$$\min_{x_1, x_2} \; (x_1-2)^2 + (x_2-2)^2$$

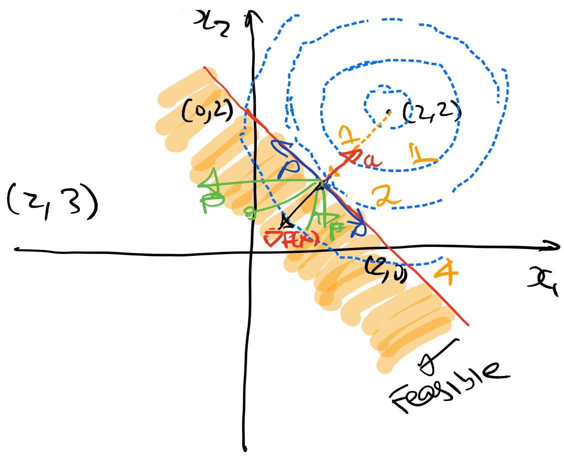
$x_1 + x_2 \le 2$ — (1)

$x_1 \ge 0; \quad x_2 \ge 0$ — (2, 3)

AP=0

$\nabla f(x) + \lambda \cdot a = 0$

$\lambda \ge 0$



min $f(x)$

$\underset{m\times n}{A}\; \underset{n\times 1}{x} \le \underset{m\times 1}{b}$ rank(A) = m;

If x^* is a

$$\nabla f(x^*) + A^T \lambda = [0]$$

$$\lambda \ge 0$$

$\mathcal{L}(x, \lambda) = f(x) + \lambda^T (Ax - b)$ $\quad \sum_{i=1}^{m} \lambda_i (a_i^T x - b_i)$

$\frac{\partial \mathcal{L}}{\partial x} = 0 \Rightarrow \nabla f(x^*) + \lambda^{*T}(Ax - b) = 0$

$Ax^* \le b;$

For active or binding constraints $\lambda \ge 0$;

For inactive or non-binding constraints $\lambda = 0$;

Let x^* be a optimal point; then x^* satisfies the following

$$\nabla f(x^*) + A^T \lambda^* = 0$$

$$\lambda^* \ge 0 \qquad \lambda_i = 0 \text{ if } i \notin \mathcal{A}(x^*)$$

$\mathcal{A}(x^*)$ = set of constraints which are active or binding at x^*.

$$Ax^* = b$$

LEP min $f(x)$

$Ax = b$

Feasible region $\subseteq$

LIP min $f(x)$

$Ax \le b$

Feasible region

$$\nabla f(x^*) + A^T \lambda^* = 0;$$

Point x^* must be optimal w.r.t. feasible directions which make binding constraint(s) non-binding

$Ap \le 0$

$Ap = 0$

$p = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ ✓

$p = \begin{bmatrix} -0.05 \\ -0.1 \\ 0 \end{bmatrix}$ ✓

$p = \begin{bmatrix} -0.05 \\ -1 \\ -2 \end{bmatrix}$ ✓

feasible directions

$p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ not at all!

$Ax^* = b$

$A(x^* + \alpha p)$ $\alpha > 0$

$a_2^T p > 0$

$a_2^T x > b_2$ ✗

Point x^* should be optimal w.r.t.

$Ap \le 0$

$Ap = 0$

$$\nabla f(x^*) + A^T \lambda^* = [0].$$

Let us try to relax a binding constraint and see what addition restriction we have to place of λ^* so that x^* is an optimal.

$f(x^* + \alpha p)$ $\quad\quad$ $\alpha > 0$

s.t. $\quad Ap \le 0$

$f(x^* + \alpha p) \ge f(x^*)$

if $f(x^* + \alpha p) < f(x^*)$ $\alpha \neq 0$ Then it means I am not at optimal!

$$\nabla f(x^*) = -A^T \lambda^*$$

$$f(x^* + \alpha p) = f(x^*) + \alpha \nabla f(x^*)^T p + \frac{1}{2}\alpha^2 p^T H(x^* + \theta \alpha p) p$$

If x^* is optimal

$$\nabla f(x^*)^T p \geq 0$$

$$\nabla f(x^*)^T p = -\lambda^* A p \geq 0$$

$$Ap \le 0 \Rightarrow -Ap \geq 0$$

$$\Rightarrow \lambda^* \geq 0$$

$b_1 - 1$

$$a_1^T p = -1 \qquad Ap = \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

does this eqn have a solur. Yes!

$$\nabla f(x^*)^T p = -[\lambda_1 \cdots \lambda_m] \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$= \lambda_1 \geq 0$$

$$\min \ (x_1 - 2)^2 + (x_2 - 2)^2$$

$x_1 + x_2 \le 2$ $\quad\quad$ $\lambda_1 \geq 0$

$x_1 \geq 0$ $\quad\quad$ $\lambda_2 = 0$

$x_2 \geq 0$ $\quad\quad$ $\lambda_3 = 0$

. . .

6 – choices of binding constraints.

1st choice is important.

$$\nabla f(x^*) = 2\begin{bmatrix}(x_1-2)\\(x_2-2)\end{bmatrix}$$

$$A = [1 \quad 1]$$

$$\nabla f(x^*) + A^T\lambda = [0]$$

$$2\begin{bmatrix}(x_1-2)\\(x_2-2)\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}\lambda_1 = \begin{bmatrix}0\\0\end{bmatrix}$$

$$x_1 + x_2 = 2$$

$$x_1 = x_2 = 1;$$

$$\begin{bmatrix}-2\\-2\end{bmatrix} + \begin{bmatrix}1\\1\end{bmatrix}2 = \begin{bmatrix}0\\0\end{bmatrix}$$

$$x_1^* = x_2^* = 1 \qquad \lambda^* = 2.$$

First order necessary conditions are met.

Second order necessary

$Z^T H(x^*) Z$ should be positive semidefinite

Z is the null space basis of active constraints

Further, following is one set of sufficiency conditions:

(1) $\lambda_i > 0 \quad i \in A(x^*)$ [strictly optimal w.r.t. non-binding perturbation]

(2) $Z^T H(x^*) Z$ should be positive definite.

$\nabla f(x^*)^T p = 0$

(1) & (2) strictly optimal w.r.t. to binding perturbations