Coordinate Frames

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- Coordinate frame for computation of velocity, position and attitude by onboard computer
- Selection of coordinate frame
  - Mission requirements
  - Worldwide navigation capability
  - Onboard computation complexity
  - Interface with other avionics subsystems
- INS configurations
  - Space-stabilized mechanization
  - Strapdown (gimablless) mechanization
  - Local-level mechanization
    - ⇒ North-slaved
    - ⇒ Unipolar
    - ⇒ Free azimuth
    - ⇒ Wander azimuth

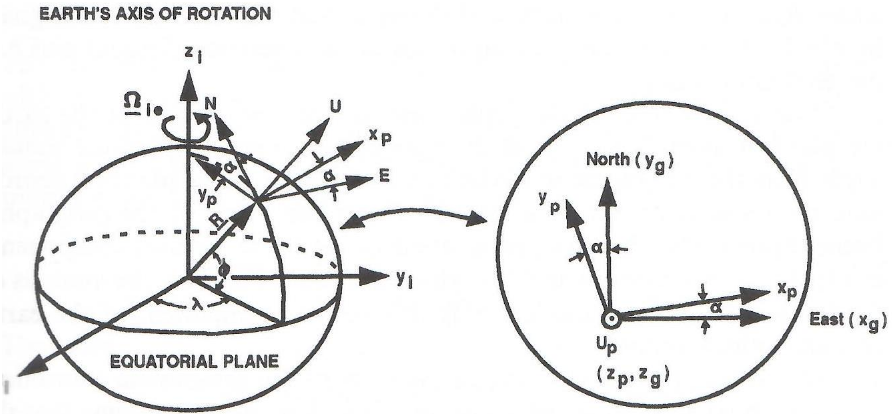
- **Space stable configuration**:
  - A constant orientation with respect to the inertial space
  - Platform frame coincides with the inertial frame
  - Sensing of accelerations in inertial reference as primary inputs
  - Relative velocity and positions relative to the center of the Earth in Cartesian coordinates: Integration of acceleration twice in inertial frame of reference
  - Standard position outputs (latitude and longitude): Derived using inertial Cartesian components.
- **Strapdown configuration**:
  - Inertial sensors are mounted directly on the host vehicle.
  - Platform frame coincides with the body frame.
  - Accelerometers rotate (w.r.t. inertial space) in the same way as the vehicle.
  - Transformation from sensor axes to inertial reference frame is computed rather than mechanized.

- Local-level systems are normally expressed in the navigation frame and measured with respect to an ECI frame.
- Distinction between the various local-level mechanization categories is in azimuth torque rate.
- These systems maintain two of accelerometer input axes in horizontal plane.
- Earth rotation rate vector

$$\boxed{\Omega_{ie} = \Omega_{ie} \cos\phi \, \boldsymbol{j} + \Omega_{ie} \sin\phi \, \boldsymbol{k}}$$

- Angular velocity of the navigation frame in inertial space is the sum of the angular velocity $\boldsymbol{\rho}$ of the navigation frame w.r.t. the rotating earth plus the angular velocity $\boldsymbol{\Omega_{ie}}$ of the earth w.r.t. inertial space.

$$\begin{aligned}
\omega_E &= \rho_E \\
\omega_N &= \rho_N + \Omega_N = \rho_N + \Omega_{ie} \cos\phi \\
\omega_Z &= \rho_Z + \Omega_Z = \rho_Z + \Omega_{ie} \sin\phi
\end{aligned}$$



EARTH'S AXIS OF ROTATION
z_I
Ω_{Ie}
N
U
x_p
α
E
y_p
α
R
ϕ
y_I
λ
EQUATORIAL PLANE
North (y_g)
y_p
α
x_p
α
East (x_g)
U_p
(z_p, z_g)

- As the vehicle moves over the surface of the earth, the platform coordinate system diverges by wander angle α from the geographic or navigation frame.
- The divergence rate $\dot{\alpha}$ is due to the vertical components of the earth's rotation and vehicle motion.
- Total angular velocity of the navigation coordinate system with respect to inertial space

$$\boxed{\boldsymbol{\omega}_{gt} = -\dot{\phi}\boldsymbol{i} + (\dot{\lambda} + \Omega_{ie})\cos\phi\boldsymbol{j} + \underbrace{(\dot{\lambda} + \Omega_{ie})\sin\phi}_{\omega_{zg}}\boldsymbol{k}}$$

- Rate of change for α is defined as

$$\boxed{\dot{\alpha} = \omega_{zp} - \omega_{zg}}$$

where ω_{zp} describes motion of the platform relative to navigation frame.

- Input torque applied to platform results into azimuth gyroscope commanded precession rate ω_{zc}, equivalent to ω_{zp}.

- Vertical platform axis of this system is torqued to maintain alignment with the navigation (geographic) axes.
- Platform must stay aligned with the geographic axes.
- In order for this to be true, the wander angle α must be zero.
- As the vehicle moves over surface of the Earth, no rotation out of the navigation axes.

$$\dot{\alpha} = 0 = \omega_{zp} - \omega_{zg}$$

- As $\omega_{zc} = \omega_{zp}$, commanded rate

$$\boxed{\omega_{zc} = \omega_{zg} = \Omega_{ie} \sin\phi + \dot{\lambda} \sin\phi = (\Omega_{ie} + \dot{\lambda}) \sin\phi}$$

- Platform vertical axis is torqued to maintain wander angle equal to longitude angle $\Rightarrow \boxed{\alpha = \lambda}$
- Platform must be torqued
  - $\Rightarrow$ To cancel the precession caused by the Earth rate and vehicle rate
  - $\Rightarrow$ To maintain the wander angle equal to the longitude

$$\boxed{\dot{\alpha} = \omega_{zc} - \omega_{zg} = \pm\dot{\lambda}}$$

- Commanded rate

$$\boxed{\omega_{zc} = \omega_{zg} + \dot{\alpha} = \Omega_{ie}\sin\phi + \dot{\lambda}\sin\phi \pm \dot{\lambda} = (\Omega_{ie} + \dot{\lambda})\sin\phi \pm \dot{\lambda}}$$

- Double sign indicates that the direction of the wander angle is reversed with respect to the longitude rate when crossing the equator.

- Vertical platform axis in this system is not torqued.
- It eliminates the torquing error associated with vertical gyroscope.

$$\boxed{\omega_{zc} = 0}$$

- Platform axes then diverge in azimuth from the geographic axes

$$\boxed{\dot{\alpha} = \omega_{zc} - \omega_{zg} = -\Omega_{ie} \sin\phi - \dot{\lambda} \sin\phi = -(\Omega_{ie} + \dot{\lambda}) \sin\phi}$$

- Non-torquing of vertical gyroscope can be considered as an advantage.
- Normally vertical gyroscope exhibits poorer drift rate characteristics than either of the horizontal gyroscopes.
- Free-azimuth coordinates would be used in aircraft INS where high platform angular rates about the vertical axis are required.

- Vertical platform axis is torqued to compensate only for the vertical component of the Earth rate.

$$\boxed{\omega_{zc} = \Omega_{ie} \sin\phi}$$

- It does not cancel the vertical component of the aircraft transport rate.
- No attempt to maintain the level axes in a preferred azimuth direction.
- Level axes are allowed to rotate freely about the vertical axis.

$$\boxed{\dot{\alpha} = \omega_{zc} - \omega_{zg} = \Omega_{ie} \sin\phi - \Omega_{ie} \sin\phi - \dot{\lambda} \sin\phi = -\dot{\lambda} \sin\phi}$$

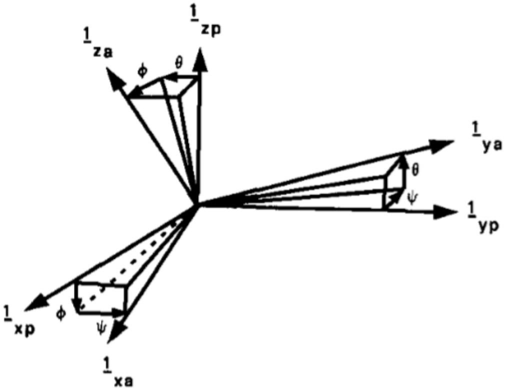
- Inertial platform is initially aligned by first driving it with torquers until the outputs of the two-level accelerometers are zero.

System	ω_c	$\dot{\alpha}$
North-slaved	$(\Omega_{ie} + \dot{\lambda}) \sin\phi$	0
Unipolar	$(\Omega_{ie} + \dot{\lambda}) \sin\phi \pm \dot{\lambda}$	$\pm\dot{\lambda}$
Free azimuth	0	$-(\Omega_{ie} + \dot{\lambda}) \sin\phi$
Wander azimuth	$\Omega_{ie} \sin\phi$	$-\dot{\lambda} \sin\phi$

Table: Local level systems

- **North-slaved**: Latitude and longitude coordinates can be read directly from the platform azimuth gimbal
- **Unipolar**: Platform azimuth is maintained equal to longitude; can obtain azimuth and longitude with a single computation
- **Free azimuth**: Needs no accurate torque electronics for the vertical or z-gyroscope
- **Wander azimuth**: No singularity at the poles. Also, the coordinate transformations are simplified considerably

- **Platform or Instrument Misalignment**: Manufacturing imperfections
    - Sensitive axes of the accelerometer triad not perfectly aligned with the platform axes
    - Causes the accelerometers to sense a specific force component due to gravity



- Unit vectors of platform coordinate frame: $\mathbf{1}_{xp}, \mathbf{1}_{yp}, \mathbf{1}_{zp}$
- Unit vectors of accelerometer triad: $\mathbf{1}_{xa}, \mathbf{1}_{ya}, \mathbf{1}_{za}$
- As misalignment angles are usually small, $\sin\theta \approx \theta$ and $\cos\theta \approx 1$.
- Transformation matrix between these two frames

$$\begin{bmatrix} \mathbf{1}_{xa} \\ \mathbf{1}_{ya} \\ \mathbf{1}_{za} \end{bmatrix} = \begin{bmatrix} 1 & \psi & -\phi \\ -\psi & 1 & \theta \\ \phi & -\theta & 1 \end{bmatrix} \begin{bmatrix} \mathbf{1}_{xp} \\ \mathbf{1}_{yp} \\ \mathbf{1}_{zp} \end{bmatrix}$$

$$\mathbf{1}_a = \boldsymbol{C}_a^p \mathbf{1}_p$$

- **Observations**:
  - Main diagonal elements of the misalignment orientation matrix are near unity.
  - Off-diagonal elements of misalignment orientation matrix are small as compared to unity.
  - Two coordinate frames are in near coincidence.

Reference

1. G. M. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*, Academic Press, Inc. 1993.

Thank you for your attention !!!

Inertial Sensors

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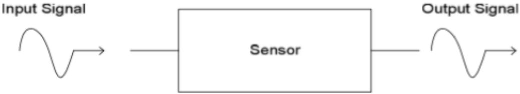
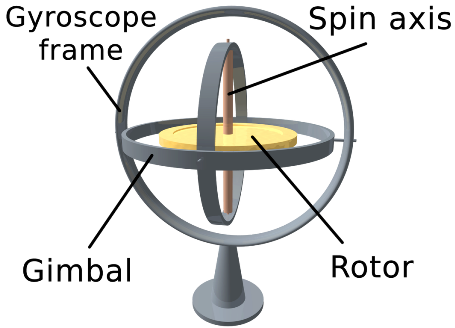
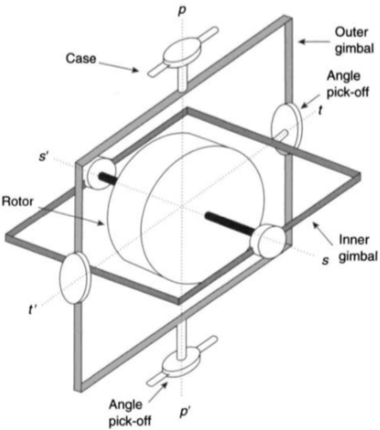
- **Sensor**: A device which provides a usable output in response to a specified measurand. (American National Standards Institute)



- A sensor acquires a physical quantity and converts it into a signal suitable for processing (e.g. optical, electrical, mechanical).
- Active element of a sensor: Transducer which converts one form of energy to another.
- When input is a physical quantity and output electrical $\Rightarrow$ Sensor
- When input is electrical and output a physical quantity $\Rightarrow$ Actuator
- Why do we need sensors ?

- **Inertial sensor**: Sensors based on inertia and relevant measuring principles
  - Gyroscope
  - Accelerometer
- **Gyroscope**: Inertial properties of a wheel or rotor spinning at high speed
  - To sense the angle turned through by a vehicle (displacement gyroscopes)
  - To sense its angular rate of turn about some defined axis (rate gyroscopes)
- Development of mechanical gyroscope: Prof. C S . Draper and his co-workers, at the Massachusetts Institute of Technology.
- Precision devices with error rates of less than 0.001°/h, to less accurate sensors with error rates of tens of degrees per hour.
- Capability to measure angular rates up to about 500°/s
- Fundamentals for gyroscope
  - Gyroscopic inertia
  - Angular momentum
  - Precession





PC: Wikipedia

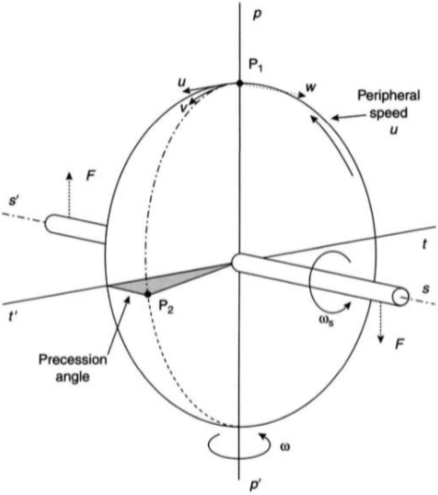
- **Gyroscopic inertia**: Defines a direction that remains fixed in inertial space
- Establishment of a fixed direction enables rotation to be detected, by making reference to this fixed direction.
- Orientation of the case of instrument with respect to direction of spin axis: measured using angle pick-off devices mounted on the gimbals
- **Angular momentum**:

$$H = I\omega_s$$

- Distribution of mass on a rotor is important.
- Angular momentum should be very high, so that the undesired torques that can act on a rotor and cause errors are virtually insignificant.
- **Drift**: Any undesired movement of the direction of the spin axis.
- How can we maximize the angular momentum with given angular velocity, mass, and size of rotor?

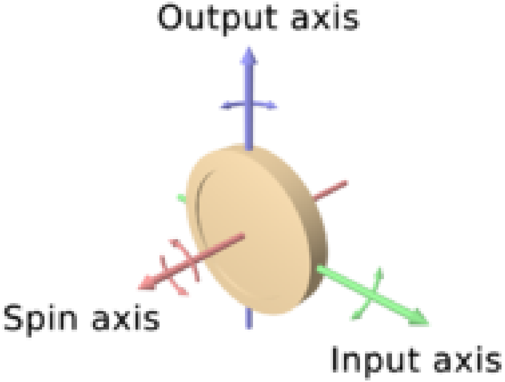
- One way to for produce a very high angular momentum is to have the majority of the mass of the rotor at its edge.
- This is due to the dependence of the moment of inertia on the square of the distance of its mass element from the centre of rotation.
- Is a high angular momentum always beneficial?
- A very high angular momentum results in negligible drift, but there could be some considerable penalties.
- Gyroscope would almost certainly be relatively large and heavy, and it may take many seconds, if not minutes, for the rotor to reach its operating speed.
- In a strapdown mode, the associated control system may not be capable of recording, or 'capturing', angular rates beyond a few tens of deg/s.
- Compromise while selecting a gyroscope for a given application.

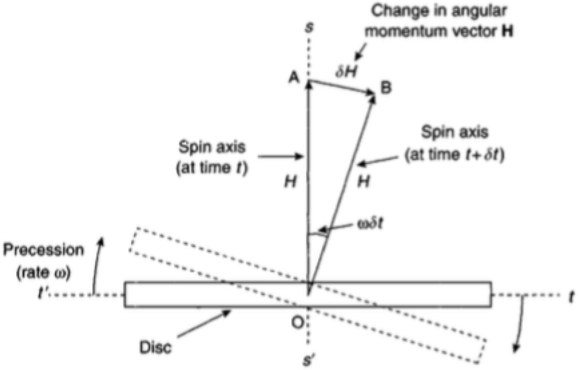
- Spin is the rotation of gyroscope rotor relative to the gimbal.
- Precession is the rotation of gimbal, relative to inertial space.
- For a freely spinning body, such as the Earth, there is not a material frame with spin bearings.
- The precession must be considered to be that of the axis system which an imaginary gimbal would have one axis through the north and south poles, and two mutually orthogonal in the plane of the Equator.



- Consider spinning of disc about ss'.
- If the disc is acted upon by a torque, about the axis tt', the spin axis of the disc will be forced to turn about the axis pp'.
- This turning is called as the precession.
- The precession axis, pp', is orthogonal to the torque axis, tt'.
- Suppose that the disc is rigid with all mass in the rim, and the rim has a peripheral speed u.
- Consider an element of mass at the highest point, P_1.
- Instantaneous velocity of the mass is changed by adding the velocity w in the same sense as the couple FF.
- Resultant velocity, v, is now in a different direction.
- Other elements of the rim change their velocities in proportion to their distance from the axis tt'.

- After the disc has spun through 90°, the element of mass arrives at the point P_2, which is not in the expected line tt', but in a plane which has precessed about the axis pp'.





- A heavy spinning disc with angular momentum $\boldsymbol{H}$ defined by vector OA, $H\boldsymbol{a}$ with unit vector $\boldsymbol{a}$.
- **Newton's law**: Angular momentum vector $\boldsymbol{H}$ remains constant unless some external torque acts on the disc.

- Suppose a torque $\boldsymbol{T}$ is applied to the disc and it causes a precession at rate $\boldsymbol{\omega} = \omega \boldsymbol{c}$.
- This axis lie in the plane of disc and may be taken to be normal to the plane of paper.
- Over time duration δt, the disc will precess through an angle $\omega \delta t$ about $\boldsymbol{c}$.
- Angular momentum vector will change to OB, that is, $(H + \delta H)\boldsymbol{b}$ where $\boldsymbol{b} = \boldsymbol{a} + \omega \delta t(\boldsymbol{c} \times \boldsymbol{a})$.
- Change in angular momentum

$$\delta \boldsymbol{H} = (H + \delta H)\boldsymbol{b} - H\boldsymbol{a} = H(\boldsymbol{b} - \boldsymbol{a}) + \delta H \boldsymbol{b} = H\omega\delta t(\boldsymbol{c} \times \boldsymbol{a}) + \delta H \boldsymbol{b}$$

- Rate of change of angular momentum

$$\frac{d\boldsymbol{H}}{dt} = \lim_{\delta t \to 0} \frac{\delta \boldsymbol{H}}{\delta t} = H\omega(\boldsymbol{c} \times \boldsymbol{a}) + \frac{dH}{dt}\boldsymbol{b} = \boldsymbol{\omega} \times \boldsymbol{H} + \frac{dH}{dt}\boldsymbol{b}$$

$$\boldsymbol{T} = \boldsymbol{\omega} \times \boldsymbol{H} + \frac{dH}{dt}\boldsymbol{b}$$

- Component of the torque, which is along the spin axis, gives rise to an acceleration in the spin rate.
- Component normal to the spin axis gives rise to a precession ω, which is normal to both the torque and the spin axes.
- The direction of precession is such as to try to align the spin axis with the torque axis.
- On neglecting component along spin axis, we obtain "Law of Gyroscopes" as

$$\boxed{\boldsymbol{T} = \boldsymbol{\omega} \times \boldsymbol{H} \Rightarrow T = \omega H}$$

- Measurements of angular rotation or rotation rate:
  - ☐ Change in angle of gimbals
  - ☐ Measurement of torque to keep rotor aligned with direction defined by instrument case

- Difficult to measure angular displacements accurately, need expensive equipments
- Easy to measure fixed defined position or zero deflection or null position
- If the spin axis of a rotor is made to precess back to the 'null' position by the application of a suitable torque, a very accurate angular measurement is possible.
- **Requirements**: Necessary torque to null the deflection can be generated and measured.
- Even most accurate of gyroscopes will appear to drift.
- This is because the angular momentum vector is fixed with respect to space axes, not the co-ordinate system defined by the Earth.
- It is sometime necessary to apply corrective torques to precess the gyroscope if it is to be used as an Earth reference.

Reference

1. G. M. Siouris, *Aerospace Avionics Systems: A Modern Synthesis*, Academic Press, Inc. 1993.
2. D. H. Titterton and J. L. Weston, *Strapdown Inertial Navigation Technology*, Progress in Astronautics and Aeronautics, Vol. 207, ed. 2, ch. 4.

Thank you for your attention !!!