Flight Mechanics/Dynamics

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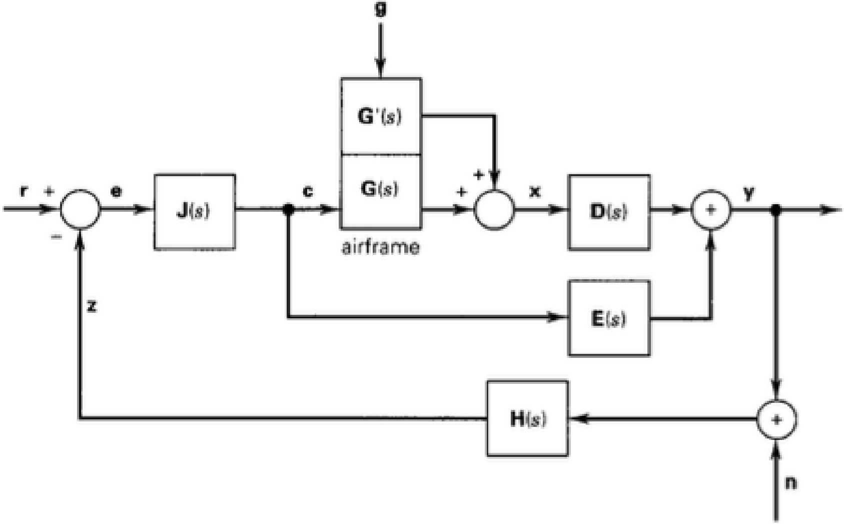
- Why do you need closed-loop control or feedback control?
- Objectives of aircraft: flight on a specified trajectory
  - $\Rightarrow$ A straight horizontal line traversed at constant speed or a turn
  - $\Rightarrow$ Transition from one symmetric flight path to another
  - $\Rightarrow$ A landing approach, following an ILS
  - $\Rightarrow$ Homing on a moving target, etc.
- Aircraft aims to follow a desired state.
- System dynamics: $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bc} + \boldsymbol{Tg}$
- Output equations: $\boldsymbol{y} = \boldsymbol{Dx} + \boldsymbol{Ec}$
- Sensor measurement: $\boldsymbol{z} = \boldsymbol{H}(s)\boldsymbol{y} + \boldsymbol{n}$
- Transfer functions: $\boldsymbol{G} = (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B}, \ \boldsymbol{G}' = (s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{T}$


g
G'(s)
G(s)
airframe
r
+
−
e
J(s)
c
+
+
x
D(s)
+
y
E(s)
z
H(s)
+
n

- Can we obtain the transfer functions G_{yr}, G_{yn}, G_{yg}?
- Output of the system

$$\begin{aligned} \boldsymbol{y} =& \boldsymbol{Dx} + \boldsymbol{Ec} = \boldsymbol{D}(\boldsymbol{Gc} + \boldsymbol{G'g}) + \boldsymbol{Ec} \\ =& (\boldsymbol{DG} + \boldsymbol{E})\boldsymbol{c} + \boldsymbol{DG'g} \\ =& (\boldsymbol{DG} + \boldsymbol{E})\boldsymbol{J}(\boldsymbol{r} - \boldsymbol{c}) + \boldsymbol{DG'g} \\ =& (\boldsymbol{DG} + \boldsymbol{E})\boldsymbol{J}(\boldsymbol{r} - \boldsymbol{H}(\boldsymbol{y} + \boldsymbol{n})) + \boldsymbol{DG'g} \end{aligned}$$

- On rearranging term, we get

$$[\boldsymbol{I} + \underbrace{(\boldsymbol{DG} + \boldsymbol{E})\boldsymbol{J}}_{\boldsymbol{F}}\boldsymbol{H}]\boldsymbol{y} = (\boldsymbol{DG} + \boldsymbol{E})\boldsymbol{Jr} - (\boldsymbol{DG} + \boldsymbol{E})\boldsymbol{JHn} + \boldsymbol{DG'g}$$

- Forward path transfer function from $\boldsymbol{e} \rightarrow \boldsymbol{y}$: $\boldsymbol{F}$
- Transfer functions

$$\boldsymbol{G}_{yr} = (\boldsymbol{I} + \boldsymbol{FH})^{-1}\boldsymbol{F}, \ \boldsymbol{G}_{yn} = -(\boldsymbol{I} + \boldsymbol{FH})^{-1}\boldsymbol{FH}, \ \boldsymbol{G}_{yg} = (\boldsymbol{I} + \boldsymbol{FH})^{-1}\boldsymbol{DG'}$$

- How can we relate the feedback control from aerodynamic point of view?
- **Feedback loop**: A method of altering one of the airplane's inherent stability derivatives
- When L_p, M_q, or N_r, is too small, or M_α or N_p is not of the desired magnitude, they can be synthetically altered by appropriate feedback.
- Let x be any nondimensional state variable, and let a control surface deflection as

$$\Delta\delta = k\Delta x, \quad k = \text{const.}$$

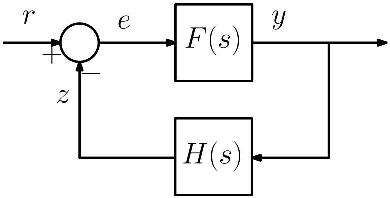
- Aerodynamic force or moment coefficient

$$\Delta C_a = C_{a\delta}\Delta\delta = C_{a\delta}k\Delta x \implies \Delta C_{a_x} = kC_{a_\delta}$$

- If x is yaw rate and δ is rudder angle then $\Delta C_{n_r} = kC_{n_{\delta_r}}$.
- If x is roll rate and δ is aileron angle then $\Delta C_{l_\phi} = kC_{n_{\delta_a}}$.

- Feedback control: Sensors to provide the states of vehicle.
- When human pilots are in control, their eyes sense, aided by the standard flight information displayed by their instruments, provide this information.
- Brains supply the required logical and computational operations needed, and their neuro-muscular systems provides all or part of the actuation.

- Automatic control system: Feedback signals
  - ⇒ Position and velocity vectors relative to a suitable reference frame.
  - ⇒ Vehicle attitude (θ, ϕ, ψ).
  - ⇒ Rotation rates (p, q, r).
  - ⇒ Aerodynamic angles (α, β).
  - ⇒ Acceleration components of a reference point in the vehicle



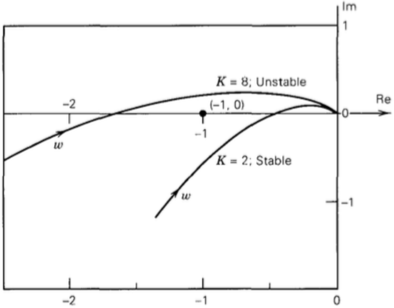
- Closed-loop transfer function

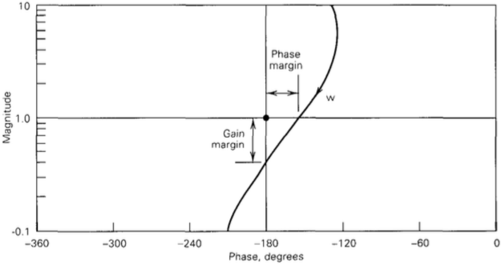
$$G_{yr}(s) = \frac{F(s)}{1 + F(s)H(s)}$$

- If $F(s) = \dfrac{N_1(s)}{D_1(s)}, H(s) = \dfrac{N_2(s)}{D_2(s)}$, then

$$G_{yr}(s) = \frac{N_1(s)D_2(s)}{D_1(s)D_2(s) + N_1(s)N_2(s)}$$

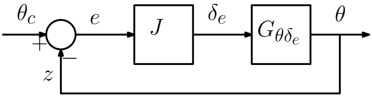
- Loop gain: $F(s)H(s)$
- How to look for the stability of closed-loop system?





If loop gain is < 1 when phase angle is 180°, or if phase is $< 180^\circ$ when gain is unity, then the system is stable.

- Phugoid: Lightly damped, low-frequency oscillation in V, θ, h
- We do not feel in actual flight, Why?
- Pilot (human or automatic) effectively suppresses them.
- Is this suppression method unique? No
- Phugoid oscillation cannot occur if θ does not change.



$$G_{yr}(s) = \frac{G_{\theta\delta_e}(s)J(s)}{1 + G_{\theta\delta_e}(s)J(s)}$$

If $G_{\theta\delta_e}(s) = \dfrac{N(s)}{D(s)}$ and $J(s) = \dfrac{N'(s)}{D'(s)}$ then characteristic equation

$$D(s)D'(s) + N(s)N'(s) = 0$$

θ: Important in both short period and phugoid modes, neither of two approximate transfer functions is sufficient for controller design.

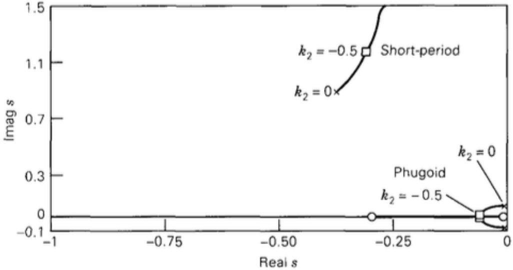
- Transfer function

$$G_{\theta\delta_e}(s) = \frac{-(1.158s^2 + 0.3545s + 0.003873)}{s^4 + 0.750468s^3 + 9.463025s^2 + 0.009463025s + 0.004195875}$$

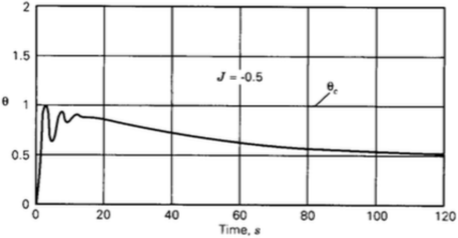
- Controller

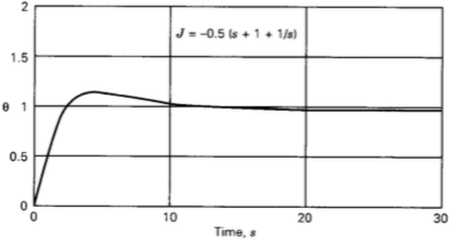
$$J(s) = \frac{k_1}{s} + k_2 + k_3 s$$

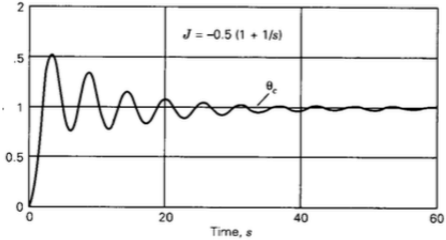
- What are the purposes of these three components?



- At a gain of about -0.5, phugoid mode is nearly critically damped, that is, it is about to split into two real roots.
- At this gain, phugoid oscillation is effectively eliminated.
- Is there any drawback or side-effect here?
- SP roots have moved in the direction of lower damping. Possible reason?
- Sum of dampings: coefficient of second largest term in characteristic equation.
- With P controller, this coefficient remains unchanged.
- Any increase in phugoid damping can only come at the expense of that of SP mode.
- How to overcome this limitation? Using derivative in controller.
- Physical reason for phugoid suppression due to proportional controller??



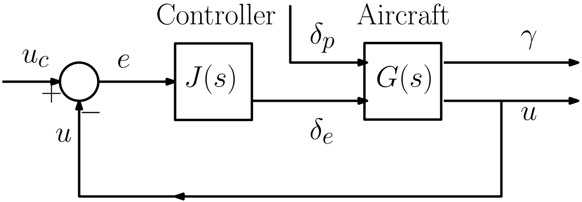




No control over altitude or speed of aircraft.

A same controller design is not valid at all operating points.

- Phugoid mode appears as transient perturbations from a steady state.
- Changing from level to climbing flight using throttle, settles in around 10 min.
- Similar techniques can be used for phugoid suppression if correct θ is known with good accuracy.
- Is there any other option? using speed information
- Both elevator and throttle influence the speed of aircraft.
- Throttle principally affects the speed only in the short term.
- For a change of steady-state speed, the elevator must be used.



- Output vector $\boldsymbol{y} = [u \quad \gamma]^T$
- Control vector $\boldsymbol{c} = [\delta_e \quad \delta_p]^T$
- Since controlled variable is u, which does not change appreciably in the short-period mode, phugoid approximation may be good for controller design.
- Transfer function from $\boldsymbol{c}$ to $\boldsymbol{y}$

$$\boldsymbol{G}(s) = \begin{bmatrix} G_{u\delta_e} & G_{u\delta_p} \\ G_{\gamma\delta_e} & G_{\gamma\delta_p} \end{bmatrix}$$

$$G_{u\delta_p} = -\frac{X_{\delta_p}}{m}\frac{u_0 M_w s}{f(s)}, \quad G_{\gamma\delta_p} = \frac{X_{\delta_p}}{m}\frac{M_w Z_u - M_u Z_w}{mf(s)}$$

- Closed-loop transfer function w.r.t. throttle input

$$\boxed{G^{\star}_{u\delta_p} = \frac{G_{u\delta_p}}{1 + G_{u\delta_e} J}, \quad G^{\star}_{\gamma\delta_p} = G_{\gamma\delta_p} - G_{\gamma\delta_e} J G^{\star}_{u\delta_p}}$$

- With transfer functions expressed as $G_{u\delta_e} = \dfrac{N_{u\delta_e}}{f(s)}$ and $J = \dfrac{N_j}{D_j}$, we have

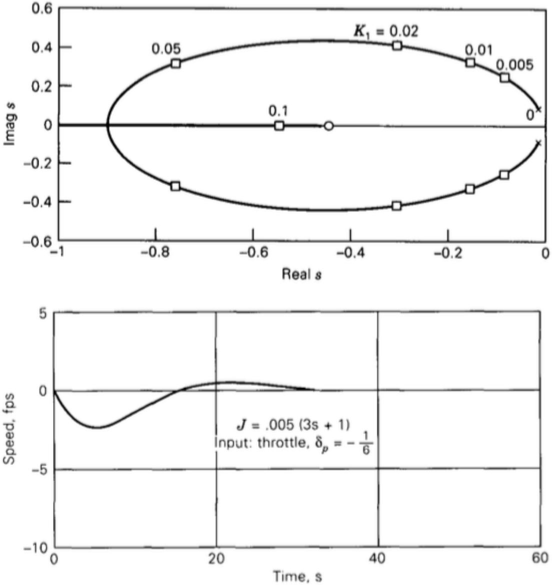
$$\boxed{G^{\star}_{u\delta_p} = \frac{D_j N_{u\delta_p}}{fD_j + N_j N_{u\delta_e}}, \quad G^{\star}_{\gamma\delta_p} = \frac{fD_j N_{\gamma\delta_p} + N_j(N_{\gamma\delta_p} N_{u\delta_e} - N_{u\delta_p} N_{\gamma\delta_e}}{f(fD_j + N_j N_{u\delta_e})}}$$

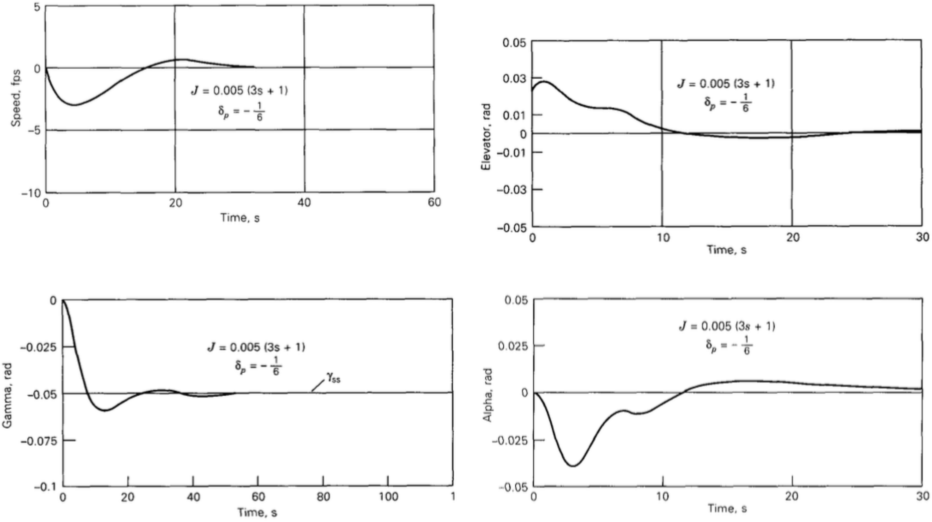
- Can linear system have two characteristic equations? No
- f in the denominator cancels out with that in the numerator.
- Second-order phugoid approximation is used for controller design.
- What is the effect of $J(s)$ on characteristic equation?

$$f(s)D_j + N_j N_{u\delta_e} = 0, \; f(s) = As^2 + Bs + C, \; N_{u\delta_e} = a_1 s + a_0$$

- With $J(s) = k_1 + k_2 s$, we have $A's^2 + B's + C' = 0$

$$A' = A + a_1 k_2, \; B' = B + a_1 k_1 + a_0 k_2, \; C' = C + a_0 k_1$$

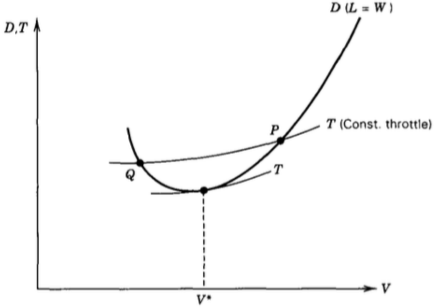

0.6
0.4
0.2
0
−0.2
−0.4
−0.6
Imag s
K_1 = 0.02
0.05
0.01
0.005
0.1
0
−1
−0.8
−0.6
−0.4
−0.2
0
Real s
5
0
−5
−10
Speed, fps
J = .005 (3s + 1)
Input: throttle, $\delta_p = -\frac{1}{6}$
0
20
40
60
Time, s

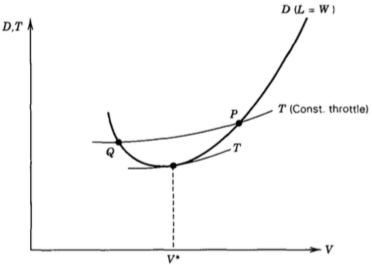

Speed, fps
Time, s
$J = 0.005\,(3s + 1)$
$\delta_p = -\frac{1}{6}$
Elevator, rad
Time, s
$J = 0.005\,(3s + 1)$
$\delta_p = -\frac{1}{6}$
Gamma, rad
Time, s
$J = 0.005\,(3s + 1)$
$\delta_p = -\frac{1}{6}$
γ_{ss}
Alpha, rad
Time, s
$J = 0.005\,(3s + 1)$
$\delta_p = -\frac{1}{6}$

- Objective of aircraft: To follow a specified line in space
- Consider a level flight at time-varying speed, with the autopilot maintaining α as required to nullify height error.
- Equation of motion

$$m\dot{V} = T - D$$

- As V can't change rapidly, effect of q and $\dot{\alpha}$ on lift and drag may be ignored.





- Reference thrust and drag: T_0, D_0
- Stability derivatives:

$$T_V = \frac{\partial T}{\partial V}, \quad D_V = \frac{\partial D}{\partial V}$$

- What would be $T - D$?

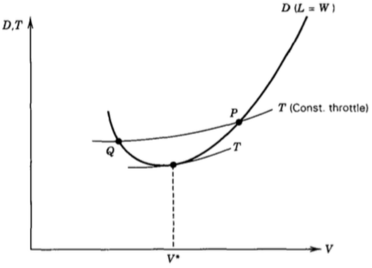
$$T - D = (T_0 + T_V \Delta V) - (D_0 + D_V \Delta V)$$

- Assuming $V = V_0 + \Delta V$,

$$m\Delta \dot{V} = (T_V - D_V)\Delta V$$

- Solution:

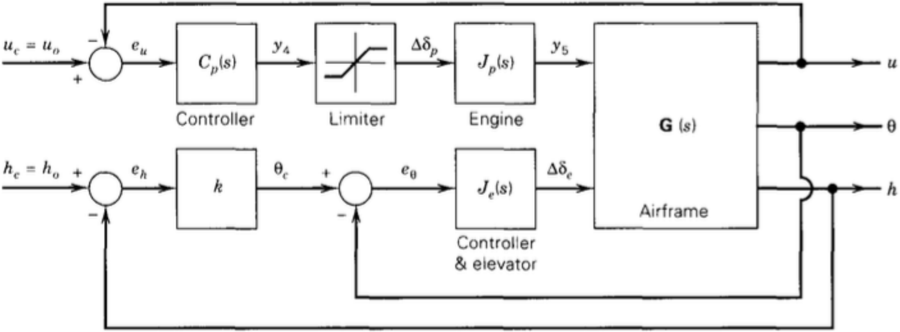
$$\Delta V = a e^{\lambda t}, \ \lambda = \frac{T_V - D_V}{m}$$



$V^{\star}$: Point where thrust and drag curves are tangent to each other.

- If the intersection is at a point P, then $T_V < D_V$, $\lambda < 0$, and the motion is stable.
- At a point like Q, $\lambda > 0$ and the motion is unstable.
- With an initial error in the speed at Q, it will either increase and reach P or decrease until airplane stalls.
- Similar results hold for other straight-line flight paths, climbing or descending.
- For $V < V^{\star}$, it is not possible for aircraft to follow a straight-line flight path, and provide stability, using elevator control alone.

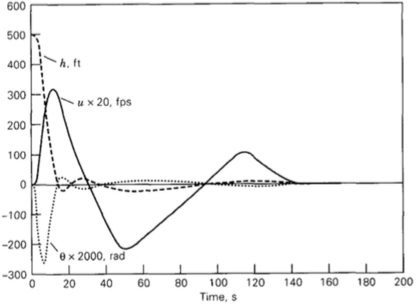
- Realistic system model: Addition of first order lag elements for control inputs ($\tau = 0.1,\ 3.5$ seconds in elevator and throttle dynamics)
- Addition of thrust limiter
- As limiter is nonlinear element, transfer function can not account for it.
- Approach using differential equation

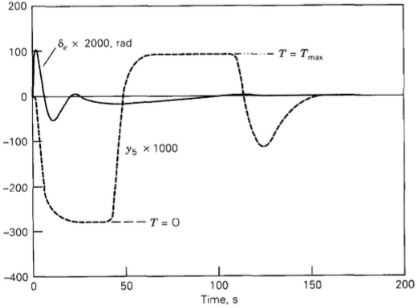


- Commanded speed and altitude: u_0, h_0
- Inner loop for θ remains same as before, with adjustment for system lag
- For initial error in h, say low altitude, γ must be deflected upward.
- Need to increase angle of attack for producing more lift, $\alpha_c = F(\Delta h)$.
- As short-term changes in θ are effectively changes in α, θ_c may also serves the purpose.
- In steady state $\Delta h = 0 \implies \theta = 0$.
- System commands $\theta_c \propto \Delta h$ and the inner loop uses δ_e to make $\theta \to \theta_c$.
- During the process, V changes due to both gravity and drag.
- V can be quickly controlled by using δ_p.

$$\boxed{J_e(s) = \left(\frac{a_0}{s} + a_1 + a_2 s\right)\frac{1}{1+\tau_e s}, \; C_p(s) = \frac{b_0}{s} + b_1 + b_2 s, \; J_p(s) = \frac{1}{1+\tau_p s}}$$

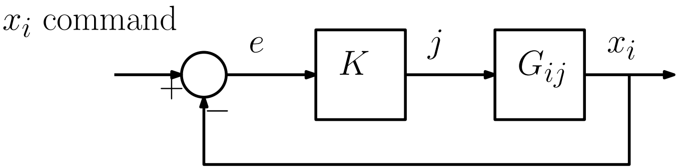
$$\boxed{a_0 = a_1 = a_2 = -0.5, \quad b_0 = 0.005, b_1 = 0.08, b_2 = 0.16}$$



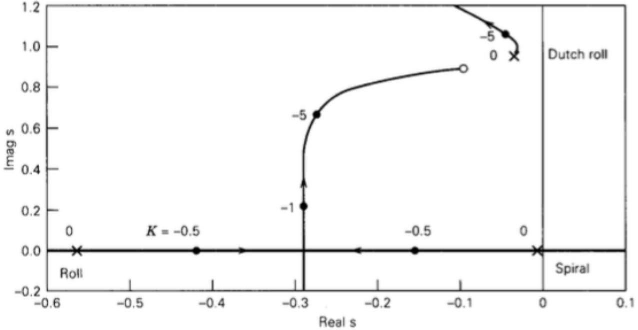


- $h \to 0$ in about 20 s, with $\theta_{\max} = 7^\circ$.
- Speed recovers in around 2 min to reference value.
- $\delta_{e_{\max}} < 3^\circ$, but the thrust drops quickly to zero, stays there for about 30 s, then increases rapidly to its maximum.
- Toward the end of the maneuver the throttle behaves linearly and reduces the speed error smoothly to zero.

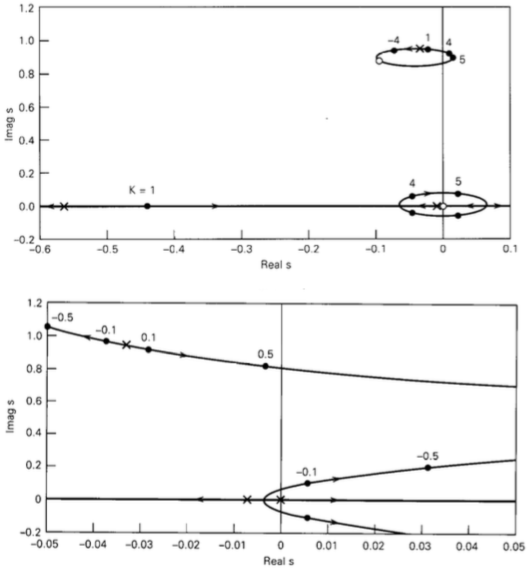
- **Lateral control**: Sources of feedback signals, v, p, r, ϕ, ψ and lateral acceleration
- Control inputs: Aileron and rudder deflections
- Synthetic modification of the inherent stability derivatives
- Analytical study using approximate transfer functions
- Closed-loop system with negative feedback
- Effect of feedback on system dynamics can be observed using root locus.



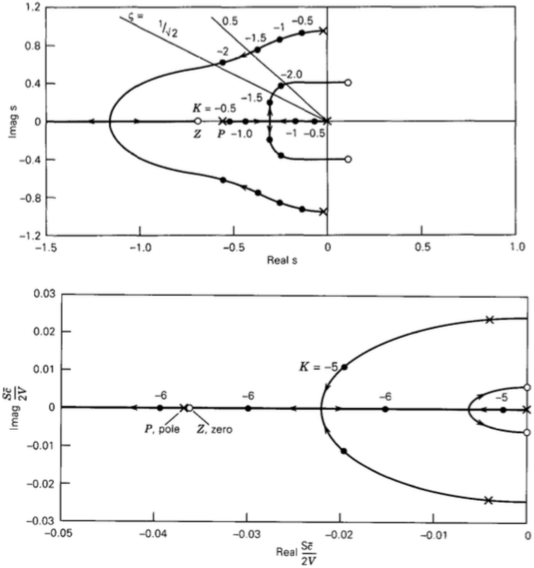
- Inherent aerodynamic rotational stiffness in pitch and yaw, but not in roll.
- Sideslip is necessary to level the wings after an initial roll upset.
- Remedy by adding the synthetic derivative, $L_\phi = L_{\delta_a} \frac{d\delta_a}{d\phi} = -KL_{\delta_a}$



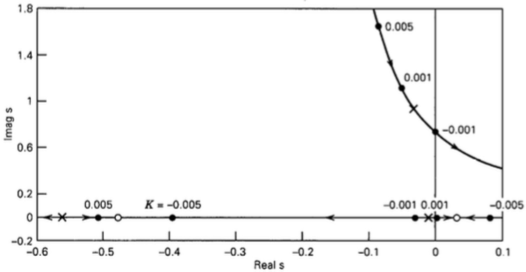
- Nonperiodic modes $\to$ low frequency, heavily damped oscillation.
- DR remains virtually unaffected.

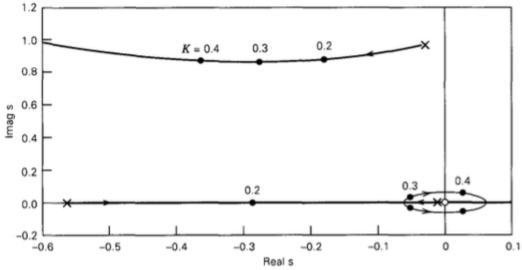


- $K > 0$ decreases λ_R, increases λ_S, slight reduction in DR damping
- $K < 0$ has opposite effect.
- Additional pole at origin.
- Spiral root diverges to instability via oscillation for $K < 0$.
- Diverges for $K > 0$
- DR slightly affected

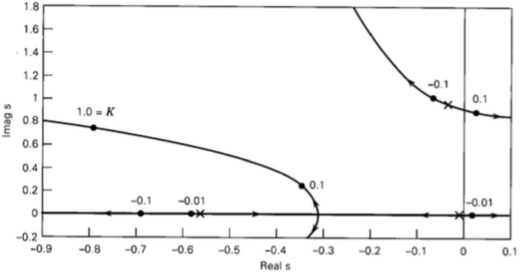


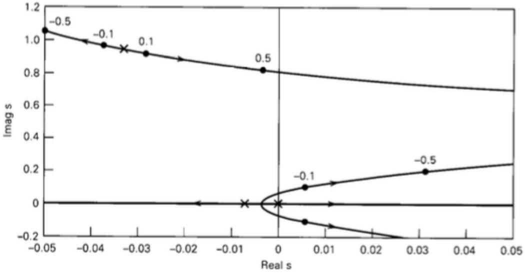
- $K < 0$ increases DR damping, with loss in roll damping.
- Higher negative gain makes real roots oscillatory and then unstable.
- Observe the effect of zero location





- $K > 0$ increases frequency of DR while simultaneously decreasing λ_S towards instability.
- $K < 0$ behaves oppositely.
- $K > 0$ increases DR and spiral dampings, but decrease roll damping
- Higher K leads to oscillation and then instability





- With $K < 0$, spiral mode diverges rapidly.
- With $K > 0$, spiral improves but DR adversely affected.
- With $K > 0$, DR is divergent.
- With $K < 0$, nonperiodic modes becomes unstable, via oscillations.

References

1. Bernard Etkin and Llyod Duff Reid, *Dynamics of Flight Stability and Control*, John Wiley and Sons, Third Edition, 1996.
2. Katsuhiko Ogata, *Modern Control Engineering*, third edition, Prentice Hall, Upper Saddle River, New Jersey 07458.
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Thank you for your attention !!!