Constant 'V' Solution

Constant 'V' Concept

Gravity turn can also be **employed**, when the **vehicle** has reached desired **velocity** but does not have **inclination.**

Such a **manoeuvre** can be done either at the **start** of the mission or **towards** the end.

These **solutions** are simpler to obtain and **implement.**

Basic Formulation

Applicable **equations** for the case of **constant 'V'** are as given below.

$$V = V_0; \quad \dot{\theta} = \frac{\tilde{g}\sin\theta}{V_0}; \quad \frac{1}{\sin\theta}d\theta = \frac{\tilde{g}}{V_0}dt$$

$$\dot{V} = -\frac{\dot{m}g_0 I_{sp}}{m(t)} - \tilde{g}\cos\theta = 0; \quad -\frac{dm}{m} = \frac{V_0\cot\theta}{g_0 I_{sp}}d\theta$$

We see that we can **obtain** the solution for **'θ'** as an **explicit** function of '**t**'. Further, **'m'** solution is obtained as function of 'θ', which **becomes** the primary **variable.**

'θ' Solution as a Function of 't'

The solution for **pitch angle profile** is as given below.

$$dt = \frac{V_0}{\tilde{g}} \frac{d\theta}{\sin\theta}; \quad \int dt = \frac{V_0}{\tilde{g}} \int \frac{d\theta}{\sin\theta} \rightarrow t = \frac{V_0}{\tilde{g}} \ln\tan\frac{\theta}{2} + C$$

$$t - t_0 = \Delta t = \frac{V_0}{\tilde{g}} \left(\ln\tan\frac{\theta}{2} - \ln\tan\frac{\theta_0}{2} \right)$$

$$\Delta t = \frac{V_0}{\tilde{g}} \ln \left(\frac{\tan\frac{\theta}{2}}{\tan\frac{\theta_0}{2}} \right) \rightarrow \tan\frac{\theta}{2} = \tan\frac{\theta_0}{2} e^{\frac{\tilde{g}\Delta t}{V_0}}$$

We see that a **higher 'Δt'** would give **higher 'θ'.**

'm' Solution as a Function of 'θ'

The **mass** solution for constant **V case** is as follows.

$$\int \frac{dm}{m} = -\frac{V_0}{g_0 I_{sp}} \int \frac{\cos\theta}{\sin\theta} d\theta \rightarrow \ln m = -\frac{V_0}{g_0 I_{sp}} \ln(\sin\theta) + C$$

$$m(\theta) = k(\sin\theta)^{-\frac{V_0}{g_0 I_{sp}}}; \quad k = m_0(\sin\theta_0)^{\frac{V_0}{g_0 I_{sp}}}; \quad \frac{m}{m_0} = \left(\frac{\sin\theta}{\sin\theta_0}\right)^{-\frac{V_0}{g_0 I_{sp}}}$$

It is found that a **higher 'θ'** results in higher **propellant** to be burnt.

'h' & 'x' Solutions

Applicable **equations** & solutions for **'h' & 'x'** profiles are as given below.

$$\frac{dh}{dt} = V_0 \cos\theta \rightarrow \int dh = \frac{V_0^2}{\tilde{g}} \int \frac{\cos\theta}{\sin\theta} d\theta; \quad h = \frac{V_0^2}{\tilde{g}} \ln \sin\theta + C$$

$$h - h_0 = \frac{V_0^2}{\tilde{g}} (\ln \sin\theta - \ln \sin\theta_0); \quad \Delta h = \frac{V_0^2}{\tilde{g}} \ln \frac{\sin\theta}{\sin\theta_0}$$

$$\frac{dx}{dt} = V_0 \sin\theta \rightarrow \int dx = \frac{V_0^2}{\tilde{g}} \int d\theta; \quad x = \frac{V_0^2}{\tilde{g}} \theta + C; \quad \Delta x = \frac{V_0^2}{\tilde{g}} \Delta\theta$$

Summary

To **summarize**, we note that constant **velocity** solutions are simpler to **obtain**.

Further, these **solutions** provide explicit control over **propellant** mass for a desired velocity and **inclination**.