Aided Inertial Navigation System

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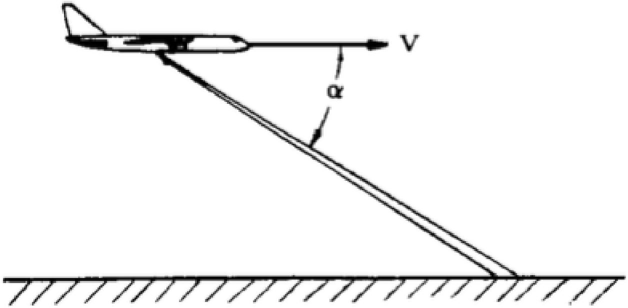
- **Doppler navigation**: Based on Doppler frequency shift phenomenon
- Developed by Austrian physicist Christian Johann Doppler
- Doppler radar transmits radio-frequency (RF) energy to ground and measures the frequency shift in the returned energy to determine ground speed.

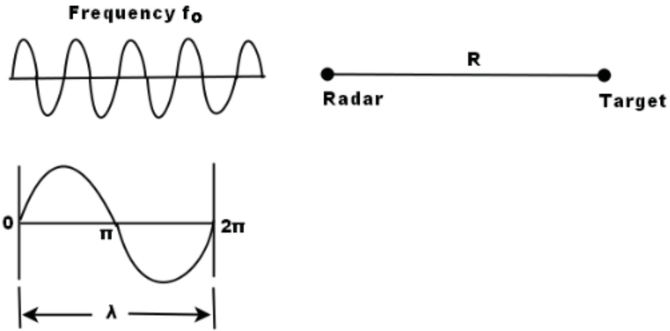


- Doppler radar: No long term degradation of error
- Doppler-INS mode gives bounded velocity error, smaller as compared to INS itself.
- Doppler suffers with high acceleration maneuvers as in fighter aircraft.
- **Doppler equation**

$$\Delta f = \underbrace{\left[\frac{2}{\lambda}\right]}_{\text{Doppler Sensitivity}} V \cos\alpha$$

where λ is wavelength of carrier frequency radiation, V is magnitude of aircraft velocity w.r.t. ground, α is angle between antenna beam and velocity vector, respectively.

Doppler effect: If there is relative motion between the source of a signal and the observer of signal, along the line joining the two, then an apparent shift in frequency will result.



- Assume that target is at distance R from source and has velocity V_r.
- Frequency of signal is f_0 and wavelength $\lambda \quad \Rightarrow \quad c = f_0\lambda$
- Total number n of wavelengths contained in to-and-fro path between radar and target

$$n = \frac{2R}{\lambda}$$

- Total angular excursion ϕ made by the electromagnetic wave during its transit to the target and back to the radar

$$\phi = \frac{2R}{\lambda} 2\pi = \frac{4\pi R}{\lambda}$$

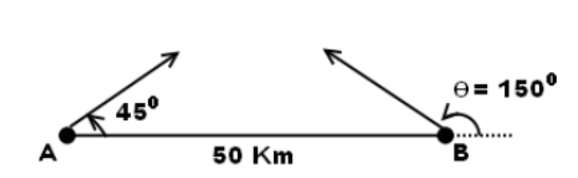
- When the target is in motion, both R and ϕ are changing.
- Doppler angular frequency, change in ϕ w.r.t. time is given by

$$W_d = 2\pi f_d = \frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dR}{dt} = \frac{4\pi V_r}{\lambda} \Rightarrow \boxed{f_d = \frac{2V_r}{\lambda} = \frac{2V_r f_0}{c}}$$

Example

Positions of the two aircraft, A and B, are as shown in the figure below. Aircraft A has a speed of 600 m/s and carries a CW radar transmitting at 300 MHz frequency and tracking aircraft B which has a speed of 800 m/s.

- What is the doppler frequency shift recorded by the radar in aircraft A?
- What should be the flight direction of aircraft B for the doppler frequency shift to be zero?



- Transmission frequency $f_0 = 300$ MHz $=3 \times 10^8$
- Relative velocity of aircraft A with respect to aircraft B along the line-of-sight is given by

$$V_r = 600 \cos 45^\circ + 800 \cos 30^\circ = 1117.08 \text{ m/s}$$

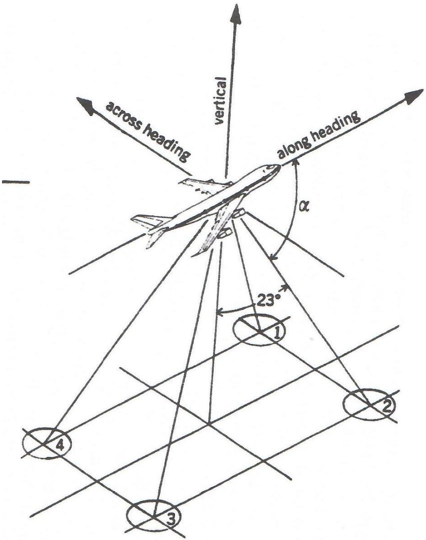
- Doppler frequency shift

$$f_d = \frac{2V_r f_0}{c} = \frac{2 \times 1117.08 \times 3 \times 10^8}{3 \times 10^8} = 2234.16 \text{ Hz}$$

- Doppler frequency shift will be zero when the relative velocity V_r is zero.

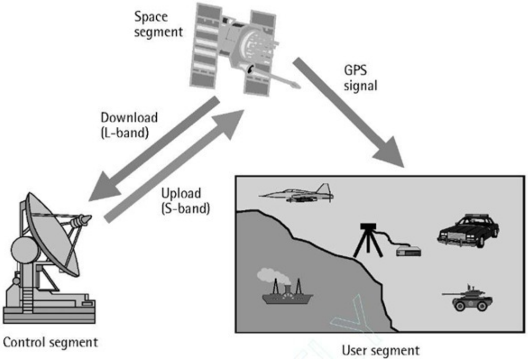
$$V_r = 600 \cos 45^\circ - 800 \cos \theta = 0 \Rightarrow \theta = \pm 57.970^\circ$$

- Change in frequency between transmitted and received signals allows received signal to be separated from transmitted signal.



- How to obtain complete information of velocity?
- Single beam provides only velocity component in the direction of beam.
- To determine complete velocity, we need at least three beams (called as Lambda configuration).
- Three non-coplanar beams provide velocity in 3D space.
- Fourth beam may be used to increase reliability and accuracy.
- What if we have only two beams?
  $\Rightarrow$ Vertical component using barometer.

- **Global positioning system (GPS)**: A space-based, pseudo-ranging navigation satellite system that will provide worldwide, nearly continuous, 3D position, velocity, and coordinate universal time to suitably equipped user.
  - Designed primarily for global navigation of a terrestrial or near-earth user.
  - System broadcasts continuously the information required for a GPS receiver to compute its own position and velocity.
  - Total worldwide coverage, all weather operations
  - Available to unlimited number of passive users at same time
  - Military applications: guidance, rendezvous, targeting operation, weapon delivery, etc.
  - Civil applications: navigation of aircraft, ships, and land vehicles, full worldwide coverage of all airport in common grid, reduce air traffic densities, reduced holding times, reduced airport congestion, saving of fuel.



Segments of GPS

- Space vehicle (SV) segment: Constellation of earth orbiting satellite.
- Ground control segment: monitoring of the orbits of all satellites and providing them with updated information several times each day.
- User segment: all air, sea, and space-based users equipped with GPS receivers.

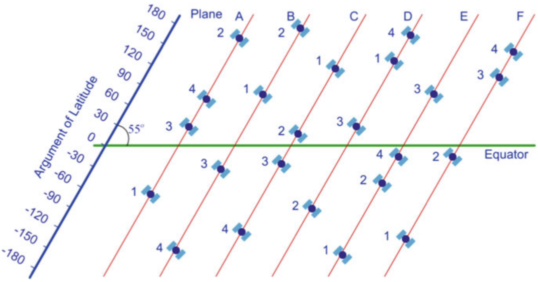


Figure: Optimized satellites constellation phasing

Orbital constellation: 21 active and 3 spare satellites.

- Six orbital planes with each inclined 55° to the equator.
- Each orbital plane has 3 or 4 satellites.
- Orbital planes are separated to each other in the longitude by 60° with a nonuniform phasing.
- Phases are chosen such that any user on earth can acquire at least 4 satellite any time.
- Each satellite is equipped with highly accurate atomic cesium clock with a know offset from GPS time.

- Each satellite transmits its information on two L-band frequencies designated as $L_1,\ L_2$. $L_1 = 1575.42$ MHz, $L_2 = 1227.60$ MHz.
- Two frequencies are required to correct for ionospheric delay uncertainties in transmission.
- L_1 frequency will be modulated by two pseudorandom codes, a coarse/acquisition (C/A) code and a precision (P) code.
- C/A code has a frequency of 1.023 MHz and repeats itself every ms.
- P code is a classified code sequence, which is created from a product of two pseudorandom codes and modulated at a frequency of 10.23 MHz.
- L_2 frequency is modulated by P code, but not by C/A code.
- Specific code sequences broadcast by each satellite are different and are used by GPS receiver to distinguish satellites from each other.
- Each frequency (L_1 and L_2) is further modulated by navigation message.

- Classes of GPS services
  - **Standard Positioning Service (SPS)**: SPS (C/A-code signal) is available to general public.
  - It will provide a horizontal position accuracy of 40 m CEP.
  - **Precise Positioning Service (PPS)**: PPS (P-code signal) is a highly accurate positioning, velocity, and timing service, only be available to authorized users.
  - PPS will provide a 3-D position RMS accuracy of 10-16 m SEP, 0.1 m/s RMS velocity accuracy, and I00 ns accuracy in time.
- Navigation message contains GPS time, satellite ephemeris data, atmospheric propagation correction data, and any other information needed by the GPS receivers.

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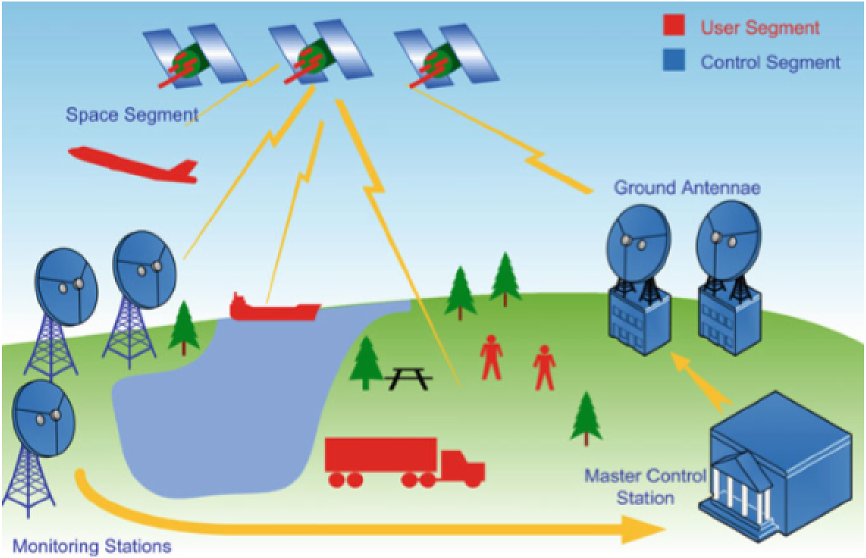
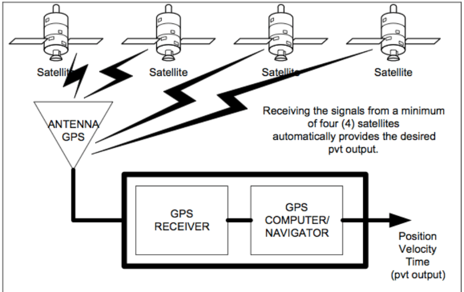
Segments for GPS



- Ground-control segment monitors satellite broadcast signals and uplinks corrections to ensure predefined accuracies.
- Responsible for monitoring and controlling orbits of satellites, for maintaining GPS system time, and for uploading necessary information to the satellites three times a day.
- Operational control segment: five monitor stations, a master control station, and three uplink antennas.
- Widely separated monitor stations: allow simultaneous tracking of full satellite constellation and relay orbital and clock information to master control station.
- Ranging data accumulated by monitor stations are processed by master control station for systematic error elimination.
- Master control station forms corrections, which are uploaded to the satellite by uplink antennas.

GPS Receiver

- ⇒ An antenna to capture GPS signals.
- ⇒ An amplifier to increase the power level of received signal.
- ⇒ A digital computer-to process the information contained in signal.



The GPS User Segment - typical

Input of GPS receivers:

- Receiver aiding signals
- Initialization inputs (e.g., position, time)
- Waypoint navigation data (if the system is used as navigator)

Output of GPS receivers:

- Position, velocity, and time
- 3-D area navigation and steering data
- Receiver status

- Satellite clocks are synchronized to GPS time by master control station.
- Signals transmitted contain information about start time of transmission.
- Due to user clock error, the measure range between user and satellite is named as pseudorange.

$$R_p = R_a + c\Delta T_b$$

where, R_p, R_a represent pseudo and actual ranges, respectively, and ΔT_b is receiver clock bias.

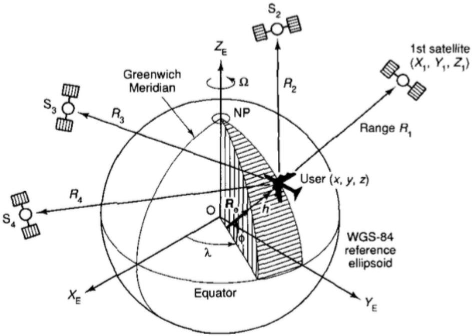
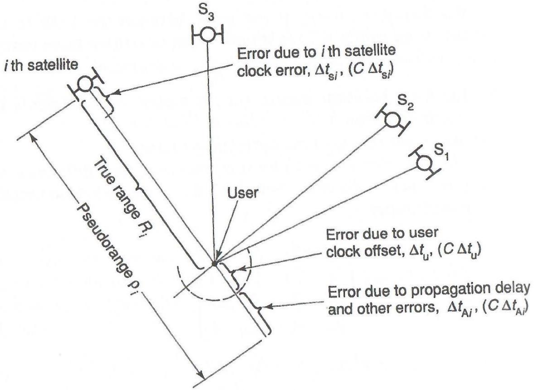


Figure: ECEF Coordinate System



- Measurement

$$\rho_i = C\,(\text{time signal received} - \text{time signal transmitted})$$
$$= R_i + C\Delta t_{Ai} + C(\Delta t_u - \Delta t_{si})$$

C = speed of light

Figure: GPS Pseudorange Concept and Error Component

- Four unknown parameters: GPS receiver location and time
- **Navigation Equations** using measurements of four satellites

$$\begin{aligned} R_i = & \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} + c\Delta T_b \\ & + I_i(f) + c\Delta T_i + c\delta_i + \gamma_i + \epsilon_i, \quad \forall\ i = 1, 2, 3, 4 \end{aligned}$$

where, (x, y, z) and (x_i, y_i, z_i) represent position of user and i^{th} satellite, respectively, ΔT_i is i^{th} satellite clock offset from GPS time, $I_i(f)$ ionospheric delay, γ_i term which accounts for any other biases in the system, and ϵ_i statistical error in the measurement.

- ΔT_i: eliminated by precise measurement by monitor station.
- $I_i(f)$: eliminated by transmission on two frequencies.
- $\delta_i, \gamma_i, \epsilon_i$: using simultaneous multiple satellite observations.

$$R_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} + c\Delta T_b, \quad \forall\ i = 1, 2, 3, 4$$

- Solution of these equations may be computationally intensive.

- How to solve these equations efficiently?
- Linear equations are easy to solve, hence linearizations a good alternative.
- Assume x_n, y_n, z_n, T_n be the nominal (a priori best-estimate) value of x, y, z, T and $\Delta x, \Delta y, \Delta z$ be their corresponding corrections.
- Let R_{ni} and ΔR_i be the nominal pseudorange measurement to i^{th} satellite and their difference between actual and nominal range measurements.

$$
\begin{aligned}
x &= x_n + \Delta x \\
y &= y_n + \Delta y \\
z &= z_n + \Delta z \\
T &= T_n + \Delta T \\
R_i &= R_{ni} + \Delta R_i \\
R_{ni} &= \sqrt{(x_n - x_i)^2 + (y_n - y_i)^2 + (z_n - z_i)^2} + T_n
\end{aligned}
$$

- With units chosen such that $c = 1$, we have

$$\sqrt{(x_n + \Delta x - x_i)^2 + (y_n + \Delta y - y_i)^2 + (z_n + \Delta z - z_i)^2} = R_{ni} + \Delta R_i - T_n - \Delta T, \quad \forall\, i = 1, 2, 3, 4$$

- We have square of LHS as

$$\begin{aligned}
(\text{LHS})^2 =& (x_n - x_i)^2 + {\color{red}(\Delta x)^2} + 2(x_n - x_i)\Delta x + (y_n - y_i)^2 + {\color{red}(\Delta y)^2} \\
&+ 2(y_n - y_i)\Delta y + (z_n - z_i)^2 + {\color{red}(\Delta z)^2} + 2(z_n - z_i)\Delta z \\
=& \underbrace{(x_n - x_i)^2 + (y_n - y_i)^2 + (z_n - z_i)^2}_{a} \\
&+ 2\underbrace{\left[(x_n - x_i)\Delta x + (y_n - y_i)\Delta y + (z_n - z_i)\Delta z\right]}_{b} + {\color{red}HOT}
\end{aligned}$$

- We know that

$$\sqrt{a + 2b} = \sqrt{a}\sqrt{1 + 2b/a} = \sqrt{a}(1 + b/a) + HOT \approx \sqrt{a} + b/\sqrt{a}$$

- Using this approximation, we get

$$
\begin{aligned}
\sqrt{a} + \frac{b}{\sqrt{a}} &= R_{ni} + \Delta R_i - T_n - \Delta T \\
&= \sqrt{(x_n - x_i)^2 + (y_n - y_i)^2 + (z_n - z_i)^2} + T_n + \Delta R_i - T_n - \Delta T \\
&= \sqrt{a} + \Delta R_i - \Delta T \\
\Rightarrow \frac{b}{\sqrt{a}} &= \Delta R_i - \Delta T
\end{aligned}
$$

- As $R_{ni} = \sqrt{a} + T_n$, we have

$$
\frac{(x_n - x_i)\Delta x + (y_n - y_i)\Delta y + (z_n - z_i)\Delta z}{R_{ni} - T_n} = \Delta R_i - \Delta T
$$

$$
\frac{x_n - x_i}{R_{ni} - T_n}\Delta x + \frac{y_n - y_i}{R_{ni} - T_n}\Delta y + \frac{z_n - z_i}{R_{ni} - T_n}\Delta z + \Delta T = \Delta R_i
$$

- Coefficients on LHS represent the direction cosines of the line-of-sight (LOS) vector from user to satellite as projected along coordinate axes.

- Linear equations in a matrix form,

$$\underbrace{\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & 1 \\ \beta_{21} & \beta_{22} & \beta_{23} & 1 \\ \beta_{31} & \beta_{32} & \beta_{33} & 1 \\ \beta_{41} & \beta_{42} & \beta_{43} & 1 \end{bmatrix}}_{\boldsymbol{B}} \underbrace{\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta T \end{bmatrix}}_{\boldsymbol{x}} = \underbrace{\begin{bmatrix} \Delta R_1 \\ \Delta R_2 \\ \Delta R_3 \\ \Delta R_4 \end{bmatrix}}_{\boldsymbol{r}}, \quad \boldsymbol{B}\boldsymbol{x} = \boldsymbol{r} \Rightarrow \boldsymbol{x} = \boldsymbol{B}^{-1}\boldsymbol{r}$$

  where, β_{ij} is the direction cosine of angle between LOS to i^{th} satellite and j^{th} coordinate.
- If $\boldsymbol{B}$ is nonsingular then the solution is possible.
- This results into $|\boldsymbol{B}| = 0$, navigation equation blows up and system outage occurs as result of poor geometry.

Reference

1. G. M. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*, Academic Press, Inc. 1993.
2. D. Ghose, *Lecture notes on Navigation, Guidance and Control*, IISc Bangalore.

Thank you for your attention !!!