→ What is a function?
→ When do we say that a function is continuous
→ Differentiability of function
Univariate Functions

→ Multivariable functions
→ partial derivatives

→ Taylor series expansion.
→ univariate
→ multivariate functions

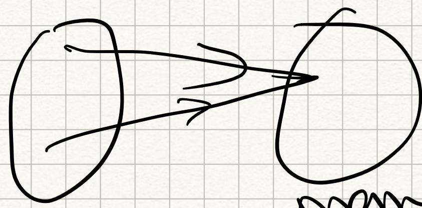
→ What is a function

univariate $f : R \rightarrow R$

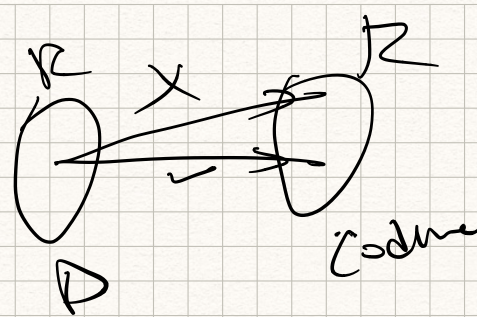
$f : D \rightarrow$ Range
$D \rightarrow$ subset of $\underline{\underline{R}}$

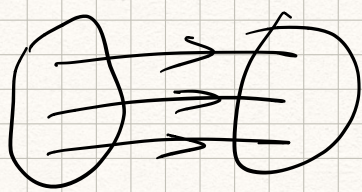
$f : R \rightarrow R$
map
assignment
$x \in D$ or R^x

$y = f(x)$
Range or codomain
$f(x_1) = \underline{\underline{y_1}}$ or $f(x_1) = \underline{\underline{y_2}}$
both x

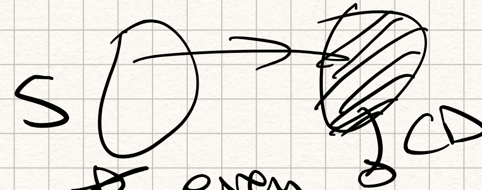


✓ many to one





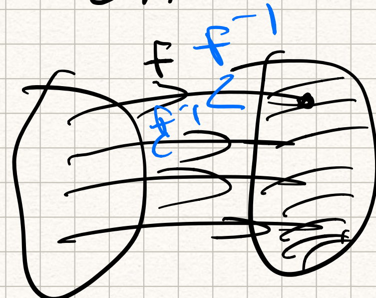
one-to-one



every
$\forall y \in CD$
$\exists x \in S$
$f(x) = y$

on-to

one-one & on-to
invertable functions.



many-to-one even if on-to cannot give you invertable functions

f^{-1} does not exist.

Continuous Functions:

$$f : \mathbb{R} \to \mathbb{R}$$

continuous at x, if given any $\varepsilon > 0$

$\exists\, \delta > 0$ s.t. $\forall\, y \in (x-\delta, x+\delta)$

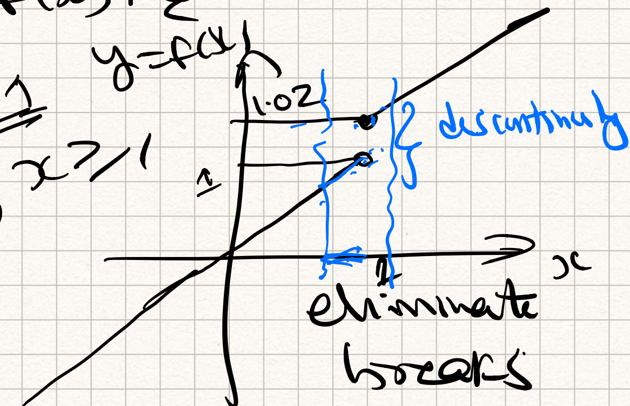
neighborhood

open interval

$f(x) - \varepsilon < f(y) < f(x) + \varepsilon$

$y = f(x) = x,\ x < 1$

$y = x + 0.02,\ x \geq 1$



choose $\varepsilon > 0$

say $\varepsilon = 0.01$

$\delta > 0$

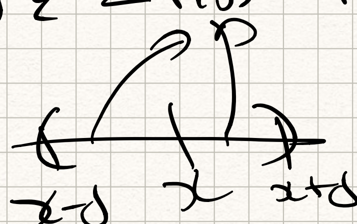
$0.99 = 1 - .01 \leq f(y) \leq 1.01$

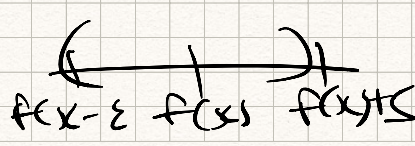
$\varepsilon = 10^{-10}, \quad \varepsilon = 10^{-20}$

$f(x), \quad \varepsilon > 0$

$f(x) - \varepsilon < f(y) < f(x) + \varepsilon$

$|y - x| < \delta$





Derivatives → function should be ~~contin~~ continuous

$$\lim_{h \to 0} \frac{f(x^* + h) - f(x^*)}{h} = f'(x^*)$$

$$f(x) = x^2$$

$$f(x+h) = (x+h)^2 = x^2 + 2hx + h^2$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{2hx + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

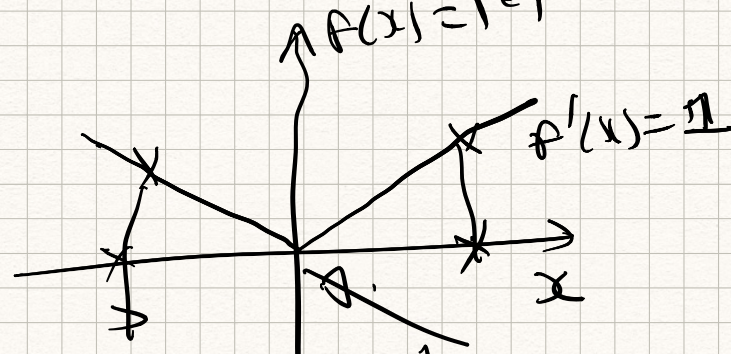
$\lim h \to 0$

$$= 2x$$

$f'(x) = 2x$ when $f(x) = x^2$!

All continuous functions need not have a derivative.

$$f(x) = |x|$$



Yes $f'(x) = -1$

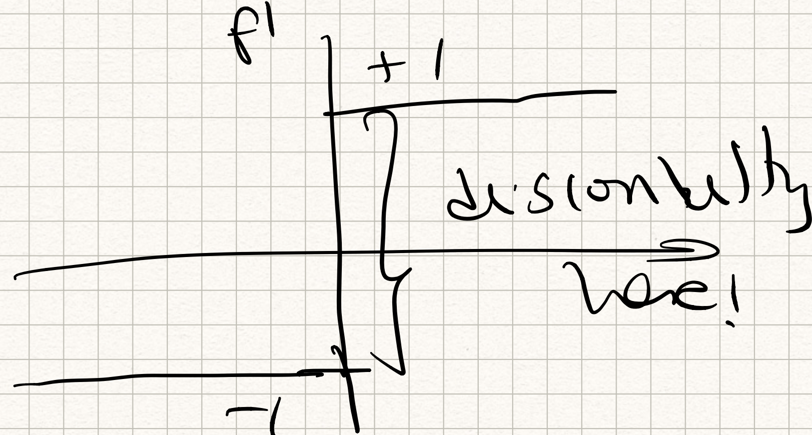
does the derivative exist

from left slope = −1   No!

from right slope = +1

does not exist! $\left\{ \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \right.$

no derivative at $f(0)$



f is continuous but f' is not

a continuous function whose f' is also continuous is a smooth function.

$f \in C^1$

$f \in C^2$ also exists & is continuous

$f, \; f', \; \underline{\underline{f''}} \quad \frac{d^2 f}{dx^2}$

Extend these definitions to multivariate case

<u>Partial</u> derivative

$$y = F(x_1, x_2, \ldots x_n)$$

$$F : \mathbb{R}^n \to \mathbb{R}$$

at

Continuous if ~~there exists~~

$\forall\, \varepsilon > 0, \quad \exists \quad N_\delta(x) = \{y : \|y - x\| < \delta\}$

$$\|y - x\| = \left[(y_1 - x_1)^2 + (y_2 - x_2)^2 \cdots + (y_n - x_n)^2\right]^{1/2}$$

norm

vs distance

Euclidean distance

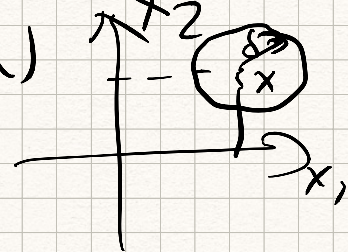
or real distance

s.t $\forall\, y \in N_\delta(x)$

$$|F(y) - f(x)| < \varepsilon$$

$$F(x) - \varepsilon \leq F(y) \leq F(x) + \varepsilon$$

$F(x_1, x_2, \dots, x_n)$



$$\frac{\partial F}{\partial x_i} = \lim_{h \to 0} \frac{F(x_1, x_2, \dots, x_i + h, \dots x_n) - F(x_1, x_2, \dots x_n)}{h}$$

exists for $i = 1 \dots n$.

$$\frac{\partial F}{\partial x_1} \quad \frac{\partial F}{\partial x_2} \quad \dots \quad \frac{\partial F}{\partial x_n} \text{ exist}$$

$$\nabla F = \begin{bmatrix} \frac{\partial F}{\partial x_1} \\ \vdots \\ \frac{\partial F}{\partial x_n} \end{bmatrix}$$