EE 659: A FIRST COURSE IN OPTIMIZATION
QUIZ-I

Marks-10. TAKE HOME EXAMINATION

Duration 48 hrs from 8:00am 13/10/2020

All Questions are compulsory

1.] Let $A = \begin{bmatrix} 10 & 5 & -1 \\ 2 & 100 & 5 \\ -3 & 1 & 1 \end{bmatrix}$.

From the first principles, compute $\|A\|_2$.

2. Let $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$; and let

$$f(x) = \left(|x_1|^p + |x_2|^p + \cdots + |x_n|^p\right)^{1/p}$$

Show from the skratch that

$$\|x\|_\infty = \lim_{p \to \infty} f(x) = \max_{1 \le i \le n} |x_i|$$

[2 marks]

3] We want to find a point x in the plane whose sum of weighted distances from a given set of points $y_1, \ldots, y_m$ is minimized. Mathematically, the problem is

$$\min \sum_{i=1}^{m} w_i \|x - y_i\| \text{ subject to } x \in \mathbb{R}^n.$$

where $w_1, \dots, w_m$ are positive scalars.

(a) Show that there exists a global minimum for this problem and that it can be realized by means of the mechanical model shown in Fig 1.

(b) Is the optimal solution always unique?

(c) Show that an optimal solution minimizes the potential energy of the mechanical model of Fig-1 defined as $\sum_{i=1}^{m} w_i h_i$ where h_i is the height of the i^{th} weight, measured from some reference point.

Note: This problem stems from Weber's work which is generally viewed as starting point of locational theory.

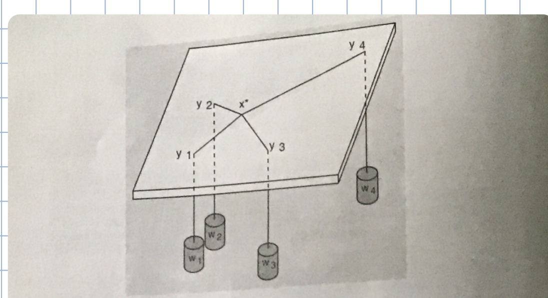

Figure 1.1.6. Mechanical model (known as the Varignon frame) associated with the Weber problem (Exercise 1.1.6). It consists of a board with a hole drilled at each of the given points y_i. Through each hole, a string is passed with the corresponding weight w_i attached. The other ends of the strings are tied with a knot as shown. In the absence of friction or tangled strings, the forces at the knot reach equilibrium when the knot is located at an optimal solution x^*.

Figure 1

[5-marks]

Reference: Problem 1.1.6 From Dimitri P. Bertsekas Nonlinear Programming, Second Edition.