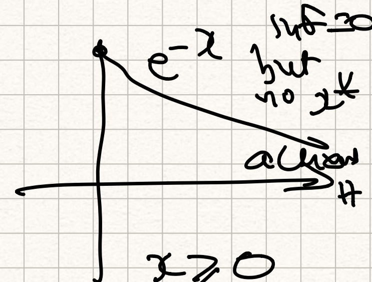
Let 'f' be a continuous function; it is continuous on set $S \subset R^n$ and 'S' is closed & bounded (compact set).

Then the problem,

$$\min_{x \in S} f(x)$$ has a soln.

or $\exists\, x^* \in S$ (there could be multiple)

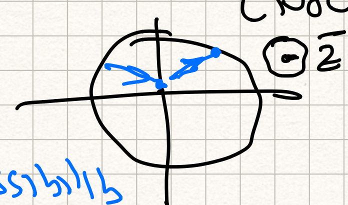
$$f(x^*) = \inf_{x \in S} f(x)$$



Proof:

Property-1 $\{z_k\} \subset S$

$\lim_{k \to \infty} z_k \to \overline{z}$ (limit, convergent) & will be in S if S is ~~compact~~ closed.



Proof: by contradiction.

Let $\overline{z} \notin S$.

$\exists\, N_\delta(\overline{z}) = \{x : \|x - \overline{z}\| < \delta\}$ open ball, $\delta > 0$

s.t. all $x \in N_\delta(\overline{z})$ have no point in S.

Possibility

1) $\overline{z} \in \text{int } S$

2) or $\overline{z} \in \partial S$

3) $\overline{z} \notin S$ (outside) rule out

- $z_k \to \bar{z}$

at some time s.t. $\|z_k - \bar{z}\| < \delta$

$k > K$

1) open ball outside rets

2) Given every $z_k \in S$

$\lim_{k \to \infty} z_k = \bar{z} \in S$ if set 'S' is closed

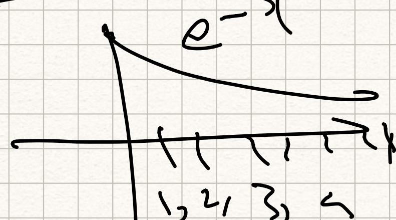
i.e, all convergent sequences in a closed set will have limit point in the set.

infimum,

$\{z_k\}$ in S

s.t. $\{f(z_k)\} \to \inf_{x \in S} f(x)$

is not convergent.



$\{z_k\}$ which has a limit point

$\lim_{n \to \infty} \{n\} \to \infty$

Every bounded set has a limit point in it.

$(f(1), f(2),$
$e^{-1}, e^{-2}, e^{-3})$
$\to 0$

$\{z_k\}_k \to \bar{z} \in S$

bounded — closed

$\{z_k\}_k \to \bar{z} = x^* \in S$

$\{f(z_k)\}_k \to \inf_{x \in S} f(x)$

p (i) $f(\bar{z}) = \inf_{x \in S} f(x)$

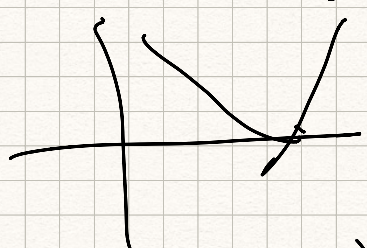
affirmatively !!

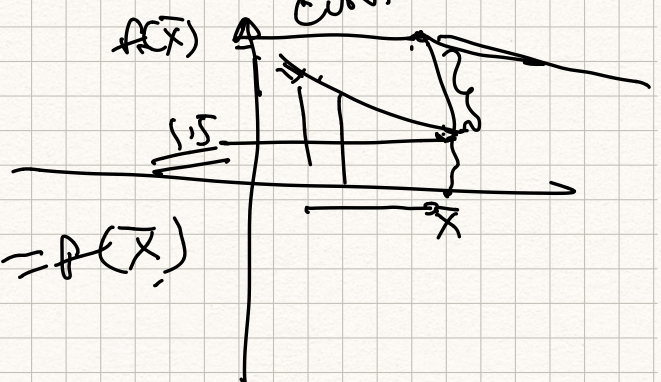
the function is continuous!

Yes $\to$ $\{x_k\} \to \bar{x}$

$\{f(x_k)\} \to f(\bar{x})$

continuous





$\lim_{k \to \infty} f(x_k) = f(\bar{x})$

$\underline{\underline{\max}}$

$$\underset{x \in S}{\min} \left(-f(x)\right) \Longleftrightarrow \underset{x \in S}{\max} f(x)$$