Teaching phase-sensitive demodulation for signal conditioning to undergraduate students

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Phase-sensitive demodulation (PSD) is important for signal conditioning and communication. This paper introduces an approach to teaching PSD for signal conditioning to undergraduate science and engineering students. The goal is to help students understand the principles of PSD for signal conditioning because they can see waveforms at various test points and the effect of the phase angle on the output of a PSD circuit using conventional laboratory equipment. Students can also gain practical skills by building electronic circuits, including a switch-based PSD circuit, a phase shifter, a switch-driving circuit, and a low-pass filter, on a breadboard using real electronic components.

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I. INTRODUCTION

Phase-sensitive demodulation (PSD) is important for signal conditioning in electronic instrument design and for communication. The device used to implement PSD is often called a "lock-in amplifier." Although the same principle of PSD is used for both signal conditioning and for communication, there are significant differences between these two applications of PSD. This paper is concerned only with PSD for signal conditioning used in instrumentation.

A typical signal conditioning circuit consists of amplifier(s), ac to dc converter(s), and filter(s). To convert an ac signal to a dc signal, either a rectifier or a PSD unit can be used. The major difference between a rectifier and a PSD unit is that a rectifier can give information only on the amplitude of the analog signal, typically a sine wave, while a PSD unit can give information on both the amplitude and the phase angle of the ac signal compared to a reference signal, which can be a sine wave or square wave. Therefore, a rectifier can provide a single measurement, but in principle a PSD unit can provide two independent measurements.

A PSD circuit has two ac input signals (sine wave or square wave) of the same frequency. Its output is a function of not only the amplitudes of both input signals but also the phase difference between the two input signals. In this paper, a PSD unit consists of a PSD circuit plus a low-pass filter. The output of a PSD circuit consists of a dc component and high-frequency harmonics. To separate out the dc component, a low-pass filter is needed to eliminate the harmonics. An example of the use of a PSD is to measure the capacitance and loss-conductance of a medium such as an oil/water mixture. Figure 1 shows a simple capacitance and lossconductance measuring circuit, where R_x and C_x represent the electrical properties of the medium that are to be determined.

The output of the circuit V_o consists of two components, one in-phase and the other in quadrature-phase, that is, with 90° phase shift,

$$V_o = -\left(j\omega C_x R_f + \frac{R_f}{R_x}\right) V_i = V_{\text{in-phase}} + jV_{\text{quadrature-phase}}, \quad (1)$$

where

$$V_{\text{in-phase}} = -\frac{R_f}{R_x} V_i, \tag{2}$$

$$V_{\text{quadrature-phase}} = -\omega C_x R_f V_i. \tag{3}$$

The in-phase component $V_{\text{in-phase}}$ is related to the unknown resistance R_x (or loss-conductance), and the quadrature-phase component $V_{\text{quatrature-phase}}$ is proportional to the unknown capacitance component C_x . If $V_{\text{in-phase}}$ and $V_{\text{quatrature-phase}}$ can be measured by a PSD unit, then R_x and C_x can be determined from the two measured components as

$$R_{x} = \frac{R_{f}}{V_{\text{in-phase}}} V_{i}, \tag{4}$$

$$C_x = \frac{V_{\text{quadrature-phase}}}{\omega R_f V_i}.$$
 (5)

This example indicates that both the capacitance and lossconductance can be measured by a single PSD unit if the phase angle of the reference signal can be programmed to be in-phase or have a 90° phase difference. If only the capacitance needs to be measured, the advantage of using a PSD unit is that the capacitance measurement would not be affected by the loss-conductance because the signal resulting from the loss-conductance and the signal resulting from the capacitance have a 90° phase difference. The circuit in Fig. 1 has been used in an ac-based electrical capacitance tomography system.

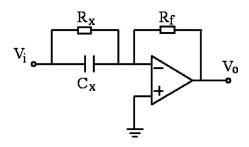


Fig. 1. Capacitance and loss-conductance measuring circuit showing the use of PSD.

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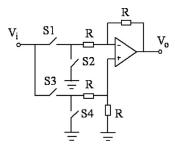


Fig. 2. Switch-based PSD with an op-amp and four analog switches controlled by two complementary square waves.

In contrast, if a rectifier is used, then the output of the rectifier will be 6

$$|V_o| = V_i \frac{R_f}{R_x} \sqrt{(\omega C_x R_x)^2 + 1}$$

$$= \sqrt{(V_{\text{in-phase}})^2 + (V_{\text{quadrature-phase}})^2}.$$
(6)

In this case, C_x and R_x cannot be separated from each other. If a rectifier is used to measure capacitance of a medium with loss-conductance, the capacitance measurement will be affected by the loss-conductance.

The motivation for teaching the principle of PSD for signal conditioning is that students find it difficult to understand PSD because of its complexity but can easily see how it works because they can see waveforms at various test points and see the effect of the phase angle on the output of a PSD circuit. Also, it is good for undergraduate science and engineering students to gain practical skills using conventional laboratory equipment and real electronic components.

II. PRINCIPLE OF PSD

There are two types of PSD for signal conditioning: Switch-based and analog multiplier-based. In principle, an analog multiplier-based PSD is simpler than a switch-based PSD. Consider two sine-wave signals input to an analog multiplier,

$$V_i = A \sin(\omega t + \alpha), \tag{7}$$

$$V_f = B \sin(\omega t), \tag{8}$$

where V_i is a signal to be measured and V_f is a reference signal.

The output of the multiplier is

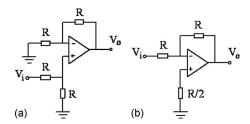


Fig. 3. Equivalent circuits for the two phases.

Table I. Four typical cases of switch-based PSD.

ϕ			
(°)	$\cos \phi$	dc output	
		2 <i>A</i>	
0	1	$\overline{\pi}$	
90	0	0	
		2A	
180	-1	$-\frac{\pi}{\pi}$	
270	0	0	

$$V_o = \frac{AB}{2} \left[\cos \alpha - \cos(2\omega t + \alpha) \right],\tag{9}$$

which consists of two components, a dc signal and an ac signal with twice the input frequency. If a low-pass filter with a sufficiently low cutoff frequency is used to eliminate the cosine-wave signal, the remaining dc signal is

$$V_o = \frac{AB}{2}\cos\alpha. \tag{10}$$

Equation (10) indicates that the output of the analog multiplier-based PSD is not only a function of the amplitude of the input signal A but also a function of the phase angle α .

The main advantage of an analog multiplier-based PSD is that it is insensitive to harmonics. The main disadvantage is that analog multipliers are expensive. In contrast, a switch-based PSD is inexpensive, but it is affected by odd harmonics.^{6,7}

For this laboratory, switch-based PSD is chosen because it is easier for students to see waveforms at various test points in a switch-based PSD circuit than an analog multiplier-based PSD circuit, and the electronic components for building a switch-based PSD circuit are less expensive than an analog multiplier-based PSD circuit.⁸

Figure 2 shows a commonly used switch-based PSD circuit. It consists of an op-amp, which is configured as a differential amplifier, and four single-pole single-throw (SPST) analog switches, which are controlled by two complementary square waves.

The circuit is operated in two phases. As indicated in Fig. 2(b), in phase 1, switches S2 and S3 are ON and S1 and S4 are OFF. In this case the circuit in Fig. 2(a) is equivalent to a noninverting amplifier with unit gain as shown in Fig. 3(a). In phase 2, S1 and S4 are ON and S2 and S3 are OFF. In this case the circuit in Fig. 2(a) is equivalent to an inverting amplifier also with unit gain as shown in Fig. 3(b). In other words, in phase 1 the gain of the circuit is 1 and in phase 2 the gain is -1. Unlike the circuits discussed by Temple² and Wolfson,³ this circuit does not have problems with loading effects.

Let's assume the input signal is a sine wave,

$$V_i = A \sin(\omega t + \phi), \tag{11}$$

where A is the amplitude and ϕ is the phase angle of the sine wave relative to a square-wave reference. Our task is to see

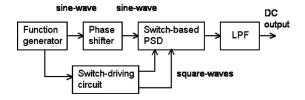


Fig. 4. Overall arrangement for laboratory.

how both the amplitude A and the phase angle ϕ affect the output of the PSD circuit. The switching function can be described by

gain =
$$\begin{cases} 1 & \text{when } 0 < \omega t < \pi \\ -1 & \text{when } \pi < \omega t < 2\pi \end{cases}$$
 (12)

so that

Fig. 6. Switch-based PSD circuit with switch-driving circuit.

$$V_o = \begin{cases} V_i = A \sin(\omega t + \phi) & 0 < \omega t < \pi \\ -V_i = A \sin(\omega t + \phi) & \pi < \omega t < 2\pi. \end{cases}$$
 (13)

The average output of the PSD circuit can be calculated by integration, ^{6,7}

$$\bar{V} = \frac{1}{2\pi} \left[\int_{0}^{\pi} V_{o} d\omega t + \int_{\pi}^{2\pi} V_{o} d\omega t \right] = \frac{1}{2\pi} \left[\int_{0}^{\pi} A \sin(\omega t + \phi) d\omega t - \int_{\pi}^{2\pi} A \sin(\omega t + \phi) d\omega t \right]
= \frac{1}{2\pi} \left\{ \left[-A \cos(\omega t + \phi) \right]_{0}^{\pi} + \left[A \cos(\omega t + \phi) \right]_{\pi}^{2\pi} \right\}
= \frac{1}{2\pi} \left\{ A \cos \phi - A \cos(\pi + \phi) + A \cos(2\pi + \phi) - A \cos(\pi + \phi) \right\} = \frac{2A}{\pi} \cos \phi.$$
(14)

Equation (14) represents the output of the PSD after the harmonics is removed by a low-pass filer with a sufficiently low cutoff frequency. This analytical result indicates that the output of the switch-based PSD unit is a function of not only the amplitude of the input sine-wave signal A, but also the phase difference ϕ between the input sine-wave signal and the reference square wave. Equation (14) for switch-based PSD is similar to that given in Eq. (10) for analog multiplier-based PSD. Table I lists four typical cases: ϕ =0°, 90°, 180°, and 270°.

In theory, if the output of the low-pass filter is positive when the phase difference between the two signals is 0° , the output must be negative when the phase difference is 180° . If the phase difference is either 90° or 270° , the output must be zero. Note that for 0° phase difference, the output is the same as an output of a full-wave rectifier.

III. LAB EQUIPMENT, ELECTRONIC COMPONENTS, AND ARRANGEMENT

Figure 4 shows the overall arrangement for the apparatus. A function generator is used to provide a sine wave and a square wave. The phase difference between the sine wave and the square wave from this function generator is inherently 90°. The sine wave goes through a phase shifter so that the phase difference between the sine wave from the phase shifter and the original sine wave or the square wave from the function generator can be adjusted.

Students are asked to build four units, which are shown in Fig. 4: a phase shifter, a switch-based PSD, a switch-driving circuit, and a low-pass filter. The other units are standard laboratory equipment, including a function generator (TTi TG315), which supplies two signals: A sine wave and a syn-

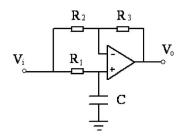


Fig. 5. Phase shifter circuit.

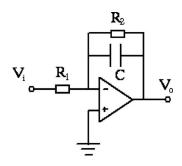


Fig. 7. Simple first-order low-pass filter circuit.

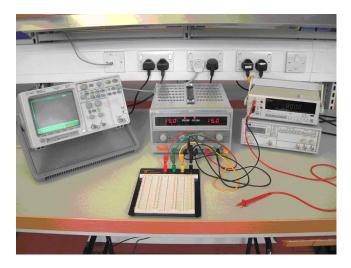


Fig. 8. Correct positions of equipment.

chronous square wave; a triple dc power supply with ± 15 and +5 V outputs (Digimess HY3003–3); a dual-trace digital oscilloscope (Agilent); and a digital multimeter (Millennium DM441B).

Figure 5 shows the detailed circuit for the phase shifter, which is constructed using an op-amp (μ A741). It has unit gain and can give a phase shift between 0° and -180° . The phase shift (when $R_2 = R_3$) can be calculated from 9,10

$$\phi = -2 \tan^{-1}(\omega CR_1), \tag{15}$$

where $\omega = 2\pi f$ and f is the input signal frequency (Hz). For the following experiment, 1 kHz is chosen to be the signal frequency.

The trimmer R_1 is used to vary the phase shift. For f = 1 kHz, C = 1 μ F, and R_1 varying from 0 to 200 k Ω , the phase angle can be calculated from Eq. (15) to be between 0° and -179.9° .

This circuit is also called an all-pass filter because in principle, a signal of any frequency can pass the circuit without reducing its amplitude if the op-amp is ideal. ^{10,11}

Figure 6 shows the detailed circuit for the switch-based PSD with the switch-driving circuit. Four CMOS switches in

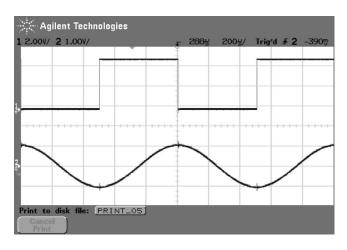


Fig. 9. Sine wave and square wave from function generator, indicating a 90° phase difference.

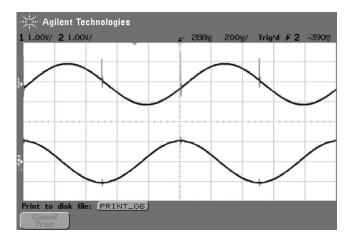


Fig. 10. Two sine waves: One from the function generator and the other from the phase shifter.

a single IC (ADG412), which can be operated by TTL-level logic devices, are used to control the demodulation process. The switch-driving circuit consists of an inverter (74LS04) and an AND gate (74LS08) and is used to provide two complementary square-wave signals from a single square wave provided by the function generator.

As discussed in Sec. II, when S2 and S3 are closed (ON) and S1 and S4 are open (OFF), the PSD circuit functions as a unit-gain noninverting amplifier. When S1 and S4 are closed (ON) and S2 and S3 are open (OFF), the PSD circuit functions as an inverting amplifier. Because this type of PSD relies on a switching function, it is called a switch-based PSD.

A low-pass filter is commonly used to eliminate high-frequency noise. Following the switch-based PSD, a simple first-order low-pass filter, as shown in Fig. 7, is used to eliminate the high-frequency harmonics produced by the PSD circuit. The cutoff frequency of this low-pass filter is ^{9,12}

$$f_o = \frac{1}{2\pi R_2 C}. ag{16}$$

If $R_1=R_2=15.8$ k Ω and C=1 μ F, the cutoff frequency calculated from Eq. (15) is 10 Hz, which is sufficiently low to

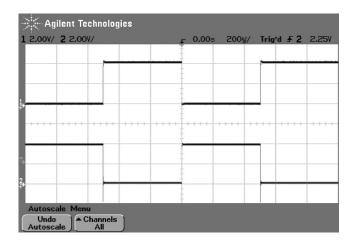


Fig. 11. Two complementary square waves.

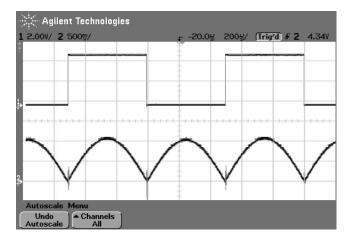


Fig. 12. Output of PSD when the sine wave and square wave are in-phase.

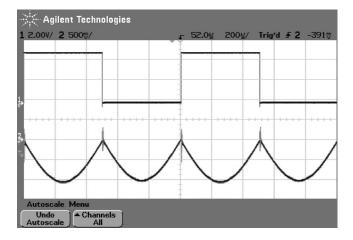


Fig. 13. Output of PSD when the phase difference between the sine wave and square wave is 180° .

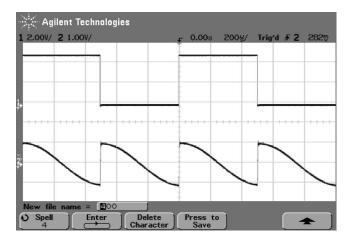


Fig. 14. Output of PSD when the phase difference between the sine wave and square wave is 90° .

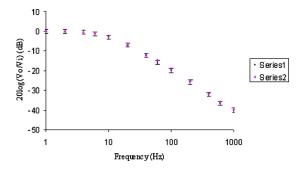


Fig. 15. Bode plots of a low-pass filter indicating a cutoff frequency of $10\ \mathrm{Hz}.$

eliminate the harmonics higher than 1 kHz. The gain of this low-pass filter in its pass band is 1. After low-pass filtering, the remained dc signal is $\bar{V}=(2A/\pi)\cos\phi$ as given in Eq. (14).

IV. PROCEDURE

Students are asked to complete all experiments in two sessions, with 3 h in each session. Laboratory demonstrators (teaching assistants in the U.S.) are arranged, and students do experiment in groups, with three students in each group. Students are asked to arrange the equipment as shown in Fig. 8 and carry out experiments according to the following steps.

- Step 1. Build circuits. Students build individual circuits, one by one on the breadboard, and test each circuit separately
- Step 2. Set function generator. Adjust the frequency of the function generator to 1 kHz. Choose a sine wave and adjust the amplitude of the sine wave to 2 V_{p-p}. A sine wave and a square wave can be observed on the oscilloscope as shown in Fig. 9.
- Step 3. Test phase shifter. Connect the sine-wave signal from the function generator to the phase shifter. Two sine-wave signals, the input signal to the phase shifter and the output signal from the phase shifter, can be observed on the oscilloscope as shown in Fig. 10. By adjusting the trimmer R_1 , the phase shift can be observed.
- Step 4. Test switch-driving circuit. Connect the TTL square-wave signal from the function generator to the switch-driving circuit. Two complementary square-wave signals with a 50/50 duty cycle from the inverter (74LS04) and the AND gate (74LS08) can be observed as shown in Fig. 11.
- Step 5. Test in-phase PSD. Connect both the sine-wave signal from the phase shifter and the two square-wave signals from the switch-driving circuit to the PSD. Adjust the phase shifter using the oscilloscope to make sure that the output sine wave is in-phase with the square wave connected to S2 and S3. The output signal from the PSD can be compared with the square wave as shown in Fig. 12. It

Table II. dc output of low-pass filter with different phase angles.

Phase difference (°)	0	90	180	270
dc output of LPF (V)	-0.637	0	0.637	0

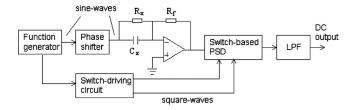


Fig. 16. Measurement of capacitance and resistance by PSD.

might be necessary to swap the square-wave signal connected to S2 and S3 with that connected to S1 and S4 to obtain an in-phase and a 180° phase shift between the sine wave and the square wave connected to S2 and S3.

- Step 6. Test out-phase PSD. Adjust the phase shifter to make sure that the phase difference between the output sine wave and the square wave is 180°. The output of the PSD and the square wave can be observed as shown in Fig. 13.
- Step 7. Test quadrature-phase PSD. Adjust the phase shifter to make sure that the phase difference between the sine wave and the square wave is 90°. The output from PSD and the square wave can be observed as shown in Fig. 14.
- Step 8. Check the output of low-pass filter circuit. Adjust the amplitude of the sine wave to 2 V_{p-p}. Connect the sine-wave signal from the function generator to the input of the low-pass filter directly without connecting to the PSD circuit. Measure the amplitude-frequency response of the low-pass filter circuit using the oscilloscope. Draw a Bode plot as shown in Fig. 15. Note that a log scale must be used for the frequency on the x-axis 20 log(output/input) must be calculated for the y-axis. Because of the log scale for frequency, the test input frequencies should be selected as 1, 2, 4, 6, 10, 20, 40, 60, 100, 200, 400, 600, and 1000 Hz, for example, rather than being linear. Find the cutoff frequency (-3 dB). As indicated in Fig. 15, the cutoff frequency in this example is 10 Hz, above which the plot is linearly reduced by 20 dB/decade. This is the basic feature of first-order low-pass filters. Note that in Fig. 15, both the calculated and measured values are shown, where Series 1 represents the calculated values with ±1 dB error bars and Series 2 represents the measured values.
- Step 9. Measure the dc output of the low-pass filter with different phase angles. Connect the low-pass filter circuit to the PSD circuit. Measure the dc output of the low-pass filter when the phase angle between the sine wave and the reference square wave is 0°, 90°, 180°, and 270°. Table II lists the measured values.

V. LABORATORY REPORT

Each student is asked to submit a report within 2 weeks after the second session is finished. The report should include the title of the experiment, aims of the experiment, brief statement of the principle of switch-based PSD, a calculation of the resistance value of the trimmer R_1 for 90° phase shift, a drawing of a complete circuit diagram showing all individual components, and all connections between them, including the power supply, all pin numbers and decoupling capacitors if any, experimental results and data analysis, including all waveforms captured and explanation, and a statement of what was gained in terms of experience, understanding of the principles of PSD and low-pass filter, and practical skills from the experiment.

Our experience has been that this approach can help science and engineering students understand the principle of phase-sensitive demodulation because they can see waveforms at various test points and see the effect of the phase angle on the output of the PSD. Our survey results show that over 99% of the students appreciate that they have gained a better understanding, which would be difficult to learn from lectures only and have acquired practical laboratory skills. A similar laboratory was run for M.Sc. students for many years and also was found to be successful. ¹³

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APPENDIX: CAPACITANCE AND LOSS-CONDUCTANCE MEASUREMENT WITH PSD

To evaluate the performance of the switch-based PSD, a simple test was done by combining the circuit in Fig. 1 and the diagram in Fig. 4 (see Fig. 16), using the following parameters: Excitation frequency—1 kHz; excitation amplitude: 2 V_{p-p}; and feedback resistor— R_f : 10 k Ω . Three sets of capacitors (C_x) and resistors (R_x) in parallel were tested, with the following nominal values: 10 nF and 5.1 k Ω , 10 nF and 10 k Ω , and 10 nF and 15.8 k Ω . The true capacitor and resistor values were measured and are given in Table III and are slightly different from the nominal values.

The phase of the sine wave from the phase shifter was adjusted to be in quadrature phase with a square-wave signal

Table III. True and measured capacitance and resistance values.

Test	True C_x (nF)	True R_x (k Ω)	$\begin{array}{c} \text{Measured} \\ V_{\text{quadrature-phase}} \\ \text{(V)} \end{array}$	Measured $V_{\text{in-phase}}$ (V)	Measured C_x (nF)	Measured R_x (k Ω)	Relative error of C_x (%)	Relative error of R_x (%)
1	10.02	5.09	0.296	0.953	9.87	5.01	1.5	1.6
2	10.02	9.93	0.295	0.488	9.83	9.78	1.9	1.5
3	10.02	15.53	0.297	0.312	9.90	15.30	1.2	1.5

from the switch-driving circuit to measure the capacitance component and in-phase to measure the resistance component. By using Eqs. (4) and (5), the capacitance and resistance values can be calculated with some correction coefficients due to the gains of the circuits. Table III gives the values, where the relative error is defined as

relative error =
$$\frac{\text{measured value - true value}}{\text{true value}} \times 100\%$$
. (A1)

This simple test demonstrates that the PSD can be used to measure both capacitance and resistance values with an error of less than 2%.

59 (6), 569–572 (1991).

⁴K. G. Libbrecht, E. D. Black, and C. N. Hirata, "A basic lock-in amplifier experiment for the undergraduate laboratory," Am. J. Phys. **71** (11), 1208–1213 (2003).

⁵W. Q. Yang, "Further developments in an ac-based capacitance tomography system," Rev. Sci. Instrum. **72** (10), 3902–3907 (2001).

⁶C. Fraser and J. Milne, *Integrated Electrical and Electronic Engineering* for Mechanical Engineers (McGraw-Hill, Maidenhead, 1994).

⁷W. Q. Yang, *Electronic Circuit Design II Teaching Notes* (The University of Manchester, Manchester, 2009); available by request from the author.

⁸W. Q. Yang, *Electronic Circuit Design II Lab Manual* (The University of Manchester, Manchester, 2009); available by request from the author.

⁹S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits* (McGraw-Hill, New York, 1998).

¹⁰ A. J. Peyton and V. Walsh, *Analog Electronics with Op Amps* (Cambridge U. P., Cambridge, 1993).

¹¹W. Q. Yang, "Non-ideal op-amp circuit analysis," Int. J. Electrical Eng. Educ. 47 (1), 122–134 (2010).

¹²H. Ahmed and P. J. Spreadbury, Analogue and Digital Electronics for Engineers (Cambridge U. P., Cambridge, 1984).

¹³ W. Q. Yang, "Organising a successful MSc course in instrumentation," in Proceedings of the Third American Society for Engineering Education International Colloquium on Eng. Educ, Sept. 7–10, 2004, Beijing, China.

BEQUEST OF PAVLOV TO THE ACADEMIC YOUTH OF HIS COUNTRY

What can I wish to the youth of my country who devote themselves to science?

Firstly gradualness. About this most important condition of fruitful scientific work I can never speak without emotion. Gradualness, gradualness and gradualness. From the very beginning of your work, school yourself to severe gradualness in the accumulation of knowledge.

Learn the ABC of science before you try to ascent to its summit. Never begin the subsequent without mastering the proceeding. Never attempt to screen an insufficiency of knowledge even by the most audacious surmise and hypothesis. Howsoever this soap-bubble will rejoice your eyes by its play it inevitably will burst and you will have nothing except shame.

School yourself to demureness and patience. Learn to inure yourselves to drudgery in science. Learn, compare, collect the facts! Perfect as is the wing of a bird, it never could raise the bird up without resting on air. Facts are the air of science. Without them you never can fly. Without them your "theories" are vain efforts.

But learning, experimenting, observing, try not to stay on the surface of the facts. Do not become the archivists of facts. Try to penetrate to the secret of the occurrence, persistently search for the laws which govern them.

Secondly, modesty. Never think that you already know all. However highly you are appraised, always have the courage to say of yourself – I am ignorant.

Do not allow haughtiness to take you in possession. Due to that you will be obstinate where it is necessary to agree, you will refuse useful advice and friendly help, you will lose the standard of objectiveness.

Thirdly, passion. Remember that science demands from a man all his life. If you had two lives that would not be enough for you. Be passionate in your work and in your searchings.

Written just before Pavlov's death, at the age of eighty-seven years, on February 27, 1936. Translated from the Russian by Professor P. Kupalov, chief assistant in the Pavlov Institute at Leningrad. Quotation suggested by Bartley Cardon.

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¹R. Taylor, *Data Acquisition for Sensor Systems* (Chapman and Hall, London, 1997).

²P. A. Temple, "An introduction to phase-sensitive amplifiers: An inexpensive student instrument," Am. J. Phys. **43** (9), 801–807 (1975).

³R. Wolfson, "The lock-in amplifier: A student experiment," Am. J. Phys.