an engagement scenario, shown in Figure 2, where missile is 10 km away in radial direction from the target and their line-of-sight (LOS) angle is 30°. Assume that the speed of missile and target are 500 m/s and 300 m/s, respectively, and the target has a flight path angle of 120°. The missile's autopilot is assumed to be perfect.

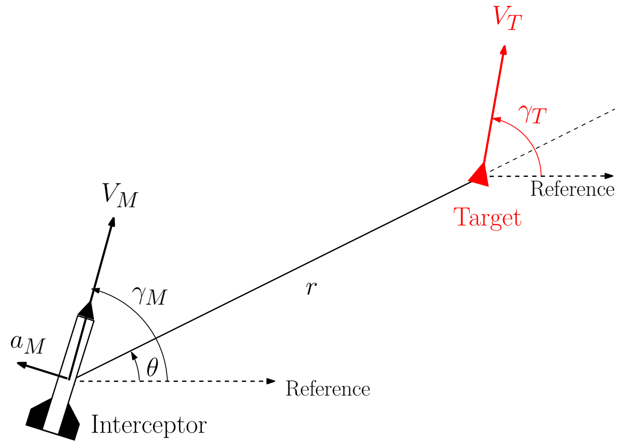

Figure 2: Planar engagement geometry between a missile and a target

(a) Plot the evolution of trajectories of missile and target (Use square and equal axes for better visualization of trajectories) when missile uses pure pursuit guidance from the launch itself. Also, plot the required guidance command with respect to time.

(b) What changes do you expect if the speed of target reduces to 250 m/s? Justify with the plot of missile's lateral acceleration, and trajectories of missile and target.

(c) Compare the plots of trajectories and guidance command for deviated pursuit guidance corresponding to deviations $\delta = 10°$; $20°$; $30°$, and comment on your observations. Note that $\delta = \gamma_M$ - θ.

(d) Suppose the missile is not in pure pursuit course from beginning but has an initial deviation of 30° anticlockwise in its velocity direction. How will you implement pursuit guidance law? Generate plots of trajectories and guidance commands accordingly.

Hint: For simulation purpose, assume the missile and the target to be the point mass vehicles. Also, use the Cartesian coordinate system for propagation of missile and target states. Assume that the position of missile and target are denoted by pairs $(X_M\ ; Y_M)$ and $(X_T\ ; Y_T\ )$, and their flight path angles are denoted by γ_M and γ_T respectively. The lateral acceleration of missile and target are denoted as a_M and a_T , respectively. The missile and target kinematics in Cartesian coordinate system is given by

$$\begin{aligned}
\dot{X}_M &= V_M \cos\gamma_M \\
\dot{Y}_M &= V_M \sin\gamma_M \\
\dot{X}_T &= V_T \cos\gamma_T \\
\dot{Y}_T &= V_T \sin\gamma_T \\
\dot{\gamma}_M &= \frac{a_M}{V_M} \\
\dot{\gamma}_T &= \frac{a_T}{V_T}
\end{aligned}$$

where V_M and V_T denote the speed of missile and target, respectively. The missile-target distance, LOS angle, and other relevant variables can be computed using the positions of missile and target. [5+5+5+5]

4. (a) Assuming that the target is stationary, for an interceptor guided by *pure proportional navigation*, show that the expression of relative range, r, as a function of lead angle, $\sigma = \gamma_M - \theta$, is given by

$$r = \frac{r_0}{(\sin\sigma_0)^{1/(N-1)}}(\sin\sigma)^{1/(N-1)} \ ,$$

where r_0, σ_0 are the initial values of relative range and the lead angle, respectively. Note that for a pure proportional guidance, the rate of change of missile's heading with respect to time is proportional to the LOS rate between the missile and the target, with some proportionality constant N.

(b) Consider an engagement scenario where missile and target are launched from ground and have a separation of 15 km and an initial LOS as 0°. Assume that the speed of missile and target are 500 m/s and 300 m/s, respectively. The target is moving at an angle of 90° from the ground. The missile's autopilot is assumed to be perfect and target is non-maneuvering. If the missile is launched with heading angle error of 20°, then plot the trajectories of missile and target using PPN guidance law. Note that the lateral acceleration for PPN guidance is applied perpendicular to the missile velocity vector.

[10+5]

5. Consider an engagement between an unguided missile and a constant velocity target. Assume that V_r and V_θ denote the relative velocity components of missile-target engagement and the initial value of relative distance between missile and target is denoted by r_0.

(a) What are the capturability (or collision) conditions for interception of target if the interception is defined as a direct hit only, that is, zero miss distance?

(b) Find out the capturability conditions if the miss distance, r_{miss}, is allowed to be within the lethal radius r_{lethal}.

(c) Verify that the capturability conditions in part (a) are same as the conditions obtained in part (b) for direct hit.

[5+5+5]

6. Among all the satellites visible to a receiver, two possible combinations A and B are selected. H_A and H_B represent the visibility matrices corresponding to A and B. If these visibility matrices are defined as

$$H_A = \begin{bmatrix} 0 & 0.996 & 0.087 & 1 \\ 0.863 & -0.498 & 0.087 & 1 \\ -0.8630 & -0.498 & 0.087 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad H_B = \begin{bmatrix} 0 & 0.98 & 0.199 & 1 \\ 0.863 & -0.498 & 0.087 & 1 \\ -0.8630 & -0.498 & 0.087 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

then

(a) Calculate covariance matrices for H_A and H_B.

(b) Calculate the values of GDOP, PDOP, HDOP, VDOP, TDOP and MDOP for both the satellite combinations.

(c) Depending on the values calculated in part (b), select the configuration of satellites that gives the most accurate calculation of 3D position of the receiver?

[5+9+1]

7. Aircraft A is moving towards another aircraft B whose corresponding speeds are 400 m/s and 600 m/s respectively,as shown in figure 3. Aircraft A carries a CW radar transmitting at 300 MHz. Assume the speed of light to be 3×10^8 m/s.

Figure 3: Motion of aircraft A and B.

(a) Calculate the Doppler frequency shift.

(b) Keeping all other parameters same, what should be the flight direction of aircraft B for the Doppler frequency shift to be zero?

(c) Assume that aircraft B has a flight path angle of 60°. What should be the flight path angle of aircraft A such that the Doppler frequency shift is equal to half of that observed in part (a). Take the LOS angle to be 0°.

[5+5+5]

8. Consider two objects M and T, moving in straight lines, as shown in Figure 4.

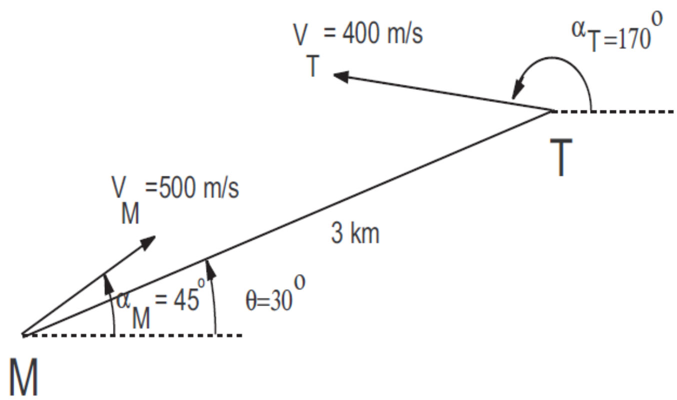

Figure 4: Engagement geometry between M and T.

(a) Analytically obtain the trajectory of the (V_θ, V_r) point in the relative velocity space. Show that the point (V_θ, V_r) always moves upwards on its locus.

(b) Plot the trajectory of $V_r(t)$ and $V_\theta(t)$ with respect to time. Moreover, plot the locus of (V_θ, V_r) point for the above engagement. What values do $V_r(t)$ and $V_\theta(t)$ converge to?

[10+10]