Aided Inertial Navigation System

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- Ranging error alone does not determine position fix error.
- Relative geometry of four satellites and user also affects fix accuracy.
- Due to linearity of navigation equation, relation between errors in pseudorange measurement and in user position and clock bias

$$\epsilon_{\boldsymbol{x}} = \boldsymbol{B}^{-1}\epsilon_{\boldsymbol{r}}$$

where, $\epsilon_{\boldsymbol{x}}$ and $\epsilon_{\boldsymbol{r}}$ are errors in user position and clock bias, and pseudorange measurement errors, respectively.

- If the covariance matrices for errors $\epsilon_{\boldsymbol{x}}$ and $\epsilon_{\boldsymbol{r}}$ as $C_{\boldsymbol{x}}$ and $C_{\boldsymbol{r}}$, respectively, then

$$C_{\boldsymbol{x}} = \boldsymbol{B}^{-1}C_{\boldsymbol{r}}\boldsymbol{B}^{-T}$$

- $\boldsymbol{B}$ is function of the direction cosines of LOS unit vectors from the user to four satellites.
- Error relationships are functions only of the satellite geometry.
- This leads to the concept of geometric dilution of precision (GDOP), a measure of how satellite geometry degrades navigation accuracy.
- To a good approximation,

$$C_{\boldsymbol{x}} = (\boldsymbol{B}^T \boldsymbol{B})^{-1}$$

- Covariance matrix depends only on the direction and is in no way dependent on the distances between the user and each satellite.
- Diagonal elements of covariance matrix are actually the variances of user position and time.

- **Geometric dilution of precision (GDOP)**: A measure of the error contributed by the geometric relationships of the GPS as seen by the receiver.

$$\text{GDOP} = \sqrt{\text{tr}(B^T B)^{-1}} = \sqrt{V_x + V_y + V_z + V_t}$$

  where, V_x, V_y, V_z, V_t are variance of x, y, z position (or velocity), and clock bias estimates, respectively.
- Diagonal of the covariance matrix contains the variances of position errors and receiver-clock bias error.

$$\text{GDOP} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + \sigma_{tt}^2}$$

  where $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{tt}$ represent 1σ error in position and time, respectively.
- GDOP is the conventional measure of overall geometric performance.

- **3D position dilution of precision (PDOP)**: A measure of geometric performance which relates only to three components of position error.
- Product of PDOP and ranging error determines fix error.
- Receiver normally attempts to track the set of four among N visible satellites that has the smallest PDOP.
- PDOP is also invariant with the coordinate system.
- In any navigation system, position accuracy is more important and knowing time is a secondary by-product.

$$\text{PDOP} = \sqrt{V_x + V_y + V_z} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2}$$

- PDOP is useful in aircraft weapon delivery applications.

- **Position accuracy**
  - ⇒ PDOP: User/navigation satellite geometries and it can be analyzed independently of system errors.
  - ⇒ Other "system" errors: Accuracy of the ephemeris data and transmitted time from the satellites, ionospheric and atmospheric effects, and various other error sources.
- Characteristic of PDOP allows separate analyses of alternative orbital configurations, user motion, and satellite losses for the purposes of comparison and choosing optimal constellation.
- What could be other alternative DOPs?
  - ⇒ Horizontal dilution of precision (HDOP)
  - ⇒ Altitude or vertical dilution of precision (VDOP)
  - ⇒ Time dilution of precision (TDOP)
  - ⇒ Maximum dilution of precision (MDOP): Larger component of horizontal position error

- Horizontal dilution of precision (HDOP)

$$\text{HDOP} = \sqrt{V_x + V_y} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2}$$

- Altitude or vertical dilution of precision (VDOP)

$$\text{VDOP} = \sqrt{V_z} = \sqrt{\sigma_{zz}^2}$$

- Time dilution of precision (TDOP)

$$\text{TDOP} = \sqrt{V_t} = \sqrt{\sigma_{tt}^2}$$

- Maximum dilution of precision (MDOP)

$$\text{MDOP} = \max\{\sqrt{V_x}.\sqrt{V_y}\} = \max\{\sqrt{\sigma_{xx}^2}, \sqrt{\sigma_{yy}^2}\}$$

- HDOP is mostly used in surface application.

- DOP depends on satellite's geometric configuration.
- How to select the optimal satellite configuration?
  - $\Rightarrow$ Satellites with most favourable geometry w.r.t. the user.
- Majority of the time, however, there will be six or more satellites in view by an earth based user and even more by a low altitude satellite user.
- Time required to compute PDOP values for all the possible combinations of satellites is excessive.
- This may not be a good option for satellite selection.
- Almost total correlation between PDOP and the volume of the tetrahedron formed by lines connecting the tips of the four unit vectors from the user toward the four satellites.
- Larger the volume of this tetrahedron, smaller the corresponding PDOP value will be for this set of satellites.

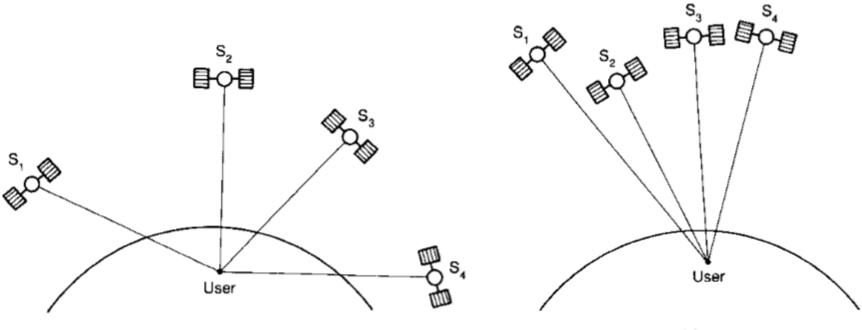


Figure: Geometry for the determination of PDOP: (a) good (low) PDOP; (b) poor (high) PDOP.

- Navigation measurements
  - ⇒ Geometry
  - ⇒ Incoming signal
  - ⇒ Noise
- Selection based on GDOP does not address the incoming signal and noise.
- An optimum satellite selection process must use every available piece of information available to determine the satellite set, which will provide the smallest navigation errors over a particular period of time.
- GDOP is given by

$$\text{GDOP} = \sqrt{\text{tr}(H^T H)^{-1}}$$

- Matrix H is formed with the direction cosines of LOS unit vector from the user to each satellite in view.

- **Visibility matrix**, H, is given by

$$H = \begin{bmatrix} h_E(1) & h_N(1) & h_u(1) & 1 \\ h_E(2) & h_N(2) & h_u(2) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ h_E(j) & h_N(j) & h_u(j) & 1 \end{bmatrix}$$

  where, j is total number of visible satellites and $h_E(.), h_n(.), h_u(.)$ are the respective direction cosines of LOS unit vectors to each satellite expressed in user local-level (east, north, up) coordinates.
- Because of variations in geographic terrain, all satellites below 5° elevation from the horizon are deleted from possible satellite selection.
- Satellite set with lowest GDOP is selected for smallest navigation errors.

- Consider vector of measurement components, $\boldsymbol{z}$, corrupted by additive noise $\boldsymbol{v}$ and linearly related to the state $\boldsymbol{x}$.

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}$$

- Least square estimator $\hat{\boldsymbol{x}}$ which minimizes the sum squared of the components of the difference $\boldsymbol{z} - \boldsymbol{H}\boldsymbol{x}$.

$$\hat{\boldsymbol{x}} = \arg_{\boldsymbol{x}} \min J, \quad J = (\boldsymbol{z} - \boldsymbol{H}\boldsymbol{x})^T (\boldsymbol{z} - \boldsymbol{H}\boldsymbol{x})$$

- On solving this optimization problem, we get

$$\hat{\boldsymbol{x}} = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\boldsymbol{1}}\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{z}$$

- In case of deterministic case, $\boldsymbol{v} = 0$, the matrix $\boldsymbol{B}$ corresponds to visibility matrix, $\boldsymbol{x}, \boldsymbol{r}$ correspond to state and measurement vectors.

- To obtain the quality of estimate, define error as $\tilde{\boldsymbol{x}} = \hat{\boldsymbol{x}} - \boldsymbol{x}$

$$\tilde{\boldsymbol{x}} = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\mathbf{1}}\boldsymbol{H}^{\boldsymbol{T}}(\boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}) - \boldsymbol{x} = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\mathbf{1}}\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{v}$$

- If the noise has zero mean then estimator error also has zero mean.
- Covariance of the estimation error

$$E\left[\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^T\right] = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\mathbf{1}}\boldsymbol{H}^{\boldsymbol{T}}E\left[\boldsymbol{v}\boldsymbol{v}^T\right]\boldsymbol{H}(\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\mathbf{1}}$$

- If all components of $\boldsymbol{v}$ are pairwise uncorrelated and have unit variance then

$$E\left[v_i v_j\right] = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}, \quad E\left[\boldsymbol{v}\boldsymbol{v}^T\right] = \boldsymbol{I}$$

- Covariance of the estimation error

$$E\left[\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^T\right] = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\mathbf{1}}\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H}(\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\mathbf{1}} = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-\mathbf{1}}$$

- Square root of tr$[(\boldsymbol{H^T H^{-1}})]$ will be recognized as the GDOP.
- All GDOP related performance measures is estimated as navigation quantity per unit of measurement noise covariance.
- In cases where certain measurement components are noisier than others, then GDOP may not be very useful due to per unit measurement noise concept.
- If choice exists between two possible sets or measurements, the set with poorer GDOP may be preferable, if they are of sufficiently higher accuracy.
- If measurement covariance matrix is not unity matrix but $E\left[\boldsymbol{v}\boldsymbol{v}^T\right] = \boldsymbol{I}\sigma^2$ then GDOP increases by scalar σ^2.
- If all potential measurements have same covariance then choice of best measurement set will be based on best GDOP.
- Measurement noise is called nonuniform when different measurements have different noise levels.

- For nonuniform noise levels, we obtain estimate by using weighted cost function

$$\hat{\boldsymbol{x}} = \arg_{\boldsymbol{x}} \min J, \quad J = (\boldsymbol{z} - \boldsymbol{H}\boldsymbol{x})^T \boldsymbol{W} (\boldsymbol{z} - \boldsymbol{H}\boldsymbol{x})$$

where

$$\boldsymbol{W} = \boldsymbol{R}^{-1}, \quad \boldsymbol{R} = E[\boldsymbol{v}\boldsymbol{v}^T]$$

- More weights to accurate measurements and less to the noisy ones.
- On solving this optimization problem, we have estimates

$$\hat{\boldsymbol{x}} = (\boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{H})^{-1} \boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{z}$$

and the covariance matrix becomes

$$E\left[\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^T\right] = (\boldsymbol{H}^T \boldsymbol{R}^{-1} \boldsymbol{H})^{-1}$$

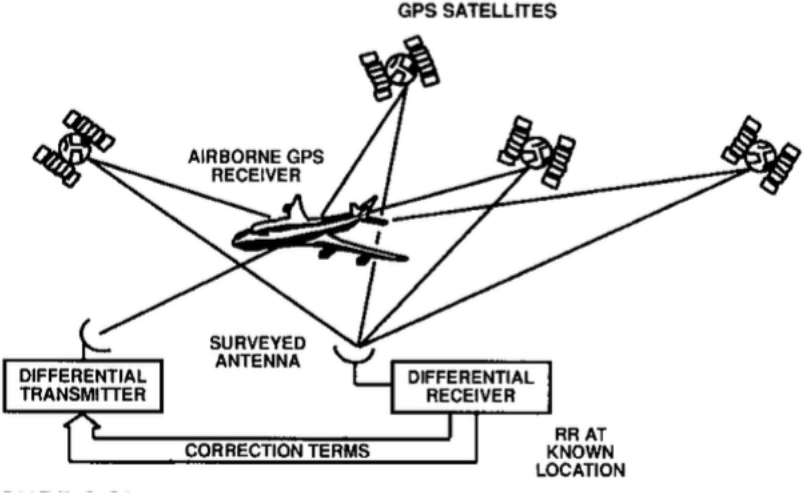


Figure: Differential GPS concept.

- Differential GPS: Improvement of performance of GPS position
  - ⇒ Use of data from GPS reference receiver (RR) in the vicinity of user.
  - ⇒ Reference receiver at a known location measures satellite location.
  - ⇒ Determination of common-mode bias errors using GPS reference receiver.
  - ⇒ Compensate for such errors to get correct user's position measurements.
- Correction of the measurements of other GPS receivers within a specified area around RR.
- Measured range includes actual range to satellite and associated errors.
- As the coordinates of RR are known, its range can be calculated by using ephemeris data.
- Other errors can be estimated from the difference between measured and calculated range, and transmitted to users.
- Schemes to accomplish differential GPS navigation
  - ⇒ Combinations of either corrections of pseudoranges, or of navigation solutions
  - ⇒ Whether corrections are made on board the user vehicle or at the RR

- To accommodate a large number of user vehicles, uplink of pseudorange corrections for participant onboard processing is used.
- RR measures pseudoranges to all visible satellites and computes differences between the calculated range to the satellite and the measured range.
- These differences are taken after the receiver clock bias has been removed .
- To compute range to each satellite, the satellite's position as defined in the ephemeris message.
- As in GPS, users select the optimum set of four satellites for measurements.
- DGPS represents a cost-effective method which significantly improves the reliability and accuracy of GPS navigation.
- Other techniques
  - $\Rightarrow$ Addition of extra sensors
  - $\Rightarrow$ Use of pseudo-satellites (pseudolites)
  - $\Rightarrow$ Assisted GPS

- A single reference station (RS) serving in small area
- Compute corrections based on differential GPS concept
- Corrections are more accurate for users nearby
- For users far way from RS, it may not be effective.

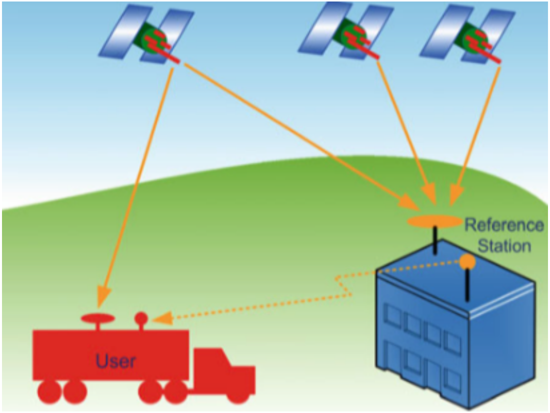


Figure: Local Area Differential GPS (LADGPS)

- Error correlation decreases with distance from RS in DGPS
- Improvement using more RS along perimeter of single RS
- Receiver weights corrections based on its proximity
- 1 Master station (MS) and several RS
- RS transfers measurements to MS and then MS estimates GPS error
- Update of corrections to users
  - ⇒ Space based augmentation systems (SBAS)
  - ⇒ Ground based augmentation systems (GBAS)

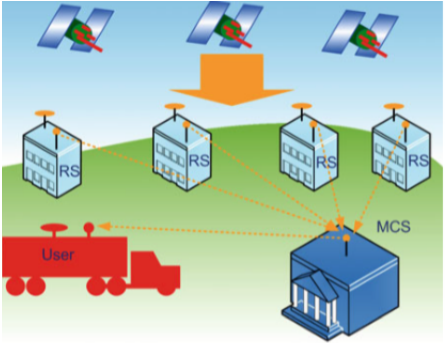


Figure: Wide Area Differential GPS (WADGPS)

- For cellular phones, GPS may not be always on.
- Requirement of fresh satellite data for position fixing.
- Receiver goes in search mode, takes longer time to get first fix.
- Longer time may not be acceptable for emergency services.

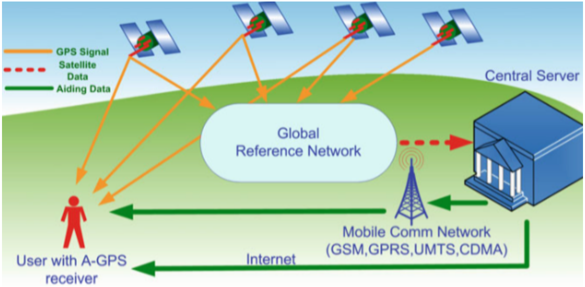
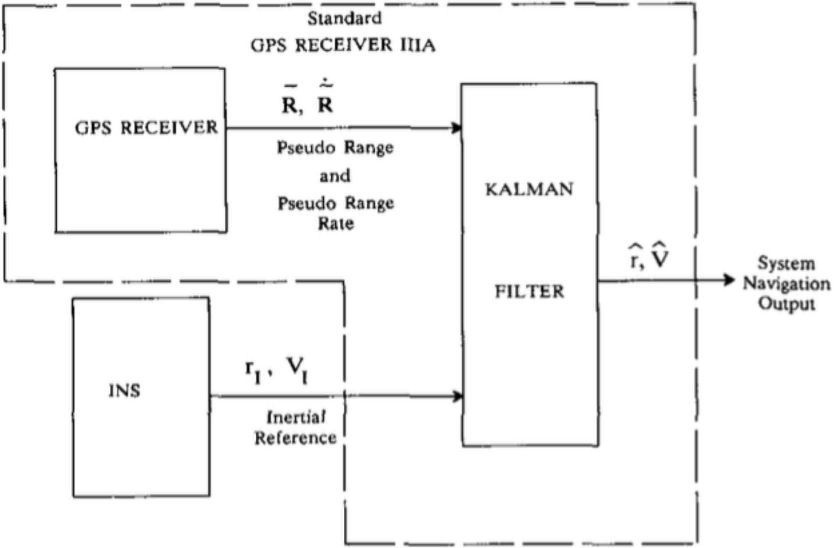


Figure: Assisted GPS (AGPS)

- GPS and INS has complementary features which allows to integrate both for improved performance.
- GPS can operate with sufficient accuracy in a stand-alone configuration.
- INS is able to provide accurate aiding data on short-term vehicle dynamics, while the GPS provides accurate data on long-term vehicle dynamics.
- GPS receiver provides pseudorange and pseudorange rate: useful for estimating errors in position, velocity, and other error parameters of the INS and GPS receiver clock.
- Estimates of INS error parameters allow GPS/INS navigation to achieve substantially smaller errors than could be achieved with either a GPS or an INS operating alone.
- Optimal integration of GPS-aided INS: Both INS and GPS jointly providing raw data to a single Kalman filter.



Standard
GPS RECEIVER IIIA
GPS RECEIVER
$\bar{R}$, $\dot{\tilde{R}}$
Pseudo Range
and
Pseudo Range
Rate
KALMAN
FILTER
$\hat{r}$, $\hat{V}$
System
Navigation
Output
INS
r_I , V_I
Inertial
Reference

Aided Inertial Navigation System

GPS/INS Integration

- INS provides GPS with a reference that is corrected by the Kalman filter.
- Estimated parameters are combined with INS measured position, velocity, and attitude to generate the best estimates.
- Accuracy of the GPS solution degrades rapidly whenever lock is lost in one or more satellites.
- It can be avoided if INS is integrated with GPS.
- Primary benefit of filtering GPS data through the INS is not to improve on the accuracy of the GPS position and velocity but to optimize the system solution in case a GPS outage occurs.
- GPS/INS have reduced sensitivity to jamming and maneuvers in military applications.

Reference

1. G. M. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*, Academic Press, Inc. 1993.