Elliptic Orbit TOF Problems

Problem No. 01

Find $\mathbf{\Delta t}$ from $\boldsymbol{\theta = 0}$ **to** $\mathbf{135^o}$ for an ellipse with $\mathbf{e = 0.9}$, **552 km** perigee altitude**.** (R_E =6,378 km, μ =3.986×10^{14}).

Solution No. 01

TOF solution for 135^o is as **follows**.

$$r_p = 6.93\times10^6 m, \quad a = 6.93\times10^7 m$$

$$T = 50.4h; \quad n = 3.46\times10^{-5} rad/s$$

$$E_B = \cos^{-1}\frac{0.9+\cos135^\circ}{1+0.9\cos135^\circ} = 57.96^\circ$$

$$M_B = 1.012 - 0.9\times\sin 57.96^\circ = 0.249 rad$$

$$TOF = \frac{0.249}{3.46\times10^{-5}} = 7193s = 1.998h$$

Problem No. 02

In the **context** of Problem No. 01, determine the **time** to go from **135^o to 220^o.**

Solution No. 02

The **solution** for 135^o to 220^o is as **follows**.

$$E_B = 57.96^o; \quad M_B = 0.249 rad$$

$$E_C = \cos^{-1}\frac{0.9+\cos 220^o}{1+0.9\cos 220^o} = -64.45^o = 295.55$$

$$M_C = 5.159 - 0.9\times\sin 295.55^o = 5.971 rad$$

$$TOF = \frac{M_C - M_B}{n} = 165370.8s = 45.94h$$

Problem No. 03

Find **$\Delta\theta$** for a travel of **30 minutes** from perigee for an ellipse with **e = 0.3,** and 400 km perigee **altitude.** (R_E =6,378 km, μ =3.986×10^{14}).

Solution No. 03

Solution for the angular travel is as **given** below.

$$r_p = 6.778\times10^6\,m, \quad a = 9.683\times10^6\,m$$

$$n = 6.626\times10^{-4}\,rad/s; \quad \Delta t = 30m = 1800s$$

$$M_B = M_A + n\times\Delta t = 0 + 1.1927 = 1.1927$$

$$E_{B1} = M_B + e\sin M_B = 1.4715; \quad E_{B2} = M_B + e\sin E_{B1} = 1.4912$$

$$E_{B3} = M_B + e\sin E_{B2} = 1.4917; \quad E_{B4} = M_B + e\sin E_{B3} = 1.4917$$

$$\tan\frac{\theta_B}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E_B}{2} = \sqrt{1.8571}\times\tan 42.73^o = 1.2588 \rightarrow \theta_B = 103.1^o$$