Flight Mechanics/Dynamics

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Homogeneity

If we scale (increase/decrease the strength of) the input $u(t)$, it is expected that the output function $y(t)$ is also scaled by the same amount.
Mathematically, for some $\alpha \in \mathbb{R}$, $\alpha u(t) \longrightarrow \alpha y(t)$.

Additivity

If $u_1(t) \longrightarrow y_1(t)$, $u_2(t) \longrightarrow y_2(t)$, ..., $u_n(t) \longrightarrow y_n(t)$, as well as $\sum_{k=0}^{n} u_k(t) \longrightarrow \sum_{k=0}^{n} y_k(t)$, that is, the output corresponding to the sum of various inputs, is the sum of the individual outputs for individual inputs.

Homogeneity + Additivity $\longrightarrow$ Superposition. Superposition $\Rightarrow$ Linearity

Example

$y(t) = tu(t)$ is linear.
Consider two inputs $u_1(t)$ and $u_2(t)$. The corresponding outputs are $y_1(t)$ and $y_2(t)$. For some arbitrary constants α, β,

$$\begin{aligned} \alpha u_1(t) + \beta u_2(t) &= \alpha y_1(t)/t + \beta y_2(t)/t \\ \Rightarrow \alpha y_1(t) + \beta y_2(t) &= t(\alpha u_1(t) + \beta u_2(t)) \\ \Rightarrow \tilde{y}(t) &= t\tilde{u}(t), \end{aligned}$$

which is the same form as $y(t) = tu(t)$. In fact $\tilde{y}(t)$ represents the linear combination of $y(t) = tu(t)$.

Example

Consider the mass-spring-damper system described by the differential equation

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = u(t),\;\; y(0) = 0,\;\; \dot{y}(0) = 0$$

We wish to obtain the response of the system, $y(t)$, subjected to input force $u(t)$.

Take the Laplace transform of both sides, resulting in

$$Ms^2Y(s) + bsY(s) + kY(s) = U(s).$$

On simplification, we have

$$\frac{Y(s)}{U(s)} = \frac{1}{Ms^2 + bs + k}.$$

- Transfer function is a frequency domain approach.
- Transfer function approach can't be used when there is nonlinearity.

- State of the system: something that characterizes the *past*, *present*, and *future* of the dynamical system.
- A minimal set of variables, such that the knowledge of these variables at any time t_0, combined with the information on the input applied, are sufficient to determine the behavior of the system at time $t > t_0$.

Example

Consider an on-off switch.

- The switch can be either on or off.
- The state of the switch can assume one of the two possible values.
- If we know the present *state* (position) of the switch at t_0, we can determine the *state* (position) of the switch at $t > t_0$ if applied input is known.

Example

Classic example: A mass-spring-damper system.

$$M\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = u(t).$$

- Position and velocity are sufficient to describe this system.
- Hence, the position and velocity are regarded as state variables for this system.
- Position $\rightarrow$ x_1 and velocity $\rightarrow$ x_2.

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -(b/M)x_2 - (k/M)x_1 + (1/M)u.\end{aligned}$$

- State variables are not necessarily physical variables.
- Quantities that are not physically measurable or observable can also be chosen as state variables.
- Choice of state variables, and hence, the state model description is not unique.
- The number of states needed to describe a system completely is unique.
- Storage/memory elements in a system can be taken as state variables.
- The output of integrators in a continuous-time system can be taken as state variables.

A system described by nonlinear differential equations

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_m) \\
\dot{x}_2 &= f_2(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_m) \\
&\vdots \\
\dot{x}_n &= f_n(x_1, x_2, \ldots, x_n; u_1, u_2, \ldots, u_m)
\end{aligned}
$$

i.e., $\dot{\mathrm{x}} = \mathrm{f}(\mathrm{x}; \mathrm{u})$, where $\dot{\mathrm{x}} = [\dot{x}_1 \ \dot{x}_2 \ \ldots \ \dot{x}_n]^{\mathrm{T}}$ and $\mathrm{f}(\mathrm{x}) = [f_1(\cdot) \ f_2(\cdot) \ \ldots \ f_n(\cdot)]^{\mathrm{T}}$.

$\mathrm{x} \in \mathbb{R}^n$, $\mathrm{u} \in \mathbb{R}^m$, and $m \leq n$.

For a very small deviation, we can expand each equation in Taylor series as

$$\begin{aligned}\frac{d}{dt}(x_i + \delta x_i) &= f_i(\mathrm{x} + \delta \mathrm{x}; \mathrm{u} + \delta \mathrm{u}) \\ &\approx f_i(\mathrm{x}; \mathrm{u}) + \frac{\partial f_i}{\partial \mathrm{x}}\delta \mathrm{x} + \frac{\partial f_i}{\partial \mathrm{u}}\delta \mathrm{u}\end{aligned}$$

where

$$\frac{\partial f_i}{\partial \mathrm{x}} = \left[\frac{\partial f_i}{\partial x_1}\ \frac{\partial f_i}{\partial x_2}\ \dots\ \frac{\partial f_i}{\partial x_n}\right].$$

As $\frac{d}{dt}(x_i) = f_i(\mathrm{x}, \mathrm{u})$, we have

$$\frac{d}{dt}(\delta x_i) \approx \frac{\partial f_i}{\partial \mathrm{x}}\delta \mathrm{x} + \frac{\partial f_i}{\partial \mathrm{u}}\delta \mathrm{u}.$$

Combining all n equations,

$$\frac{d}{dt}(\delta \mathsf{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial \mathsf{x}} \\ \frac{\partial f_2}{\partial \mathsf{x}} \\ \vdots \\ \frac{\partial f_n}{\partial \mathsf{x}} \end{bmatrix} \delta \mathsf{x} + \begin{bmatrix} \frac{\partial f_1}{\partial \mathsf{u}} \\ \frac{\partial f_2}{\partial \mathsf{u}} \\ \vdots \\ \frac{\partial f_n}{\partial \mathsf{u}} \end{bmatrix} \delta \mathsf{u} = \mathsf{A}(t)\delta \mathsf{x} + \mathsf{B}(t)\delta \mathsf{u},$$

where

$$\mathsf{A}(t) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ & & \vdots & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial u_2} & \dots & \frac{\partial f_n}{\partial u_m} \end{bmatrix} \quad \text{and} \quad \mathsf{B}(t) = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \dots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \dots & \frac{\partial f_2}{\partial u_m} \\ & & \vdots & \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \dots & \frac{\partial f_n}{\partial u_m} \end{bmatrix}.$$

For nonlinear measurement equation, $y = g(x; u)$, we can proceed in a similar way such that

$$\delta y = \begin{bmatrix} \frac{\partial g_1}{\partial x} \\ \frac{\partial g_2}{\partial x} \\ \vdots \\ \frac{\partial g_p}{\partial x} \end{bmatrix} \delta x + \begin{bmatrix} \frac{\partial g_1}{\partial u} \\ \frac{\partial g_2}{\partial u} \\ \vdots \\ \frac{\partial g_p}{\partial u} \end{bmatrix} \delta u = C(t)\delta x + D(t)\delta u,$$

With slight abuse of notation, the state and the output equations can be written in matrix form as

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), \\ y(t) &= C(t)x(t) + D(t)u(t). \end{aligned}$$

Note: If the system is operated around only one set point (LTI system), then the matrices $A(t), B(t), C(t), D(t)$ are constant.

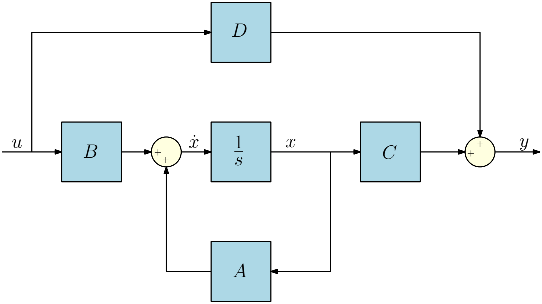


Figure: Illustrative block diagram of LTI state-space model.

$x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$.

A: System matrix (dim[A] $= n \times n$), B: Input matrix (dim[B] $= n \times m$),
C: Output matrix (dim[C] $= p \times n$), D: Feedforward matrix (dim[D] $= p \times m$).

- Eigenvalues of $\mathsf{A} \in \mathbb{R}^{n\times n}$: Roots of the characteristic equation $|\lambda \mathsf{I} - \mathsf{A}| = 0$.
- Eigenvalues are also known as characteristic roots.
- Eigenvalues of the system matrix A $\implies$ the poles of the system.
- For a system to be stable, poles of the system must have negative real parts (poles must lie in left half complex plane).
- Similarly, eigenvalues of the system matrix must have negative real parts for a system to be stable.
- If $\mathsf{A} \in \mathbb{R}^{n\times n}$, then a nonzero vector $\boldsymbol{x} \in \mathbb{R}^n$ is called an eigenvector of A if $\mathsf{A}\boldsymbol{x}$ is a scalar multiple of $\boldsymbol{x}$, i.e., $\mathsf{A}\boldsymbol{x} = \lambda \boldsymbol{x}$ for some scalar λ.
- This scalar λ is called an eigenvalue of A, and $\boldsymbol{x}$ is referred as an eigenvector corresponding to λ.
- Any nonzero scalar multiple of an eigenvector is another eigenvector.

- If A has linearly independent eigenvectors, then there exists an invertible matrix T that diagonalizes A.
- Resulting diagonal matrix: $\tilde{\mathsf{A}} = \mathsf{T}^{-1}\mathsf{A}\mathsf{T}$
- Determinant of A and $\tilde{\mathsf{A}}$ are same. How?
  Proof: $\det(\tilde{\mathsf{A}}) = \det(\mathsf{T}^{-1}\mathsf{A}\mathsf{T}) = \det(\mathsf{T}^{-1})\det(\mathsf{A})\det(\mathsf{T}) = \det(\mathsf{A})$.
- Eigenvalues of A and $\tilde{\mathsf{A}}$ are same. How?

$$
\begin{aligned}
|\lambda \mathsf{I} - \mathsf{T}^{-1}\mathsf{A}\mathsf{T}| &= |\lambda \mathsf{T}^{-1}\mathsf{T} - \mathsf{T}^{-1}\mathsf{A}\mathsf{T}| \\
&= |\mathsf{T}^{-1}(\lambda \mathsf{I} - \mathsf{A})\mathsf{T}| \\
&= |\mathsf{T}^{-1}|\,|\lambda \mathsf{I} - \mathsf{A}|\,|\mathsf{T}| \\
&= |\mathsf{T}^{-1}|\,|\mathsf{T}|\,|\lambda \mathsf{I} - \mathsf{A}| \\
&= |\mathsf{T}^{-1}\mathsf{T}|\,|\lambda \mathsf{I} - \mathsf{A}| \\
&= |\lambda \mathsf{I} - \mathsf{A}|.
\end{aligned}
$$

- Matrices A and $\tilde{A}$ are similar if there exists an invertible matrix T such that $\tilde{A} = T^{-1}AT$.
- T can be formed by stacking the columns of T by linearly independent eigenvectors of A.
- If A has fewer than *n* linearly independent eigenvectors, then it can't be diagonalized.

Example

Let $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. The characteristic eq. is $(\lambda - 1)(\lambda - 2)^2 = 0$. However, there are three linearly independent eigenvectors $v_1 = [-1\ 0\ 1]^T$, $v_2 = [0\ 1\ 0]^T$ and $v_3 = [-2\ 1\ 1]^T$. Thus, $T = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $T^{-1}AT = \tilde{A} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

- Linear homogeneous state equation, $\dot{\mathsf{x}} = \mathsf{A}\mathsf{x}$.
- Let $\mathbf{\Phi}(t)$ be a $n \times n$ matrix that satisfies the linear homogeneous equation,

$$\frac{d\mathbf{\Phi}(t)}{dt} = \mathsf{A}\mathbf{\Phi}(t).$$

- $\mathbf{\Phi}(t)$ is known as the state transition matrix, i.e, it describes the evolution of state as time progresses.
- If $\mathsf{x}(0)$ denotes the initial state at $t = 0$, then

$$\mathsf{x}(t) = \mathbf{\Phi}(t)\mathsf{x}(0)$$

  is the solution of the linear homogeneous equation for $t \geq 0$.
- How do we compute $\mathbf{\Phi}(t)$?

- Take the Laplace transform of $\dot{\mathrm{x}} = \mathrm{A}\mathrm{x}$ to get

$$sX(s) - x(0) = AX(s) \Rightarrow X(s) = (sI - A)^{-1}x(0)$$
$$\Rightarrow x(t) = \mathcal{L}^{-1}[(sI - A)^{-1}]x(0) = \mathbf{\Phi}(t)x(0).$$

- Scalar differential equation, $\dot{x} = ax$.
- Solution? $x(t) = e^{at}x(0)$.
- Draw analogy to the vector-matrix differential equation, $\dot{\mathrm{x}} = \mathrm{A}\mathrm{x}$.
- Assume solution of the form $\mathrm{x}(t) = \mathrm{e}^{\mathrm{A}t}\mathrm{x}(0)$, where

$$\mathrm{e}^{\mathrm{A}t} = \sum_{k=0}^{\infty} \frac{\mathrm{A}^k t^k}{k!}.$$

- Verify that $\mathrm{e}^{\mathrm{A}t}$ is indeed a solution to $\dot{\mathrm{x}} = \mathrm{A}\mathrm{x}$.

$$\frac{d}{dt}\mathrm{e}^{\mathrm{A}t} = \mathrm{A}\mathrm{e}^{\mathrm{A}t} = \mathrm{A}\mathrm{x}$$

Example

Compute the state transition matrix ($\Phi(t)$) for the given state model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Here, the system matrix $\mathrm{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$.

<u>First method</u>: $\Phi(t) = \mathcal{L}^{-1}[(s\mathrm{I} - \mathrm{A})^{-1}]$.

$s\mathrm{I} - \mathrm{A} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$

$\Rightarrow (s\mathrm{I} - \mathrm{A})^{-1} = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$

$\therefore\ \Phi(t) = \mathcal{L}^{-1}[(s\mathrm{I} - \mathrm{A})^{-1}] = \begin{bmatrix} 2e^{t} - e^{2t} & e^{t} - e^{2t} \\ -2e^{t} + 2e^{2t} & -e^{t} + 2e^{2t} \end{bmatrix}$

Example

Second method: Power series expansion of matrix exponential.

$\mathbf{\Phi}(t) = \mathrm{e}^{\mathrm{A}t} = \sum_{k=0}^{\infty} \frac{\mathrm{A}^k t^k}{k!} = \mathrm{I} + \mathrm{A}t + \frac{\mathrm{A}^2 t^2}{2} + \frac{\mathrm{A}^3 t^3}{6} + \ldots$

$$\begin{aligned}\mathbf{\Phi}(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ -2t & -3t \end{bmatrix} + \begin{bmatrix} -t^2 & -3t^2/2 \\ 3t^2 & 7t^2/2 \end{bmatrix} + \cdots \\ &= \begin{bmatrix} 2e^t - e^{2t} & e^t - e^{2t} \\ -2e^t + 2e^{2t} & -e^t + 2e^{2t} \end{bmatrix}\end{aligned}$$

- The power series method may be cumbersome sometimes.
- Can we compute $\mathrm{e}^{\mathrm{A}t}$ by another method?
- Cayley-Hamilton theorem: every square matrix satisfies its own characteristic equation.

$$f(\lambda) = \lambda^n + \alpha_1 \lambda^{n-1} + \alpha_2 \lambda^{n-2} + \ldots + \alpha_n = 0$$

$$\Delta(\mathrm{A}) = \mathrm{A}^n + \alpha_1 \mathrm{A}^{n-1} + \alpha_2 \mathrm{A}^{n-2} + \ldots + \alpha_n \mathrm{I} = 0.$$

Properties of Φ(t)

1. $\Phi(t)$ represents the free response of the system, i.e., response due to initial conditions only.
2. $\Phi(t)$ is only dependent on A.
3. $\Phi(0) = \mathrm{I}$. How?

   Proof: $\Phi(t) = e^{\mathrm{A}t} \Rightarrow \Phi(0) = e^{\mathrm{A}0} = \mathrm{I}$.
4. $\Phi^{-1}(t) = \Phi(-t)$. How?

   Proof: $\Phi(t) = e^{\mathrm{A}t} \Rightarrow \Phi^{-1}(t) = e^{-\mathrm{A}t} = \Phi(-t)$.
5. $\Phi(t_2 - t_1)\Phi(t_1 - t_0) = \Phi(t_2 - t_0)$ for any t_0, t_1, t_2. How?

   Proof: $\Phi(t_2 - t_1)\Phi(t_1 - t_0) = e^{\mathrm{A}(t_2 - t_1)}e^{\mathrm{A}(t_1 - t_0)} = e^{\mathrm{A}(t_2 - t_0)} = \Phi(t_2 - t_0)$.
6. $\Phi^k(t) = \Phi(kt)$. How?

   Proof: $\Phi^k(t) = e^{(\mathrm{A}t)^k} = e^{k\mathrm{A}t} = \Phi(kt)$.

- Consider LTI state equation $\dot{\mathrm{x}} = \mathrm{Ax}(t) + \mathrm{Bu}(t)$.
- Taking Laplace transform on both sides,

$$\begin{aligned}
s\mathrm{X}(s) - \mathrm{x}(0) &= \mathrm{AX}(s) + \mathrm{BU}(s) \\
\Rightarrow \mathrm{X}(s) &= (s\mathrm{I} - \mathrm{A})^{-1}\mathrm{x}(0) + (s\mathrm{I} - \mathrm{A})^{-1}\mathrm{BU}(s) \\
\Rightarrow \mathrm{x}(t) &= \mathcal{L}^{-1}[(s\mathrm{I} - \mathrm{A})^{-1}]\mathrm{x}(0) + \mathcal{L}^{-1}[(s\mathrm{I} - \mathrm{A})^{-1}\mathrm{BU}(s)] \\
\Rightarrow \mathrm{x}(t) &= \mathbf{\Phi}(t)\mathrm{x}(0) + \underbrace{\int_0^t \mathbf{\Phi}(t-\tau)\mathrm{Bu}(\tau)d\tau}_{\text{convolution integral}}
\end{aligned}$$

- In general, if $t_0 \neq 0$, then the total response is given by

$$\underbrace{\mathrm{x}(t)}_{\text{total response}} = \underbrace{\mathbf{\Phi}(t-t_0)\mathrm{x}(t_0)}_{\text{free response}} + \underbrace{\int_{t_0}^t \mathbf{\Phi}(t-\tau)\mathrm{Bu}(\tau)d\tau}_{\text{forced response}}.$$

- Relation between state-space model and transfer function

$$X(s) = (sI - A)^{-1}x(0) + (sI - A)^{-1}BU(s)$$

$$X(s) = (sI - A)^{-1}\cancelto{0}{x(0)} + (sI - A)^{-1}BU(s)$$

$$\Rightarrow X(s) = (sI - A)^{-1}BU(s).$$

- Output equation is given as $y(t) = Cx(t) + Du(t)$.
- In Laplace domain,

$$\begin{aligned} Y(s) &= C{\color{blue}X(s)} + DU(s) \\ &= C{\color{blue}(sI - A)^{-1}BU(s)} + DU(s) \\ &= \left\{C(sI - A)^{-1}B + D\right\}U(s) \\ \Rightarrow \frac{Y(s)}{U(s)} &= C(sI - A)^{-1}B + D \end{aligned}$$

- Suppose A has distinct eigenvalues, $\lambda_1, \lambda_2, \ldots, \lambda_n$, so that it has a set of linearly independent eigenvectors, $v_1, v_2, \ldots, v_n$.
- If $x(0) = x_0 = v_j$, then

$$x(t) = e^{At}x_0 = e^{At}v_j = \sum_{k=0}^{\infty} \frac{A^k t^k}{k!} v_j = \sum_{k=0}^{\infty} \frac{\lambda_j^k t^k}{k!} v_j = e^{\lambda_j t} v_j$$

- If we start with initial condition along an eigenvector of A, the solution $x(t)$ will stay in the direction of the eigenvector, with length being stretched or shrunk by $e^{\lambda_j t}$.
- If the eigenvectors $v_1, v_2, \ldots, v_n$ are linearly independent,

$$x_0 = \sum_{j=1}^{n} \xi_j v_j, \quad \xi_j = x_0^T v_j$$

- For some $\xi = [\xi_1, \ldots, \xi_n]^\top$, and $P = [v_1, v_2 \ldots, v_n]^\top$, we have

$$x_0 = \sum_{j=1}^{n} \xi_j v_j = P\xi$$

- Solutions of homogenous system

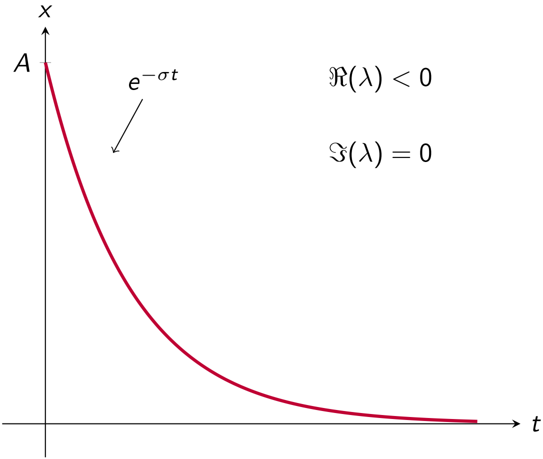
$$x(t) = e^{At} x_0 = e^{At} \sum_{j=1}^{n} \xi_j v_j = \sum_{j=1}^{n} \xi_j e^{At} v_j = \sum_{j=1}^{n} \xi_j e^{\lambda_j t} v_j$$

- Solution $x(t)$: A (time-varying) linear combination of the eigenvectors of A.

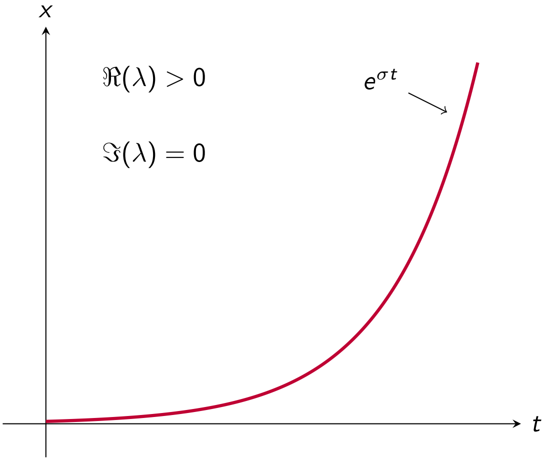
- Note that P diagonalizes A, that is, $P^{-1}AP = \Lambda$.
- Since P diagonalizes A, we have

$$
\begin{aligned}
e^{At}x_0 &= Pe^{\Lambda t}P^{-1}x_0 = Pe^{\Lambda t}\xi \\
&= P\begin{bmatrix} \xi_1 e^{\lambda_1 t} \\ \xi_2 e^{\lambda_2 t} \\ \vdots \\ \xi_n e^{\lambda_n t} \end{bmatrix} \\
&= \sum_j \xi_j e^{\lambda_j t} v_j
\end{aligned}
$$

- Same result as the previous one!
- These representations of the solution $x(t)$ in terms of the eigenvectors are known as the modal decomposition of $x(t)$.



- Purely real eigenvalue, $\lambda = \sigma \pm j\,0$.
- This mode is statically stable.



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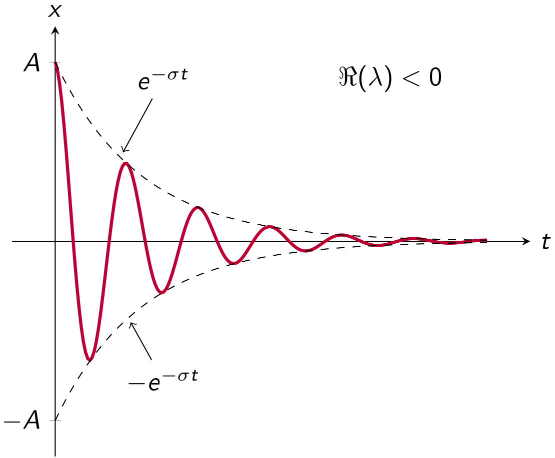
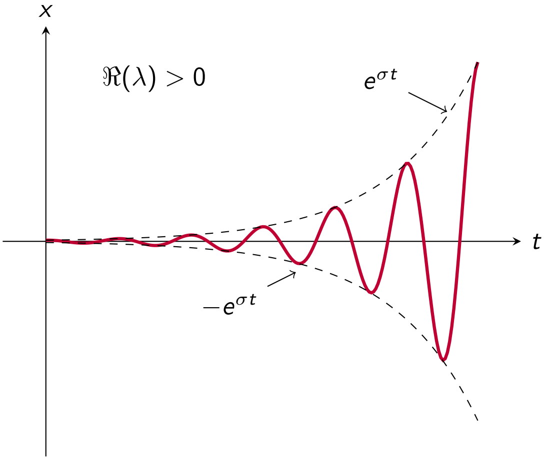


- Complex conjugate eigenvalues, $\lambda = \sigma \pm j\omega$, with negative real part.
- This mode is dynamically stable/damped/convergent.



- Complex conjugate eigenvalues, $\lambda = \sigma \pm j\omega$, with positive real part.
- This mode is dynamically unstable/undamped/divergent.

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Thank you for your attention !!!