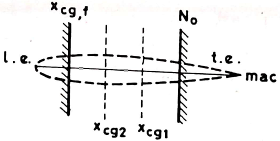
Flight Mechanics/Dynamics

Dr. Shashi Ranjan Kumar

Assistant Professor
Department of Aerospace Engineering
Indian Institute of Technology Bombay
Powai, Mumbai, 400076 India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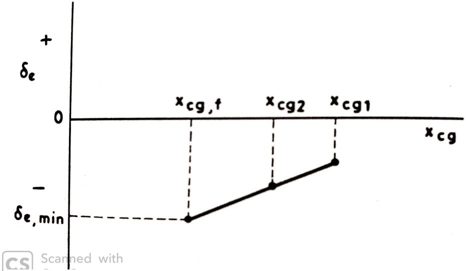
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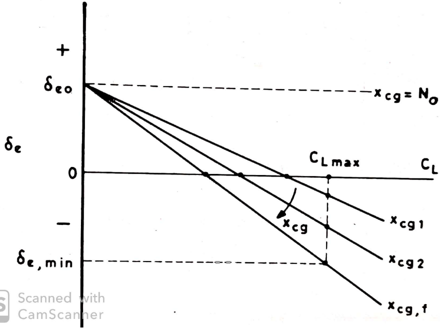
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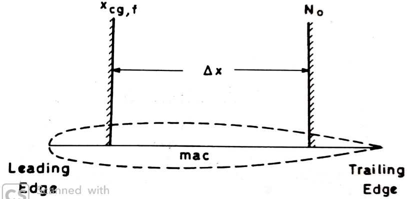
$x_{cg,f}$
N_o
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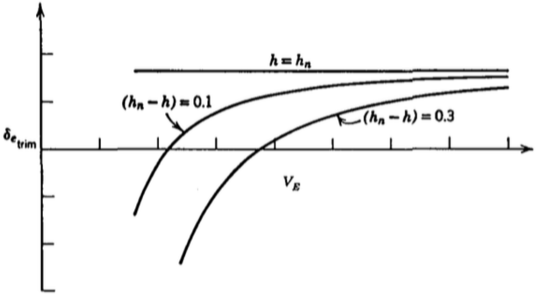
Leading Edge
Trailing Edge

- Lift coefficient in trim condition, with equivalent airspeed V_E,

$$C_{L_{\text{trim}}} = \frac{W}{(1/2)\rho_0 V_E^2 S}$$

- $\delta_{e_{\text{trim}}}$ is a unique function of $C_{L_{\text{trim}}}$ and thus V_E.
- For any CG position, an increase in trim speed $\Rightarrow \delta_e > 0$.



- Gradient $\partial\delta_{e_{\text{trim}}}/\partial V_E$ decreases with rearward movement of the CG.
- $\partial\delta_{e_{\text{trim}}}/\partial V_E = 0$ at $h = h_n$.

- What if aerodynamic coefficients vary with speed?
- δ_p throttle control
- Throttle for horizontal flight, at varying speed, must be a function of V that is compatible with $T = D$.
- In normal range of angles of climb or descent, $L = W$ is a reasonable approximation.
- For trimmed steady flight,

$$C_m = 0, \;\; L = C_L \frac{1}{2} \rho V^2 S = W$$

- Consider $C_m = C_m(\alpha, V, \delta_e, \delta_p)$ and $C_L = C_L(\alpha, V, \delta_e, \delta_p)$
- For constant air density ρ,

$$dC_m = 0, \quad C_L V^2 = \text{constant}$$

- At trimmed flight, for small change, we have

$$V_e^2 dC_L + 2C_{L_e} V_e dV = 0 \implies dC_L = -2C_{L_e}\frac{dV}{V_e} = -2C_{L_e} d\hat{V}$$

where $\hat{V} = V/V_e$.

- On taking differential of C_L, C_m,

$$C_{L_\alpha} d\alpha + C_{L_{\delta_e}} d\delta_e = -\left[C_{L_{\delta_p}} d\delta_p + (C_{L_V} + 2C_{L_e}) d\hat{V}\right]$$
$$C_{m_\alpha} d\alpha + C_{m_{\delta_e}} d\delta_e = -\left[C_{m_{\delta_p}} d\delta_p + C_{m_V} dV\right]$$

where $C_{L_V} = \dfrac{\partial C_L}{\partial \hat{V}}$ and $C_{m_V} = \dfrac{\partial C_m}{\partial \hat{V}}$.

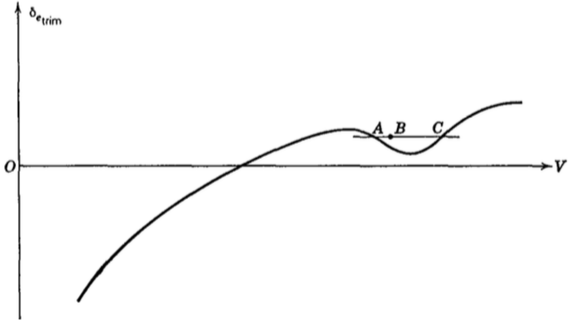
- On solving,

$$d\delta_e = \frac{[(C_{L_V} + 2C_{L_e})C_{m_\alpha} - C_{m_V} C_{L_\alpha}]d\hat{V} + [C_{L_{\delta_p}} C_{m_\alpha} - C_{m_{\delta_p}} C_{L_\alpha}]d\delta_p}{\Delta}$$

- For constant δ_p,

$$\boxed{\left.\frac{d\delta_{e_{\text{trim}}}}{d\hat{V}}\right|_{\delta_p} = \frac{[(C_{L_V} + 2C_{L_e})C_{m_\alpha} - C_{m_V} C_{L_\alpha}]}{\Delta}}$$

- A true criterion of stability $\left.\frac{d\delta_{e_{\text{trim}}}}{d\hat{V}}\right|_{\delta_p} > 0$.
- C_{L_V} or C_{m_V} are large, resulting in reversal of the slope.



- Critical CG position for zero elevator trim slope

$$\left.\frac{d\delta_{e_{\text{trim}}}}{d\hat{V}}\right|_{\delta_p} = \frac{[(C_{L_V} + 2C_{L_e})C_{m_\alpha} - C_{m_V} C_{L_\alpha}]}{\Delta} = 0$$

- On substituting $C_{m_\alpha} = C_{L_\alpha}(h - h_n)$, we get

$$C_{m_\alpha} = \frac{C_{m_V} C_{L_\alpha}}{(C_{L_V} + 2C_{L_e})} \Rightarrow h - h_n = \frac{C_{m_V}}{(C_{L_V} + 2C_{L_e})}$$

- Static stability limit h_s

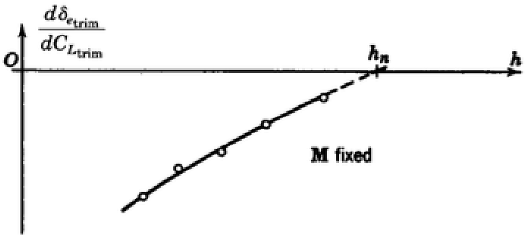
$$\boxed{h_s = h_n + \frac{C_{m_V}}{(C_{L_V} + 2C_{L_e})}}$$

- $h_s > h_n$ or $h_s < h_n$ based on sign of C_{m_V}
- Elevator trim slope in terms of h_s

$$\boxed{\left.\frac{d\delta_{e_{\text{trim}}}}{d\hat{V}}\right|_{\delta_p} = \frac{C_{L_\alpha}}{\Delta}(C_{L_V} + 2C_{L_e}) \underbrace{(h - h_s)}_{\text{stability margin}}}$$

- Measurement of h_n, requires the measurements of C_{m_α} and C_{L_α}.
- Can we find it using other means?
- In absence of complications, it can be found using the elevator trim curve.

$$\boxed{\frac{d\delta_{e_{\text{trim}}}}{dC_{L_{\text{trim}}}} = -\frac{C_{m_\alpha}}{\Delta} = -\frac{C_{L_\alpha}(h-h_n)}{\Delta}}$$

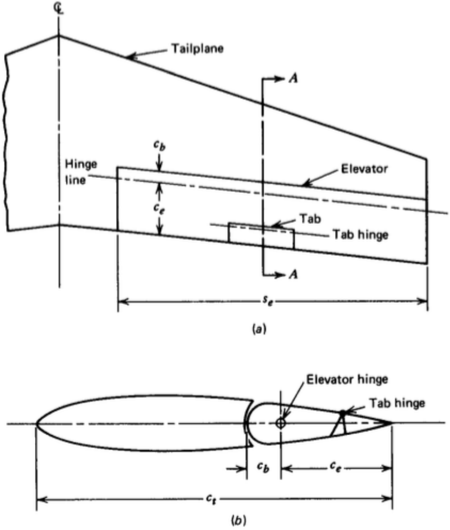


- Similarly, $\left.\frac{d\delta_{e_{\text{trim}}}}{d\hat{V}}\right|_{\delta_p}$ gives us stability limit h_s.

- Rotation of aerodynamic control surfaces: Requirement of force to overcome the aerodynamic pressures.
- Application of force
  - ⇒ A human pilot through a mechanical system of cables, pulleys, rods, and levers
  - ⇒ Partly by powered actuator
  - ⇒ Fly-by-wire
- Amount of force must be known with precision for design purpose.
- Aerodynamic hinge moment, H_e: Moment produced by aerodynamic forces on any control surface.
- Elevator hinge moment coefficient

$$C_{he} = \frac{H_e}{(1/2)\rho V^2 S_e \bar{c}_e}$$

where S_e is area of that portion of elevator and tab which lies aft of elevator hinge line, and $\bar{c}_e$, is a mean chord of the same portion of elevator and tab.



- C_{he} is most difficult to determine with precision among all the aerodynamic parameters
- Dependence on large number of geometrical parameters
- 2D airfoil theory: H_e is linear with α and δ_e in both subsonic and supersonic flow.

- For finite surfaces,

$$C_{he} = b_0 + b_1\alpha_s + b_2\delta_e + b_3\delta_t$$

where α_s is angle of attack of the surface to which control is attached, and δ_t is angle of deflection of tab, and

$$\begin{aligned} b_1 &= \frac{\partial C_{he}}{\partial \alpha_s} = C_{he_{\alpha_s}} \\ b_2 &= \frac{\partial C_{he}}{\partial \delta_e} = C_{he_{\delta_e}} \\ b_3 &= \frac{\partial C_{he}}{\partial \delta_t} = C_{he_{\delta_t}} \end{aligned}$$

- Computation of $C_{he} \Rightarrow$ determination of b_0, b_1, b_2, and b_3.
- Force required to hold elevator at desired angle $\propto$ hinge moment.
- For finite surfaces,

$$C_{he} = b_0 + C_{he_{\alpha_s}}\alpha_s + C_{he_{\delta_e}}\delta_e + C_{he_{\delta_t}}\delta_t$$

- For tailless aircraft, $\alpha_s = \alpha$, but for aircraft with tails, $\alpha_s = \alpha_t$.
- C_{he} w.r.t α can be written as

$$\boxed{C_{he} = C_{he_0} + C_{he_\alpha}\alpha + C_{he_{\delta_e}}\delta_e + C_{he_{\delta_t}}\delta_t}$$

- For tailless aircraft, $C_{he_0} = b_0,\ C_{he_\alpha} = b_1$.
- For tailed aircraft, $\alpha - \alpha_{wb} = -\dfrac{a_t S_t}{aS}(i_t + \epsilon_0),\ \alpha_t = \alpha_{wb} - i_t - \epsilon$

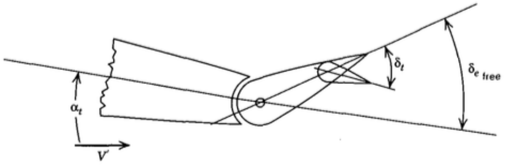
$$\alpha_t = \alpha\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) - (\epsilon_0 + i_t)\left[1 - \frac{a_t}{a}\frac{S_t}{S}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right] \quad \text{How?}$$

- For tailed aircraft, with symmetrical airfoil sections in the tail, $b_0 = 0$

$$C_{he_\alpha} = b_1\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)$$

$$C_{he_0} = -b_1(\epsilon_0 + i_t)\left[1 - \frac{a_t}{a}\frac{S_t}{S}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\right]$$

- Pitch stiffness of an airplane with fixed controls
- However, fixed control is not possible with human pilot.
- What about the stability of airplane with free elevator?
- Stability in the control-free condition is less than with fixed controls.
- Small difference is desirable.
- Free control is never realized in practice due to friction.
- Two extremes, free control and fixed control
- What would be deflection δ_e with free elevator?



$$C_{he} = 0 \Rightarrow \boxed{\delta_{e_{\text{free}}} = -\frac{C_{he_0} + C_{he_\alpha}\alpha + b_3\delta_t}{b_2}}$$

- Lift and moment coefficients

$$C_{L_{\text{free}}} = C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_{e_{\text{free}}}$$
$$C_{M_{\text{free}}} = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_{e_{\text{free}}}$$

- On substituting for $\delta_{e_{\text{free}}}$,

$$C_{L_{\text{free}}} = C'_{L_0} + C'_{L_\alpha}\alpha, \quad C_{M_{\text{free}}} = C'_{m_0} + C'_{m_\alpha}\alpha$$

where

$$C'_{L_0} = -\frac{C_{L_{\delta_e}}(C_{he_0} + b_3\delta_t)}{b_2}, \quad \boxed{a' = C'_{L_\alpha} = C_{L_\alpha} - \frac{C_{L_{\delta_e}}C_{he_\alpha}}{b_2}}$$

$$C'_{m_0} = C_{m_0} - \frac{C_{m_{\delta_e}}(C_{he_0} + b_3\delta_t)}{b_2}, \quad \boxed{C'_{m_\alpha} = C_{m_\alpha} - \frac{C_{m_{\delta_e}}C_{he_\alpha}}{b_2}}$$

- Reduced magnitude of C_{L_α} and C_{m_α}, leading to reduction of stability.

- For tailless aircraft with free elevator, $a' = a\left[1 - \frac{C_{L_{\delta_e}} b_1}{a b_2}\right]$
- Free elevator factor, $\boxed{F = 1 - \frac{C_{L_{\delta_e}} b_1}{a b_2}} < 1.$
- What about the tailed aircraft?
- If elevator is a part of tail, with $b_0 = 0$, $\delta_{e_{\text{free}}}$ is related to α_t

$$C_{h_e} = b_1\alpha_t + b_2\delta_{e_{\text{free}}} + b_3\delta_t = 0 \Rightarrow \delta_{e_{\text{free}}} = -\frac{b_1\alpha_t + b_3\delta_t}{b_2}$$

- Tail lift coefficient, $C'_{L_t} = a_t\alpha_t + a_e\delta_{e_{\text{free}}} = a_t \underbrace{\left[1 - \frac{a_e b_1}{a_t b_2}\right]}_{F} \alpha_t - \frac{a_e b_3}{b_2}\delta_t$
- Effective lift-curve slope, $\frac{\partial C'_{L_t}}{\partial \alpha_t} = F a_t$
- If $a_t \to F a_t$ and $a \to a'$, all results will hold for aircraft with a free elevator.

- For free elevator, NP is given by

$$\boxed{h_n' = h_{n_{wb}} + \frac{Fa_t}{a'}\bar{V}_H\left(1-\frac{\partial \epsilon}{\partial \alpha}\right) - \frac{1}{a'}\frac{\partial C_{m_p}}{\partial \alpha}}$$

- Can we obtain elevator-free NP alternatively?

$$C'_{m_\alpha} = C'_{L_\alpha}(h - h_n') \Rightarrow h_n' = h - \frac{C'_{m_\alpha}}{C'_{L_\alpha}}$$

- As $C'_{m_\alpha} = C_{m_\alpha} - \dfrac{C_{m_{\delta_e}} C_{he_\alpha}}{b_2}$,

$$h - h_n' = \frac{1}{C'_{L_\alpha}}\left[C_{m_\alpha} - \frac{C_{m_{\delta_e}} C_{he_\alpha}}{b_2}\right] = \frac{1}{a'}\left[a(h-h_n) - \frac{C_{m_{\delta_e}} C_{he_\alpha}}{b_2}\right]$$

- For tailed aircraft,

$$C_{m_{\delta_e}} = C_{L_{\delta_e}}(h - h_{n_{wb}}) - a_e\bar{V}_H$$

- To obtain NP,

$$\begin{aligned}
h - h_n' &= \frac{1}{a'}\left[a(h-h_n) - \frac{(C_{L_{\delta_e}}(h - h_{n_{wb}}) - a_e\bar{V}_H)C_{he_\alpha}}{b_2}\right] \\
&= \frac{1}{a'}\left[a(h-h_n) - \frac{C_{L_{\delta_e}}C_{he_\alpha}(h - h_{n_{wb}})}{b_2} + \frac{a_e\bar{V}_H C_{he_\alpha}}{b_2}\right] \\
&= \frac{a}{a'}(h - h_n) - \frac{C_{L_{\delta_e}}C_{he_\alpha}(h - h_{n_{wb}})}{a'b_2} + \frac{a_e\bar{V}_H C_{he_\alpha}}{a'b_2} \\
&= \frac{h}{a'}\left(a - \frac{C_{L_{\delta_e}}C_{he_\alpha}}{b_2}\right) - \frac{1}{a'}\left(ah_n - \frac{C_{L_{\delta_e}}C_{he_\alpha}h_{n_{wb}}}{b_2}\right) + \frac{a_e\bar{V}_H C_{he_\alpha}}{a'b_2} \\
&= h - \frac{1}{a'}\left(ah_n - \frac{C_{L_{\delta_e}}C_{he_\alpha}h_{n_{wb}}}{b_2}\right) + \frac{a_e\bar{V}_H C_{he_\alpha}}{a'b_2}
\end{aligned}$$

- On simplifying,

$$
\begin{aligned}
h_n' &= \frac{1}{a'}\left(ah_n - \frac{C_{L_{\delta_e}}C_{he_\alpha}\left[(h_{n_{wb}} - h_n) + h_n\right]}{b_2}\right) - \frac{a_e\bar{V}_H C_{he_\alpha}}{a'b_2} \\
&= \frac{h_n}{a'}\left(a - \frac{C_{L_{\delta_e}}C_{he_\alpha}}{b_2}\right) + \frac{C_{L_{\delta_e}}C_{he_\alpha}(h_n - h_{n_{wb}})}{a'b_2} - \frac{a_e\bar{V}_H C_{he_\alpha}}{a'b_2} \\
&= h_n + \frac{C_{L_{\delta_e}}C_{he_\alpha}(h_n - h_{n_{wb}})}{a'b_2} - \frac{a_e\bar{V}_H C_{he_\alpha}}{a'b_2}
\end{aligned}
$$

- As $C_{L_{\delta_e}} = a_e\dfrac{S_t}{S},\ C_{he_\alpha} = b_1\left(1 - \dfrac{\partial\epsilon}{\partial\alpha}\right)$

$$
h_n' = h_n + \frac{C_{he_\alpha}}{a'b_2}\left[C_{L_{\delta_e}}(h_n - h_{n_{wb}}) - a_e\bar{V}_H\right]
$$

$$
\boxed{h_n' = h_n - \frac{a_e b_1}{a'b_2}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right)\left[\bar{V}_H - (h_n - h_{n_{wb}})\frac{S_t}{S}\right]}
$$

- For tailless aircraft, $C_{m_{\delta_e}} = \dfrac{\partial C_{m_{ac_{wb}}}}{\partial \delta_e} + C_{L_{\delta_e}}(h - h_n),\ C_{he_\alpha} = b_1$
- To obtain NP,

$$
\begin{aligned}
h - h_n' =& \frac{1}{a'}\left[a(h-h_n) - \frac{b_1}{b_2}\left(\frac{\partial C_{m_{ac}}}{\partial \delta_e} + C_{L_{\delta_e}}(h-h_n)\right)\right] \\
=& \frac{h-h_n}{a'}\left[a - \frac{C_{L_{\delta_e}} b_1}{b_2}\right] - \frac{b_1}{a'b_2}\frac{\partial C_{m_{ac}}}{\partial \delta_e} \\
=& h - h_n - \frac{b_1}{a'b_2}\frac{\partial C_{m_{ac}}}{\partial \delta_e}
\end{aligned}
$$

- Elevator-free NP $\boxed{h_n' = h_n + \dfrac{b_1}{a'b_2}\dfrac{\partial C_{m_{ac}}}{\partial \delta_e}}$
- Control-free static margin $\boxed{K_n' = h_n' - h}$, $h_n - h_n' \approx 0.08$
- A substantial forward movement of the NP, with reduction of static margin, pitch stiffness, and stability.

- Why do we need trim tab?
- To fly at a given speed, or C_L, a certain elevator angle $\delta_{e_{\text{trim}}}$ is required.
- When $\delta_{e_{\text{trim}}} \neq \delta_{e_{\text{free}}}$, a force is required to hold the elevator.
- For long period flight at a constant speed, it is difficult for pilot to maintain such a force.
- Trim tabs: To relieve pilot of this load by causing $\delta_{e_{\text{free}}} = \delta_{e_{\text{trim}}}$.
- How to ensure this condition? By using trim tab deflection
- How much deflection for trim tab?
- When airplane is at trim with free elevator

$$C_m = 0, \; C_{he} = 0 \implies \boxed{\delta_{t_{\text{trim}}} = -\frac{C_{he_0} + C_{he_\alpha}\alpha_{\text{trim}} + C_{he_{\delta_e}}\delta_{e_{\text{trim}}}}{b_3}}$$

- At trim,

$$\alpha_{\text{trim}} = \frac{C_{m_0}C_{L_{\delta_e}} + C_{m_{\delta_e}}C_{L_{\text{trim}}}}{\Delta},\ \delta_{e_{\text{trim}}} = -\frac{C_{m_0}C_{L_\alpha} + C_{m_\alpha}C_{L_{\text{trim}}}}{\Delta}$$

where $\Delta = C_{L_\alpha}C_{m_{\delta_e}} - C_{m_\alpha}C_{L_{\delta_e}}$.

- Trim tab angle

$$\begin{aligned}
\delta_{t_{\text{trim}}} =& -\frac{1}{b_3}\left[C_{he_0} + C_{he_\alpha}\left(\frac{C_{m_0}C_{L_{\delta_e}} + C_{m_{\delta_e}}C_{L_{\text{trim}}}}{\Delta}\right)\right. \\
& \left. - C_{he_{\delta_e}}\left(\frac{C_{m_0}C_{L_\alpha} + C_{m_\alpha}C_{L_{\text{trim}}}}{\Delta}\right)\right] \\
=& -\frac{1}{b_3}\left[C_{he_0} + C_{m_0}\left(\frac{C_{he_\alpha}C_{L_{\delta_e}} - C_{he_{\delta_e}}C_{L_\alpha}}{\Delta}\right)\right. \\
& \left. + C_{L_{\text{trim}}}\left(\frac{C_{he_\alpha}C_{m_{\delta_e}} - C_{he_{\delta_e}}C_{m_\alpha}}{\Delta}\right)\right]
\end{aligned}$$

- Linear relation with $C_{L_{\text{trim}}}$ for constant cg location

- For free elevator, we have

$$h - h'_n = \frac{1}{C'_{L_\alpha}}\left[C_{m_\alpha} - \frac{C_{m_{\delta_e}} C_{he_\alpha}}{b_2}\right]$$

$$\Rightarrow C_{he_\alpha} C_{m_{\delta_e}} - C_{he_{\delta_e}} C_{m_\alpha} = -C'_{L_\alpha} b_2 (h - h'_n)$$

- Trim tab angle

$$\delta_{t_{\text{trim}}} = -\frac{1}{b_3}\left[C_{he_0} + C_{m_0}\left(\frac{C_{he_\alpha} C_{L_{\delta_e}} - C_{he_{\delta_e}} C_{L_\alpha}}{\Delta}\right) - C_{L_{\text{trim}}}\left(\frac{C'_{L_\alpha} b_2 (h - h'_n)}{\Delta}\right)\right]$$

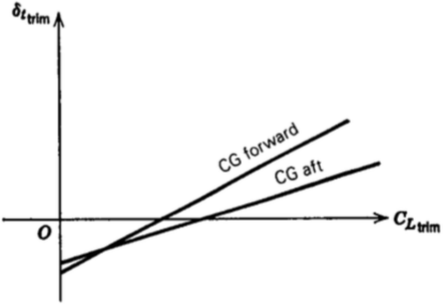
- Applicable to both tailed and tailless aircraft, with appropriate values of the coefficients.
- What would be the slope of $\delta_{t_{\text{trim}}}$ vs $C_{L_{\text{trim}}}$ curve with constant coefficients?

$$\boxed{\frac{\partial \delta_{t_{\text{trim}}}}{\partial C_{L_{\text{trim}}}} = \frac{b_2 C'_{L_\alpha}(h - h'_n)}{b_3 \Delta} = -\frac{b_2 C'_{L_\alpha} K'_n}{b_3 \Delta}}$$

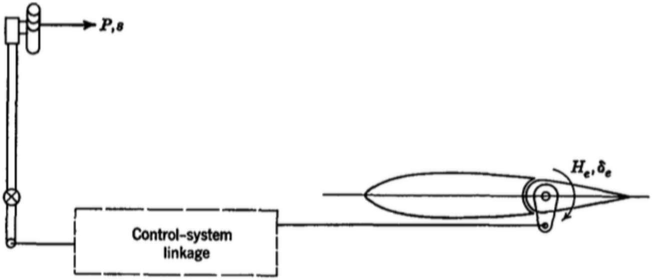
- Slope of $\delta_{t_{\text{trim}}}$ vs $C_{L_{\text{trim}}}$ curve $\propto$ control-free static margin.

$$\frac{\partial \delta_{t_{\text{trim}}}}{\partial C_{L_{\text{trim}}}} \propto K_n'$$

- Trim-tab slope bears the same relation to control-free NP as the elevator angle slope does to control-fixed NP.
- Flight determination of h_n' from measurements of $\dfrac{\partial \delta_{t_{\text{trim}}}}{\partial C_{L_{\text{trim}}}}$



- Elements of linkage and structure to which it is attached are ideally rigid $\implies$ no strain energy
- Assumption of no friction
- P: Force applied by pilot
- s: Displacement of hand grip
- W_b: Work done by power boost system



- From conservation of energy

$$Pds + dW_b + H_e d\delta_e = 0 \Rightarrow P = -\frac{dW_b}{ds} - \frac{H_e d\delta_e}{ds}$$

- We can write

$$P = (\underbrace{G_1}_{\text{elevator gearing}} - \underbrace{G_2}_{\text{boost gearing}})H_e = GH_e, \; G_1 = -\frac{d\delta_e}{ds}, \; G_2 = \frac{dW_b/ds}{H_e}$$

- For a fixed G_1, what is the effect of power boost?
- Power boost reduces G and thus P.
- Force applied by the pilot

$$P = GH_e = \frac{1}{2} G C_{he} S_e \bar{c}_e \rho V^2$$

- Hinge moment coefficient

$$C_{he} = C_{he_0} + C_{he_\alpha}\alpha + C_{he_{\delta_e}}\delta_e + C_{he_{\delta_t}}\delta_t \Rightarrow \boxed{C_{he} = \ C_{he_{\delta_t}}(\delta_t - \delta_{t_{\text{trim}}})}$$

- Using $\delta_{t_{\text{trim}}}$, we can rewrite

$$C_{he} = C_{he_{\delta_t}}\delta_t + C_{he_0} + C_{m_0}\left(\frac{C_{he_\alpha}C_{L_{\delta_e}} - C_{he_{\delta_e}}C_{L_\alpha}}{\Delta}\right) - C_{L_{\text{trim}}}\left[\frac{C'_{L_\alpha}b_2(h - h'_n)}{\Delta}\right]$$

- Lift balances out the weight in horizontal flight,

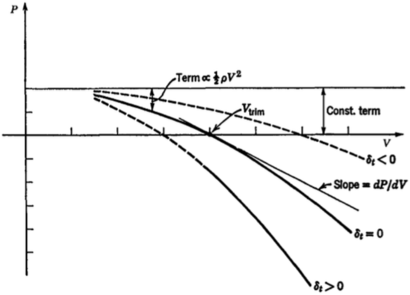
$$C_{L_{\text{trim}}} = \frac{W}{(1/2)\rho V^2 S} = \frac{w}{(1/2)\rho V^2}$$

where wing loading is given by $w = W/S$.

- Force applied by the pilot

$$
\begin{aligned}
P =& \frac{1}{2} G S_e \bar{c}_e \rho V^2 \left[C_{he_{\delta_t}} \delta_t + C_{he_0} + C_{m_0} \left(\frac{C_{he_\alpha} C_{L_{\delta_e}} - C_{he_{\delta_e}} C_{L_\alpha}}{\Delta} \right) \right. \\
& \left. - \frac{w}{(1/2)\rho V^2} \left\{ \frac{C'_{L_\alpha} b_2 (h - h'_n)}{\Delta} \right\} \right] \\
=& \left(\frac{1}{2} \rho V^2 \right) \underbrace{G S_e \bar{c}_e \left[C_{he_{\delta_t}} \delta_t + C_{he_0} + C_{m_0} \left(\frac{C_{he_\alpha} C_{L_{\delta_e}} - C_{he_{\delta_e}} C_{L_\alpha}}{\Delta} \right) \right]}_{B} \\
& \underbrace{- \frac{G S_e \bar{c}_e w C'_{L_\alpha} b_2 (h - h'_n)}{\Delta}}_{A} \\
=& A + B \left(\frac{1}{2} \rho V^2 \right)
\end{aligned}
$$

- $P \propto S_e \bar{c}_e$, i.e., cube of airplane size
- $P \propto G$
- CG location affect constant term, forward movement $\implies$ upward translation
- Wing loading has same effect as CG
- ρV^2 decreases with height and increases with V^2
- All terms in B are built-in except δ_t
- Trim tab changes the curvature of parabola.
- Trim tab controls intercept on V axis and thus trim speed with zero control force.



- Gradient of control force applied at $P=0$ is important parameter.

$$P = A + B\frac{1}{2}\rho V^2 \Rightarrow \frac{dP}{dV} = B\rho V$$

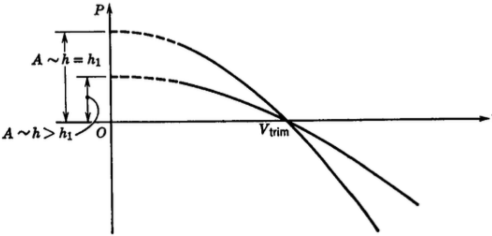
- At $V_{\rm trim}$, $P=0 \Rightarrow B = -\dfrac{A}{(1/2)\rho V_{\rm trim}^2}$

$$\frac{dP}{dV} = -\frac{A}{(1/2)\rho V_{\rm trim}^2}\rho V_{\rm trim} = -\frac{2A}{V_{\rm trim}}$$

- On substituting for A,

$$\frac{dP}{dV} = \frac{2GS_e\bar{c}_e w C'_{L_\alpha} b_2(h-h'_n)}{\Delta V_{\rm trim}}\boxed{\propto \frac{S_e\bar{c}_e w}{V_{\rm trim}}}$$

- Also, $\boxed{\dfrac{dP}{dV} \propto K'_n}$



Elevator control: Heaviest at sea-level, low-speed, forward CG, maximum weight

- Trim tab is assumed to be set so as to keep V_{trim} the same.
- Gradient $dP/dV \downarrow$ as the CG moves backward.
- What would happen if CG is at control-free NP?
- At control-free NP, $A = 0$, P/V graph becomes a straight line lying on the V axis.
- At control-free NP, no force is required to change the trim speed.
- Independent of height for a given true airspeed, but decreases with height for a fixed V_E.

Reference

1. John Anderson Jr., *Introduction to Flight*, McGraw-Hill Education, Sixth Edition, 2017.
2. Bernard Etkin and Llyod Duff Reid, *Dynamics of Flight Stability and Control*, John Wiley and Sons, Third Edition, 1996.

Thank you for your attention !!!