Constrained Flyby Solution

Planetary Flyby With Constraints

There can be **many** objectives for the **flyby** mission.

In cases when it is just a **one-time** flyby, actual helio-centric **trajectory** after flyby is of no **value** and we can **enter** planet SOI at any **point.**

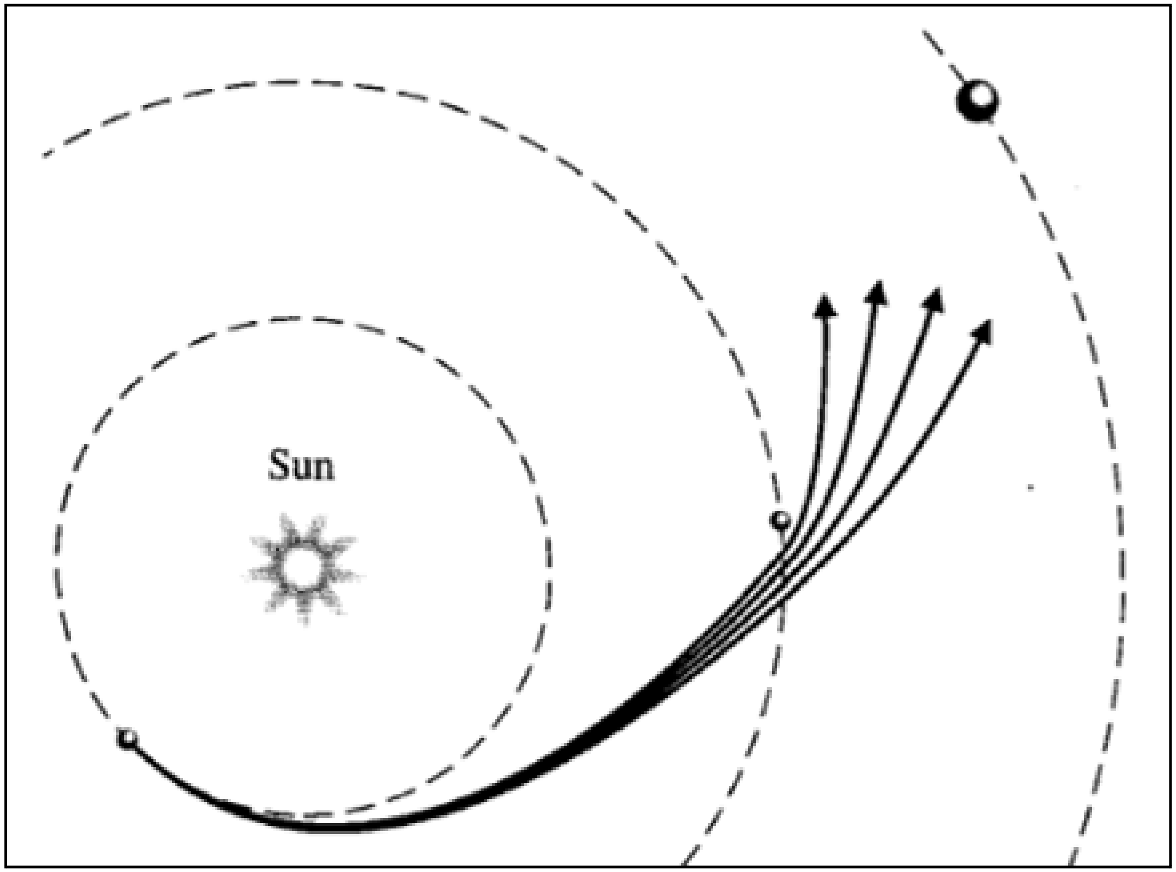
Planetary Flyby With Constraints

However, if **flyby** is to go **somewhere**, then it is used to **position** the exit suitably for the planned **destination.**

This requires **accurate** determination of the **planetary** positions so that spacecraft can **enter** at the right **point** and exit at the **right** point during the **flyby.**

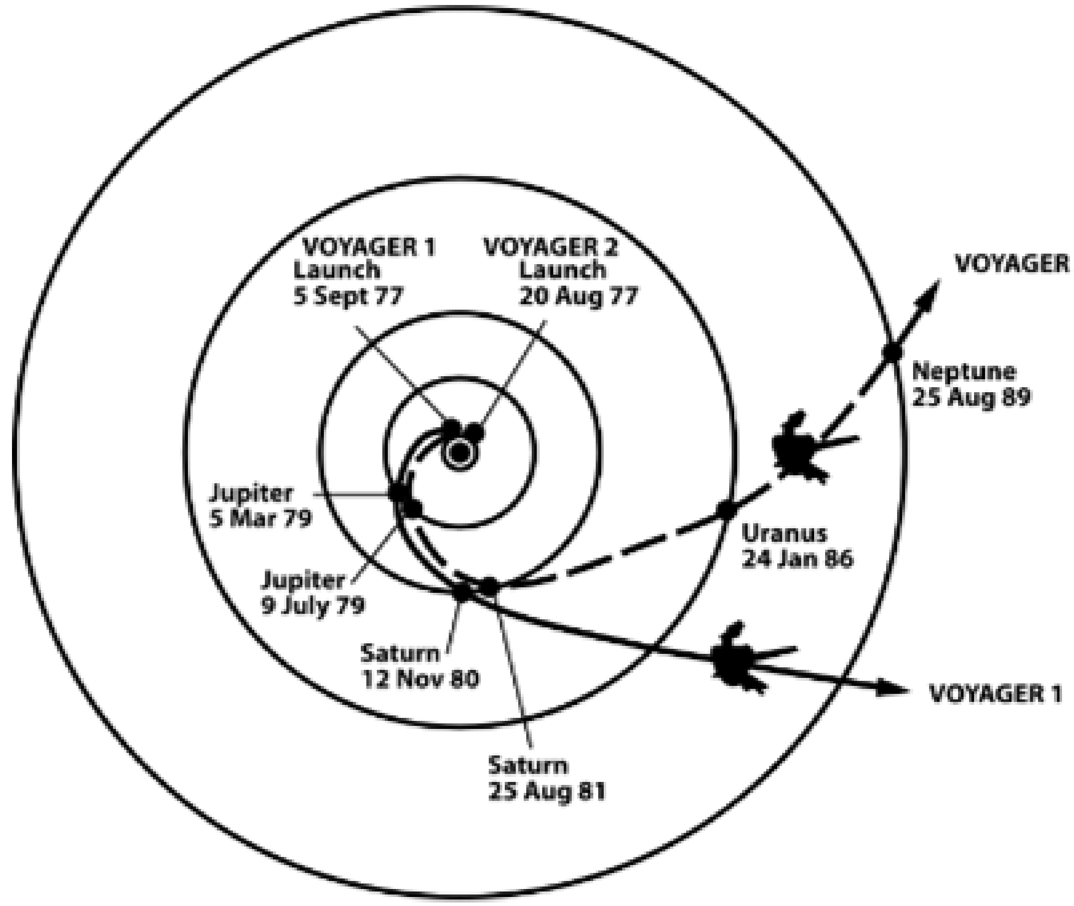
Destination Driven Flyby Solutions

The following **diagram** shows the many **possibilities**.



Voyager Flyby Manoeuvres

Trajectories that enabled NASA's **twin** Voyager spacecrafts to **tour** four giant **planets** & achieve velocity to escape **Solar** System, were a result of **flybys**.



Other Planetary Flyby Scenarios

Planetary probes generally fly by **Venus** on way to **mercury** and fly by mars or **Jupiter** on way to outer **planets** and Asteriods.

A **flyby** of Jupiter can also be **used** to execute a large **plane** change out of the **ecliptic** to place spacecraft in a **polar** orbit around the **sun.**

Other Planetary Flyby Scenarios

A **mercury** flyby produces the largest **energy** changes because of its **closeness** to sun, while a **Jupiter** flyby gives the **largest** trajectory deflection due to its **mass**.

VEGA is flyby that **uses** Earth also for **flyby.**

Planetary Capture Solutions

Planetary Capture

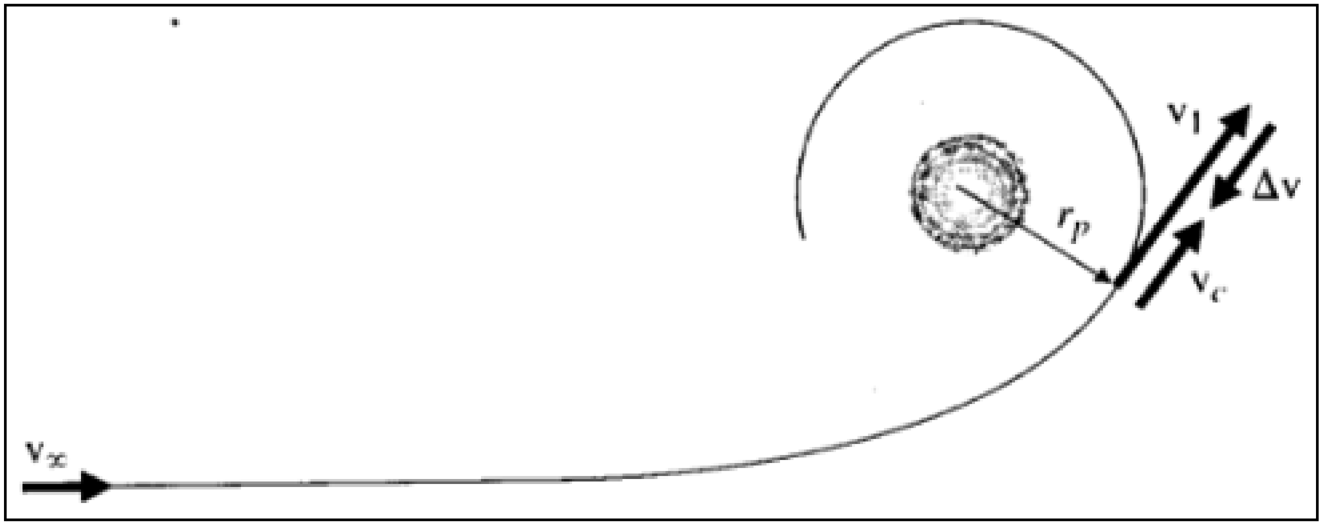
Capture is an important aspect of **interplanetary** travel, wherein spacecraft **forms** an orbit around **another** planet.

Since, the **spacecraft** crosses the planet **SOI** with a non-zero '$\mathbf{v}_\infty$', it will always be on a **hyperbolic** path, with respect to the **planet**.

Thus, to **avoid** impact and form an **orbit**, we need to reduce **velocity** sufficiently so that **planeto-centric** eccentricity is < 1.

Planetary Capture

One way to **achieve** capture is to **allow** spacecraft to **reach** its periapsis and then **impulsively** reduce velocity to enable the **capture,** as shown schematically below.



The **desirable** orbit generally is a **circular** one.

Planetary Capture Solution

Given below is the **minimum** ΔV required for **capture** in a **circular** orbit.

$$v_c = \sqrt{\frac{\mu_p}{r_{periapsis}}}; \quad \varepsilon = \frac{1}{2}v_\infty^2 - \frac{\mu_{pl}}{r_{SOI}} \approx \frac{1}{2}v_\infty^2; \quad v_1 = \sqrt{v_\infty^2 + \frac{2\mu_{pl}}{r_{periapsis}}}$$

$$\Delta V = v_1 - v_c = \sqrt{v_\infty^2 + \frac{2\mu_{pl}}{r_{periapsis}}} - \sqrt{\frac{\mu_{pl}}{r_{periapsis}}}; \quad \text{Optimal radius: } \frac{d\Delta V}{dr_{periapsis}} = 0$$

$$\frac{d}{dr_{periapsis}}\Delta V = -\left(v_\infty^2 + \frac{2\mu_{pl}}{r_{periapsis}}\right)^{-\frac{1}{2}} \times \mu_{pl} r_{periapsis}^{-2} + \frac{1}{2}\sqrt{\mu_{pl}} \cdot r_{periapsis}^{-\frac{3}{2}} = 0$$

$$4\mu_{pl}^2 = \mu_{pl} v_\infty^2 r_{periapsis} + 2\mu_{pl}^2 \rightarrow r_{periapsis} = \frac{2\mu_{pl}}{v_\infty^2}; \quad \Delta V_{min} = \frac{v_\infty}{\sqrt{2}} \text{ (for circular orbit)}$$

In reality, **lowest ΔV** is when the capture is **parabolic.**

Planet Motion Implication

Planet Motion during Capture/Flyby

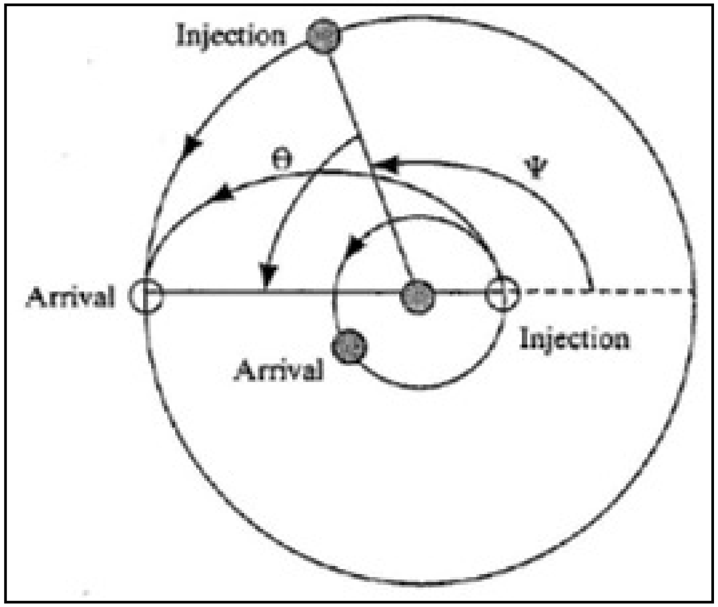
An **important** aspect that is to be considered for **capture** is the fact that all planets are **moving** in their orbits.

If the spacecraft is to intercept the **SOI** of a planet at a designated point, the **departure** & arrival must have the appropriate **angular** relationships**.**

This **relationship** is called **'lead angle',** which is an angle between **origin** & **target** planets at departure, which depends on target planet **'n' & Δt** for the spacecraft.

Planet Motion Model

Given below is an example of **intercept** with Mars along Hohmann **Transfer** ellipse**.**



$$n_M = \frac{2\pi}{P_M} = \frac{2\pi}{687} = 0.524^\circ / day$$

$$TOF = \frac{P_{t-Ellipse}}{2} = 259 days$$

$$\theta = n_M \cdot TOF = 135.7^\circ; \quad \psi = 44.3^\circ$$

Synodic Time Period Concept

In case a particular **time** slot is **missed**, either we need to compute a **new** trajectory with a **different** lead angle, or **wait** for the next **window** to appear.

This **requires** knowing the **return** period of **window** for the calculated **lead** angle**,** which is called the **synodic** time period. In case of Mars, this value is **2.135 years.**

Thus, if we are **planning** a return trip to **mars**, we would need around **971** days (or 2.66 years) to **complete** the mission, including a **wait** period of 454 days (**1.24** years).

Planetary Synodic Time Periods

Synodic time period of **other** planets with respect to **earth** can be obtained through the following **relation.**

$$\Delta t = \frac{360^\circ}{|n_P - n_E|} \quad \text{(For Hohmann Transfer)}$$

Planet	n (rad/yr)	Synodic period (yr)
Mercury	26.11	0.1369
Venus	10.21	1.600
Mars	3.340	2.135
Jupiter	0.5297	1.092
Saturn	0.2132	1.035
Uranus	0.0748	1.012
Neptune	0.0381	1.006
Pluto	0.0254	1.004

n_E = 6.283 rad/yr.

Summary

In **summary**, planetary capture requires **reduction** in object velocity and is **generally** on an elliptic path.

Synodic time period is an important **concept** that helps in synchronizing the **arrival** and departure instants.