Flight Mechanics/Dynamics

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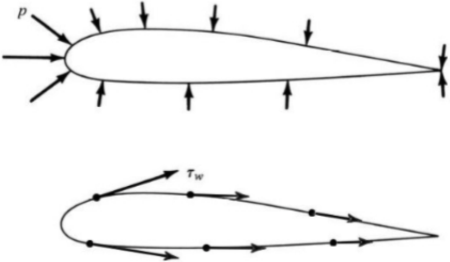
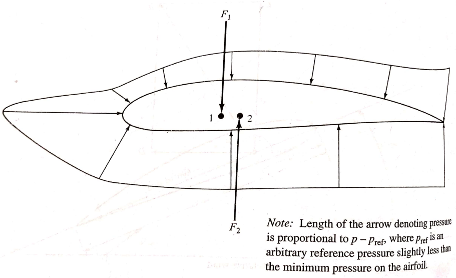
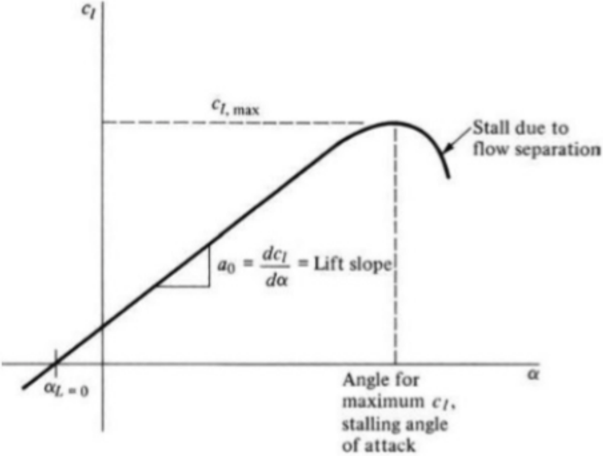
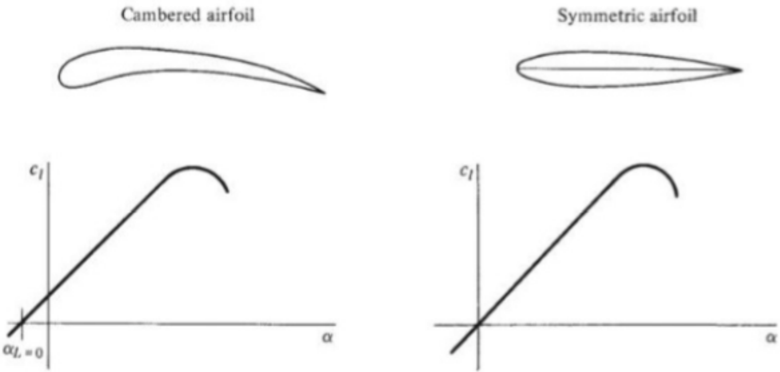

p
τ_w


F_1
1
2
F_2
Note: Length of the arrow denoting pressure is proportional to p − p_ref, where p_ref is an arbitrary reference pressure slightly less than the minimum pressure on the airfoil.

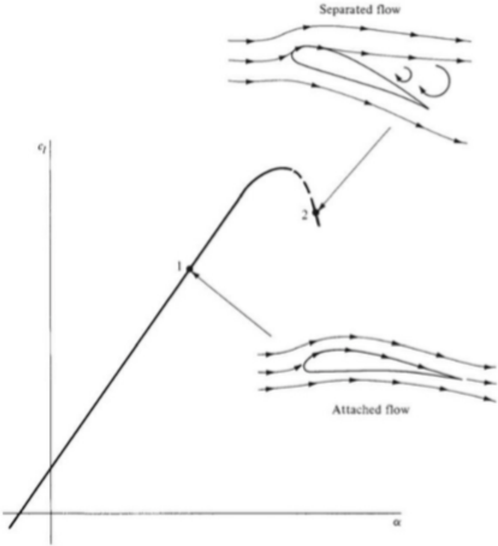


- c_l varies linearly with α, having lift slope a_0.
- Lift at $\alpha = 0$ is due to positive camber.
- Zero-lift angle of attack, $\alpha_{L=0}$.



- Linearity breaks down at some α, leading to maximum value of c_l, $c_{l,\max}$.
- Lift decreases significantly at high α, leading to stall of the airfoil.
- What is the cause of airfoil stall?

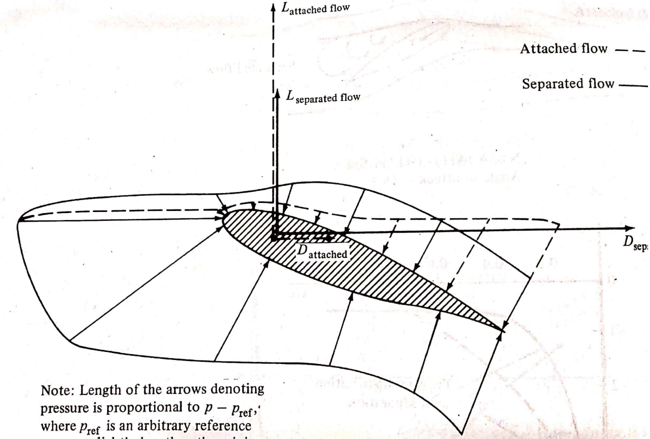
- Flow separation causes stalling
- With high α, adverse pressure gradient becomes stronger.
- Flow separation at stalling angle of attack



- **Drag**: Component of aerodynamic force parallel to relative wind
- **Sources of drag**:
  - ⇒ Skin friction drag (D_f): Shear stress at wall
  - ⇒ Pressure or form drag (D_p): Flow separation
  - ⇒ Wave drag (D_w): Shock wave at supersonic speed
- Total drag due to viscous effect

$$\boxed{D = \underbrace{D_f + D_p}_{\text{Profile drag}} + D_w}$$

- Profile drag due to nature of source (shape and size or profile of body).
- Skin friction drag: More for turbulent and less for laminar
- Pressure drag: Less for turbulent and more for laminar



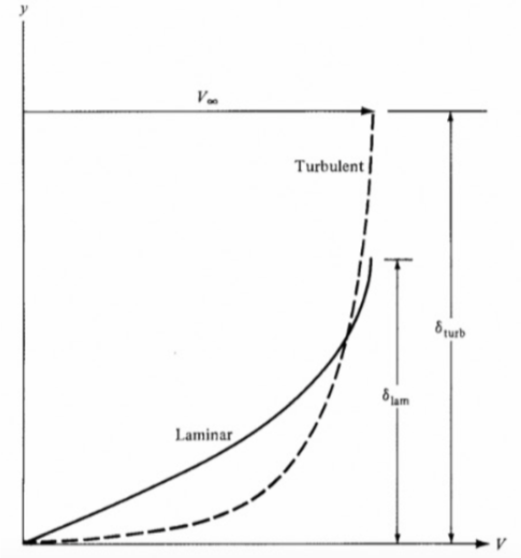
Note: Length of the arrows denoting pressure is proportional to $p - p_{\text{ref}}$, where p_{ref} is an arbitrary reference pressure slightly less than the minimum pressure on the airfoil

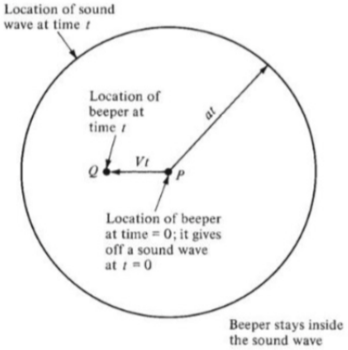
- Flow separation
  - $\Rightarrow$ Loss of lift
  - $\Rightarrow$ Increment in drag, caused by pressure drag due to separation
- Separation does not affect the bottom surface pressure distribution.
- Pressure on top surface is more when flow is separated.
- Geometric effect of top surface of airfoil being approximately horizontal.
- High pressure acts vertically and aids to reduction of lift with full effect.
- Pressure on trailing edge is smaller for separated flow.
- These pressures have strong effect in horizontal direction.
- Net pressure drag: Difference between pressure acting on front and back of airfoil.
- For separated flow, net force toward left is less and thus net drag is increased.
- Turbulent boundary layer prevent flow separation. Why?

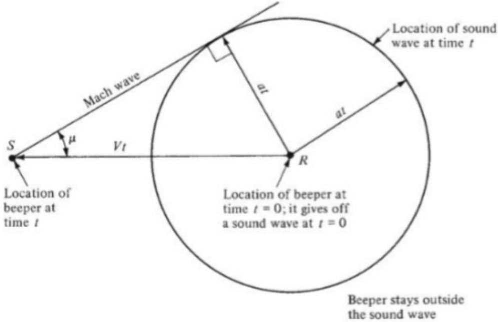
- Fluid has to overcome adverse pressure gradient, thus slow down and reverse its direction.
- At given distance, velocity of fluid is higher in turbulent flow, thus less tendency to separate.
- Shear stress

$$\tau_w = \mu \frac{dV}{dy}\bigg|_{y=0}$$

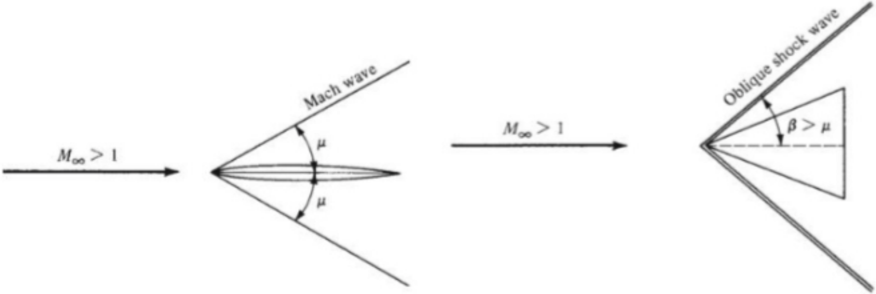
- $\left.\frac{dV}{dy}\right|_{y=0}$ is less for laminar, leading to less shear stress.

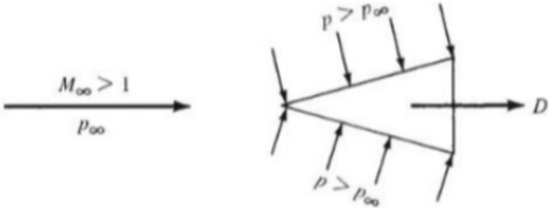




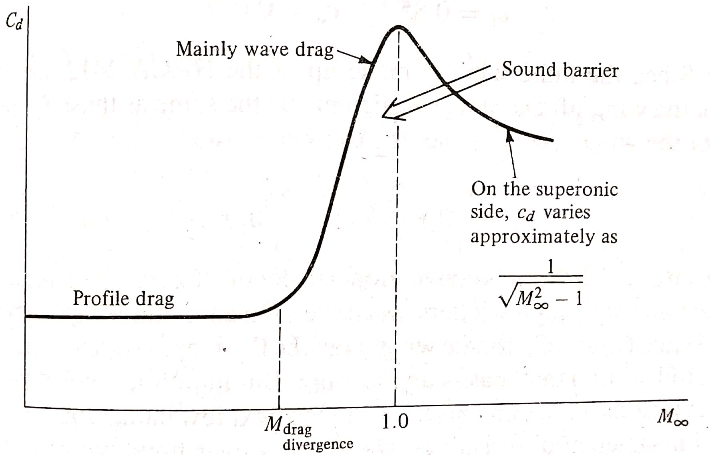


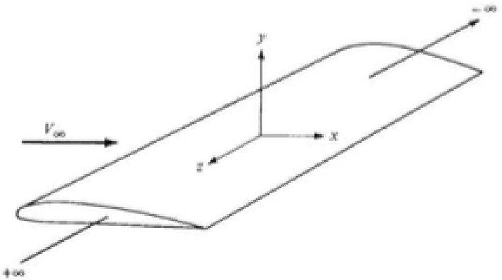
$$\boxed{\text{Mach angle} = \mu = \sin^{-1}\frac{1}{M}}$$

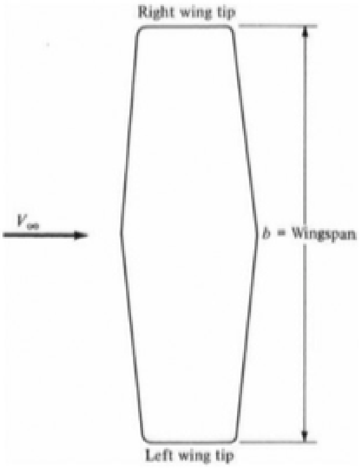




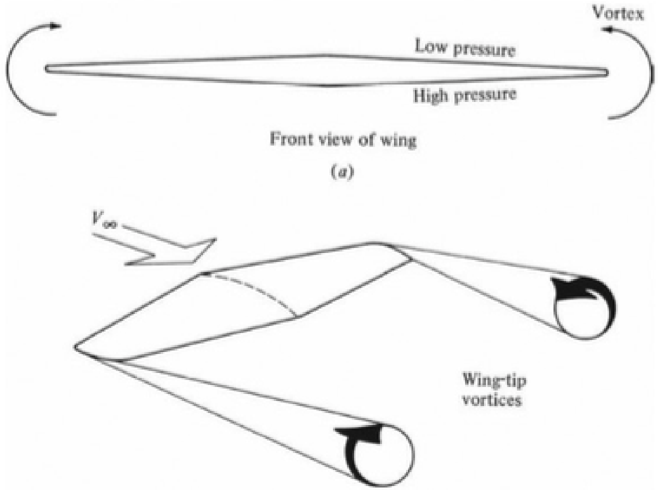
Wave drag –
net drag due to
higher pressure
behind the
shock wave


C_d
Mainly wave drag
Sound barrier
On the superonic side, c_d varies approximately as
$$\frac{1}{\sqrt{M_\infty^2 - 1}}$$
Profile drag
$M_{\text{drag divergence}}$
1.0
M_∞

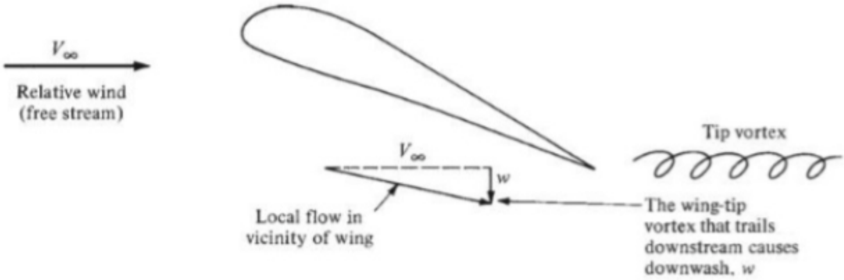




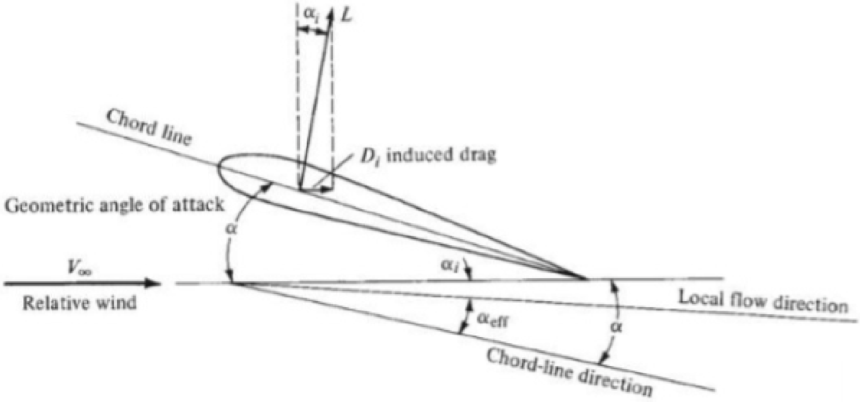
$$\text{Aspect ratio} = \text{AR} = \frac{b^2}{S}$$



Vortex
Low pressure
High pressure
Front view of wing
(a)
V_∞
Wing-tip
vortices



- **Vortex**: Trailing circulatory motion, as a result of pressure difference between upper and lower surfaces of wing.
- **Downwash**: Downward component of air velocity
- Local relative wind is changed.



- **Geometric angle of attack**: α between mean chord and V_∞
- **Induced angle of attack**: Difference between local and free stream flow directions
- **Effective angle of attack**: $\boxed{\alpha_{\text{eff}} = \alpha - \alpha_i}$

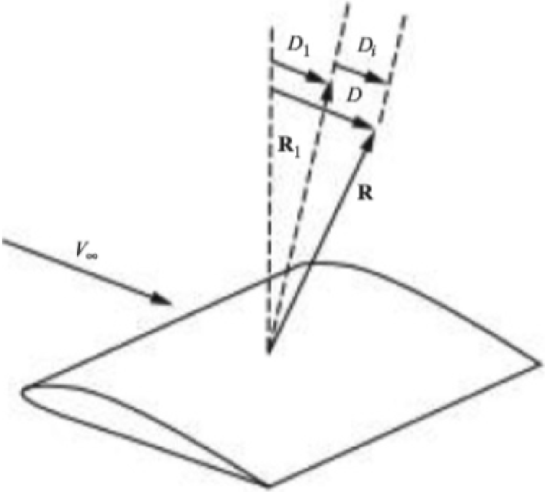
- Reduced angle of attack of the airfoil sections of the wing
- Increase in the drag, resulting in induced drag.
- What are the physical interpretations?

Wing-tip vortices alter the flow field about wing to change the surface pressure distributions in the direction of increased drag.

Local relative wind is canted downward, L itself is "tilted back", contributing a certain component of force $\parallel V_\infty$ (*a drag force*).

Wing-tip vortices contains a certain amount of kinetic energy, supplied by aircraft propulsion system. Extra power needs to be added to overcome this increment in drag due to induced drag.

- Decrease in lift coefficient and increase in drag coefficient



How do we get the expression for induced drag?

- Induced drag:

$$D_i = L \sin \alpha_i \approx L\alpha_i$$

- How to compute induced angle of attack, α_i?
- According to incompressible flow theory, for elliptical lift distribution,

$$\boxed{\alpha_i = \frac{C_L}{\pi \mathsf{AR}}}$$

where C_L is lift coefficient of finite wing and AR is aspect ratio.

- Induced drag

$$D_i = L\alpha_i = \frac{LC_L}{\pi \mathsf{AR}} = \frac{q_\infty S C_L^2}{\pi \mathsf{AR}}$$

- Induced drag coefficient

$$\boxed{C_{D,i} = \frac{D_i}{q_\infty S} = \frac{C_L^2}{\pi \mathsf{AR}}}$$

- In general, induced drag coefficient,

$$\boxed{C_{D,i} = \frac{C_L^2}{\pi e \mathsf{AR}}}$$

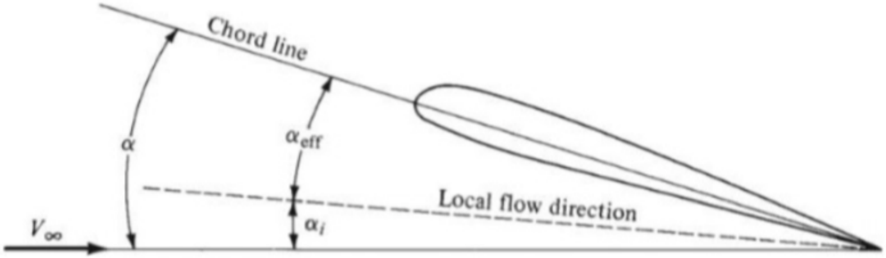
where e is span efficiency factor.

- For elliptical planforms, $e = 1$ and $e < 1$ for others.
- $C_{D,i}$ is minimum for elliptical planforms.
- At high lift, induced drag is also large.
- Induced drag (*drag due to lift*) can be reduced by increasing AR.
- What would be $C_{D,i}$ for infinite wing?
- Total drag coefficient

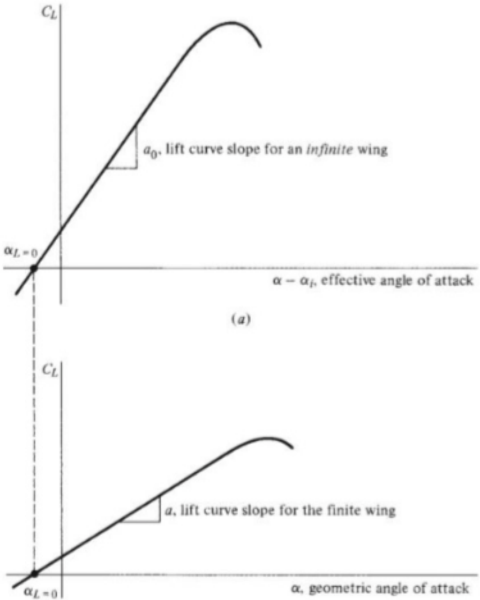
$$\boxed{C_D = \underbrace{C_d}_{\text{Profile drag}} + \underbrace{\frac{C_L^2}{\pi e \mathsf{AR}}}_{\text{Induced drag}}}$$

- Finite wing
  - ⇒ Induced drag
  - ⇒ Change in lift-slope
- Induced angle of attack

$$\alpha_i = \underbrace{\frac{C_L}{\pi e_1 \mathsf{AR}}}_{\text{Radians}} = \underbrace{\frac{57.3 C_L}{\pi e_1 \mathsf{AR}}}_{\text{Degrees}}$$



Change in Lift-slope



- When lift is zero then $\alpha_i = 0 \Rightarrow \alpha = \alpha_{\text{eff}}$.
- Angle of attack for zero lift, $\alpha_{L=0}$, is the same for both finite and infinite wings.
- Lift slope

$$\frac{dC_L}{d(\alpha - \alpha_i)} = a_0 \Rightarrow C_L = a_0(\alpha - \alpha_i) + \text{constant}$$

- On substituting for α_i,

$$C_L = a_0 \left(\alpha - \frac{57.3 C_L}{\pi e_1 \mathsf{AR}} \right) + \text{constant}$$

$$C_L = \frac{a_0 \alpha}{1 + 57.3 a_0/(\pi e_1 \mathsf{AR})} + \frac{\text{constant}}{1 + 57.3 a_0/(\pi e_1 \mathsf{AR})}$$

- Lift-slope for finite wing

$$\boxed{a = \frac{dC_L}{d\alpha} = \frac{a_0}{1 + 57.3 a_0/(\pi e_1 \mathsf{AR})}}$$

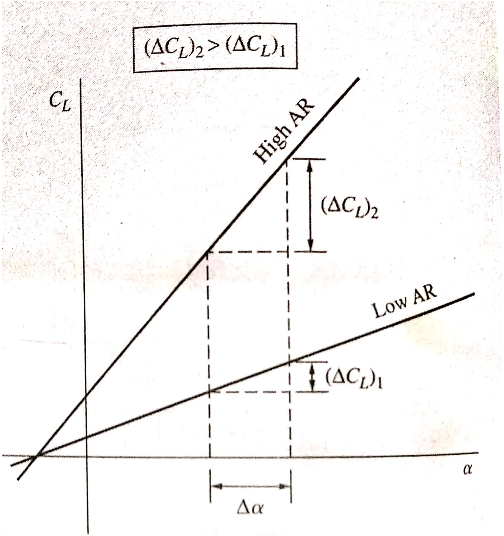
- If the AR is high then induced drag is less.
- High AR also ensure higher lift-slope.
- Can we make AR very high?
- Lift acting on each wing acts to bend the wing upward, creating a bending moment at the joint of wing and fuselage.
- Wing and fuselage structures must be strong enough to resist this bending moment.
- Increase in wing stiffness $\Rightarrow$ Increased wing structural weight.
- AR design is a compromise between competing values in aerodynamics and structures.
- Values of AR: 5-7
- An example of high AR 14.3 is Lockheed U-2, high altitude reconnaissance airplane.

- What would be the purpose for such high AR airplane design?
- In steady flight,

$$L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$$

- ρ_∞ decreases with altitude.
- For airplane at high altitude, C_L must be large.
- Airplane stalls at high α, so it can fly at maximum altitude, where C_L reaches its maximum value.
- A high value of C_L results in high induced drag, which can be reduced if AR is chosen to be high.
- Highest velocity allowed by drag divergence and the lowest velocity allowed by stalling were almost the same.
- Can there be a low AR airplane also useful?

- Consider a subsonic military aircraft designed for low-altitude, high-speed penetration of an enemy's defense, flying close enough to the ground.
- High density at sea-level results in low C_L and induced drag.
- A low AR wing, with a relatively small surface area, will reduce the profile drag.
- Aircraft with low AR is less sensitive to atmospheric turbulence encountered at low altitudes.
- Lift slope is smaller for a low AR wing.
- If α is changed by an atmospheric gust then ΔC_L will be less for low AR wing.
- Smaller change in C_L due to a gust for the low AR wing results in a smoother ride, which is good for both the flight crew and the structure of airplane.


$(\Delta C_L)_2 > (\Delta C_L)_1$
C_L
High AR
$(\Delta C_L)_2$
Low AR
$(\Delta C_L)_1$
α
$\Delta\alpha$

- **Stalling speed**: Slowest speed at which an airplane can fly in straight and level flight.

$$L = q_\infty S C_L \Rightarrow V_\infty = \sqrt{\frac{2L}{\rho_\infty S C_L}}$$

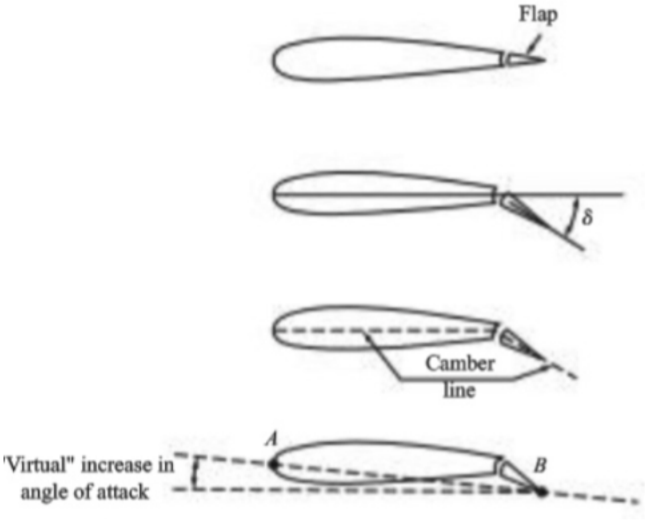
- For level and steady flight, $W = L$

$$V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

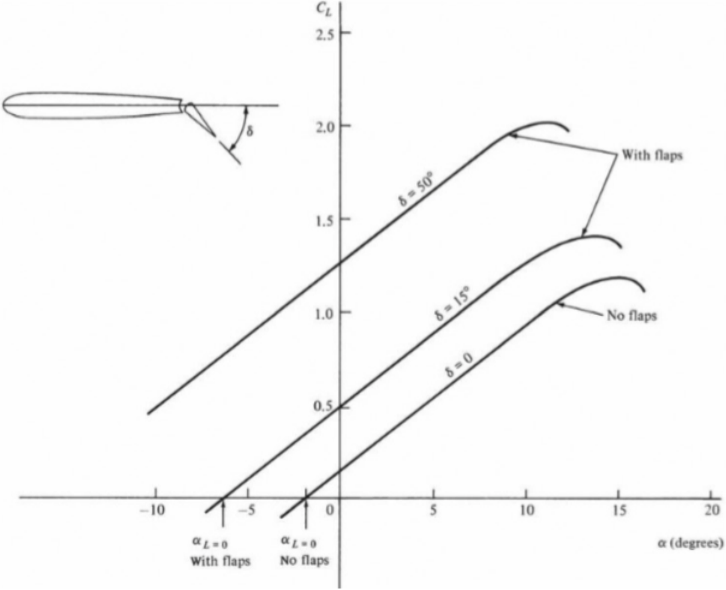
- Stalling speed correspond to α resulting in $C_{L,\max}$.

$$V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_{L,\max}}}$$

- How to achieve enhanced lifting property for a given airfoil?



Flap
δ
Camber line
A
B
"Virtual" increase in angle of attack



C_L
2.5
2.0
1.5
1.0
0.5
δ
δ = 50°
δ = 15°
δ = 0
With flaps
No flaps
−10
−5
0
5
10
15
20
$\alpha_{L=0}$ With flaps
$\alpha_{L=0}$ No flaps
α (degrees)

Reference

1. John Anderson Jr., *Introduction to Flight*, McGraw-Hill Education, Sixth Edition, 2017.

Thank you for your attention !!!