AE 240/713/240M **End-semester Examination** **April 28, 2021**
Weight: 20% **Time: 0900-1100 Hrs.** **Venue: Online – Moodle**
Exam is open notes (including hand-written). **Only calculator is allowed.**

Model Solutions

Q.1 A spacecraft with $m_* = 100$ kg needs to form a circular orbit at 200 km altitude using a two-stage rocket. If the rocket uses same propellant of $I_{sp} = 300$s as well as same structural ratio, 'ε', of 0.1 for both the stages, determine stage-wise mass configuration, m_{si}, m_{pi} & lift-off mass, m_0 to achieve the desired V_*, assuming a 10% loss of ideal energy due to gravity. ($R_E = 6.378\times10^6$m, $\mu = 3.986\times10^{14}$ m^{3}s^{-2}, $g_0 = 9.81$ m/s^2). (4)

$$V_c = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986\times10^{14}}{6.578\times10^6}} = 7784.3; \quad \varepsilon_g = \frac{V_c^2}{2} + gh = \frac{(7784.3)^2}{2} + 9.81\times2\times10^5 = 3.225966\times10^7$$

$$0.9\varepsilon_0 = \varepsilon_g = 3.225966\times10^7 \to V_* = 8466.9m/s; \quad m_* = 100kg; \quad \text{Optimal Payloads: } \pi_1 = \pi_2 = \pi$$

$$V_* = -2g_0 I_{sp}\ln\left[\varepsilon + (1-\varepsilon)\pi\right] \to 0.1 + 0.9\pi = e^{-1.4385} = 0.237287 \to \pi = 0.15254; \quad \pi_* = 0.023269$$

$$m_0 = \frac{m_*}{\pi_*} = \frac{100}{0.023269} = 4297.5kg; \quad \pi_2 = \frac{m_*}{m_{02}} = 0.15254 \to m_{02} = 655.56 = m_{s2} + m_{p2} + m_*$$

$$m_{s2} + m_{p2} = 555.56 = \frac{m_{s2}}{\varepsilon} \to m_{s2} = 55.56; \quad m_{p2} = 500; \quad \pi_1 = \frac{m_{02}}{m_{01}} = 0.15254$$

$$m_{01} = 4297.6 = m_{02} + m_{s1} + m_{p1} \to m_{s1} + m_{p1} = 3642.04 = \frac{m_{s1}}{\varepsilon} \to m_{s1} = 364.20; \quad m_{p1} = 3277.8$$

Q.2 A spacecraft weighing 2500 kg and flying at an angle of 3.0° with local horizon during its terminal phase needs to become parallel to local horizon over a horizontal distance of 30 km. Determine the constant velocity (V_0) at which the required gravity turn manoeuvre must be performed, along with the time taken for it. Also determine the propellant burn rate at the start of the manoeuvre, if a fuel of $I_{sp} = 300$s is to be used and approximate fuel mass that must be carried for this manoeuvre. (You may assume flat earth. Also, $g_0 = 9.81$ m/s^2) (3)

$$\dot{\theta} = \frac{g_0\sin\theta}{V} \to dt = \frac{V_0}{g_0\sin\theta}d\theta \to \Delta t = \frac{V_0}{g_0}\ln\left[\frac{\tan\left(\theta/2\right)}{\tan\left(\theta_0/2\right)}\right]; \quad \dot{x} = V\sin\theta; \quad dx = V_0\sin\theta dt$$

$$dx = \frac{V_0^2}{g_0}d\theta \to \Delta x = \frac{V_0^2}{g_0}(\theta - \theta_0); \quad V_0 = \sqrt{\frac{30000\times9.81}{0.052365}} = 2370.7m/s; \quad \Delta t = \frac{2370.7}{9.81}\ln\left[\frac{1}{0.948}\right]$$

$$\Delta t = 12.66s; \quad \frac{\dot{m}_0 g_0 I_{sp}}{m_0} = -g_0\cos\theta \to \dot{m}_0 = -\frac{m_0\cos\theta}{I_{sp}} = 0.436kg/s; \quad \Delta m_p \approx 2.76kg$$

Q.3 A satellite with perigee of 522 km altitude and apogee of 20022 km altitude needs to make a 90° plane change when the spacecraft reaches its apogee. Determine the 'ΔV' required for the above manoeuvre and compare it with velocity impulse if a parabola is used for the inclination change? ($R_E = 6.378\times10^6$m, $\mu = 3.986\times10^{14}$ m^{3}s^{-2}). (3)

$$r_p = a(1-e) = 6.9\times10^6; \quad r_a = a(1+e) = 20.4\times10^6; \quad a = \frac{r_p + r_a}{2} = 13.65\times10^6$$

$$\frac{1}{2}v_a^2 - \frac{\mu}{r_a} = -\frac{\mu}{2a} \to v_a = \sqrt{\frac{2\mu}{r_a} - \frac{\mu}{a}} = 3142.76m/s; \quad \Delta v_a = \sqrt{2}v_a = 4444.5m/s$$

$$v_p = v_a\frac{r_a}{r_p} = 9291.6; \quad \Delta v_1 = \sqrt{\frac{2\mu}{r_p}} - v_p = 1457.18; \quad \Delta v_{parabola} = 2\times\Delta v_1 = 2917.36m/s$$

Q.4 A Spacecraft has an orbit around Earth with perigee altitude of 500 km and apogee altitude of 40,000 km and its communication range is less than 1000 km. Determine the time interval during which it is in contact with the ground stations and the corresponding mean and actual angular travel that it has done during this interval. ($R_E = 6.378\times10^6$m, $\mu = 3.986\times10^{14}$ m^{3}s^{-2}). (5)

$$r_p = a(1-e) = 6.878\times10^6; \quad r_a = a(1+e) = 46.378\times10^6; \quad a = 26.628\times10^6 m; \quad e = 0.7417$$

$$r_{comm} = 7.378\times10^6 = \frac{a\left(1-e^2\right)}{1+e\cos\theta} = \frac{26.628\times10^6\times0.44988}{1+0.7417\cos\theta} \rightarrow \cos\theta = 1.3482\times(1.6237-1) = 0.8409$$

$$\theta = \pm32.76^o \rightarrow \Delta\theta = 65.52^o \quad n = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{3.986\times10^{14}}{\left(26.628\times10^6\right)^3}} = 0.000145 rad/\sec; \quad \tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}$$

$$E_A = 2\times\tan^{-1}\left(0.3851\times0.29394\right) = 12.92^o = 0.2254 rad; \quad E_B = -E_A = 12.92^o = -0.2254 rad$$

$$M_A = E_A - e\sin E_A = 0.0596 rad; \quad M_B = -M_A = -0.0596 rad; \quad \Delta M = 0.11913 rad$$

$$TOF\,(\text{Communication Interval}) = \frac{\Delta M}{n} = \frac{0.11913}{0.000145} = 821.56\sec = 13.69\min$$

Q.5 Two geocentric ellipses have aligned major axes and their perigees are on the same side of earth. For first ellipse $r_p = 7\times10^6$ m and e = 0.3, while for the second ellipse $r_p = 32\times10^6$ m and e = 0.5. Find the total impulse required to complete the transfer as well as the time taken to complete it. ($R_E = 6.378\times10^6$m, μ = 3.986×10^{14} m^{3}s^{-2}). (5)

$$r_{p1} = 7\times10^6; \quad e_1 = 0.3; \quad a_1 = \frac{r_{p1}}{1-e_1} = \frac{7\times10^6}{1-0.3} = 1\times10^7 m$$

$$r_{p2} = 32\times10^6; \quad e_2 = 0.5; \quad a_2 = \frac{r_{p2}}{1-e_2} = \frac{32\times10^6}{1-0.5} = 6.4\times10^7 m$$

$$r_{a2} = a_2(1+e_2) = 6.4\times10^7\times1.5 = 9.6\times10^7 m; \quad r_{aTO} = 9.6\times10^7, \quad r_{pTO} = 0.7\times10^7$$

$$a_{TO} = \frac{r_{p1}+r_{a2}}{2} = 5.15\times10^7; \quad v_{pTO} = \sqrt{\frac{2\mu}{r_{p1}} - \frac{\mu}{a_{TO}}} = \sqrt{1.056316\times10^8} = 10227.7 m/s;$$

$$v_{p1} = \sqrt{\frac{2\mu}{r_{p1}} - \frac{\mu}{a_1}} = \sqrt{7.351143\times10^7} = 8573.9 m/s; \quad \Delta V_1 = 1653.8 m/s$$

$$v_{aTO} = \sqrt{\frac{2\mu}{r_{aTO}} - \frac{\mu}{a_{TO}}} = \sqrt{5.6436\times10^5} = 751.2 m/s; \quad v_{a2} = \sqrt{\frac{2\mu}{r_{a2}} - \frac{\mu}{a_2}} = \sqrt{2076041.7} = 1440.8$$

$$\Delta V_2 = v_{a2} - v_{aTO} = 689.6 m/s; \quad \Delta V = \Delta V_1 + \Delta V_2 = 2343.4 m/s$$

$$TOF = \pi\sqrt{\frac{a_{TO}^3}{\mu}} = \pi\times\sqrt{\frac{\left(5.15\times10^7\right)^3}{3.986\times10^{14}}} = 18511.5\sec = 308.5\min = 5.14h$$

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