Hohmann Manoeuvre

Single Impulse Change Restrictions

The **single** impulse based orbit change **solution**, seen previously, does not **provide** complete freedom due to **under-**actuation.

I.e., **we** have only one **input** impulse, but **want** to change **three** degrees-of-freedom, ‘a’, ‘e’ and ‘ω’.

Unconditional Orbit Changes

Thus, we find that only **one** degree-of-freedom will be as per our **requiremen**t, while the other **two** would tak**e** **specific** values, as dictated by the **constraint.**

This situation can be **remedied** by increasing the number of **inputs.**

Two-impulse Orbit Changes

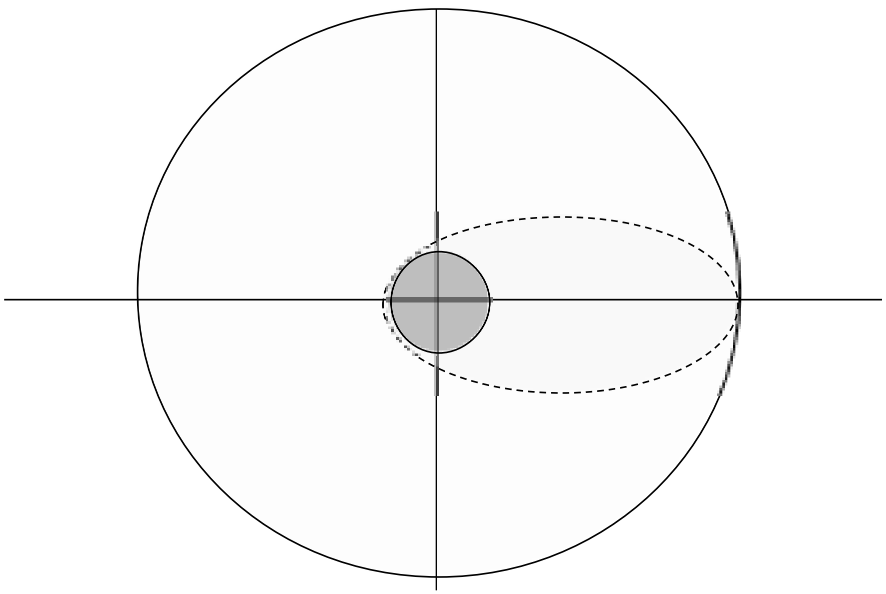
It should be noted that while **we** need three degrees-of-freedom to **change** all unconditionally, in general there is a **greater** need for change in **shape** and size of the **orbit.**

Thus, we can **restric**t ourselves to only **two** velocity impulses for the task of changing **'a' and 'e'.**

Among the **many** possibilities, we consider the **one** which preserves 'ω' and is also a **minimum** time solution, called '**Hohmann**' transfer.

Hohmann Transfer Concept

Hohmann strategy involves two **impulses**, first one given at **perigee** and second one at **apogee**, as shown below.



Two-Impulse Transfer Strategy

In this case, **first** impulse is given at the **perigee** to put the **satellite** on a transfer **ellipse.**

Second **impulse** is given once the satellite **reaches** apogee of the transfer **ellipse** (after half a cycle), so that orbit becomes the outer circle.

Thus, total **tangential** velocity impulse is the **sum** of the two velocity **impulse** magnitudes.

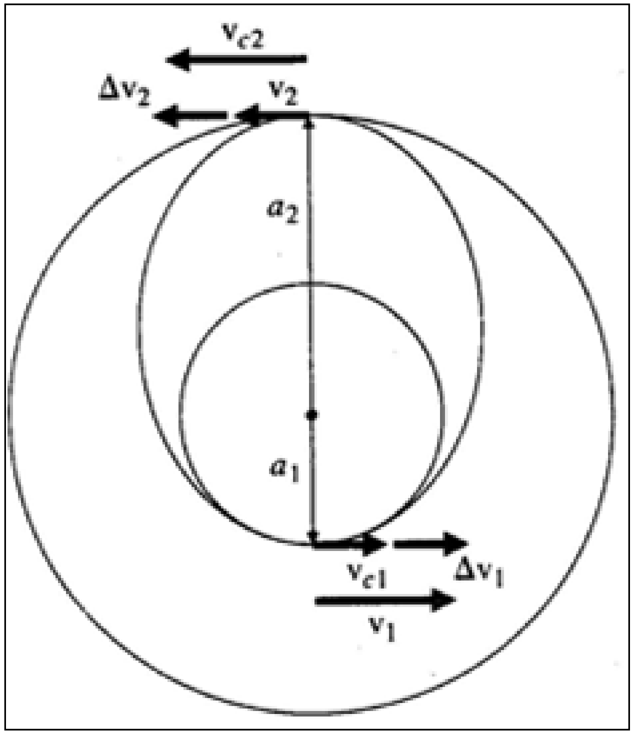
Two-Impulse Transfer Strategy

This **manoeuvre** is found to be most **fuel** efficient and also is a **good** solution for total '**ΔV**'.

Hohmann transfer can be used between **any** two orbits, and can also be **employed** for inter-planetary **transfers.**

Hohmann Transfer Solution - Circles

Hohmann transfer between **circles** is as shown below.



$$a = \frac{a_1 + a_2}{2}, \quad V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

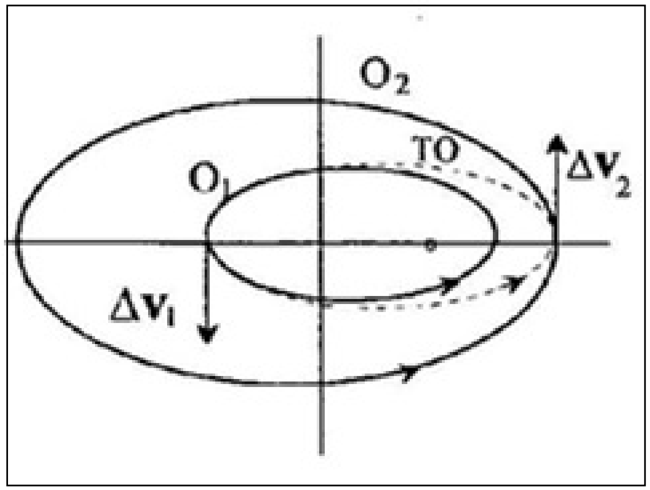
$$\Delta V_1 = \sqrt{\frac{2\mu}{a_1} - \frac{2\mu}{a_1 + a_2}} - \sqrt{\frac{\mu}{a_1}}$$

$$\Delta V_2 = \sqrt{\frac{\mu}{a_2}} - \sqrt{\frac{2\mu}{a_2} - \frac{2\mu}{a_1 + a_2}}$$

$$\Delta V = \Delta V_1 + \Delta V_2; \quad \Delta t = \pi\sqrt{\frac{a^3}{\mu}}$$

Hohmann Transfer Solution - Ellipses

Solution for **Hohmann** transfer between two **ellipses**, is as shown below.



$$r_{aTO} = r_{a2}, \quad r_{pTO} = r_{p1}, \quad a_{TO} = \frac{r_{p1} + r_{a2}}{2}$$

$$\Delta V_1 = \sqrt{\frac{2\mu}{r_{pTO}} - \frac{\mu}{a_{TO}}} - \sqrt{\frac{2\mu}{r_{pTO}} - \frac{\mu}{a_1}}$$

$$\Delta V_2 = \sqrt{\frac{2\mu}{r_{TO}} - \frac{\mu}{a_2}} - \sqrt{\frac{2\mu}{r_{aTO}} - \frac{\mu}{a_{TO}}}$$

$$\Delta V = \Delta V_1 + \Delta V_2; \quad \Delta t = \pi\sqrt{\frac{a_{TO}^3}{\mu}}$$

Low Thrust Orbital Transfer

Impulsive Manoeuvre Drawback

Impulsive orbital manoeuvres, discussed previously, require **large** thrusts, to achieve even a modest '**ΔV**'.

However, for **small** orbital drifts, **low** thrust orbit corrections are **preferred**, as time is not a major **issue.**

Low Thrust Orbit Transfer Concept

Such manoeuvres can be **performed** using either **small** plasma **jets** or even solar **pressure**.

However, it can take **large** time as **energy** per transfer cycle is quite **small**.

Transfer path for such **manoeuvres** is an approximate circular **spiral**.

Low Thrust Formulation & Solution

Consider the **transfer** between two **circular** orbits.

If 'A' is **acceleration** applied in the **direction** of instantaneous **velocity**, we can obtain **ΔV** as follows.

$$\varepsilon(t) = -\frac{\mu}{2a(t)} \rightarrow \frac{d\varepsilon}{dt} = \frac{\mu}{2a^2} \cdot \frac{da}{dt} = \bar{A} \cdot \bar{v}(t); \quad v(t) = \sqrt{\frac{\mu}{a(t)}}$$

$$\frac{da}{dt} = \frac{2}{\sqrt{\mu}} a^{3/2} \cdot A \rightarrow \int_{t_0}^{t_1} A \cdot dt = \frac{\sqrt{\mu}}{2} \int_{a_0}^{a_1} \frac{da}{a^{3/2}} = \frac{\sqrt{\mu}}{2} \left[-2a^{-1/2} \right]_{a_0}^{a_1}$$

$$A \cdot \Delta t = \Delta V = \sqrt{\frac{\mu}{a_0}} - \sqrt{\frac{\mu}{a_1}}; \quad \Delta t = \frac{\Delta V}{A}$$

Low Thrust Orbit Transfer Attributes

We **see** that in case of **low** thrust transfer, required velocity **impulse** is exactly equal to the **difference** between the velocities in **respective** circular orbits.

However, for the **same** orbit change, ΔV for **Hohmann** transfer is as given **below**.

$$\Delta V = \left(1 - \frac{a_0}{a_1}\right)\sqrt{\frac{2\mu a_1}{a_0\left(a_0 + a_1\right)}} - \left(\sqrt{\frac{\mu}{a_0}} - \sqrt{\frac{\mu}{a_1}}\right)$$

Summary

Hohmann transfer strategy is an **elegant** two-impulse model that is **able** to achieve all kinds of **orbits** in a reasonably efficient **manner**.

We also see that low thrust **orbital** transfer provides an **alternative** method to change orbits, where **time** is not a major **concern**.