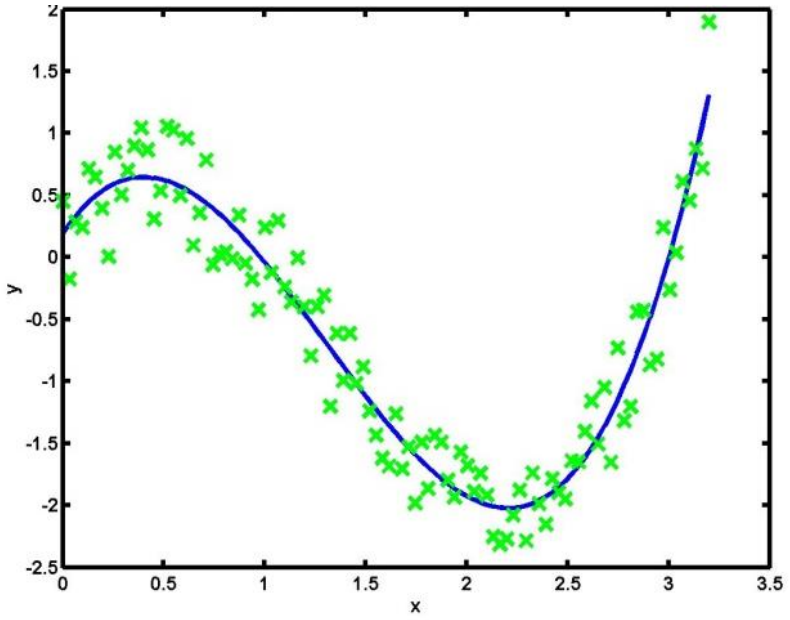
Noise Modeling of Sensors: The Allan Variance Method



Siddharth Tallur, 31/08/2020 EE617 Sensors in Instrumentation

Modifications to slides found here: https://eecs.wsu.edu/~taylorm/16_483/Jerath.pptx

You should be able to answer these questions…

PART I: MOTIVATION

- What is noise?
- What is noise modeling and why is it required?

PART II: BASICS

- How is noise characterized?
- How is noise in sensors quantified?

PART III: ALLAN VARIANCE ANALYSIS

- What is Allan Variance?
- How can it be used to specify sensor characteristics?

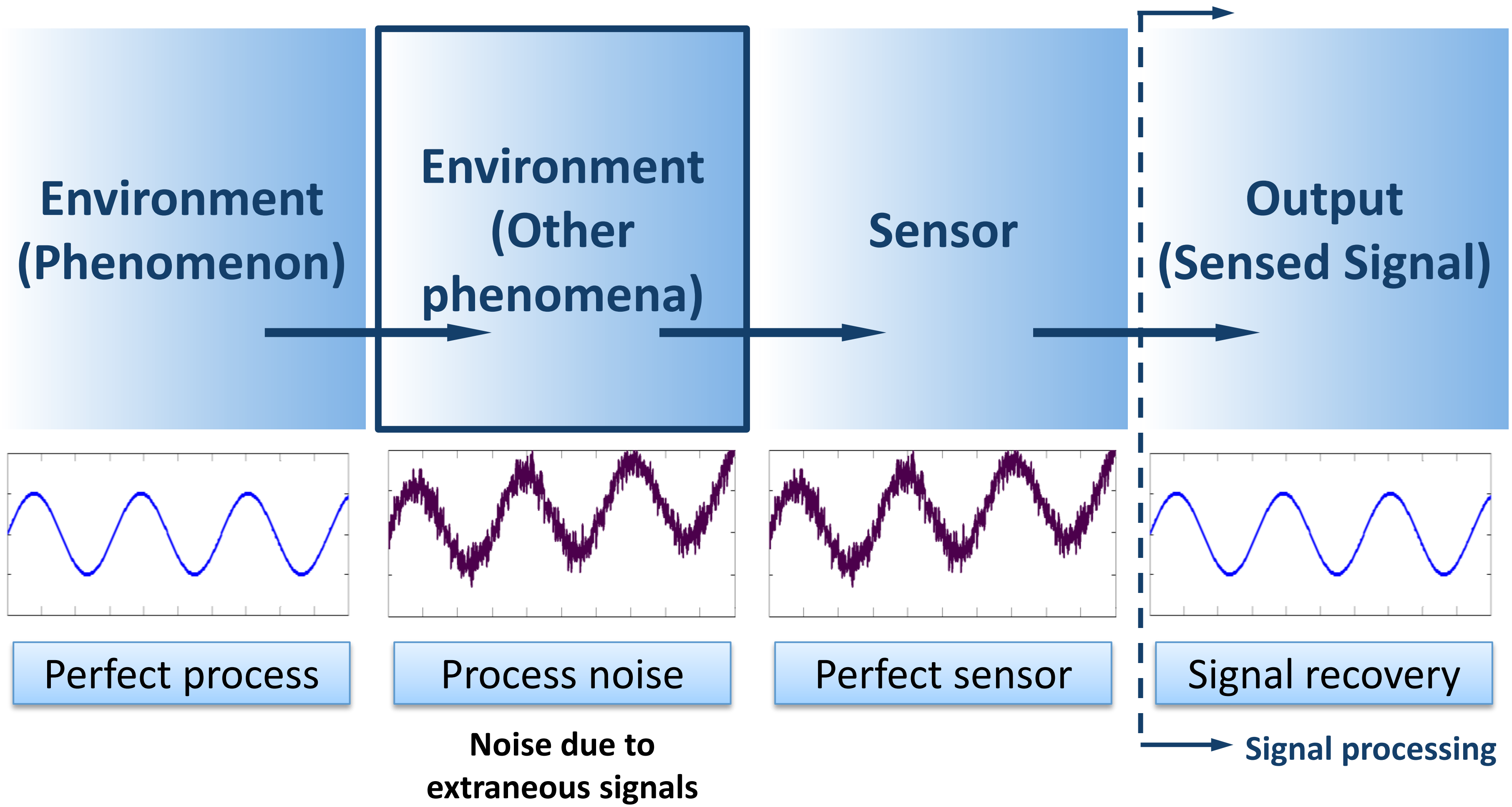
Noise models can be useful

Think of the sensing process

Environment (Phenomenon) → Environment (Other phenomena) → Sensor → Output (Sensed Signal)

Noise models are useful in signal filtering

- **Signal filtering or recovery**: E.g. Noise canceling hardware may utilize knowledge of noise models for signal filtering

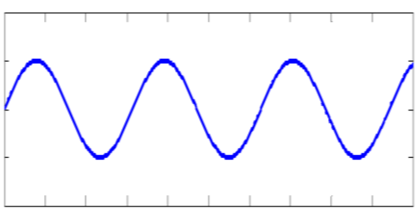


Noise models are useful in sensor selection

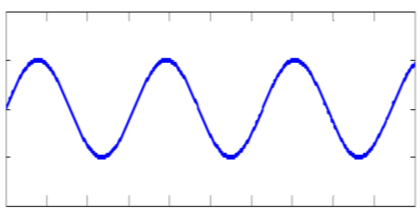
Sensor design, selection and performance mapping:

Identifying performance of various “virtual” sensors for a given sensing task requires knowledge of noise models

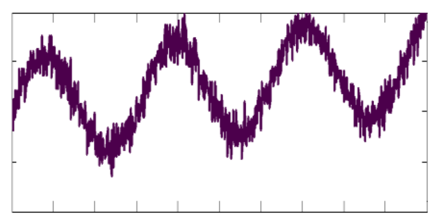
Environment (Phenomenon) → **Environment (Other phenomena)** → **Sensor 1 / Sensor 2 / Sensor 3 / Sensor N** → **Output (Sensed Signal)**



Perfect process



Perfect environs



‘Virtual’ sensors

Varying degrees of measurement noise

Performance metric

You should be able to answer these questions…

PART I: MOTIVATION

- What is noise?
- What is noise modeling and why is it required?

PART II: BASICS

- How is noise characterized?
- How is noise in sensors quantified?

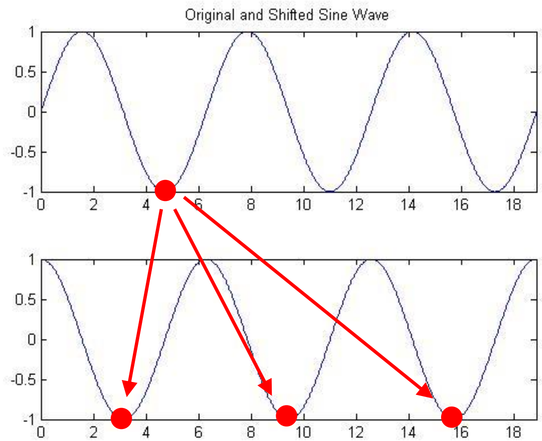
PART III: ALLAN VARIANCE ANALYSIS

- What is Allan Variance?
- How can it be used to specify sensor characteristics?

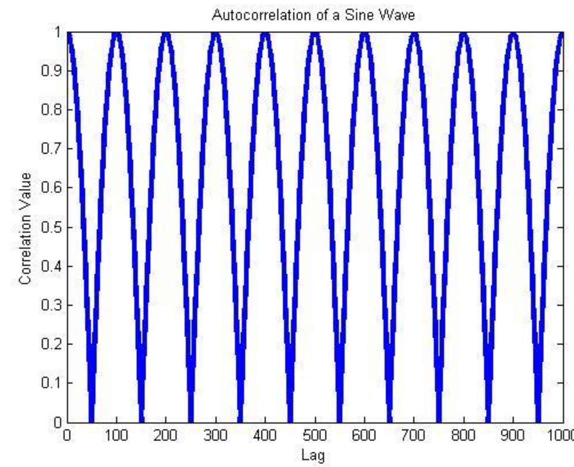
Question…

Can you think of some tools used to ***characterize noise***?

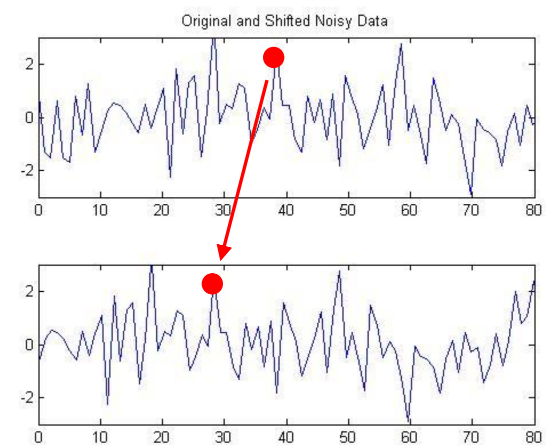
What is autocorrelation?



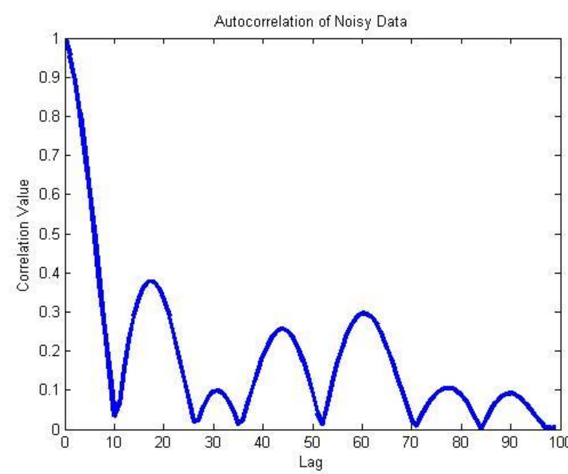
Infinite Perfect Correlations



As you change the data's phase, how probable is correlation with the original data?



Only 1 Perfect Correlation



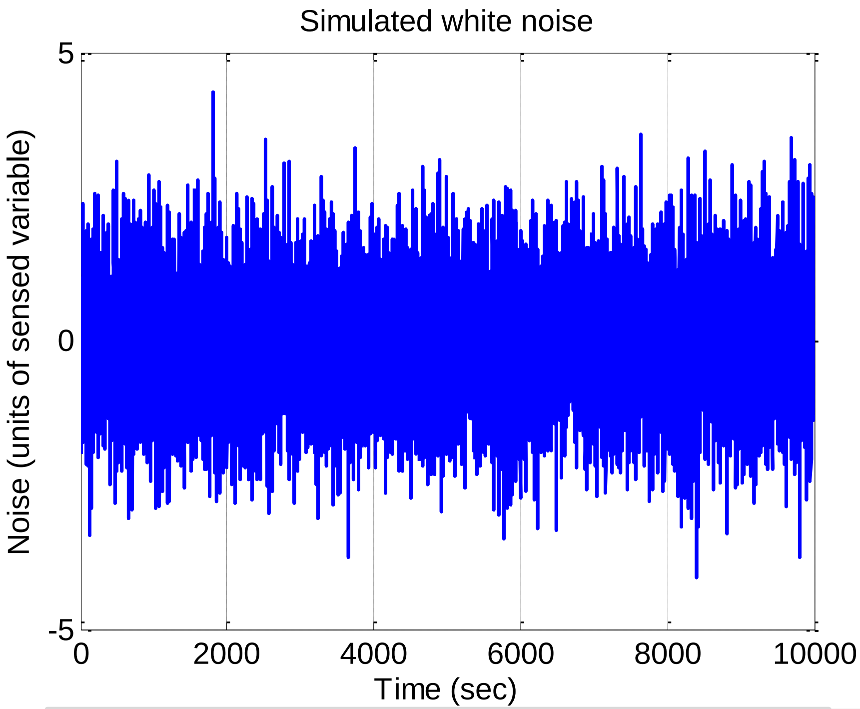
Noise characterization tools: Autocorrelation

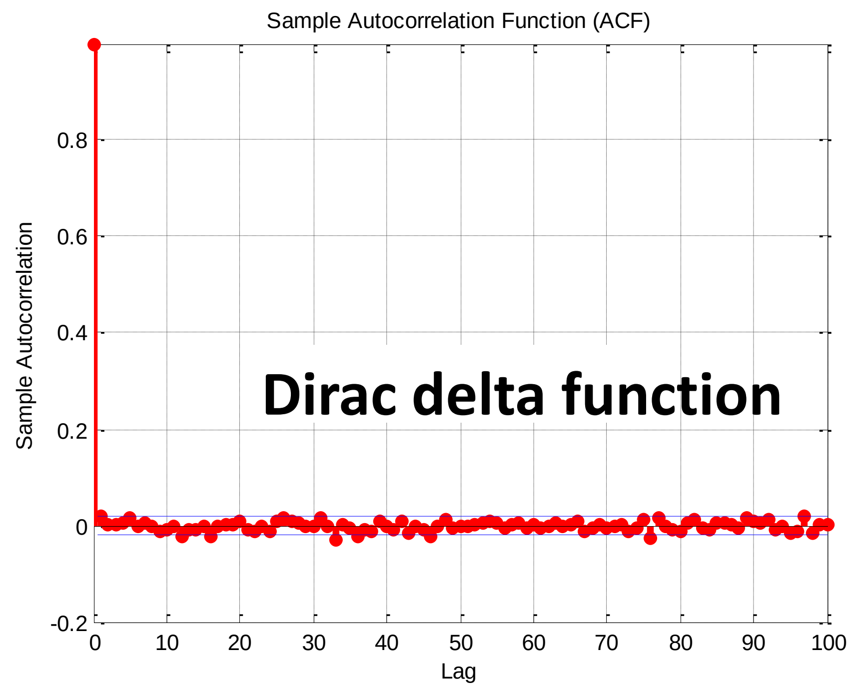
- Time-domain representation
- Let $X(t)$ be a stochastic variable:

$$R_{XX}(t,\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(\tau)\bar{f}(t-\tau)d\tau$$

Noise characterization tools: Autocorrelation

- If $X(t)$ represents white noise, $R_{XX}(t_1, t_2) = \delta(t_1 - t_2)$





MATLAB code

```
%% White noise autocorrelation
clear all
clc
close all

N = 10000;
w = randn(N,1);
t = linspace(0,N,N);

[c,lags] = xcorr(w,w);
plot(lags,abs(c./max(c)),'Linewidth',2);
```

Autocorrelation of colored noise

212 PROCEEDINGS OF THE IEEE, VOL. 70, NO. 3, MARCH 1982

1/f Noise

MARVIN S. KESHNER

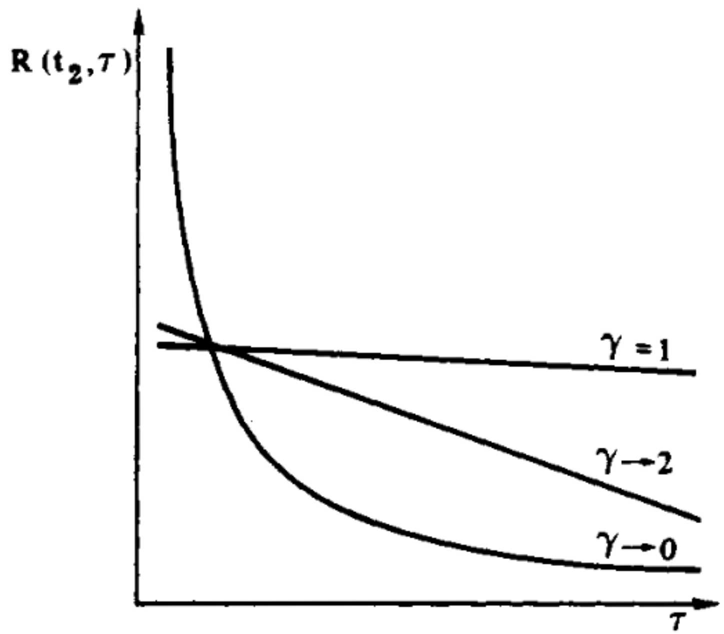


Fig. 6. Autocorrelation functions for (1/f) ** γ noise. For γ = 1, the recent and the distant past have almost equal correlation with the present.

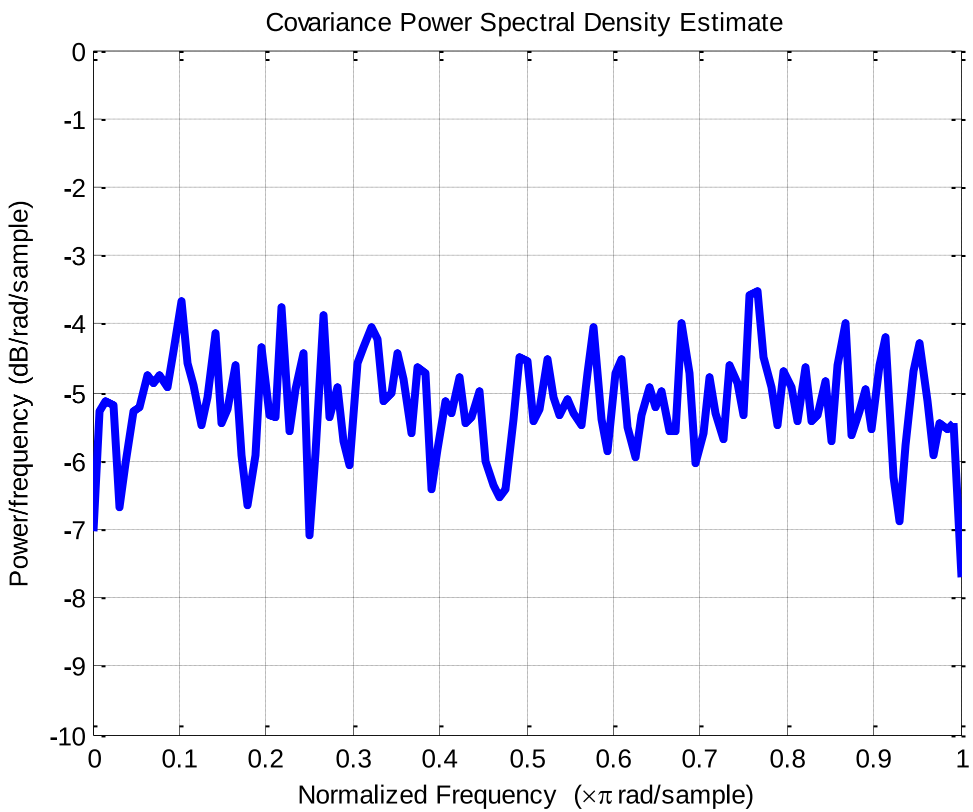
Noise characterization tools: PSD

- Power Spectral Density (PSD) is frequency-domain representation of noise
- PSD is the Fourier Transform of the autocorrelation function

$$S_X(f) = \mathcal{F}(R_{XX}(\tau)) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-2\pi i f \tau} d\tau$$

Noise characterization tools: PSD

- If $X(t)$ represents white noise, $S_X(f) = N_0$



MATLAB code

```
%% Power Spectral density

N = 10000;
w = randn(N,1);

n = 256;
W = fft(w,n);
% Method 1
Pww = W.*conj(W)/n;
% Method 2
Pww_welch = pwelch(w);
% Method 3
Pww_cov = pcov(w,3);
% See Mathworks help for more details
```

Sensor characterization: Check the datasheet!

ANALOG DEVICES

Linear Output Magnetic Field Sensor

AD22151

OUTPUT NOISE FIGURE (6 kHz BW)	2.4	mV/rms

NOISE

The principal noise component in the sensor is thermal noise from the Hall cell. Clock feedthrough into the output signal is largely suppressed with application of a supply bypass capacitor.

Figure 12 shows the power spectral density (PSD) of the output signal for a gain of 5 mV/Gauss. The effective bandwidth of the sensor is approximately 5.7 kHz, as shown in Figure 13. The PSD indicates an rms noise voltage of 2.8 mV within the 3 dB bandwidth of the sensor. A wideband measurement of 250 MHz indicates 3.2 mV rms (see Figure 14a).

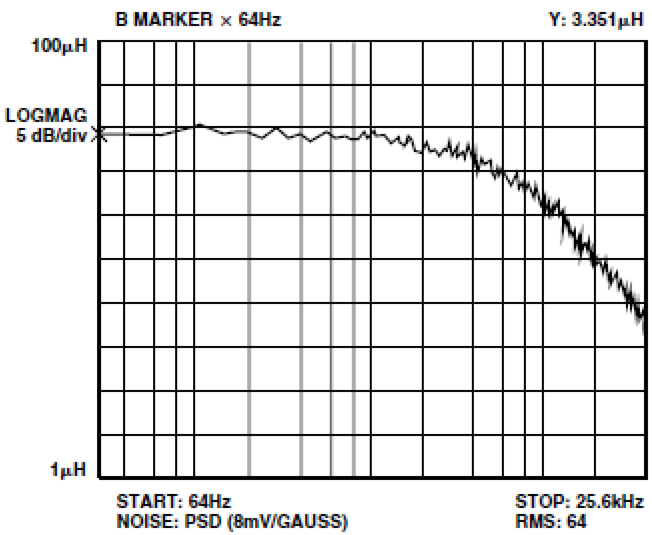


Figure 12. Power Spectral Density (5 mV/G)

ANALOG DEVICES

Four Degrees of Freedom Inertial Sensor

ADIS16300

Parameter	Min	Typ	Max	Unit
GYROSCOPE				
Dynamic Range	±300	±375		°/sec
Initial Sensitivity	0.0495	0.05	0.0505	°/sec/LSB
		0.025		°/sec/LSB
		0.0125		°/sec/LSB
Sensitivity Temperature Coefficient		400		ppm/°C
Misalignment		0.1		Degrees
		±0.5		Degrees
Nonlinearity		0.1		% of FS
Initial Bias Error		±3		°/sec
In-Run Bias Stability		0.007		°/sec
Angular Random Walk		1.9		°/√hr
Bias Temperature Coefficient		0.1		°/sec/°C
ACCELEROMETERS				
Dynamic Range	±3	±3.6		g
Initial Sensitivity	0.594	0.6	0.606	mg/LSB
Sensitivity Temperature Coefficient		250		ppm/°C
		300		ppm/°C
Misalignment		±0.25		Degrees
		±0.5		Degrees
Nonlinearity		±0.3		% of FS
Initial Bias Error		±60		mg
		±110		mg
In-Run Bias Stability		0.048		mg
		0.054		mg
Velocity Random Walk		0.118		m/sec/√hr
		0.164		m/sec/√hr
Bias Temperature Coefficient		2.5		mg/°C
		4.5		mg/°C

You should be able to answer these questions…

PART I: MOTIVATION

- What is noise?
- What is noise modeling and why is it required?

PART II: BASICS

- How is noise characterized?
- How is noise in sensors quantified?

PART III: ALLAN VARIANCE ANALYSIS

- What is Allan Variance?
- How can it be used to specify sensor characteristics?

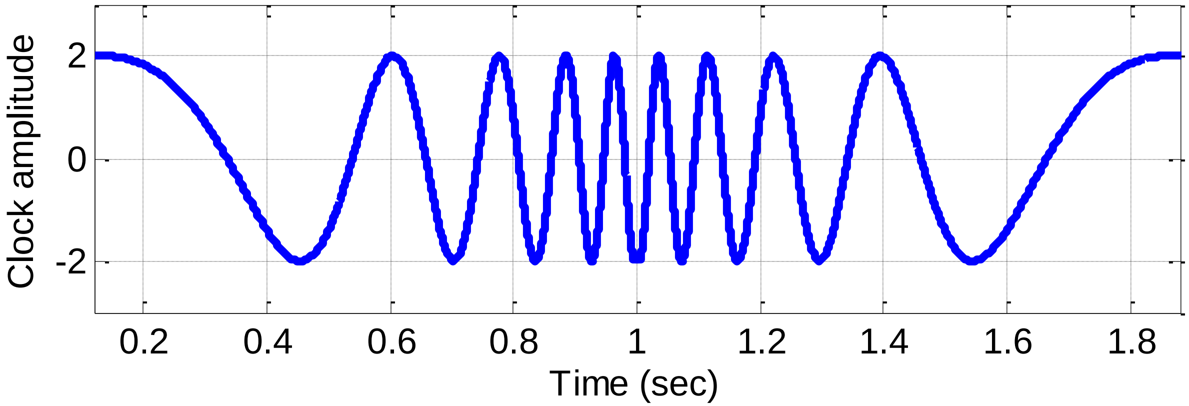
Make an educated guess…

Which of the following technologies in the 1950s ***necessitated*** the development of a new measure of variance, i.e. the Allan variance?

a) Solar-powered batteries
b) Precision atomic clocks
c) Leak-free ball point pens
d) Flexible optical fibers

Origins: Frequency stability of atomic clocks

- 1950s-1960s: Development of precise atomic clocks – issues pertaining to frequency stability arise



- Preliminary work: D W Allan, "*Statistics of Atomic Frequency Standards*", Proceedings of the IEEE, 1966

Allan Variance

- Allan variance is defined as one half of the **time average** of the squares of the differences between successive readings of the **frequency deviation** sampled over the sampling period.

$$\sigma^2(\tau) = \frac{1}{2}\langle(\overline{\Omega}_{\mathrm{k+m}} - \overline{\Omega}_k)^2\rangle$$

$$\sigma^2(\tau) = 4\int_0^\infty S_X(f)\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}df$$

NOTE:

- Allan variance analysis is **always performed for zero input** to the sensor. In this situation, any sensor output is due to noise arising from the sensor.
- See next slide to understand variables in the equation

How is Allan Variance computed?

Partition the data such that each section has 'm' samples

Compute the mean of data points in each partition

Evaluate difference of the means of adjacent sections

Square the value each difference

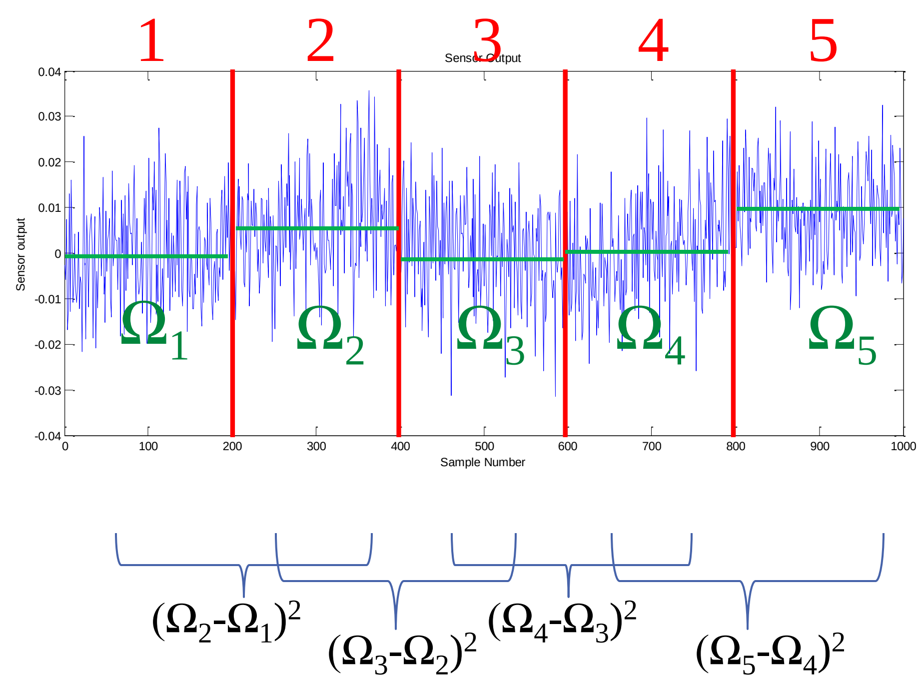
Compute the average of these squared values

Allan Variance: $\sigma^2(m)$ = ½(Average Value)

Repeat for different values of m

Allan Variance ($\sigma^2(m)$) is a function of a parameter m.

Ω_k … Compute the … For example … m = 200 … k^{th} partition … $\Omega_k - \Omega_{k-1}$

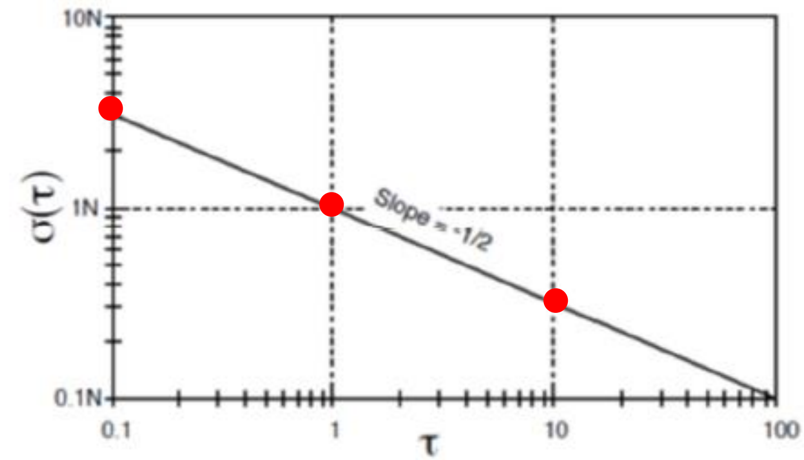


$<(\Omega_{i+1}-\Omega_i)^2> = [(\Omega_2-\Omega_1)^2 + (\Omega_2-\Omega_1)^2 + (\Omega_2-\Omega_1)^2 + (\Omega_2-\Omega_1)^2] /4$

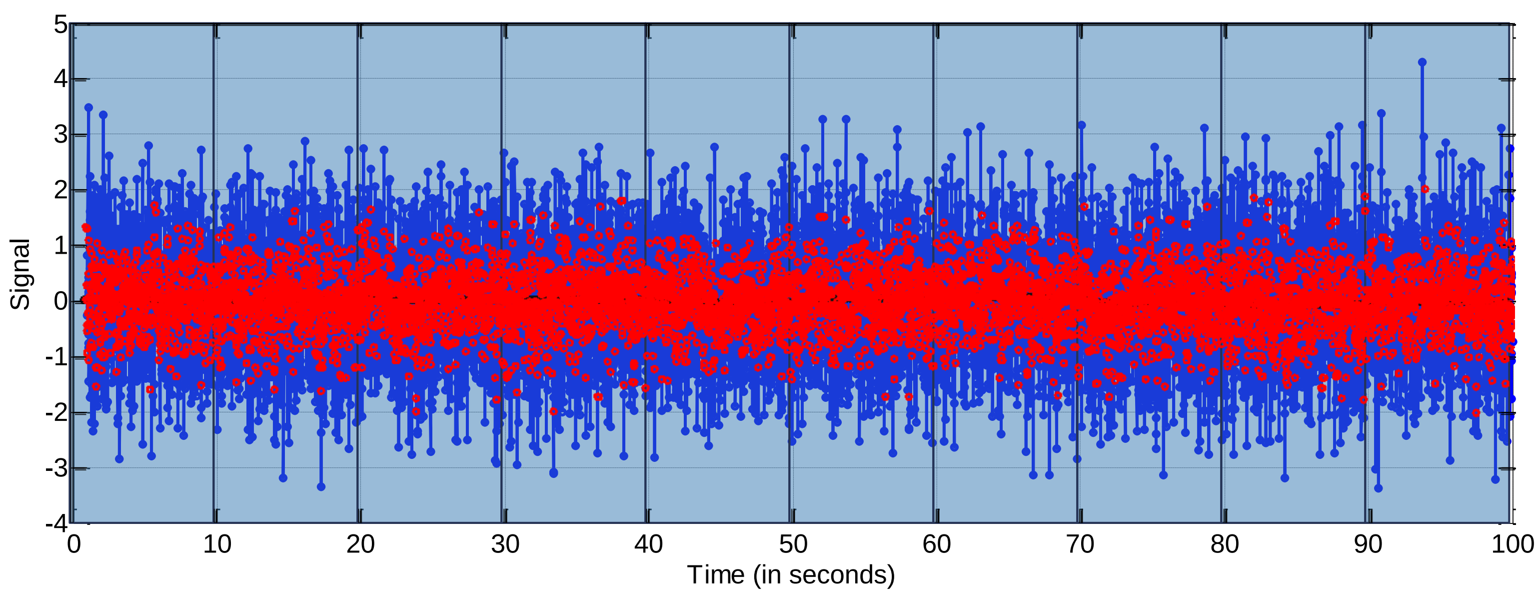
Also, known as the Expected value.

Allan Variance – Angle random walk

$$\sigma^2(\tau) = \frac{1}{2}\langle(\overline{\Omega}_{\mathrm{k+m}} - \overline{\Omega}_k)^2\rangle$$



Simulated white noise

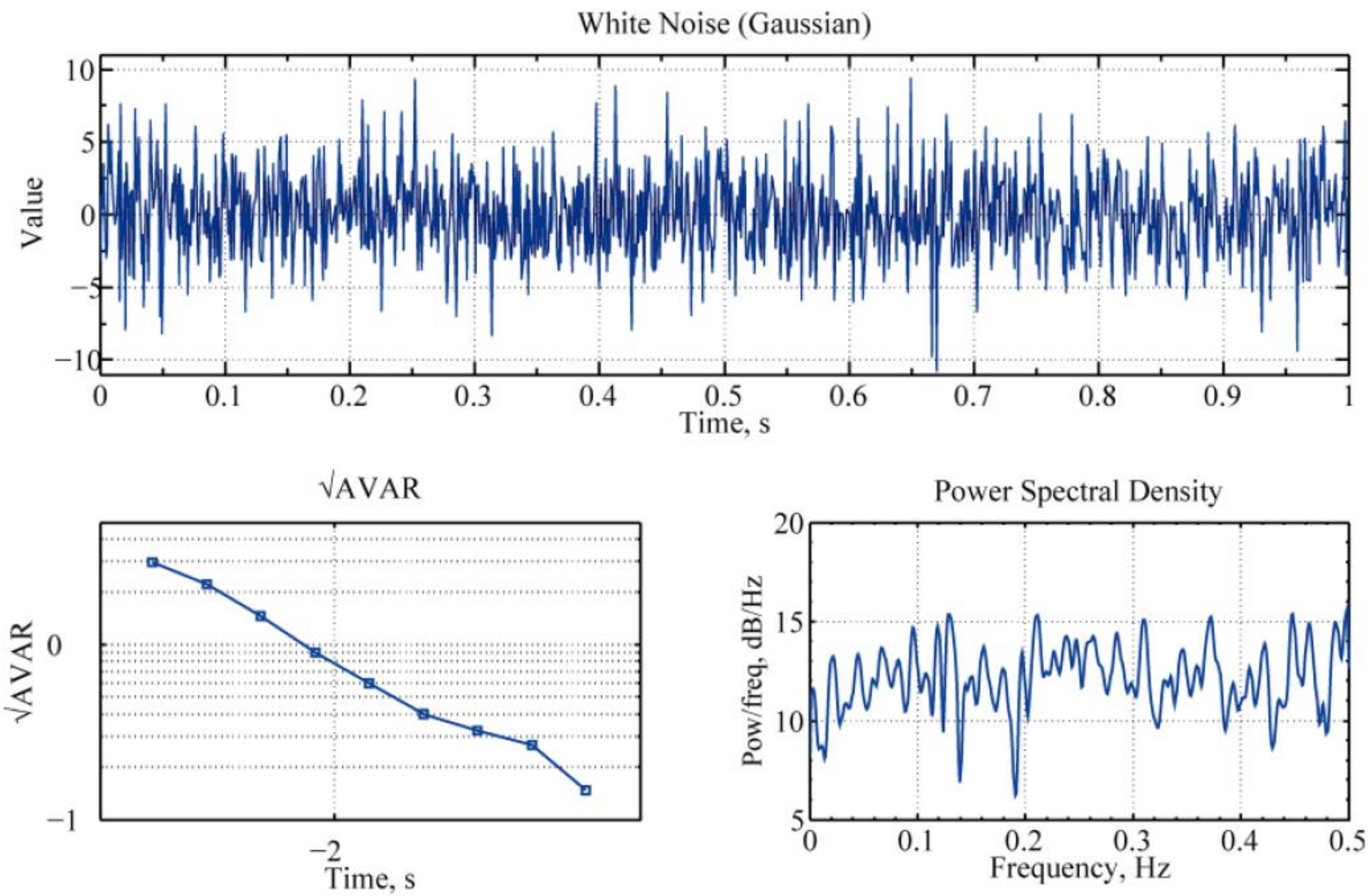


Correlation time = 0.02 seconds

Allan Variance – Noise sources

PSD(f) ∝ f $^{\alpha}$ is equivalent to, σ(τ) ∝ τ $^{\beta}$,

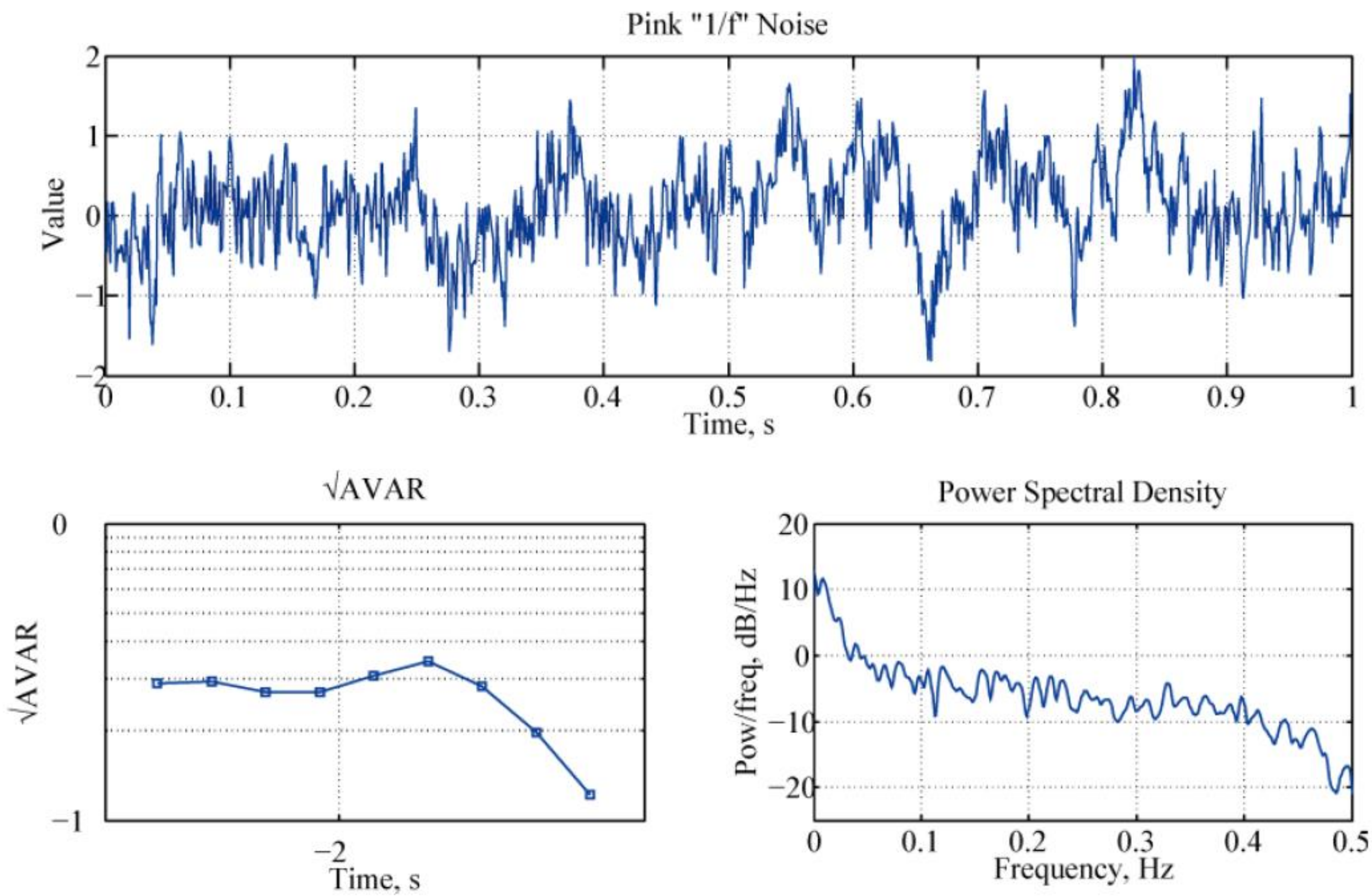
where β = -(α+1)/2 and f=1/τ.



Allan Variance – Noise sources

$PSD(f) \propto f^{\alpha}$ is equivalent to, $\sigma(\tau) \propto \tau^{\beta}$,

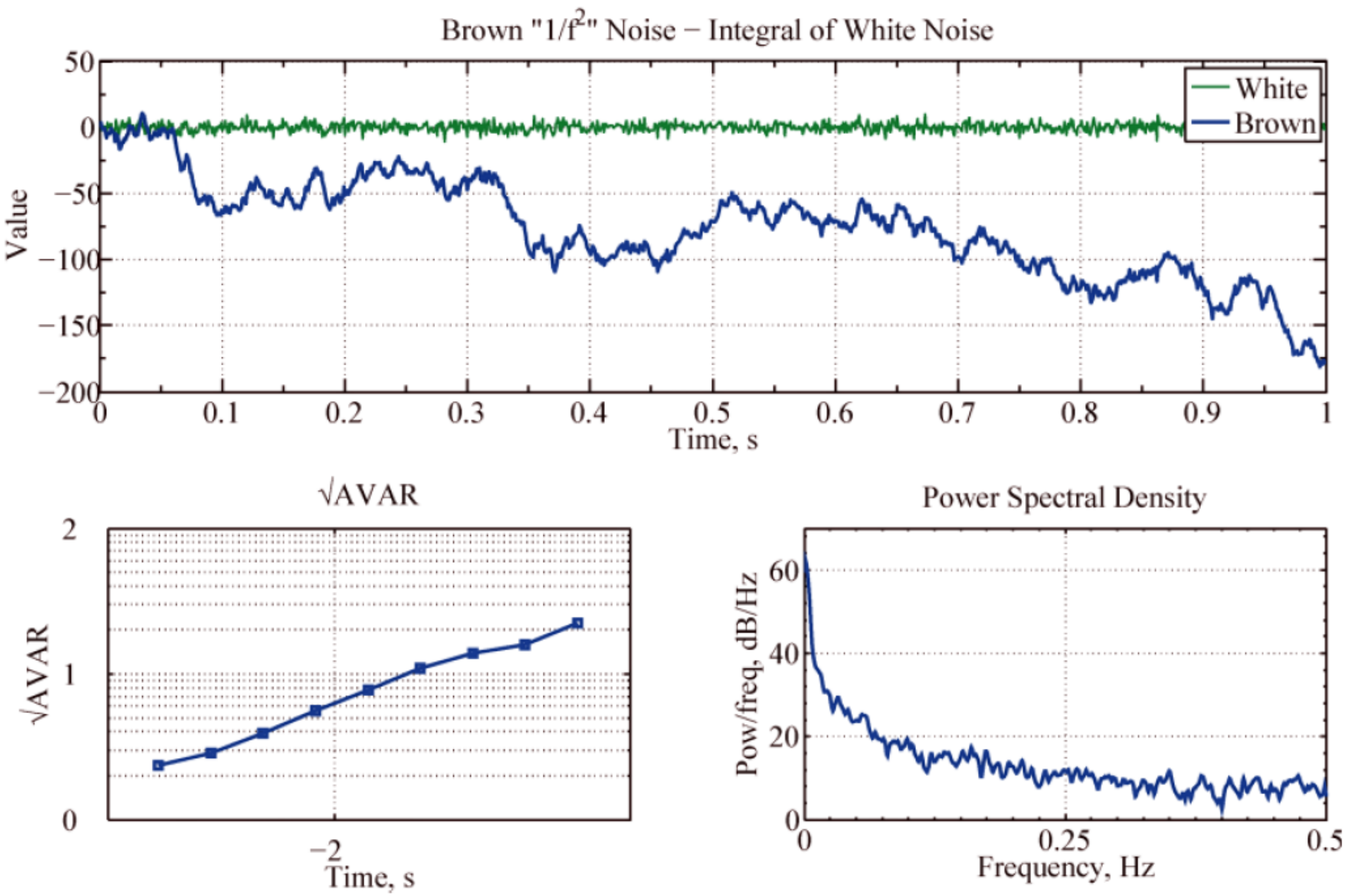
where $\beta = -(\alpha+1)/2$ and $f=1/\tau$.



Allan Variance – Noise sources

PSD(f) $\propto$ ***f*** $^{\alpha}$ ***is equivalent to, σ(τ)*** $\propto$ ***τ*** $^{\beta}$***,***

where β = -(α+1)/2 and f=1/τ.



Allan Variance – Noise sources

Spectral type	Example sources	PSD(*f*) power law f^{α}	$\sigma(\tau)$ power law τ^{β}	Averaging over time	Associated gyroscope parameters
White	Johnson-Nyquist thermal noise	f^{0}	$\tau^{-1/2}$	Good	Rate resolution in deg/s/√Hz, deg/hour/√Hz Angle Random Walk (ARW) in deg/√hour
Pink	Electronics flicker	f^{-1}	τ^{0}	Neutral	Flicker, Bias Instability in deg/s, deg/hour
Red (Brown)	White noise accumulation	f^{-2}	$\tau^{+1/2}$	Bad	Angle Rate Random Walk (ARRW) in deg/s*√Hz, deg/hour*√Hz, deg*√hour

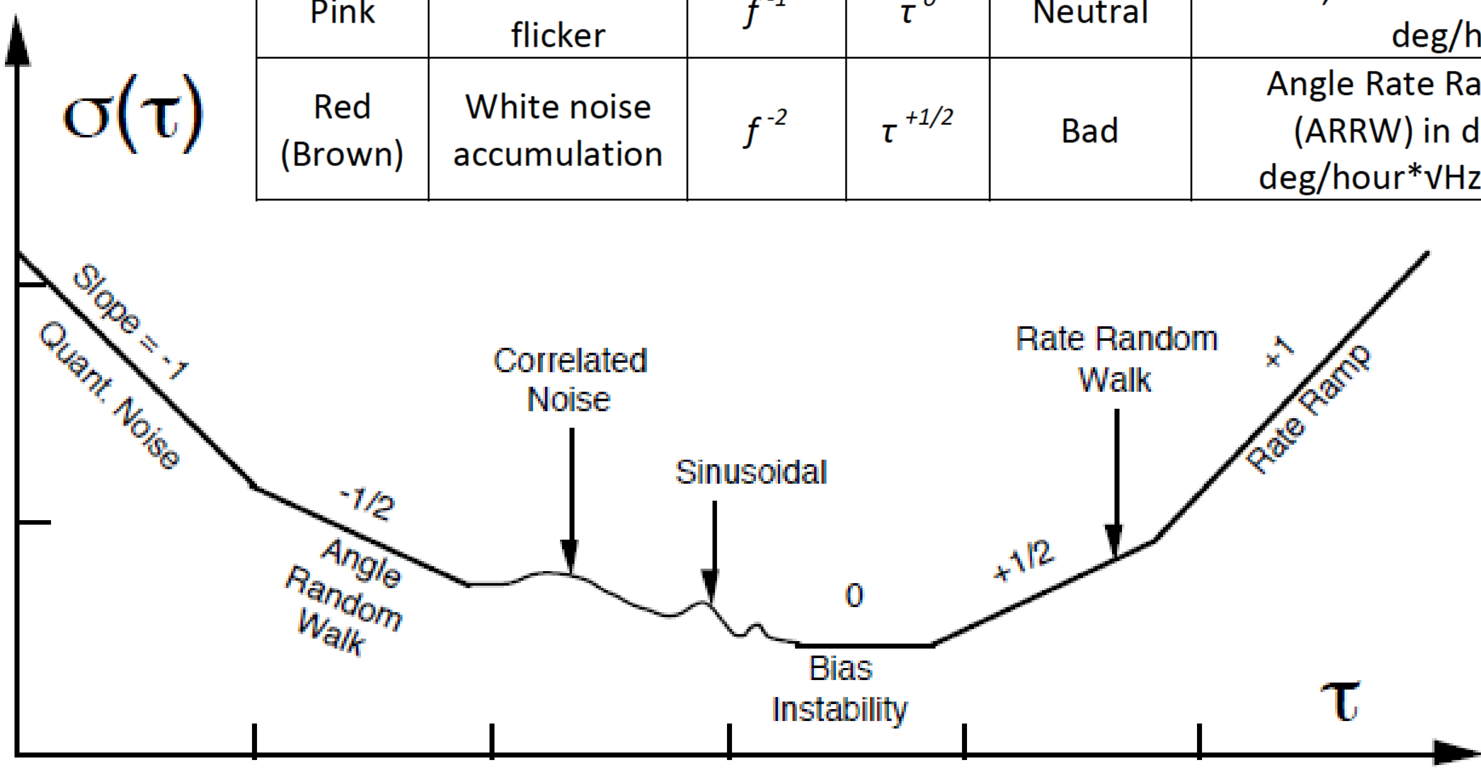


Figure C.8 — σ(τ) Sample plot of Allan variance analysis results