Flight Mechanics/Dynamics

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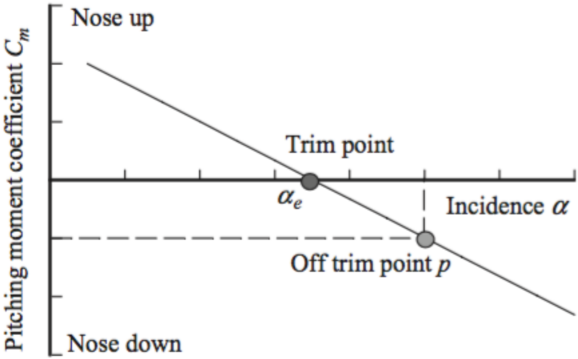
- Notions of stability: Static or Dynamic
- **Static stability**: If the forces and moments on the body caused by a disturbance tend initially to return the body toward its equilibrium position, the body is statically stable.
  - ⇒ No requirement of actual return of vehicle to equilibrium.
  - ⇒ If the forces and moments are such that the body continues to move away from its equilibrium position after being disturbed, the body is statically unstable.
- **Dynamic stability**: A body is dynamically stable if, of its own accord, it eventually returns to and remains at its equilibrium position over time.
- Static stability is related to initial tendency while dynamic stability focus on final state.

- An airplane can continue in steady unaccelerated flight only when the resultant external force and moment about the CG both vanish.
- Longitudinal balance: Zero pitching moment
- Nonzero pitching moment $\Longrightarrow$ Rotation in direction of unbalanced moment

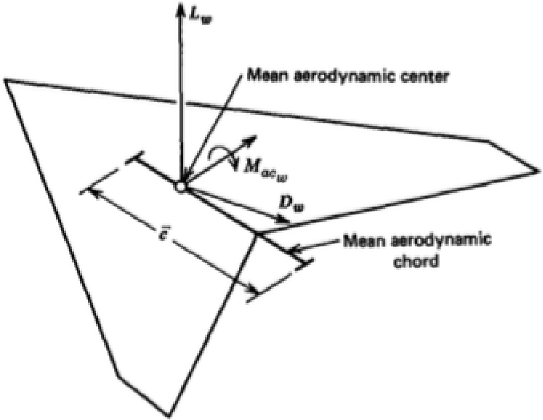


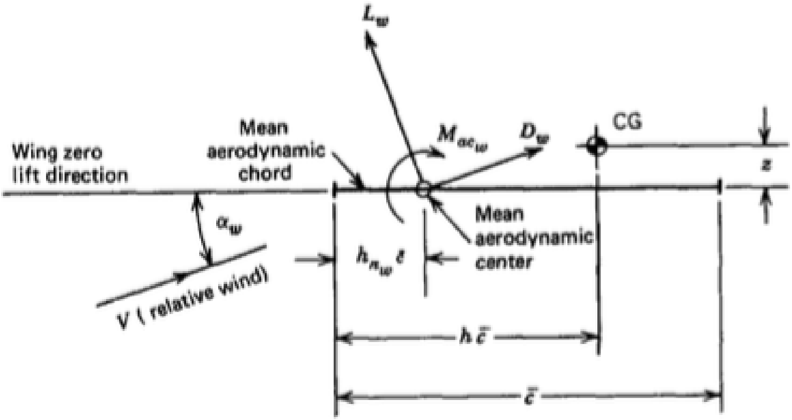
How most of airplanes are using positive camber airfoil?

Any other possible configurations?

What about angle of attack for both configurations?

- How to obtain lift and pitching moment?
- Depends on α, ρ, δ_i, M, T_i, etc.
- Through wind tunnel test
- Contributions of various parts of airplane
- Aerodynamic forces and moment





- Pitching moment about CG

$$\boxed{M_w = M_{ac_w} + (L_w \cos\alpha_w + D_w \sin\alpha_w)(h - h_{n_w})\bar{c} + (L_w \sin\alpha_w - D_w \cos\alpha_w)z}$$

- For small α_w

$$M_w = M_{ac_w} + (L_w + D_w\alpha_w)(h - h_{n_w})\bar{c} + (L_w\alpha_w - D_w)z$$

- On dividing by $(1/2)\rho V^2 S\bar{c}$,

$$\boxed{C_{m_w} = C_{m_{ac_w}} + (C_{L_w} + \underbrace{C_{D_w}\alpha_w}_{\ll C_{L_w}})(h - h_{n_w}) + \underbrace{(C_{L_w}\alpha_w - C_{D_w})z/\bar{c}}_{\text{negligible}}}$$

- On simplification,

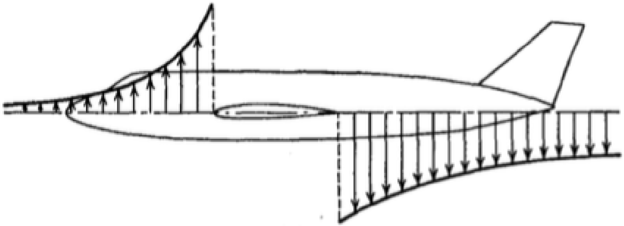
$$\begin{aligned} C_{m_w} &= C_{m_{ac_w}} + C_{L_w}(h - h_{n_w}) \\ &= C_{m_{ac_w}} + \alpha_w a_w(h - h_{n_w}) \end{aligned}$$

where a_w is the lift-curve slope of the wing.

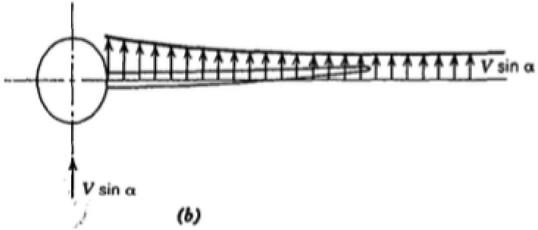
- What is the effect of adding body and nacelles to the wing, on a moment coefficient?
- Will superposition of aerodynamic forces hold here?
- When wing and body are put together, however, a simple superposition of the aerodynamic forces that act upon them separately does not give a correct result.
- For wing-body combination,

$$\begin{aligned} C_{m_{wb}} &= C_{m_{ac_{wb}}} + C_{L_{wb}}(h - h_{n_{wb}}) \\ &= C_{m_{ac_{wb}}} + \alpha_{wb} a_{wb}(h - h_{n_{wb}}) \end{aligned}$$

where a_{wb} is the lift-curve slope of the wing-body-nacelle combination.



(a)



(b)

Example 1

For a given wing-body combination, the aerodynamic center lies 0.05 chord length ahead of CG. Moment coefficient about the aerodynamic center is -0.016. If the lift coefficient is 0.45, calculate the moment coefficient about the CG.

- Moment coefficient about the CG

$$\begin{aligned} C_{m_{wb}} &= C_{m_{ac_{wb}}} + C_{L_{wb}}(h - h_{n_{wb}}) \\ &= -0.016 + 0.45 \times 0.05 = 0.0065 \end{aligned}$$

Example 2

A wing-body model is tested in a subsonic wind tunnel. The lift is found to be zero at geometric $\alpha = -1.5^\circ$. At $\alpha = 5^\circ$ the lift coefficient is measured as 0.52. Also, at $\alpha = 1^\circ$ and 7.88°, the moment coefficients about CG are measured as -0.01 and 0.05, respectively. The CG is located at $0.35c$. Calculate the location of ac $h_{n_{wb}}$ and moment coefficient about aerodynamic center, $C_{m_{ac,wb}}$.

- Lift slope

$$a_{wb} = \frac{dC_L}{d\alpha} = \frac{0.52 - 0}{5 - (-1.5)} = 0.08/\text{deg}$$

- Moment coefficient about the CG

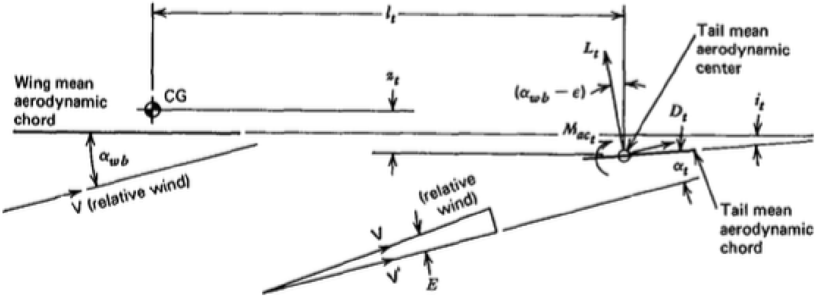
$$C_{m_{wb}} = C_{m_{ac_{wb}}} + a_{wb}\alpha_{wb}(h - h_{n_{wb}})$$

- On substituting values, we obtain

$$\begin{aligned} -0.01 =& C_{m_{ac_{wb}}} + 0.08(1 + 1.5)(h - h_{n_{wb}}) \\ 0.05 =& C_{m_{ac_{wb}}} + 0.08(7.88 + 1.5)(h - h_{n_{wb}}) \end{aligned}$$

- As $h = 0.35$, $h_{n_{wb}} = 0.24$ and $C_{m_{ac_{wb}}} = -0.032$.

- Forces on tail alone is same as that of wing.
- But when mounted on airplane, interference occurs.
- A downward deflection of the flow, mean downwash angle ϵ at the tail.
- A reduction of the relative wind when the tail lies in the wing wake.



- Lift and drag $\perp$ and parallel to V'.
- Tail angle i_t is positive.
- Contribution of tail lift to the airplane lift $\perp$ V is $L_t \cos\epsilon - D_t \sin\epsilon$.
- As ϵ is always small, contribution reduces to L_t.
- Lift coefficient of the tail

$$C_{L_t} = \frac{L_t}{\frac{1}{2}\rho V^2 S_t}$$

- What about difference in V and V' magnitude?
  - $\Rightarrow$ Accounted in tail lift-slope a_t
  - $\Rightarrow$ Tail efficiency factor η_t
- Total lift of airplane $L = L_{wb} + L_t$, thus total lift coefficient

$$\boxed{C_L = C_{L_{wb}} + \frac{S_t}{S} C_{L_t}}$$

- Pitching moment about CG

$$M_t = M_{ac_t} - l_t[L_t\cos(\alpha_{wb}-\epsilon) + D_t\sin(\alpha_{wb}-\epsilon)] - z_t[D_t\cos(\alpha_{wb}-\epsilon) - L_t\sin(\alpha_{wb}-\epsilon)]$$

- For small α_{wb}, with only dominant term

$$M_t = -l_t L_t = -l_t C_{L_t}\frac{1}{2}\rho V^2 S_t$$

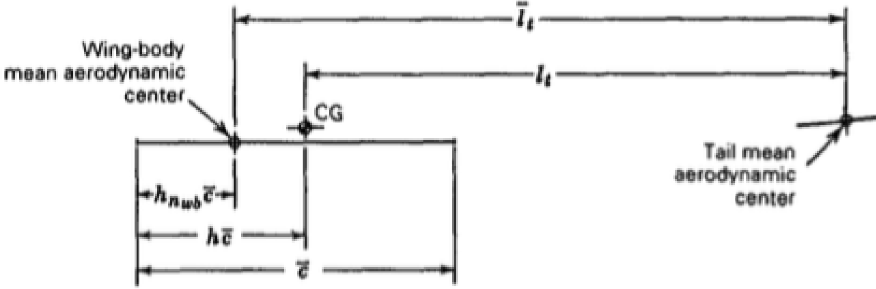
- On dividing by $(1/2)\rho V^2 S\bar{c}$,

$$C_{m_t} = \frac{M_t}{(1/2)\rho V^2 S\bar{c}} = -\frac{l_t}{\bar{c}}\frac{S_t}{S}C_{L_t}$$

- On defining $V_H = \dfrac{l_t S_t}{\bar{c}S}$, we have

$$\boxed{C_{m_t} = -V_H C_{L_t}}$$

- What does V_H represent?
- A ratio of two volume characteristic of airplane geometry, called "horizontal tail volume ratio" or "tail volume"



- CG is not fixed so better to compute M_t about fixed point, mean aerodynamic center of wing-body combination.

- Define

$$\bar{V}_H = \frac{\bar{l}_t S_t}{\bar{c}S}$$

- We have the relation between two volume ratios

$$\boxed{V_H = \bar{V}_H - \frac{S_t}{S}(h - h_{n_{wb}})}$$

- Moment of the tail about the wing-body mean aerodynamic center

$$\bar{C}_{m_t} = -\bar{V}_H C_{L_t}$$

- Moment of tail about the CG

$$\boxed{C_{m_t} = -\bar{V}_H C_{L_t} + C_{L_t}\frac{S_t}{S}(h - h_{n_{wb}}) = \bar{C}_{m_t} + C_{L_t}\frac{S_t}{S}(h - h_{n_{wb}})}$$

- Pitching moment due to propulsive system, C_{m_p}

- Total pitching moments

$$\boxed{C_m = C_{m_{ac_{wb}}} + C_L(h - h_{n_{wb}}) - \bar{V}_H C_{L_t} + C_{m_p}}$$

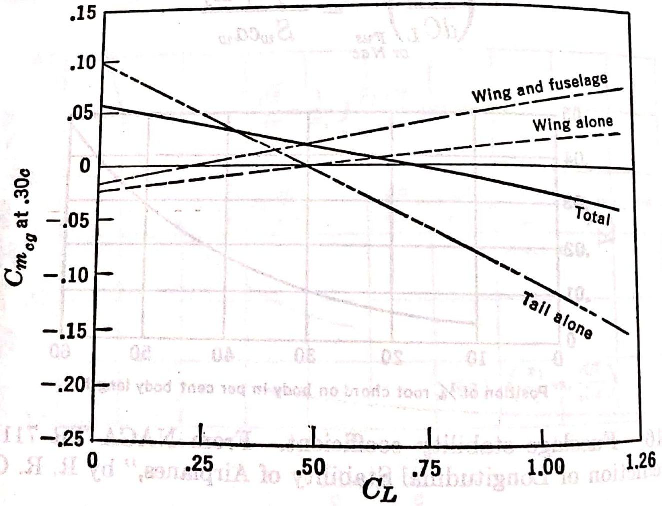
- Pitch stiffness is given by $-C_{m_\alpha}$.

$$C_{m_\alpha} = \frac{\partial C_{m_{ac_{wb}}}}{\partial \alpha} + C_{L_\alpha}(h - h_{n_{wb}}) - \bar{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}$$

- As $\dfrac{\partial C_{m_{ac_{wb}}}}{\partial \alpha} = 0$, we have

$$\boxed{C_{m_\alpha} = C_{L_\alpha}(h - h_{n_{wb}}) - \bar{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}}$$

- As C_{L_α} is large, magnitude of C_{m_α} is dictated by h.
- C_{m_α} can always be made negative by a suitable choice of h, CG location.
- When is $C_{m_\alpha} = 0$?



.15
.10
.05
0
−.05
−.10
−.15
−.20
−.25
$C_{m_{cg}}$ at .30c
Wing and fuselage
Wing alone
Total
Tail alone
0
.25
.50
.75
1.00
1.26
C_L

- **Neutral point (NP)**: A CG position h, for which $C_{m_\alpha} = 0$. Also known as "vehicle aerodynamic center".
- A boundary between positive and negative pitch stiffness
- NP is given by

$$\boxed{h_n = h_{n_{wb}} - \frac{1}{C_{L_\alpha}}\left[\frac{\partial C_{m_{ac_{wb}}}}{\partial \alpha} - \bar{V}_H \frac{\partial C_{L_t}}{\partial \alpha} + \frac{\partial C_{m_p}}{\partial \alpha}\right]}$$

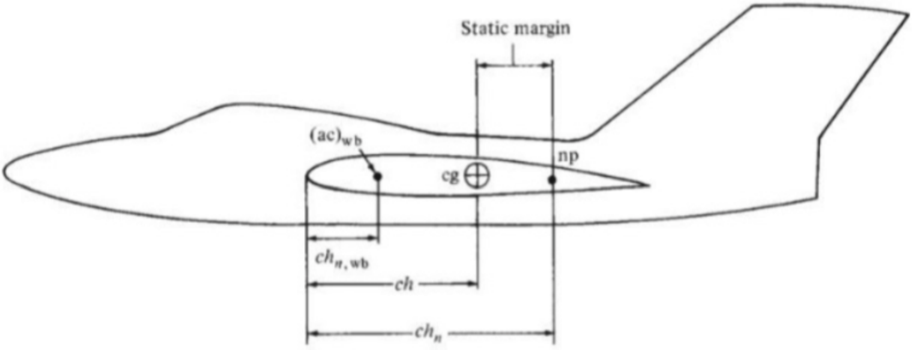
- C_{m_α} in terms of NP

$$\boxed{C_{m_\alpha} = C_{L_\alpha}(h - h_n)}$$

- **Static margin**: Difference between CG position and NP

$$\boxed{K_n = h_n - h}$$

- $K_n > 0 \implies C_{m_\alpha} < 0$, a positive stiffness
- For positive stiffness, CG must be forward to NP



Static margin
$(ac)_{wb}$
np
cg
$ch_{n,wb}$
ch
ch_n

- Farther forward CG $\Longrightarrow$ greater K, and in the sense of "static stability" a more stable vehicle.
- NP is sometimes defined as the CG location for which $\dfrac{dC_m}{dC_L} = 0$.
- Why is it so?
- C_L is a unique function of α under some restrictions and $\dfrac{dC_m}{dC_L} = \dfrac{\dfrac{\partial C_m}{\partial \alpha}}{\dfrac{\partial C_L}{\partial \alpha}}$
- Both $\dfrac{dC_m}{dC_L} = 0 = \dfrac{\partial C_m}{\partial \alpha}$ simultaneously.

Reference

1. John Anderson Jr., *Introduction to Flight*, McGraw-Hill Education, Sixth Edition, 2017.
2. Bernard Etkin and Llyod Duff Reid, *Dynamics of Flight Stability and Control*, John Wiley and Sons, Third Edition, 1996.

Thank you for your attention !!!