Tutorial 2

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Problem 1

Question

Consider an airplane patterned after the Fairchild Republic A-10, a twin-jet attack aircraft. The airplane has the following characteristics: wing area $=$ 47 m^2 , aspect ratio $= 6.5$, Oswald efficiency factor $= 0.87$, weight $=$ $103,047$ N, and parasite drag coefficient $= 0.032$. The airplane is equipped with two jet engines with $40,298$ N of static thrusts *each* at sea level.

- Calculate and plot the power-required curve at sea level.
- Calculate the maximum velocity at sea level.
- Calculate and plot the power-required curve at 5 km altitude.
- Calculate the maximum velocity at 5 km altitude. (Assume the engine thrust varies directly with freestream density.)

Problem 1
Solution - 1 (a)

At sea level $\rho_\infty = 1.225\text{kg}/\text{m}^3$

- Free stream velocity: $V_\infty = 100$ m/s
- Calculate dynamic pressure :

$$q_\infty = \frac{1}{2}\ \rho_\infty\ V_\infty^2 = 6125\ \mathrm{N}/\mathrm{m}^2$$

- Calculate coefficient of lift :

$$C_L = \frac{W}{q_\infty S} = 0.358$$

- Calculate total drag coefficient :

$$C_D = C_{D_0} + \frac{C_L^2}{\pi\ e\ A_R} = 0.0392$$

- Calculate thrust and power required :

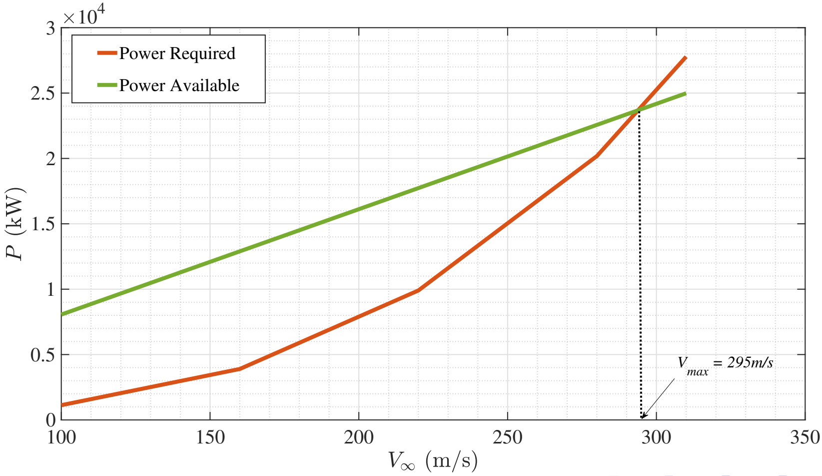
$$T_R = \frac{W}{C_L/C_D} = 11287 \text{ N} \qquad \text{and} \quad P_R = T_R \; V_\infty = 1.129 \times 10^6 \text{W}$$

V_∞ (m/s)	C_L	C_D	C_L/C_D	P_R (kW)
100	0.358	0.0392	9.13	1129
160	0.140	0.0331	4.23	3898
220	0.074	0.0323	2.29	9900
280	0.046	0.0321	1.43	20,180
310	0.037	0.0321	1.15	27,780

Table: Power required at sea level conditions

Problem 1
Solution - 1 (b)

Power available : $P_A = T_A \ V_\infty = 2(40298) \ V_\infty$



At altitude of 5 km, $\rho_\infty = 0.7364 \text{kg/m}^3$.
Hence,

- $\left(\dfrac{\rho_0}{\rho_{alt}}\right)^{1/2} = \left(\dfrac{1.225}{0.7364}\right)^{1/2} = 1.29$
- $V_{alt} = \left(\dfrac{\rho_0}{\rho_{alt}}\right)^{1/2} V_0 = 1.29 V_0$
- $P_{R_{alt}} = \left(\dfrac{\rho_0}{\rho_{alt}}\right)^{1/2} P_{R_0} = 1.29 P_{R_0}$

Problem 1

Solution - 1 (c)

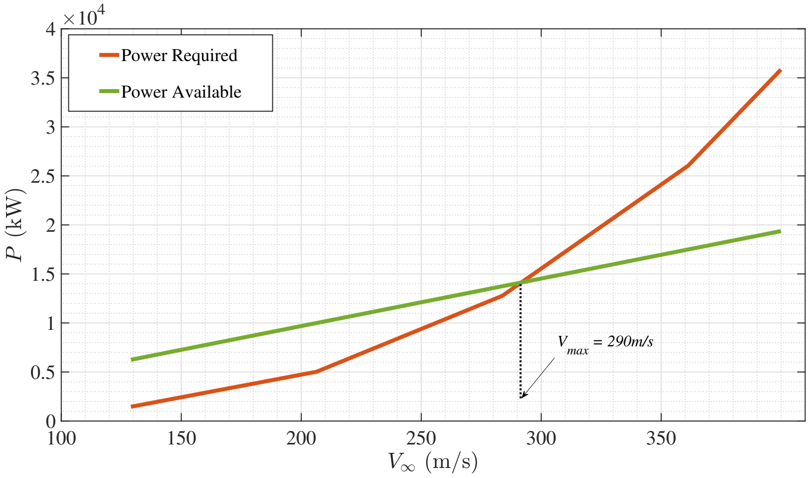
V_0 (m/s)	V_{alt}	P_{R_0}	$P_{R_{alt}}$
100	129	1129	1456
160	206	3898	5028
220	284	9900	12771
280	361	20,180	26032
310	400	27,780	35836

Table: Power required at an altitude of 5 km

- $T_{A_{alt}} = \frac{\rho_{alt}}{\rho_0} T_{A_0} = 0.601\, T_{A_0}$
- $P_{A_{alt}} = 0.601 \;\; T_{A_0} V_\infty = 48438\, V_\infty$

Problem 1

Solution - 1 (d)



Problem 2

Question

The following data apply to a $\frac{1}{25}$ scale wind tunnel model of a transport airplane. The full-scale mass of the aircraft is $22,680$ kg. Assume that the aerodynamic data can be applied at full-scale. For level unaccelerated flight at $V = 123$ m/s of the full-scale aircraft, under the assumption that propulsion effects can be ignored, find the limits on tail angle i_t, and CG position h imposed by the conditions $C_{m_0} > 0$ and $C_{m_\alpha} < 0$.

Geometric Data: Wing area, $S = 0.139$ m^2, Wing mean aerodynamic chord, $\bar{c} = 15.61$ cm, $\bar{l}_t = 38.84$ cm, Tail area $S_t = 0.0342$ m^2

Aerodynamic Data : $a_{wb} = 0.077$/deg , $a_t = 0.064$/deg, $\epsilon_0 = 0.72^\circ$, $\dfrac{\partial \epsilon}{\partial \alpha} = 0.30$, $C_{m_{ac_{wb}}} = -0.018$, $h_{n_{wb}} = 0.25$, $\rho = 1.225$kg/m^3

Problem 2
Solution - 2

- Ignoring propulsive effect,

$$C_{m_\alpha} = a\ (h - h_{wb}) - a_t\ \bar{V}_H\ \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

$$C_{m_0} = C_{m_{ac_{wb}}} + a_t\ \bar{V}_H\ (\epsilon_0 + i_t)\left[1 - \frac{a_t}{a}\frac{S_t}{S}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right]$$

- Calculating tail volume : $\bar{V}_H = \dfrac{l_t\ S_t}{\bar{c}\ S} = 0.6121$
- Lift slope : $a = a_{wb}\left[1 + \dfrac{a_t}{a_{wb}}\dfrac{S_t}{S}\left(1 - \dfrac{\partial \epsilon}{\partial \alpha}\right)\right] = 0.0889/\text{deg}$
- From $C_{m_\alpha} < 0$ we get $h < 0.5616$
- From $C_{m_0} > 0$ we get $i_t > -0.2344$

Problem 3

Question

A model of a wing-body shape is mounted in a wind tunnel having flow conditions in the test section equal to standard sea-level properties with a velocity of 100 $\mathrm{m/s}$. The wing area and chord are 2 m^2 and 0.5 m, respectively. The moment about the center of gravity when the lift is zero is found to be $-15\ \mathrm{N\cdot m}$. When the model is pitched to another angle of attack, the lift and moment about the center of gravity are measured to be 4125 N and 22.45 $\mathrm{N\cdot m}$, respectively. Calculate the value of the moment coefficient about the aerodynamic center and the location of the aerodynamic center.

- Dynamic Pressure : $q_\infty = \frac{1}{2}\ \rho_\infty\ V_\infty^2 = 6125\ \mathrm{N/m^2}$
- At zero lift, the moment coefficient about c.g. is,

$$C_{M_{cg}} = \frac{M}{q_\infty\ S\ c} = \frac{-15}{6125 \times 2 \times 0.5} = -0.00245$$

- However, at zero lift, this is also the value of the moment coefficient about the a.c. $C_{M_{cg}} = C_{M_{ac}} = -0.00245$
- At the other angle of attack,

$$C_L = \frac{L}{q_\infty\ S} = \frac{4125}{6125 \times 2} = 0.3367$$

- And

$$C_{M_{cg}} = \frac{M_{cg}}{q_\infty \ S \ c} = \frac{22.45}{6125 \times 2 \times 0.5} = 0.003665$$

- Also $C_{M_{cg}} = C_{M_{ac}} + C_L(h - h_{ac})$
- Thus,

$$h - h_{ac} = \frac{C_{M_{cg}} - C_{M_{ac}}}{C_L} = \frac{0.003665 + 0.00245}{0.3367} = 0.018$$

- The aerodynamic center is 1.8 percent of the chord length ahead of the center of gravity.

Problem 4

Question

Assume that a horizontal tail with no elevator is added to the wing body model in above problem. The distance from the airplane's center of gravity to the tail's aerodynamic center is 1 m. The area of the tail is 0.45 m^2, and the tail-setting angle is 1.5°. The lift slope of the tail is 0.1 per degree. From experimental measurement, $\epsilon_0 = 0$ and $\dfrac{\partial \epsilon}{\partial \alpha} = 0.4$.

1. If the absolute angle of attack of the model is 5° and the lift at this angle of attack is 4624 N, calculate the moment about the center of gravity.
2. Does this model have longitudinal stability and balance?
3. Calculate the neutral point and static margin for h $= 0.3$.

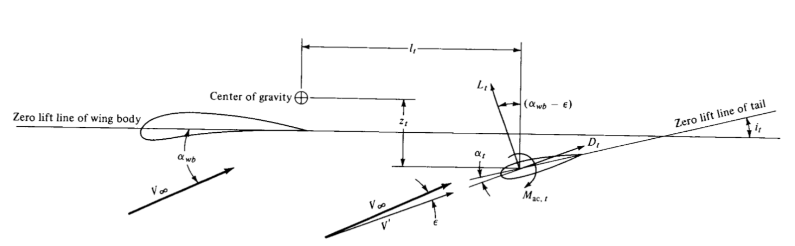


Figure: Geometry of wing-tail combination

- The sum of moments due to L_t, D_t and M_{ac_t} of the tail is,

$$M_{cg_t} = -l_t \left[L_t \cos(\alpha_{wb} - \epsilon) + D_t \sin(\alpha_{wb} - \epsilon)\right] + z_t \left[L_t \sin(\alpha_{wb} - \epsilon) - D_t L_t \cos(\alpha_{wb} - \epsilon)\right] + M_{ac_t}$$

$$M_{cg_t} = -l_t \; L_t = -l_t \; (C_{L_t} \; q_\infty \; S_t)$$

- Dividing above equation by $q_\infty Sc$ gives,

$$\frac{M_{cg_t}}{q_\infty Sc} = C_{M_{cg_t}} = \frac{l_t \ S_t}{c \ S} C_{L_t} = -V_H C_{L_t}$$

- $C_{M_{cg_t}}$ represents total contribution of the tail to moments about the airplane's center of gravity.
- If a_t denotes the lift slope of the tail,

$$C_{L_t} = a_t \alpha_t = a_t \left(\alpha_{wb} - i_t - \epsilon\right)$$

- The value of downwash ϵ is obtained as,

$$\epsilon = \epsilon_0 + \frac{\partial \epsilon}{\partial \alpha} \alpha_{wb}$$

where ϵ_0 is the downwash angle when the wing body combination is at zero lift.

- Hence, $C_{L_t} = a_t \alpha_{wb} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) - a_t(i_t + \epsilon_0)$
- Also, the pitching moment coefficient due to the tail about the c.g is : $C_{M_{cg_t}} = -V_H \left[a_t \alpha_{wb} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) - a_t(i_t + \epsilon_0)\right]$
- The total M_{cg} of the airplane as a whole is

$$C_{M_{cg}} = C_{M_{cg_{wb}}} + C_{M_{cg_t}} = C_{M_{ac_{wb}}} + C_{L_{wb}}(h - h_{ac_{wb}}) - V_H \; C_{L_t}$$

- Substituting the corresponding values,

$$C_{M_{cg}} = C_{M_{ac}} + a \; \alpha_a \; \left[(h - h_{ac}) - V_H \frac{a_t}{a} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right] + V_H \; a_t \; (i_t + \epsilon_0)$$

Equations for Longitudinal Static Stability

- For zero lift condition $\alpha_a = 0$. Thus,

$$C_{M_0} = C_{M_{cg}}{}_{L=0} = C_{M_{ac}} + V_H \ a_t \ (i_t + \epsilon_0)$$

- The slope of the moment coefficient curve is obtained by,

$$\frac{\partial C_{M_{cg}}}{\partial \alpha_a} = a \ \left[(h - h_{ac}) - V_H \frac{a_t}{a} \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right]$$

From Problem 3, we know $q_\infty = 6125\ \mathrm{N/m^2}$, $C_{M_{ac}} = -0.00245$, $h - h_{ac} = 0.18$

- Coefficient of lift can be obtained as:

$$C_L = \frac{L}{q_\infty\ S} = \frac{4624}{6125 \times 2} = 0.3775$$

- Absolute angle of attack $\alpha_a = 5^\circ$, hence,

$$a = \frac{dC_L}{d\alpha} = \frac{0.3775}{5} = 0.075/\mathrm{deg}$$

- Also,

$$V_H = \frac{l_t\ S_t}{\bar{c}\ S} = \frac{1 \times 0.45}{2 \times 0.5} = 0.45$$

Problem 4

Solution - 4 (a)

- The moment coefficient at c.g. is given by,

$$
\begin{aligned}
C_{M_{cg}} &= C_{M_{ac}} + a\ \alpha_a\ \left[(h - h_{ac}) - V_H \frac{a_t}{a}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right] + V_H\ a_t\ (i_t + \epsilon_0) \\
&= -0.00245 + 0.075 \times 5 \times \left[0.018 - 0.45 \frac{0.1}{0.075}(1 - 0.4)\right] \\
&\qquad + 0.45 \times 0.1 \times 1.5 \\
&= -0.0632
\end{aligned}
$$

- Hence, the moment is,

$$
\begin{aligned}
M_{cg} &= q_\infty S c C_{M_{cg}} \\
&= 6125 \times 2 \times 0.5 \times (-0.0632) \\
M_{cg} &= -387.1\ \mathrm{Nm}
\end{aligned}
$$

Problem 4

Solution - 4 (b)

- The slope of moment coefficient curve is given by,

$$\frac{\partial C_{M_{cg}}}{\partial \alpha_a} = a \left[(h - h_{ac}) - V_H \frac{a_t}{a} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \right]$$

$$= 0.075 \left[0.018 - 0.45 \frac{0.1}{0.075} (1 - 0.4) \right] = -0.02565 < 0$$

- Also,

$$C_{M_0} = C_{M_{ac}} + V_H \ a_t \ (i_t + \epsilon_0)$$

$$= -0.00245 + 0.45 \times 0.1 \times (1.5 + 0) = 0.06505 > 0$$

- Both the stability conditions are satisfied. Hence the aircraft model is statically stable.

Problem 4

Solution - 4 (b)

- To check if the model is balanced, calculate the trim angle of attack.
- Hence, from

$$C_{M_{cg}} = C_{M_0} + \frac{\partial C_{M_{cg}}}{\partial \alpha} \alpha_e = 0$$

$$\alpha_e = \frac{-C_{M_0}}{\partial C_{M_{cg}} / \partial \alpha_a} = \frac{-0.06505}{-0.02565} = 2.5360^\circ$$

- This is a reasonable angle of attack, falling within the normal flight range. Hence, the airplane model is also balanced.

Location of Neutral Point

- At neutral point $\dfrac{\partial C_{M_{cg}}}{\partial \alpha} = 0$
- Thus,

$$\frac{\partial C_{M_{cg}}}{\partial \alpha_a}_{\ (h=h_n)} = a \left[(h_n - h_{ac}) - V_H \frac{a_t}{a}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)\right] = 0 \implies$$

$$h_n = h_{ac} + V_H \frac{a_t}{a}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

- The difference between the location of neutral point and center of gravity of the airplane is defined as static margin.

$$\text{Static Margin} = h_n - h$$

Problem 4

Solution - 4 (c)

From Problem 3 solution, $h - h_{ac_{wb}} = 0.018$

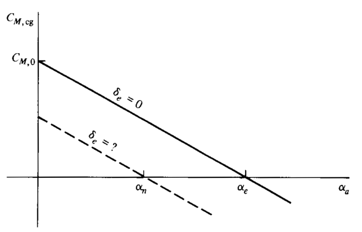
- Thus, $h_{ac_{wb}} = h - 0.018 = 0.3 - 0.018 = 0.282$
- Position of neutral point is given by

$$
\begin{aligned}
h_n &= h_{ac_{wb}} + V_H \frac{a_t}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) \\
&= 0.282 + 0.45\frac{0.1}{0.075}(1 - 0.4) \\
h_n &= 0.642
\end{aligned}
$$

- By definition static margin is given as,

$$h_n - h = 0.642 - 0.3 = 0.342$$

Elevator Angle of Trim



At $\delta_e = 0$,

$$C_{M_{cg}} = C_{M_0} + \frac{\partial C_{M_{cg}}}{\partial \alpha_a}\alpha_a$$

- Elevator deflection $= \delta_e$

$$C_{M_{cg}} = C_{M_0} + \frac{\partial C_{M_{cg}}}{\partial \alpha_a}\alpha_a + \Delta C_{M_{cg}}$$

$$C_{M_{cg}} = C_{M_0} + \frac{\partial C_{M_{cg}}}{\partial \alpha_a}\alpha_a - V_H \frac{\partial C_{M_{cg}}}{\partial \delta_e}\delta_e$$

- At $\alpha_a = \alpha_n$ and $\delta_e = \delta_{trim}$ we need $C_{M_{cg}} = 0$,

$$\delta_{trim} = \frac{C_{M_0} + (\partial C_{M_{cg}}/\partial \alpha_a)\alpha_n}{V_H\ (\partial C_{L_1}/\partial \delta_e)}$$

Problem 5

Question

Assume that an elevator is added to the horizontal tail of the configuration given in Problem 4. The elevator control effectiveness is 0.04. Calculate the elevator deflection angle necessary to trim the configuration at an angle of attack of 8°.

Solution - 5 :

- The elevator deflection angle is given by

$$\delta_{trim} = \frac{C_{M_0} + (\partial C_{M_{cg}}/\partial \alpha_a)\alpha_n}{V_H\ (\partial C_{L_1}/\partial \delta_e)}$$

$$= \frac{0.06505 + (-0.02565)(8)}{0.45(0.04)}$$

$$\delta_{trim} = -7.786^\circ$$

Thank You!