CS 747, Autumn 2020: Week 11, Lecture 1

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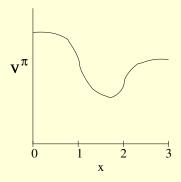
Autumn 2020

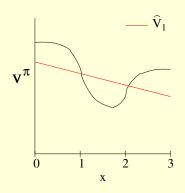
Reinforcement Learning

- 1. Tile coding
- 2. Issues in control with function approximation
- 3. Policy search
- Case studies
 - Humanoid robot soccer
 - Railway scheduling

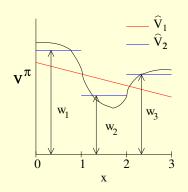
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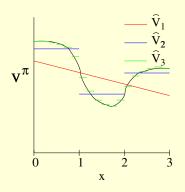




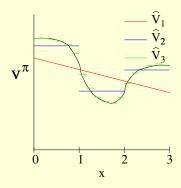
$$\hat{V}_1(x)=w_1x+w_2.$$



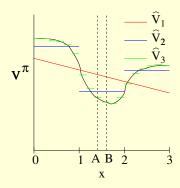
$$b_1 = egin{cases} 1 & ext{if } 0 \leq x < 1, \ 0 & ext{otherwise}. \end{cases}$$
 $b_2 = egin{cases} 1 & ext{if } 1 \leq x < 2, \ 0 & ext{otherwise}. \end{cases}$ $b_3 = egin{cases} 1 & ext{if } 2 \leq x < 3, \ 0 & ext{otherwise}. \end{cases}$ $\hat{V}_2(x) = w_1 b_1 + w_2 b_2 + w_3 b_3. \end{cases}$



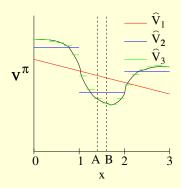
 $\hat{V}_3(x)$: 18 piece-wise constants.



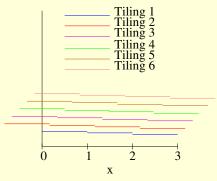
• Is \hat{V}^3 the obvious choice?



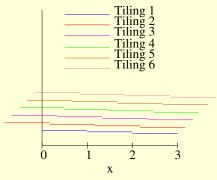
- Is \hat{V}^3 the obvious choice?
- \hat{V}^3 has the highest resolution, but does not generalise well.



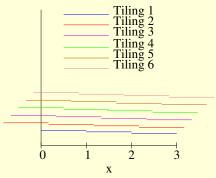
- Is \hat{V}^3 the obvious choice?
- \hat{V}^3 has the highest resolution, but does not generalise well.
- How to achieve high resolution along with generalisation?



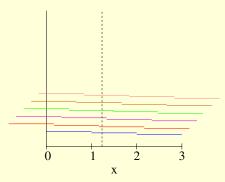
• A tiling partitions *x* into equal-width regions called tiles.



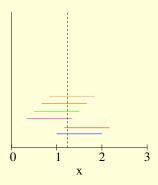
- A tiling partitions *x* into equal-width regions called tiles.
- Multiple tilings (say m) are created, each with an offset (1/m tile width) from the previous.



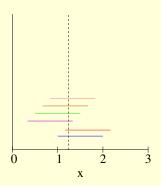
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- Each tile is a binary feature.
- Tile width and the number of tilings determine generalisation, resolution.
- Observe that two points more than (tile width / number of tilings) apart can be given arbitrary function values.

Representing \hat{Q}

• Given a feature value x as input, the corresponding set of tilings $T : \mathbb{R} \to \mathbb{R}$ returns the sum of the weights of the tiles activated by x.

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$$\hat{Q}(s,a) = \sum_{j=1}^{d} T_{aj}(x_j(s)).$$

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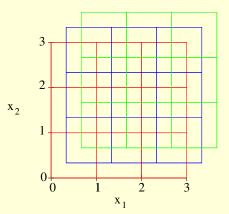
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$$\hat{Q}(s,a) = \sum_{j=1}^{d} T_{aj}(x_j(s)).$$

 Usually, tile widths and the number of tilings are configured specifically for each feature. For example, in soccer, could use 2m as tile width for "distance" features, and 10° as tile width for "angle" features.

2-d Tile coding

 For representing more complex functions, can also have tilings on conjunctions of features (see below for 2 features).



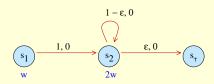
Introduces more parameters—which could help or hurt.

Tile Coding: Summary

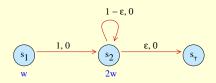
- Linear function approximation does not restrict us to a representation that is linear in the given/raw features.
- Tile coding a standard approach to discretise input features and tune both resolution and generalisation.
- Enjoys many empirical successes, especially in conjunction with Linear Sarsa(λ).
- Common to store weights in a hash table (collisions don't seem to hurt much), whose size is set based on practical constraints.
- 1-d tilings most common; rarely see conjunction of 3 or more features.

Reinforcement Learning

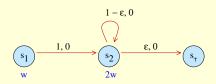
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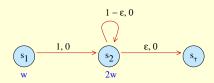
- Prediction problem (policy π).
- Episodic, start state is s_1 .
- Observe that $V^{\pi}(s_1) = V^{\pi}(s_2) = 0$.
- Linear function approximation with single parameter w: $x(s_1) = 1, x(s_2) = 2$; hence $\hat{V}(s_1) = w, \hat{V}(s_2) = 2w$.



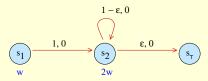
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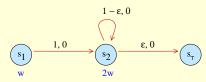
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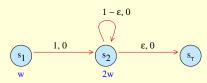
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- w = 0 gives the exact answer!
- We design an iteration $w_0 \rightarrow w_1 \rightarrow w_2 \rightarrow ...$, and see if it converges to 0 (due to Tsitsiklis and Van Roy, 1996).



• From state s, let s', r be the (random) next state, reward.

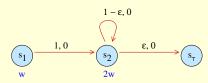


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- We begin with arbitrary w_0 , and for $k \ge 0$ set

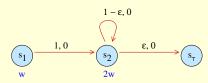
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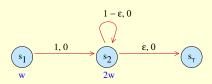
• Is $\lim_{k\to\infty} w_k = 0$?



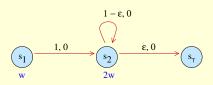
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• Is $\lim_{k\to\infty} w_k = 0$? Let's see.

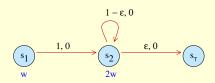


$$\begin{aligned} w_{k+1} &= \underset{w \in \mathbb{R}}{\operatorname{argmin}} \sum_{s} \left(\mathbb{E}_{\pi}[r + \gamma \hat{V}(w_k, x(s'))] - \hat{V}(w, x(s)) \right)^2 \\ &= \underset{w \in \mathbb{R}}{\operatorname{argmin}} \left((2\gamma w_k - w)^2 + (2\gamma(1 - \epsilon)w_k - 2w)^2 \right) = \gamma \frac{6 - 4\epsilon}{5} w_k. \end{aligned}$$



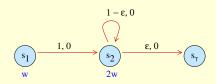
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- The failure owes to the combination of three factors: off-policy updating, generalisation, bootstrapping.
- But these are almost always used together in practice!

Summary of Theoretical Results*

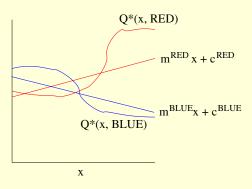
Method	Tabular	Linear FA	Non-linear FA
TD(0)	C, O	С	NK
$TD(\lambda),\lambda\in(0,1)$	C, O	С	NK
TD(1)	C, O	C, "Best"	C, Local optimum
Sarsa(0)	C, 0	Chattering	NK
Sarsa(λ), $\lambda \in (0,1)$	NK	Chattering	NK
Sarsa(1)	NK	NK	NK
Q-learning(0)	C, 0	NK	NK

(C: Convergent; O: Optimal; NK: Not known.)

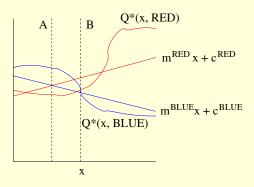
★: to the best of your instructor's knowledge.

Reinforcement Learning

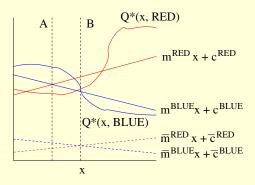
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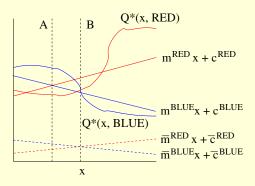
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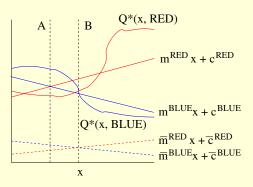
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- Perhaps we found $(m^{RED}, c^{RED}, m^{BLUE}, c^{BLUE})$ by Q-learning.
- How to find $(\bar{m}^{\text{RED}}, \bar{c}^{\text{RED}}, \bar{m}^{\text{BLUE}}, \bar{c}^{\text{BLUE}})$?

An abstract model:

Input
$$w = (w_1, w_2, \dots, w_d) \longrightarrow \boxed{\text{System}} \longrightarrow \text{Output } f(w).$$

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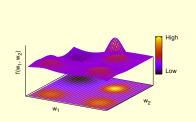
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- Is finding the optimal *w* easy? Why is this approach called black box optimisation?

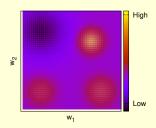
Typical Context for Black Box Optimisation

- Little/nothing known/assumed about f—can be discontinuous, non-linear, "erratic".
- Given w, evaluating f(w) is relatively efficient.
- Calculating f(w) usually involves a computer simulation.

Typical Context for Black Box Optimisation

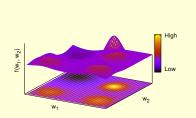
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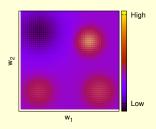




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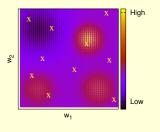
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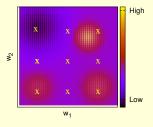


• How to find a "relatively good" w?

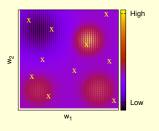
Random weight guessing



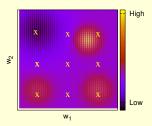
Grid search



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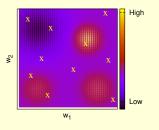


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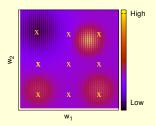


• These approaches work for small dimensions d (say $d \le 5$).

Random weight guessing

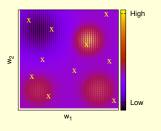


Grid search

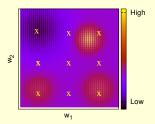


- These approaches work for small dimensions d (say $d \le 5$).
- No method can be expected to work well for very large d (1000's or higher).

Random weight guessing



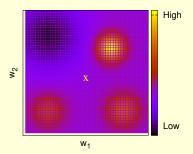
Grid search



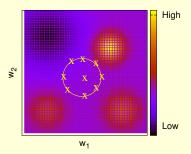
- These approaches work for small dimensions d (say $d \le 5$).
- No method can be expected to work well for very large d (1000's or higher).
- Local search works for intermediate *d* (10's, 100's).

Illustrative method: Hill Climbing.

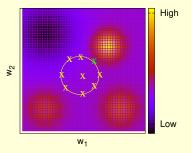
Illustrative method: Hill Climbing.



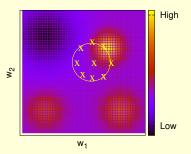
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- Called Policy search when applied on the RL problem.

Reinforcement Learning

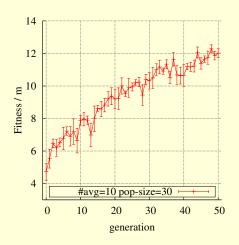
- 1. Tile coding
- 2. Issues in control with function approximation
- 3. Policy search
- 4. Case studies
 - Humanoid robot soccer
 Joint work with Patrick MacAlpine, Yinon Bentor, Daniel
 Urieli, and Peter Stone (UT Austin Villa robot soccer team).
 - Railway scheduling

Gait Optimisation: Policy Parameters

Notation	Description
maxStep*	Maximum step sizes allowed for x , y , and θ
y _{shift}	Side to side shift amount with no side velocity
Z [*] torso	Height of the torso from the ground
Z*torso Z*step	Maximum height of the foot from the ground
	Fraction of a phase that the swing
f_g^*	foot spends on the ground before lifting
f _a	Fraction that the swing foot spends in the air
$f_{\mathcal{S}}^*$	Fraction before the swing foot starts moving
f _m	Fraction that the swing foot spends moving
ϕ^*_{length}	Duration of a single step
δ^*	Factors of how fast the step sizes change
Уѕер	Separation between the feet
X* offset	Constant offset between the torso and feet
	Factor of the step size applied to
X* factor	the forwards position of the torso
err*	Maximum COM error before the steps are slowed
err*	Maximum COM error before all velocity reach 0

Design and Optimization of an Omnidirectional Humanoid Walk: A WinningApproach at the RoboCup 2011 3D Simulation Competition. Patrick MacAlpine, Samuel Barrett, Daniel Urieli, Victor Vu, and Peter Stone. In Proceedings of the Twenty-Sixth AAAI Conference on Artificial Intelligence (AAAI 2012), AAAI Press, 2012.

Progress of Forward Speed Optimisation



On Optimizing Interdependent Skills: A Case Study in Simulated 3D Humanoid Robot Soccer. Daniel Urieli, Patrick MacAlpine, Shivaram Kalyanakrishnan, Yinon Bentor, and Peter Stone. In Proceedings of the Tenth International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2011), pp. 769–776, IFAAMAS. 2011.

RoboCup 2011 3D Simulation Competition

• UT Austin Villa combined score: 136-0 (over 24 games).

Rank	Team	Goal Difference			
3	apollo3d	1.45 (0.11)			
5-8	boldhearts	2.00 (0.11)			
5-8	robocanes	2.40 (0.10)			
2	cit3d	3.33 (0.12)			
5-8	fcportugal3d	3.75 (0.11)			
9-12	magmaoffenburg	4.77 (0.12)			
9-12	oxblue	4.83 (0.10)			
4	kylinsky	5.52 (0.14)			
9-12	dreamwing3d	6.22 (0.13)			
5-8	seuredsun	6.79 (0.13)			
13-18	karachikoalas	6.79 (0.09)			
9-12	beestanbul	7.12 (0.11)			

. . .

UT Austin Villa 2011: A Champion Agent in the RoboCup 3D Soccer Simulation Competition. Patrick MacAlpine, Daniel Urieli, Samuel Barrett, Shivaram Kalyanakrishnan, Francisco Barrera, Adrian Lopez-Mobilia, Nicolae Ştiurcă, Victor Vu, and Peter Stone. In Proceedings of the Eleventh International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2012), pp. 129–136, IFAAMAS, 2012.

Reinforcement Learning

- 1. Tile coding
- 2. Issues in control with function approximation
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 - Humanoid robot soccer
 - Railway scheduling
 Joint work with Rohit Prasad and Harshad Khadilkar.

Railways and the Economy



[1]

- Indian Railways, 20000+ trains, 9.1 billion yearly ridership.
- Delay incurs economic costs

[1] https://pixabay.com/photos/transportation-system-travel-vehicle-3351330/.

Railway Rescheduling Problem

Trai	n Station	Timetable Arrival Time	Timetable Departure Time	Minimum Halt Time	Minimum Run Time	Scheduled Arrival Time	Scheduled Departure Time	Delay
10	1 Alpha	4:30	4:40	10	30			
102	2 Alpha	0:00	0:05	5	10			
60	1 Echo	9:15	9:16	1	60			
40	1 Delta	3.50	3:35	15	20			

Given an initial timetable compute a feasible timetable subject to resource allocation and time constraints minimising

Priority-weighted departure delay

$$= \sum_{\text{Train t. Resource r}} \frac{\text{Delay of train t at resource r}}{\text{Priority of train t}}$$

Railway Rescheduling Problem

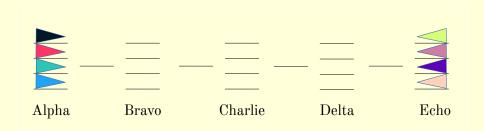
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A hypothetical railway line with 8 trains and 5 stations



A hypothetical railway line with 8 trains and 5 stations



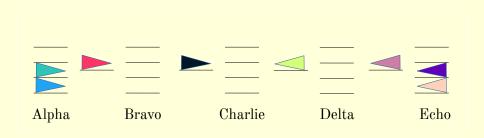
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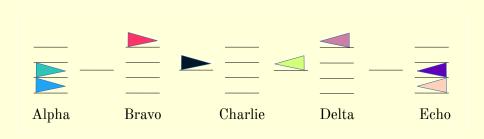
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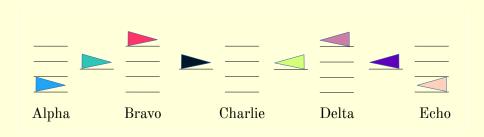
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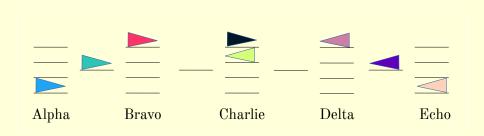


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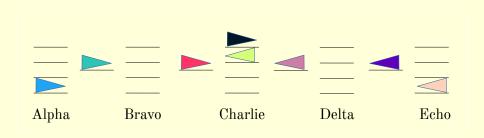
Moving trains

A hypothetical railway line with 8 trains and 5 stations

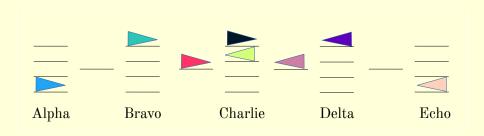


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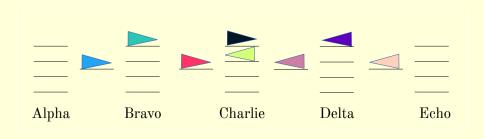


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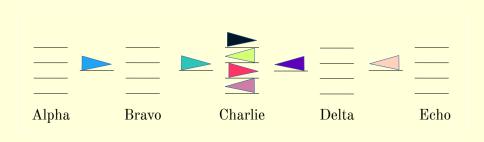


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Intermediate state

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Deadlock!

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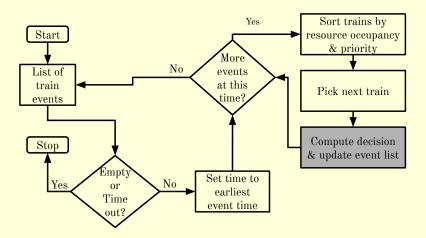
Intermediate state

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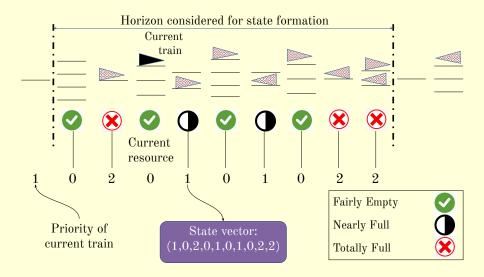
No Deadlock!

Our Solution

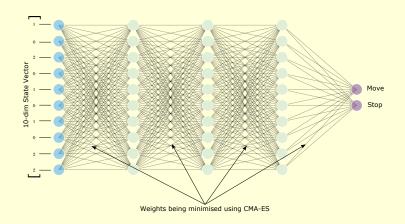


- The wrapper layer picks a potential train to move.
- We optimise the module that decides MOVE / STOP.

State Space



Policy Representation

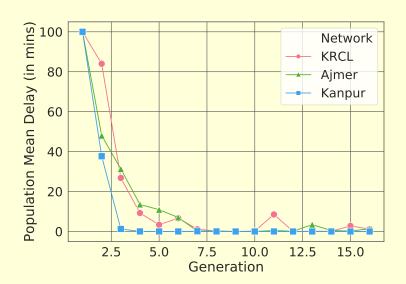


$$d = 352$$
.

Benchmark Railway Lines

Scenario	Type	Stations	Trains	Events	Timetable span
HYP-2	Line	11	60	1320	4 hours
HYP-3	Line	11	120	2640	7 hours
KRCL	Line	59	85	5418	3 days
Kanpur	Line	27	190	7716	3 days
Ajmer	Line	52	444	26258	7 days

Results



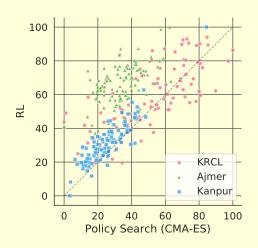
Summary of Results

Priority-weighted departure delay.

	Policy search	RL	TAH-FP	TAH-CF	Naive	GWP	PTD
HYP-2	4.28 (0)	4.78 (0)	4.58 (0)	5.93 (0)	11.16 (2)	4.35 (0)	714.00 (0)
HYP-3	15.50 (0)	18.54 (0)	61.89 (97)	140.14 (95)	- (100)	16.35 (0)	2003.98 (0)
KRCL	42.34 (0)	43.04 (0)	46.41 (8)	47.02 (0)	- (100)	42.40 (0)	4714.08 (0)
Ajmer	3.92 (0)	4.65 (0)	10.76 (3)	5.99 (0)	9.25 (76)	3.99 (3)	8304.84 (0)
Kanpur	1.54 (0)	1.66 (0)	2.19 (0)	2.28 (0)	1.85 (0)	1.54 (0)	313.60 (0)

Comparison with RL (Q-learning)

Priority-weighted departure delay.



Summary

- Initial lectures assumed finite state spaces.
 Seldom seen in practice!
- Need to generalise over states (sometimes actions).
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- Next week: policy gradient methods.