Approximate Staging Solution

Approximate Staging Concept

Approximate staging philosophy is an **alternative** to the Lagrange' technique in which, **we** drop one equation and **solve** the residual $N \times N$ system of **equations**.

While, this **methodology** provides sub-optimal **solutions**, in many cases we can **use** the solutions, so obtained, to **initiate** a more rigorous design **iteration**.

Approximate Staging Features

While, there can be **many** options for **dropping** one equation, this can be **achieved** exactly, if one of the **partial** derivatives is **zero** throughout the design **space.**

However, as this is **not** necessarily true all the **tim**e, we assume that there is **only** one point in design **space** where objective **function** is a maximum.

Approximate Staging Concept

In such a case, the **point** automatically represents **optimal** design solution, at which all **partial** derivatives go to zero.

Of course, as **sensitivity** of objective function to design **variables** may not be **same** for all **variable**s, we may remain in the **close** vicinity of the exact **optimal** point.

Such **solutions**, while approximate in nature, provide a good **starting** point for a more **rigorous** design exercise.

Approximate Staging Strategy

In this method, as **constraint** needs to be satisfied **exactly,** the corresponding **equation** is used to express **one** design variable in **terms** of all other remaining variables.

This **solution** is then substituted in **(N-1)** equations corresponding to the **(N-1)** partial derivatives.

Approximate Staging Strategy

Further, the **derivative** corresponding to **selected** design **variable** is ignored, resulting in **(N-1)** $\times$ **(N-1)** system**.**

Once this system is **solved**, these are substituted back into the **constraint** and the N^{th} variable is **solved** for.

Approximate Staging Formulation

The **basic** formulations for both the **cases** of constraints, are as given **below.**

$$\ln\left[\varepsilon_1+(1-\varepsilon_1)\pi_1\right]=-\left\{\frac{V_*}{I_{sp_1}g_0}+\sum_{i=2}^{N}\frac{I_{sp_i}}{I_{sp1}}\ln\left[\varepsilon_i+(1-\varepsilon_i)\pi_i\right]\right\}$$

$$\ln\pi_*=\sum_{i=1}^{N}\ln\pi_i \rightarrow \frac{\partial\pi_*}{\partial\pi_j}=0 \quad j=2,N; \quad V_* \text{ Constraint Solution}$$

$$\ln\pi_1=\ln\pi_*-\sum_{i=2}^{N}\ln\pi_i; \quad V_*=\sum_{i=1}^{N}g_0 I_{spi}\ln\left[\varepsilon_i+(1-\varepsilon_i)\pi_i\right]$$

$$\frac{\partial V_*}{\partial\pi_j}=0 \quad j=2,N; \quad \pi_* \text{ Constraint Solution}$$

Summary

To **summarize**, approximate staging solution **method** simplifies the solution **steps**.

However, it also **results** in the loss of **accuracy**, which is also dependent on the **equation** that is ignored.