



## ***Rocket Thrust Model***



## ***Propulsion Models***

**Thrust** of a rocket **engine** is based on Newton's **3rd** law.

It also is **derivable** from conservation of **momentum**, as per the following **interpretation**.

**Typically**, a rocket engine **burns** the propellant and resulting **hot** gases are ejected in the **opposite** direction at a very **high** speed.



## *Propulsion Models*

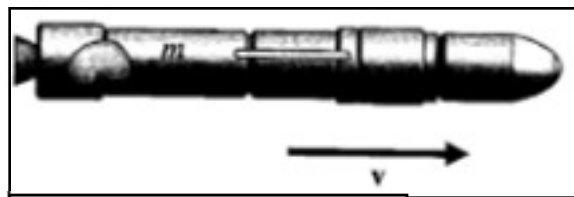
The **momentum**, so generated, creates equal & **opposite** momentum on the **rocket**, and appears as a **force**.

While **accurate** thrust models **require** detailed thermodynamic **laws**/ internal flow models, basic **thrust model** is derived from **rocket** engine firing on test **bed**.

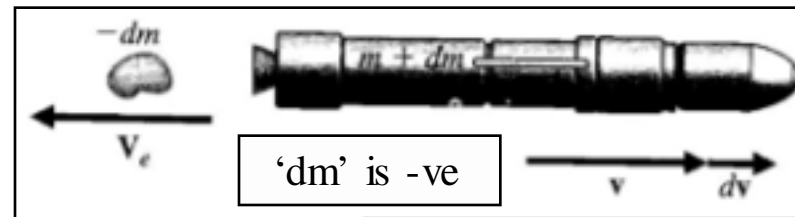


## *Reaction Jet Representation*

Following **schematic** shows test bed scenario for a **rocket** burning propellant and **ejecting** hot gases.



Snapshot at time 't'



Snapshot at time 't+dt'

Thus, rocket '**loses**' a mass '**dm**' with the relative velocity ' **$-V_e$** ' and **gains** a net forward velocity '**dV**'.



## *Thrust Expression*

**Following** is the corresponding momentum **conservation** equation.

$$\begin{aligned}
 (m - dm)(\vec{V} + d\vec{V}) + dm(\vec{V} - \vec{V}_e) &= m\vec{V} \\
 \cancel{m\vec{V}} + md\vec{V} - \cancel{dm\vec{V}} - \cancel{dm d\vec{V}} + \cancel{dm\vec{V}} - dm\vec{V}_e &= \cancel{m\vec{V}} \\
 md\vec{V} - dm\vec{V}_e = 0 \rightarrow -\vec{T}dt - dm\vec{V}_e &= 0 \rightarrow \vec{T} = -\dot{m}\vec{V}_e
 \end{aligned}$$

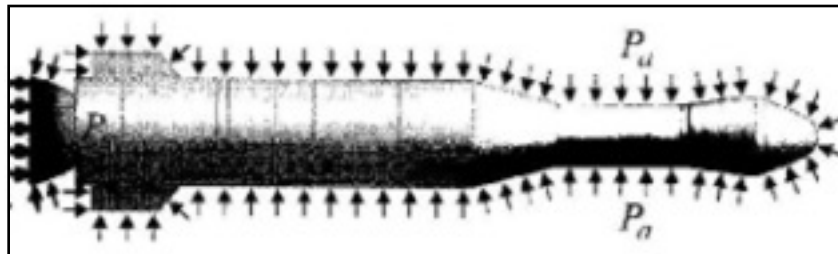
On test **bed**, as both  $V$ ,  $dV$  are **zero**, a net loss of **momentum** ' $dmV_e$ ' appears as incremental **impulse** ' $Tdt$ ', where ' $T$ ' is **thrust** in the direction opposite to  $V_e$ .



## *Pressure Thrust Concept*

The **thrust** component so obtained is **purely** due to exhaust **velocity** and, hence, is called 'velocity ' **thrust**.

In addition, **there** is generally a difference in the **static** pressure ( $P_a$ ) and pressure at nozzle **exit** ( $P_e$ ), resulting in a **force**, as shown in the **schematic** below.





## ***Pressure Thrust Concept***

This **force** is also in the axial **direction** and is called ‘pressure’ **thrust**, as it is due to pressure **difference**.

Therefore, the **resultant** total thrust (takes as **+ve** along the direction of  $V$ ) expression for a **rocket** is as follows.

$$\vec{T} = -\dot{m}\vec{V}_e + A_e (P_e - P_a)$$

Here,  $A_e$  is the **cross-sectional** area of the **nozzle**.



## *Total Thrust Features*

**It** is to be noted that the **1<sup>st</sup>** term is positive as **( $dm/dt$ )** is taken as a **negative** quantity.

However, **2<sup>nd</sup>** term can be both **positive** or negative, depending on the **sign** of '**( $P_e - P_a$ )**'.

In **this** regard, we note that for **same**,  $P_e$ , we get slightly **better** performance from the **rocket** motor in **vacuum** (i.e.  $P_a = 0$ ), than at **sea-level**.





## *Specific Impulse Concept*

Conventionally, **thrust** is treated as a **combined** effect of propellant & **nozzle**, and is converted to a mechanical ‘**figure of merit**’ called specific impulse or  $I_{sp}$ .

**Given** below is the definition of **specific** impulse.

$$I_{sp} \text{ (seconds)} = \frac{F_t}{|\dot{m}| g_0} = \frac{v_e}{g_0} - \frac{\Delta p A_e}{|\dot{m}| g_0}; \quad g_0 = 9.80665 \text{ m/s}^2$$



## *Typical Specific Impulse Values*

**$I_{sp}$**  is normally attributed to a **rocket** motor and depends on the **calorific** value of the propellant **used**.

**Typical** values for different **propellants** are as follows.

**Solid:** APCP (AP + HTPB/PBAN + Al) – 170 to 220

**Liquid:** (UDMH, Kerosene) +  $\text{LO}_2/\text{N}_2\text{O}_4$  – 200 to 350

**Cryogenic:** ( $\text{LH}_2$  +  $\text{LO}_2$ ), LNG ~ 450

**Nuclear:** 300 – 500



## *Summary*

Therefore, to **summarize**, thrust model, employed in **ascent** mission, is simplified **version** of the more accurate, but more **complex** internal flow based **models**.

However, it is **found** that we can still get a **fairly** good estimate of the **engine** performance using the **simplified** thrust and specific impulse **relations**.