Orbit Determination Problems

Problem No. 01

A **satellite** is launched from **Earth** & attains a velocity of **8.5 km/s** which is parallel to **local horizon** at an altitude of **400 km.** (R_E = 6,371 km, $\mu_E = 3.986\times10^{14}$ m^3/s^2).

Determine the **trajectory parameters.**

Solution No. 01

The **orbital** solution is as follows.

$$h = 6.771\times10^{6}\times8.5\times10^{3} = 5.755\times10^{10}$$

$$\varepsilon = \frac{1}{2}(8500)^{2} - \frac{3.986\times10^{14}}{6.771\times10^{6}} = -2.274\times10^{7}$$

$$e = \sqrt{1+\frac{2\times\left(-2.274\times10^{7}\right)\times\left(5.755\times10^{10}\right)^{2}}{\left(3.986\times10^{14}\right)^{2}}} = 0.227$$

$$a = -\frac{3.986\times10^{14}}{\left(2\times-2.274\times10^{7}\right)} = 8.725\times10^{6} \rightarrow \text{Ellipse}$$

Problem No. 02

For the **initial** conditions defined in **problem** No. 01, determine the **maximum** altitude of satellite as well as the **time** taken from perigee to **apogee**.

Solution No. 02

The **maximum** altitude and time taken for **moving** from perigee to apogee are as **follows**.

$$r_a = 8.725\times10^6 \times(1+0.227) = 10.705\times10^6$$

$$\text{Height} = r_a - R_E = 4.334\times10^6 m$$

$$T = \frac{2\times\pi}{\sqrt{3.986\times10^{14}}}\left(8.725\times10^6\right)^{3/2} = 8110s = 2.25h$$

$$T_a - T_p = \frac{T}{2} = 4055s = 1.126h$$

Problem No. 03

A satellite is **released** at 400 km above **Earth's** surface, with velocity **parallel** to local horizon, and forms a **circular** orbit. (R_E = 6,371 km, $\mu_E = 3.986\times10^{14}$ m^3/s^2).

Determine the **injection velocity.**

Solution No. 03

The **solution** for injection velocity is as **follows**.

$$V = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986 \times 10^{14}}{6.771 \times 10^{6}}} = 7672.6 m/s$$

Problem No. 04

Based on the data of problem No. **03**, determine whether or not it is **possible** to get a circular **orbit** at any other altitude. in case the **injection** velocity is **higher** by 5%.

Solution No. 04

The **altitude** solution in respect of 5% **higher** velocity is as follows.

$$V' = 1.05 \times 7672.6 = 8055 = \sqrt{\frac{3.986 \times 10^{14}}{r'}}$$

$$r' = 6.143 \times 10^{6} < R_E \rightarrow \text{ Not possible.}$$