RECAP:

Formulation 1:

$$\max_x \ \bar{c}^T x$$
$$Ax \le b$$
$$x \ge 0$$

Description:

(1) Maximize average profit subject to resource availability constraint

(2) Linear Programming formulation

Formulation 2:

$$\min_x \ x^T V x = \sigma^2$$
$$Ax \le b$$
$$x \ge 0$$

Description:

(1) Minimize variance of the profit $\sigma^2 = x^T \underset{n \times n}{V} x \ge 0$

(2) V is covariance matrix.

(3) Covariance matrix is always positive semidefinite.

i.e, $x^T V x \ge 0 \ \forall x \ne 0$.

Formulation 3:

$$\min_x \ x^T V x$$
$$Ax \le b$$
$$x \ge 0$$
$$z = \bar{c}^T x \ge z_{asp}$$

Description:

(1) Modifies formulation 2 by adding a minimum aspiration level constraint on profit

(2) Still not that neat formulation.

Risk Aversion Model

→ Concept of utility function

→ Law of diminishing marginal utility

→ Risk Aversion and risk aversion constant

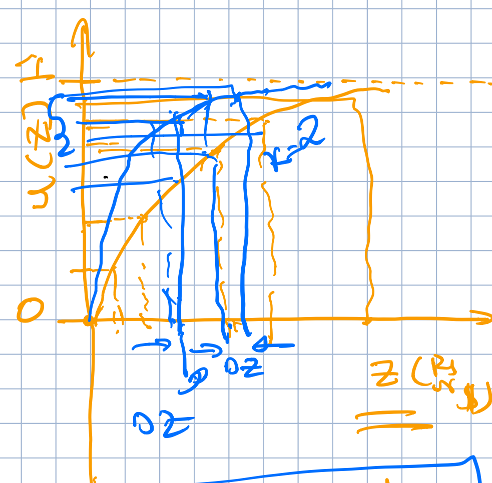
→ $u(z) \in [0, 1]$

→ montonically increasing and $u(0) = 0$;

$$u(z) = 1 - e^{-kz}$$

$u(0) = 0$; $u(\infty) = 1$;

$u'(z) = k e^{-kz}$



$$\boxed{u'(z_1) > u'(z_2) \text{ when } z_1 < z_2}$$

$k \to$ risk aversion constant

Formulation 4:

Maximize the mean utility.

$k=2$; $\Delta z = +1$, $\Delta u = 0.1$

$\Delta z = -1$; $\Delta u = -0.12$

$k=3$ $\Delta z = +1$, $\Delta u = 0.05$

$\Delta z = -1$; $\Delta u = -0.1$

Let pdf, of profit or worth z, be normal with mean, $\bar{z}$ and standard deviation σ;

$E(u(z))$

$$\phi(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{z-\bar{z}}{\sigma}\right)^2}$$

Recall $\sigma^2 = x^T V x$

$\bar{z} = \bar{c}^T x$.

$$E(u(z)) = \int_{-\infty}^{+\infty} (1 - e^{-kz})\phi(z)\,dz$$

$\int_{-\infty}^{+\infty} u(z)\phi(z)\,dz$

$$= \int_{-\infty}^{+\infty} \phi(z)\,dz - \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{+\infty} e^{-\left[kz + \frac{1}{2}\left(\frac{z-\bar{z}}{\sigma}\right)^2\right]} dz$$

$(a+b)^2 = a^2 + 2ab + b^2$

$$= 1 - \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}\left[\left(\frac{z-\bar{z}}{\sigma}\right)^2 + 2\sigma k\left(\frac{z-\bar{z}+\bar{z}}{\sigma}\right)\right]} dz$$

a α

$$= 1 - \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}(\alpha^2 + 2k\alpha)} \cdot e^{-k\bar{z}}\,dz$$

$$= 1 - \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}(\underbrace{\alpha^2}_{a^2} + \underbrace{2k\sigma\alpha}_{2ab} + \underbrace{k^2\sigma^2}_{b^2} - \underbrace{k^2\sigma^2}_{-b^2})} e^{-k\bar{z}}\,dz$$

$$= 1 - \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{+\infty} e^{-\frac{1}{2}(\alpha + k\sigma)^2} e^{-\left(k\bar{z} - \frac{1}{2}k^2\sigma^2\right)}\,dz$$

$\bar{z} = \bar{c}^T x$

$\sigma^2 = x^T V x$

$$= 1 - e^{-(k\bar{z} - \frac{1}{2}k^2\sigma^2)} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{z-\bar{z}}{\sigma} + k\sigma\right)^2} dz$$

$$= 1 - e^{-(k\bar{z} - \frac{1}{2}k^2\sigma^2)} \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}\left(\frac{z - (\bar{z} - k\sigma^2)}{\sigma}\right)^2} dz$$

normal distribution with $\bar{z} - k\cdot\sigma^2$ and std. dev. σ

1 at

$E(u(z))$

$$= \boxed{1 - e^{-(k\bar{z} - \frac{1}{2}k^2\sigma^2)}}$$

So, goal is to maximize argument of $e^{-(\,)}$ (i.e.)

min e^{-ay}

$$\max_{x} \; k\bar{z} - \frac{1}{2}k^2\sigma^2$$

σ^2

$$\text{or} \quad \min_{x} \; \frac{1}{2}k^2 x^T V x - k\,\bar{c}^T x$$

$\bar{z}$

max $f(x)$

min $-f(x)$

$k=1$

$k=3$

$k=2$

$$\text{s.t.} \quad Ax \le b; \quad x \ge 0;$$

Notes

(1) Quadratic minimization problem

programming problem.

(2) Has a minimum because 'V' is positive semidefinite.

(3) Larger k, i.e., higher risk aversion leads to more weightage to risk aversion & vice-versa.

(4) Range of $k = 1 - 3$

$K=1$ $\quad \min \frac{1}{2} x^T V x - c^T x$

$K=2$ $\quad \min (K) \frac{1}{2} x^T V x - 2 c^T x$

$\mu = 0$

$\sigma = 1$

$K=2$ lower σ, lower $\bar{z}$

$K=1$ higher σ, higher $\bar{z}$

$\int_{-\infty}^{+\infty} \phi(z) dz = 1$

-100

-500