ARPIT Course on

Electronic Systems for Sensor Applications

Lecture 1: Measurements: Accuracy, Standards

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Learning objectives

- Understand measurement terminology
- Datasheet formulation
- Measurement uncertainty
  - Systematic and random error
  - How repeatability and reproducibility can overshadow true variation
  - Methods to estimate repeatability and reproducibility (Gage R&R)
- Measurement standards

Relevance of this lecture

- Measurements are an inherent part of sensor systems – understanding how to make and interpret measurements is crucial
- Understanding how to obtain information from datasheets to construct sensors is an essential part of building systems

Wonder where these numbers come from?

ADXL335

SPECIFICATIONS

T_A = 25°C, V_S = 3 V, C_X = C_Y = C_Z = 0.1 μF, acceleration = 0 *g*, unless otherwise noted. All minimum and maximum specifications are guaranteed. Typical specifications are not guaranteed.

Table 1.

Parameter	Conditions	Min	Typ	Max	Unit
SENSOR INPUT	Each axis				
Measurement Range		±3	±3.6		*g*
Nonlinearity	% of full scale		±0.3		%
Package Alignment Error			±1		Degrees
Interaxis Alignment Error			±0.1		Degrees
Cross-Axis Sensitivity[1]			±1		%
SENSITIVITY (RATIOMETRIC)[2]	Each axis				
Sensitivity at X_{OUT}, Y_{OUT}, Z_{OUT}	V_S = 3 V	270	300	330	mV/*g*
Sensitivity Change Due to Temperature[3]	V_S = 3 V		±0.01		%/°C
ZERO *g* BIAS LEVEL (RATIOMETRIC)					
0 *g* Voltage at X_{OUT}, Y_{OUT}	V_S = 3 V	1.35	1.5	1.65	V
0 *g* Voltage at Z_{OUT}	V_S = 3 V	1.2	1.5	1.8	V
0 *g* Offset vs. Temperature			±1		m*g*/°C
NOISE PERFORMANCE					
Noise Density X_{OUT}, Y_{OUT}			150		μ*g*/√Hz rms
Noise Density Z_{OUT}			300		μ*g*/√Hz rms
FREQUENCY RESPONSE[4]					
Bandwidth X_{OUT}, Y_{OUT}[5]	No external filter		1600		Hz
Bandwidth Z_{OUT}[5]	No external filter		550		Hz
R_{FILT} Tolerance			32 ± 15%		kΩ
Sensor Resonant Frequency			5.5		kHz

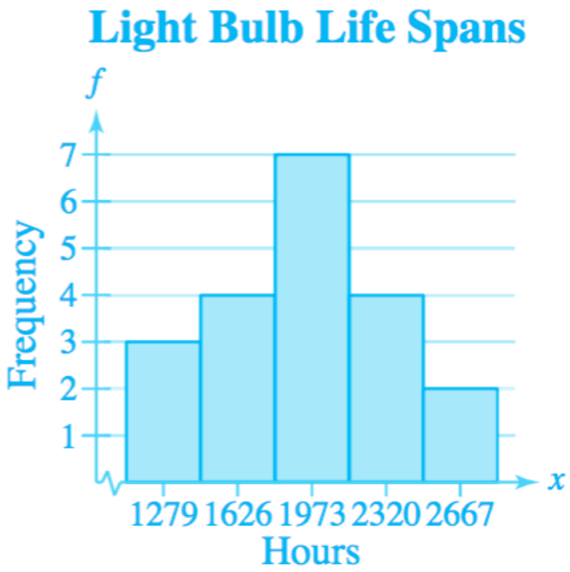
Offset and noise: understanding the difference

- Systematic error in a measurement is also referred to as offset
  - Origins of offset are often difficult to pin-point
  - Offset could be inherent to the sensor, caused due to signal conditioning, or an artefact of measurement
- Random error in a measurement is also referred to as noise
  - Origins of noise are often easier to pin-point
  - Noise could be inherent to the sensor, caused due to signal conditioning, or an artefact of measurement
- For samples of a normal distributed sensor output
  - Offset is specified by the mean
  - Noise is specified by standard deviation (RMS noise)
  - Peak noise = 5.5*RMS noise

Exercise

Manufacturer Claims You work for a consumer watchdog publication and are testing the advertising claims of a light bulb manufacturer. The manufacturer claims that the life span of the bulb is normally distributed, with a mean of 2000 hours and a standard deviation of 250 hours. You test 20 light bulbs and get the following life spans.

2210, 2406, 2267, 1930, 2005, 2502, 1106, 2140, 1949, 1921,
2217, 2121, 2004, 1397, 1659, 1577, 2840, 1728, 1209, 1639



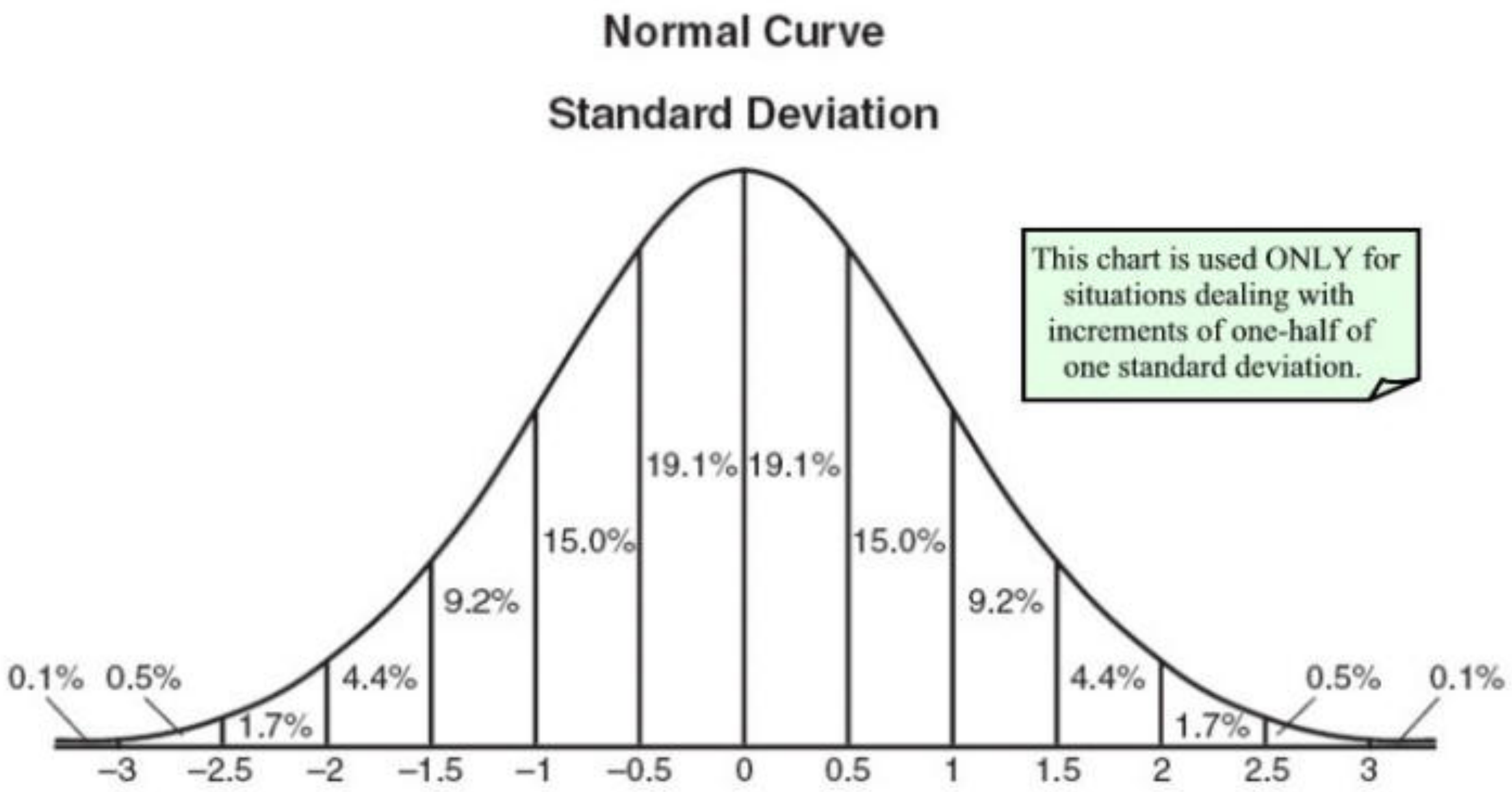
Mean (μ): 1941.3
Std. dev. (σ): 432.4

Explain the discrepancy between your measurement and manufacturer's spec

Scenarios:

1. Manufacturer has made a false claim
2. Your measurement method has errors
3. You obtained a defective batch
4. Test conditions are different from manufacturer recommendations
5. …

Think of distributions in terms of probability



https://mathbits.com/MathBits/TISection/Statistics2/normaldistribution.htm

Conventions

- ‘Typ’ specification may usually be $\mu \pm 3\sigma$
- ‘Min/max’ specification may usually be $\mu \pm 5.5\sigma$
- Definitions may vary by manufacturer; large population chosen with ‘skew lots’ to obtain guaranteed numbers that account for variation over life of the component
- How many measurements must be made to project an accurate estimate of the specification?

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Package Alignment Error			±1		Degrees
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Cross-Axis Sensitivity[1]			±1		%
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Sensitivity at X_{OUT}, Y_{OUT}, Z_{OUT}	$V_S = 3$ V	270	300	330	mV/*g*
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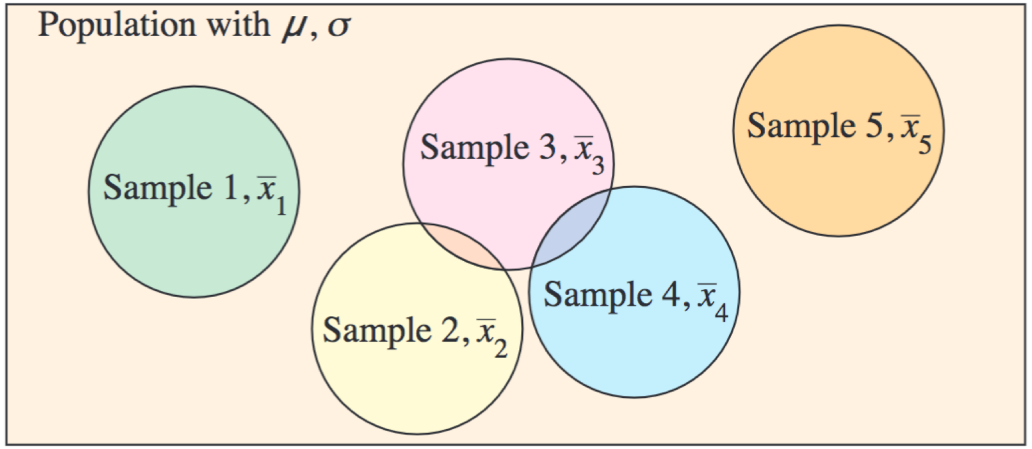
Sampling distributions

DEFINITION

A **sampling distribution** is the probability distribution of a sample statistic that is formed when samples of size n are repeatedly taken from a population. If the sample statistic is the sample mean, then the distribution is the **sampling distribution of sample means.**



Properties of Sampling Distributions of Sample Means

1. The mean of the sample means $\mu_{\bar{x}}$ is equal to the population mean μ.

$$\mu_{\bar{x}} = \mu$$

2. The standard deviation of the sample means $\sigma_{\bar{x}}$ is equal to the population standard deviation σ divided by the square root of n.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

The standard deviation of the sampling distribution of the sample means is called the **standard error of the mean.**

Central limit theorem

The Central Limit Theorem

1. If samples of size n, where $n \geq 30$, are drawn from any population with a mean μ and a standard deviation σ, then the sampling distribution of sample means approximates a normal distribution. The greater the sample size, the better the approximation.
2. If the population itself is normally distributed, the sampling distribution of sample means is normally distributed for *any* sample size n.

In either case, the sampling distribution of sample means has a mean equal to the population mean.

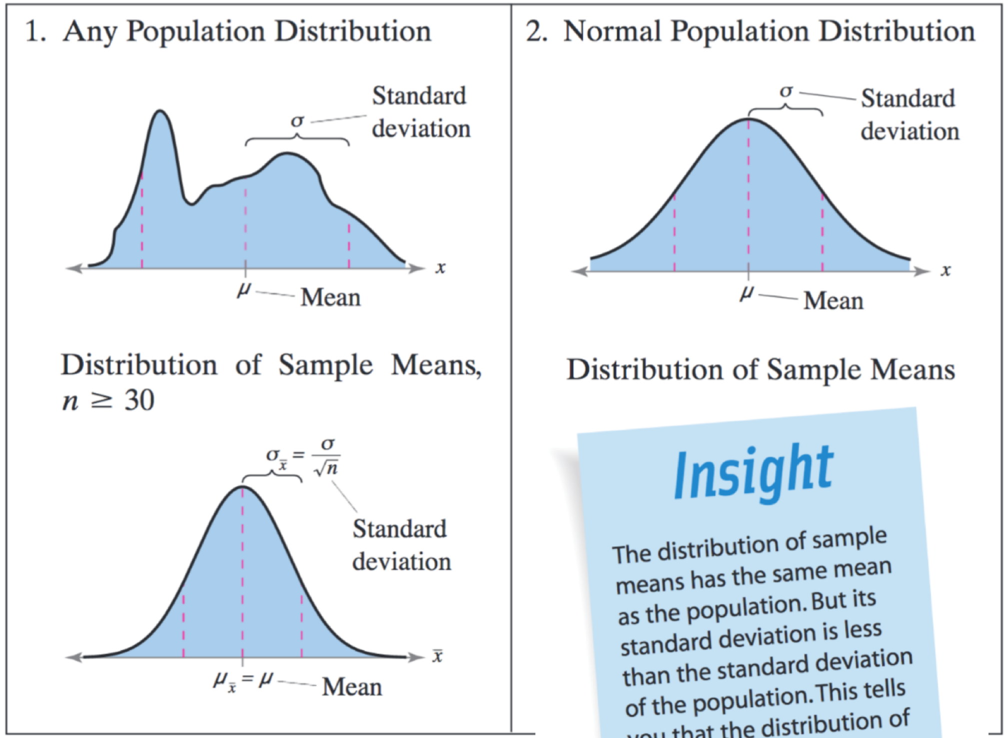
$$\mu_{\bar{x}} = \mu \qquad \text{Mean}$$

The sampling distribution of sample means has a variance equal to $1/n$ times the variance of the population and a standard deviation equal to the population standard deviation divided by the square root of n.

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \qquad \text{Variance}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad \text{Standard deviation}$$

The standard deviation of the sampling distribution of the sample means, $\sigma_{\bar{x}}$, is also called the **standard error of the mean.**



Insight

The distribution of sample means has the same mean as the population. But its standard deviation is less than the standard deviation of the population. This tells you that the distribution of sample means has the same center as the population, but it is not as spread out.

Moreover, the distribution of sample means becomes less and less spread out (tighter concentration about the mean) as the sample size n increases.

Measuring offset and noise

- What do you think offset and noise measurements (variation) could be impacted by?
  - Inherent (stimulus + sensor + circuit) variation
  - Variation due to different operators (reproducibility)
  - Variation of measurement due to equipment error (repeatability)
- <u>Repeatability</u> refers to the measurement variation obtained when one person repeatedly measures the same item with the same gage.
- <u>Reproducibility</u> refers to the variation due to different operators using the same gage measuring the same item.
- $\sigma_m^2 = \sigma_p^2 + \sigma_o^2 + \sigma_e^2$

Variance due to equipment error

Variance of actual measurement

Variance due to operator

Variance of true product characteristic

Summary

- A measurement specification cannot be quoted based on a few measurements – there is a methodology to be followed in order to correctly specify the performance of a system
- Offset and noise are often the most confused specifications – understand the difference and use appropriate term
- Wherever possible, account for repeatability and reproducibility (R&R) in your measurements and inferences