Orbital Solution in Geographical Frame

Conic Solution Features

Orbital **solutions** in terms of **conic** sections have the **limitation** that spacecraft position is **fixed** in relation to the **perigee**, which is defined quite **arbitrarily.**

Thus, all **orbits** having same **conic** section geometry as **solution** are considered to be **same** in orbital frame.

Conic Solution Limitations

However, to **observers** on earth at different **locations**, same position of **spacecraft** will appear to be **different.**

As practical **usefulness** of orbit solution **lies** in its connection with **earth-fixed** observations, we also need to **fix** the conic solution in the **geographical** context.

Geographical Frame Features

In this **regard**, it is worth noting that **earth** rotates on its **axis** as well as **revolves** around the **sun.**

Therefore, we need to **fix** the conic section (or **orbit**) in relation to a **point** on earth, which is typically **referenced** through an inertial **location** with respect to **sun.**

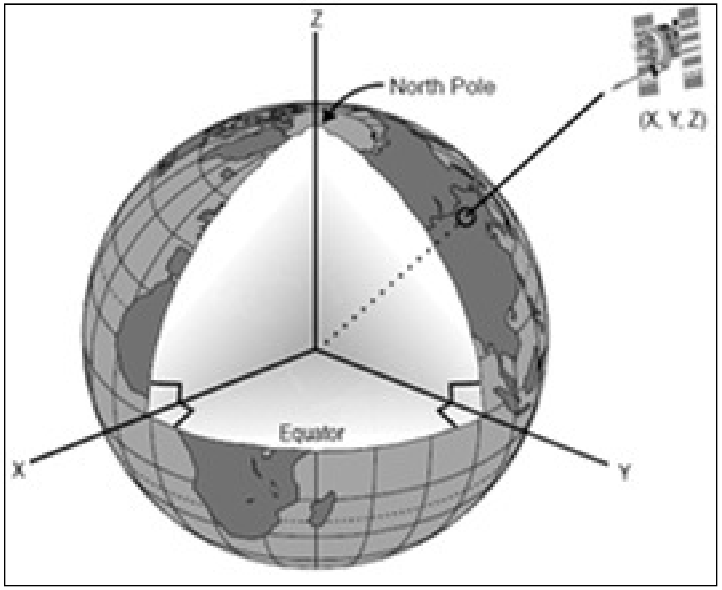
Geographical Frame Definition

For this purpose, we **employ** a point on surface of **earth**, called 'vernal equinox' or '**1st point of Aries**', which is nearly **invariant** with respect to **sun.**

Based on this choice, we **define** the origin of an **inertial** frame at the centre of **earth**, as described next.

Coordinate Fixing

Consider the **ECI** frame, as shown **below.**



'X' outwards from **centre along** line joining **centre** & '1st point of Aries' on **equator.**

'Z' towards geographical **north** & 'Y' normal to **X-Z** plane.

Points on **earth** as well as **spacecraft** are fixed with respect to this **frame.**

ECI Frame Features

Spacecraft is uniquely located in **ECI** frame with respect to **stars** through the radius **vector** from origin.

However, in **most** cases, satellite also **needs** to be fixed with respect to **tracking** locations on earth's **surface.**

This has **brought** into focus two useful **time** references; T_0 - time of **perigee** passage & UTC - **inertial** time.

Space – Time Mapping

In general, **‘T_0’** provides an important **time** reference for the spacecraft location in **orbit** with respect to **perigee.**

Further, **UTC**, is used for **fixing** the time coordinate of the **spacecraft** in orbit in the inertial context.

‘T_0’ in Practical Context

In most **orbital** missions, spacecraft **injection** occurs at ‘**perigee**’, so that ‘T_0’ is also the **time** of separation, which is **available** as part of ascent **mission.**

However, there are **cases** where the injection **point** is not the **perigee**, due to difference in the **injection** conditions.

This can be **addressed** by locating ‘**perigee**’, through ‘$\Delta\theta$’ estimates for injection **point**, via time solutions.

Spacecraft Inertial Parameters

Fixing of spacecraft in **geographical** frame is completed when its **location** is connected to UTC and **ECI** frame.

In **such** a case, we need **six** quantities to completely **describe** spacecraft position, called **orbital** parameters.

Orbital Parameters Definition

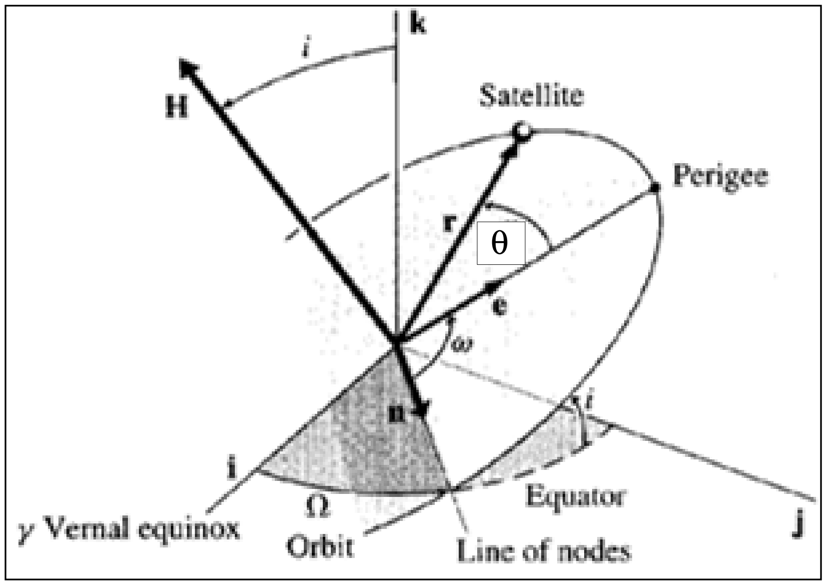
Orbital parameters are geometrical **quantities** that are defined by **determining** the orientation of **orbit** with respect to the **ECI** frame.

This is **done** in a simple **manner** by defining the **intersection** of the orbital **plane** with the geographical **frame**, as shown in figure **next.**

Orbital Parameters

Orbital Parameters

The **resulting** intersection is shown **below.**



Intersection Features

Intersection generates a **useful** feature, called '**line of nodes**', which is a **result** of intersection of two **planes.**

This line is **common** to both orbital & **equatorial** planes and is at an angle 'Ω' with respect to Vernal Equinox, called '**Right Ascension**'.

It is now **possible** to uniquely fix the spacecraft **location** based on other **features**, which are divided in two **groups**.

Parameter Groups Definition

Group-1: (Orbit)

a – Semi-major axis

e – Orbit Eccentricity

T_0–Perigee Passage Time

Group-2: (ECI Orientation)

Ω - Right Ascension of LoN

i – Inclination of Orbit

ω - Argument of Perigee

Orbital Parameters from 'r' & 'H'

Inertial orientation of **orbit** is defined through **three** angles about three **inertial** axes that fix the orbital **plane** orientation with **respect** to the equatorial **plane.**

We also **see** that these are **related** to the position and angular momentum **vectors** in the ECI **fram**e through a number of vector and scalar **products.**

Orbit Parameters

Orbit parameters are **obtained** as follows.

$$a = -\frac{\mu}{2\varepsilon}; \quad \varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

$$\vec{e} = \frac{1}{\mu}\left(\dot{\vec{r}} \times \vec{H} - \frac{\mu\vec{r}}{r}\right); \quad \vec{H} = \vec{r} \times \vec{v}$$

$$\theta = \cos^{-1}\left(\frac{\vec{e} \cdot \vec{r}}{|\vec{e} \cdot \vec{r}|}\right)$$

Inertial Orientation

Inertial orientation angles are **obtained** as follows.

$$\hat{n} = \frac{\vec{k} \times \vec{H}}{|\vec{k} \times \vec{H}|} \text{ (or } \vec{n} = \vec{k} \times \vec{H}) \rightarrow \text{Ascending Node Vector}$$

$$\Omega = \cos^{-1}\left(\hat{i}.\hat{n}\right) = \cos^{-1}\left(\frac{\hat{i}.\vec{n}}{|\vec{n}|}\right) \rightarrow \text{Right Ascension}$$

$$i = \cos^{-1}\left(\frac{\vec{k} \cdot \vec{H}}{|\vec{H}|}\right) \rightarrow \text{Orbit Inclination}$$

$$\omega = \cos^{-1}\left(\frac{\hat{n} \cdot \vec{e}}{|\vec{e}|}\right) = \cos^{-1}\left(\frac{\vec{n} \cdot \vec{e}}{|\vec{n}||\vec{e}|}\right) \rightarrow \text{Argument of Perigee}$$

Orbital Parameter Features

Inclination 'i' is in the range $0 \le i \le 180^o$. where $\mathbf{i = 0^o} \rightarrow$ Equatorial Orbit and $\mathbf{i = 90^o} \rightarrow$ Polar Orbit.

$0^o \le i \le 90^o \rightarrow$ **Prograde**. $i > 90^o \rightarrow$ **Retrograde**.

$i = 0^o$ or $180^o \rightarrow$ **singularity** in right ascension, $\mathbf{\Omega}$.

For a perfectly **circular** orbit, ω is **indeterminate**.

Summary

In **conclusion**, we see that orbital **parameters** are important quantities that **fix** the orbit of a spacecraft in the **geographical** context.

These **definitions** help us to set up mission **related** tasks that are generally **carried** out from earth-bound **stations**.