$$\min_{x} f(x)$$

$$g_1(x)=0;\ g_2(x)=0;\ \cdots;\ g_p(x)=0$$

$$h_1(x)\leq 0;\ h_2(x)\leq 0;\ \cdots;\ h_r(x)\leq 0$$

$$x=(x_1, x_2, \cdots; x_n)^T \qquad x \in R^n;$$

→ f would be differentiable.

& derivatives functions would also be continuous.

→ something similar for g & h's as well

$$\min_{x,y} (x-2)^2 + \left(y-\frac{5}{2}\right)^2 ✓$$

Nonlinear optimization problem.

$$✓ \quad \frac{x^2}{2}+\frac{y^2}{8}\leq 1$$

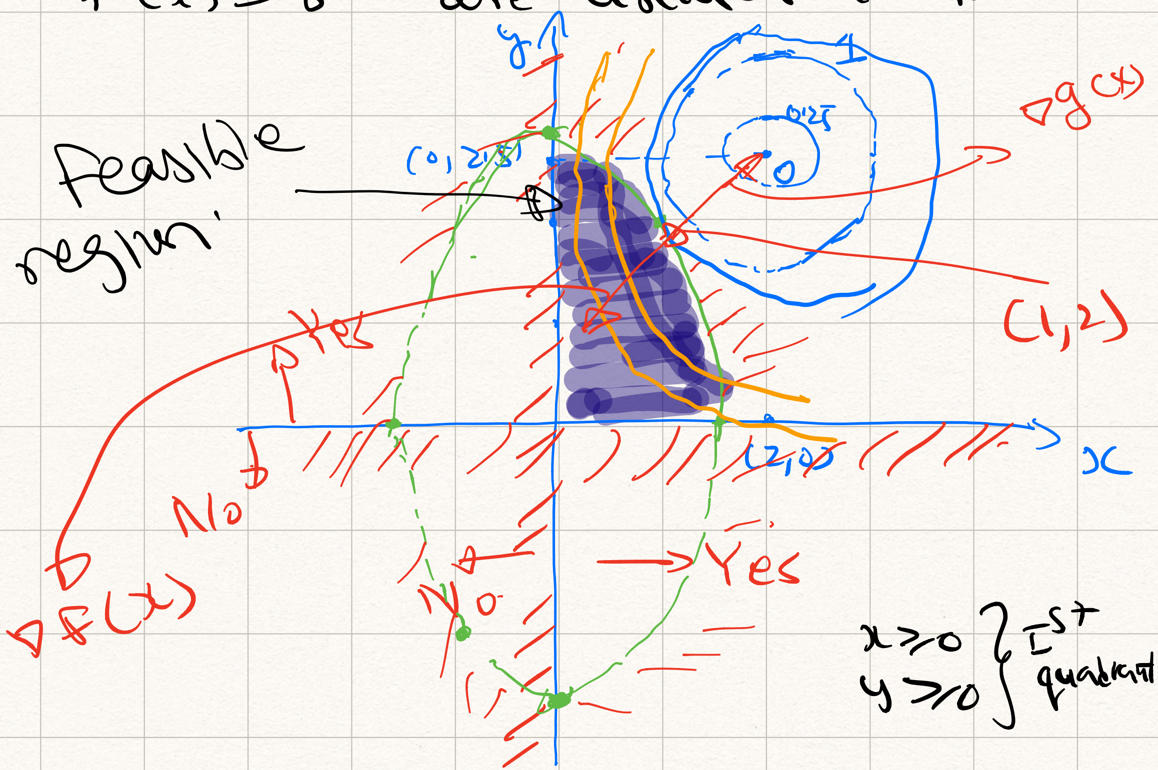
$x \geq 0$ ✓
$y \geq 0$ ✓

simple linear constraints

$$f(x) = \underbrace{(x-2)^2+(y-2.5)^2 = r^2}_{\text{circle.}}$$

$$\nabla f(x^*) = 2\begin{bmatrix}(x-2)\\(y-2.5)\end{bmatrix}$$

$f(x) = r^2$ are circular contours:




$x \geq 0$, $y \geq 0$ } I$^{st}$ quadrant

$$\frac{x^2}{2} + \frac{y^2}{8} \leq 1; \quad \frac{x^2}{2} + \frac{y^2}{8} = 1$$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1; \quad a = \sqrt{2}, \; b = 2\sqrt{2};$$

$(\pm 1, \pm 2)$ also satisfy the ellipse eqn.

Feasible region, pts inside of it are admissible during optimization & vice versa.

unconstrained mini $(2, 2\cdot5)$

obj func' $f(x) = 0$.

e] Is this admissible pt?

We get a minimum when the circles (contours of obj function) just touch, the feasible region which in this case, touching the ellipse in
o $I^{st}$ quardrant.

$x^* = (1, 2)$

$f(1,2) = (1-2)^2 + (2-2.5)^2$

$= 1 + 0\cdot25$

$f(1,2) = 1.25$ @ $(1,2)$

Check:

if ellipse & circle just touch at (1, 2), the $\nabla g(x^*)$ & $\nabla f(x^*)$ should be collinear.

$$\nabla f(x^*) + \lambda \nabla g(x^*) = 0$$

$$\underline{\underline{\nabla f(x^*)}} = -\lambda \underbrace{\underline{\underline{\nabla g(x^*)}}}_{+ve}$$

$$\nabla f(x^*) = 2\begin{bmatrix} (x^*-2) \\ (y^*-2.5) \end{bmatrix} \qquad x^* = (1,2)$$

$$\nabla g(x^*) = \begin{bmatrix} x^* \\ \frac{y^*}{4} \end{bmatrix}$$

$$\frac{x^2}{2} + \frac{y^2}{8} = 1$$

$$\nabla g(x^*) = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$\nabla f(x^*) = 2\begin{bmatrix} -1 \\ -0.5 \end{bmatrix}$$

$$\lambda = 2$$

$$\nabla f(x^*) + \lambda \nabla g(x^*) = 0$$

$$2\begin{bmatrix} -1 \\ -0.5 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = 0$$

lagrange multiplier