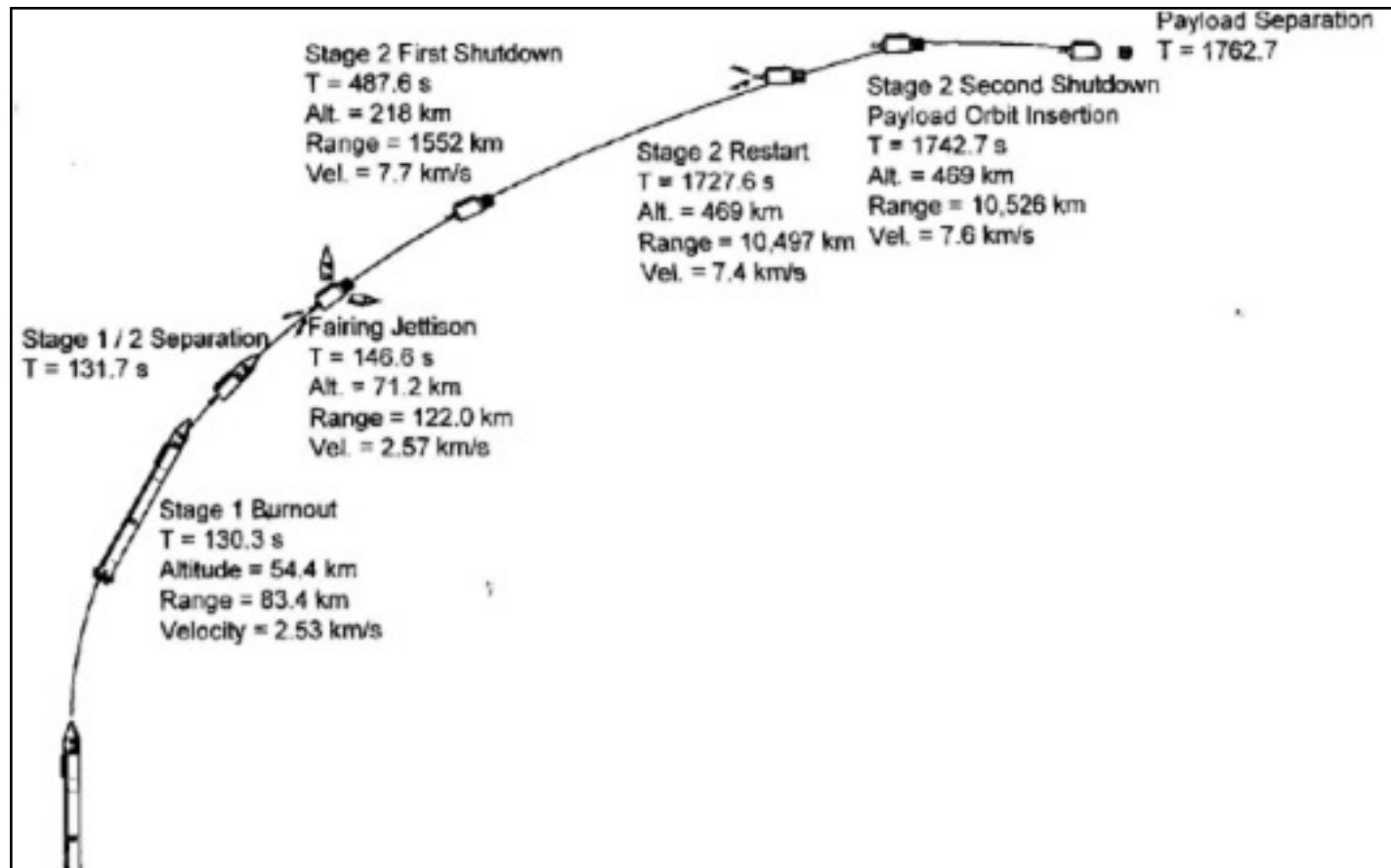




Inclined / Curvilinear Trajectories



Typical Ascent Trajectory





Inclined Motion Concept

In reality, **vertical** motion is used **only** for a very small **part** of the overall ascent **mission** and for the most part, **ascent** trajectory is inclined and **curvilinear** in nature.

This is mainly because one of the **terminal** constraint is that the **inclination** should be zero or **close** to it, with respect to the **local** horizon.



Inclined Motion Concept

Further, considering **Earth's** curvature, the rocket **needs** to undergo large **flight** path angle changes ($>90^\circ$).

This requirement calls for a **different** methodology of **trajectory** design & solution.



Effect of Inclination

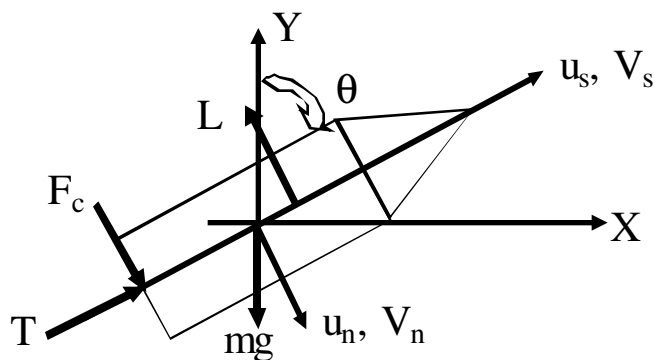
A **curvilinear** flight path requires **motion** in a plane and therefore, needs a **2-D** model for the motion.

Also, as **thrust** is used mainly for $\Delta \mathbf{V}$ and is always along the **flight** path, a normal **force** is needed to produce the **curvilinear** path.



Effect of Inclination

Consider a rocket having **inclination** with the vertical, as shown below.

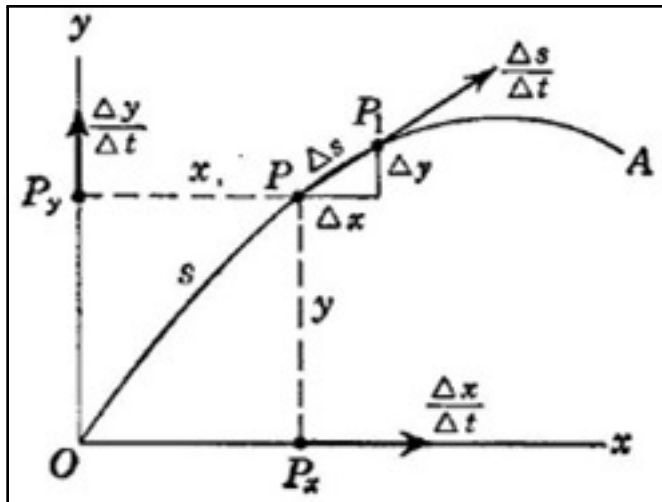


$u_s, u_n \rightarrow$ Unit Vectors along s, n directions



Curvilinear Velocity Model

The **schematic** of a planar **motion**, along with the **expression** for velocity are as **follows**.

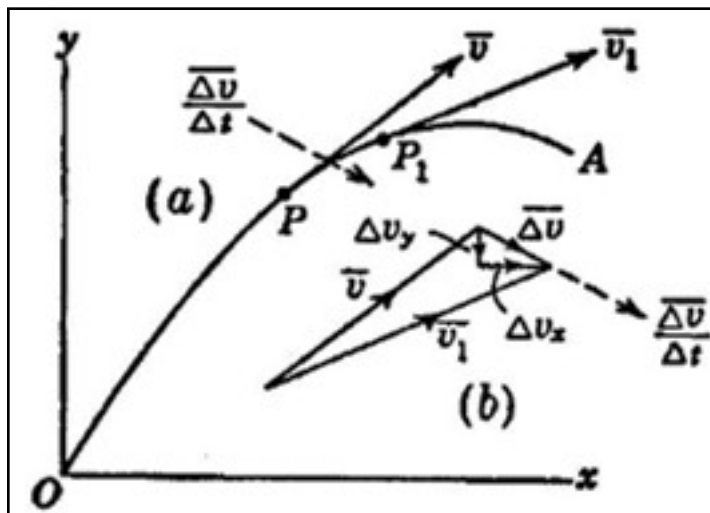


$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \rightarrow \vec{V} = \dot{s} \cdot \hat{u}_s$$



Curvilinear Acceleration Model

The **corresponding** acceleration model and its **expression** are as given **below**.



$$\begin{aligned}\bar{a} &= \frac{d}{dt}(\bar{V}) = \frac{d}{dt}(V\hat{u}_s) \\ \bar{a} &= \dot{V}\bar{u}_s + V \frac{d\hat{u}_s}{dt} = \dot{V}\bar{u}_s + V\dot{\theta}\bar{u}_n \\ \bar{a} &= \bar{a}_s + \bar{a}_n\end{aligned}$$



Planar Motion Equations

The **scalar** equations of planar **motion**, in the absence of ‘**L**’ and ‘**F_c**’, are as follows.

$$a_z = \frac{dV}{dt} = -\frac{\dot{m}}{m} g_0 I_{sp} - \tilde{g} \cos \theta; \quad \tilde{g} \rightarrow \text{Average Constant Gravity}$$
$$a_n = V \frac{d\theta}{dt} = \tilde{g} \sin \theta; \quad V, \theta \rightarrow \text{Trajectory Parameters}$$



Trajectory Features

In this case, the resulting **trajectory** is called ‘**gravity turn**’ trajectory, as ‘**g**’ alone is **responsible** for turning of the velocity **vector**.

Conceptually, we can **solve** for any **2** of the **3** quantities (V , m (or dm/dt), θ), provided **third** one is given.



Gravity Turn Solution Aspects

Thus, **gravity** turn equations can be **solved** either as trajectory **design** or as a vehicle **design** problem.

However, the problem is **complex** because we only have initial & **terminal** conditions, which can be **met** from many different profile **geometries**.

Further, we **note** that as equations are **time** varying and nonlinear, **most** such solutions are **numerical** in nature, as analytical **solutions** are not feasible in **general** case.



Conceptual Design Solutions

However, in the **early** design stage, **analytical** solutions are found to be more **useful** for gross vehicle **sizing**.

It should be noted that such **closed** form solutions help **greatly** in quick **assessment** of performance of **many** concepts, before selecting **best** for detailed **analyses**.

However, **such** solutions are **possible** only under simplifying **assumptions** that limit their **applicability**.



Constrained Closed Form Scenarios

Among the **many** possibilities of generating approximate **closed** form solutions, there are **three** which are simple, **elegant** and also have practical **utility**.

In this regard, we note that as **most** launch vehicles use **some** kind of control during **ascent**, it is possible to **specify** a constant value for either $(d\theta/dt)$, (T/m) or V .



Constrained Solution Features

It is to be **noted** that constant ($d\theta/dt$) helps manage the mission **time** better.

Further, constant (T/m) helps to manage the **structural** weight better.

Lastly, constant V case is used to manage **propellant**.



Summary

To **summarize**, gravity based rotation of the **velocity** vector is a convenient **way** of achieving the **desired** trajectory profile, while **minimizing** the propellant.

Further, **simplified** scenarios help in **arriving** at closed form solutions that **aid** in quick performance **assessment**.