EE 622 : Optimal Control Systems

Assignment 3

1. Derive the relations between optimal control $u(t)$ and state $x(t)$ for the system
$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{1}$$
and the performance measure is such that we want the state $x(t)$ to track $r(t)$, i.e.
$$\begin{aligned} J = [x(t_f) - r(t_f)]^T H[x(t_f) - r(t_f)]+ \\ \int_{t_0}^{t_f} \{[x(t) - r(t)]^T Q(t)[x(t) - r(t)] + u^T(t)R(t)u(t)\}dt \end{aligned} \tag{2}$$
Final time t_f is fixed, $x(t_f)$ is free and states and controls are not bounded. H and Q are real symmetric positive semi-definite matrices and R is real symmetric and positive definite.

2. Determine the optimal control law for the system
$$\dot{x}(t) = -x(t) + u(t) \tag{3}$$
to be transferred to origin from an arbitrary initial state. The performance measure to be minimised is
$$J = \int_0^1 \frac{3x^2(t) + u^2(t)}{2} dt \tag{4}$$
Given that $u(t)$ is not bounded.

3. Repeat the problem above for $x(1)$ free. Determine the optimal control law and resulting performance measure.

4. Find the optimal control law for the system
$$\dot{x_1}(t) = x_2(t) \tag{5}$$
$$\dot{x_2}(t) = -x_2(t) + u(t) \tag{6}$$
which transfers the system from $x(0) = 0$ to the line $x_1(t) + 5x_2(t) = 15$ and minimizes
$$J[u] = 0.5[x_1(2) - 5]^2 + 0.5[x_2(2) - 2]^2 + \int_0^2 0.5u^2(t)dt \tag{7}$$

5. Consider the system

$$\dot{x_1}(t) = -x_2(t) + u(t) \tag{8}$$

$$\dot{x_2}(t) = 2x_1(t) - x_2(t) - 2u(t) \tag{9}$$

with initial condition $x_1(0) = 1$ and $x_2(0) = -1$ and performance index (PI) given by

$$J(u) = \frac{1}{2}[x_1^2(t_f) + 2x_1(t_f)x_2(t_f) + 2x_2^2(t_f)]+ \\ \frac{1}{2}\int_0^{t_f} \{[x_1^2(t) + x_1(t)x_2(t) + 2x_2^2(t) + 2u^2(t)]\}dt \tag{10}$$

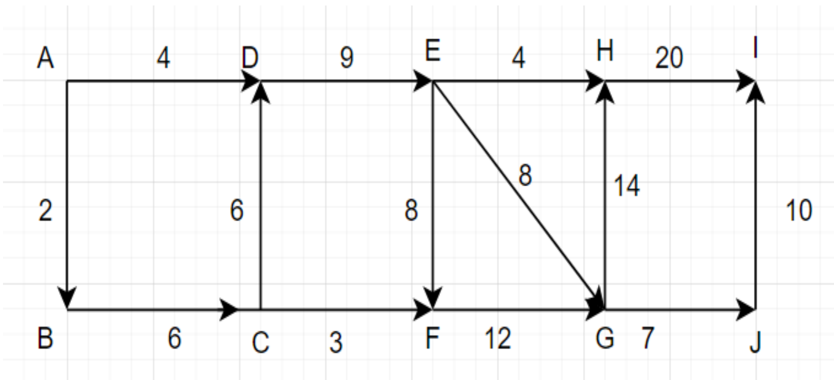
where $t_f = 10s$. Find the control u^* that minimizes the performance index, the corresponding optimal state trajectory x^* and the minimum cost J^*.(make suitable assumption if required)

6. Consider the same system as the last problem, with the same initial condition. The new performance index is

$$J(u) = \frac{1}{2}\int_0^{\infty} \{[x_1^2(t) + x_1(t)x_2(t) + 2x_2^2(t) + 2u^2(t)]\}dt \tag{11}$$

Find the control u^* that minimizes the performance index, the corresponding state trajectory x^* and the minimum cost J^* (make any suitable assumption that is required).

7. From the given figure cost of each path shown.what will be the min cost to reach point I.(starting point A)



8. Consider the dynamical system.

$$\ddot{x} = -x + u \tag{12}$$

where $u \in [-1, 1]$. Find the shortest path from an arbitrary point $p \in \mathbb{R}\backslash\{0\}$ to origin and the corresponding u^*.

9. Consider the dynamical system,

$$\ddot{x} = u \tag{13}$$

where $u \in [-1, 1]$. Find the shortest path to the circle $x^2 + \dot{x}^2 = R^2$ from any initial point p (outside the circle) and the corresponding u^*.

10. Consider a plant,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2x_1 + 4x_2 + u \end{aligned} \tag{14}$$

where $u \in [-1, 1]$. Find the optimal control law u^* to transfer the states from an arbitrary initial state x_{t_0} to origin such that it minimizes the performance measure,

$$J = \int_{t_0}^{t_f} \left(\lambda + u^2\right) dt, \quad \lambda > 0 \tag{15}$$