Inertial Sensors

Dr. Shashi Ranjan Kumar

Assistant Professor
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Powai, Mumbai, 400076 India

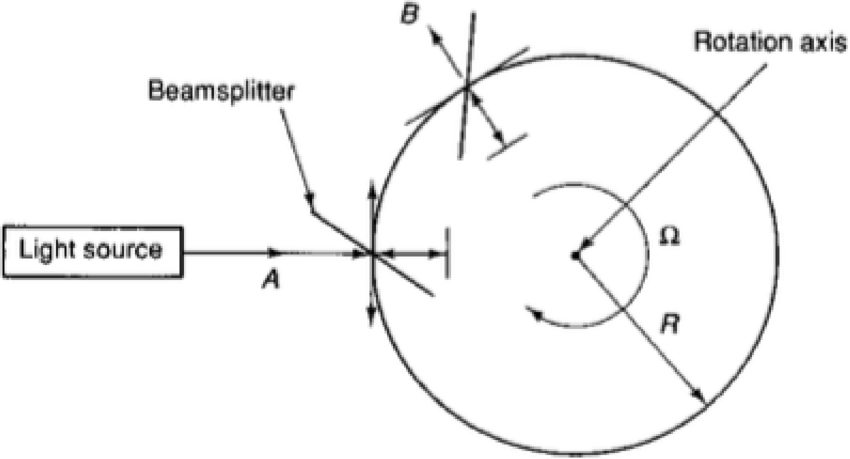
- Factors affecting selection of sensors
- Environmental factors
  - Temperature change
  - Humidity effect
  - Corrosion
  - Size
  - Susceptibility to EM interference
  - Ruggedness
  - Power consumption
  - Self-test capability
- Economical factors
  - Cost
  - Availability
  - Lifetime
- Sensor characteristics
  - Sensitivity
  - Range
  - Stability
  - Repeatability
  - Linearity
  - Error
  - Response time

- **Fixed bias**: Sensor output even in the absence of an applied input rotation
  - Residual torques from flexible leads within the sensor
  - Spurious magnetic fields and temperature gradients
  - Acceleration-independent bias
- **Acceleration-dependent bias (g-dependent bias)**: Biases proportional to the magnitude of applied acceleration.
  - Mass unbalance in the rotor suspension
  - Non-coincidence of the rotor centre of gravity and the centre of the suspension mechanism
  - Fixed bias in the measured rate for a steady acceleration
- **Anisoelastic bias (g^2-dependent bias)**: Biases proportional to product of acceleration along orthogonal pairs of axes
  - Unequal finite compliances of gyroscope rotor suspension structure in different directions.

- **Anisoinertia errors**: Biases owing to inequalities in gyroscope moments of inertia about different axes
  - Proportional to the product of angular rates applied about pairs of orthogonal axes
- **Scale-factor errors**: Errors in the ratio relating the change in the output signal to a change in the input rate
  - Scale-factor non-linearity: deviations from the least-squares straight line or non-linear function fitted to the measurements
  - Scale-factor asymmetry: differences in the magnitude of the output signal for equal rotations of the sensor in opposite directions
- **Cross-coupling errors**: Gyroscope sensitivity to turn rates about axes normal to the input axis
  - Non-orthogonality of the sensor axes

- Fixed or repeatable terms:
  - A bias component which is predictable
  - Always present whenever the sensor is switched on
  - Can therefore be corrected
- Temperature induced variations:
  - A temperature-dependent bias component
  - Can be corrected with suitable calibration
- Switch-on to switch-on variations:
  - A random bias which varies from gyroscope switch-on to switch-on
  - Constant for any one run
- In-run variations:
  - An in-run random bias which varies throughout a run
  - Precise form of this error varies from one type of sensor to another
- First two may be corrected to large extent, but later two types of errors persists.

- **Optical sensor**: Sensors using properties of EM radiation to sense rotation.
- **Ring Laser Gyro (RLG)**: Principles of general relativity
  - A resonant optical cavity containing two oppositely directed traveling light waves generated by stimulated emission of radiation
  - An inherent capability to operate in the strapdown mode
  - Scale factor linearity over the full dynamic range
  - Easy interface with digital systems
- Detection of rotation with light was demonstrated by Sagnac in 1913.
- **Principle**: Two light waves acquire a phase difference by propagating in opposite directions around a loop interferometer
- These beams travel the same path in opposite directions around a closed ring.
- Solid-state sensor with no moving parts.
- It detects and measures differential angular rotations by measuring the frequency difference between the two contrarotating beams.



B
Rotation axis
Beamsplitter
Light source
A
Ω
R

- Consider a circular interferometer.
- Assume the light beam is splitted into two beams rotating in opposite directions.
- After one rotation they combine at the beamsplitter.
- In absence of rotation, transit time taken by light to complete rotation

$$t = \frac{2\pi R}{c}$$

  where, c is the speed of light, and R is the radius of circular path.
- If the interferometer is rotated with constant angular velocity Ω then the travel time for both the beams will be different.
- Note that the beamsplitter is moved to new location at B.
- With respect to inertial space, the light moving in the direction of rotation must travel a longer distance than the light traveling in the opposite direction.

- Let X be the inertial space distance between points A and B.
- Positive $(+)$ and negative $(-)$ signs refer to the beam traveling in the direction of rotation and opposite to the direction of rotation, respectively.
- Total closed-path transit for the light

$$ct_\pm = 2\pi R \pm X_\pm, \quad X_\pm = R\Omega t_\pm$$

- On solving above equations,

$$t_\pm = \frac{2\pi R}{c} \pm \frac{R\Omega t_\pm}{c} \quad \Rightarrow t_\pm = \frac{2\pi R}{c \mp R\Omega}$$

- Transit time Δt

$$\begin{aligned}\Delta t &= t_+ - t_- = \frac{2\pi R}{c - R\Omega} - \frac{2\pi R}{c + R\Omega} \\ &= 2\pi R\left[\frac{2R\Omega}{c^2 - R^2\Omega^2}\right] = \frac{4\pi R^2\Omega}{c^2 - R^2\Omega^2}\end{aligned}$$

- Transit time Δt

$$\Delta t = \frac{4\pi R^2\Omega}{c^2 - R^2\Omega^2} = \frac{\left(\frac{4\pi R}{c}\right)\left(\frac{R\Omega}{c}\right)}{1 - \left(\frac{R\Omega}{c}\right)^2}$$

- On neglecting smaller terms, we get equation called as "Sagnac effect"

$$\Delta t = \frac{4\pi R^2\Omega}{c^2 - R^2\Omega^2} = \left(\frac{4\pi R}{c}\right)\left(\frac{R\Omega}{c}\right) = \frac{4\pi\Omega R^2}{c^2} = \frac{4A\Omega}{c^2}$$

where, $A = 4\pi R^2$ is the area of circular optical path.

- Optical path difference ΔL

$$\Delta L = c\Delta t = \frac{4c\pi\Omega R^2}{c^2} = \frac{4\Omega\pi R^2}{c} = \frac{4A\Omega}{c}$$

- This result holds in general for any geometric closed path.
- Issue: The path difference is small even with a large area.
- If Ω is very low then it is difficult to measure these angular rates as required closed area is very large.
- Ratio of total enclosed area to the wavelength must be large to sense low angular rate.
- Lack of sensitivity because the path difference for light traveling in the two directions is much less than wavelength.

- Improvement of sensitivity
  - By replacing the beamsplitter with a mirror
  - Form a resonant circuital optical cavity supporting traveling-wave modes for the counterrotating beams.
- These modes could be made self-sustaining by placing the lasing medium in the cavity.
- Laser frequency is dependent on the cavity length.
- Two oppositely directed traveling waves oscillate independently, each with its own frequency and amplitude.
- Fractional difference between these two frequencies corresponds to the fractional difference in optical path lengths traveled by each wave and, therefore, is proportional to the angular velocity.

- To sustain oscillation, there must be enough gain in the medium to overcome losses in the cavity.
- Optical length of beam also need to satisfy

$$N\lambda_{\pm} = L_{\pm}$$

where, $L_{\pm}$ is the optical length of each beam, $\lambda_{\pm}$ is the wavelength, and N is large integer $(10^5$ to $10^6)$.

- Cavity geometry determines the wavelengths of a given mode.
- Fractional frequency shift equals the fractional path length

$$\frac{\Delta\nu}{\nu} = \frac{\Delta L}{L} \quad \text{(Proof?)}$$

- As $\lambda = c/\nu$, we have beat frequency Δt given by

$$\Delta\nu = \frac{\Delta L\nu}{L} = \frac{4A\Omega}{c}\frac{c}{\lambda L} = \left(\frac{4A}{L\lambda}\right)\Omega$$

- Ideal RLG equation

$$\Delta\nu = \underbrace{\left(\frac{4A}{L\lambda}\right)}_{\text{Geometric or ideal scale factor } S} \Omega$$

where, λ and L are the wavelength of laser light and optical path length or cavity length, respectively.

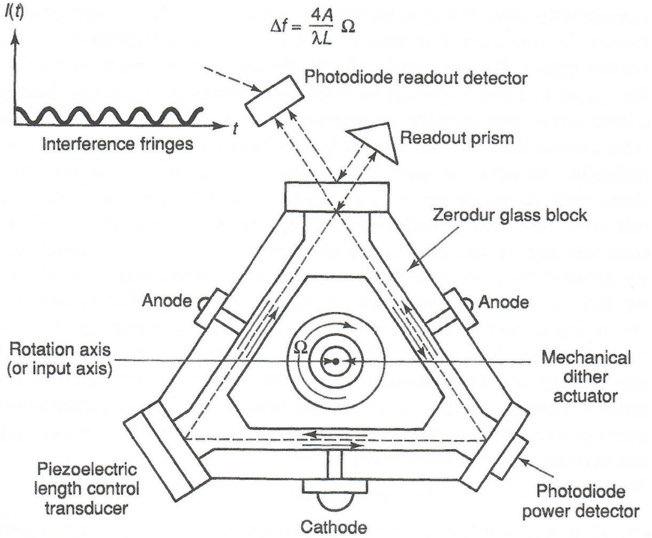
- On integration of ideal RLG equation

$$\int_{t_1}^{t_2} \Delta\nu dt = S \int_{t_1}^{t_2} \Omega dt \Rightarrow N = S\theta$$

where, N is total phase shift or beats counted during measurement time and θ is total angle of rotation.

- Laser gyro is rate integrating gyro as it gives N counts when turned through angle θ.

- RLG is a triangular or square cavity filled with gas, in which two oppositely traveling light waves are generated by stimulated emission of radiation.
- Active laser gyro: A two-mode, continuous wave (cw)
- Passive laser gyro: Lasing medium is external to the cavity.
- Laser gyro combines the properties of an optical oscillator and general relativity to produce the function of the conventional mechanical gyroscopes.
- Components of RLG
  - Block material
  - Mirrors
  - Gain medium (He-Ne plasma cavity)
  - Readout mechanism
  - Associated electronics



I(t)
$\Delta f = \frac{4A}{\lambda L}\,\Omega$
t
Interference fringes
Photodiode readout detector
Readout prism
Zerodur glass block
Anode
Anode
Ω
Rotation axis
(or input axis)
Mechanical
dither
actuator
Piezoelectric
length control
transducer
Cathode
Photodiode
power detector

RLG: Example

Consider an equilateral triangular RLG with its side length and height given by 7.239 and 6.2687 cm. Assume input angular velocity is 1 deg/h. The operating wavelength is assumed to be 0.6328 μm. Compute measurable beat frequency.

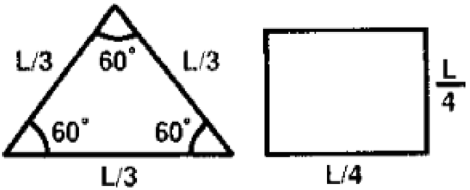
- Total optical length = 3(7.29)=21.717 cm
- Area of triangle $A = (1/2)bh$ = 3.1695(6.2687) = 22.6895 cm^2
- $\Omega = 1$ deg/h = 4.85$\times 10^{-6}$ rad/s
- Measurable beat frequency

$$\Delta\nu = \left(\frac{4A}{L\lambda}\right)\Omega = \frac{4(2.27 \times 10^{-3})(4.85 \times 10^{-6})}{2.172 \times 10^{-1}(0.6328 \times 10^{-6})}$$
$$= 0.32 \text{ Hz}$$

- How can we increase scale factor sensitivity of RLG?
- Ideal RLG equation

$$\boxed{\Delta\nu = S\Omega = \left(\frac{4A}{L\lambda}\right)\Omega}$$

- Sensitivity of scale factor
  - Increase enclosed area A
  - Decrease wavelength λ or optical length L
- How does geometric form of the closed path affect scale factor?



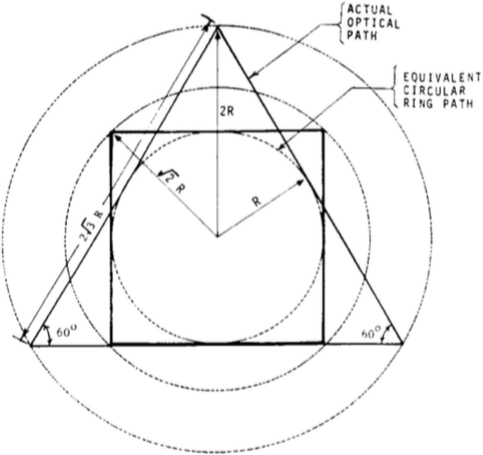
- For equilateral triangle and square shape

$$S_{ET} = \frac{4}{L\lambda}\frac{\sqrt{3}}{4}\left(\frac{L}{3}\right)^2 = \frac{1}{\lambda}\left(\frac{L}{3\sqrt{3}}\right)$$
$$S_{SQ} = \frac{4}{L\lambda}\frac{L}{4}\frac{L}{4} = \frac{1}{\lambda}\left(\frac{L}{4}\right)$$

- For a general shape of closed path

$$\boxed{S = \frac{1}{\lambda}(\text{Diameter of inscribed circle})}$$

- Accuracy of RLG $\propto$ Area enclosed by the path and inversely proportional to path length.
- A square laser gyro encloses a greater area for a given path length than triangular one, thus having greater potential accuracy.
- Square configuration: package into a smaller-sized inertial navigation unit.



Equivalent circular RLG: One that gives the same ideal scale factor as the actual polygonal ring laser.

Reference

1. G. M. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*, Academic Press, Inc. 1993.
2. D. H. Titterton and J. L. Weston, *Strapdown Inertial Navigation Technology*, Progress in Astronautics and Aeronautics, Vol. 207, ed. 2, ch. 4.

Thank you for your attention !!!