CS 747, Autumn 2020: Week 2, Q&A

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- Typo on slide 4 in Week 2 Lecture 1.

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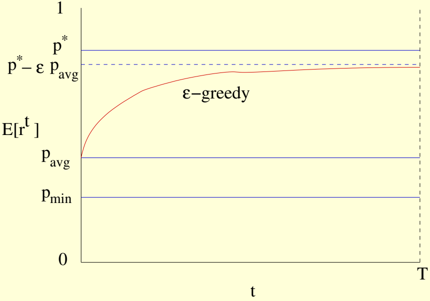
  ϵ-greedy: On each step explore (uniform sampling) w.p. ϵ, exploit w.p. $1 - \epsilon$.



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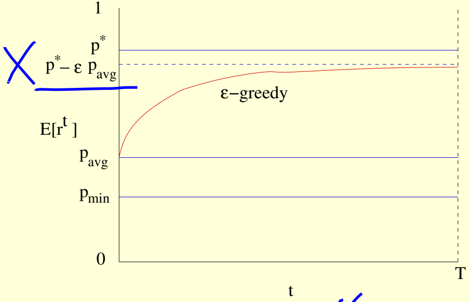

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Must be $p^*(1-\varepsilon) + \varepsilon p_{avg} = p^* - \varepsilon(p^* - p_{avg})$.

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$$exploit^t = \begin{cases} 1 & \text{if for all } a \in A : \hat{p}^t_{a^t} \geq \hat{p}^t_a, \\ 0 & \text{otherwise.} \end{cases}$$

$$exploit(T) = \sum_{t=0}^{T-1} exploit^t.$$

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- Let E_1 be the event that $\hat{p}_1^{100} = 0$ and $\hat{p}_2^{100} > 0$.
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  $\mathbb{P}\{E_2\} = 1 - \mathbb{P}\{\exists t \geq 100, \neg exploit^t\} \geq 1 - \sum_{t=100}^{\infty} \mathbb{P}\{\neg exploit^t\}$

$$= 1 - \sum_{t=100}^{\infty} \frac{1}{2(t+1)^2} = C_2 > 0.$$

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  $$= 1 - \sum_{t=100}^{\infty} \frac{1}{2(t+1)^2} = C_2 > 0.$$
- $\mathbb{P}\{E_1 \wedge E_2\} > C_1 C_2 = C_3 > 0 \implies$ linear regret.

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No! What about that algorithm that always pulls arm 3?

Question 4

- Why did we exclude bandit instances with optimal mean reward of 1 in our result that GLIE $\iff$ sub-linear regret?

- Recall $\bar{\mathcal{I}} = [0, 1)^n$. We showed that GLIE($L$, I) $\forall I \in \bar{\mathcal{I}} \iff$ subLinearRegret(L, I) $\forall I \in \bar{\mathcal{I}}$.

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- Let $\mathcal{I} = [0,1]^n$.
- It is **true** that GLIE(L, I) $\forall I \in \mathcal{I} \implies$ subLinearRegret(L, I) $\forall I \in \mathcal{I}$.
- It is **not true** that subLinearRegret(L, I) $\forall I \in \mathcal{I} \implies$ GLIE(L, I) $\forall I \in \mathcal{I}$. Why? Consider L such that

$$L(h^t) = a^t = \begin{cases} a^{t-1} & \text{if } r^{t-1} = 1, \\ UCB(h^{t-1}) & \text{otherwise.} \end{cases}$$

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- Here's the formula given for ucb-kl$_a^t$:

$$\text{ucb-kl}_a^t = \max(S) \text{ where}$$
$$S = \{q \in [\hat{p}_a^t, 1] \text{ s.t. } u_a^t KL(\hat{p}_a^t, q) \leq \ln(t) + c\ln(\ln(t))\},$$

where $c \geq 3$.

Can we instead write

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Yes!