



Trade-off Ratio Problems



Problem No. 01

Consider a rocket with the following **mass** configuration.

$$\begin{aligned} m_{p1} &= 21087kg; & m_{s1} &= 1296kg; & I_{sp1} &= 261s \\ m_{p2} &= 3854kg; & m_{s2} &= 360kg; & I_{sp2} &= 324s \\ m_* &= 668kg \end{aligned}$$

Obtain the **trade-off** ratios.



Solution No. 01

The **trade-off** ratios are as follows.

$$m_{01} = 27256; \quad m_{f1} = 6169$$

$$m_{02} = 4882; \quad m_{f2} = 1028$$

$$\frac{\delta m_*}{\delta m_{s1}} = -0.116; \quad \frac{\delta m_*}{\delta m_{s2}} = -1$$

$$\frac{\delta m_*}{\delta m_{p1}} = 0.034; \quad \frac{\delta m_*}{\delta m_{p2}} = 0.119$$



Parallel Staging Problems



Problem No. 02

Consider the booster and first **stage** of PSLV, as defined below.

$$\begin{aligned} m_0 &= 295T; & m_{p0} &= 9T; & m_{s0} &= 2T; & I_{sp1} &= 262s; & t_{b0} &= 44s \\ m_{p1} &= 138T; & m_{s1} &= 30T; & I_{sp2} &= 269s; & t_{b1} &= 105s \end{aligned}$$

Assuming that there are 6 **boosters** and that all 6 boosters and the **first** stage fire together, **determine** the ideal burnout velocity at the **end** of the first stage burnout.



Solution No. 02

The **solution** at the end of 44s is as follows.

$$\dot{m}_0 = \sum_{i=1}^n \dot{m}_{0-i} = 6 \times 0.2045 + 1.3143 = 2.5415 T / s$$

$$I_{sp-0} = \frac{\sum_{i=1}^n \dot{m}_{0-i} I_{sp0-i}}{\sum_{i=1}^n \dot{m}_{0-i}} = \frac{1.2272 \times 262 + 1.3143 \times 269}{2.5415} = 265.6 s$$

$$\begin{aligned} V_{b-0} &= g_0 I_{sp-0} \ln \frac{m_0}{m_{0-1} - \dot{m}_0 t_{b0}} = 9.81 \times 265.6 \times \ln \frac{295}{295 - 2.5415 \times 44} \\ &= 1241.7 m / s \end{aligned}$$



Solution No. 02

The **solution** at the end of 105s is as follows.

$$\begin{aligned} V_{b-1} &= V_{b-0} + g_0 I_{sp-0} \ln \frac{m_{0-1}}{m_{0-1} - \dot{m}_{0-2} (t_{b1} - t_{b0})} \\ &= 1241.7 + 9.81 \times 269 \times \ln \frac{183.2}{183.2 - 1.3143 \times (105 - 44)} \\ &= 1241.7 + 1518.9 = 2760.6 \text{ m / s} \end{aligned}$$