Indian Institute of Technology Bombay

Introduction to Navigation and Guidance

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Tutorial 3

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Objectives

- To familiarize yourselves with various terminologies used in the guidance literature.
- Basic concepts of autonomous guidance in two-dimensions.
- Concept of *unguided* versus *guided* vehicle engagement kinematics.

1. **Terminologies**: Pursuer, evader, line-of-sight, heading angle error, miss distance, time-to-go, relative velocity components, homing (endgame).
2. **Unguided vehicle**: Vehicle has no intelligence, follows some manual routine, probably human experience is involved
3. **Guided vehicle**: Vehicle is commanded via a computer, either installed on it or via telemetry.
4. **Kinematics of engagement**: Set of equations describing motions of the pursuer and the evader.

Consider the planar engagement geometry between a pursuer and an evader (both treated as point mass vehicles) as shown in Figure 1, whose kinematics, expressed in polar coordinates fixed to the pursuer's body, is governed by the set of nonlinear equations given by

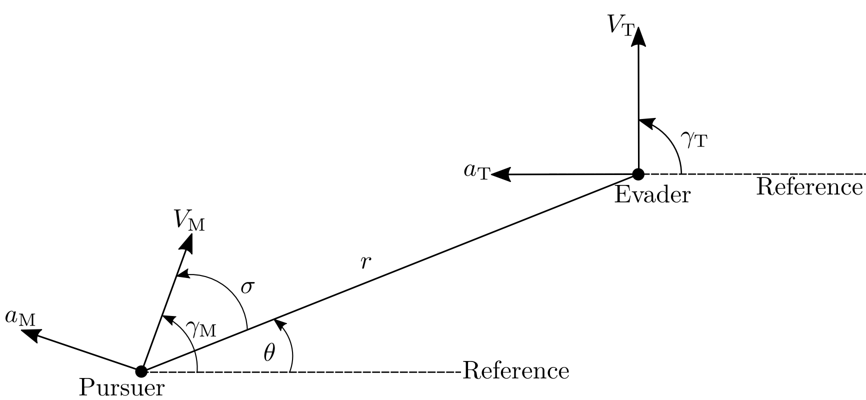

Figure 1: Pursuer-evader engagement geometry in a plane.

$$
\begin{aligned}
V_r = \dot{r} =& V_{\mathrm{T}} \cos(\gamma_{\mathrm{T}} - \theta) - V_{\mathrm{M}} \cos\sigma, && \text{(1a)}\\
V_\theta = r\dot{\theta} =& V_{\mathrm{T}} \sin(\gamma_{\mathrm{T}} - \theta) - V_{\mathrm{M}} \sin\sigma, && \text{(1b)}\\
\dot{\gamma}_{\mathrm{M}} =& \frac{a_{\mathrm{M}}}{V_{\mathrm{M}}}, && \text{(1c)}\\
\dot{\gamma}_{\mathrm{T}} =& \frac{a_{\mathrm{T}}}{V_{\mathrm{T}}}, && \text{(1d)}\\
\sigma =& \gamma_{\mathrm{M}} - \theta, && \text{(1e)}
\end{aligned}
$$

where V_{M}, V_{T} denote the speed of the pursuer and the evader, respectively, while γ_{M} and γ_{T} are their respective flight path angles. The relative distance and the line-of-sight (LOS) between the adversaries are represented by r and θ, respectively, while the pursuer's look angle is denoted by σ. The components of relative velocities along and across the LOS are V_r and V_θ, respectively, while the lateral accelerations of the pursuer and the evader are a_{M} and a_{T}, respectively. Usually, it is reasonable to assume that speeds of the pursuer and the evader remain invariant during the course of engagement, and the adversaries have acceleration capabilities in lateral direction only.

Note that throughout our discussion, we shall stick to the viewpoint of the pursuer. That said, we study the problem of interception, however, the analogous problem of avoidance may also be studied in similar context.

Problem 1. Suppose $V_{\mathrm{M}} = 400$ m/s, $V_{\mathrm{T}} = 300$ m/s, $\gamma_{\mathrm{M}} = 40°$, and $\gamma_{\mathrm{T}} = 30°$. If the pursuer captures the evader successfully, what would be the value of LOS?

Solution. If the evader is captured, then the LOS rate, $\dot{\theta}$, must be zero. Then,

$$V_{\mathrm{T}} \sin(\gamma_{\mathrm{T}} - \theta) - V_{\mathrm{M}} \sin\sigma = 0 \implies V_{\mathrm{T}} \sin(\gamma_{\mathrm{T}} - \theta) = V_{\mathrm{M}} \sin(\gamma_{\mathrm{M}} - \theta).$$

Substitute the given values to obtain θ.

Problem 2. For $\theta = 0°$, what would be the heading angle of the pursuer if the evader has a flight path angle of 90°. Assume $V_{\mathrm{M}} = 500$ m/s, $V_{\mathrm{T}} = 300$ m/s. What is the pursuer's the closing speed?

Solution. If the evader is captured, then the LOS rate, $\dot{\theta}$, must be zero. Then,

$$V_{\mathrm{T}} \sin(\gamma_{\mathrm{T}} - \theta) - V_{\mathrm{M}} \sin\sigma = 0 \implies V_{\mathrm{T}} \sin(\gamma_{\mathrm{T}} - \theta) = V_{\mathrm{M}} \sin(\gamma_{\mathrm{M}} - \theta),$$

which gives $\gamma_{\mathrm{M}} \approxeq 37°$. The closing speed is given by $-V_r$. Substitute the given values to obtain $-V_r$, which is the closing speed.

Problem 3. If the pursuer's flight path angle is given by the relation

$$\gamma_{\mathrm{M}} = \theta + \cot^{-1}\left[\cot\left(\gamma_{\mathrm{T}} - \theta\right) + \csc\left(\gamma_{\mathrm{T}} - \theta\right)\right],$$

and it is certain that the evader is captured, then can you determine the closing speed?

Solution. Check that $-V_r = V_{\mathrm{T}}$.

Problem 4. A pursuer and a target are moving with speeds of 500 m/s and 250 m/s, respectively. If their look angles, σ and $\gamma_{\mathrm{T}} - \theta$, are given as 150° and 90°, respectively, then check whether the pursuer is able to capture the target.

Solution. Here $V_\theta = 0$ but $V_r > 0$, thus interception will not occur.

Problem 5. Does the condition, $\cos(\gamma_{\mathrm{M}} - \theta)\sin(\gamma_{\mathrm{T}} - \theta) = \cos(\gamma_{\mathrm{T}} - \theta)\sin(\gamma_{\mathrm{M}} - \theta)$, imply an interception of the evader?

Solution. This implies that $V_r = V_\theta = 0$, that is, the vehicles are moving parallel to each other.

Problem 6. Consider the planar engagement scenario in Figure 1 where the pursuer is launched with a flight path angle, γ_{M}, while that of the evader (target) is γ_{T}. The relative separation, r, between them is 10 km with LOS angle, $\theta = 30°$. The speed of the evader is half of the pursuer's speed, and is also equal to the closing velocity of the pursuer-evader engagement. Assume that the evader is non-maneuvering with its heading angle, given by $\gamma_{\mathrm{T}} = 90°$.

(a) Obtain the possible pursuer's flight path angles with which it is launched, corresponding to above mentioned conditions.

(b) Is interception guaranteed with above launch angles, if the pursuer is unguided?

(c) Compute the possible heading angle errors corresponding to the above obtained pursuer launch angles.

Solution. It is given that $\gamma_{\mathrm{T}} = 90°, \theta = 30°, r = 10$ Km, $V_{\mathrm{T}} = \dfrac{V_{\mathrm{M}}}{2} = V_c$, where V_c is the closing speed.

(a) We know that the closing speed of the pursuer, V_c, is given by

$$
\begin{aligned}
& V_c = -V_r = V_{\mathrm{M}} \cos(\gamma_{\mathrm{M}} - \theta) - V_{\mathrm{T}} \cos(\gamma_{\mathrm{T}} - \theta) \\
\implies & \frac{V_c}{V_{\mathrm{T}}} = \frac{V_{\mathrm{M}}}{V_{\mathrm{T}}} \cos(\gamma_{\mathrm{M}} - \theta) - \cos(\gamma_{\mathrm{T}} - \theta) \\
\implies & 1 = 2\cos(\gamma_{\mathrm{M}} - 30^\circ) - \cos(90^\circ - 30^\circ) \implies \cos(\gamma_{\mathrm{M}} - 30^\circ) = 0.75 \\
\implies & \gamma_{\mathrm{M}} - 30^\circ = \pm 41.4096^\circ. \\
\text{Thus,} \quad & \gamma_{\mathrm{M}} = +41.4096^\circ + 30^\circ = 71.4096^\circ, \\
\text{or} \quad & \gamma_{\mathrm{M}} = -41.4096^\circ + 30^\circ = -11.4096^\circ.
\end{aligned}
$$

These are the two possible launch angles for the given initial conditions.

(b) For guaranteed interception, $\dot{\theta} = 0$ and $\dot{r} < 0$. For the given scenario, $V_c = V_{\mathrm{T}}$ which is a positive constant value. This ensures $\dot{r} < 0$. Now, for interception to occur

$$
\dot{\theta} = \frac{1}{r}\left[V_{\mathrm{T}} \sin(\gamma_{\mathrm{T}} - \theta) - V_{\mathrm{M}} \sin(\gamma_{\mathrm{Md}} - \theta)\right] = 0 \implies V_{\mathrm{T}} \sin(\gamma_{\mathrm{T}} - \theta) = V_{\mathrm{M}} \sin(\gamma_{\mathrm{Md}} - \theta),
$$

resulting in

$$
\frac{1}{2}\sin 60^\circ = \sin(\gamma_{\mathrm{Md}} - 30^\circ) \implies \sin(\gamma_{\mathrm{Md}} - 30^\circ) = \frac{\sqrt{3}}{4} \implies \gamma_{\mathrm{Md}} = 55.6589^\circ.
$$

The desired missile heading angle for interception, γ_{Md}, is not same as the obtained launch angle, γ_{M}. Hence, interception is not guaranteed if the pursuer is unguided.

(c) Heading angle error is given as $\gamma_{\mathrm{M}} - \gamma_{\mathrm{Md}}$. Thus, for $\gamma_{\mathrm{M}} = 71.4096^\circ$, one has heading angle error $71.4096^\circ - 55.6589^\circ = 15.7507^\circ$. Similarly, for $\gamma_{\mathrm{M}} = -11.4096^\circ$, heading angle error is obtained as $-11.4096^\circ - 55.6589^\circ = -67.0685^\circ$.