Flight Mechanics/Dynamics

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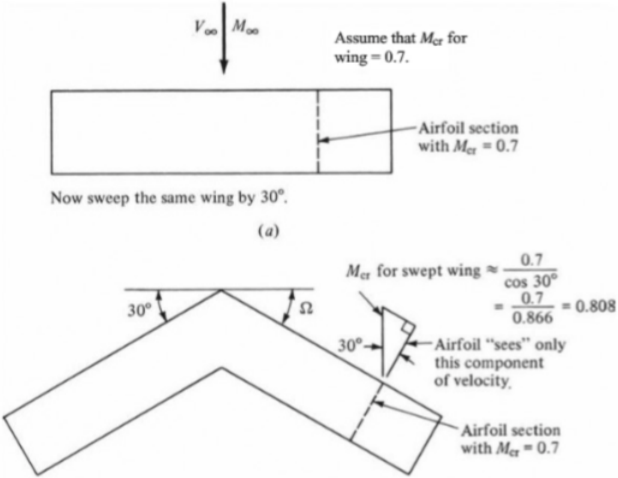
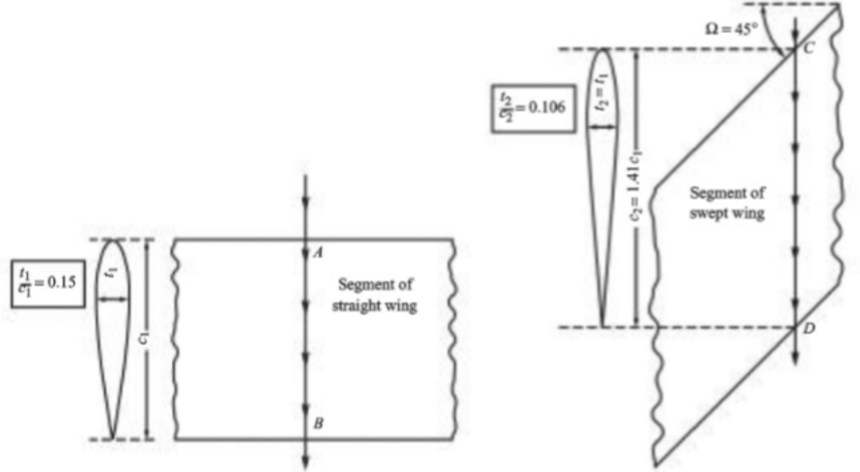
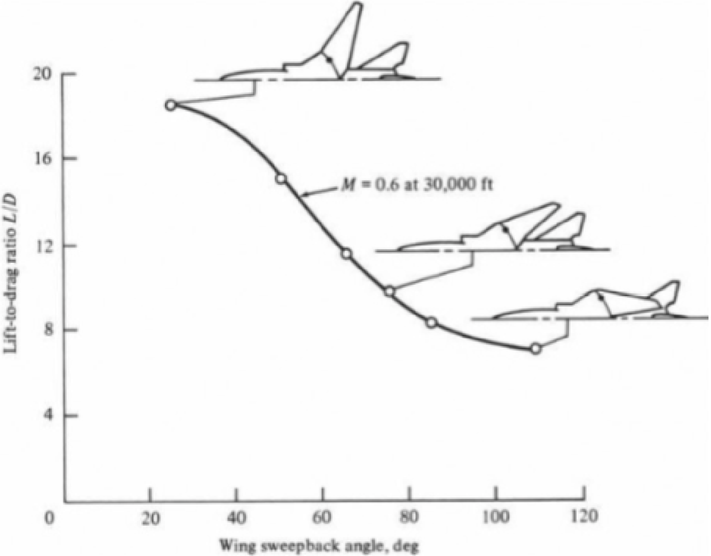
- Why most of the high speed airplanes are having swept wing?
- Delay of drag divergence to high Mach numbers.
- Effective airfoil is thiner than in straight wing.

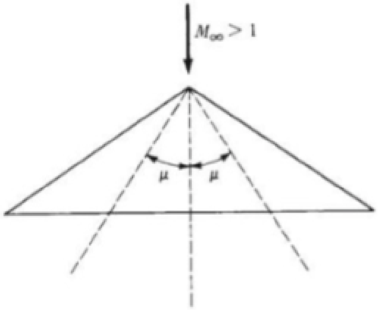

V_∞ M_∞
Assume that M_{cr} for wing = 0.7.
Airfoil section with M_{cr} = 0.7
Now sweep the same wing by 30°.
(a)
M_{cr} for swept wing $\approx \frac{0.7}{\cos 30^\circ} = \frac{0.7}{0.866} = 0.808$
30°
Ω
30°
Airfoil "sees" only this component of velocity.
Airfoil section with M_{cr} = 0.7

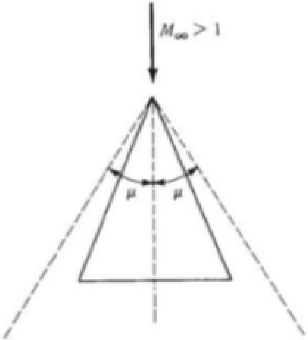


What is the price we are paying for using swept wing?


20
16
12
8
4
0
20
40
60
80
100
120
Lift-to-drag ratio L/D
M = 0.6 at 30,000 ft
Wing sweepback angle, deg

- Swept wing is advantageous for supersonic flight also. How?





- For supersonic flight, if the swept wing is outside Mach cone then leading edge of wing is supersonic.
- If swept wing is within Mach cone then leading edge of wing is subsonic and thus wave drag is less.

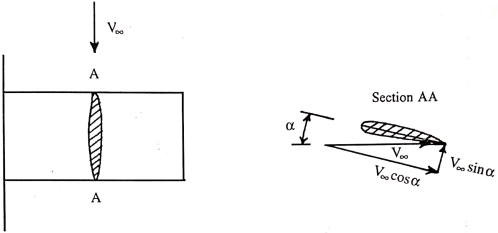
- Let V_n be the flight velocity normal to straight wing aircraft.
- Assume that two identical aircraft is at the same lift, drag and normal dynamic pressure.
- For a wing with sweep angle, with same normal dynamic pressure, flow velocity is $V_n/\cos\Lambda$.
- For straight and swept wing aircraft,

$$L = \frac{1}{2}\rho V_n^2 S C_l, \quad L_s = \frac{1}{2}\rho\left(\frac{V_n}{\cos\Lambda}\right)^2 S C_{ls}$$

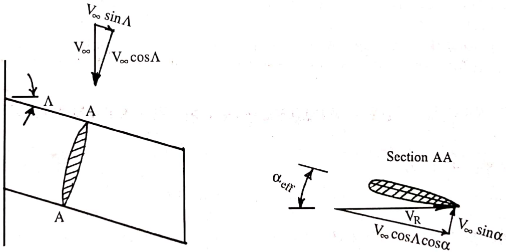
- On equating these two,

$$\boxed{C_{ls} = C_l \cos^2\Lambda}$$

- Similarly, $\boxed{C_{ds} = C_d \cos^2\Lambda}$
- For same lift and drag force (same engine power), aircraft can fly at higher speed.



a) Straight wing



b) Swept-back wing

- Consider aircraft operating at angle of attack α.
- Effective angle of attack

$$\tan \alpha_{\text{eff}} = \frac{V_\infty \sin \alpha}{V_\infty \cos \Lambda \cos \alpha} \implies \alpha_{\text{eff}} \approx \alpha \sec \Lambda$$

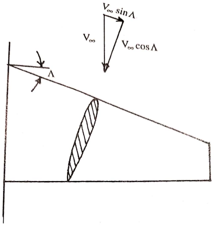
- Let a be the lift curve slope of wing.

$$L = \left(\frac{1}{2}\rho V_\infty^2 \cos^2 \Lambda\right) Sa(\alpha \sec \Lambda) = \frac{1}{2}\rho V_\infty^2 \cos \Lambda S a \alpha = \frac{1}{2}\rho V_\infty^2 S C_{Ls}$$

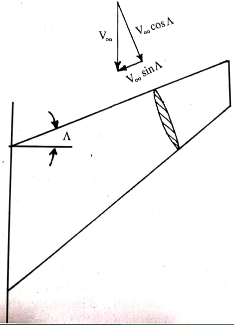
- Lift coefficient for swept wing,

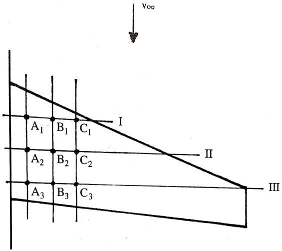
$$C_{Ls} = a\alpha \cos \Lambda \implies a_s = a \cos \Lambda; \quad a_s = \frac{\partial C_{Ls}}{\partial \alpha}$$

- For delaying the drag divergence both swept back and forward are same.

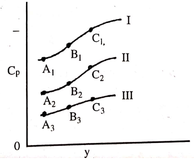


a) Swept-back wing





a)



- Drag polar for complete airplane

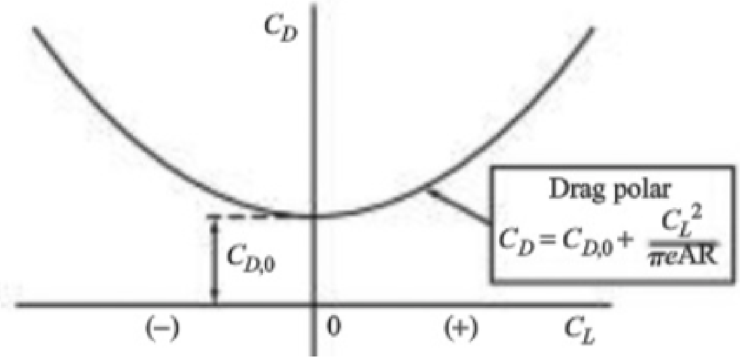
$$C_D = \underbrace{C_{d,e}}_{\text{Parasite drag coefficient}} + \frac{C_L^2}{\pi e AR}$$

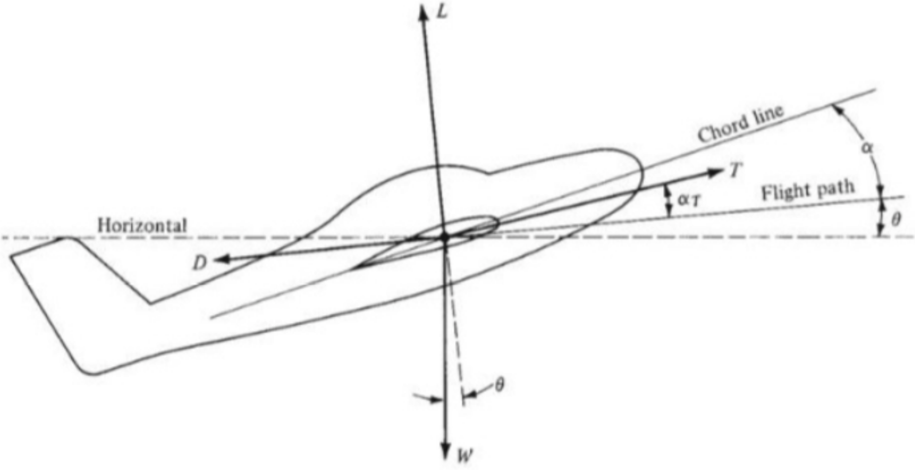
- Parasite drag include profile drag, and friction and pressure drag to other parts of airplane.
- Parasite drag coefficient

$$C_{D,e} = C_{D,0} + rC_L^2 \Rightarrow C_D = C_{d,e} + \left(r + \frac{1}{\pi e AR}\right) C_L^2$$

- Drag polar for complete airplane, with e redefined as Oswald efficiency factor

$$\boxed{C_D = \underbrace{C_{d,0}}_{\text{Zero-lift drag coeff.}} + \underbrace{C_{D,i}}_{\text{Drag coeff. due to lift}}, \quad C_{D,i} = \frac{C_L^2}{\pi e AR}}$$


C_D
$C_{D,0}$
Drag polar
$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR}$
(−)
0
(+)
C_L



L
Chord line
α
T
α_T
Flight path
θ
Horizontal
D
θ
W

- For a curvilinear flight

$$\sum F_{\|} = m\frac{dV}{dt}, \qquad \sum F_{\perp} = m\frac{V^2}{r_c}$$

- Total forces in parallel and perpendicular directions

$$\sum F_{\|} = T\cos\alpha_T - D - W\sin\theta$$

$$\sum F_{\perp} = L + T\sin\alpha_T - W\cos\theta$$

- Equations of motion in translational flight

$$\boxed{T\cos\alpha_T - D - W\sin\theta = m\frac{dV}{dt}}$$

$$\boxed{L + T\sin\alpha_T - W\cos\theta = m\frac{V^2}{r_c}}$$

- **Static performance**: Performance of airplane in unaccelerated flight condition
- For unaccelerated level flight, we have $\theta = 0$.
- Also, there are no accelerations.

$$\boxed{T\cos\alpha_T = D, \quad L + T\sin\alpha_T = W}$$

- With assumption of $\alpha_T \approx 0$, $\sin\alpha_T = 0, \;\; \cos\alpha_T \approx 1$.
- EOM for level, and unaccelerated flight

$$\boxed{T = D, \quad L = W}$$

- Thrust and lift balance aerodynamic drag and weight, respectively.

- For steady flight,

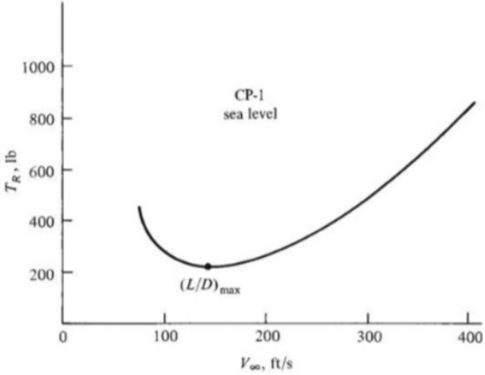
$$T = D = q_\infty S C_D$$

$$L = W = q_\infty S C_L$$

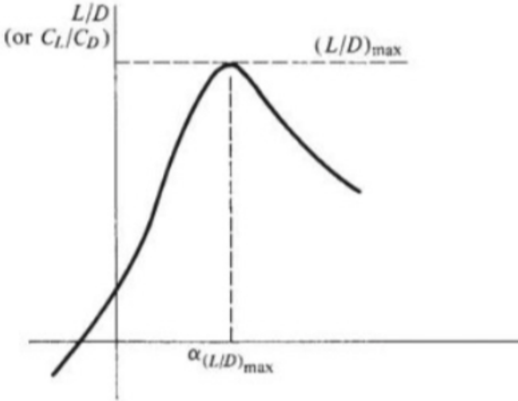
$$\Rightarrow \frac{T}{W} = \frac{C_D}{C_L}$$

- Thrust required for airplane to fly at given velocity, unaccelerated flight

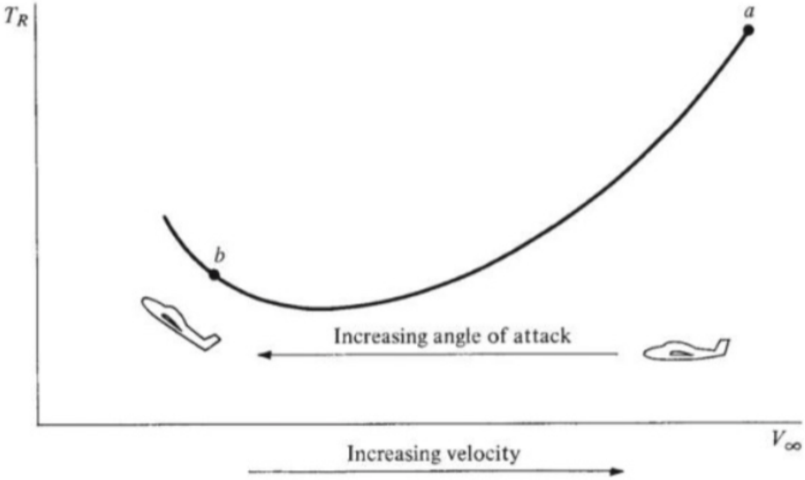
$$\boxed{T_R = \frac{W}{C_L/C_D} = \frac{W}{L/D}}$$



- L/D is a measure of aerodynamic efficiency.
- Maximum aerodynamic efficiency$\Rightarrow$ Minimum thrust
- L/D is a function of α, and maximum at around 2°-5°.



Airplane flying at the velocity for minimum $T_R \Rightarrow$ flying at α for maximum L/D.

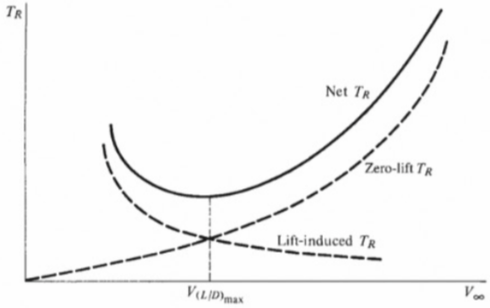


T_R
a
b
Increasing angle of attack
V_∞
Increasing velocity

- Required thrust

$$
\begin{aligned}
T_R =& D = q_\infty S(C_{D,0} + C_{D,i}) \\
=& \underbrace{q_\infty S C_{D,0}}_{\text{Zero-lift } T_R} + \underbrace{q_\infty S \frac{C_L^2}{\pi e \mathsf{AR}}}_{\text{Lift-induced } T_R}
\end{aligned}
$$

- Zero-lift thrust increase while lift induced thrust decreases with increase in velocity.
- Where will be the minimum required thrust?



- Thrust required

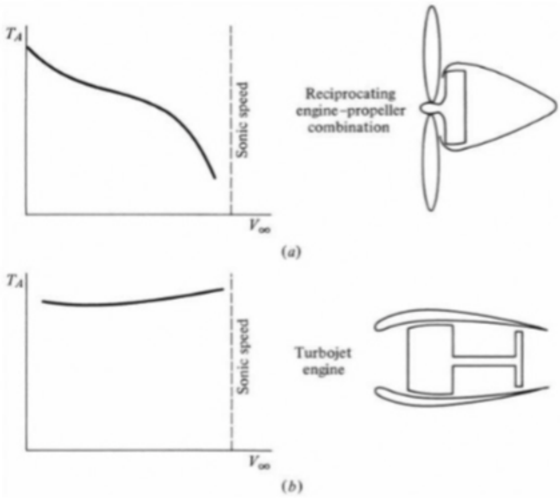
$$T_R = q_\infty S C_{D,0} + q_\infty S \frac{C_L^2}{\pi e \mathsf{AR}} = q_\infty S C_{D,0} + \frac{W^2}{q_\infty S \pi e \mathsf{AR}}$$

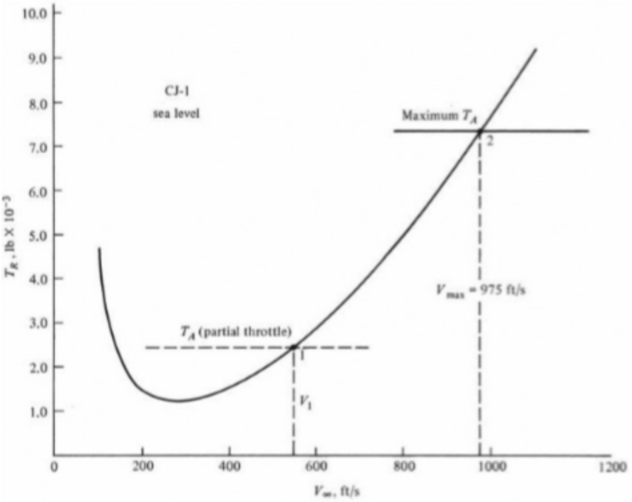
- Point of minimum thrust T_R correspond to $\dfrac{dT_R}{dV_\infty} = 0.$
- As $\dfrac{dT_R}{dq_\infty} = \dfrac{dT_R}{dV_\infty}\dfrac{dV_\infty}{dq_\infty}$, a minimum thrust also implies $\dfrac{dT_R}{dq_\infty} = 0.$
- On differentiation,

$$\frac{dT_R}{dq_\infty} = S C_{D,0} - \frac{W^2}{q_\infty^2 S \pi e \mathsf{AR}} = 0 \Rightarrow C_{D,0} = \frac{W^2}{q_\infty^2 S^2 \pi e \mathsf{AR}}$$

- On simplifying,

$$\boxed{C_{D,0} = \frac{C_L^2}{\pi e \mathsf{AR}} = C_{D,i} \Rightarrow \text{Zero-lift drag} = \text{Drag due to lift}}$$


T_A
Sonic speed
V_∞
Reciprocating engine–propeller combination
(a)
T_A
Sonic speed
V_∞
Turbojet engine
(b)



CJ-1
sea level
Maximum T_A
T_A (partial throttle)
V_{max} = 975 ft/s
V_1
1
2
T_R, lb × 10^{-3}
V_∞, ft/s
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
200
400
600
800
1000
1200

- Power required by airplane

$$\boxed{P_R = T_R V_\infty = \frac{W V_\infty}{C_L/C_D}}$$

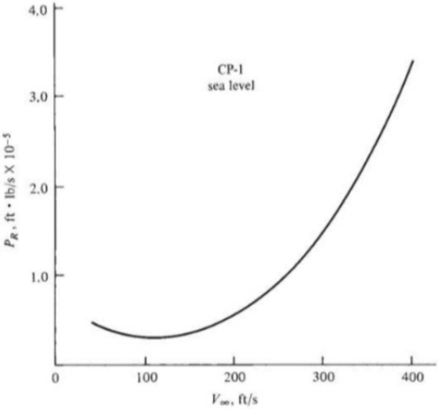
- In steady flight,

$$L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$\Rightarrow V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

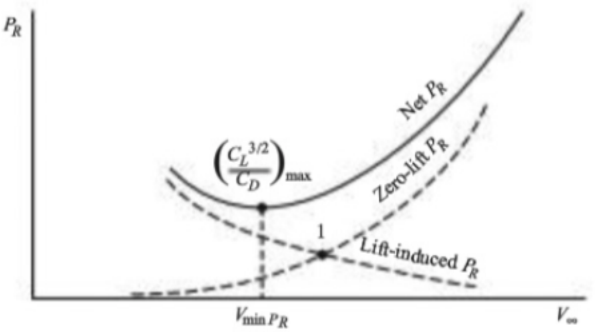
- Power required

$$\boxed{P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}} \propto \frac{1}{C_L^{3/2}/C_D}}$$



- Power required by airplane

$$P_R = T_R V_\infty = D V_\infty = q_\infty S C_D V_\infty$$
$$= \underbrace{q_\infty S C_{D,0} V_\infty}_{\text{Zero lift power req.}} + \underbrace{q_\infty S V_\infty \frac{C_L^2}{\pi e \mathsf{AR}}}_{\text{Lift-induced power req.}}$$



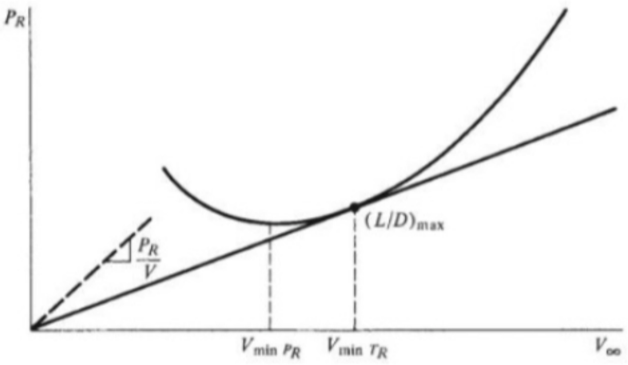
- Power required by airplane,

$$
\begin{aligned}
P_R &= \frac{1}{2}\rho_\infty C_{D,0} V_\infty^3 S + \frac{1}{2}\rho_\infty S V_\infty^3 \frac{1}{\pi e \mathsf{AR}} \left(\frac{W}{(1/2)\rho_\infty V_\infty^2 S}\right)^2 \\
&= \frac{1}{2}\rho_\infty C_{D,0} V_\infty^3 S + \frac{1}{\pi e \mathsf{AR}} \frac{W^2}{(1/2)\rho_\infty V_\infty S}
\end{aligned}
$$

- On differentiation,

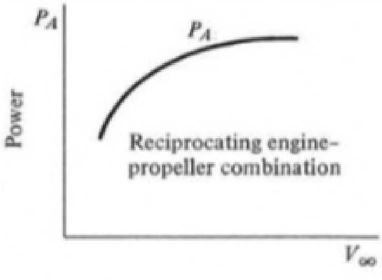
$$
\begin{aligned}
\frac{dP_R}{dV_\infty} &= \frac{3}{2}\rho_\infty C_{D,0} V_\infty^2 S - \frac{1}{\pi e \mathsf{AR}} \frac{W^2}{(1/2)\rho_\infty V_\infty^2 S} \\
&= 3\rho_\infty V_\infty^2 S \left(C_{D,0} - \frac{C_L^2/3}{\pi e \mathsf{AR}}\right) \\
&= 3\rho_\infty V_\infty^2 S \left(C_{D,0} - \frac{1}{3} C_{D,i}\right)
\end{aligned}
$$

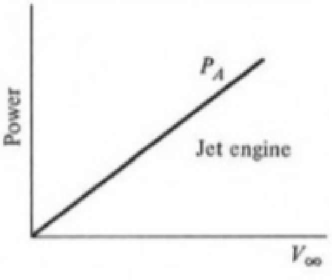
- For minimum power, $\dfrac{dP_R}{dV_\infty} = 0 \Rightarrow \boxed{C_{D,0} = \dfrac{1}{3} C_{D,i}}$



- Point of tangency corresponds to minimum T_R (and hence $(L/D)_{\max}$). How?
- At tangent point, P_R/V_∞ is minimum.

$$\frac{d(P_R/V_\infty)}{dV_\infty} = \frac{dT_R}{dV_\infty} = 0 \implies T_{R\,\min} \text{ and } (L/D)_{\max}$$



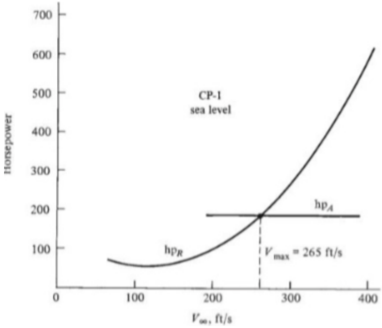


- For reciprocating engine-propeller combination, with shaft brake power P,

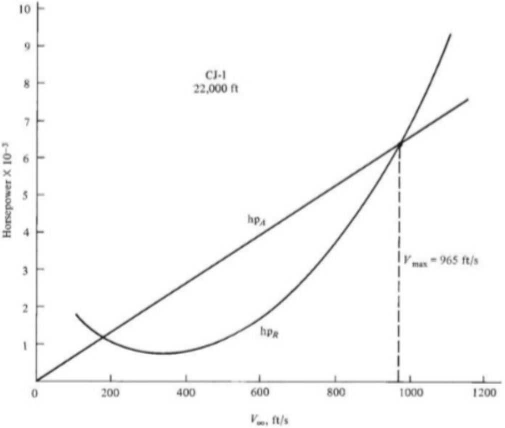
$$\boxed{P_A = \eta P, \ \eta < 1}$$

- For jet engine,

$$\boxed{P_A = T_A V_\infty}$$



CP-1
sea level
Horsepower
V_∞, ft/s
hp_R
hp_A
V_{max} = 265 ft/s



CJ-1
22,000 ft
Horsepower × 10^{-3}
V_∞, ft/s
hp_A
hp_R
V_{max} = 965 ft/s

Reference

1. John Anderson Jr., *Introduction to Flight*, McGraw-Hill Education, Sixth Edition, 2017.

Thank you for your attention !!!