Lecture 1: The first course in optimization.

$\min f(x)$ or $\max -(x)$

S $x \in$. $x \in S$.

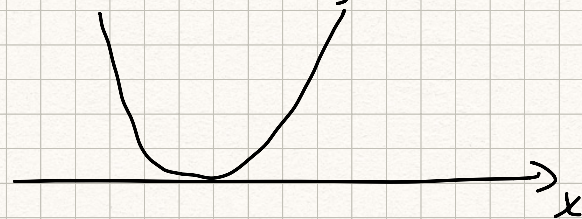
→ Naturally in the life

$\min_x f(x)$

$x^* = 0$. !

minimizer of the minimum.

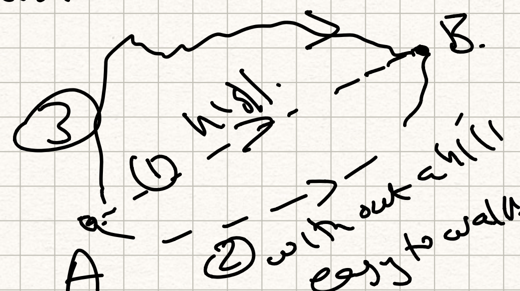
→ objective function.



→ walk

→ bike

→ car



→ objective would be minimize effort.

choose (2) over (1)

effort in (2) is less than (1)

$(R_1, R_2, R_3) \in S$ $S = \{R_1, R_2 \dots R_n\}$

$$\min_{x \in S} f(x)$$

Humans are optimizers.

max. welfare.

max. earnings.

· 24 hrs a day.

→ 8 hrs to sleep

→ 8 hrs to exercise, eat, cleanings, interacting with friends,

→ 8 hrs to work

(A) you could take work now!

→ skills

drive uber car

work in a restaurant

Study at IIT

→ earn a deg.

→ then work

→ later on when you earn over compensate for option (A).

→ long horizon!

→ complex in time!

Restaurant Owner

10 kg. of wheat
5 lit of milk
1 kg of potato
$\frac{1}{2}$ kg of salt
500 g paneer.
- - - - - -

m resources available with him.

menu card → has n items which I call as activities.

→ Paneer butter masala Rs 20 per plate
→ samosa → Rs 10 per ~~plate~~ piece.
→ Chai → Rs 5 per cup.
- - -
→ n - activities.

1 samosa → 100 g of potato -
→ 100 ml of oil -
→ 5 g of salt -
→ 10 g of masala -
→ cooking energy 500 cal -

production cost → Rs 5 per piece.
profit of Rs 5 per piece.

$$\max_{(x_1, x_2, \ldots, x_n)} \; Pr_1 x_1 + Pr_2 x_2 \cdots + Pr_n x_n .$$

$$\max \; c^T x$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad c = \begin{bmatrix} pr_1 \\ pr_2 \\ \vdots \\ pr_n \end{bmatrix}.$$

Objective function

(linear!)

$$\max \; c^T x .$$

$$\begin{array}{l} \\ \\ \text{potato.} \\ \text{milk.} \\ \vdots \\ \vdots \\ \text{milk} \end{array}
\begin{array}{c} \text{samosa} \\ \text{item-1} \\ \left[\begin{array}{c} 100 \\ 0 \\ \\ \\ - \end{array}\right. \end{array}
\begin{array}{c} \\ \text{item2-} \\ \begin{array}{c} 10 \\ 5 \\ \\ \\ - \end{array} \end{array}
\begin{array}{c} \\ \text{--} \\ \begin{array}{c} \\ --- \\ \\ \\ - \end{array} \end{array}
\begin{array}{c} \text{\sout{Food} curry} \\ \text{item-n-} \\ \left.\begin{array}{c} 0.01 \\ 25 \\ \\ \\ - \end{array}\right] \end{array}
\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \leq \begin{bmatrix} 100 \\ 5000 \\ \vdots \\ \vdots \end{bmatrix}$$

$$\underline{100 x_1 + 10 x_2 \; -- + 0.01 x_n} \leq \underline{\underline{100}}$$

$$\underline{0 \cdot x_1 + 5 x_2 \; -- + 25 x_n} \leq 5000.$$

$$\boxed{\begin{array}{c} \max \; c^T x \\ A x \leq b. \\ x \geq 0 \end{array}}$$

linear programming problem.

→ objective function is linear
→ constraints are also linear.

no. of samosa → +ve integer.

no. of cup is an integer.

discrete nature or combinatorial nature.

$\min f(x)$
$x \in S$

$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ → real no.

$x \in \mathbb{R}^n$

$\min_x f(x)$ ← continuous differentiable twice "

$$\left.\begin{array}{l} g_1(x) = 0 \\ g_2(x) = 0 \\ \vdots \\ g_p(x) = 0 \\ h_1(x) \le 0 \\ h_2(x) \le 0 \\ \vdots \\ h_r(x) \le 0 \end{array}\right\} S$$

$x_1^2 + x_2^2 \ge 5$

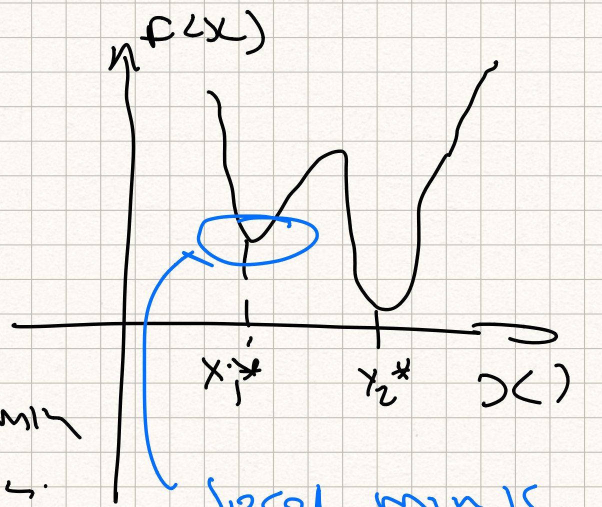
$\Rightarrow -x_1^2 - x_2^2 \le -5$

→ global min.

$$f(x^*) \le f(x) \quad \forall\, x \in S$$

$x^* \in S$

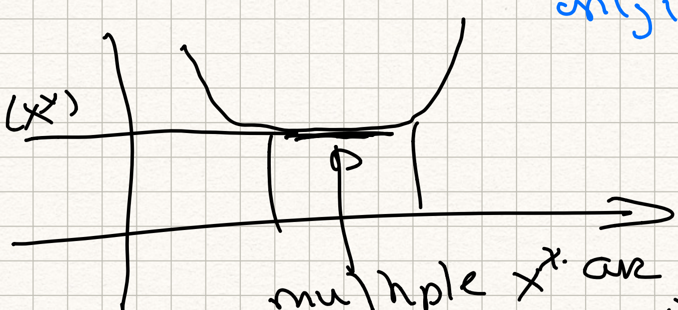
feasible



$f(x_2^*)$ is global min

$f(x_1^*)$ is local min.

local min is only in a neighborhood



multiple x^* are possible with same cost.

→ uniqueness of the minimizer!