Departure Concept

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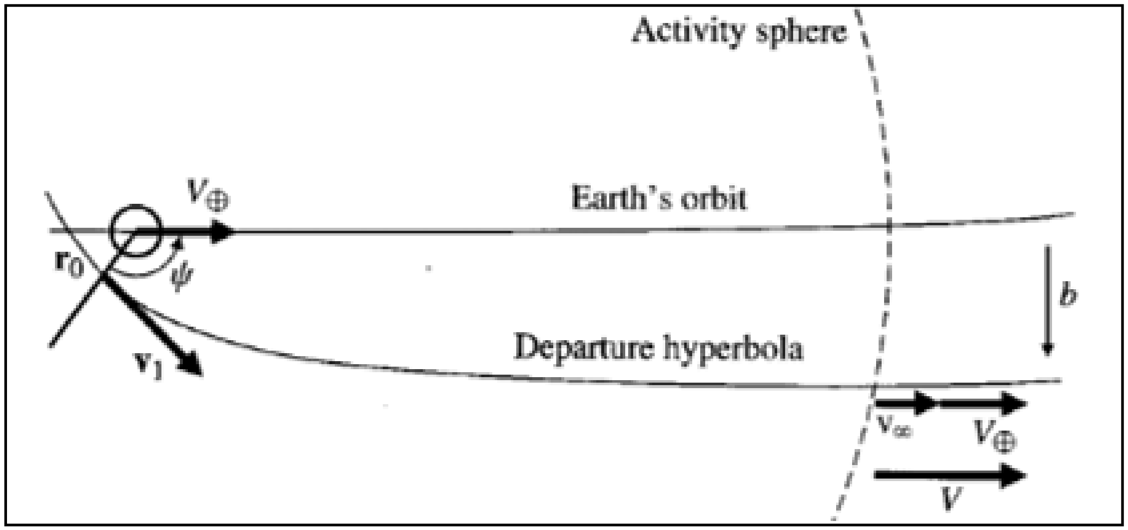
Consider the schematic of **departure** as given below.



Departure Features

'v_∞' is earth-centric **velocity** at ∞, (or the **edge of SOI**).

Departure from a planet needs a **change** of reference **frame** at the boundary of **SOI** (i.e. from v_∞ to V).

Departure Formulation

Beyond **SOI**, space object is considered **helio-centric** till it is **captured** by another planet's **gravitational** field.

However, we also know that the **object** must possess a minimum **velocity** to be in a **circular** orbit around **sun**

Departure Patch Condition

The **departure** patch condition is as **shown** below.

$$V_{cir\odot} = \sqrt{\frac{\mu_{\odot}}{R}}; \quad \vec{R} = \vec{R}_{\oplus} + \vec{r}$$

Thus, **'v_∞ + $V_\oplus$'** must be at least equal to '**V_{cirO}**' in order to **ensure** that the object does not **fall** on sun's surface.

Departure Relations

However, as we **need** to go to another **planet**, we need to set up a **transfer** mechanism which allows the **object** to travel to a specific **destination.**

In this context, we make use of the **Hohmann** transfer ellipse as a **means** of travel from SOI of **earth** to SOI of another **planet**, in the helio-centric **frame** of reference.

Departure Problem Definition

This is a **minimum 'Δv'** transfer mechanism which can be done by **ensuring** the requisite velocity on the **departing** hyperbolic asymptote.

Problem is setup by assuming that '$\mathbf{R}_{\oplus}$' is the perihelion and '$\mathbf{R}_{\otimes}$' is the aphelion of the **transfer** ellipse, where these are the **orbital** radii of earth and target **planet**.

Departure Solution

The **relations** for departure **hyperbola** are as follows.

$$a_{TO} = \frac{R_{\oplus} + R_{\otimes}}{2}; \quad V_{perihelion} = \sqrt{\left(\frac{2\mu_{\odot}}{R} - \frac{\mu_{\odot}}{a_{TO}}\right)} \rightarrow v_{\infty} = V_{perihelion} - V_{\oplus}$$

$$\varepsilon_{dep} = \frac{1}{2}v_{dep}^2 - \frac{\mu_{\oplus}}{r_{dep}} = \frac{1}{2}v_{\infty}^2; \quad v_{dep} = \sqrt{v_{\infty}^2 + \frac{2\mu_{\oplus}}{r_0}}; \quad a_{hyperbola} = -\frac{\mu_{\oplus}}{2\varepsilon}$$

$$e = 1 - \frac{r_{dep}}{a_{hyperbola}}; \quad \text{For } \theta = \psi \text{ at } r = \infty \rightarrow \psi = \cos^{-1}\frac{-1}{e}$$

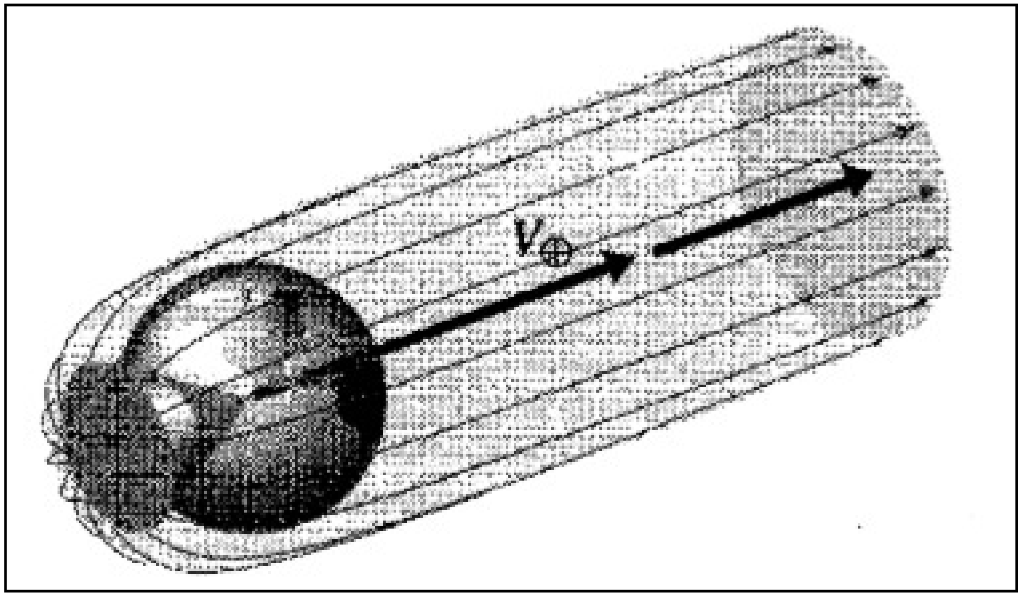
Departure Solution Features

The **departure** solution ensures that at $\mathbf{r} \approx \infty$ (r_{SOI}), the object **velocity** is parallel to the **earth's** orbital velocity.

As **departure** can be in three **dimensions**, we can have a **family** of hyperbolas as **departure** trajectories.

Departure Solution Features

Given below is a **3-d** view of all such possible **departure** trajectories from all **locations** on earth.



Arrival Concept

Arrival at a Planet

Once a **spacecraft** is put on a **heliocentric** Hohmann transfer ellipse, it **arrives** at target **planet** on this ellipse.

The first **point** of contact with **planet** is its **SOI** at which point, **spacecraft** comes under the influence of the **gravity** of the target **planet.**

Arrival at a Planet

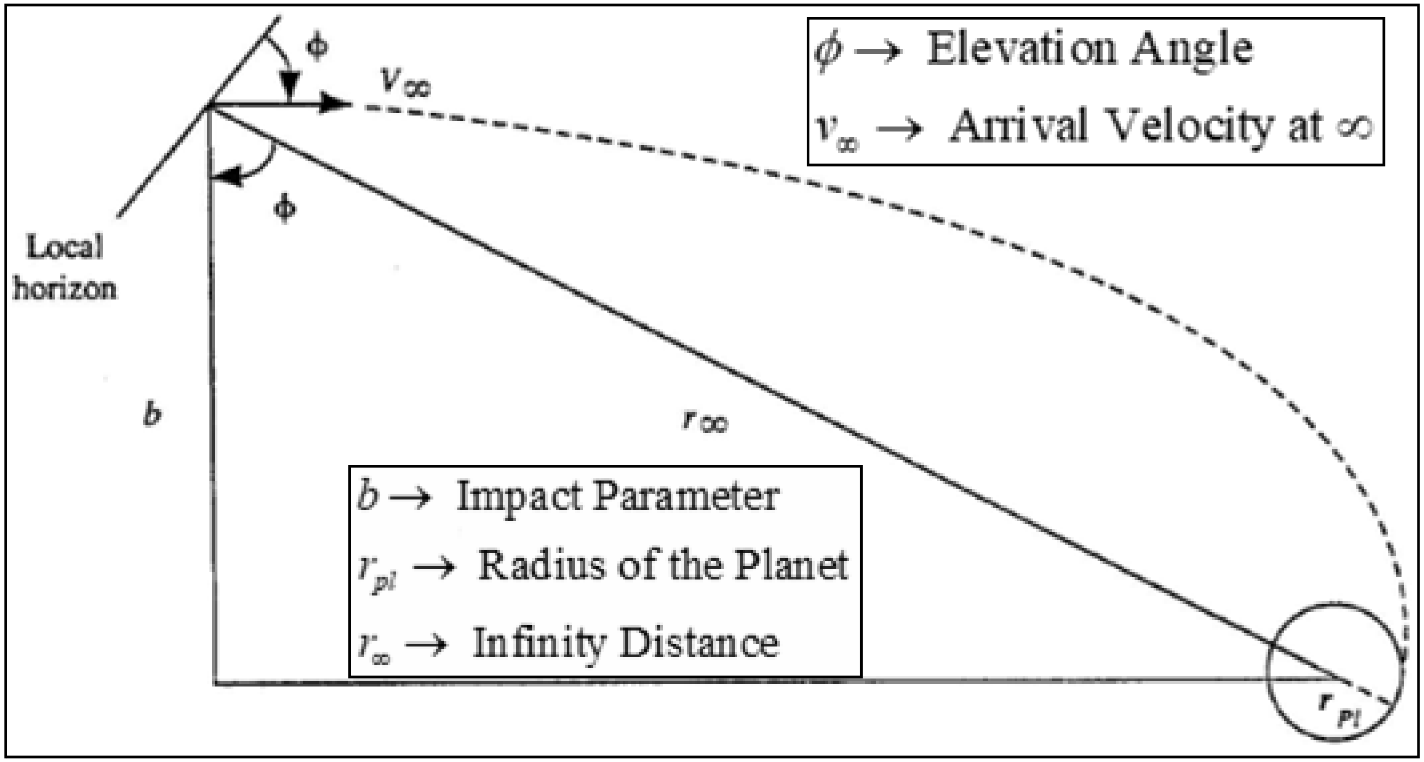
From this point onwards, **planeto-centric** analysis is required to **determine** whether the spacecraft has a **flyby**, forms an orbit or **impacts** its surface**.**

Usually, our **interest** is for a **flyby** or an orbit (called **capture**), so we first **examine** the impact **condition.**

Impact Concept & Analysis

Conditions for Impact on Planet

Consider the **case** when spacecraft just **grazes** planet's surface at its **periapsis**, as shown **below.**



Impact Parameter Solution

The **solution** for the impact parameter **'b'**, for the case of surface **graze,** is as follows.

$$\cos\phi = \frac{b}{r_\infty}; \quad h = v_\infty r_\infty \cos\phi = v_\infty b = v_{pl} r_{pl}$$

$$\frac{v_{pl}^2}{2} - \frac{\mu_\otimes}{r_p} = \frac{v_\infty^2}{2} \rightarrow v_{pl}^2 = \frac{2\mu_\otimes}{r_{pl}} + v_\infty^2; \quad v_{esc} = \sqrt{\frac{2\mu_\otimes}{r_{pl}}}$$

$$v_{pl} = \sqrt{v_{esc}^2 + v_\infty^2}; \quad b = r_\infty \cos\phi = \frac{h}{v_\infty} = r_{pl}\sqrt{1 + \frac{v_{esc}^2}{v_\infty^2}}$$

Stand-off Distance Concept

We see that **‘b’** is the **minimum** stand-off distance that is **permitted** at ∞ so that there is no **impact**.

However, spacecraft **approaches** a planet at a **distance** defined by its **SOI** and hence we need to define a **stand-off** distance with respect to the **SOI**.

Stand-off Distance Formulation

In this context, we **assume** that arrival at a planet would be with a **velocity 'v_∞'** with respect to the **planet** and calculate the actual **stand-off** distance, 'd' as follows.

$$\boxed{d = r_{SOI} \cos\phi}$$

Impact Condition Results

Following **relations** provide conditions for **impact** (or no impact).

$d > b \rightarrow$ there will be a flyby

$d = b \rightarrow$ there will be surface graze

$d < b \rightarrow$ there will be an impact

$$\lim_{V_\infty \to \infty} b = r_{pl}; \quad \lim_{V_\infty \to 0} b = \infty$$

Summary

To **conclude**, departure is defined as the **condition** at which the object becomes **helio-centric** and arrival is the condition at which the **object** becomes planeto-centric.

We **see** that stand-off distance plays a **crucial** role in deciding whether or not an **object** will impact the surface.