Inertial Sensors

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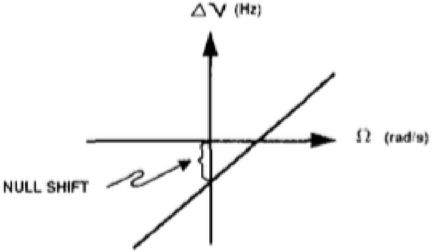
- **Error source**: Any effect that causes the input/output relationship to deviate from the ideal case.
  - Null shift
  - Lock-in
  - Mode pulling
- Potential causes: inherent EM properties, optical instrument imperfections, properties of materials used to construct the instrument, and detector limitations

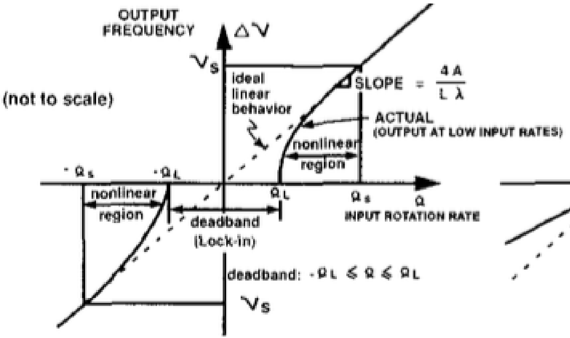


- **Backscatter**: Reflection of light from one laser beam back along the path where it interacts with the oppositely traveling beam
- **Lock-in**: A zero beatnote for a nonzero rotation
- A fundamental physical phenomenon associated with all oscillating devices
- Scale factor S becomes a function of the rotation rate Ω

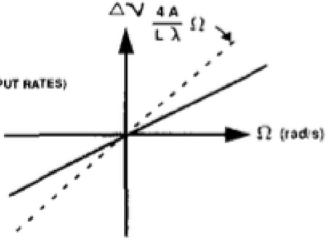
$$\Delta\nu = \begin{cases} 0 & \Omega^2 \leq \Omega_L^2 \\ \dfrac{4A}{L\lambda}\sqrt{\Omega^2 - \Omega_L^2} & \Omega^2 > \Omega_L^2 \end{cases}$$

  where, Ω_L is lock-in threshold (typically 0.1 deg/s).
- A bias must be added to achieve sufficient frequency difference for low input angular rate
- With added bias Ω_B for zero input rate,

$$\Delta\nu = S(\Omega_{\text{in}} - \Omega_B)\sqrt{1 - \left(\frac{\Omega_L}{\Omega_{\text{in}} - \Omega_B}\right)^2}, \quad \Omega_L < (\Omega_{\text{in}} - \Omega_B)$$



OUTPUT FREQUENCY
ΔV
Vs
(not to scale)
ideal linear behavior
SLOPE = 4A / Lλ
ACTUAL (OUTPUT AT LOW INPUT RATES)
nonlinear region
-Ωs
-ΩL
ΩL
Ωs
Ω
INPUT ROTATION RATE
nonlinear region
deadband (Lock-in)
deadband: -ΩL ≤ Ω ≤ ΩL
Vs



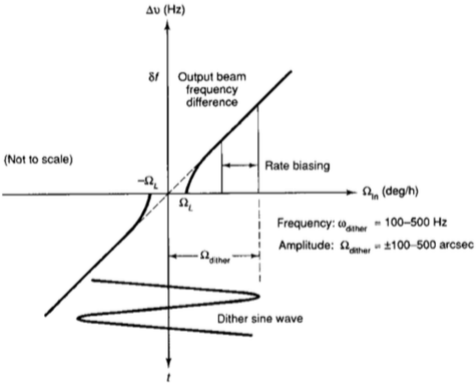
ΔV
4A / Lλ Ω
Ω (rad/s)

(c)

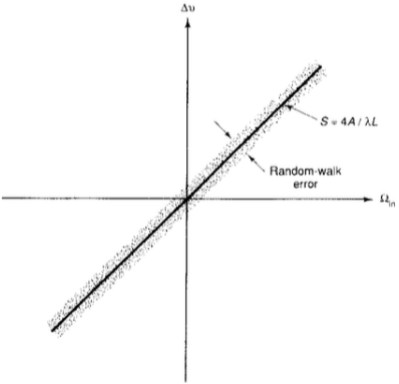
- **Dither**: Used to improve the performance by introducing a rotational bias so that the gyro operates outside the lock-in band or region.
- Mechanical dither is a sinusoidal, symmetric signal, alternating the bias about the zero-input rate.
- Bias averages to zero over each cycle.
- The frequency and amplitude of the dither are such that the laser gyro now becomes responsive to very low angular input rates.
- Typical values of dither-frequency are on the order of 100-500 Hz and amplitudes are between ± 100 and 500 arcsec.
- Simplified expression for output

$$\dot{\psi} = \Omega - \Omega_L \sin(\psi + \beta)$$

where, ψ is the instantaneous phase difference between the two counterrotating beams (including input and null shifts), Ω is input rotation rate, Ω_L is the lock-in rate, and β is the effect of backscattering phase shift.



Δυ (Hz)
δf
Output beam frequency difference
(Not to scale)
−Ω_L
Ω_L
Rate biasing
Ω_in (deg/h)
Frequency: ω_dither = 100–500 Hz
Amplitude: Ω_dither = ±100–500 arcsec
Ω_dither
Dither sine wave
t



Δυ
S = 4A / λL
Random-walk error
Ω_in

- Mathematical model of the laser gyro, including dithering effects

$$\dot{\psi} = \Omega - \Omega_L \sin(\psi + \beta) + \Omega_D \sin \omega_D t$$

where, $\dot{\psi}$, Ω_L, ω_D are beat frequency, bias magnitude, and dither rate.
- Easiest approach: Rotation with constant frequency above lock-in threshold.
- Why don't people use this to avoid lock-in?
- Dither eliminates the deadband and nonlinearity associated with lock-in.
  - Rotation rate periodically passes through zero where lock-in occurs, resulting in random-walk angular position error.
  - Vibration of the gyro body may affect other components or the gyro may be affected by other vibration sources.

- **RLG error model**

$$\Omega_{\text{out}} = \Omega_B + (1+\epsilon)[\Omega_{\text{in}} + \gamma_z \Omega_y - \gamma_y \Omega_z]$$
$$\epsilon = \epsilon_0 + f(|\Omega_{\text{in}}|) + g(\Omega_{\text{in}})$$
$$\Omega_B = B_0 + n_1 + n_2$$

where,

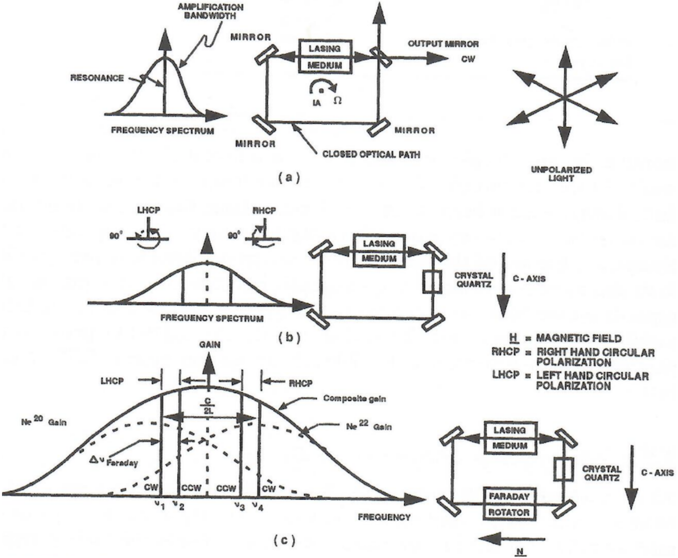
Ω_{out},Ω_{in}	Laser gyro output and input rate
Ω_B,ϵ	Gyro bias and scale factor errors
Ω_y, Ω_z	Angular rate normal to input axis
γ_y, γ_z	Misalignment of gyro lasing plane relative to nominal gyro input axis
B_0, ϵ_0	Fixed bias and scale factor errors
$f(\|\Omega_{\text{in}}\|), g(\Omega_{\text{in}})$	Symmetrical and general linearity errors
n_1, n_2	Random bias errors with bounded and unbounded integral values

- Simplest error model: $\Omega_{\text{out}} = S\Omega_{\text{in}} + \Omega_{\text{bias}}$
- Other RLG error model

$$\begin{aligned}\Omega_{\text{out}} &= B + \epsilon\Omega_{\text{in}} + \gamma\Omega_a \\ B &= B_0 + B_t\Delta T + B_\nabla \nabla T + B_m M + B_a A + N_{RW}(t)\sqrt{t} \\ \gamma &= \gamma_0 + \gamma_t\Delta T + \gamma_\nabla \nabla T + \gamma_m M + \gamma_a A + N_{RW}(t)\sqrt{t}\end{aligned}$$

Ω_{out},Ω_{in}	Laser gyro output and input rate
B,ϵ, γ	Gyro bias, scale factor errors and input axis misalignment
Ω_a	Rotation rate about input axis $\perp$ gyro input axis
$(.)_0$	Fixed component
$(.)_t$	error due to temperature difference from nominal
$(.)_\nabla, (.)_m, (.)_a$	Error due to temperature gradients, magnetic field, acceleration along input axis
M, ∇	Magnetic field strength and temperature gradient
$N_{RW}(t)$	Random component consisting of angular random walk and bias random walk

- Is there a way to avoid lock-in without using compensation?
- Two independent, two-frequency laser gyros sharing the same cavity and having a common optical path, but biased in opposite senses
- Multi-oscillator laser gyros produce four laser beams at four separate frequencies.
- Two of the beams, one traveling in each direction, are left circularly polarized, while the other two are right circularly polarized.
- Due to orientation of the quartz, the beams of left-hand circular polarization (LHCP) and right-hand circular polarization (RHCP) see nominally different path lengths, and consequently different optical lasing frequencies.
- Addition of a Faraday polarization rotator introduces a reciprocal (direction-independent) polarization anisotropy.



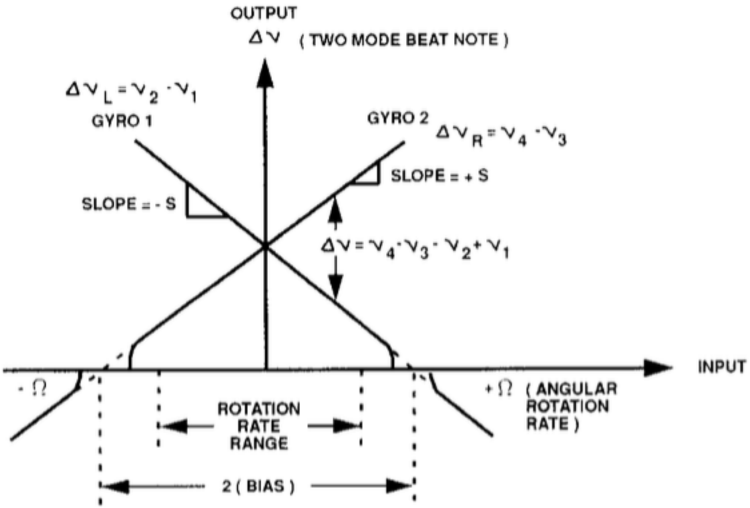
AMPLIFICATION BANDWIDTH
RESONANCE
FREQUENCY SPECTRUM
MIRROR
LASING MEDIUM
IA
Ω
OUTPUT MIRROR
CW
MIRROR
MIRROR
CLOSED OPTICAL PATH
UNPOLARIZED LIGHT
(a)
LHCP
RHCP
90°
90°
FREQUENCY SPECTRUM
LASING MEDIUM
CRYSTAL QUARTZ
C - AXIS
(b)
H = MAGNETIC FIELD
RHCP = RIGHT HAND CIRCULAR POLARIZATION
LHCP = LEFT HAND CIRCULAR POLARIZATION
GAIN
LHCP
RHCP
Composite gain
c/2L
Ne 20 Gain
Ne 22 Gain
Δν Faraday
CW
CCW
CCW
CW
ν1 ν2 ν3 ν4
FREQUENCY
LASING MEDIUM
CRYSTAL QUARTZ
C - AXIS
FARADAY ROTATOR
N
(c)

- Orthogonally polarized modes propagating in the same direction experience slightly different optical path lengths during traversal of the same path.
- Polarization rotator splits the frequencies of the resulting RHCP and LHCP modes.
- Individual laser gyro equations, with ideal scale factor S

$$\begin{aligned}\Delta\nu_L = \nu_2 - \nu_1 &= -S\Omega + \text{Faraday Bias}\\ \Delta\nu_R = \nu_4 - \nu_3 &= \quad S\Omega + \text{Faraday Bias}\end{aligned}$$

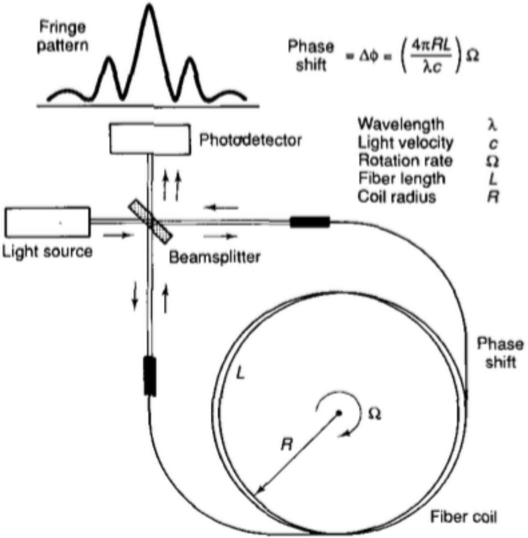
- Bias is normally chosen to be positive and at the same time much greater than $|S\Omega|$, so that the gyro operates far from the lock-in region.
- Measured beat frequency

$$\Delta\nu = \Delta\nu_R - \Delta\nu_L = (\nu_4 - \nu_3) - (\nu_2 - \nu_1) = 2S\Omega = \frac{8A}{L\lambda}\Omega$$



OUTPUT
Δν (TWO MODE BEAT NOTE)
Δν L = ν2 - ν1
GYRO 1
GYRO 2
Δν R = ν4 - ν3
SLOPE = + S
SLOPE = - S
Δν = ν4 - ν3 - ν2 + ν1
INPUT
- Ω
+ Ω (ANGULAR ROTATION RATE)
ROTATION RATE RANGE
2 (BIAS)

- Fiber-optic gyros (FOG) use optical fiber as the light path, in contrast to RLGs where light is beamed around a cavity.
- FOG is an angular rate sensor.
- Paths of clockwise and counterclockwise traveling beams are identical.
- **Sagnac effect**: Differential phase shift is induced by rotating an optical system in which light travels clockwise and counterclockwise directions.
- Only possible source of the phase difference is inertial rotation of the fiber.
- FOG measures rotation rates about an axis $\perp$ to the plane of fiber-optic coil by means of ring interferometry.
- This type of gyros are called as interferometric FOG (IFOG).
- Typical length: 50 m-1 km.



Fringe pattern
Phase shift $= \Delta\phi = \left(\frac{4\pi RL}{\lambda c}\right)\Omega$
Photodetector
Wavelength λ
Light velocity c
Rotation rate Ω
Fiber length L
Coil radius R
Light source
Beamsplitter
Phase shift
L
Ω
R
Fiber coil

- With fiber ring rotation, the transit times are different, resulting in relative phase shift between the beams.
- Phase difference causes the interference of the two beams to change in intensity.
- Change in the intensity of the interference signal is converted into an electrical signal by photodetector.
- Digital electronics process this analog signal into a digital signal proportional to the rotation rate.
- Relative phase shift

$$\Delta\phi = \frac{2\pi\Delta L}{\lambda} = \frac{2\pi}{\lambda}\left(\frac{4A\Omega}{c}\right) = \frac{8\pi A}{\lambda c}\Omega = \left[\frac{4\pi RL}{\lambda c}\right]\Omega$$

  where, L, λ, R, Ω, and c are the fiber length, diode laser-wavelength, coil radius, inertial rotation rate and velocity of light, respectively.
- IFOG: A rate gyroscope unlike RLG, which is a rate-integrating gyro.

- IFOG avoids difficulties with RLG, lock-in and scale factor limitations
  - In IFOGs, the light source is normally placed outside the sensing cavity.
  - Gain medium does not amplify any effects from backscatter resulting in the lock-in phenomenon.
  - Scale factor can be raised by simply adding more turns of fiber and increasing the mean area enclosed by each.
- Signal losses in the fiber impose a practical limit to the fiber coil's length.
- FOG achieve optimum performance using light source dominated by spontaneous rather than stimulated emission.
- FOG can use cheaper and more rugged solid-state light sources, such as a superradiant diode.
- Phase shift in an IFOG is analogous to the frequency shift in an RLG.
- IFOG has advantage in terms of size, weight, power, packaging flexibility, and assembly.

- Requirements for navigation
  - Coordinate frames
  - Gyroscope
  - Accelerometers
  - Navigation equations
- Accelerometers provide measurements (component of acceleration in different directions) required for navigation.
- Can we directly measure the velocity or position of a vehicle?
- Velocity and position measurements require an external reference whilst the acceleration can be measured internally.
- Sensor types
  - Mechanical
  - Solid-state

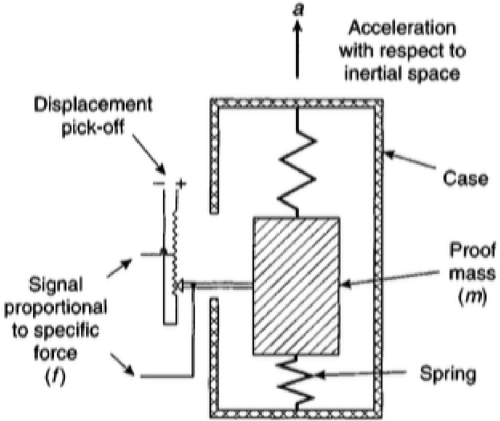
- Acceleration of a body of mass m w.r.t. inertial space, subjected to force F

$$F = ma$$

- Not practical to determine the acceleration of a vehicle by measuring the total force acting upon it.
- Possible to measure force acting on a proof mass contained within vehicle, which is constrained to move with vehicle.
- When instrument case is subjected to an acceleration along its sensitive axis, the proof mass tends to resist the change owing to its own inertia.
- Total force (F) acting on a proof mass in space

$$F = ma = mf + mg$$

where, f is acceleration produced by forces other than gravity and g is the gravitational acceleration.


a
Acceleration with respect to inertial space
Displacement pick-off
Case
Proof mass (m)
Signal proportional to specific force (f)
Spring

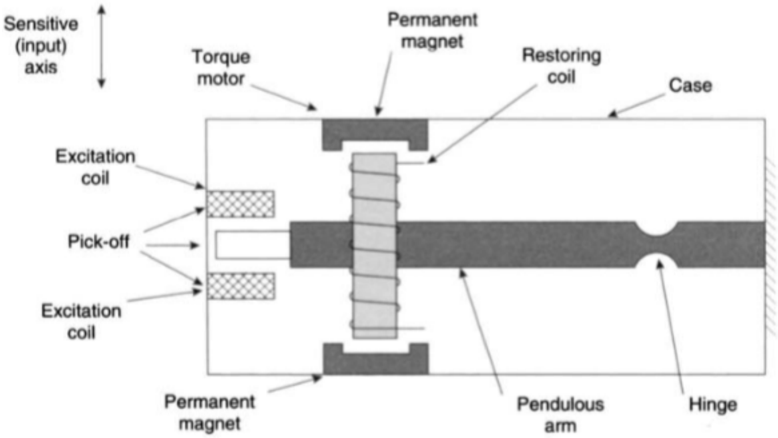
- An accelerometer is insensitive to the gravitational acceleration.
- It provides an output proportional to the non-gravitational force per unit mass to which the sensor is subjected along its sensitive axis.
- Consider an accelerometer falling freely within a gravitational field.
- Output of the instrument will remain at zero. (why?)
- Acceleration of the instrument w.r.t. an inertially fixed set of axes, $a = g$ and $f = 0$.
- If instrument is held stationary, $a = 0$, the accelerometer will measure the force acting to stop it from falling.

$$mf + mg = 0 \Rightarrow f = -g$$

- Knowledge of the gravitational field is essential to enable the measurement provided by the accelerometer to be related to the inertial acceleration.

- Types of accelerometers
  - ☐ Open loop
  - ☐ Closed loop
- A proof mass is suspended in a case and confined to a zero position by means of a spring.
- Damping may be applied to give mass and spring system a realistic response corresponding to a proper dynamic transfer function.
- With applied accelerations, the proof mass is deflected w.r.t. its zero or null position and the resultant spring force provides necessary acceleration of proof mass.
- For a single-axis sensor, the displacement of proof mass w.r.t. its null position $\propto$ the specific force applied along its input or sensitive axis.

- **Closed loop sensor**: Works on principle of nulling the displacement of pendulum.
- Null positions can be measured more accurately than displacements.
- With a closed loop accelerometer, the spring is replaced by an electromagnetic device that produces a force on the proof mass to maintain it at its null position.
- A pair of coils is mounted on the proof mass within a strong magnetic field.
- When a deflection is sensed, an electric current is passed through the coils in order to produce a force to return the proof mass to its null position.
- Magnitude of current in coils $\propto$ the specific force sensed along the input axis.
- Force-feedback type is far more accurate than the open-loop devices.



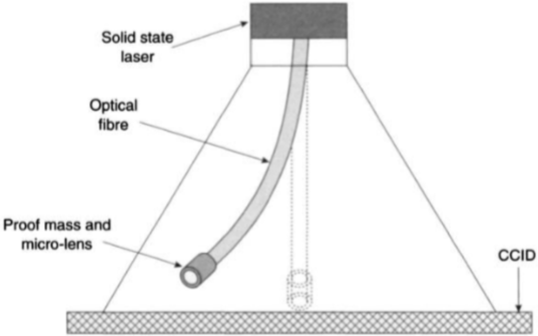
Sensitive (input) axis
Torque motor
Permanent magnet
Restoring coil
Case
Excitation coil
Pick-off
Excitation coil
Permanent magnet
Pendulous arm
Hinge

- Sensor components
  - Pendulum
  - Suspension mechanism or hinge element
  - Pick-off device
  - Force re-balance mechanism
- **Pick-off device**: optical, inductive or capacitive techniques
  - **Optical system**: A detector measuring the change in transmittance of a light beam through a slit in the pendulum.
  - **Inductive system**: Measurement of the differential current in coils fixed to the case interacting with a plate on the pendulum
  - **Capacitive system**: Movement of pendulum causes a change in capacitance between the faces of the pendulum and two electrodes in close proximity to the pendulum.
- Advantage of force re-balance mechanism?
- Greater accuracy, no bending stress for hinge

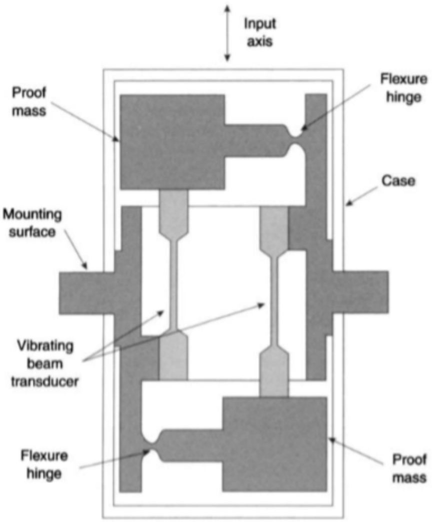
- Sensor errors
  - Fixed bias
  - Scale-factor errors
  - Cross-coupling errors
  - Vibro-pendulous error: When the sensor is subject to vibration along the sensitive and pendulum axes simultaneously.
- Broad categories of errors
  - Repeatability errors
  - Temperature dependent errors
  - Switch-on to switch-on variations
  - In-run errors
- Measured acceleration

$$\tilde{a}_x = (1 + S_x)a_x + M_y a_y + M_z a_z + B_f + B_v a_x a_y + n_x$$

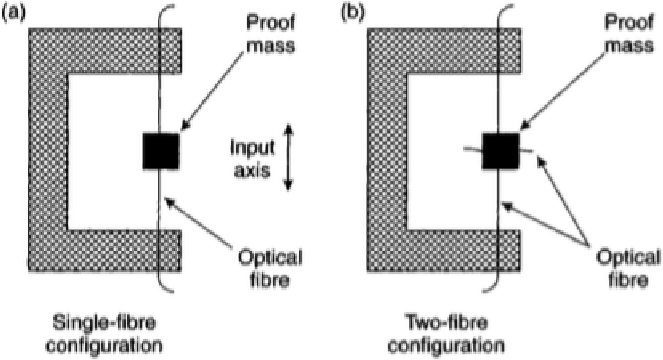
where, S_x,M_y, M_z,B_f,B_v and n_x denote scale-factor error, cross axis coupling factors, measurement bias, vibro-pendulous error coefficient and random bias, respectively.



- **Optical fibres**: excellent mechanical strength, elastic modulus characteristics, negligible thermal expansion
- Selection based on isoelastic properties
- Sense of displacement using laser light passing through optical fibre and 2D photo-sensitive array
- Provide accelerations in two dimensions
- Factors affecting range of sensor

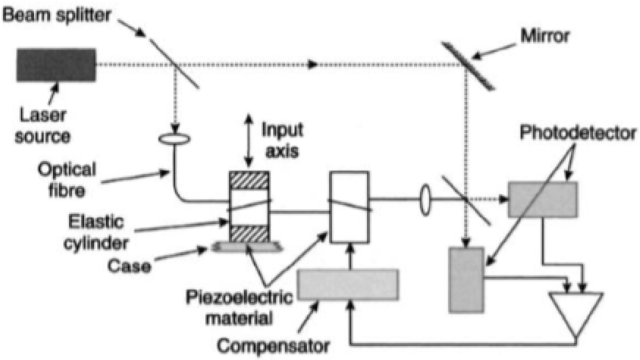


- Each beam vibrates at same resonant frequency with no acceleration.
- With acceleration along the sensitive axis, one beam experiences compression whilst the other is stretched
- Compression and tension result in decrease and increase in frequency, respectively
- Difference in frequency: Directly proportional to the applied acceleration
- Symmetrical arrangement: Cancellation of several errors that exist if only one beam is used

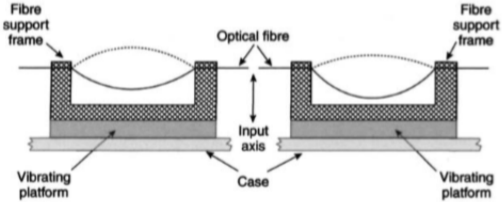


(a)
Proof mass
Input axis
Optical fibre
Single-fibre configuration
(b)
Proof mass
Optical fibre
Two-fibre configuration

- With applied acceleration along the axis of optical fibre, a small proportional change in length occurs.
- Change in length can be detected by interferometric techniques similar to FOG.
- Two optical fibres allows each fibre to form an arm of the interferometer.
- Nulling techniques enables greater sensitivity to be achieved, along with compensation for temperature changes in the fibres.
- It is necessary to constrain the proof mass to move only along the sensitive axis of the instrument.
- Sensitivity can be increased by using binding of coils $\propto$ number of coils
- Intensities of the two light beams in the interferometer are detected separately and compared in a differential amplifier.



- Output of the differential amplifier $\propto$ applied acceleration.
- Output signal can then be used to drive a piezoelectric device to null the phase change introduced by distortion of sensing element.



(a) No applied acceleration (symmetrical deflection)

(b) Applied acceleration (asymmetrical deflection)

- An optical fibre is fastened and tensioned between two pivot points in a rigid structure.
- This structure is vibrated so that the optical fibre oscillates at its fundamental frequency.
- With no acceleration, displacements are symmetrical and maximum stretch occurs at maximum displacement with relaxation as it passes the centre line.
- With applied acceleration, the displacement of the fibre will now be asymmetrical.

Reference

1. G. M. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*, Academic Press, Inc. 1993.
2. D. H. Titterton and J. L. Weston, *Strapdown Inertial Navigation Technology*, Progress in Astronautics and Aeronautics, Vol. 207, ed. 2, ch. 4.

Thank you for your attention !!!