Flight Mechanics/Dynamics

(Course Code: AE 305/305M/717)

Department of Aerospace Engineering

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Time: **120 Minutes**

Mid-Semester Examination

Total Points: 100

Instructions

- All questions are mandatory.
- In case a question is missing some data/information, assume the same suitably and clearly mention it in your answer sheet.
- You are only allowed to open lecture slides of the course, any other form of help/reference is not permitted.
- In cases where the answers of two students are found to be copied, both of them will be awarded zero marks for that particular question.
- Answer sheets need to be submitted in a single "Roll_Number.pdf" format on Moodle.
- You will get 15 minutes duration for submission of your answer sheet on Moodle after the exam time.

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1. Table 1 shows the variation of the pitching moment coefficient, C_m, about 0.67 chord ahead the trailing edge, with the lift coefficient C_l for a certain airfoil section. Obtain the location of aerodynamic center and the centre of pressure for $C_l = 0.5$, with respect to the leading edge of airfoil. [5+5]

C_l	0.20	0.40	0.60	0.80
C_m	-0.02	0	0.02	0.04

Table 1: C_m variation for different C_l values

Solution: The moment reference point (MRP) is given to be 0.67 chord length ahead of the trailing edge. This implies that the MRP is located at 0.33 chord length from the leading edge. The location of aerodynamic center w.r.t. moment reference point (0.33 from leading edge)

$$x_{ac} = -\frac{dC_m}{dC_l} = -\frac{0.04 - 0.02}{0.80 - 0.60} = -0.1.$$

Thus, the aerodynamic center is at 0.23 chord from the leading edge.

Furthermore, the location of center of pressure w.r.t. moment reference point (0.33 from leading edge) is given by

$$x_{cp} = -\frac{C_{m_0}}{C_l} - \frac{dC_m}{dC_l} = -\frac{-0.04}{0.5} - 0.1 = -0.02.$$

Thus, the center of pressure is at 0.31 chord from the leading edge.

2. Consider a wing-body configuration with lift coefficient, drag coefficient, zero-lift drag coefficient, Oswald's efficiency factor and the aspect ratio denoted by C_L, C_D, C_{D_0}, e and AR, respectively.

   (a) Show that the ratio $C_L^{3/2}/C_D$ is maximum when $C_{D_0} = (1/3)C_{D_i}$, where C_{D_i} is the induced drag coefficient.

   (b) Derive an expression for $\left[\frac{C_L^{3/2}}{C_D}\right]_{\max}$ in terms C_{D_0}, e and AR.

[10+5]

Solution:

(a) The ratio $C_L^{3/2}/C_D$ can be written as

$$\frac{C_L^{3/2}}{C_D} = \frac{C_L^{3/2}}{C_{D_0} + \dfrac{C_L^2}{\pi e AR}}.$$

Differentiating the above expression w.r.t to the lift coefficient and equating it to 0, we get the following

$$\frac{d}{dC_L}\left[\frac{C_L^{3/2}}{C_D}\right] = \frac{(3/2)C_L^{1/2}\left(C_{D_0} + \dfrac{C_L^2}{\pi e AR}\right) - \dfrac{2C_L^{5/2}}{\pi e AR}}{\left(C_{D_0} + \dfrac{C_L^2}{\pi e AR}\right)^2} = 0$$

$$\implies \frac{3}{2}\left(C_{D_0} + \frac{C_L^2}{\pi e AR}\right) = \frac{2C_L^2}{\pi e AR}$$

$$\implies C_{D_0} = \frac{1}{3}\frac{C_L^2}{\pi e AR} = \frac{1}{3}C_{D_i}$$

Hence, proved.

(b) From the previous result, we get

$$C_L = \sqrt{3C_{D_0}\pi eAR}.$$

Therefore, $\left[\frac{C_L^{3/2}}{C_D}\right]_{\max}$ is given by

$$\left[\frac{C_L^{3/2}}{C_D}\right]_{\max} = \frac{(3C_{D_0}\pi eAR)^{3/4}}{4C_{D_0}}$$

3. Consider an aircraft with canard-wing-tail configuration, with the setting angle for tail and canard surfaces are i_t and i_c, respectively. The aerodynamic center of the canard and tail are located at a distance of l_c and l_t from the CG of the aircraft. The canard and tail have a surface area of S_c and S_t, respectively. Assume the other necessary variables and mention the same. Derive expression for the location of the neutral point, by assuming the small angle of attack and neglecting the effects of downwash and upwash. [20]

Solution: The moment contribution of the canard will be in the clockwise sense expressed as

$$M_c = l_c L_c \implies C_{m_c} = \frac{l_c S_c}{S\bar{c}} C_{L_c} = V_{H_c} C_{L_c},$$

where V_{H_c} is the canard volume ratio. Now, we know that the moment about the CG for a wing-body-canard configuration is given by

$$C_m = C_{m_{ac,wb}} + C_{L_{wb}}(h - h_{n_{wb}}) - V_{H_t} C_{L_t} + C_{m_p}$$

where $C_{m_p} = C_{m_{p_0}} + \frac{\partial C_{m_p}}{d\alpha}\alpha$ and α is the absolute angle of attack experienced by the aircraft.

Adding the contribution of the canard, similar to canard, we get

$$C_m = C_{m_{ac,wb}} + C_{L_{wb}}(h - h_{n_{wb}}) - V_{H_t} C_{L_t} + V_{H_c} C_{L_c} + + C_{m_p}\,.$$

Now, substituting $C_{L_{wb}} = a_{wb}\alpha_{wb}$, $C_{L_c} = a_c\alpha_c$ and $C_{L_t} = a_t\alpha_t$ in the expression for C_m, we get

$$C_m = C_{m_{ac,wb}} + a_{wb}\alpha_{wb}(h - h_{n_{wb}}) - V_{H_t} a_t\alpha_t + V_{H_c} a_c\alpha_c + C_{m_p}\,.$$

Neglecting the down and upwash faced by the tail and canard, respectively, we can express the angle of attack faced by the canard and tail, in terms of their corresponding setting angles and the angle of attack of wing, as

$$\alpha_c = \alpha_{wb} + i_c\,,$$
$$\alpha_t = \alpha_{wb} - i_t\,.$$

 Question 3 continues on the next page...

Substituting for α_c and α_t in the expression for C_m, we get

$$\begin{aligned}
C_m &= C_{m_{ac,wb}} + a_{wb}\alpha_{wb}(h - h_{n_{wb}}) - V_{H_t}a_t(\alpha_{wb} - i_t) + V_{H_c}a_c(\alpha_{wb} + i_c)\,, \\
&= C_{m_{ac,wb}} + V_{H_t}a_t i_t + V_{H_c}a_c i_c + \left[a_{wb}(h - h_{wb}) - V_{H_t}a_t + V_{H_c}a_c\right]\alpha_{wb} + C_{m_p}\,, \\
&= \bar{C}_{m_0} + C_{m_\alpha}\alpha_{wb}\,,
\end{aligned}$$

where, $\bar{C}_{m_0}$ and C_{m_α} are given by

$$\begin{aligned}
\bar{C}_{m_0} &= C_{m_{ac,wb}} + V_{H_t}a_t i_t + V_{H_c}a_c i_c + C_{m_{p_0}} + (\alpha - \alpha_{wb})\frac{\partial C_{m_p}}{\partial \alpha} \\
C_{m_\alpha} &= a_{wb}(h - h_{wb}) - V_{H_t}a_t + V_{H_c}a_c + \frac{\partial C_{m_p}}{\partial \alpha}
\end{aligned}$$

If the location of the aerodynamic centers of the canard and tail from the wing aerodynamic center are denoted by $\bar{l}_c$ and $\bar{l}_t$, then we define the

$$\bar{V}_{H_c} = \frac{\bar{l}_c S_c}{S\bar{c}}, \quad \bar{V}_{H_c} = \frac{\bar{l}_t S_t}{S\bar{t}}$$

which leads to

$$V_{H_c} = \bar{V}_{H_c} + \frac{S_c}{S}(h - h_{wb}),\; V_{H_t} = \bar{V}_{H_t} - \frac{S_t}{S}(h - h_{wb})\,.$$

Furthermore, the lift coefficient of the entire aircraft is given by

$$\begin{aligned}
C_L &= C_{L_{wb}} + \frac{S_t}{S}C_{L_t} + \frac{S_c}{S}C_{L_c}\,, \\
&= a_{wb}\alpha_{wb} + \frac{S_c}{S}a_c\alpha_c + \frac{S_t}{S}a_t\alpha_t\,, \\
&= \underbrace{a_{wb}\left[1 + \frac{S_c a_c}{S a_{wb}} + \frac{S_t a_t}{S a_{wb}}\right]}_{a}\alpha_{wb} + \underbrace{\frac{S_c a_c}{S}i_c - \frac{S_t a_t}{S}i_t}_{C_{L_0}}
\end{aligned}$$

If the overall lift coefficient is expressed as $C_L = a\alpha$, where α is overall absolute angle of attack experienced by the aircraft. Therefore, we get a relationship between α and α_{wb} as

$$\alpha_{wb} = \alpha - \frac{S_c a_c}{Sa}i_c + \frac{S_t a_t}{Sa}i_t$$

Substituting for α_{wb}, V_{H_c} and V_{H_t} in the expression of C_m, we get

$$\begin{aligned}
C_m &= C_{m_{ac,wb}} + \bar{V}_{H_t}a_t i_t + \bar{V}_{H_c}a_c i_c + \left[a(h - h_{wb}) - \bar{V}_{H_t}a_t + \bar{V}_{H_c}a_c\right]\alpha + C_{m_p}\,, \\
&= C_{m_0} + C_{m_\alpha}\alpha
\end{aligned}$$

where C_{m_0} and C_{m_α} are given by

$$C_{m_0} = C_{m_{ac,wb}} + \bar{V}_{H_t} a_t i_t + \bar{V}_{H_c} a_c i_c + C_{m_{p0}} \,,$$
$$C_{m_\alpha} = a(h - h_{wb}) - \bar{V}_{H_t} a_t + \bar{V}_{H_c} a_c + \frac{\partial C_{m_p}}{\partial \alpha}$$

Now recall that the neutral point is defined as the point where the moment becomes independent of the angle of attack. In other words, at neutral point the coefficient C_{m_α} vanishes. Therefore, the neutral point for a canard-wing-tail configuration is given by

$$h_n = h_{wb} + \frac{\bar{V}_{H_t} a_t}{a} - \frac{\bar{V}_{H_c} a_c}{a} - \frac{1}{a}\frac{\partial C_{m_p}}{\partial \alpha}$$

OR

$$h_n = h_{wb} + \frac{V_{H_t} a_t}{a_{wb}} - \frac{V_{H_c} a_c}{a_{wb}} - \frac{1}{a_{wb}}\frac{\partial C_{m_p}}{\partial \alpha}$$

4. For the data shown in Fig. 1, find the stick fixed neutral point location. Also, if we wish to fly at a speed of 125 ft/s at sea-level then what would be the trim-lift coefficient and elevator angle to trim? Assume air density at sea-level to be 0.002377 slug/ft^3.

[5+10 Points]

Solution: The slope of given $C_m - C_L$ curve from Fig. 1, we get

$$\frac{dC_m}{dC_L} = -0.15.$$

The neutral point for the flying vehicle can be found by utilizing the relationship $C_{m_\alpha} = C_{l_\alpha}(h - h_n)$, where h_n is the neutral point for the aircraft. Simplifying this relationship, we get

$$\frac{C_{m_\alpha}}{C_{l_\alpha}} = \frac{dC_m}{dC_L} = -0.15 = h - h_n \implies h_n = h + 0.15 = 0.25 + 0.15 = 0.40.$$

For aircraft to trim at 125 ft/s, trim lift coefficient can be computed as

$$C_{L_{\text{trim}}} = \frac{2W}{\rho V^2 S} = \frac{2 \times 2750}{0.002377 \times 180 \times 125^2} = 0.82269.$$

The coefficient $C_{m_{\delta_e}}$ can be obtained using lines corresponding $\delta = -15^\circ$ and $\delta_e = 5^\circ$, we get

$$C_{m_{\delta_e}} = \frac{0.15}{-15 - 5} = -0.0075.$$

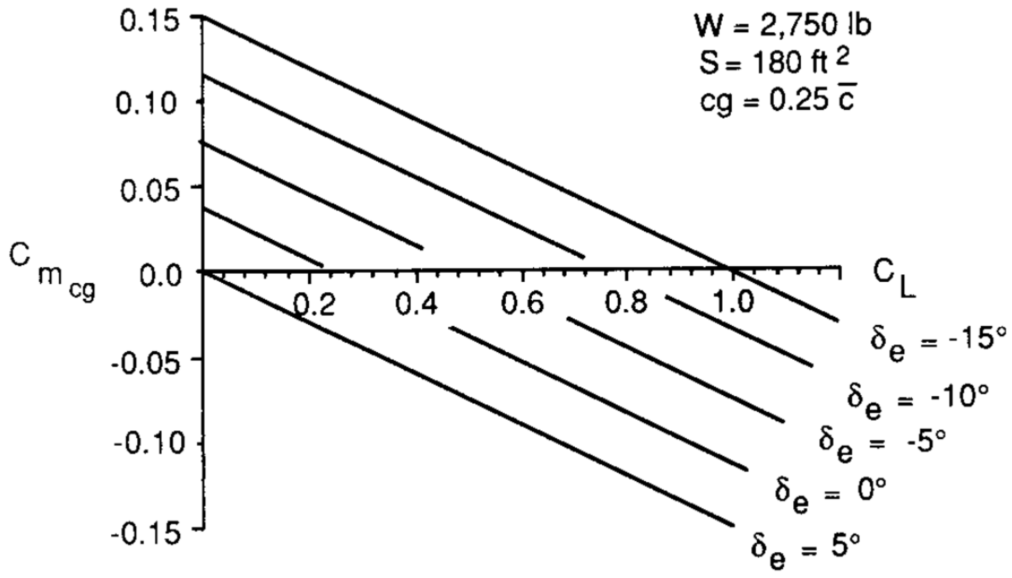

Figure 1: $C_{\mathrm{m_{cg}}}$ vs C_L curve.

Note that the moment coefficient can be written as

$$\begin{aligned} C_m &= C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e \\ &= C_{m_0} + \frac{dC_m}{dC_L}\frac{dC_L}{d\alpha}\alpha + C_{m_{\delta_e}}\delta_e \\ &= C_{m_0} + \frac{dC_m}{dC_L}C_L + C_{m_{\delta_e}}\delta_e. \end{aligned}$$

We can note from the Fig. 1 that for $\delta_e = 5°$, $C_m = 0$ for $C_L = 0$. Thus, substituting the same we get

$$C_{m_0} = -C_{m_{\delta_e}}\delta_e = -(-0.0075 \times 5) = 0.0375.$$

As the $C_m = 0$ at trim condition, the elevator angle to trim the aircraft at $C_{L_{\mathrm{trim}}}$ can be computed as,

$$\delta_{e_{\mathrm{trim}}} = -\frac{C_{m_0} + \dfrac{dC_m}{dC_L}C_{L_{\mathrm{trim}}}}{C_{m_{\delta_e}}} = -\frac{0.0375 - 0.15 \times 0.82269}{-0.0075} = -11.45°.$$

5. For a tailed aircraft with weight of 22700 N and wing area of 19 m^2 which is flying at a trim speed of 61 m/s at sea-level. The value of zero-lift moment coefficient is 0.06,

while the values of other parameters are as given below.

$$C_{L_\alpha} = 0.08/\text{deg},\ C_{L_{\delta_e}} = 0.016/\text{deg},\ C_{m_\alpha} = -0.0133/\text{deg},\ C_{m_{\delta_e}} = -0.0176/\text{deg}.$$

The coefficients related to the elevator hinge moment are given by

$$C_{he_0} = 0,\ C_{he_\alpha} = -0.003/\text{deg},\ C_{he_{\delta_e}} = \ -0.006/\text{deg}, C_{he_{\delta_t}} = -0.003/\text{deg}$$

(a) Calculate the trim tab deflection necessary to trim the aircraft such that the pilot does not need to apply any control force.

(b) If the trim tab deflection is $-5°$, compute the control force required by the pilot to maintain a trimmed flight? Assume that the value of $G = G_1 - G_2 = 5$ rad/m, where G_1 and G_2 denote elevator and boost gearings, respectively. The elevator chord length and surface area are $\bar{c}_e = 0.61$ m and $S_e = 3.72$ m^2, respectively.

[10+5 Points]

Solution: The trim value of lift coefficient is computed as

$$C_{L_{\text{trim}}} = \frac{2W}{\rho V^2 S} = \frac{2 \times 22700}{1.225 \times 61^2 \times 19} = 0.52$$

The trim values of angle of attack and elevator deflection can be computed as

$$\begin{aligned}
\alpha_{\text{trim}} &= \frac{C_{m_0} C_{L_{\delta_e}} + C_{m_{\delta_e}} C_{L_{\text{trim}}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}} \\
&= \frac{0.06 \times 0.016 - 0.0176 \times 0.52}{-0.08 \times 0.0176 + 0.0133 \times 0.016} = 6.8541°, \\
\delta_{e_{\text{trim}}} &= -\frac{C_{m_0} C_{L_\alpha} + C_{m_\alpha} C_{L_{\text{trim}}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}} \\
&= -\frac{0.06 \times 0.08 - 0.0133 \times 0.52}{-0.08 \times 0.0176 + 0.0133 \times 0.016} = -1.7704°
\end{aligned}$$

The trim tab deflection can be obtained as

$$\delta_{t_{\text{trim}}} = -\frac{C_{he_0} + C_{he_\alpha} \alpha_{\text{trim}} + C_{he_{\delta_e}} \delta_{e_{\text{trim}}}}{C_{he_{\delta_t}}} = -3.3133°$$

As the trim tab deflection is given as $-5°$, the hinge moment coefficient is computed as

$$\begin{aligned}
C_{h_e} &= b_3(\delta_t - \delta_{t_{\text{trim}}}) \\
&= C_{h_{\delta_t}}(\delta_t - \delta_{t_{\text{trim}}}) \\
&= -0.003 * (-5 - (-3.3133)) = 0.005061
\end{aligned}$$

The control force required by the pilot can be obtained as

$$\begin{aligned}
P &= GH_e = GC_{he} S_e \bar{c}_e q_\infty \\
&= 5 \times 0.005061 \times 0.61 \times 3.72 \times 0.5 \times 1.225 \times 61^2 = 130.871\,\text{N}
\end{aligned}$$

6. For a large jet transport airplane (wing body plus tail configuration), C_m vs α curve is given in Fig. 2. The lift coefficient for the airplane is given as $C_L = 0.03 + 0.08\alpha$, where α is in degrees. Consider the limits of elevator deflection as: $-25° \le \delta_e \le 20°$ and $\alpha_{\text{stall}} = 20°$, the CG is located at 0.29 chord (that is $h = 0.29$). Determine the location of neutral point, $C_{m_{\delta_e}}$, $C_{L_{\delta_e}}$, $\dfrac{d\delta_{e_{\text{trim}}}}{dC_{L_{\text{trim}}}}$, and most forward CG location.

   Hint: The forward most CG location corresponds to the trim at stall angle of attack and maximum up elevator deflection. [4+5+5+4+7 Points]

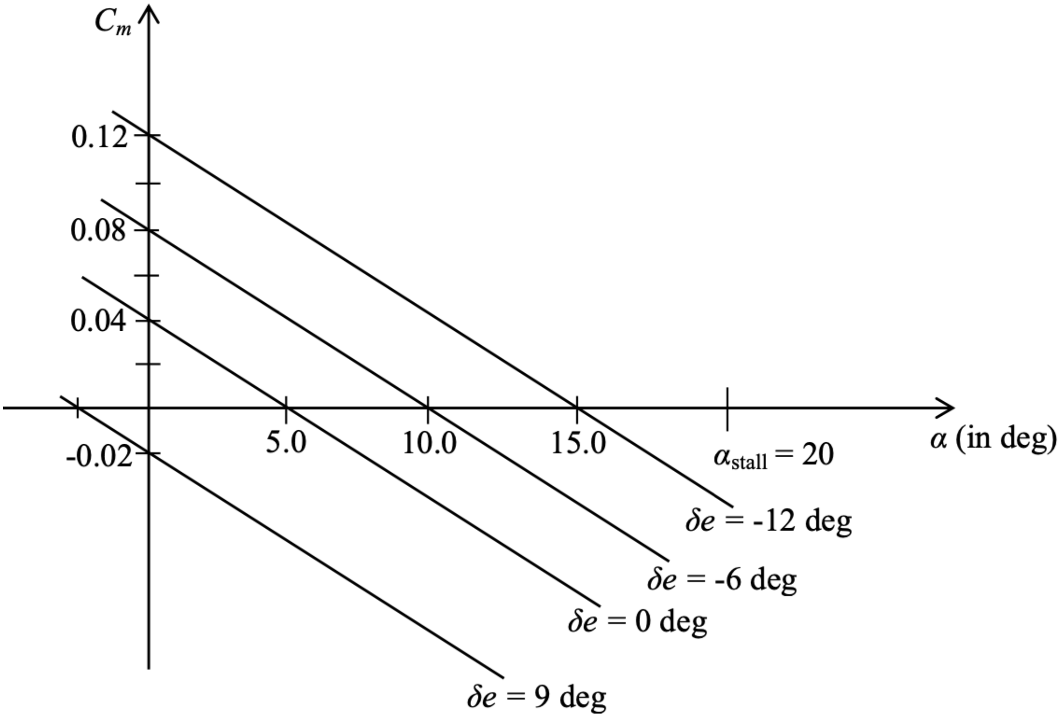

Figure 2: C_m vs α curve.

Solution: From $C_L = 0.03 + 0.08\alpha$, we can conclude that $C_{L_\alpha} = 0.08$. The CG is located at 29% of the chord from the leading edge.

From Fig. 2, we can compute C_{m_α} as

$$C_{m_\alpha} = -\frac{0.12}{15} = -0.008/°.$$

Therefore, we know that C_{m_α} can be written as

$$C_{m_\alpha} = C_{L_\alpha}(h - h_n) \implies h_n = h - \frac{C_{m_\alpha}}{C_{L_\alpha}} = 0.29 + 0.1 = 0.39$$

 Question 6 continues on the next page...

The coefficient $C_{m_{\delta_e}}$ can be computed, using lines corresponding to $\delta_e = -12°$ and $\delta_e = -6°$, as

$$C_{m_{\delta_e}} = \frac{0.12 - 0.08}{-12 - (-6)} = -0.0067/°.$$

Furthermore, the coefficient $C_{L_{\delta_e}}$ can be computed as follows

$$C_{L_{\delta_e}} = \frac{dC_L}{d\delta_e} = \frac{dC_L}{d\alpha} \times \frac{d\alpha}{dC_m} \times \frac{dC_m}{d\delta_e} = -\frac{0.08 \times 0.00667}{-0.008} = 0.066/°$$

The variation in elevator trim angle w.r.t to the trim lift coefficient is given by

$$\frac{d\delta_{e_{\text{trim}}}}{dC_{L_{\text{trim}}}} = -\frac{C_{m_\alpha}}{C_{L_\alpha}C_{m_{\delta_e}} - C_{m_\alpha}C_{L_{\delta_e}}} = \frac{0.008}{-0.08 \times 0.0067 + 0.008 \times 0.066} \approx -1000$$

Note that a large value of $\frac{d\delta_{e_{\text{trim}}}}{dC_{L_{\text{trim}}}}$ corresponds to the higher sensitivity of the trim elevator angle w.r.t to the trim lift coefficient. Such a configuration may not be desirable.

The forward-most CG location can be calculated, using the hint provided, as

$$0 = C_{m_0} + C_{L_\alpha}(h - h_n)\alpha_{\text{stall}} + C_{m_{\delta_e}}\delta_{e_{\text{up,max}}}$$

$$\implies h_{FMCG} = h_n - \frac{C_{m_0} + C_{m_{\delta_e}}\delta_{e_{\text{up,max}}}}{C_{L_\alpha}\alpha_{\text{stall}}} = 0.26$$