Basic Concepts of Guidance

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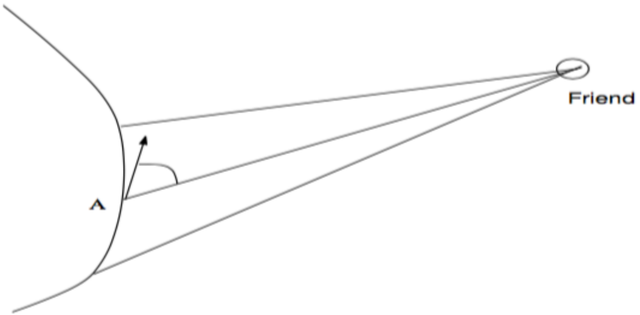
- Flight control system: Autopilot
  - $\Rightarrow$ Stabilize the missile
  - $\Rightarrow$ Control of missile in its flight
  - $\Rightarrow$ Ensure that missile airframe responds effectively to guidance commands.
- Roll, pitch and yaw autopilots
- Pitch and yaw autopilots: lateral autopilots
- Nature of gains in autopilots
  - $\Rightarrow$ Change in altitude for SAMs.
  - $\Rightarrow$ Change in missile's velocity and weight
  - $\Rightarrow$ Atmospheric temperature, pressure.

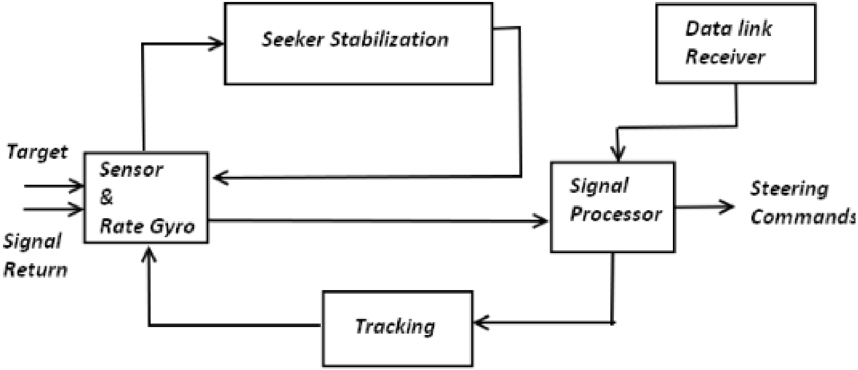
Basic Concepts of Guidance

Guidance Systems in Tactical Missiles

- Guidance system
  - ⇒ Target acquisition
  - ⇒ Target tracking
  - ⇒ Seeker stabilization
  - ⇒ Steering command generation

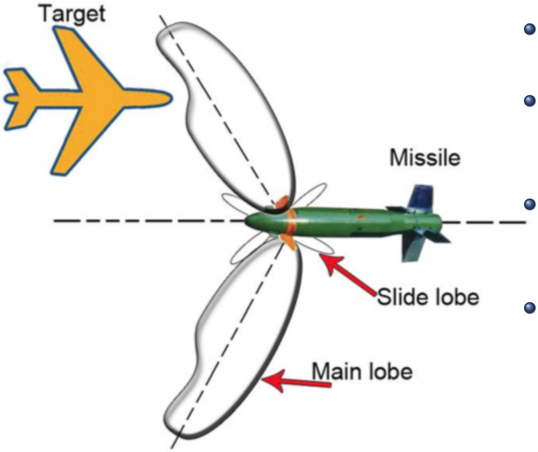


- Seeker stabilization: Decoupling of seeker from body motion.



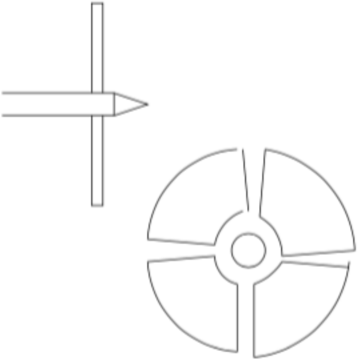
Data link receiver is not always necessry (as in a purely homing guidance system). similarly, the seeker in a command guidance system may not be present onboard

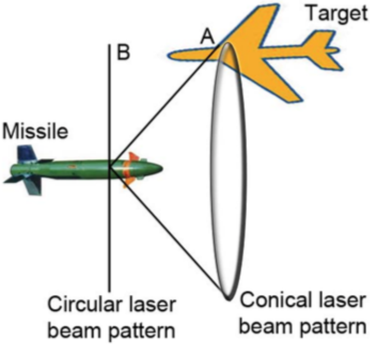
- Direct hit of target happens rarely.
- Usually missile comes very close to target, the missile senses this event and detonate its fuze.
- Proximity fuze:
  - $\Rightarrow$ RF proximity fuze
  - $\Rightarrow$ Laser proximity fuze
- **RF proximity fuze**:
  - $\Rightarrow$ Two CW radars placed diametrically opposite on two sides of missile behind guidance system.
  - $\Rightarrow$ When target enters this pattern, receiving antennas receives reflected energy.
  - $\Rightarrow$ Doppler frequency is extracted from this signal and is used to generate the fuze pulse.
  - $\Rightarrow$ An in-built range cut-off which suppresses reflected signals from objects at larger distance.
  - $\Rightarrow$ Avoidance of detonation due to signals reflected from ground or sea or nearby objects like buildings, etc.



- Two other checks to prevent a false alarm.
- Doppler signal is passed through a bandpass filter to check signal bandwidth.
- A threshold detector to check the minimum level of reflected signal.
- Threshold detector also eliminates the possibility of ambiguous range measurements.

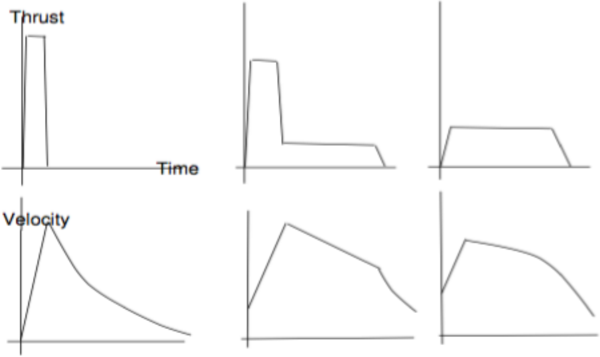
- Laser proximity fuze uses a laser source as the active transmitter and an infrared detector as the receiver.
- High frequency energy helps in obtaining very accurate information about the target position.
- Four emitters and receivers are mounted on the missile at 90 degrees from each other.
- Each produces a sector-shaped pattern with 90 degrees angular spread.
- Combination of four patterns produces a circular pattern of a definite radius and very small thickness.
- Received signal is passed through some simple circuitry to extract the necessary information to generate the fuze pulse.
- Possibility of false alarm in laser fuzes is very small compared to the RF fuze.



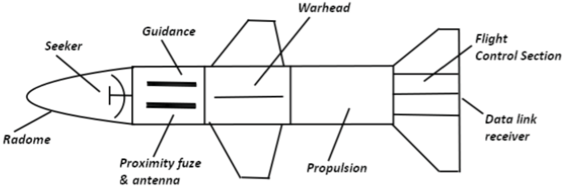


What is the advantage of using circular conical beam pattern?

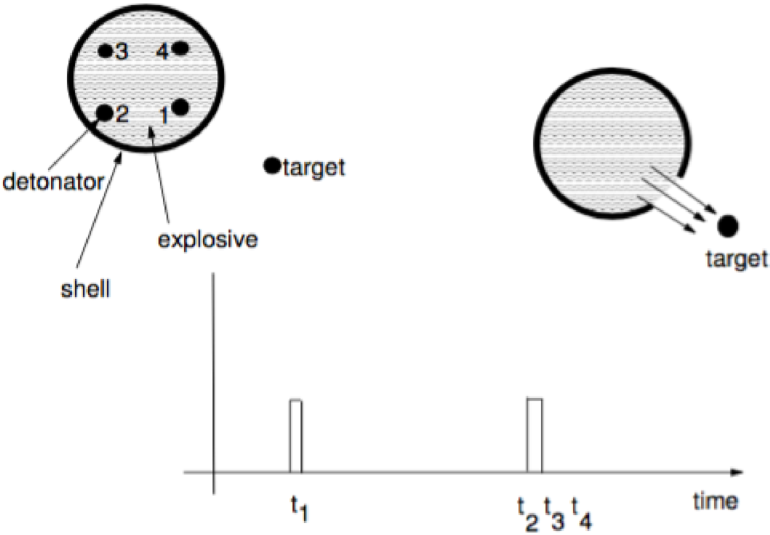
- Propulsion system: Provides required initial thrust to missile and make it fly with sufficient velocity during subsequent engagement period.
- Phases in missile propulsion
  - ⇒ Boost: high level of missile acceleration over a relatively short period of time.
  - ⇒ Sustain: maintain missile at a desired speed for majority of remaining flight.



- **Warhead**: payload of the missile
  - ⇒ A shell
  - ⇒ Explosives
  - ⇒ Detonator



- Weight of the warhead depends on the size of the missile.
- Fuze pulse activates detonator, which triggers the explosive.
- Shell breaks into numerous fragments, propelled outward in a 60-90 degrees spread and achieves target kill by penetrating target components.



3
4
2
1
detonator
shell
explosive
target
target
t_1
t_2 t_3 t_4
time

- **Guidance**: To direct one object to move in such a way as to enable it to come as close as possible to another object.
- **Interception**: An interception or capture is said to have taken place if the positions of the two objects coincide.
- Can we still call it an interception if the exact positions of two objects do not coincide?
- If the closest distance is within the lethal radius then target can be destroyed.
- This results into "target interception".
- Effect of guidance on relative motion of an object with respect to another object: study of kinematics of engagement
- Interception and avoidance: Two sides of similar concept

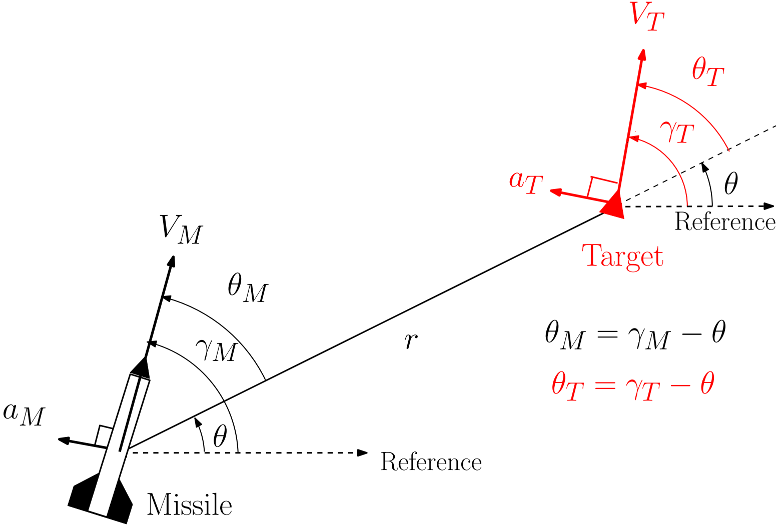
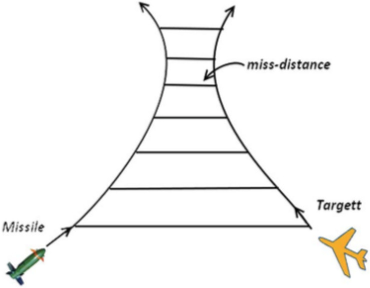


Figure: Planar missile-target engagement

- **Missile lateral acceleration**: Acceleration that missile needs to apply in order to achieve a desired turn rate.
- It is usually applied in a direction close to the normal to the missile velocity vector.
- In fact, the guidance command generated by the guidance law is usually expressed as a lateral acceleration.
- It is also called as commanded lateral acceleration and is given as an input to the missile autopilots.
- As the autopilot are dynamical systems, there may be a difference in achieved and commanded lateral accelerations.
- Difference in acceleration may also result due to saturation effect.
- Why saturation of lateral acceleration occurs in missile?

- **Line-of-sight**: Imaginary line joining the missile and the target at any given instant in time is called the instantaneous line-of-sight (LOS).
- LOS may change in both length and orientation as the engagement proceeds.
- The change in its angular orientation is given by its angular velocity or rate of turn, usually expressed in units of radians/sec.
- This rate is called as line-of-sight rate (LOS rate).

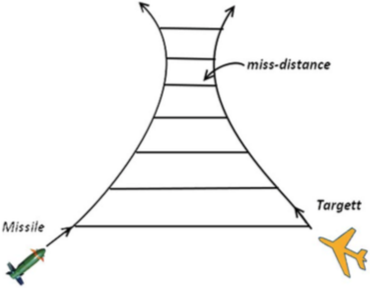


- **Closing velocity**: It is the velocity with which the missile closes on to the target.
- This is measured as the rate at which the length of the LOS or the LOS separation shrinks.
- Hence, it is the negative of the rate of change of the LOS separation or range rate.

$$\boxed{V_c = -\dot{r}}$$

- It is also the doppler relative velocity of the target with respect to the missile along the line-of-sight (LOS).

- **Miss distance**: Distance of closest approach of the missile to the target.
- When missile directly hits target, the miss distance is zero.
- When missile passes close to target, the miss distance is non-zero.
- Proximity fuze detonates the warhead and the engagement comes to an end.
- Guidance system tries to achieve direct hit.
- If it is not possible to obtain direct hit then try to minimize miss-distance.



- Can miss distance be a negative quantity?

- The distance between the missile and target is denoted by r and is also referred to as the LOS separation.
- Angles are measured w.r.t. some fixed reference, called as LOS angle, θ.
- Engagement dynamics

$$
\begin{aligned}
\dot{r} &= V_r = V_T \cos(\gamma_T - \theta) - V_M \cos(\gamma_M - \theta) \\
r\dot{\theta} &= V_\theta = V_T \sin(\gamma_T - \theta) - V_M \sin(\gamma_M - \theta) \\
\dot{\gamma}_M &= \frac{a_M}{V_M} \\
\dot{\gamma}_T &= \frac{a_T}{V_T}
\end{aligned}
$$

- Consider the engagement when there is no guidance system and target is assumed to have constant velocity.

$$a_M = 0, \quad a_T = 0.$$

- V_r and V_θ are components of relative velocity of target w.r.t. missile.
- Speeds of missile and target are constant during engagement.
- On differentiating V_r with respect to time

$$\begin{aligned}\dot{V}_r =& -V_T\sin(\gamma_T-\theta)(\dot{\gamma}_T-\dot{\theta})+V_M\sin(\gamma_M-\theta)(\dot{\gamma}_M-\dot{\theta})\\ =& -V_T\sin(\gamma_T-\theta)(-\dot{\theta})+V_M\sin(\gamma_M-\theta)(-\dot{\theta})\\ =&\dot{\theta}\left[V_T\sin(\gamma_T-\theta)-V_M\sin(\gamma_M-\theta)\right]\\ =&\dot{\theta}V_\theta\end{aligned}$$

- On differentiating V_θ with respect to time

$$\begin{aligned}\dot{V}_\theta =& V_T\cos(\gamma_T-\theta)(\dot{\gamma}_T-\dot{\theta})-V_M\cos(\gamma_M-\theta)(\dot{\gamma}_M-\dot{\theta})\\ =& V_T\cos(\gamma_T-\theta)(-\dot{\theta})-V_M\cos(\gamma_M-\theta)(-\dot{\theta})\\ =& -\dot{\theta}\left[V_T\cos(\gamma_T-\theta)-V_M\cos(\gamma_M-\theta)\right]\\ =& -\dot{\theta}V_r\end{aligned}$$

- From $\dot{V}_r$ and $\dot{V}_\theta$, we can write

$$\dot{\theta} = \frac{\dot{V}_r}{V_\theta} = -\frac{\dot{V}_\theta}{V_r}$$

- We can obtain the relation

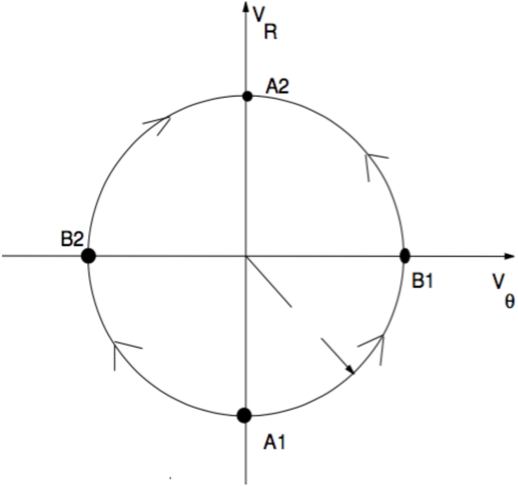
$$V_r \dot{V}_r = -V_\theta \dot{V}_\theta \Rightarrow V_r \dot{V}_r + V_\theta \dot{V}_\theta = 0$$

- On integrating above equation, we get

$$V_r^2 + V_\theta^2 = c^2 (\text{constant})$$

- Equation of a circle in (V_r, V_θ) space with radius c and centre at origin.
- If initial values of $V_r,\ V_\theta$ are

$$V_r(0) = V_{r_0}, \quad V_\theta(0) = V_{\theta_0} \Rightarrow c = \sqrt{V_{r_0}^2 + V_{\theta_0}^2}$$



How to find direction of movement?

Missile-Target Engagement

- Dynamics of V_r and V_θ

$$\dot{V}_r = \dot{\theta} V_\theta, \quad \dot{V}_\theta = -\dot{\theta} V_r$$

- On multiplying LOS separation, r, on both equations, we get

$$r\dot{V}_r = r\dot{\theta} V_\theta = V_\theta^2, \quad r\dot{V}_\theta = -r\dot{\theta} V_r = -V_r V_\theta$$

- As LOS separation is always positive, that is, $r > 0$,

$$r\dot{V}_r = V_\theta^2 > 0 \Rightarrow \dot{V}_r > 0$$

- If V_r and V_θ are of opposite signs

$$\dot{V}_\theta > 0 \text{ if } V_r V_\theta < 0$$

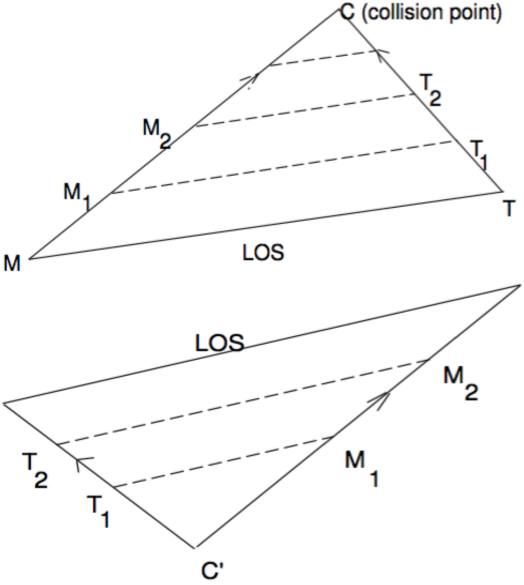
- If V_r and V_θ are of same signs

$$\dot{V}_\theta < 0 \text{ if } V_r V_\theta > 0$$

- When $V_\theta = 0$, we also have $\dot{V}_\theta = 0$.
- These conditions correspond to the stationary points A_1, A_2.

$$\dot{V}_r = 0, \quad \dot{V}_\theta = 0$$

- At point A_1, $V_r < 0$. What does it mean?
  - $\Rightarrow$ LOS separation shrinks at constant rate if state is on A_1 and after finite time it becomes zero leading to the collision of missile and target.
  - $\Rightarrow$ **Collision condition**: $\boxed{V_\theta = 0, \quad V_r < 0}$.
- At point A_2, $V_r > 0$, LOS separation increases with constant rate.
- For $V_\theta = r\dot{\theta} = 0$, LOS angle θ remains constant with time.
  - $\Rightarrow$ LOS does not rotate in space.
- **Collision triangle**: A triangle which the trajectories of missile and target make in case of non-rotating LOS.
- Point A_2 corresponds to inverse collision triangle.



C (collision point)
T_2
M_2
T_1
M_1
T
M
LOS
LOS
M_2
T_2
M_1
T_1
C'

- At points B_1, B_2, we have $V_r = 0, V_\theta \neq 0$. What does it mean?
- In the neighbourhood of these points, $V_r < 0$ just prior to these points and $V_r > 0$ immediately after these points.
- It defines the distance of closest approach, that is, miss distance.
- How to get expressions for the time and the distance of closest approach?
- Consider the trajectories in (V_r, V_θ)-space.

$$V_r^2 + V_\theta^2 = \; c^2$$

- Using $r\dot{V}_r = V_\theta^2$,

$$V_r^2 + r\dot{V}_r = c^2 \implies \dot{r}^2 + r\ddot{r} = \; c^2$$

- How to proceed further?

- Using the fact that $\dfrac{dr\dot{r}}{dt} = \dot{r}^2 + r\ddot{r}$, we can write

$$\frac{dr\dot{r}}{dt} = c^2 \Rightarrow r\dot{r} = c^2 t + b,\ b = r_0 V_{r_0},$$

where r_0 is the initial separation between missile and target and V_{r_0} is initial value of range rate.

- At points B_1 and B_2, range rate $V_r = 0,\ r = r_{\text{miss}},\ t = t_{\text{miss}}$

$$c^2 t_{\text{miss}} + b = 0 \Rightarrow t_{\text{miss}} = -\frac{b}{c^2}$$

- On substituting for b and c, we get $\boxed{t_{\text{miss}} = -\dfrac{r_0 V_{r_0}}{c^2} = -\dfrac{r_0 V_{r_0}}{V_{r_0}^2 + V_{\theta_0}^2}}$

- How to obtain miss distance?

Miss Distance

- As we know that

$$r\dot{r} = c^2 t + b$$

- On integration, we get

$$\frac{r^2}{2} = \frac{c^2 t^2}{2} + bt + a, \quad a = \frac{r_0^2}{2}$$

- Expression for range

$$r = \sqrt{c^2 t^2 + 2bt + 2a}$$

- Miss distance

$$r_{\text{miss}} = \sqrt{c^2 t_{\text{miss}}^2 + 2bt_{\text{miss}} + 2a} = r_0 \sqrt{\frac{V_{\theta_0}^2}{V_{r_0}^2 + V_{\theta_0}^2}}$$

Reference

1. D. Ghose, *Lecture notes on Navigation, Guidance and Control*, Indian Institute of Science, Bangalore.