



# *Multi-stage Solution Methodology*



## *Multi-stage Solution Strategy*

It is possible to use  $\mathbf{m}_*$ ,  $\varepsilon_i$ , and  $\pi_i$ , to determine **stage configuration**,  $(m_{si}, m_{pi})$  and  $\mathbf{m}_0$ , as shown below.

$$\varepsilon_i = \frac{m_{si}}{m_{si} + m_{pi}}; \quad m_{si} + m_{pi} = \frac{1}{\varepsilon_i} m_{si}$$
$$\frac{1}{\pi_i} = \frac{m_{si} + m_{pi} + m_{0i+1}}{m_{0i+1}}; \quad m_{si} + m_{pi} = \left( \frac{1 - \pi_i}{\pi_i} \right) m_{0i+1}$$



## *Multi-stage Solution Steps*

These are **recursive** relations, and are **solved** as follows.

$$m_{zi} = \varepsilon_i m_{0i+1} \left( \frac{1 - \pi_i}{\pi_i} \right); \quad m_{pi} = (1 - \varepsilon_i) m_{0i+1} \left( \frac{1 - \pi_i}{\pi_i} \right)$$
$$m_0 = m_* + \sum_{i=1}^n (m_{zi} + m_{pi})$$

As  $m_*$  is known, **solution** proceeds from top **downwards**.



## *Design Exercise Features*

Thus, we note that a simple **algebraic** strategy is able to provide a **reasonable** configuration in the context of **multi-stage** rockets.

However, the **methodology** requires the values of the **stage** parameters, which are actually **unknowns** and are required to be **determined** from a separate **exercise**.



## ***Multi-stage Design Problem***

In this **context**, as has been **noted** earlier, rocket mass **configuration**, including its stages, is a **function** of payload ratios,  $\pi_i$ 's.

Further, **we** have also **seen** that different **values** of these ratios **result** in different **lift-off** mass as well as stage-wise masses, for **same** payload, propulsion and **structure**.



## *Multi-stage Design Strategy*

Further, from a **practical** view point, we **want** the mission **cost** to be as small as possible, without **affecting** safety and accuracy.

Therefore, the **design** task is to choose among the **many** combinations, the one **which** results in the best **rocket**.

**Optimal** techniques aim to achieve this **objective**.



## *Summary*

Thus, to summarize, simple algebraic strategy presented here is able to provide a fairly realistic stage-wise configuration for multi-stage rockets.

However, **we** see that the configuration so **obtained** strongly depends on the **stage-wise** payload ratios, which are **generally** obtained from an optimization **procedure**.