Introduction to Navigation & Guidance

(Course Code: AE 410/641)

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Tutorial - 1

Learning Goals

- To be very explicit about the choice of coordinate system
- When a coordinate transformation is needed and how to do that
- Understanding of frames and vectors

1. If the consecutive rotations are performed in the order ψ, θ and ϕ i.e., (yaw, pitch and roll) on reference frame **XYZ** then we obtain the another reference frame *xyz*. This rotation can be performed using a transformation matrix D. For such a transformation:
   (a) What are the range for the angles ψ, θ and ϕ?
   (b) Can $\pi/2 < |\theta| < \pi$? If not, Why?
   (c) If $|\theta| < \pi$, what would be the resultant transformation matrix $\hat{D}$?
2. Prove that the sum of the squares of direction cosines is 1.
3. Consider two right hand orthogonal, unit vectors, ($\vec{m}1$,$\vec{m}2$,$\vec{m}3$) and ($\vec{n}1$,$\vec{n}2$,$\vec{n}3$), associated with a DCM, $\mathbf{C} = [C_{ij}]$.

$$\begin{bmatrix} \vec{m}1 \\ \vec{m}2 \\ \vec{m}3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \vec{n}1 \\ \vec{n}2 \\ \vec{n}3 \end{bmatrix}$$

   (a) Check for Orthonormality of the DCM
   (b) Prove the correctness of $|\ \mathbf{C}\ | = 1$
4. Recall that a conic section is represented by the general formula

$$\mathcal{C}:\ ax^2 + bxy + cy^2 + dx + ey + f = 0,\ \ a, b, c, d, e, f \in \mathbb{R}.$$

   (a) Suppose that a rotation of 45° is applied to the above conic in counter-clockwise direction. What would be the representation (equation) of new conic so obtained? Denote this by $\mathcal{C}'$.

(b) Suppose $\mathcal{C}'$ is again rotated by 45° in counter-clockwise direction. What would be the representation (equation) of new conic so obtained? Denote this by $\mathcal{C}''$.

(c) Suppose a counter-clockwise rotation of 90° is directly applied to the conic $\mathcal{C}$ to obtain $\mathcal{C}^\star$. What would be the representation (equation) of new conic so obtained?

(d) Are $\mathcal{C}''$ and $\mathcal{C}^\star$ identical? Why/Why not?

5. **Food for thought!**

Can we have trigonometric and hyperbolic representations of quaternions?

Put differently, how can we formally *define*

$$\sin(\mathbf{q}),\ \cos(\mathbf{q}),\ \tan(\mathbf{q}),\ \cosh(\mathbf{q}),\ \sinh(\mathbf{q}),\ \tanh(\mathbf{q})$$

for a quaternion $\mathbf{q} = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k} = q_0 + \mathbf{v}$?

Think of this in polar form, $\mathbf{q} = re^{\theta\mathbf{u}}$, where $r = |q|$ denotes the magnitude evaluated using $|\mathbf{q}|^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2$ and $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|}$. The angle, θ, is a convex angle ($\theta \in [0, \pi)$). Note that this form is unique for $\mathbf{q} \neq 0, \pm 1$.
