Indian Institute of Technology Bombay

Flight Mechanics/Dynamics

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Tutorial 5

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Objectives

- To have a linear systems perspective in studying aircraft flight dynamics.

Problem 1. Consider the linearized unforced longitudinal dynamics of an aircraft

$$\begin{bmatrix} \delta\dot{u} \\ \dot{w} \\ \dot{q} \\ \delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.0212 & 0.0466 & 0 & -32.174 \\ -0.2229 & -0.5839 & 262.472 & 0 \\ 0.0001 & -0.0018 & -0.5015 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \delta u \\ w \\ q \\ \delta\theta \end{bmatrix}.$$

(a) Find the characteristic equation governing the longitudinal dynamic stability.

(b) Determine the damping coefficients and natural frequencies of phugoid and short-period modes.

Solution. The above system can be written compactly as $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$.

(a) The characteristic polynomial for the above system can be computed as $\det\{\boldsymbol{A}-\lambda\boldsymbol{I}\} = 0$. Substituting for $\boldsymbol{A}$ and simplifying, we get

$$\lambda^4 + 1.1066\lambda^3 + 0.7987\lambda^2 + 0.0234\lambda + 0.0148 = 0$$

(b) Note that $E = 0.0148 > 0$ and $R = D(BC - AD) - B^2E = 0.002 > 0$, therefore, all the modes of the above dynamics are stable. The eigenvalues of the matrix $\boldsymbol{A}$ are

$$\lambda_{1,2} = -0.5518 \pm 0.6869i, \quad \lambda_{3,4} = -0.0015 \pm 0.1380i$$

As $\lambda_{1,2}$ is heavily damped, it corresponds to short period mode. Whereas, $\lambda_{3,4}$ correspond to phugoid mode. For a complex pair of eigenvalues $n \pm mi$, the natural frequency is $\omega_n = \sqrt{n^2 + m^2}$ and damping coefficient can be computed as $\zeta = |n|/\omega_n$. Using the eigenvalues derived above, ω_n and ζ are mentioned in Table 2.

	Damping coefficient	Natural frequency
Short period	0.6267	0.8810
Phugoid-mode	0.0108	0.1380

Table 1: damping coefficients and natural frequencies of phugoid and short-period modes

Problem 2. Consider the linearized unforced lateral dynamics of an aircraft

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{\phi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -0.0999 & 0 & 32.174 & -279.10 \\ -0.0057 & -1.0932 & 0 & 0.2850 \\ 0 & 1 & 0 & 0 \\ 0.0015 & -0.0395 & 0 & -0.2454 \end{bmatrix} \begin{bmatrix} v \\ p \\ \phi \\ r \end{bmatrix}.$$

(a) Find the characteristic equation governing the lateral dynamic stability.

(b) Determine the exponential decay rate for spiral and roll modes.

(c) Determine the damping coefficients and natural frequencies of dutch roll mode.

Solution. The above system can be written compactly as $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$.

(a) The characteristic polynomial for the above system can be computed as $\det\{\hat{\boldsymbol{A}} - \lambda \boldsymbol{I}\} = 0$. Substituting for $\hat{\boldsymbol{A}}$ and simplifying, we get

$$\lambda^4 + 1.4385\lambda^3 + 0.8319\lambda^2 + 0.7318\lambda + 0.0312 = 0$$

(b) Note that $E = 0.0312 > 0$ and $R = D(BC - AD) - B^2E = 0.2756 > 0$, therefore, all the modes of the above dynamics are stable. The eigenvalues of the matrix $\hat{\boldsymbol{A}}$ are

$$\lambda_1 = -1.2292, \quad \lambda_2 = -0.0448, \quad \lambda_{3,4} = -0.0822 \pm 0.7487i$$

The eigenvalues λ_1 and λ_2 correspond to rolling convergence and spiral modes, respectively. Whereas, $\lambda_{3,4}$ correspond to Dutch roll mode.

(c) For a complex pair of eigenvalues $n \pm mi$, the natural frequency is $\omega_n = \sqrt{n^2 + m^2}$ and damping coefficient can be computed as $\zeta = |n|/\omega_n$. Using the eigenvalues derived above, ω_n and ζ for the Dutch roll mode are mentioned in Table 2.

	Damping coefficient	Natural frequency
Dutch-roll	0.0109	0.7531

Table 2: damping coefficients and natural frequencies for Dutch-Roll mode

Problem 3. Consider a system defined by the following state and output equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -25 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Note that this system involves two inputs and two outputs.

(a) How many input-output transfer functions can be realized?

(b) Compute all the input-output transfer functions.

Solution. (a) Since there are two inputs and two outputs, four transfer functions may be realized.

(b) We respectively identify the system, the input, the output, and the feedforward matrices as

$$A = \begin{bmatrix} 0 & 1 \\ -25 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Since this is a multi-input multi-output system, we have a transfer matrix, $\mathbf{G}(s)$ whose entries are respective transfer functions, that is,

$$\mathbf{G}(s) = \begin{bmatrix} \dfrac{s+4}{s^2+4s+25} & -\dfrac{25}{s^2+4s+25} \\ \dfrac{s+5}{s^2+4s+25} & -\dfrac{s-25}{s^2+4s+25} \end{bmatrix}.$$

Note that $\mathbf{G}_{11}$ and $\mathbf{G}_{12}$ denote to the transfer functions between u_1 & y_1, & u_1 and y_2, respectively. Similarly, $\mathbf{G}_{21}$ and $\mathbf{G}_{22}$ respectively represent the transfer functions between u_2 & y_1, & u_2 and y_2.

Problem 4. The lateral-directional characteristic equation for the Douglas DC-8 aircraft in a low altitude cruise flight conditions is

$$\Delta(s) = s^4 + 1.326s^3 + 1.219s^2 + 1.09s - 0.015 = 0.$$

What can you say about the stability of the aeroplane?

Solution. The characteristic equations is given as

$$\Delta(s) = s^4 + 1.326s^3 + 1.219s^2 + 1.09s - 0.015 = 0.$$

On constructing the Routh-array to determine the number of sign changes in the first column of the Routh-array, one obtains

$$\begin{array}{c|rrr} s^4 & 1 & 1.219 & -0.015 \\ s^3 & 1.326 & 1.09 & 0 \\ s^2 & 0.3970 & -0.015 & 0 \\ s^1 & 1.1401 & 0 & 0 \\ s^0 & -0.015 & 0 & 0 \end{array}$$

From the above tabulation, one may infer that the system is unstable since there is one sign change in the first column of the Routh-array.

Alternately, from the method given in slides, one may compare the characteristic equation with $\Delta(s) = As^4 + Bs^3 + Cs^2 + Ds + E$. The conditions to check the stability of the system described by such a quartic characteristic polynomial are

$$\begin{aligned} A, B, D, E &> 0, \\ R = D(BC - AD) - B^2E &> 0. \end{aligned}$$

Clearly, $E < 0$, so the given system is unstable.