Idealized Ascent Mission Analysis

Ideal Burnout Performance Concept

Ideal burnout analysis is the **1st step** to capture primary **effect** due to propellant and is **obtained** under the force-free **assumption**.

Objective of this analysis is to **establish** payload mass **fraction**, for a specified space **mission** through the terminal **velocity** and altitude **requirements**.

Ideal Burnout Formulation & Solution

Basic **equations** for a force-free **motion** are as follows.

$$\frac{d\vec{V}}{dt} = -\frac{\dot{m}}{m} g_0 I_{sp} \hat{u}_V ; \quad \frac{d\vec{s}}{dt} = \vec{V}$$

The **applicable** solution is as given **below**.

$$\frac{dm}{m} = -\frac{dV}{g_0 I_{sp}} \rightarrow \ln m = -\frac{V}{g_0 I_{sp}} + C \rightarrow \frac{m_b}{m_0} = e^{-\frac{\Delta V_b}{g_0 I_{sp}}} ; \quad m_b = m_0 - m_p$$

$$V(t) = V_0 - g_0 I_{sp} (\ln m - \ln m_0) ; \quad s(t) = V_0 t - \int g_0 I_{sp} (\ln m - \ln m_0)\, dt + C$$

Ideal Burnout Features

Ideal burnout velocity is the **maximum** velocity that a rocket will **generate** from given $\mathbf{m_p}$ **&** $\mathbf{m_0}$**.**

Similarly, final **mass** fraction is also the **maximum** that a rocket can **provide** for a given velocity **increment.**

It is **interesting** to note that these **values** do not **depend** on the **way** the propellant is **burnt** (i.e. m(t)).

Ideal Burnout Solution Features

A **drawback** is that distance **solution** is a function of **m(t)** and hence, is **multi-valued.**

Lastly, we see that as **time** of flight is related to **m(t)**, it is also **multi-valued.**

Ideal Burnout Example

A **rocket** has the following **configuration**. m_0 = 80T, m_p = 60T, I_{sp} = 240s, g_0 = 9.81m/s^2. Determine ideal $\mathbf{V_b}$**.**

$V_b = g_0\, I_{sp} \ln\{m_0/(m_0 - m_p)\} = 9.81 \times 240 \times \ln(80/20)$

$= \mathbf{3.264\ km/s}$ $(m_b/m_0 = 0.25)$

What is $\mathbf{V_b}$ if burnout mass ratio i.e. (m_b/m_0) is **0.15?**

$V_b = 9.81 \times \ln(1/0.15) = \mathbf{4.46\ km/s}$

Ideal Burnout Solution Benefit

We note from earlier **discussion** that ideal burnout **performance**, which is a measure of **total** mechanical energy that can be **imparted**, is also related to rocket $\mathbf{m_0}$.

Further, as total **desired** mechanical energy is **normally** a design specification, **derived** from spacecraft mission, **ideal** burnout analysis can **help** in overall rocket **sizing**.

Summary

Therefore, to **summarize**, the ideal burnout **performance** is an important **parameter** that helps us to give **us** an initial sizing of the **required** launch vehicle.