Advanced Manoeuvre Problems – 2

Problem No. 01

A spacecraft is put in a **circular** orbit at 400 km **altitude**. Determine ΔV to transfer it into **geosynchronous** orbit of 36000 km **altitude**. ($R_E = 6378$, $\mu = 3.986\times10^{14}$).

Also determine the **time** taken to complete the **manoeuvre.**

Solution No. 01

Orbit **change** result for circular **orbits** is as follows**.**

$$r_p = a_0 = 6778km; \quad r_a = a_1 = 42378km; \quad a_{te} = \frac{r_a + r_p}{2} = 24578km$$

$$v_0 = 7668.6m/s; \quad v_{p1} = 10069.6m/s; \quad \Delta V_1 = 2401.3m/s$$

$$v_{a1} = 1610.5m/s; \quad v_1 = \sqrt{\frac{\mu}{a_1}} = 3066.9m/s; \quad \Delta V_2 = 1456.4m/s$$

$$\Delta V = \Delta V_1 + \Delta V_2 = 3858m/s; \quad \Delta t = \frac{\pi}{\sqrt{\mu}} a_{te}^{3/2} = 19173s = 5.32h$$

Problem No. 02

Determine **Δv,** Δt required to **change** from an orbit with **h_p = 552 km, h_a = 2208 km** to a circular orbit of **35, 860 km** altitude. (1st apogee change, then circularization).

Solution No. 02

Result for **circularization** manoeuvre is as given **below**.

$$r_p = 6930km; \quad r_a = 8586km; \quad V_p = 7980m/s; \quad r_{ate} = 42{,}238km$$

$$a_{te} = 24{,}584km; \quad \frac{V_{pte}^2}{2} - \frac{\mu}{r_p} = -\frac{\mu}{2a_{te}} \rightarrow V_{pte} = 9941m/s$$

$$\Delta v_p = 1961m/s; \quad V_c = \sqrt{\frac{\mu}{r_{ate}}} = 3072m/s; \quad V_{ate} = V_{pte} \times \frac{r_p}{r_a} = 1631m/s$$

$$\Delta v_a = 1441m/s; \quad \Delta v = 3402m/s; \quad \Delta t = \frac{\pi}{\sqrt{\mu}} a_{te}^{3/2} = 19180s = 5.33h$$

Solution No. 02

The **Δv solution** for reverse order (1st perigee change, then circularization), is as **follows.**

$$r_p = 6930km; \quad r_a = 8586km; \quad V_p = 7980m/s; \quad r_{ate} = 42,238km$$

$$V_a = 6454.4; \quad a_{te} = 24,584km; \quad \frac{V_{ate}^2}{2} - \frac{\mu}{r_{ate}} = -\frac{\mu}{2a_{te}} \rightarrow V_{ate} = 8795m/s$$

$$\Delta v_a = 2340.6m/s; \quad V_c = \sqrt{\frac{\mu}{r_{ate}}} = 3072m/s; \quad V_{ate} = V_{pte} \times \frac{r_p}{r_a} = 1783.4m/s$$

$$\Delta v_p = 1288.6m/s; \quad \Delta v = \Delta v_a + \Delta v_p = 3629.2m/s$$

Problem No. 03

A spacecraft in circular orbit at **400 km** altitude is to be put in circular orbit at **36000 km** altitude, using low thrust jet.

Determine total **impulse** used and **time** taken to perform the manoeuvre if spacecraft generates **1 mili 'g'** acceleration in **instantaneous** velocity direction.

Solution No. 03

Low thrust solution for orbit change is as **follows**.

$$a_0 = 6778km; \quad a_1 = 42378km; \quad \Delta V = \sqrt{\frac{\mu}{a_0}} - \sqrt{\frac{\mu}{a_1}} = 4601.7 m/s$$

$$\Delta t = \frac{\Delta V}{A} = \frac{4601.7}{0.00981} = 469085.9 s = 130.3 hours = 5.43 days$$

$$\Delta V_{Hohmann} = 3858 m/s < \Delta V (?)$$