Fast Transfers Concept

General Orbital Transfers

We know that **time** taken to complete the orbital **transfers** through Hohmann technique **requires** at least one half **cycle** time to achieve the **mission**.

This can be **quite** large in case of far away **planets** and may need to be **reduced.**

Also, in the cases of **missions** involving humans / other time **critical** requirements, there is a **need** for faster **mechanisms** of orbital transfers.

General Orbital Transfers

Another **important** spacecraft **manoeuvre** is the requirement of going to another **spacecraft** in a different **orbit** (e.g. space shuttle mission to **space station**).

These are **particularly** needed for **transporting** material and humans to **space** & back.

General Orbital Transfers

It is found that in **such** cases it is better to **directly** launch into the **desired** orbit, also called launch to **orbit**.

However, such **missions** need special **conditions** to be satisfied, to create **efficient** transfers.

Launch to Orbit Concept

Fast transfer is **energy** efficient if spacecraft is **launched** in a **'chase'** orbit with destination **vertically** overhead.

In this case, we typically **wait** for the destination to be **overhead** and then launch the **spacecraft** so that it **intersects** the desired **orbit** behind the destination.

Launch to Orbit Concept

This requires a **higher** energy launch in comparison to the **energy** of the destination **orbit**.

SSTO (Singe-stage-to-orbit) and **TSTO** (Two-stage-to-orbit) missions are **common** examples of such **transfers.**

Launch to Orbit Strategy

In this **context**, it should be noted that all **spacecraft,** visible from earth, create a **path** on the surface of the **earth**, which is called the **ground** trace**.**

This is nothing but the **locus** of all points due to **intersection** of radius **vector** with earth's **surface**, over one orbital **cycle.**

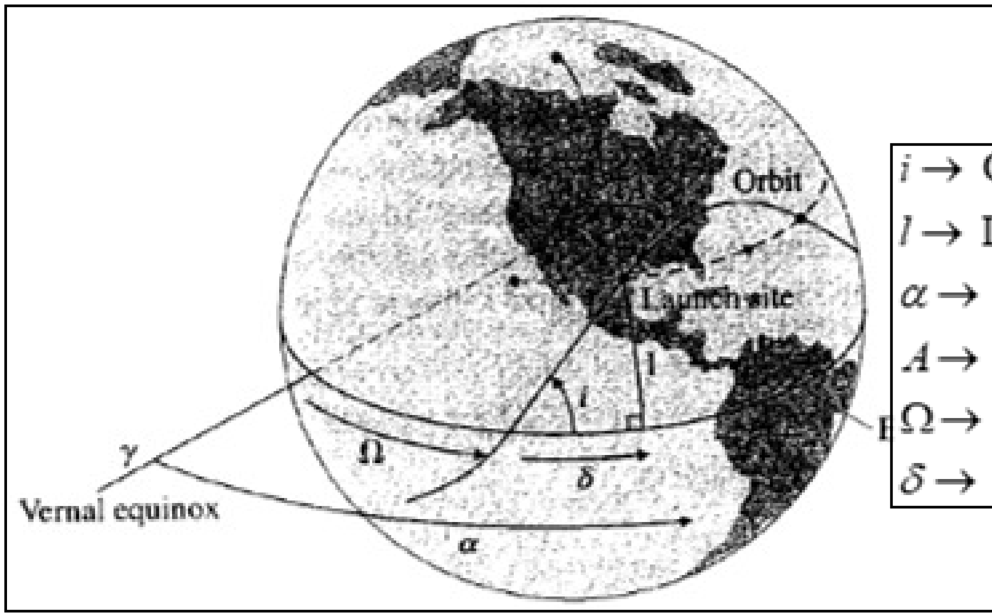
Launch to Orbit Strategy

This ‘**ground** trace’ appears like a **sine** wave as spacecraft moves in its **orbit.**

We further **find** that if it is **visible** above the launch latitude, it will be **visible** overhead at launch **site** at only two time **instants**, leading to launch time **restrictions.**

Launch Instant Constraint

Consider the **ground** trace in relation to the **launch** site, as shown in the **following** figure**.**



$i \rightarrow$ Orbit Inclination

$l \rightarrow$ Launch Site Latitude

$\alpha \rightarrow$ Launch Sidereal Time

$A \rightarrow$ Launch Inertial Azimuth

$\Omega \rightarrow$ Orbit Right Ascension

$\delta \rightarrow \alpha - \Omega$

Launch Instant Solutions

Satellite orbiting earth appears **overhead** twice a day when $\alpha = \Omega+\delta$ & $\Omega+(180^o-\delta)$.

The **resulting** solution for time **window** for launch can be **obtained** as follows.

$$\sin\delta = \tan l \cot i; \quad -90^o \le \delta \le 90^o; \quad \omega_\ominus = 360^o / 23^h 56^m 4.0905^s$$

$\alpha_g \rightarrow$ Right ascension of Greenwich at t_0 (from Almanacs)

$\lambda_E \rightarrow$ East longitude of launch site; $\quad \alpha = \alpha_g + \lambda_E + \omega_\ominus (t - t_0)$

$$t_1 = t_0 + \frac{\Omega+\delta-\alpha_g-\lambda_E}{\omega_\ominus}; \quad t_0 + \frac{\Omega+180^o-\delta-\alpha_g-\lambda_E}{\omega_\ominus}; \quad \sin A = \frac{\cos i}{\cos l}$$

Chase Orbit Design

It should be **noted** that while orbiting **satellite** typically has **dwell** time of only a few seconds over the **launch** site, no launch can actually be **instantaneous.**

Therefore, **'chase'** orbits are created with slightly **smaller** **'a'** so that once the **correct** angular relation is achieved, a small **Hohmann** transfer achieves the desired **orbit**.

The actual **orbital** landing point is **usually** some distance **away** from the **orbiting** satellite due to **safety** concerns.

Docking Manoeuvre

Docking Manoeuvre

'Docking' is defined as the **final** mating of the two **spacecrafts**, with physical **connections** established.

This **requires** both relative **velocity** and relative **distance** to be driven to **zero**.

Docking Modelling Strategy

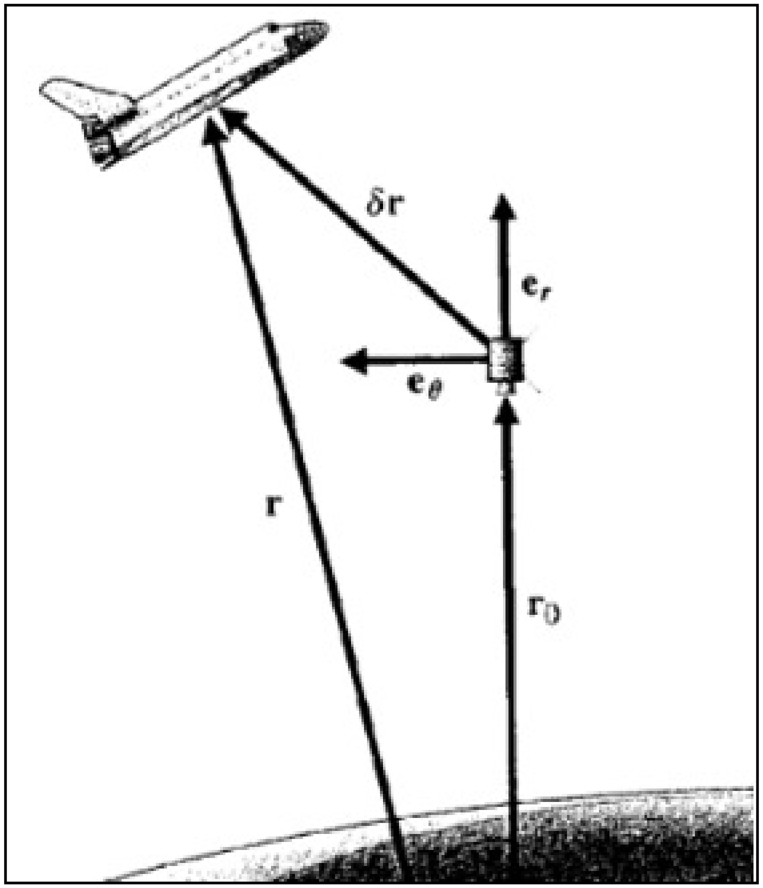
A **simple** mathematical model called **'Clohessy-Wiltshire'** equations is commonly **used** to represent the relative **motion** dynamics of the two **docking** objects**.**

These are **applicable** when at least **one** of the objects is in a **circular** orbit**.**

Docking Problem

Consider the **schematic** of a space **station** moving in circular **orbit** of radius 'r_0', mean motion 'n' and relative motion **coordinate** system δr, δθ & δz, **shown** along side.

Motion of **docking** vehicle is described in **terms** of the above relative distance **parameters.**



Docking Formulation

We can write the **applicable** linear governing **equations** of relative **motion**, as follows.

$$n=\sqrt{\frac{\mu}{r_0^3}};\quad \vec{r}=(r_0+\delta r)\hat{e}_r+r_0\delta\theta\hat{e}_\theta+\delta z\hat{e}_z;\quad \vec{\omega}^{ei}=n\hat{e}_z$$

$$\frac{{}^i d^2}{dt^2}\vec{r}=\frac{{}^e d^2}{dt^2}\vec{r}+2\vec{\omega}^{ei}\times\frac{{}^e d}{dt}\vec{r}+\vec{\omega}^{ei}\times\left(\vec{\omega}^{ei}\times\vec{r}\right);\quad a_g=-\frac{\mu}{r^3}\vec{r}$$

$$a_g\approx-\frac{\mu}{r_0^2}\hat{e}_r-\frac{\mu}{r_0^3}(-2\delta r\hat{e}_r+r_0\delta\theta\hat{e}_\theta+\delta z\hat{e}_z);\quad a_g=\frac{{}^i d^2}{dt^2}\vec{r}$$

$$\delta\ddot{r}-2nr_0\delta\dot{\theta}-3n^2\delta r=0;\quad r_0\delta\ddot{\theta}+2n\delta\dot{r}=0;\quad \delta\ddot{z}+n^2\delta z=0$$

Docking Motion Features

Docking motion is **harmonic**, as shown below.

$$\delta z = \delta z_0 \cos nt + \frac{\delta \dot{z}_0}{n} \sin nt; \quad \delta \dot{\theta} = \delta \dot{\theta}_0 + \frac{2n}{r_0 \left(\delta r_0 - \delta r \right)}$$

$$\delta r = -\left(\frac{2}{n} r_0 \delta \dot{\theta}_0 + 3 \delta r_0 \right) \cos nt + \frac{\delta \dot{r}_0}{n} \sin nt + 4 \delta r_0 + \frac{2}{n} r_0 \delta \dot{\theta}_0$$

$$\delta \theta = \delta \theta_0 - \left(3 \delta \dot{\theta}_0 + \frac{6 n \delta r_0}{r_0} \right) t + \left(\frac{4 \delta \dot{\theta}_0}{n} + \frac{6 \delta r_0}{r_0} \right) \sin nt + \frac{2 \delta \dot{r}_0}{n r_0} \left(\cos nt - 1 \right)$$

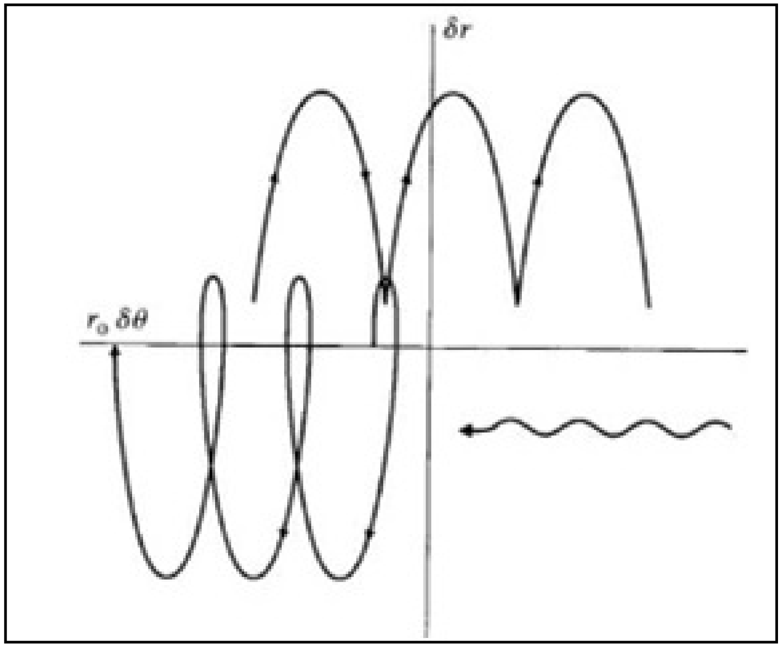
Docking Motion Features

‘δz’ reflects a slightly **different** orbital plane and is **oscillatory** with same **period** as that of the **original** orbit.

‘δ**r**’ is also **oscillatory**, with slight eccentricity, while **‘$\delta\theta$’** motion, though **oscillatory**, represents a **drift**.

Typical Docking Motion

The above **solution** is commonly represented as **relative** trajectories over several **time** periods, as shown below.



Summary

To **conclude**, launch and arrival time **windows** are important parameters that **help** in setting up inter-planetary **missions**.

Docking is a critical manoeuvre that **requires** precise modelling of the relative **motion** of orbiting objects.