Flight Mechanics/Dynamics

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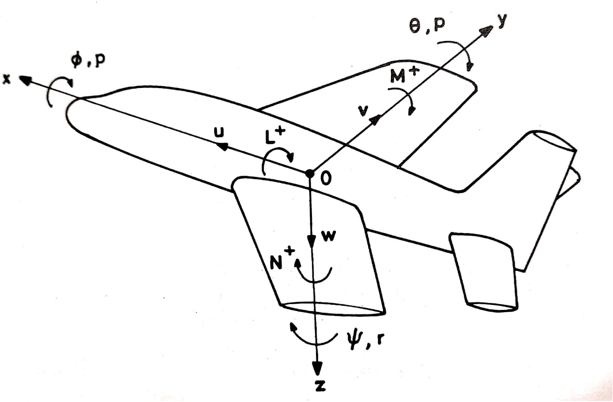
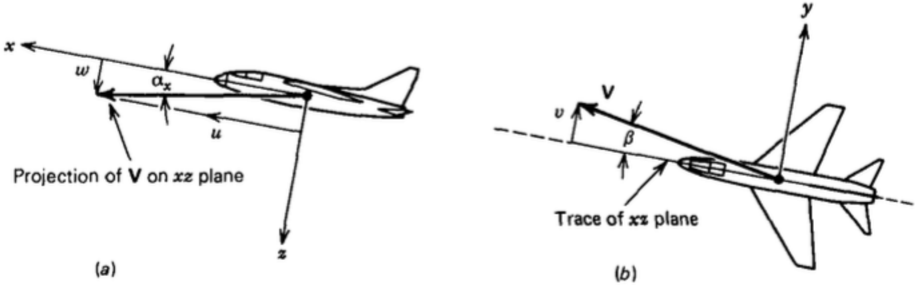
- In symmetrical configurations, aircraft fly with its velocity vector in the plane of symmetry.
- Only nonzero motion variables: V, α, and q
- Nonzero forces and moments: T, D, L, and M.
- In lateral motion, we do not have V in plane of symmetry
- Side force, rolling and yawing moments are also present.
- Longitudinal motion: Rotation about one axis, making rotational stiffness critical parameter for dynamic behavior.
- Lateral motions: Rotations about two axes which are coupled.
- Secondary trimming function: geometric or inertial asymmetries
- CG does not play important role for lateral motions as in longitudinal case.
- CG limits: governed by longitudinal motion

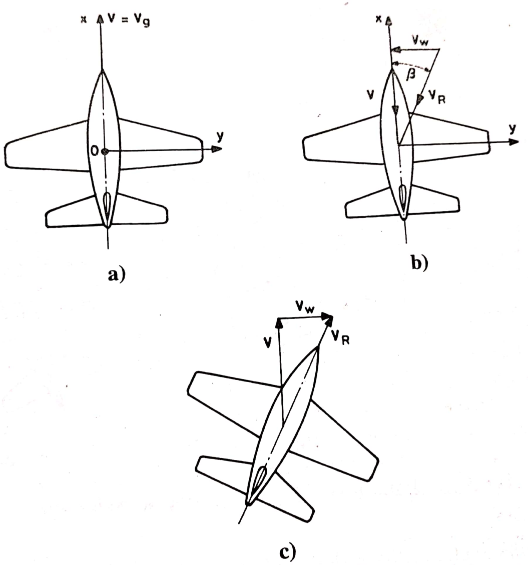


- Static directional stability: A measure of aircraft's ability to realign itself along direction of resultant wind.

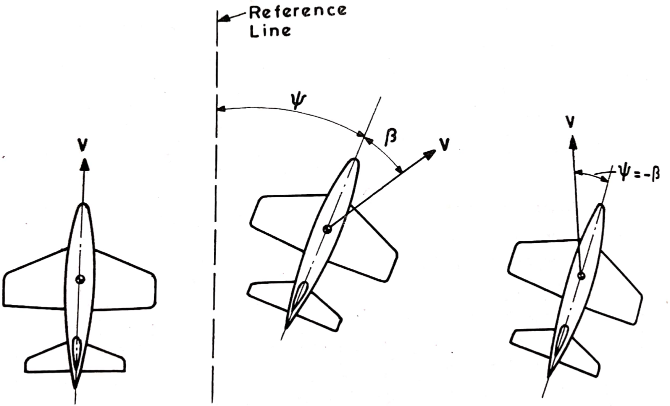


- Components of V: u, v, w
- Angle of attack and sideslip angle

$$\alpha_x = \tan^{-1}\left[\frac{w}{u}\right], \quad \beta = \sin^{-1}\left[\frac{v}{V}\right]$$

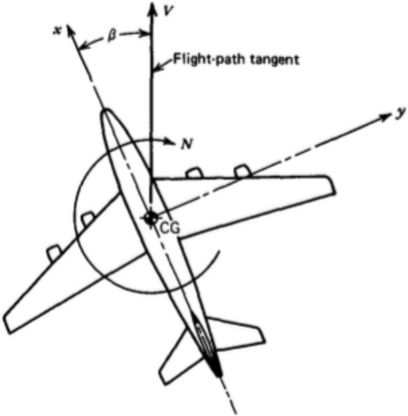


- Aircraft in steady level flight
- Disturbance (horizontal gust) v_w from right wing side
- Realign to have zero sideslip but orientation in space changed
- Aircraft orientation changed but heading remains same.
- Orientation restored if disturbance vanishes



a) b) c)

- Yaw angle: Angle between airplane's plane of symmetry and reference plane fixed in space. (direction of motion unchanged but yawed)



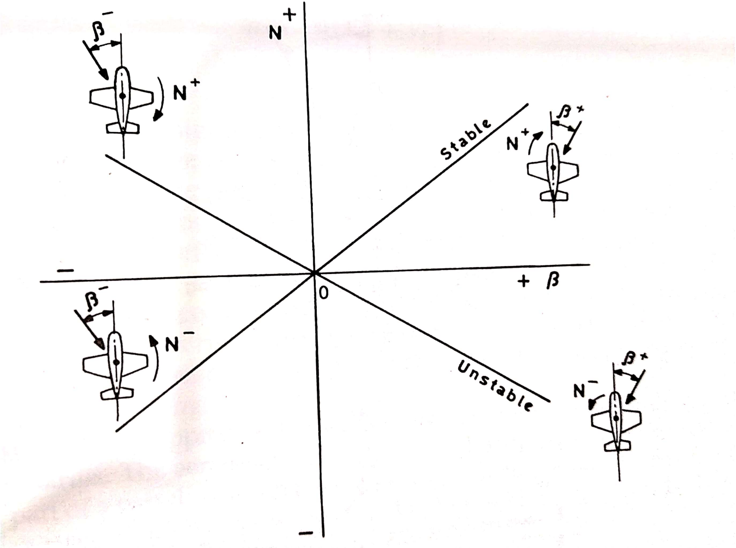
- Yaw stiffness: When airplane is at an angle of sideslip β relative to its flight path, yawing moment should be such as to tend to restore it.
- Required for static stability about z-axis.
- N is positive clockwise.
- For positive stiffness, what should be the sign of $\dfrac{\partial N}{\partial \beta}$?

$$\frac{\partial N}{\partial \beta} > 0$$

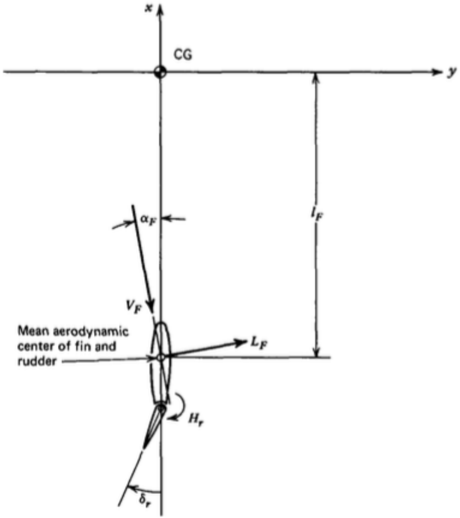
- Moment coefficient corresponding to yawing moment N

$$\boxed{C_n = \frac{N}{(1/2)\rho V^2 S b}}$$

- For positive stiffness, $\boxed{C_{n_\beta} = \frac{\partial C_n}{\partial \beta} > 0}$
- Similar to C_{m_α} for longitudinal case.
- Estimation in similar way of C_{m_α}, component-wise
- Negligible influence of wing and CG position
- Actual stability is decided by dynamic analysis.


N+
β−
N+
Stable
β+
N+
−
+ β
0
β−
N−
Unstable
β+
N−
−

- If tail is alone in an airstream, V_F equals to free stream velocity and $\alpha_F = -\beta$.
- On an airplane, both magnitude and direction of local flow at tail change.
- Reasons of change in flow
- Sidewash angle σ is induced similar to downwash earlier.
- $\sigma > 0$ when it corresponds to a flow in the y direction, i.e, when it tends to increase α_F.



- What should be the angle of attack for vertical tail surface?

$$\alpha_F = -\beta + \sigma$$

- Lift coefficient of vertical tail surface

$$C_{L_F} = a_F(\sigma - \beta) + a_r\delta_r \Rightarrow \boxed{L_F = \frac{1}{2}\rho V_F^2 S_F C_{L_F}}$$

where δ_r is rudder angle and a_F is lift-slope, respectively.

- Yawing moment and coefficient

$$N_F = -\frac{1}{2}\rho V_F^2 S_F C_{L_F} l_F \Rightarrow \boxed{C_{n_F} = -\left(\frac{V_F}{V}\right)^2 \underbrace{\frac{S_F l_F}{Sb}}_{V_V} C_{L_F}}$$

where V_V is vertical-tail volume ratio.

- Contribution to yaw stiffness

$$\frac{\partial C_{n_F}}{\partial \beta} = -V_V\left(\frac{V_F}{V}\right)^2 \frac{\partial C_{L_F}}{\partial \beta} = V_V a_F \left(\frac{V_F}{V}\right)^2 \left(1 - \frac{\partial \sigma}{\partial \beta}\right)$$

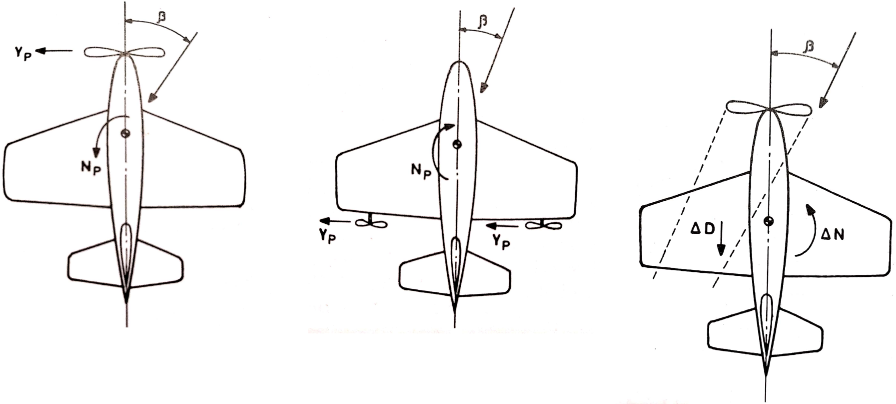
- When the vertical tail is not in a propeller slipstream, velocity ratio

$$\boxed{\frac{V_F}{V} = 1}$$

- When it is in a slipstream, effective velocity increment may be dealt with as for a horizontal tail.
- Yawing moment produced by normal force that acts on yawed propeller can be computed as earlier.
- Propeller fin effect

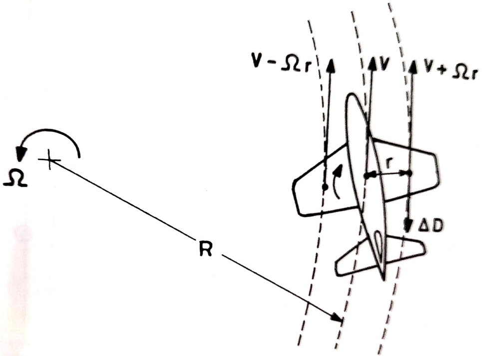
$$\boxed{\Delta \frac{\partial C_n}{\partial \beta} = -\frac{x_p}{b}\frac{S_p}{S}\frac{\partial C_{N_p}}{\partial \alpha_p}}$$

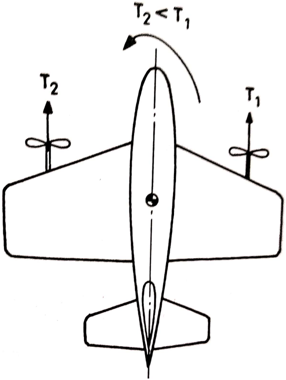
- It is negative (i.e., destabilizing) when the propeller is forward of the CG, but is usually positive for pusher propellers.



β
Y_P
N_P
β
N_P
Y_P
Y_P
β
ΔD
ΔN

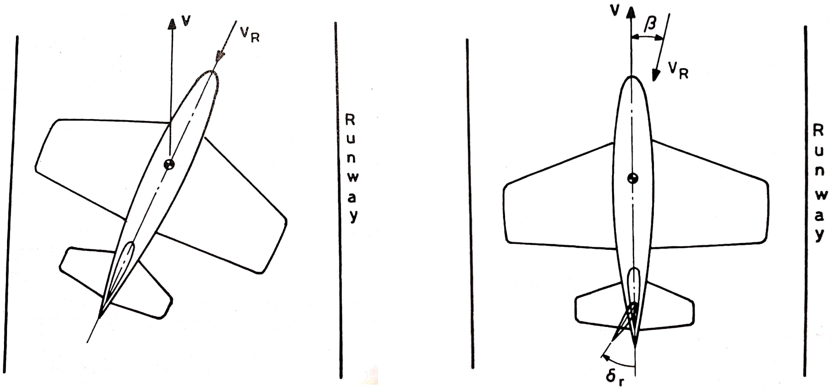
- In most flight conditions, it is desired to maintain the sideslip angle at zero.
- If the airplane has positive yaw stiffness, and is truly symmetrical, then it will tend to fly in this condition.
- Yawing moments in unsymmetrical flight
  - ⇒ Unsymmetrical thrust (e.g., one engine inoperative)
  - ⇒ Unsymmetrical flow field associated with turning flight.
- Zero sideslip using control moment with rudder
- Another condition for using rudder
  - ⇒ Steady sideslip
  - ⇒ A maneuver, particularly with light aircraft, to increase the drag and hence the glide path angle.
- Difference between rudder and elevator: trimming of airplane is a secondary, not a primary function of rudder.





Rudder must generate yawing moment to prevent adverse yaw

Rudder must generate enough moment to counter moment due to asymmetric thrust



- Aircraft with positive directional stability aligns to resultant wind
- To avoid safety issues, rudder must generate yawing moment to ensure zero sideslip with aircraft orientation along runway

- How to judge rudder control?
  - $\Rightarrow$ Rudder power
  - $\Rightarrow$ Steady β that can be maintained by a given rudder angle
- Rudder power: Rate of change of yawing moment with rudder deflection

$$\boxed{\frac{\partial C_n}{\partial \delta_r} = -V_V \left(\frac{V_F}{V}\right)^2 \frac{\partial C_{L_F}}{\partial \delta_r} = -a_r V_V \left(\frac{V_F}{V}\right)^2}$$

- Rudder power must be large enough to make it possible to maintain $\beta = 0$ under the most extreme conditions.
- Total yawing moment during steady sideslip

$$C_n = C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r$$

- For steady motion, the moment must be zero, $C_n = 0$, resulting in

$$\boxed{\beta = -\frac{C_{n_{\delta_r}}}{C_{n_\beta}}\delta_r}$$

- Rudder hinge-moment coefficient

$$C_{hr} = b_1\alpha_F + b_2\delta_r$$

- Rudder pedal force, with given rudder system gearing G,

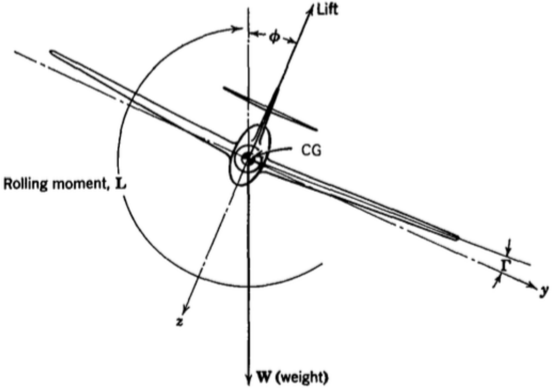
$$P = G\frac{1}{2}\rho V_F^2 S_r \bar{c}_r C_{hr} = \frac{1}{2}\rho V_F^2 G S_r \bar{c}_r (b_1 \alpha_F + b_2\delta_r)$$

- On substituting for α_F, $\boxed{P = G\frac{1}{2}\rho V_F^2 S_r c_r [b_1(\sigma - \beta) + b_2\delta_r]}$

- For free rudder, $C_{hr} = 0 \Rightarrow \boxed{\delta_{r_{\text{free}}} = -\frac{b_1}{b_2}\alpha_F}$

- Vertical-tail lift coefficient with free rudder

$$\boxed{C'_{LF} = a_F\alpha_F - a_r\frac{b_1}{b_2}\alpha_F = a_F\alpha_F\left(1 - \frac{a_r b_1}{a_F b_2}\right)}$$



- A vehicle constrained to rolling about the x-axis.
- Motion with fixed ϕ is different from that of rotations α, β.
- If x- axis coincides with V then no aerodynamic change.
- Symmetrical flow field w.r.t. plane of symmetry

$$\frac{\partial C_l}{\partial \phi} = C_{l_\phi} = 0$$

- Suppose x-axis does not coincide with V and assume angle of attack be α_x.
- Velocity vector with roll angle of 0 and ϕ be V_1 and V_2

$$V_1 = \begin{bmatrix} V\cos\alpha_x \\ 0 \\ V\sin\alpha_x \end{bmatrix}, \quad V_2 = R(\phi)V_1 = \begin{bmatrix} V\cos\alpha_x \\ V\sin\alpha_x\sin\phi \\ V\sin\alpha_x\cos\phi \end{bmatrix}$$

- Sideslip angle

$$\beta = \sin^{-1}\left[\frac{v}{V}\right] = \sin^{-1}\left(\sin\alpha_x\sin\phi\right)$$

- Rolling moment, with small angle assumptions, for α_x, ϕ

$$\begin{aligned} \Delta C_l &= C_{l_\beta}\beta = C_{l_\beta}\sin^{-1}\left(\sin\alpha_x\sin\phi\right) \\ &\approx C_{l_\beta}\sin^{-1}\left(\alpha_x\sin\phi\right) \approx C_{l_\beta}\alpha_x\sin\phi \\ &\approx C_{l_\beta}\alpha_x\phi \end{aligned}$$

- For roll stability, $C_{l_\beta} < 0$

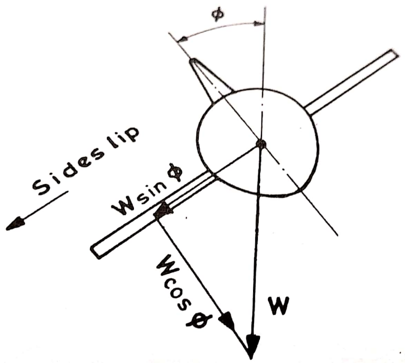
- Stiffness derivatives for rolling about x-axis

$$\frac{\partial C_l}{\partial \phi} = C_{l_\beta} \frac{\sin \alpha_x \cos \phi}{\sqrt{1 - \sin^2 \alpha_x \sin^2 \phi}}$$

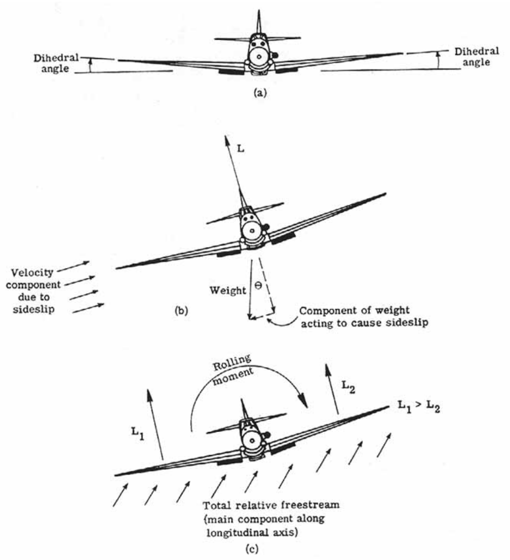
- With small angle assumptions for α_x, ϕ,

$$\boxed{C_{l_\phi} = \frac{\partial C_l}{\partial \phi} \approx \ C_{l_\beta} \alpha_x \cos \phi \approx C_{l_\beta} \alpha_x}$$

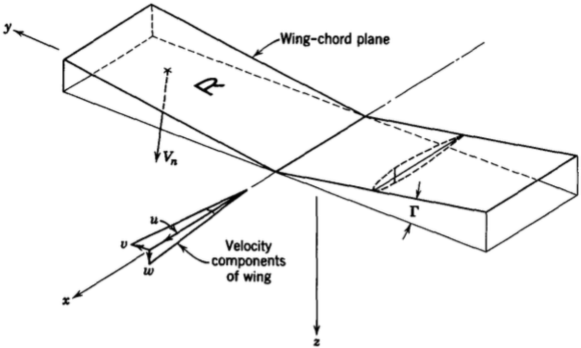
- What can you say about roll stiffness?
- Resistive roll stiffness if $\alpha_x > 0$ which tends to keep wing level.
- For $\alpha_x = 0$, zero stiffness and no preferred roll angle.
- If $\alpha_x < 0$, stiffness is negative.



- Control over rolling moment: Primarily with ailerons, secondary through sideslip angle
- Neutrally stable: Zero induced rolling moment
- Stable and unstable: Depending on nature of induced moment
- What do you mean by dihedral effect?
- Dihedral effect: generation of rolling moment due to sideslip
- Stable dihedral effect: Negative or restoring rolling moment for positive sideslip.



- Dihedral angle: Angle between plane of wing and horizontal plane.
- Positive if wing tip is above wing root.
- In dihedral wing, angle of attack during sideslip is different on both sides.
- Different lift forces on both sides.
- Restoring moment

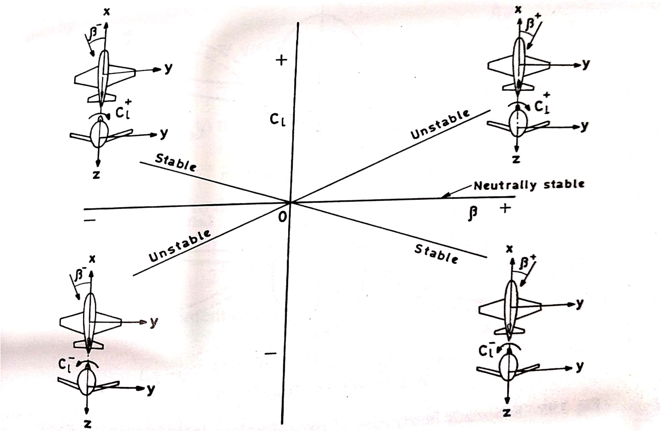


- Normal velocity component V_n

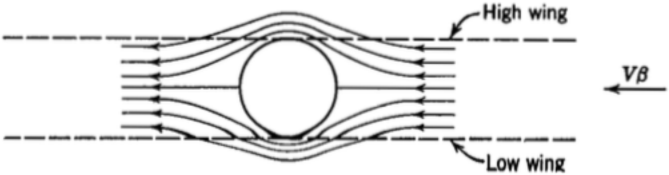
$$
\begin{aligned}
V_n|_{\text{right}} &= w\cos\Gamma + v\sin\Gamma \\
&\approx w + v\Gamma \\
V_n|_{\text{left}} &= w\cos\Gamma - v\sin\Gamma \\
&\approx w - v\Gamma
\end{aligned}
$$

- $\pm v\Gamma/V = \pm\beta\Gamma$
- Opposite change in angle of attack and lift of two panels
- Rolling moment linear in β, Γ
- Can we say C_{l_β} is fixed for given Γ?

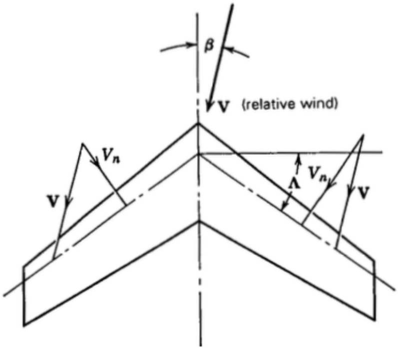
For a rectangular wing and constant lift-curve slope a_0, $C_{l_\beta} = -a_0\Gamma/4$.



Is there any effect of location of wing w.r.t. fuselage on lateral stability?



- Cross-flow component of the stream of magnitude $V\beta$
- Why high-wing airplanes usually have less wing dihedral than low-wing airplanes?
- For high wing, the angle-of-attack distribution is such as to produce a negative rolling moment, i.e, enhanced dihedral effect.
- For low wing, dihedral effect is diminished.
- Magnitude of the effect is dependent upon fuselage length ahead of wing, its cross-section shape, and planform and location of wing.

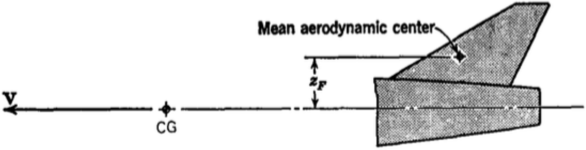


- According to simple sweep theory, $V_\perp$ to a wing reference line determines the lift.
- Lift is greater on right half than left half.
- A negative rolling moment

For small β, rolling moment $\propto$

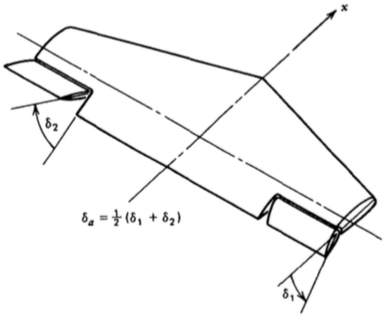
$$C_L \left[V_n^2|_{\text{right}} - V_n^2|_{\text{left}}\right] = C_L V^2 \left[\cos^2(\Lambda - \beta) - \cos^2(\Lambda + \beta)\right]$$
$$= 2C_L \beta V^2 \sin 2\Lambda$$

What would happen if wing is swept forward? Destabilizing effect



- Sideslipping airplane generate a side force on the vertical tail.
- When the mean ac of vertical surface is offset from rolling axis, rolling moment gets generated.
- With zero rudder angle, $C_{L_F} = a_F(\sigma - \beta) + a_r\delta_r = a_F(\sigma - \beta)$
- Moment generated would be $a_F(\sigma - \beta)\dfrac{1}{2}\rho V_F^2 S_F z_F$
- Moment coefficient

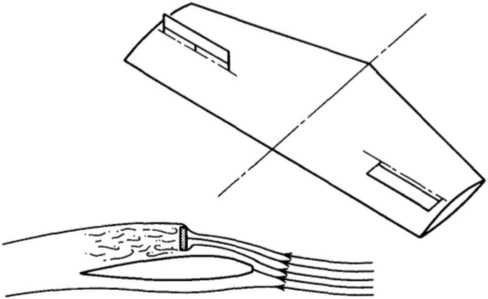
$$\Delta C_l = a_F(\sigma - \beta)\frac{S_F z_F}{Sb}\left(\frac{V_F}{V}\right)^2 \Rightarrow \Delta C_{l_\beta} = -a_F\left(1 - \frac{\partial \sigma}{\partial \beta}\right)\frac{S_F z_F}{Sb}\left(\frac{V_F}{V}\right)^2$$



Sign convention: right downward and left upward positive

- Roll motion is achieved by deflecting ailerons in opposite directions.
- Aileron angle deflection: average of deflections of two aileron surfaces
- What would be the moment direction when right aileron is deflected downward?
- Increase in lift on right and decrease in lift on left.
- Coefficient $C_{l_a} < 0$.
- What would be the effect on drag?
- Nose right yawing moment
- Any issue with such yawing moment?

- Aileron adverse yaw: Yawing moment opposite to the intended motion with aileron deflection.
- Difficulty in yaw control
- How to avoid such problems?
- By using spoilers, how?



Section through spoiler

What is the functional difference between elevator, rudder and aileron controls?

Ailerons are rate controls while other two are displacement controls

- Static stability is inherent stability (open loop stability) to counter a disturbance in angle of attack or sideslip.
- Stability w.r.t. angle of attack is longitudinal stability while directional and lateral stabilities were w.r.t. sideslip.
- CG location has significant effect on longitudinal stability while little effect on other two.
- For a safe flight, aircraft must be able to trim.
- Relations of these stabilities with different aspects such as CG location, effect of control surfaces, etc.
- For high performance, we often compromise on inherent stabilities, but to ensure safety, augmented control system is necessary.
- Controllability of aircraft inversely proportional to stability.
- Elevator is primary control for longitudinal while rudder and aileron are the same for yaw and roll.
- Pedal force must be within limit for a better feel to pilot.

Reference

1. John Anderson Jr., *Introduction to Flight*, McGraw-Hill Education, Sixth Edition, 2017.
2. Bernard Etkin and Llyod Duff Reid, *Dynamics of Flight Stability and Control*, John Wiley and Sons, Third Edition, 1996.

Thank you for your attention !!!