AE 240/240M/713 Practice Problem Set No. 03 (Part-1) Basic Orbital Solutions

1. Geosynchronous circular orbit is used for communication satellites which appear stationary with respect to a fixed point on earth's surface. Determine its altitude above the surface of the earth.
2. Determine the eccentricity of an elliptic orbit, which has (r_p/r_a) = 0.5.
3. Determine the velocity at perigee for an orbit with a = 7.0×10^6 m and e = 0.35.
4. A spacecraft is injected at an altitude of 400 km above earth's surface, with a velocity of 8.5 km/s, which is at an angle of 5° with the local horizon. Determine the semi-major axis and eccentricity of the orbit formed. In what way would the orbit change if the angle with the local horizon is zero?
5. Halley's Comet passed its perihelion last time on February 9, 1986. It has its semi-major axis of 17.834 AU and eccentricity of 0.96714. Solve Kepler's equation and determine eccentric, mean and true angles as well as the radial distance from sun for March 24, 2020.

Astronomical Constants

R_E = 6,378 km; 1 AU = 1.495978×10^11 m;

Sidereal Day = 23h 56m 4.09s = 23.94 hrs

1TU=806.8s; μ_E =1DU^3 / 1TU^2 = 1 μ_{Earth} = 3.986005x10^14 m^3 s^-2;

μ_{Sun} = 1.32713x10^20 m^3 s^-2; 1DU=6,378km 1SU=7,905m/s;

Solutions:

1.

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$$a = \left[\mu\left(\frac{T}{2\pi}\right)^2\right]^3$$

$$\text{Put} \quad T = 1440s; a = 42.164km$$

$$\text{Answer} \quad h = 35,786km$$

2.

$$\frac{r_p}{r_a} = \frac{(1-e)}{1+e} = 0.5$$
$$e = 1/3$$

3.

$$\begin{aligned}
v_p &= \sqrt{2\left(\varepsilon + \frac{\mu}{r_p}\right)} \\
&= \sqrt{2\left(-\frac{\mu}{2a} + \frac{\mu}{r_p}\right)} \\
&= \sqrt{2\left(-\frac{\mu}{2a} + \frac{\mu}{a(1-e)}\right)} \\
&= \sqrt{\frac{\mu}{a}\frac{(1+e)}{(1-e)}} \\
&= 10.875 km/s
\end{aligned}$$

4.

$$\begin{aligned}
h &= rv\sin(90^\circ - \theta) \\
&= 5.73 \times 10^{10} m^2/s^2; \\
\varepsilon &= \frac{v^2}{2} - \frac{\mu}{r} \\
&= -2.2743 \times 10^7; \\
e &= \sqrt{1 + \frac{2\varepsilon h^2}{\mu^2}} \\
&= 0.24493 \\
a &= \frac{h^2}{\mu}\frac{1}{1-e^2} \\
&= 10.90 \times 10^6 m
\end{aligned}$$

5.

$$
\begin{aligned}
M_p = E_p = \theta_p &= 0 \\
\Delta t &= 12462\text{days} = 1.076 \times 10^9 s; \\
n &= \sqrt{\frac{\mu}{a^3}} \\
&= 2.643 \times 10^{-9} rad/s; \\
M_{22\text{mar}2020} - M_p &= n \times \Delta t \\
M_{22\text{mar}2020} &= 2.8445 rad = 162.804^\circ; \\
M &= E - e \sin E \\
E_{22\text{mar}2020} &= 2.9902 rad \quad \text{solve Newton-Raphson} \\
\tan\frac{\theta}{2} &= \sqrt{\frac{1+e}{1-e}} \tan\frac{E}{2} \\
\theta_{22mar2020} &=?
\end{aligned}
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