Flight Mechanics/Dynamics

(Course Code: AE 305/305M/717)

Department of Aerospace Engineering
Indian Institute of Technology Bombay

Instructor: Shashi Ranjan Kumar
6 May 2021

Time: **180 Minutes**

End-Semester Examination

Total Points: 100

Instructions

- All questions are mandatory.
- In case a question is missing some data/information, assume the same suitably and clearly mention it in your answer sheet.
- You are only allowed to open lecture slides of the course, any other form of help/reference is not permitted.
- In cases where the answers of two students are found to be copied, both of them will be awarded zero marks for that particular question.
- Answer sheets need to be submitted in a single "Roll_Number.pdf" format on Moodle.
- You will get 15 minutes duration for submission of your answer sheet on Moodle after the exam time.

1. The coefficients of the characteristic polynomial corresponding to lateral–directional stability of an aircraft are

$$A = 1,\ B = 9.42,\ C = 9.48 + N_v,\ D = 10.29 + 8.4N_v,\ E = 2.24 - 0.39N_v\,.$$

Find the range of values of N_v for which the aircraft will be laterally dynamically stable.

[10]

2. Consider an aircraft equipped with accelerometers and gyroscopes to measure accelerations and body rates.

(a) If the aircraft is undergoing a steady rotation with angular velocity components in body axes system $p = 10$ deg/s, $q = 2$ deg/s, and $r = 5$ deg/s, then determine the corresponding Euler angle rates at the time instant where the Euler angles are $\psi = -30$ deg, $\theta = 10$ deg, and $\phi = 15$ deg.

(b) If the aircraft is flying at an angle of attack of 10°, sideslip of 5°, and a bank angle of 10° and the onboard accelerometers record $a_{xb} = 10$ ft/s^2, $a_{yb} = 5$ ft/s^2, and a_{zb} = -5 ft/s^2, then determine the acceleration components in the wind axes system.

[7.5+7.5]

3. Answer the following:

   (a) Determine the missing elements of the following direction cosine matrix:

$$C = \begin{bmatrix} 0.1587 & c_{12} & 0.4858 \\ 0.8595 & -0.1218 & c_{23} \\ c_{31} & 0.4963 & 0.7195 \end{bmatrix}$$

   (b) Consider a rotation of a vector, using quaternion, about an axis defined by the vector $(1, 0, 0)$ through an angle of $2\pi/3$.

      (i) Obtain the quaternion Q to perform this rotation.

      (ii) Compute the effect of rotation on the basis vector $\mathbf{k} = (0, 0, 1)$.

[5+10]

4. Consider the speed controller, as shown in Fig. 1, with the system output, $\boldsymbol{y} = [u \quad \gamma]^T$ and the control vector $\boldsymbol{c} = [\delta_e \quad \delta_p]^T$. Note that only the elevator input is in feedback loop. The desired and actual speeds are denoted by u_c and u, respectively. Transfer function matrix $\boldsymbol{G}(s)$ which relates the control vector, $\boldsymbol{c}$, to the output, $\boldsymbol{y}$, is represented as

$$\boldsymbol{G}(s) = \begin{bmatrix} G_{u\delta_e} & G_{u\delta_p} \\ G_{\gamma\delta_e} & G_{\gamma\delta_p} \end{bmatrix}.$$

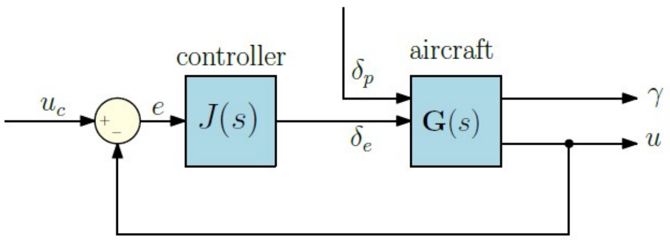

Figure 1: Speed controller

Explicitly derive the *closed-loop transfer functions*, denoted by $G^\star_{u\delta_p}$ and $G^\star_{\gamma\delta_p}$, corresponding to the throttle input and the outputs, i.e., $\delta_p \to u$ and $\delta_p \to \gamma$.

[10+10]

5. Show that the small-disturbance equations, with $\theta_0 = 0$ and neglecting all Y force derivatives, yields the following approximation for the lateral displacement:

$$\Delta y_E(t) = g \int_0^t \int_0^t \phi(\tau) d\tau dt\,.$$

[10]

6. Consider a stable system whose transfer function is

$$\boldsymbol{G}(s) = \frac{Y(s)}{U(s)} = \frac{N(s)}{f(s)} = \frac{1}{(s^2+3s+2)(s^2+7s+12)}\,.$$

The roots of the characteristic equation are denoted as $\lambda_1, \lambda_2, \lambda_3$ and λ_4. Find these roots and derive the expression for output $y(t)$ of the system in time domain, using the partial fraction expansion, for an input given by $u(t) = e^{i2t}$. Also, show that the *steady state* output becomes a scaled version of the input along with some phase change depending on the transfer function.

[10]

7. Consider the following approximate system of equations, corresponding to the short period mode of an aircraft, given by

$$\begin{aligned}
\dot{w} &= \frac{Z_w}{m} w + u_0 q\,,\\
\dot{q} &= \frac{1}{I_y}\left[M_w + \frac{M_{\dot{w}} Z_w}{m}\right] w + \frac{1}{I_y}\left[M_q + M_{\dot{w}} u_0\right] q\,,
\end{aligned}$$

where u_0 is the nominal speed and m is the mass of the aircraft.

(a) Find the natural frequency and damping ratio corresponding to the short period mode as a function of $m, u_0, I_y, M_w, M_q, M_{\dot{w}}$ and Z_w.

(b) If an aircraft weighing 2×10^6 N is moving at a nominal speed of 230 m/s has the following structural and aerodynamic parameters: $I_y = 0.5 \times 10^8$ kg m^2, $M_w = -1.563 \times 10^4$ Nm, $M_q = -1.521 \times 10^7$ Nm, $M_{\dot{w}} = -1.702 \times 10^4$ Nm and $Z_w = -9.030 \times 10^4$ N, then use the relationships derived in the previous part to compute the natural frequency and damping ratio corresponding to short-period mode of the aircraft. Assume the gravitational acceleration to be 9.81 m/s^2.

[20]