Basic Concepts of Guidance

Dr. Shashi Ranjan Kumar

Assistant Professor
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Powai, Mumbai, 400076 India

- Engagement dynamics

$$
\begin{aligned}
\dot{r} &= V_T \cos(\gamma_T - \theta) - V_M \cos(\gamma_M - \theta) \\
r\dot{\theta} &= V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta) \\
\dot{\gamma}_M &= \frac{a_M}{V_M} \\
\dot{\gamma}_T &= \frac{a_T}{V_T}
\end{aligned}
$$

- Absence of guidance system and constant velocity target

$$a_M = 0, \quad a_T = 0.$$

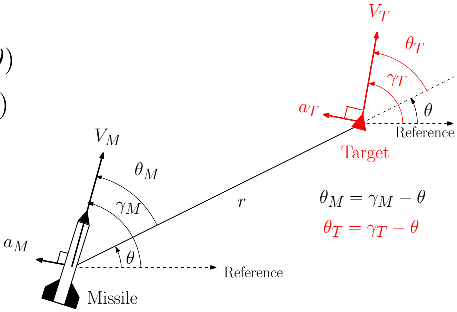
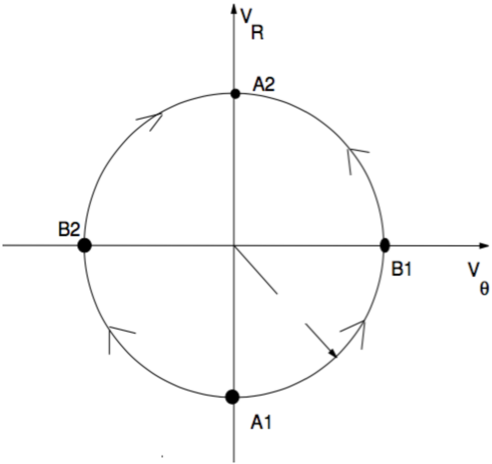


Figure: Planar missile-target engagement



- Trajectories in (V_r, V_θ)-space

$$V_r^2 + V_\theta^2 = \; c^2$$

- Time of miss

$$t_{\text{miss}} = -\frac{r_0 V_{r_0}}{c^2} = -\frac{r_0 V_{r_0}}{V_{r_0}^2 + V_{\theta_0}^2}$$

- Miss distance

$$r_{\text{miss}} = \sqrt{c^2 t_{\text{miss}}^2 + 2b t_{\text{miss}} + 2a}$$

$$= r_0 \sqrt{\frac{V_{\theta_0}^2}{V_{r_0}^2 + V_{\theta_0}^2}}$$

- Collision conditions: $\dot{\theta} = 0, V_r < 0$

- In practice, direct interceptions rarely occur. However, missile passes close to target then the warhead detonates.
- Interception is assumed to occur if $\boxed{r_{\text{miss}} \leq r_{\text{lethal}}}$

$$r_0 \sqrt{\frac{V_{\theta_0}^2}{V_{r_0}^2 + V_{\theta_0}^2}} \leq r_{\text{lethal}} \quad \Rightarrow r_0^2 V_{\theta_0}^2 \leq r_{\text{lethal}}^2 \left(V_{r_0}^2 + V_{\theta_0}^2\right)$$

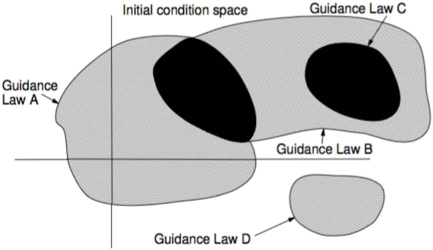
- On rearrangement, we get

$$\left(r_0^2 - r_{\text{lethal}}^2\right) V_{\theta_0}^2 \leq r_{\text{lethal}}^2 V_{r_0}^2 \Rightarrow V_{\theta_0}^2 \leq \frac{r_{\text{lethal}}^2}{\left(r_0^2 - r_{\text{lethal}}^2\right)} V_{r_0}^2$$

- Condition for capturability

$$\boxed{|V_{\theta_0}| \leq |V_{r_0}| \sqrt{\frac{r_{\text{lethal}}^2}{\left(r_0^2 - r_{\text{lethal}}^2\right)}}}$$

- **Capture Region**: Region in the state space or initial condition space, from which the missile is able to capture the target with a given guidance law
- Collection of those points in the initial condition space which satisfy the requirements of capturability.
- Capture regions of two guidance laws may intersect, may form a subset or superset of each other, or may be totally disjoint.



- For zero miss distance as intercept,

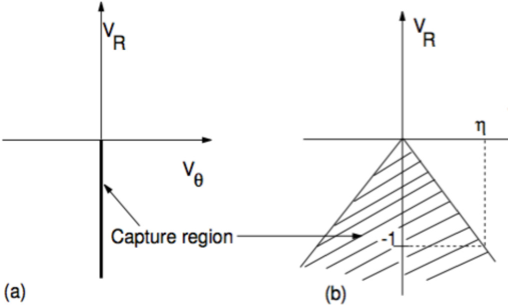
$$V_{\theta_0} = 0, \quad V_{r_0} < 0$$

- For miss distance less than the lethal radius r_{lethal},

$$|V_{\theta_0}| \leq \eta \, |V_{r_0}| \, , V_{r_0} < 0$$

where

$$\eta = \sqrt{\frac{r_{\text{lethal}}^2}{(r_0^2 - r_{\text{lethal}}^2)}}$$



- How the trajectories of V_r, V_θ vary with respect to time?
- Using $r\dot{r} = c^2 t + b$, we have

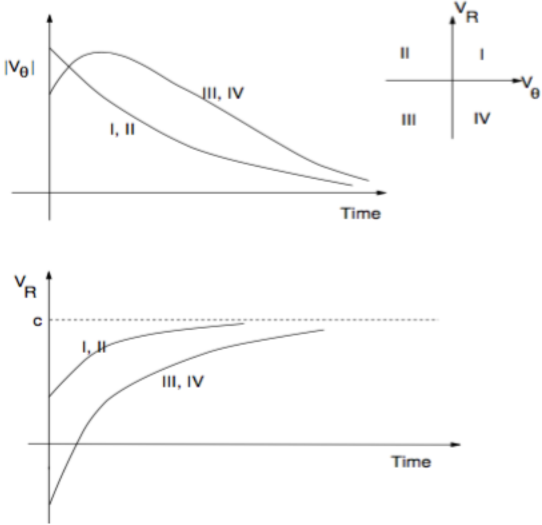
$$V_r = \frac{c^2 t + b}{r} = \frac{c^2 t + b}{\sqrt{c^2 t^2 + 2bt + 2a}}$$

- As $V_r^2 + V_\theta^2 = c^2$,

$$V_\theta = \sqrt{c^2 - V_r^2}$$

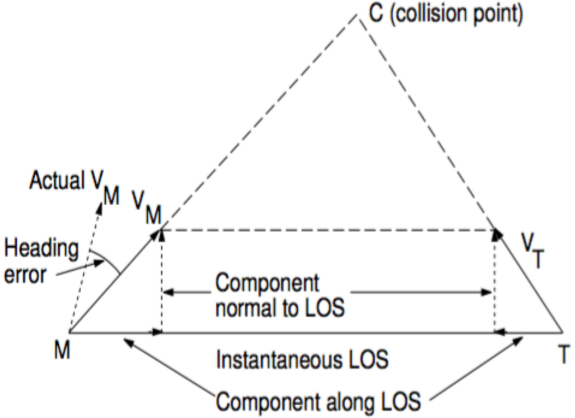
- Asymptotic behavior of V_r, V_θ

$$\lim_{t\to\infty} V_r = \lim_{t\to\infty} \frac{c^2 t + b}{\sqrt{c^2 t^2 + 2bt + 2a}} = c$$
$$\lim_{t\to\infty} V_\theta = \lim_{t\to\infty} \sqrt{c^2 - V_r^2} = 0$$



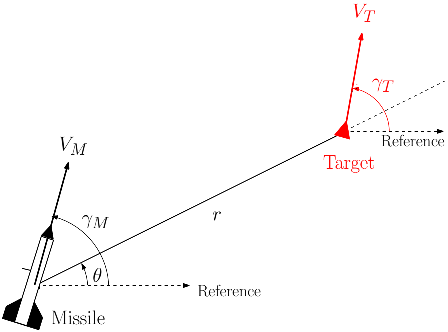
How to justify such behavior of V_r and V_θ?

Collision triangle defines the trajectories of missile and target when they culminate in an intercept, while both the vehicles are moving with constant speeds and in straight-line trajectories.



- **Heading error**: Difference in the angle between the actual missile velocity vector and the angle required by the missile velocity vector to satisfy the collision geometry conditions.

$$\mathsf{HE} = \Delta\gamma_M = \gamma_M - \gamma_{Md}$$



- How to compute heading angle error for a given flight path angles of target and missile, and the LOS angle?

- The desired flight path angle of missile can be obtained using the collision triangle condition, $\dot{\theta} = 0$.

$$r\dot{\theta} = V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta) = 0$$

- Assume desired flight path angle of missile as γ_{Md}, then we have

$$V_M \sin(\gamma_{Md} - \theta) = V_T \sin(\gamma_T - \theta)$$

- Using trigonometric manipulations

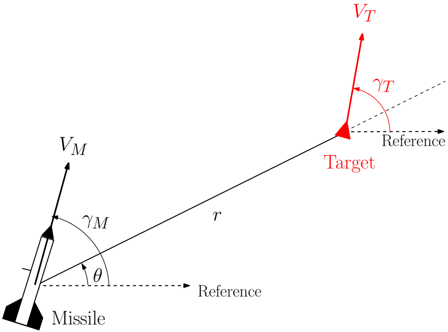
$$\gamma_{Md} = \theta + \sin^{-1}\left[\frac{V_T}{V_M}\sin(\gamma_T - \theta)\right]$$

- Heading error in terms of speed ratio of target to the missile $\nu = V_T/V_M$

$$\boxed{HE = \gamma_M - \gamma_{Md} = (\gamma_M - \theta) - \sin^{-1}\left[\nu \sin(\gamma_T - \theta)\right]}$$

Heading Angle Errors: Example

Assume that a missile is launched from surface of the earth while an aerial target is located at a radial distance of 6 km and at a height of 3 km above the surface. The target is moving in horizontal direction receding away from missile at speed of 500 m/s. Assume that the missile is launched at a speed of 1000 m/s at an angle of 60° with the horizontal reference. Compute the LOS rate and closing speed of missile-target engagement. How much is the heading angle error of missile?



- What kind of engagement is there in this example? Tail Chase
- LOS rate

$$
\begin{aligned}
\dot{\theta} =& \frac{V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta)}{r} \\
=& \frac{500 \sin(0^\circ - 30^\circ) - 1000 \sin(60^\circ - 30^\circ)}{6000} \\
=& \frac{-750}{6000} = -0.1250 \quad \text{rad/s}
\end{aligned}
$$

- It means that the missile and target are not on the collision triangle.
- Closing speed $V_c = -V_r$

$$
\begin{aligned}
V_c =& -\left[V_T \cos(\gamma_T - \theta) - V_M \cos(\gamma_M - \theta)\right] \\
=& -\left[500 \cos(0^\circ - 30^\circ) - 1000 \cos(60^\circ - 30^\circ)\right] \\
=& 433.0127 \text{ m/s}
\end{aligned}
$$

- Desired flight path angle γ_{Md}

$$1000 \sin(\gamma_{Md} - 30^\circ) = 500 \sin(0^\circ - 30^\circ)$$

- On solving above equation

$$\begin{aligned}\gamma_{Md} =& 30^\circ + \sin^{-1}\left[\frac{500}{1000}\sin(0^\circ - 30^\circ)\right] = 30^\circ - 14.4775^\circ \\ =& 15.5225^\circ\end{aligned}$$

- Heading angle error of missile

$$HE = 60^\circ - 15.5225^\circ = 44.4775^\circ$$

- What would happen if the target is in head-on engagement and moves in the opposite direction to the one in current example?

- LOS rate

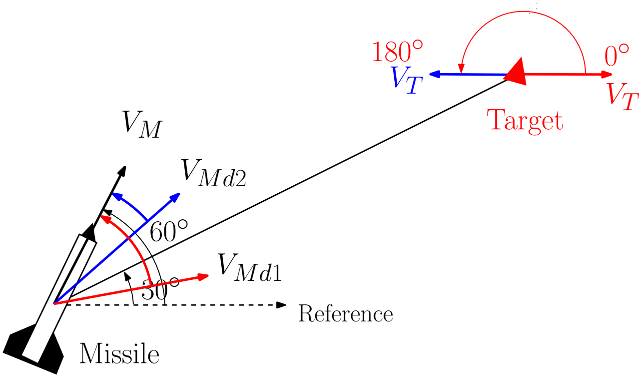
$$
\begin{aligned}
\dot{\theta} =& \frac{V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta)}{r} \\
=& \frac{500 \sin(180^\circ - 30^\circ) - 1000 \sin(60^\circ - 30^\circ)}{6000} \\
=& \frac{-250}{6000} = -0.0417 \quad \text{rad/s}
\end{aligned}
$$

- Closing speed $V_c = -V_r$

$$
\begin{aligned}
V_c =& - \left[V_T \cos(\gamma_T - \theta) - V_M \cos(\gamma_M - \theta) \right] \\
=& - \left[500 \cos(180^\circ - 30^\circ) - 1000 \cos(60^\circ - 30^\circ) \right] \\
=& 1299 \text{ m/s}
\end{aligned}
$$

- Heading error

$$
\begin{aligned}
HE =& (60^\circ - 30^\circ) - \sin^{-1} \left[\frac{500}{1000} \sin(180^\circ - 30^\circ) \right] \\
=& 30^\circ - 14.4775^\circ = 15.5225^\circ
\end{aligned}
$$



180°
V_T
0°
V_T
Target
V_M
V_{Md2}
60°
V_{Md1}
30°
Reference
Missile

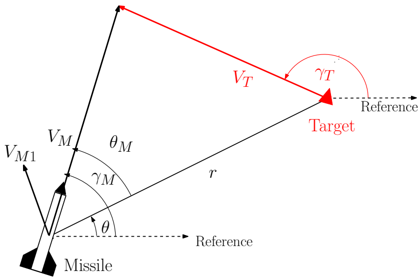
- **Interception**: When either missile directly hits target or comes at point of closest approach.
- **Time-to-go**: Time-to-go is defined as the time remaining till interception at any given instant in time t during the engagement.

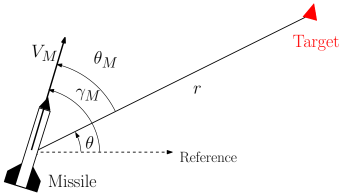
$$t_{\text{go}} = t_f - t$$

  where t_f is the final time and t is the current time.
- This value is actual time-to-go, known only after the engagement is over.
- Can we get some estimate of the time-to-go? How??
- What about the ratio of range and missile's speed?

$$t_{\text{go}} = \frac{r}{V_M}$$

- In what condition, will it provide accurate estimate?





⇒ What about the ratio of range and closing speed?

$$t_{\text{go}} = \frac{r}{V_c} = \frac{r}{-\dot{r}}$$

⇒ In what condition, will it provide accurate estimate?

⇒ Good if missile and target are on collision course. Why??

⇒ Time-to-go estimate

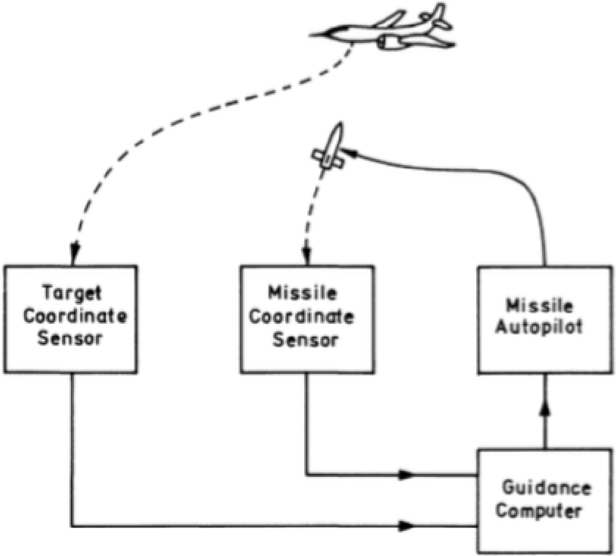
$$t_{\text{go}} = \frac{r}{V_M}\left[1 + \frac{\theta_M^2}{2(2N-1)}\right]$$

where N is a positive constant and proportional gain of PN guidance.

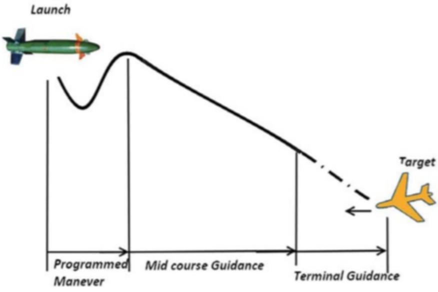
- Free-flight weapons system
  - ⇒ Location and any movement of the target are measured as accurately as possible up to the moment of firing
  - ⇒ A point of impact is calculated, based upon prediction of target movement during the time of flight of projectile.
- Sources of major errors
  - ⇒ Change of course or speed of target during the time of flight
  - ⇒ Deviation from expected trajectory due to errors in velocity and direction of projection, and unpredictable variations in meteorological conditions prevailing over flight path.
- A guided weapon system avoids both these sources of error by monitoring continuously flight of missile w.r.t. position and movement of target, thereby ensuring a much higher kill probability.
- Basic difference between guided and unguided missiles: Generation of guidance commands to change the missile's flight direction.

Basic Concepts of Guidance

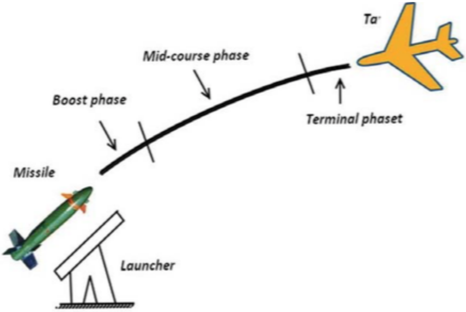
Guidance Principle



- Guidance Phases:
  - ⇒ **Programmed maneuver phase**: Independent of target information and is executed solely to ensure that missile clears launch aircraft.
  - ⇒ **Midcourse guidance phase**: To place missile within terminal acquisition range of its seeker with the missile seeker pointed at the target.
  - ⇒ **Terminal phase**: Missile locks on to the target and attempts to close the distance to the target as quickly as possible under the constraints of energy availability and maneuver limitations. (Crucial for success of mission)



- Guidance Phases:
  - ⇒ Missile trajectory for SAMs is almost the same except the initial phase, called the boost phase.
  - ⇒ In this phase, the missile's booster provides the required velocity to the missile.
  - ⇒ Since this phase occurs for a very short time during which the missile is marginally stable and has high longitudinal acceleration, no guidance commands are normally given to the missile.



- Guidance Phases: Classification based on nature of guidance scheme
  - $\Rightarrow$ **Command guidance** scheme
  - $\Rightarrow$ **Homing guidance** scheme
- Command guidance scheme
  - $\Rightarrow$ Target's and missile's positions and velocities are measured by a tracking radar situated at the ground station.
  - $\Rightarrow$ This information is processed and given to the guidance computer which generates the steering or guidance commands using some guidance law.
  - $\Rightarrow$ Guidance computer is also situated at the ground station.
  - $\Rightarrow$ Guidance commands are then communicated to missile via a data uplink.
  - $\Rightarrow$ Based on these inputs, the missile flight control system takes action.
  - $\Rightarrow$ This kind of guidance is also known as three-point guidance.
- Homing guidance
  - $\Rightarrow$ Does not depend on any ground station for guidance commands generation.
  - $\Rightarrow$ Two-point guidance.

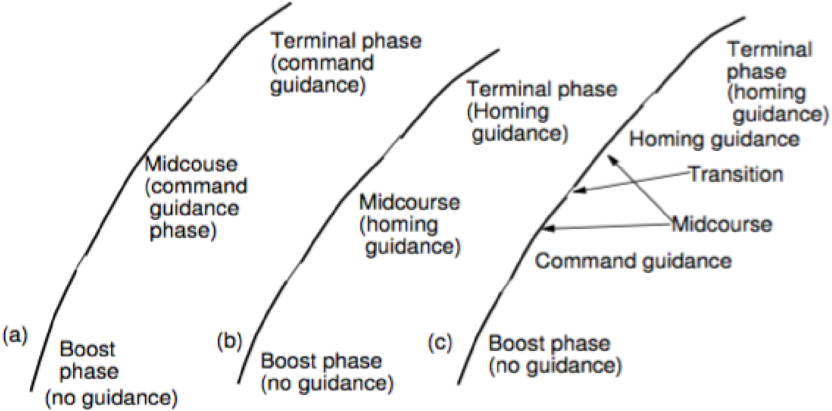


Figure: Guidance Phases: Command, homing, and mixed guidance

- Which one is better, homing or command guidance?
- Advantages of homing compared with the other methods
  - ⇒ Precision of tracking improves as the missile closes upon the target and the range shortens,
  - ⇒ Probability of success largely independent of the initial range of the target from the parent installation of the missile.
  - ⇒ Missile is potentially autonomous, that is, once launched it guides itself towards the target without further assistance from the parent installation, leaving the latter free to engage another target.
- Are there issues?
- Disadvantages
  - ⇒ An expensive guidance package is lost with each missile fired.
  - ⇒ Guidance package occupies the prime location within the missile, displacing the warhead.
  - ⇒ Missile nose has to be a vulnerable and expensive radome.

Reference

1. D. Ghose, *Lecture notes on Navigation, Guidance and Control*, Indian Institute of Science, Bangalore.

Thank you for your attention !!!