Tutorial: 1

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Q: Prove that the maximum value of the ratio C_L/C_D is independent of attitude and free stream velocity, rather, they depend only on the aerodynamic design of the aircraft.

A: The ratio of C_L/C_D is given by

$$\frac{C_L}{C_D} = \frac{C_L}{C_{D_0} + C_{D_i}} = \frac{C_L}{C_{D_0} + \frac{C_L^2}{\pi e AR}}\,.$$

Differentiating the above w.r.t to C_L, we get

$$\frac{d}{dC_L}\left(\frac{C_L}{C_D}\right) = \frac{1}{C_{D_0} + \frac{C_L^2}{\pi e AR}} - \frac{C_L}{\left(C_{D_0} + \frac{C_L^2}{\pi e AR}\right)^2}\left(\frac{2C_L}{\pi e AR}\right) = 0\,.$$

$$\implies \quad C_{D_0} + \frac{C_L^2}{\pi e AR} = \frac{2C_L^2}{\pi e AR} \quad \implies C_{D_0} = C_{D_i} \quad \therefore \left.\frac{C_L}{C_D}\right|_{max} = \frac{C_L}{2C_{D_0}}$$

Hence, proved.

Q: Consider an airplane with zero-lift drag coefficient of 0.025, an aspect ratio of 6.72, and span efficiency factor (Oswald efficiency factor) of 0.9. Using the results of part (a), calculate the value of $(L/D)_{max}$.

A: Using the result of previous slide, we get

$$\left.\frac{C_L}{C_D}\right|_{max} = \frac{C_L}{2C_{D_0}}\,.$$

Note that for the maximum C_L/C_D ratio

$$C_{D_0} = C_{D_i} \implies C_L = \sqrt{C_{D_0}\pi e AR}\,.$$

Therefore,

$$\left.\frac{C_L}{C_D}\right|_{max} = \frac{\sqrt{C_{D_0}\pi e AR}}{2C_{D_0}} = 13.78$$

- The rate-of-climb RC is given by

$$RC = \frac{V_\infty}{W}(T - D) = \frac{V_\infty}{W}\left(T - 0.5\rho S C_D V_\infty^2\right)$$

- Differentiating w.r.t to V_∞, we get

$$\frac{d\ RC}{dV_\infty} = \frac{1}{W}\left(T - 0.5\rho S C_D V_\infty^2\right) - \frac{V_\infty}{W}\left(\rho S C_D V_\infty\right) = 0$$

$$\implies \quad T = \frac{3\rho S C_D V_\infty^2}{2} \quad \implies V_\infty = \sqrt{\frac{2T}{3\rho C_D S}}.$$

- Substituting the above, we get

$$RC_{max} = \sqrt{\frac{8T^3}{27W^2\rho S C_D}}$$

$\boxed{\text{Assumption that induced drag is negligible.}}$

Q: Consider an aircraft with maximum lift coefficient (with no flap deflection) of 1.5, wing area of 15 squared meter, weight of 25000 N. Furthermore, the aircraft can structural bear a maximum positive lift force of 100,000 N. Is it possible for the aircraft to maneuver at the a sea-level corner velocity of 100 m/s? Justify your answer.

A: The positive load factor limit can be calculated as

$$n_{+_{limit}} = \frac{L_{+_{max}}}{W} = \frac{100000}{25000} = 4.$$

The maximum Load factor at corner velocity is

$$n_{max} = \frac{\rho S V_\infty^2 C_{L_{max}}}{2W} = 5.5125.$$

Therefore as $n_{max} > n_{+_{limit}}$, the aircraft needs to operate at a lower lift coefficient to be able to maneuver at sea level ar the corner velocity.

Q: The maximum lift coefficient for an aircraft is 1.5 along with a zero-lift drag coefficient of 0.3. Neglecting the induced drag on the aircraft, what is the equilibrium glide velocities corresponding to minimum glide angle and a wing loading of 3000 N/m^2. Furthermore, if the aircraft is in flight at 1500 m of altitude when the engine fails, how far can it glide in terms of distance measured along the ground?

A: The minimum glide angle is given by

$$\theta_{min} = \tan^{-1}\left(\frac{1}{(L/D)_{max}}\right) = 11.30^\circ.$$

Therefore, the minimum glide velocity is

$$V_\infty = \sqrt{\frac{2\cos\theta W}{\rho C_L S}} = 61.12 \quad m/s$$

The maximum range is given by

$$R_{max} = \frac{h}{\tan\theta_{min}} = 1500 \times 5 = 7.5km.$$

Reference

1. John Anderson Jr., *Introduction to Flight*, McGraw-Hill Education, Sixth Edition, 2017.

Thank you for your attention !!!