Parabolic & Hyperbolic TOF Problems

Problem No. 01

Determine **Δt** for a spacecraft on a **parabolic** path to reach **r** = 920,000 km, & corresponding **v,** having earth's **surface** as perigee**.**

$$\mu = 3.986 \times 10^{14} m^2/s^2; \quad R_E = 6378 km$$

Solution No. 01

The **applicable** solution is as given below.

$$r_p = 6.378\times10^6 m; \quad v_p = \sqrt{\frac{2\mu}{r_p}} = 11{,}180 m/s; \quad h = r_p v_p = 7.13\times10^{10} m^2/s^2$$

$$p = \frac{h^2}{\mu} = 12.75\times10^6 m; \quad \theta = \cos^{-1}\left(1-\frac{p}{r}\right) = 170.48^\circ$$

$$\Delta t = \frac{h^3}{2\mu^2}\left(\tan\frac{\theta}{2} + \frac{1}{3}\tan^3\frac{\theta}{2}\right) = 655800 s = 182.2 h; \quad v_r = 930.9 m/s$$

Problem No. 02

An **object** is launched from **earth's** surface with **12,190 m/s** along local horizon. Find 'Δt' and 'v' for r = **9.2×10^8 m**. Compare this with '$\mathbf{v}_\infty$'. ($R_E = 6{,}378$, $\mu = 3.986 \times 10^{14}$).

Solution No. 02

The **TOF** and velocity values are as **follows**.

$$r_p = 6.378\times10^6 m, \quad h = 7.775\times10^{10}; \quad p = 15.16\times10^6 m$$

$$\varepsilon = 11.8\times10^6; \quad a = -16.89\times10^6 m, \quad e = 1.38, \quad \theta(r) = 135.5^\circ$$

$$F = 4.378 rad, \quad M = 51.08 rad, \quad T = \sqrt{\frac{(-a)^3}{\mu}} = 3477 s/rad$$

$$TOF = 3477\times51.08 = 177600 s = 49.3 h = 2.06 d,$$

$$v_r = 4946 m/s, \quad v_\infty = \sqrt{2\varepsilon} = 4858 m/s$$