# Aided Inertial Navigation System

Dr. Shashi Ranjan Kumar

Assistant Professor
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Powai, Mumbai, 400076 India

- INS: vertical velocity and position using integration of vertical channel accelerometer output.
- Inherently divergent: correction using an externally measured altitude reference to stabilize the INS vertical channel.
- Vertical channel: damped with measurements of altitude using Central Air Data Computer (CADC) outputs.
- Barometric and inertial altitude systems are complementary to each other.
- Barometric system provides good altitude rate in nearly level flight.
- INS needs to be bounded by an external reference for sustained periods, but provides direct information about vertical acceleration and a good short-term reference for use during climbing or diving in high-performance aircraft.
- To avoid vertical channel instability, INS vertical channel is frequently not implemented.
- In systems for high-performance aircraft, vertical channel is usually slaved to barometric height data.

- Consider a simple vertical accelerometer with its input axis along $Z$ axis.
- Dynamics of altitude $Z$ is given by

$$\ddot{Z} = A - g$$

- From Newton's law of gravitation

$$g = g_0 \left( \frac{Z_0}{Z} \right)^2$$

where $Z_0$ is an arbitrary initial point and $g_0$ is the gravity at $Z_0$.

- If $Z = Z_0 + h$ then

$$g = g_0 \left( 1 + \frac{h}{Z_0} \right)^{-2} = g_0 \left( 1 - \frac{2h}{Z_0} \right) = g_0 - \frac{2g_0}{Z_0} h$$

- Dynamics of $h$

$$\ddot{h} = A - g_0 + \frac{2g_0}{Z_0} h \Rightarrow \boxed{\ddot{h} - \frac{2g_0}{Z_0} h = A - g_0}$$

- If there is an error $\Delta A$ in $A$, then the dynamics of error in $h$

$$\Delta\ddot{h} - \frac{2g_0}{Z_0}\Delta h = \Delta A, \text{ What about stability?}$$

- This dynamics has a pole in the RH plane giving rise to the instability.
- General solution of this equation

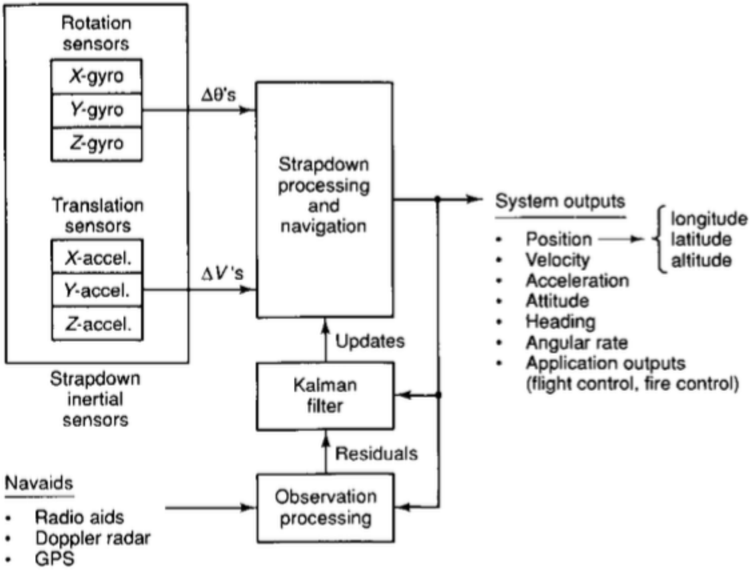
$$\Delta h = A\cosh\sqrt{\frac{2g_0}{Z_0}}t + B\sinh\sqrt{\frac{2g_0}{Z_0}}t - \frac{\Delta A}{2g_0/Z_0}$$

- If $\Delta h(0) = \Delta\dot{h}(0)$ then

$$\Delta h = \frac{Z_0}{2}\frac{\Delta A}{g_0}\left[\cosh\sqrt{\frac{2g_0}{Z_0}}t - 1\right]$$

- Pure INS can be used for measurement of altitude only for short period.
- Requirement of aided inertial navigation system

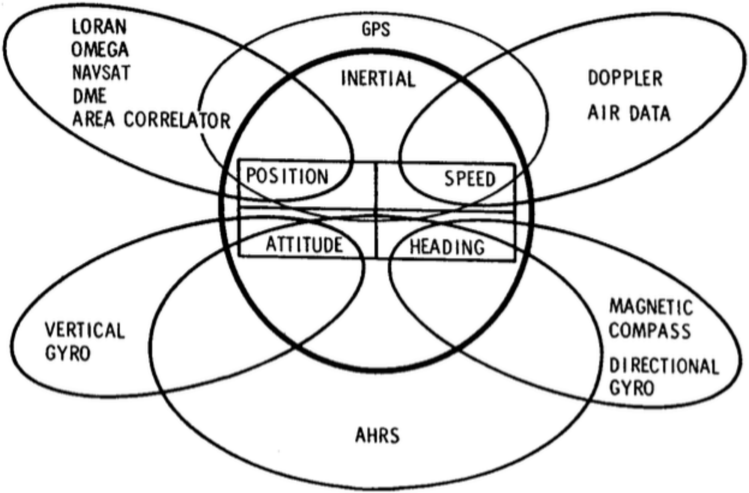
- Navigation-aided (Navaid) systems use one or more different sources of navigation information integrated into a single system.
- Kalman filter integration of the inertial and noninertial navigation sensor information provides a method of utilizing these updates in optimal manner.
- Motivation of using integrated system: size, weight, and power savings.
- Navaid systems:
  - Integration of various sensors
  - Military applications
  - Perform even in hostile terrain, jamming environment
  - Capability to operate when some or all ground/satellite navigation systems are disabled.
  - Automatic navigation function

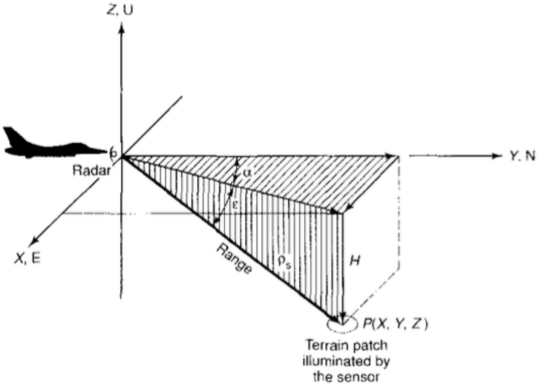


Rotation sensors
X-gyro
Y-gyro
Z-gyro
Translation sensors
X-accel.
Y-accel.
Z-accel.
Strapdown inertial sensors
Δθ's
ΔV's
Strapdown processing and navigation
Updates
Kalman filter
Residuals
Observation processing
Navaids
• Radio aids
• Doppler radar
• GPS
System outputs
• Position
• Velocity
• Acceleration
• Attitude
• Heading
• Angular rate
• Application outputs (flight control, fire control)
longitude
latitude
altitude

# Aided Inertial Navigation System







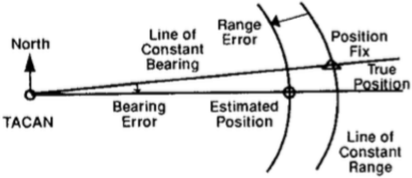

- Ground mapping radar equation

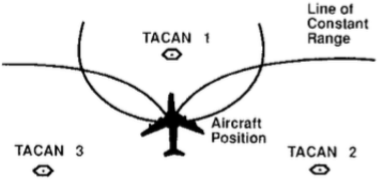
$$\boldsymbol{\rho}_s = -H\mathbf{1}_U + \sqrt{R_s^2 - H^2} \times [\sin\alpha\mathbf{1}_E + \cos\alpha\mathbf{1}_N]$$

  where, $\boldsymbol{\rho}_s$ is slant range from aircraft to ground checkpoint and $R_s$ is the slant range distance, $H$ is measured baroinertial altitude and $\alpha$ is measured azimuth angle.

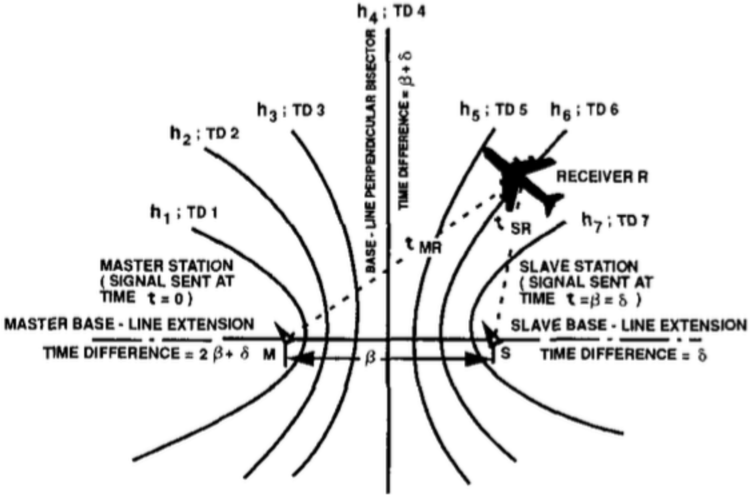
- Range of 92.6 km (50 NM)

- TACtical Air Navigation (TACAN)
  - By transmitting pulses to a ground station and timing the delay before receiving response pulses, the system measures the distance between aircraft and beacon.
  - System also displays bearing from the beacon to the aircraft based on variations in the beacon's rotating antenna pattern.



- LOng RAnge xNavigation (LORAN)
  - Low-frequency, pulsed, radio-navigation position-fixing system
  - Hyperbolic navigation
- Hyperbolic navigation: Based on time-difference in time of arrival of signals from the various transmitters
- Amount of time taken by a radio pulse to travel a certain distance is a measure of the distance.
- Easy to measure time difference with clock rather than distance
- Hyperbolic navigation is more common as compared to circular navigation.
- In free space, the time difference between reception of signal from separate stations at the vehicle is proportional to the difference of distance traveled.
- What does constant time-difference mean here?



h4 ; TD 4
BASE - LINE PERPENDICULAR BISECTOR
TIME DIFFERENCE = β + δ
h3 ; TD 3
h2 ; TD 2
h5 ; TD 5
h6 ; TD 6
RECEIVER R
h1 ; TD 1
t SR
h7 ; TD 7
t MR
MASTER STATION
( SIGNAL SENT AT
TIME t = 0 )
SLAVE STATION
( SIGNAL SENT AT
TIME t = β = δ )
MASTER BASE - LINE EXTENSION
SLAVE BASE - LINE EXTENSION
TIME DIFFERENCE = 2 β + δ
M
β
S
TIME DIFFERENCE = δ

- Model for time difference

$$\Delta T = \frac{\rho_S - \rho_M}{c} + \epsilon$$

where, $\epsilon$ is error in emission delay between master and slave stations, and $\rho_S, \rho_M$ are the distances travelled by signal.

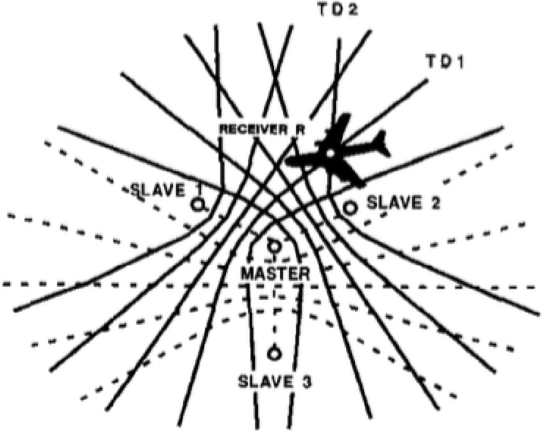
- In a hyperbolic navigation system, an LOP represents a constant range difference from two transmitters.
- Time difference may also be given by

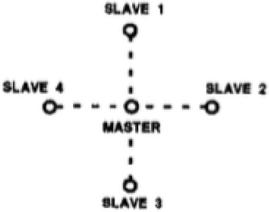
$$TD = (\beta + \delta) + t_{SR} - t_{MR},$$

where, $t_{SR}$ and $t_{MR}$ are times required for signal to travel between $S$ and $M$ to $R$, $\beta$ the time required to travel from $M$ to $S$, and $\delta$ is the time after which slave transmit its own signal.

- What would the range of TD? $\delta \leq TD \leq 2\beta + \delta$

- Two transmitters are not enough.
- At least three transmitters are necessary to get a position fix.
- Third transmitting station must be coupled with the master transmitter to generate the second set of LOPs.
- Exact location based on the intersection of the two different hyperbolic LOPs.
- What about coverage area?
- Coverage area: Depends on geometry of chain
  - Triad: one master and two slaves
  - Wye: one master and three slaves
  - Star: one master and four slaves
- Two time-difference measurements using either a triad or star transmitting station configuration provides two LOPs, and their intersection gives a position fix for the receiver.

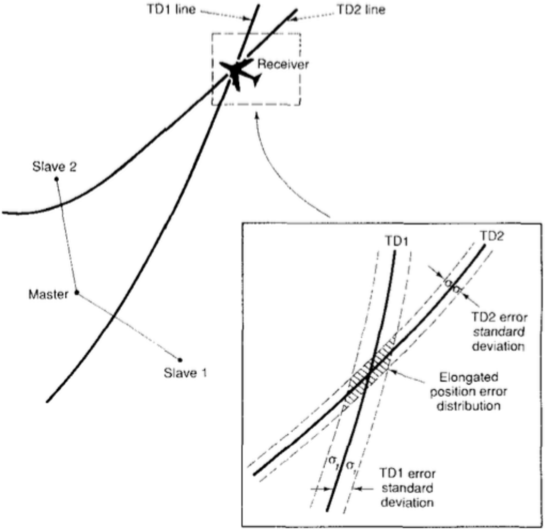


Star Configuration

# Aided Inertial Navigation System

## Long Range Navigation



- Accuracy of position solution: vehicle's position relative to the transmitters
- Geometric effect:
  - Crossing angle of the hyperbolic LOPs
  - Proximity to the baseline extension
- In regions sufficiently far from baseline extensions, Rms position error $\propto$ Rms time-difference measurement error.
- Near baseline extension, the position error does not vary linearly with the measurement errors.
- Types of accuracy
  - **Predictable accuracy**: Accuracy of position w.r.t. geographic or geodetic coordinates of the earth.
  - **Repeatable accuracy**: Accuracy with which a user can return, again and again, to a position whose coordinates have been measured at a previous time and with the same navigation system.
  - **Relative accuracy**: Accuracy with which a user can measure position relative to another user or to some reference point.

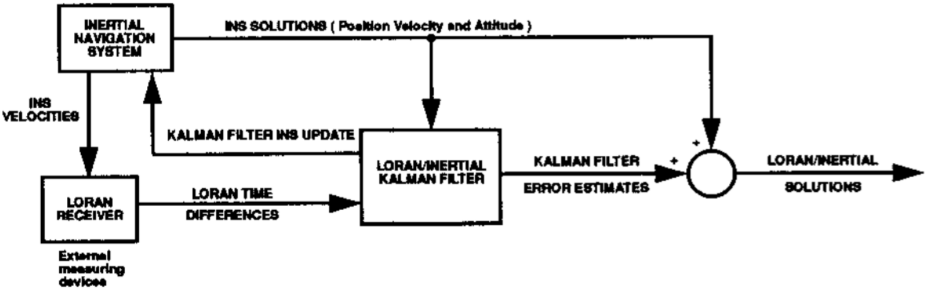



Figure: Integrated LORAN/INS

- A long-range (14816 km or 8000 NM), worldwide, radio navigation system
- Established by Prof. I. A Pierce in the late 1940
- A network of eight transmitter stations worldwide which transmit signal at precise time and exact intervals.
- At least three of the Omega transmitter signals can be received any place on Earth.
- A hyperbolic very-low-frequency navigation system using phase difference of continuous-wave radio signal (10-14 KHz)
- As signals are propagated within the waveguide formed by the earth and ionosphere, changes in propagation velocity of the signal occur as a result of changes in the ionosphere or differences on the earth's surface such as mountains, oceans, and plains.

- Phase angles for a pair of transmitting stations

$$\phi_1 = \phi_{10} + 2\pi f \left(\frac{\rho_1}{c}\right), \quad \phi_2 = \phi_{20} + 2\pi f \left(\frac{\rho_2}{c}\right)$$

  where, $f$ is frequency of transmitted signal, $\rho_1, \rho_2$ are distances from transmitters to receiver, $\phi_{10}, \phi_{20}$ are initial phases of transmitters, respectively.

- Phase difference

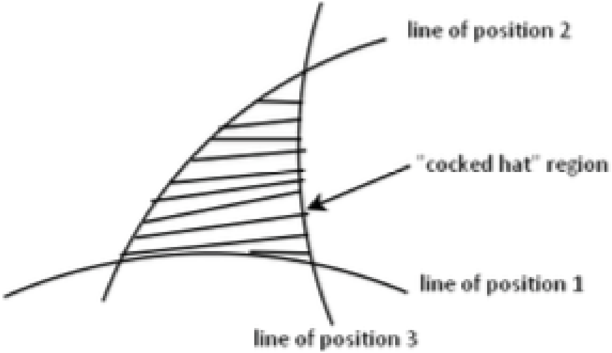
$$\Delta\phi = \phi_1 - \phi_2 = (\phi_{10} - \phi_{20}) + 2\pi f \left(\frac{\rho_1 - \rho_2}{c}\right)$$

- How to extract relative distance from this phase change?
- Due to synchronization of transmitters, $\phi_{10} = \phi_{20}$,

$$\Delta\phi = \phi_1 - \phi_2 = \left(\frac{2\pi f}{c}\right)(\rho_1 - \rho_2)$$

- Many of Omega receivers work in ranging mode.
- Advantage:
  - Requirement of only two ground stations
- Disadvantages:
  - Requirement of a very accurate and stable clock
  - Clock drift affect the range measurement
- At least three ground stations for position fixing in hyperbolic navigation
- Clock bias error do not affect measurement due to cancellation while taking difference.
- Omega errors do not accumulate with time.
- Omega has long term accuracy for position measurements.
- It can be combined with INS to use its short term accuracy.
- How to avoid ambiguity in position fixing? 3 LOPs

- What would happen if three LOPs are used?
- Will there always be an intersection if clock bias errors are present?




- Can we estimate clock bias using these measurements?

- Sources of errors
  - Seasonal activity
  - Variations of ionosphere
  - Earth geometry and magnetic field
  - Ground conductivity
- Improvement of Omega navigation using Differential Omega
- Ground unit with a monitor receiver at a fixed, precisely known location with an uplink transmitter
- Ground receiver with known location measures Omega signal phases and compares them with the nominal phase characteristic of known receiver.
- Differences between the nominal and actual phase measurements are used to generate signal correction data.
- Corrections are uplinked to airborne differential receiver within area of coverage.

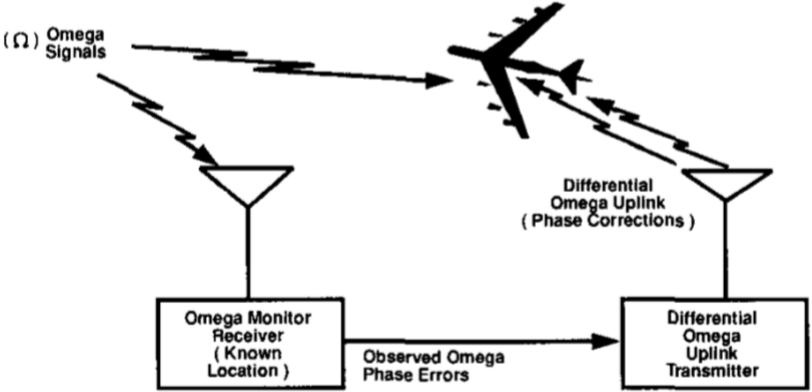



Figure: Differential Omega Concept

Disadvantages of differential Omega ?

Maintenance of large number of surveyed reference locations

Communication facility requirements to uplink corrections

## Reference

1. G. M. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*, Academic Press, Inc. 1993.

Thank you for your attention !!!