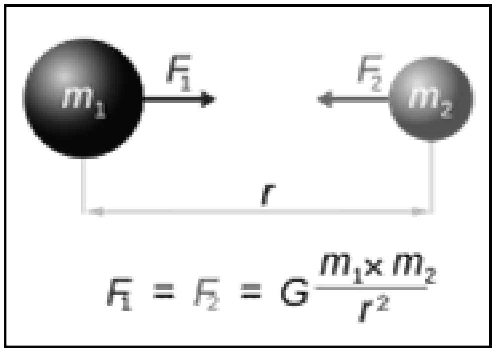
Gravitational Force Model

Earth's Gravity Model Basics

Earth's **gravity** model is based on the **universal** law of **gravitation**, which is defined as **follows**.



$$\vec{F}_g = -\frac{G m_1 m_2}{r^3}\vec{r} = -\frac{\mu m_2}{r^3}\vec{r};$$

$$\mu = G m_1; \quad \Phi_g = -\frac{\mu}{r};$$

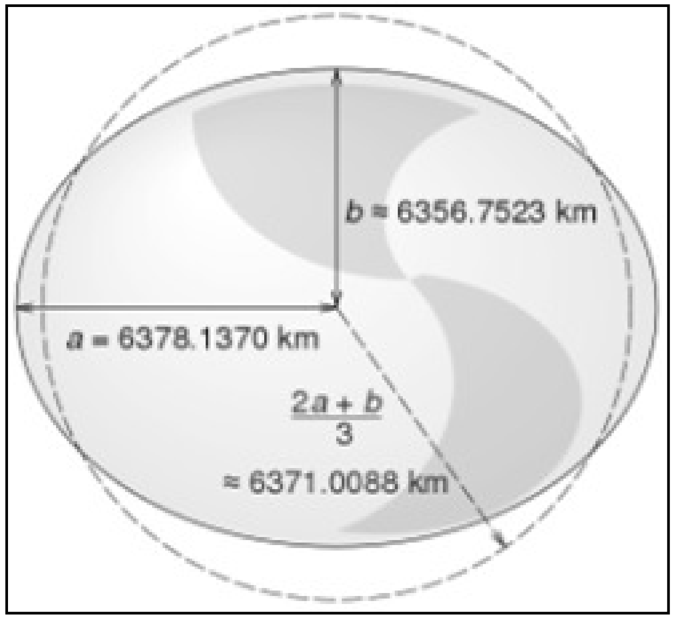
$$g = \mathrm{grad}(\Phi_g)$$

For **spherically** symmetric mass distribution, **acceleration** due to gravity **reduces** to,

$$\vec{g} = -\frac{\mu \vec{r}}{r^3}; \quad \text{for } r \geq r_0; \quad r_0 \rightarrow \text{Radius of the body}$$

WGS84 Gravitational Model

Of course, earth is not **spherical** and is generally modelled as an **ellipsoid**, as shown below.

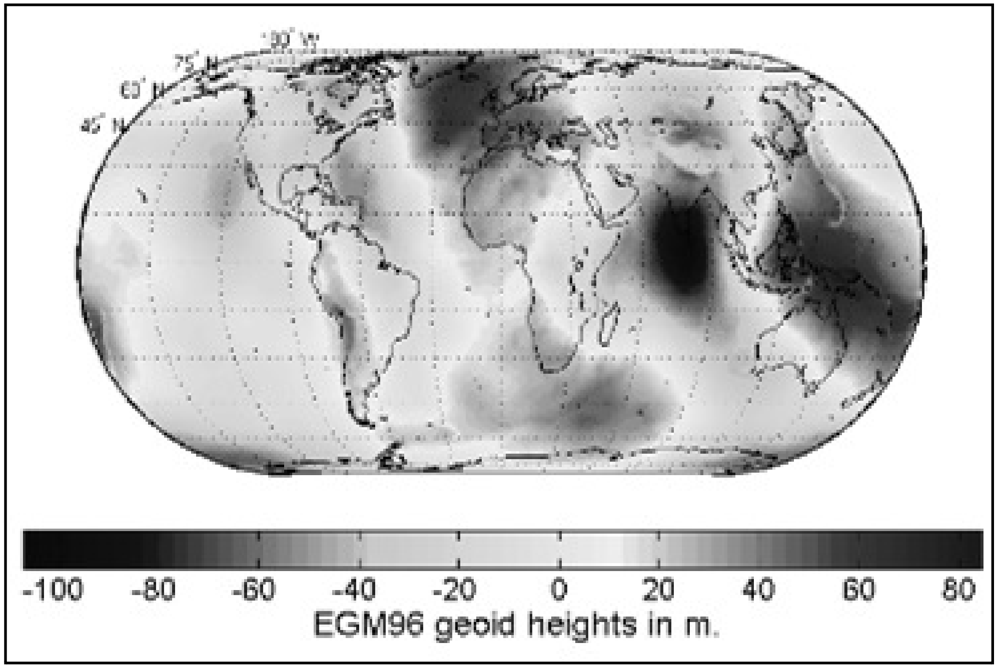


WGS84

Figures Courtesy Wikipedia

EGM96 Gravitational Model

In reality, **Earth** is not even an ellipsoid, but, a **Geoid**, or an equipotential **surface**, as shown below.



Latest is EGM2008

Figure Courtesy Wikipedia

Simplified Gravity Model

In the context of **ascent** mission, it is **possible** to use **spherical** symmetry for initial **design**.

$$a \approx G\frac{M}{r^2}; \quad G - \text{Gravitational Constant}; \quad R_E = 6371 km; \quad \mu = 3.986\times 10^{14} m^3 s^{-2}$$

M - Mass of the Earth; r - distance from Earth's Centre

$$a \approx G\frac{M}{(R_E+h)^2} \approx \frac{\mu}{R_E{}^2}\left(\frac{1}{1+(h/R_E)}\right)^2 \approx g_0\left(\frac{1}{1+(h/R_E)}\right)^2; \quad g_0 = 9.81$$

For h = 100 km, a = $0.97 g_0$; For h = 400 km, a = $0.88 g_0$

End point of ascent **mission** is usually **180 – 400 km.**

Aerodynamic Force Model

Aerodynamic Force Models

Most **missions** fly with nearly '0' **angle** of attack so that the only **aerodynamic** force of importance is **drag,** which represents an energy **loss** and needs to be modelled**.**

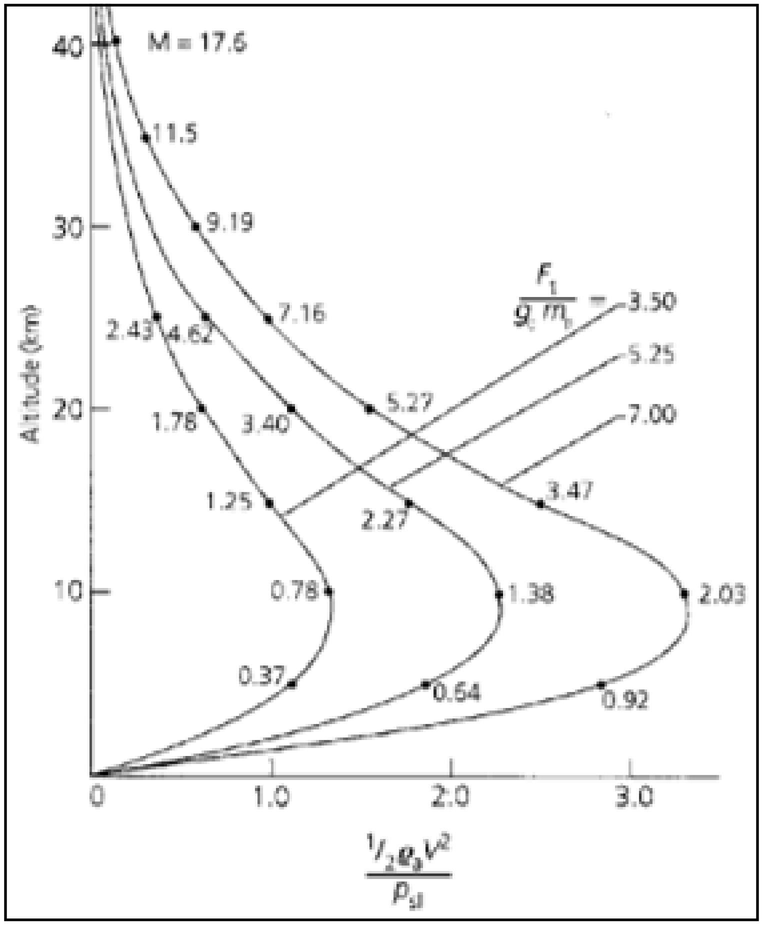
Normally rockets are **axi-symmetric** bluff bodies, and hence, experience (a) **wave** drag due to **normal** shocks and (b) **viscous** drag due to skin **friction**.

Drag Formulation

Drag, which depends on **dynamic** pressure, generally has a variation with **altitude**, as shown alongside.

Also, for **initial** rocket sizing, a **bluff** body value of **~ 1.0** for C_D is commonly **employed**.

We note that for **altitudes** beyond 40 km, **drag** effect on ascent **trajectory** is **negligible.**



Summary

Therefore, to **summarize**, similar to thrust, gravity and drag force **models** are also simplified **versions** that are considered to be **adequate** for modelling ascent **mission**.