



## *Constant Pitch Rate Problems*



## ***Problem No. 01***

**Consider** the following rocket **configuration**.

$m_0 = 74$  Tons,  $m_p = 54$  Tons,  $I_{sp} = 240$  s,  $g_0 = 9.81 \text{ m/s}^2$ ,  $V_0 = 85.4$  m/s,  $\theta_0 = 5^\circ$ ,  $h_0 = 415$  m.

Determine all the **terminal** parameters in case **rocket** executes the above manoeuvre for another **90s**, assuming sea-level **gravity** and vacuum condition.



## ***Solution No. 01***

The  $V$ ,  $\theta$  **solutions** are as follows.

$$q_0 = \frac{\tilde{g} \sin \theta_0}{V_0} = \frac{9.81 \times \sin 5^\circ}{85.4} = 0.01 \text{ rad} / s = 0.573^\circ / s$$
$$\theta_{i=90} = \theta_0 + q_0 \Delta t = 5 + 0.573 \times 90 = 56.6^\circ$$
$$V_{i=90} = \frac{\tilde{g} \sin \theta_{90}}{q_0} = \frac{9.81 \times 0.835}{0.01} = 819 \text{ m} / s$$



## ***Solution No. 01***

The mass, x and y **solutions** are as follows.

$$\ln \frac{m_0}{m_{t=90}} = \frac{2\tilde{g}}{q_0 g_0 I_{sp}} (\sin \theta_{t=90} - \sin \theta_0) = 0.62 \rightarrow m_{90} = 39.7T$$

$$h_{t=90} = 415 + \frac{9.81}{4 \times 0.01^2} (0.9848 + 0.3939) = 33813 \text{ m}$$

$$x_{t=90} = \frac{9.81}{2 \times 0.01^2} \left[ 0.900 - \frac{(\sin 2\theta_{t=90} - \sin 2\theta_0)}{2} \right] = 25862m$$



## ***Problem No. 02***

Can all the **propellant** can be burnt to **reach**  $90^\circ$ ? If yes, **give** final burnout **parameters**. If no, give the **trajectory** parameters when all the **propellant** is consumed.



## ***Solution No. 02***

**No.** All the propellant **cannot** be consumed while reaching 90°. The burnout **trajectory** parameters as are follows.

$$t_{\theta=90} = \frac{90-5}{0.573} = 148.3s; \quad V_{90} = \frac{\tilde{g} \sin 90}{q_0} = \frac{9.81}{0.01} = 981m/s$$

$$\ln \frac{m_0}{m_{\theta=90}} = \frac{2\tilde{g}}{q_0 g_0 I_{sp}} (\sin 90 - \sin \theta_0) = 0.7607 \rightarrow m_{\theta=90} = 34.6T$$

$$h_{\theta=90} = 415 + \frac{9.81}{4 \times 0.01^2} (0.9848 + 1.000) = 49092 \text{ m}$$

$$x_{\theta=90} = \frac{9.81}{2 \times 0.01^2} \left[ 1.4837 - \frac{(\sin 180 - \sin 2\theta_0)}{2} \right] = 77034m$$



## ***Problem No. 03***

**What** should be  $\theta_0$  and  $q_0$ , for the same  $V_0$ , if all **fuel** is to be burnt for  $\theta_b = 90^\circ$ ? Also, determine the **mission** terminal parameters in this case?



## ***Solution No. 03***

Here, we know mass **fraction** and final angle so we can **obtain** initial angle through **solution** of two nonlinear equations, as **follows**.

$$V_0 = 85.4 = \frac{9.81 \times \sin \theta_0}{q_0}; \quad \ln \frac{m_{t=5}}{m_{\theta=90}} = 1.308 = \frac{2}{q_0 \times 240} (1.000 - \sin \theta_0)$$

$$\sin \theta_0 = 8.705 q_0; \quad \frac{2}{q_0 \times 240} (1.000 - 8.705 q_0) = 1.308; \quad q_0 = 0.00604$$

$$\sin \theta_0 = 8.705 \times 0.00604 = 0.00526 \rightarrow \theta_0 = 3.012^\circ; \quad t_b = 5 + \frac{90 - 3.012}{0.00604 \times 57.29} = 251.4s$$

$$V_b = \frac{9.81 \times \sin 90}{0.00604} = 1624.2 m/s; \quad h_b = 415 + \frac{9.81}{4 \times 0.00604^2} (0.9945 + 1.000) = 134080 m$$

$$x_b = \frac{9.81}{2 \times 0.00604^2} \left[ 1.518 - \frac{(\sin 180 - \sin 2\theta_0)}{2} \right] = 211147 m$$