Introduction to Navigation & Guidance

(Course Code: AE 410/641)

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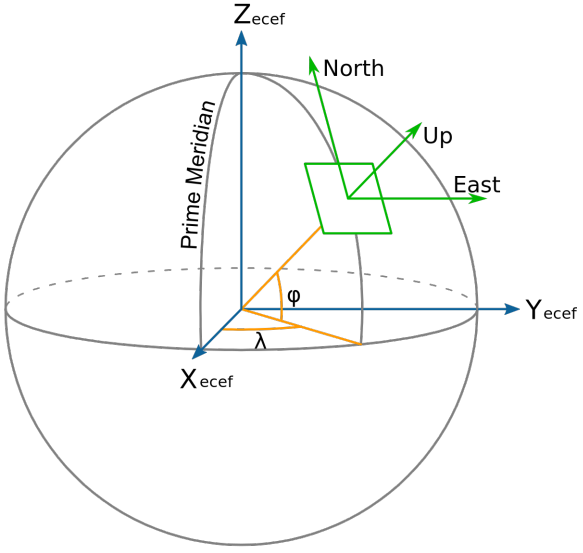
Tutorial - 2

1. Consider two coordinate systems, one a fixed rectangular system with unit vectors $I,\ J,\ K$ and the other rotating on a local tangent plane of a sphere. The latter has unit vectors $i,\ j,\ k$. The sphere is rotating at a constant rate of Ω. In the inertial coordinate system the rotation vector is $\Omega\ K$. In the rotating coordinate system, the unit vector i is to the east, the unit vector j is to the north and the unit vector k is vertically normal to the surface. Based on this information,



   (a) Transform the unit vectors $i,\ j,\ k$ of rotating frame in fixed frame.

   (b) Show that :

$$\frac{di}{dt} = \Omega \times i$$
$$\frac{dj}{dt} = \Omega \times j$$
$$\frac{dk}{dt} = \Omega \times k$$

2. Consider a square shaped RLG with input angular velocity equal to 1°/hr. The operating wavelength is assumed to be 0.6328μm. If the measurable beat frequency is obtained to be 0.35 Hz,

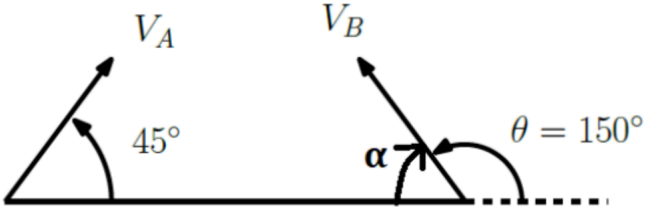
(a) Calculate the scale factor, side length and optical path length of given RLG.

(b) Calculate the radius for the circular RLG equivalent to given square shaped RLG.

(c) If ν represents beat frequency and L represents optical path length for a RLG, prove that :

$$\frac{\Delta\nu}{\nu} = \frac{\Delta L}{L}$$

3. Consider the positions of two aircraft, A and B, as shown in the figure below. Aircraft A has a speed of 400 m/s and carries a radar transmitting signal at frequency of 300 MHz. Aircraft A is tracking aircraft B which has a speed of 300 m/s. The direction of aircraft are the same as shown in figure below.



(a) How much is the doppler frequency shift observed by the radar in aircraft A?

(b) What should be the flight directions θ of aircraft B for the doppler frequency shift to be equal to 2000 Hz and 0 Hz?

(c) Find the value of maximum doppler shift for the above given speeds of aircraft if it is allowed to change directions of both the aircraft ?

4. A position vector in navigation frame is obtained as $r_n = 0.6124\hat{i} - 0.7071\hat{j} - 0.3536\hat{k}$. If the corresponding position vector in earth frame is obtained as $r_e = \hat{i}$, find out the location of INS with respect to earth frame.

5. A vehicle is elevating in earth atmosphere, consisting of the INS devices to measure different parameters. A vertical accelerometer is mounted on the vehicle to obtain the acceleration values.

   Derive the relation between altitude error (Δh) and time (t), where h is altitude from the surface of the earth. Use the radius of earth as $Z_0 = 6378$ km and the gravitational acceleration at earth's surface as $g_0 = 9.8$ m/s^2. Assume the error in acceleration measurement to be $10^{-5}g_0$, and initial conditions $\Delta h(0) = \dot{\Delta h}(0)$. Also,

   (a) Comment on stability of the vehicle.

   (b) Plot the relation derived between Δh and t, to draw the relevant inference.
