Steering and Basics of Guidance

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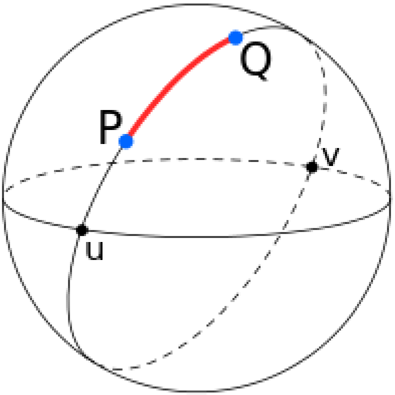
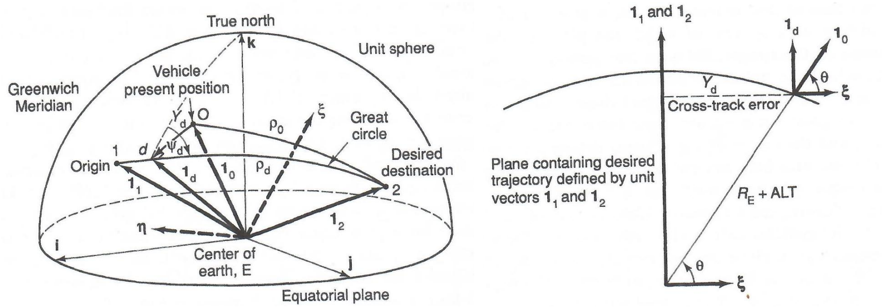
- Shortest path between two points on Earth: Path along great circle passing through two points
- **Great circle**: A circle on surface of sphere, of which the plane of circle passes through center of sphere.
- A unique great circle between two points unless they are diametrically opposite.
- Pole of great circle
- Small circle: Circle which does not pass through center, lower radius than great circle





True north
k
Unit sphere
Vehicle present position
Greenwich Meridian
O
ξ
Great circle
ρ0
1
d
Origin
ψd
1d
10
ρd
Desired destination
2
11
12
η
i
Center of earth, E
j
Equatorial plane
11 and 12
1d
10
θ
ξ
Cross-track error
Plane containing desired trajectory defined by unit vectors 11 and 12
RE + ALT
θ
ξ

- For ECI frame, position unit vector

$$\mathbf{1}_n = \cos\phi_n \cos\lambda_n \boldsymbol{i} + \cos\phi_n \sin\lambda_n \boldsymbol{j} + \sin\phi_n \boldsymbol{k}$$

  where, $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are unit vector along x, y, z axes, respectively.
- Unit vectors: $\mathbf{1}_0, \mathbf{1}_1, \mathbf{1}_2$, for $n = 0, 1, 2$.
- Angular separation μ between two positions

$$\mathbf{1}_1.\mathbf{1}_2 = \|\mathbf{1}_1\|\|\mathbf{1}_2\| \cos\mu \Rightarrow \mu = \cos^{-1}(\mathbf{1}_1 \cdot \mathbf{1}_2)$$

- Arc length ρ_d between two points 1 and 2

$$\rho_d = (R_E + h)\cos^{-1}(\mathbf{1}_1 \cdot \mathbf{1}_2)$$

- Similarly, arc length ρ_0 between two points, present position and destination

$$\rho_0 = (R_E + h)\cos^{-1}(\mathbf{1}_0 \cdot \mathbf{1}_2)$$

- Assume $\boldsymbol{\xi}$ as a unit vector $\perp$ plane defined by $\mathbf{1}_1$ and $\mathbf{1}_2$.

$$\boldsymbol{\xi} = \frac{\mathbf{1}_1 \times \mathbf{1}_2}{\|\mathbf{1}_1 \times \mathbf{1}_2\|}$$

- Angle between $\mathbf{1}_0$ and $\boldsymbol{\xi}$

$$\cos\theta = \frac{\mathbf{1}_0 \cdot \boldsymbol{\xi}}{\|\mathbf{1}_0\|\|\boldsymbol{\xi}\|} = \mathbf{1}_0 \cdot \boldsymbol{\xi}$$

- Desired position vector $\mathbf{1}_d$ and cross track error Y_d

$$\mathbf{1}_d = \mathbf{1}_0 - (\mathbf{1}_0 \cdot \boldsymbol{\xi})\boldsymbol{\xi}, \ Y_d = \ (R_E + h)[\mathbf{1}_0 \cdot \boldsymbol{\xi}] = (R_E + h)\cos\theta$$

- Heading angle ψ_d, angle between planes formed by $\mathbf{1}_1 \& \mathbf{1}_2$, and the meridian

$$\psi_d = \tan^{-1}\left[\frac{\boldsymbol{\eta} \times \boldsymbol{\xi}}{\boldsymbol{\eta} \cdot \boldsymbol{\xi}}\right], \qquad \boldsymbol{\eta} = \mathbf{1}_d \times \boldsymbol{k}$$

- Range-to-go and time-to-go, with ground speed V_g

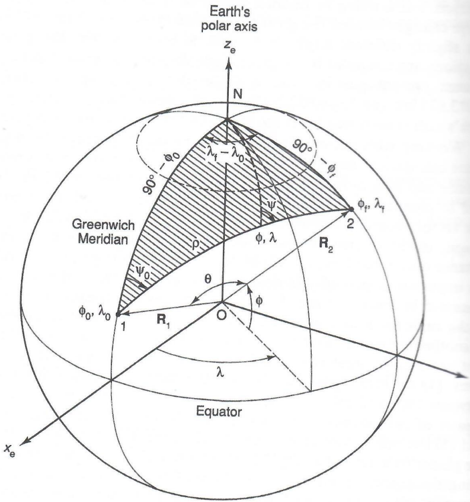
$$R_{\text{go}} = \rho_0 = \gamma(R_E + h), \quad \gamma = \tan^{-1}\left[\frac{\mathbf{1}_0 \times \mathbf{1}_2}{\mathbf{1}_0 \cdot \mathbf{1}_2}\right], \quad t_{\text{go}} = \frac{\rho}{V_g}$$

- Heading needs to be updated at every new position.
- Computation using vectors may be cumbersome.
- Relations can be derived using spherical triangle concept.
- Average radius of Earth

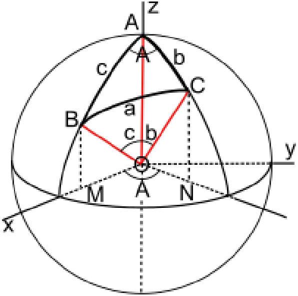
$$a = \frac{R_1 + R_2}{2}$$

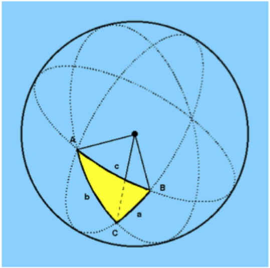
- For small angle θ, $\rho = a\theta$

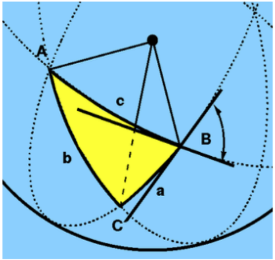
$$\cos\theta = \cos\left(\frac{\rho}{a}\right)$$



- **Spherical angle**: Angle on the surface of a sphere, formed by the intersection of two great circles.
- **Spherical triangle**: Triangle on the surface of a sphere, formed by the intersection of three great circles.





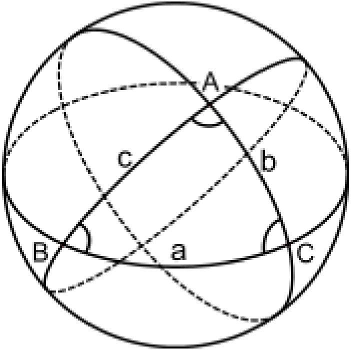


- Law of cosine

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

- Law of sine

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$



- Using law of cosines on $\triangle\, 1N2$,

$$\cos\left(\frac{\rho}{a}\right) = \cos(90^\circ - \phi_1)\cos(90^\circ - \phi_2) + \sin(90^\circ - \phi_1)\sin(90^\circ - \phi_2)\cos(\lambda_1 - \lambda_2)$$
$$= \sin\phi_1 \sin\phi_2 + \cos\phi_1 \cos\phi_2 \cos(\lambda_1 - \lambda_2)$$

- Great circle path distance between points 1 and 2 can be obtained as

$$\rho = a\cos^{-1}\left[\sin\phi_1 \sin\phi_2 + \cos\phi_1 \cos\phi_2 \cos(\lambda_1 - \lambda_2)\right]$$

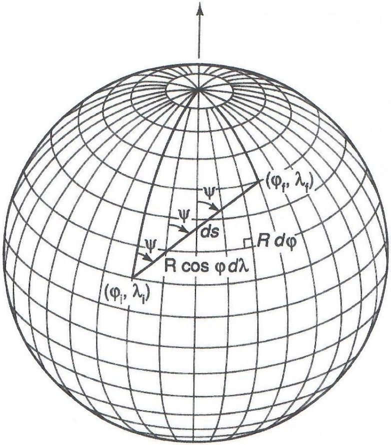
- Similarly, using law of cosine we also have

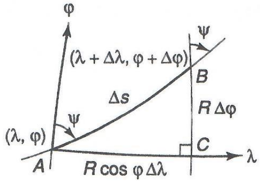
$$\cos(90^\circ - \phi_2) = \cos(90^\circ - \phi_1)\cos(\rho/a) + \sin(90^\circ - \phi_1)\sin(\rho/a)\cos\psi_0$$
$$\sin\phi_2 = \sin\phi_1 \cos(\rho/a) + \cos\phi_1 \sin(\rho/a)\cos\psi_0$$

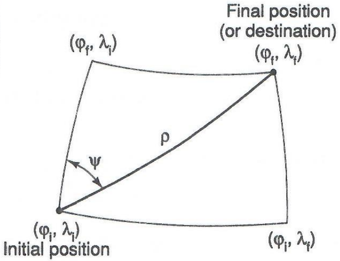
- Heading angle can be obtained as

$$\psi_0 = \cos^{-1}\left[\frac{\sin\phi_2 - \sin\phi_1 \cos(\rho/a)}{\cos\phi_1 \sin(\rho/a)}\right]$$

- Shortest path between two point on earth: Path along great circle passing through two points
- For both points on equator and meridian, paths along them are shortest.
- Angle of path w.r.t. meridian is constant for both cases.
- What are the angle of intersection for path along equator or meridian??
- For any great circle with points other than equator or same meridian, angle changes constantly.
- **Rhumb line**: Line on Earth's surface which intersects all meridians at same angle.
- Difference between rhumb line and great circle are more near pole and less near equator.
- For smaller distance, navigation along rhumb line is preferred over great circle.
- How to obtain Rhumb line distance and heading angle?


ψ
ψ
ψ
ds
R dφ
R cos φ dλ
(φ_i, λ_i)
(φ_f, λ_f)


φ
ψ
$(\lambda + \Delta\lambda, \varphi + \Delta\varphi)$
B
Δs
R Δφ
ψ
(λ, φ)
C
λ
A
R cos φ Δλ


Final position
(or destination)
(φ_f, λ_i)
(φ_f, λ_f)
ρ
ψ
(φ_i, λ_i)
Initial position
(φ_i, λ_f)

- For small $\Delta\phi$ and $\Delta\lambda$, right angle triangle ABC is assumed to be spherical.
- Relationship in terms of ϕ, λ for point on path

$$\tan\psi = \frac{\Delta\lambda\cos\phi}{\Delta\phi}, \quad \cos\psi = \frac{R\Delta\phi}{\Delta s}$$

- For $\Delta\phi \to 0$, we get

$$\frac{d\lambda}{d\phi} = \frac{\tan\psi}{\cos\phi}, \quad \frac{ds}{d\phi} = \frac{R}{\cos\psi}$$

- On integrating above equation,

$$d\lambda = \tan\psi\frac{d\phi}{\cos\phi}, \quad ds = \frac{R}{\cos\psi}d\phi$$

$$\int_{\lambda_i}^{\lambda_f} d\lambda = \tan\psi\int_{\phi_i}^{\phi_f}\frac{d\phi}{\cos\phi}, \quad \int_0^s ds = \int_{\phi_i}^{\phi_f}\frac{R}{\cos\psi}d\phi$$

- On performing integration of first integral equation,

$$\begin{aligned}\lambda_f - \lambda_i &= \tan\psi\left[\ln\tan\left(\frac{\pi}{4}+\frac{\phi_f}{4}\right) - \ln\tan\left(\frac{\pi}{4}+\frac{\phi_i}{4}\right)\right] \\ &= \tan\psi\left[\ln\left\{\tan\left(\frac{\pi}{4}+\frac{\phi_f}{4}\right)/\tan\left(\frac{\pi}{4}+\frac{\phi_i}{4}\right)\right\}\right]\end{aligned}$$

- To obtain heading angle ψ,

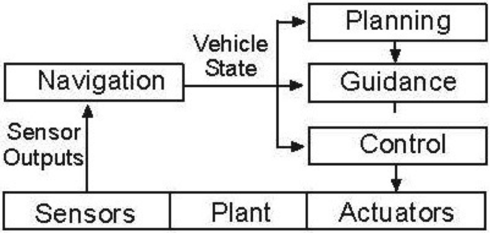
$$\psi = \tan^{-1}\left[\frac{\lambda_f - \lambda_i}{\ln\left\{\tan\left(\frac{\pi}{4}+\frac{\phi_f}{4}\right)/\tan\left(\frac{\pi}{4}+\frac{\phi_i}{4}\right)\right\}}\right]$$

- On integration of second integral equation, path length

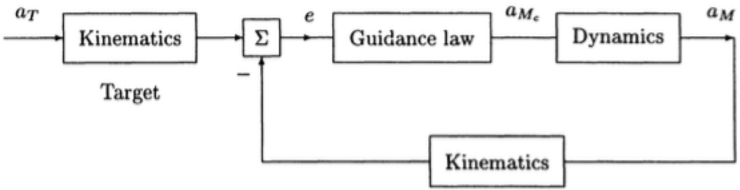
$$s = \frac{R(\phi_f - \phi_i)}{\cos\psi}$$

- Path length from equator to north pole?

- Guidance: Process for guiding the path of an object towards a given point, which in general may be moving.
- Guidance system: Brain of an aerospace vehicle
- Generation of guidance commands to achieve desired objectives of aerospace vehicle.
- It utilizes the vehicle's current states and their deviations from desired values to nullify the errors in controlled variables.



- What information do we need to collect from the environment?
- How do we go about collecting this information?
- How much can we trust the correctness of this information?
- How do we use this information to achieve our goal?
- Is our capability sufficient to meet our goal?
- How do we know if we have reached our goal or not?
- Are there some secondary goals that we must keep in mind while trying to achieve our primary goal? If yes, then how do we go about doing it?



When one object, based on information gathered from its environment, moves in such a way that it comes closer and closer to another stationary or moving object (a goal point) then we say that the object is guiding itself toward its goal point.

Guided missile

- A guided missile is a space-traversing unmanned vehicle which carries within itself the means for controlling its flight path.
- A guided missile is one which is usually fired in a direction approximately toward the target and subsequently receives steering commands from the guidance system to improve its accuracy.

Classifications of missiles:

- Surface-to-Surface Missiles (SSM)
- Surface-to-Air Missiles (SAM)
- Air-to-Surface Missiles (ASM)
- Air-to-Air Missiles (AAM)

- Launched from surface of earth or ships
- Usually large and stationary targets (small factory or a big city)
- Range of missile and type of warhead depends on the kind of targets.
- Range could be as low as a few kms to as high as thousands of kms
- Accuracy of missile : Accuracy of determining the target's position
- Offensive missiles
- Prithvi (600 km) and Agni (5000 km) missiles

- Launched from earth's surface against aerial targets (moving or maneuvering)
- Targets: small in size, move at high speeds, and/or are capable of executing complicated maneuvers
- Accurate guidance system and short engagement time
- Ability of guidance system to take appropriate actions within a short time.
- Defensive missiles
- Akash (30 km), Barak 8 (90 km), and Trishul missiles

- Launched from an aircraft to destroy targets on the surface of the earth
- Targets could be moving, but are normally stationary.
- Possibility and mechanism to search and seek out targets whose positions or movements are not known beforehand.
- Improvement of accuracy if missile comes closer to target.
- Velocity and other dynamic properties of the aircraft must be taken into account in the guidance system.
- Primarily offensive but may be defensive
- Helina (7-8 km), and Brahmos (400 km)

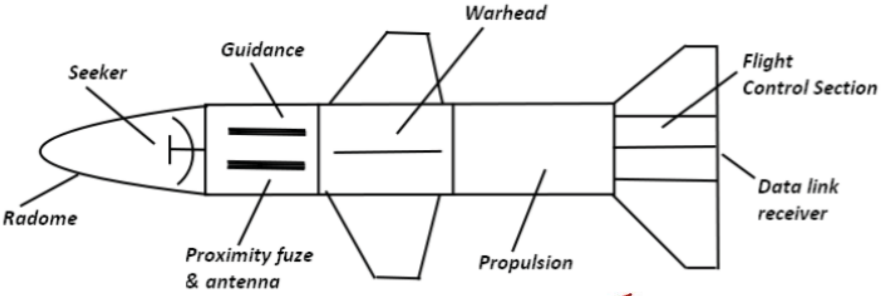
- Launched from air against aerial targets (small and difficult to locate)
- Most difficult to design and build
- High speed launcher and targets with high maneuverability
- Include challenges for SAMs w.r.t. target end, and ASMs at launcher.
- Guidance system should not prevent the launcher aircraft from taking evasive actions for its own survival.
- Offensive and defensive missiles
- Astra missile (80-110 km in head-on, 20 km in tail-chase)

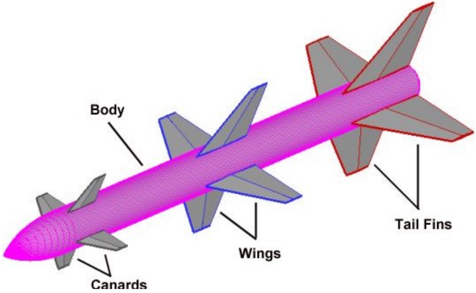
Tactical missiles

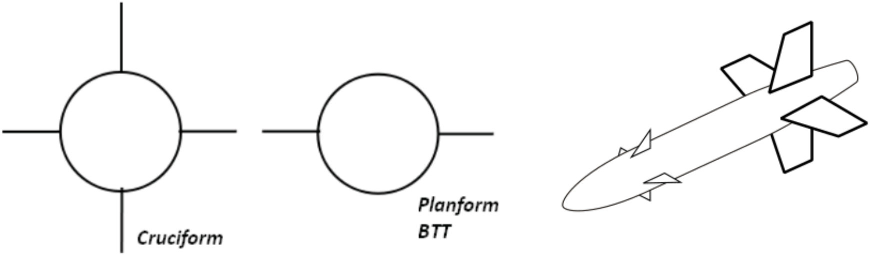
- Used for achieving some short-term missions.
- Small in size, have short ranges, and have limited destructive power.
- Short missions, destroying a penetrating enemy aircraft using a SAM or an AAM, destroying a small ground target (for example, an enemy tank or an enemy supply base) using an ASM.
- Large SSMs rarely used for this kind of missions.
- Mainly defensive weapons with limited capability for offence.

Strategic missiles

- Long term goals which make a strategic difference to the outcome of a war.
- Large in size, have long ranges, and have vast destructive capabilities.
- Mainly SSMs and their missions could be destruction of a huge military-industrial complex in the enemy territory or even big cities.
- Mainly offensive weapons


Warhead
Guidance
Seeker
Flight
Control Section
Data link
receiver
Radome
Proximity fuze
& antenna
Propulsion

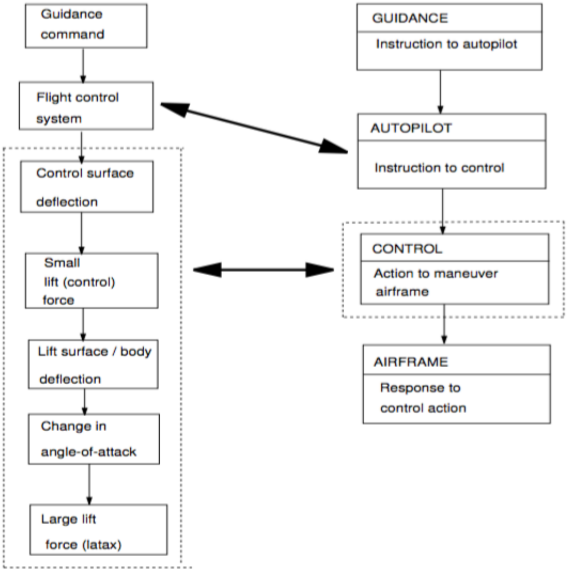

Body
Tail Fins
Wings
Canards



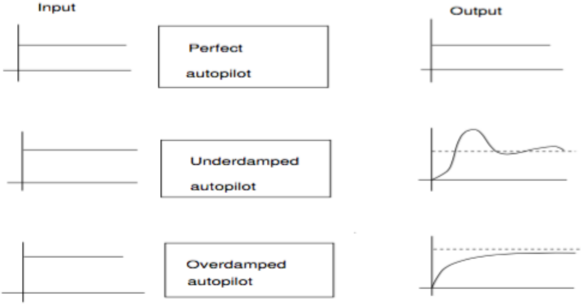
- Cruciform: Control surfaces/lifting surfaces at 90° from each other
- Planform: Control surfaces/lifting surfaces at 180° from each other
- Classification based on source of lift and location of control surfaces
  - Body (B), Tail (T), Canard (C), Wing (W), lift (l), control (c)
  - Examples: $B_l W_l T_c$, $B_l W_l C_c$

Steering and Basics of Guidance

Cause and Effect in Tactical Missiles



- Guidance command is usually expressed as missile's lateral acceleration.
- Achieved by generating lift by the airframe in proper direction.
- Guidance command is influenced by the dynamics of the autopilot, the flight control system, and the airframe.
- Achieved lateral acceleration may contain oscillations.
- For usual amount of damping, the response is quite fast.



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Thank you for your attention !!!