A FIRST COURSE IN OPTIMIZATION
EE 659
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1. We begin with modelling examples for following problems:
   - Stochastic resource allocation problem
   - Facility location problem
   - Diet Planning problem.

2. Then, we review concepts like continuity, differentiability, smoothness of univariate & multivariate functions. We review mean value theorem & Taylor series. We introduce concept of open & closed sets, vectors, convergence & vector norms. This is followed by Weirstrass Theorem, which gives sufficiency conditions for existence of optimal solution. We also introduce notion of infimum & supremum.

3. Next, we dive into theory of unconstrained optimization. We derive first & second order necessary conditions for optimality of differentiable functions. Then, we discuss some sufficiency conditions. In the process we learn, terms like gradient, Hessian, positive definiteness of matrices etc. LU decomposition will be reviewed.

4. We repeat the above (item-3) for linearly constrained optimization problem. In the process, we introduce Lagrangian function, projected Hessian etc. This will also provide us, opportunity to review linear algebra fundamentals, vector spaces like range space & null space.

5. We will now revert and take up a very interesting, important & yet simple problem, called linear least squares. We will derive, optimality conditions, given by normal equations. This helps us to appreciate weirstrass theorem as well. We introduce generalized inverse, in particular Penrose Moore inverse. Projection matrices will also be introduced. Some examples & numericals will be taken.

6. Next, we shift attention to equality constrained non-linear optimization. We will derive necessary first order and second order conditions for x^* to be optimal. We solve following examples from physics & economics

   (a) Law of reflection in optics
   (b) Law of refraction in optics
   (c) Markowitz criteria for portfolio optimization.

7. We will dive deeper, now, into inequality constrained, non-linear optimization problems. We will derive necessary conditions for optimality and optimality conditions for sufficiency, both first order and second order. Some examples as tutorial will be worked out.

8. Next, we introduce, a very interesting class of optimization problems called convex programming. Here in, necessary conditions are also sufficient for optimality! These problems have a strong geometrical interpretation as well.

9. We will close the course with a simpler, yet very important class of problems called, linear programming problems where in both, objective function and constraints are linear. Notion of duality gap will be shown. Simplex method will be introduced.

REFERENCES.

1. Bazara, Sherali & Shetty, Nonlinear Programming, theory and algorithms, IInd Edition

2. Philip. E. Gill, Walter Murray and Margaret. H. Wright : Practical Optimization

3. Dimitri P. Bertsekas, Non-linear Programming, IInd Edition .