Deviated Pursuit Guidance

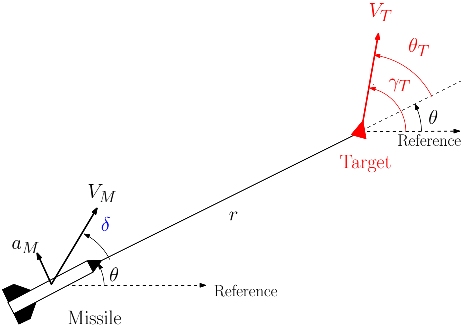
Dr. Shashi Ranjan Kumar

Assistant Professor
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Powai, Mumbai, 400076 India

Deviated Pursuit Guidance

Philosophy of Deviated Pursuit

- At all instants in time, V_M should be directed towards a point that deviates from the current target position by a constant angle δ.



- Target is assumed to be a non-maneuvering.
- Equations of relative motion

$$\dot{r} = V_r = V_T \cos(\gamma_T - \theta) - V_M \cos\delta$$

$$r\dot{\theta} = V_\theta = V_T \sin(\gamma_T - \theta) - V_M \sin\delta$$

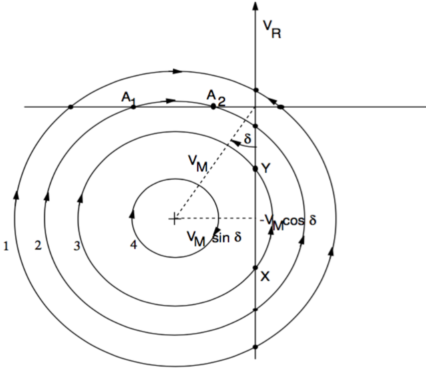
- Trajectories in (V_θ, V_r) space

$$
\begin{aligned}
V_r + V_M \cos\delta &= V_T \cos(\gamma_T - \theta) \\
V_\theta + V_M \sin\delta &= V_T \sin(\gamma_T - \theta)
\end{aligned}
$$

- Locus of V_θ, V_r as

$$(V_\theta + V_M \sin\delta)^2 + (V_r + V_M \cos\delta)^2 = V_T^2$$

- Equation of a circle with radius V_T and centered at $(-V_M \sin\delta, -V_M \cos\delta)$.
- What about the direction of movement of the point on (V_θ, V_r)-space?



Deviated Pursuit Guidance

Direction of Movement

- On differentiating V_r, V_θ

$$\dot{V}_r = -V_T \sin(\gamma_T - \theta)\left(\dot{\gamma}_T - \dot{\theta}\right) = \dot{\theta}(V_\theta + V_M \sin\delta)$$
$$\dot{V}_\theta = V_T \cos(\gamma_T - \theta)\left(\dot{\gamma}_T - \dot{\theta}\right) = -\dot{\theta}(V_r + V_M \cos\delta)$$

- On multiplying r on both sides,

$$r\dot{V}_r = r\dot{\theta}(V_\theta + V_M \sin\delta) = V_\theta(V_\theta + V_M \sin\delta)$$
$$r\dot{V}_\theta = -r\dot{\theta}(V_r + V_M \cos\delta) = -V_\theta(V_r + V_M \cos\delta)$$

- Observations: As $r > 0$

$$\dot{V}_r = \begin{cases} \text{Positive} & \text{if } V_\theta(V_\theta + V_M \sin\delta) > 0 \\ \text{Negative} & \text{if } V_\theta(V_\theta + V_M \sin\delta) < 0 \end{cases}$$

$$\dot{V}_r > 0 \Rightarrow V_\theta > 0 \;\&\; V_\theta > -V_M \sin\delta \text{ or } V_\theta < 0 \;\&\; V_\theta < -V_M \sin\delta$$
$$\dot{V}_r < 0 \Rightarrow V_\theta > 0 \;\&\; V_\theta < -V_M \sin\delta \text{ or } V_\theta < 0 \;\&\; V_\theta > -V_M \sin\delta$$

- We have

$$r\dot{V}_\theta = -r\dot{\theta}(V_r + V_M\cos\delta) = -V_\theta(V_r + V_M\cos\delta)$$

- Observations: As $r > 0$

$$\dot{V}_\theta = \begin{cases} \text{Positive} & \text{if } V_\theta(V_r + V_M\cos\delta) < 0 \\ \text{Negative} & \text{if } V_\theta(V_r + V_M\cos\delta) > 0 \end{cases}$$

$$\dot{V}_\theta > 0 \Rightarrow V_\theta > 0 \;\&\; V_r < -V_M\cos\delta \text{ or } V_\theta < 0 \;\&\; V_r > -V_M\cos\delta$$
$$\dot{V}_\theta < 0 \Rightarrow V_\theta > 0 \;\&\; V_r > -V_M\cos\delta \text{ or } V_\theta < 0 \;\&\; V_r < -V_M\cos\delta$$

- Points where the circle cuts the V_r-axis are stationary points.
- At these points $V_\theta = 0$ and $\dot{V}_\theta = 0$ and $\dot{V}_r = 0$.
- Points on the negative V_r axis correspond to the collision triangle and those on the positive V_r axis correspond to the inverse collision triangle.

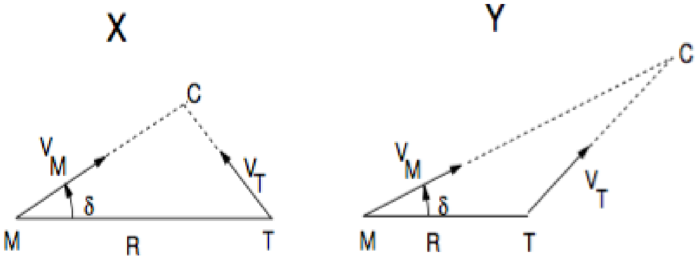
- How to find collision triangle for deviated pursuit guidance?
- Missile has to always point at an angle deviated by δ from the current LOS.
- For missile and target to be on a collision course

$$V_T \sin(\gamma_T - \theta) = V_M \sin\delta, \quad V_r < 0$$

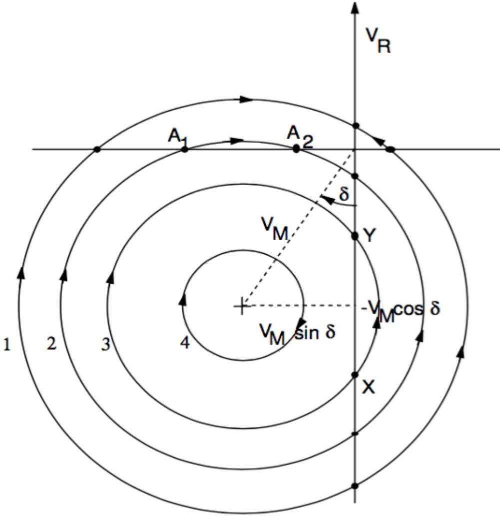
- For $\delta = 0$, there are two possibilities:

$$\gamma_T - \theta = 0, \ \pi$$

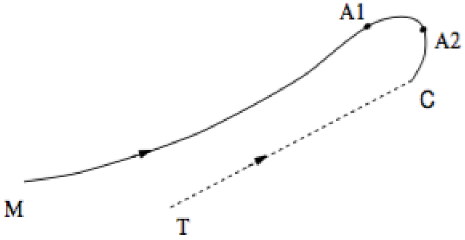
- Head-on and tail-chase scenarios



- Consider $\delta < \frac{\pi}{4}$.
- Capture regions
  - $\Rightarrow$ $V_T > V_M$
  - $\Rightarrow$ $V_M \cos\delta < V_T < V_M$
  - $\Rightarrow$ $V_M \sin\delta < V_T < V_M \cos\delta$
  - $\Rightarrow$ $V_T < V_M \sin\delta$
- Largest circle corresponds to $V_T > V_M$.
- Except for those initial conditions which are on negative V_r axis, all other points end up on positive V_r axis.
- No interception.



- Points corresponding to Circle 2 lead to interception because they end up on the negative V_r axis.
- However, initial conditions in 3^{rd} quadrant first move into positive V_r region and then come back to negative V_r region before hitting negative V_r axis.
- Point A_1 on the trajectory is the point of closest approach before the missile overshoots the target, turns at point A_2, and then intercepts the target.

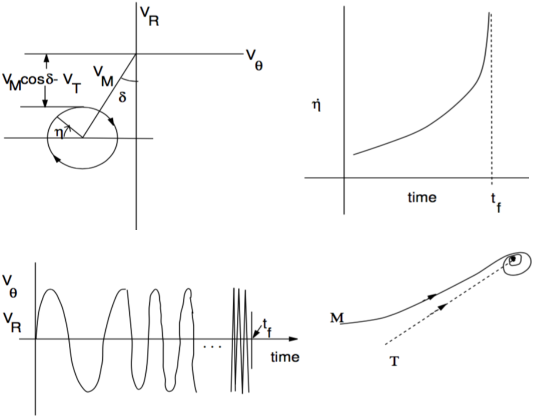


- Points corresponding to Circle 3 also lead to interception, but the trajectory remains in the negative V_r region.

Deviated Pursuit Guidance

Pursuit Guidance: Capture Regions

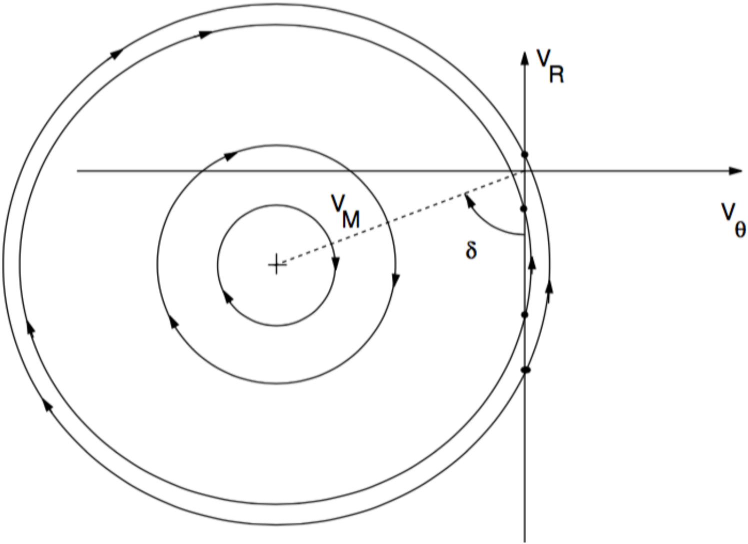
- Points corresponding to Circle 4 also lead to interception, but in this case the interception is somewhat different from the previous cases.



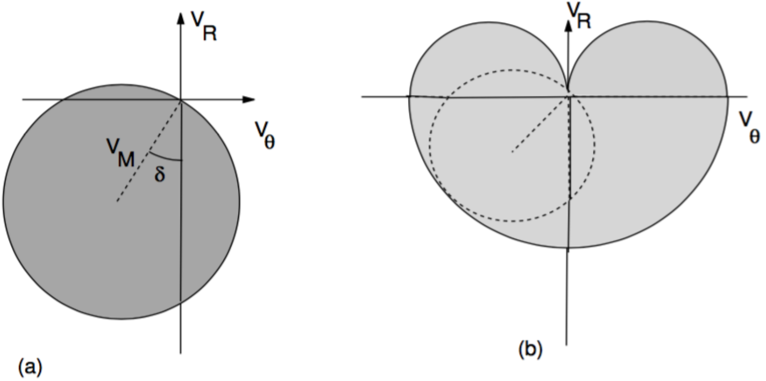
$$V_r \leq -V_M \cos\delta + V_T, \quad t_f \leq \frac{r}{V_M \cos\delta - V_T}$$

Deviated Pursuit Guidance

Pursuit Guidance for $\delta > \pi/4$



- Even in this case, if the initial geometry does not satisfy the collision triangle condition the capture is possible if and only if $V_T < V_M$.
- Capture region for the deviated pursuit guidance law for a fixed δ.



Deviated Pursuit Guidance

Capture Regions

- For a fixed δ, the capture region for the deviated pursuit guidance law is of the same size as the pure pursuit guidance law.
- Capture circle is now rotated by an angle δ clockwise.
- Capture region includes a portion of the positive V_r region.
- Deviated pursuit guidance performs better than pure pursuit guidance.
- deviation angle δ: A guidance parameter
- With $\delta \in (-\pi/2, \pi/2)$, total capture region will be union of all individual capture regions for each δ.
- For $\delta < 0$, capture region is obtained by rotating capture region for pure pursuit guidance law ($\delta = 0^\circ$) anti-clockwise.
- If we consider δ to be a freely selected guidance parameter then the capture region expands considerably.
- Can you guess why we do not consider $\delta > \pi/2$ or $\delta < -\pi/2$ to expand the capture region even further?

- How to compute time of interception in deviated pursuit guidance?
- Trajectories of V_θ, V_r

$$(V_r + V_M \cos\delta)^2 + (V_\theta + V_M \sin\delta)^2 = V_T^2$$
$$V_r^2 + V_M^2 \cos^2\delta + 2V_rV_M\cos\delta + V_\theta^2 + V_M^2\sin^2\delta + 2V_\theta V_M \sin\delta = V_T^2$$

- On rearranging we get

$$V_r^2 + 2V_rV_M\cos\delta + V_\theta^2 + 2V_\theta V_M\sin\delta = V_T^2 - V_M^2$$
$$V_r^2 + 2V_rV_M\cos\delta + V_\theta(V_\theta + V_M\sin\delta) + V_\theta V_M\sin\delta = V_T^2 - V_M^2$$
$$\underbrace{\dot{r}^2 + (r\dot{V}_r)}_{\frac{d(r\dot{r})}{dt}} + \underbrace{2\dot{r}V_M\cos\delta}_{2V_M\cos\delta\frac{dr}{dt}} + V_\theta V_M\sin\delta = V_T^2 - V_M^2$$

- How to express third term in terms of derivatives?

Deviated Pursuit Guidance

Time of Interception

- We know that

$$r\dot{V}_\theta = -V_\theta V_r - V_\theta V_M \cos\delta \Rightarrow V_\theta V_M = -\frac{r\dot{V}_\theta + V_\theta V_r}{\cos\delta}$$

- We can rewrite

$$V_\theta V_M \sin\delta = -\left(r\dot{V}_\theta + V_\theta V_r\right)\tan\delta = -\frac{d(rV_\theta)}{dt}\tan\delta$$

- Using this result,

$$\frac{d(rV_r)}{dt} + 2V_M \cos\delta \frac{dr}{dt} - \frac{d(rV_\theta)}{dt}\tan\delta = V_T^2 - V_M^2$$

- On integration,

$$r(V_r + 2V_M\cos\delta - V_\theta \tan\delta) = (V_T^2 - V_M^2)t + c$$
$$c = r_0 V_{r_0} + 2V_M \cos\delta r_0 - r_0 V_{\theta_0}\tan\delta$$

- Time of interception or time-to-go for deviated pursuit guided missile

$$t_{\mathrm{go}} = -\frac{c}{V_T^2 - V_M^2} = -\frac{r_0[V_{r_0} + 2V_M \cos\delta - V_{\theta_0}\tan\delta]}{V_T^2 - V_M^2}$$

- If interception occurs, then the terminal value of a_M is given by

$$a_M \rightarrow \begin{cases} \text{Finite} & 1 < \nu \leq \dfrac{2}{\sqrt{1 + 3\sin^2\delta}} \\ \\ \infty & \nu > \dfrac{2}{\sqrt{1 + 3\sin^2\delta}} \end{cases}$$

- Engagement dynamics

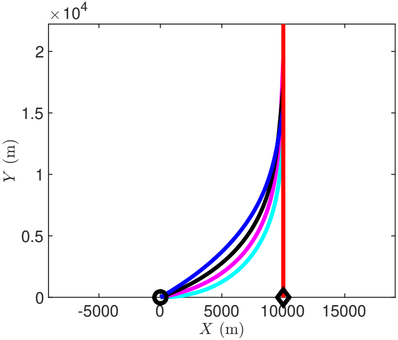
$$\dot{r} = V_T \cos\theta_T - V_M \cos\delta, \quad \dot{\theta}_T = -\dot{\theta} = \frac{-V_T \sin\theta_T + V_M \sin\delta}{r}$$

$$\implies \frac{dr}{d\theta_T} = \frac{r(\cos\theta_T - \nu\cos\delta)}{-\sin\theta_T + \nu\sin\delta}$$

- For $\nu > 1$ and $|\nu \sin\delta| < 1$, we may obtain

$$r(\theta_T) = C\frac{\sin^{\mu-1}\left(\frac{\theta_T - \beta}{2}\right)}{\cos^{\mu+1}\left(\frac{\theta_T + \beta}{2}\right)}, \ \beta = \sin^{-1}\left[\nu\sin\delta\right], \mu = \nu\frac{\cos\delta}{\cos\beta} = \frac{\nu\cos\delta}{\sqrt{1-\nu^2\sin^2\delta}}$$

- Missile acceleration for deviated pursuit: $a_M = V_M\dot{\theta}$.
- Analyze the expression for a_M when it remains finite.
- For bounded lateral acceleration, $\nu \leq \dfrac{2}{\sqrt{1+3\sin^2\delta}}$



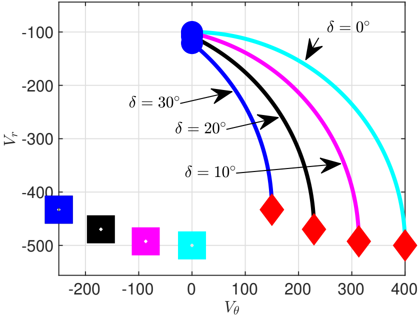


Figure: Target interception using deviated pursuit guidance

- Deviation angles of $0°, 10°, 20°, 30°$
- Different V_r at interception, different time of interception, different centre of relative trajectories

Pursuit Guidance

Implementations: different deviation angles

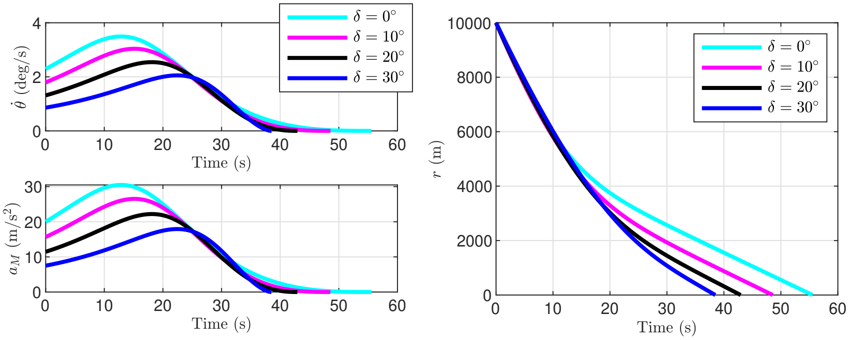


Figure: Target interception using deviated pursuit guidance

- Acceleration demand and LOS rate converge to zero at interception

- For pursuit guidance, missile is assumed to be initially on a pursuit course.
- What happens if missile points in a direction different from pursuit geometry (pure or deviated) initially?
  - $\Rightarrow$ Missile applies maximum lateral acceleration till it is on a pursuit course and then applies pursuit lateral acceleration.
  - $\Rightarrow$ If there is no bound on missile lateral acceleration, then missile can turn instantaneously and then apply pursuit acceleration.
- What are the problems with such approach?
  - $\Rightarrow$ Open-loop in nature.
  - $\Rightarrow$ Errors in measurements, and mismatch between missile flight angle and LOS angle, will lead to large miss-distances.

Deviated Pursuit Guidance

Engagement Scenarios

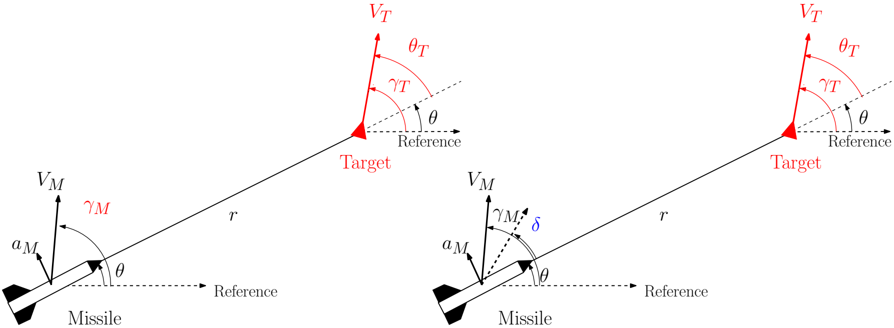


Figure: General engagement geometries for pure and deviated pursuit guidance

- A practical implementation: Use of a feedback law.

$$a_M = -K(\gamma_M - \theta),\ K > 0$$

- This may not be enough and we need to also ensure $\dot{\gamma}_M = \dot{\theta}$.
- Missile lateral acceleration required to maintain turn rate

$$\dot{\gamma}_M = \frac{a_M}{V_M} \Rightarrow a_M = V_M \dot{\theta}$$

- An implementable pure pursuit guidance law would have a form

$$a_M = V_M \dot{\theta} - K(\gamma_M - \theta)$$

- Implementable deviated pursuit guidance law

$$a_M = V_M \dot{\theta} - K(\gamma_M - \theta - \delta)$$

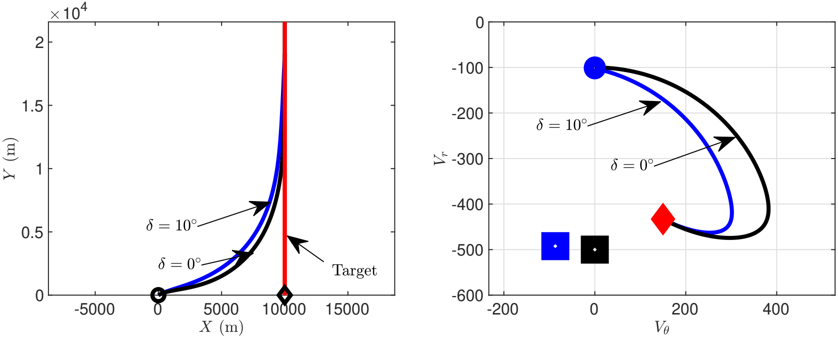


Figure: Target interception using pure and deviated pursuit guidance

- $\gamma_T = 90°$,$\gamma_M = 30°$,$\theta = 0°$, V_M= 500 m/s, V_T= 400 m/s
- Gain K=300

Pursuit Guidance

Pursuit Guidance: pure and deviated pursuit

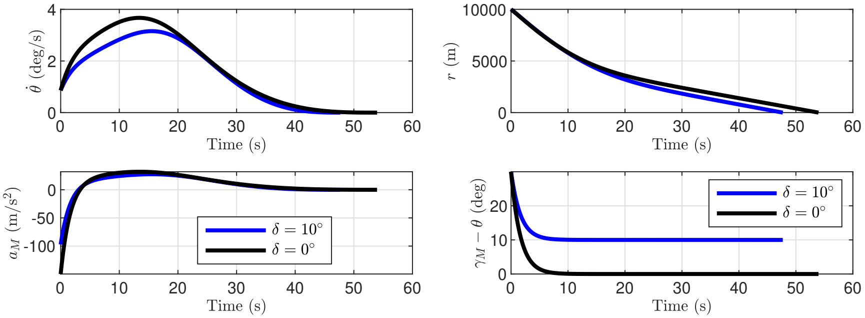


Figure: Target interception using pure and deviated pursuit guidance

- Initial deviation angle of 30° and Desired deviation angles of $0^\circ, 10^\circ$
- Acceleration demand and LOS rate converge to zero at interception

Reference

1. D. Ghose, *Lecture notes on Navigation, Guidance and Control*, Indian Institute of Science, Bangalore.
2. N. A. Shneydor, *Missile Guidance and Pursuit: Kinematics, Dynamics, and Control*, Woodhead Publishing, 1998.

Thank you for your attention !!!