Tutorial: 3

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Tutorial 3
Question-1

Q: Consider the wing-body alone configuration of a general aviation airplane with the following properties-

$$C_{mac_{wb}} = -0.04, \quad h_{ac_{wb}} = 0.25, \quad C_{L\alpha_{wb}} = 4.5/rad, \quad h_{CG} = 0.4.$$

The zero lift angle of attack for the positively cambered wing is given as $\alpha_0 = -2^\circ$. Answer the following:

1. Determine the trim angle of attack for the aircraft.
2. What will be the trim angle of attack if the CG of the airplane is shifted ahead of the AC to $h_{CG} = 0.1$? Determine the stability of the airplane in this new trim condition.
3. What should be the $C_{mac_{wb}}$ if the airplane is required to trim at $\alpha_{trim} = 5^\circ$ for the new location of the CG at $h_{CG} = 0.1$?

A:

- The angle of attack by $C_M = 0$ as

$$\alpha_a = \alpha_{trim} + \alpha_0 = -\frac{C_{M_{ac}}}{C_{L_\alpha}(h_{CG} - h_{ac})} = 3.39^\circ \implies \alpha_{trim} = 5.39^\circ$$

- For $h_{CG} = 0.1$, α_{trim} can be computed as

$$\alpha_a = \alpha_{trim} + \alpha_0 = -\frac{C_{M_{ac}}}{C_{L_\alpha}(h_{CG} - h_{ac})} = -3.39^\circ \implies \alpha_{trim} = -1.39^\circ$$

Since the CG is ahead of AC, the configuration may be stable.

- The $C_{M_{ac}}$ for $\alpha_{trim} = 5^\circ$ and $h_{CG} = 0.1$ is

$$C_{M_{ac}} = -C_{L_\alpha}(h_{CG} - h_{ac}) \times (\alpha_{trim} + \alpha_0) = 0.082$$

Q: Figure 1 shows the variation of the moment coefficient around CG with respect to α. It can be seen that the aircraft trims at $\alpha = 5^\circ$, for which the CG is located at 0.25 the chord length along with a static margin of 15 percent. The airplane is required to change the trim angle to $\alpha = 10^\circ$ by changing the CG location. Find the new CG location and corresponding static margin of the airplane.

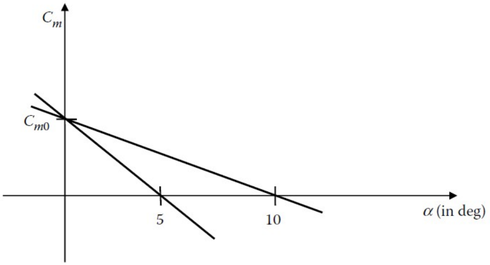

Figure: C_M vs. α curve for a general aviation airplane

A:

- The NP is computed as

$$h_{NP} = K_n + h_{CG} = 0.15 + 0.25 = 0.4$$

- Equations of the two lines

$$C_{m_1} = C_{m_0} + C_{M_{\alpha_1}}\alpha, \quad C_{m_2} = C_{m_0} + C_{M_{\alpha_2}}\alpha$$

- The trim point corresponding to either profiles

$$C_{M_{\alpha_1}} = -C_{m_0}/5^\circ, \quad C_{M_{\alpha_2}} = -C_{m_0}/10^\circ \implies C_{M_{\alpha_1}} = 2C_{M_{\alpha_2}}$$

- Simplifying further

$$h_{CG_1} - h_n = 2(h_{CG_2} - h_n) \implies h_{CG_2} = 0.325$$

- The stability resultant margin $K_n' = 0.075$

- **Q:** Elevator hinge moment is given by the expression $H_e = 0.5\rho V^2 S_e c_e C_{he}$, where S_e is the elevator area behind hinge, c_e is the corresponding chord length and C_{he} is the hinge moment coefficient. This is the moment that a pilot needs to overcome using stick force F_s and the stick arm length l_s. The relation between the stick force and the hinge moment is defined by $F_s = GH_e$. The proportionality factor $G = \delta e/(l_s\delta_s)$, known as the gear ratio, is a function of the elevator deflection δe, stick arm length l_s and angular displacement of the stick about its own hinge point, δ_s. A pilot pulls a 0.75 m long stick towards himself $(\delta_s = 5^\circ)$ to create an elevator up deflection of -15°. Determine the hinge moment if the stick force applied by the pilot is $2N$.

- **A:**

$$F_e = GH_e = \frac{\delta e H_e}{l_s\delta_s} \implies H_e = \frac{F_e l_s \delta_s}{\delta e} = 0.5\ N\ m$$

Q: The elevator control force to trim a particular airplane at a speed of $154\ m/s$ is zero. Using the following data estimate the force required to change the trim speed to $159\ m/s$. Assume that $C_{L_{\delta_e}} = 0$.

Geometric Data:

$G = 1.18^\circ/cm$, $S_e = 3.72\ m^2$, $c_e = 0.61m$, $\bar{V}_H = 0.56$, $h_{CG} = 0.38$,
wing loading $= 2395\ Pa$

Aerodynamic Data:

$\frac{\partial C_{h_e}}{\partial \delta_e} = -0.005/deg$, $a_e = 0.025/deg$, Free elevator neutral point, $h'_n =$ 0.45

A:

- Note that $C_{L_{\delta_e}} = 0$, hence, $a' = a$ and $\Delta = -aa_e\bar{V}_H$
- The control force is given by $P = A + B\left(0.5\rho V_\infty^2\right)$ where

$$A = -GS_e\bar{c}_e \frac{Wb_2}{Sa_e\bar{V}_H}\left(h - h_n'\right) = 279.348\ N$$

- Given: $P = 0$ for $V = 154\ m/s$. We need to find P for $V = 159\ m/s$, therefore,

$$0 = 279.348 + \frac{1}{2}B\rho(154)^2, \quad P = 279.348 + \frac{1}{2}B\rho(159)^2$$

- Eliminating $\frac{1}{2}B\rho$, we get

$$P = 279.348 - 279.348\left(\frac{159}{154}\right)^2 = -18.949\ N \quad \text{(Forward push)}$$