EE 622 : Optimal Control Systems

Assignment 2

1. Find the necessary condition that must be satisfied by an extremal of the functional
$$J(x) = \int_{t_0}^{t_f} g(\dot{x}(t), x(t), t)dt \tag{1}$$
where $x(t_0) = x_0$, $x(t_f) = x_f$, t_0 are specified and t_f is free.

2. For the given system, Find the optimal control input $u^*(t)$.
$$\dot{x}_1 = x_2 \tag{2}$$
$$\dot{x}_2 = -2x_1 + u \tag{3}$$
With $x_1(0) = 3$, $x_2(0) = 5$, $x_1(2) = 0$ and $x_2(2) = 1$ which minimize
$$J(x) = \int_0^2 ({x_1}^2 + u^2)dt \tag{4}$$

3. Minimize
$$J(x) = \int_0^{\pi/2} (\dot{x_1}^2 + \dot{x_2}^2 + 2\dot{x_1} + 2\dot{x_2} + x_1x_2)dt \tag{5}$$
with boundary condition $x_1(0) = 0$, $x_2(0) = 0$, $x_1(\pi/2) = 1$ and $x_2(\pi/2)$ is free.

4. Find the extremal curves for the functional
$$J(x) = \int_0^{t_f} \frac{\sqrt{1+\dot{x}^2}}{x(t)}dt \tag{6}$$
Given $x(0) = 0$, and $x(t_f)$ must lie on $\theta(t) = t - 5$

5. Show that Euler-Lagrange equation for
$$J(x) = \int_{t_0}^{t_f} h(x(t), \dot{x}(t), \ddot{x}(t)...., \frac{d^r x(t)}{dt^r}, t)dt \tag{7}$$
is
$$\sum_{k=0}^{r} (-1)^k \frac{d^k}{dt^k}\left(\frac{\partial h}{\partial x^{(k)}}\left(x^*(t), ..., \frac{d^r x^*(t)}{dt^r}, t\right)\right) = 0 \tag{8}$$
With t_0 and t_f specified, and $x(t_0)$, $x(t_f)$ and $(r-1)$ derivatives of x at t_0 and t_f given.

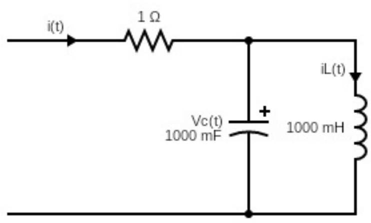

Figure 1: Circuit

6. For the given circuit, Find the input current $i(t)$ such that the energy loss across resistor is minimized.
   Given $i_L(0) = 0A$, $V_c(0) = 2V$, $i_L(t_f) = 0A$ and $V_c(t_f) = 0V$

7. Find the extremum of the functional,

$$J(x, \dot{x}) = \int_0^{\frac{\pi}{4}} \left(2\dot{x}^2 + 6\dot{x}x - x^2\right) dt \tag{9}$$

   satisfying the boundary conditions $x(0) = 0$ and $x(\frac{\pi}{4}) = 1$.

8. Find the extremal for the functional,

$$J(X) = \int_0^{\frac{\pi}{2}} \left(x_1^2 + \dot{x}_1\dot{x}_2 + 4\dot{x}_2^2\right) dt \tag{10}$$

   where $X = [x_1, x_2]^T$, functions x_1, x_2 are independent and the boundary conditions are,

$$\begin{aligned} x_1(0) &= 1 \qquad x_1\left(\frac{\pi}{2}\right) = 2 \\ x_2(0) &= \frac{3}{2} \qquad x_2\left(\frac{\pi}{2}\right) \text{ is free.} \end{aligned} \tag{11}$$

9. Mechanical systems do evolve in a fashion such that the equations of motion of such systems can be derived from the *action* principle. For instance, a system at static equillibrium has its potential energy at its minimum. Similarly, mechanical systems in motion do have their "action" at its extremum. In configuration space, let q_is and $\dot{q}_i$s be their generalised coordinates and generalised velocities, respectively, then the *action* is given by,

$$A = \int_{t_0}^{t_f} \mathcal{L}\left(q, \dot{q}, t\right) dt \tag{12}$$

   where $\mathcal{L}$ is in general the difference between the kinetic and the potential energy. Motions of mechanical systems do coincide with the extremals of the aforementioned functional. Using the *action* principle, derive the equation of motions of a simple and inverted pendulum. Mention all your assumptions explicitly.