Rocket Configuration Design Problem

Problem No. 01

A **rocket** is to be designed to **inject** a spacecraft of 1.5 Tons **mass** in a sun-synchronous circular **polar** orbit at 650 km altitude above **earth's** surface. ($R_E = 6371$ km).

Further, it is **known** that spacecraft will need **7535** m/s of speed parallel to **local** horizon at the injection **point**.

Assuming that all the losses are restricted to **20%** of ideal mechanical energy, **determine** the lower bound on m_0 if $\mathbf{I_{sp}}$ is 350s.

Lastly, if 14.5 Tons of inert mass is **unavoidable**, what would be $\mathbf{m_0}$?

Solution No. 01

The **applicable** solution is as **follows**.

$$\frac{1}{2}V^2 + g_0 h = \frac{1}{2}\times 7535^2 + 9.81\times 7.021\times 10^6 = 9.7264\times 10^7$$

$$\frac{1}{2}V_0^2 = \frac{9.7264\times 10^7}{0.8} = 12.158\times 10^7 \rightarrow V_0 = 15593$$

$$m_b = m_0 e^{-\frac{1589.6}{350}} = 0.01066 m_0 \rightarrow m_0 > 140.7 Tons$$

$$m_b = 1.5 + 14.5 = 16 \rightarrow m_0 > 1509.4T$$

Multi-stage Performance Problem

Problem No. 02

Consider a rocket of following **configuration**.

$\mathbf{m_0 = 80T,}$ $m_p = 60T$, $I_{sp} = 240$ s, $g_0 = 9.81 m/s^2$.

Assume equal stages and generate **ideal** velocity solutions for, 1, 2 and 3-stage **operation**. (Hint: $m_{inert} = 18T$, $m_* = 2T$).

Solution No. 02

The **applicable** solution is as follows.

$$V_{b-ideal-1stage} = g_0 I_{sp} \ln\frac{80}{20} = 3264 m/s$$

$$V_{b-ideal-2stage} = g_0 I_{sp}\left[\ln\frac{80}{50} + \ln\frac{41}{11}\right] = 4204 m/s$$

$$V_{b-ideal-3stage} = g_0 I_{sp}\left[\ln\frac{80}{60} + \ln\frac{54}{34} + \ln\frac{28}{8}\right] = 4716 m/s$$

Multi-stage Methodology Problem

Problem No. 03

Angara 1.2, a 2-stage rocket, has a **payload** of 4T, with the following stage-wise **parameters**.

1-Stage: $I_{sp1} = 310s; \quad \varepsilon_1 = 0.072, \quad \pi_1 = 0.188$

2-Stage: $I_{sp2} = 342.5s; \quad \varepsilon_2 = 0.089, \quad \pi_2 = 0.124$

Determine stage-wise mass **distribution,** the total lift-off **mass** and the ideal burnout velocity.

Solution No. 03

The **solution** is as follows.

$$m_{s2} = 0.089 \times 4 \times 7.064 = 2.515T$$

$$m_{p2} = 0.911 \times 4 \times 7.064 = 25.74T; \quad m_{02} = 32.26T$$

$$m_{s1} = 0.072 \times 32.26 \times 4.319 = 10.03T$$

$$m_{p1} = 0.928 \times 32.26 \times 4.319 = 129.3T; \quad m_0 = m_{01} = 171.6T$$

$$V_* = -9.81 \times \begin{bmatrix} 310 \times \ln(0.072 + 0.928 \times 0.188) + \\ 342.5 \times \ln(0.089 + 0.911 \times 0.124) \end{bmatrix} = 9633.9m/s$$