Flight Mechanics/Dynamics

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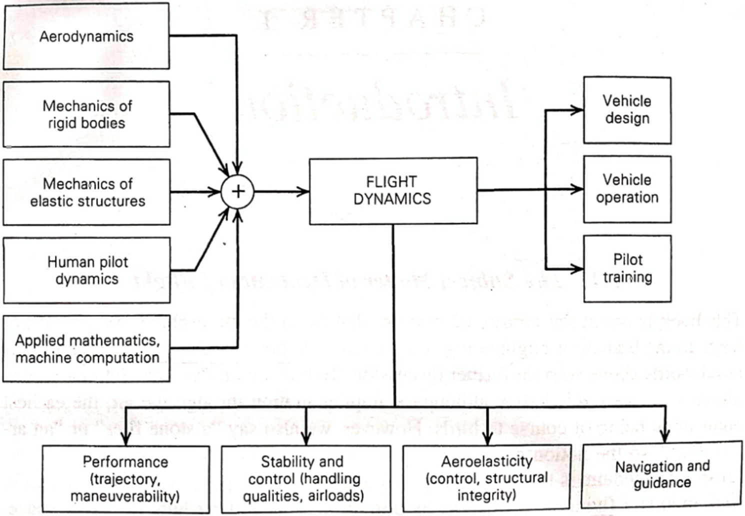
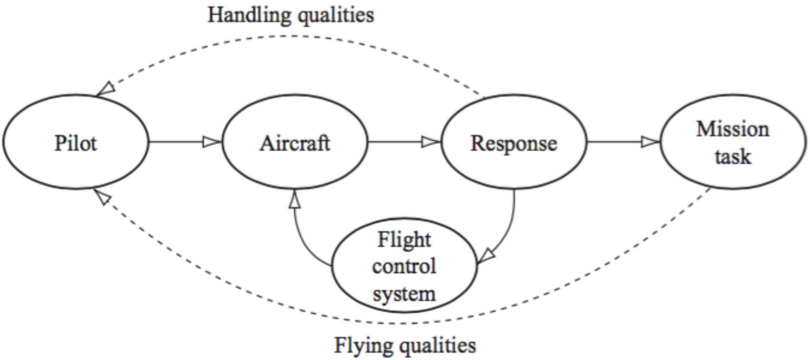
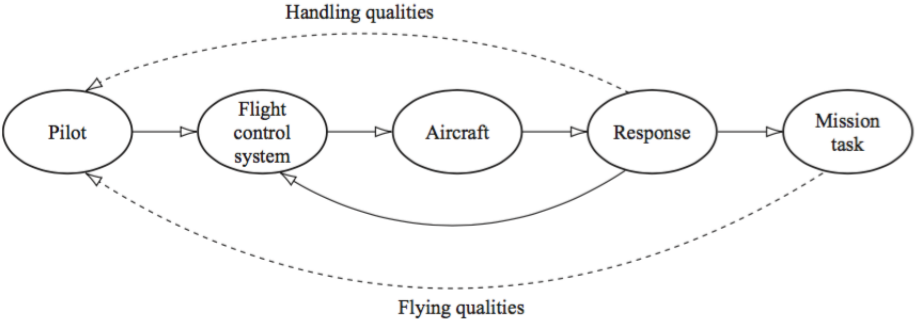


Aerodynamics
Mechanics of rigid bodies
Mechanics of elastic structures
Human pilot dynamics
Applied mathematics, machine computation
+
FLIGHT DYNAMICS
Vehicle design
Vehicle operation
Pilot training
Performance (trajectory, maneuverability)
Stability and control (handling qualities, airloads)
Aeroelasticity (control, structural integrity)
Navigation and guidance



Flying quality: How well the airplane carries a commanded task?

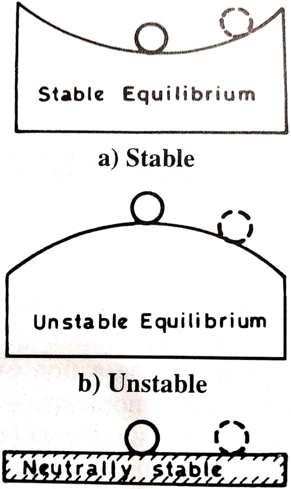
Handling quality: Adequacy of short term dynamic response to control in execution of flight task



Flying and handling quality deficiencies: Aerodynamic design of aircraft

Necessity of augmented flight control system (AFCS)

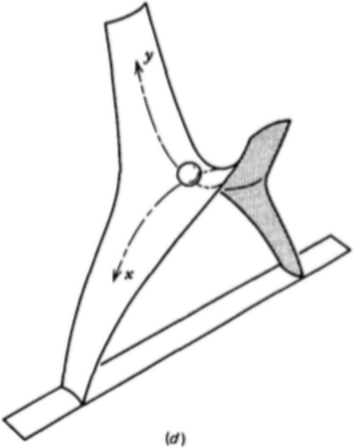
- Notions of stability: Static or Dynamic
- **Static stability**: If the forces and moments on the body caused by a disturbance tend initially to return the body toward its equilibrium position, the body is statically stable.
  - ⇒ No requirement of actual return of vehicle to equilibrium.
  - ⇒ If the forces and moments are such that the body continues to move away from its equilibrium position after being disturbed, the body is statically unstable.
- **Dynamic stability**: A body is dynamically stable if, of its own accord, it eventually returns to and remains at its equilibrium position over time.
- Static stability is related to initial tendency while dynamic stability focus on final state.



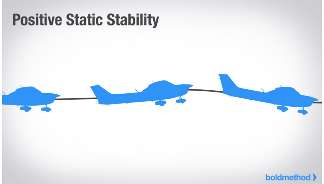
a) Stable

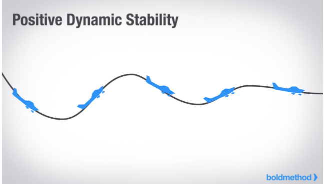
b) Unstable

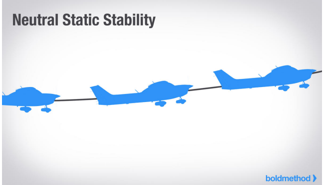
c) Neutrally stable

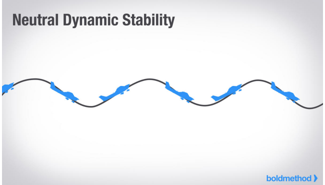


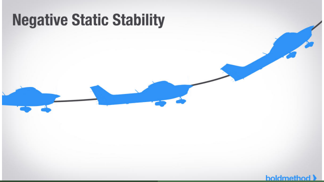
(d)

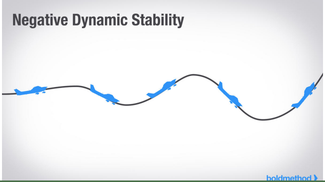

Positive Static Stability
boldmethod

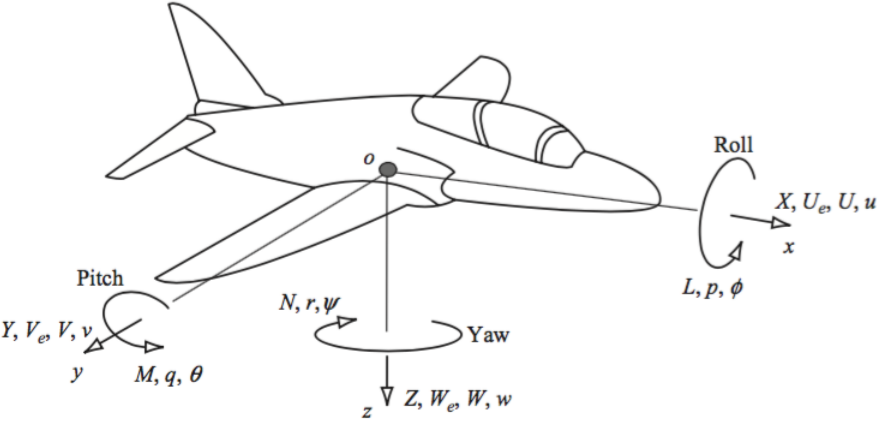

Positive Dynamic Stability
boldmethod


Neutral Static Stability
boldmethod

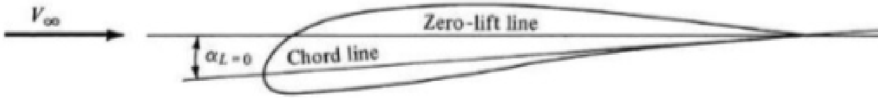

Neutral Dynamic Stability
boldmethod


Negative Static Stability
boldmethod

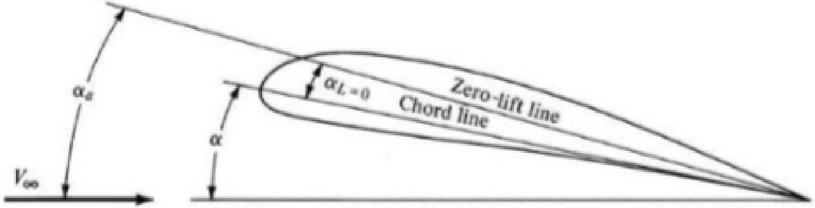

Negative Dynamic Stability
boldmethod


o
Roll
X, U_e, U, u
x
L, p, ϕ
Pitch
Y, V_e, V, v
y
M, q, θ
N, r, ψ
Yaw
z
Z, W_e, W, w

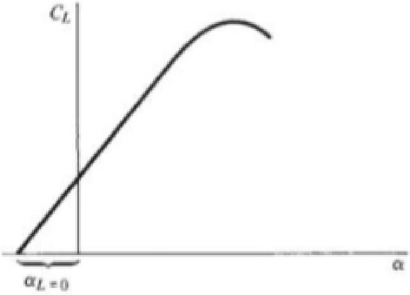
- **Static Stability**: Study about equilibrium points, not the dynamic motion
- Unsteady motion:
  - Longitudinal (symmetric) motion: level wings, C.G moves in vertical
  - Lateral (asymmetric) motion: Yawing, rolling, and sideslipping
- Separation valid for both static and dynamic stability
- Two aspects of the equilibrium state: stability and control
- **Stability**: Whether or not the pitching moment acts in such a sense as to restore the airplane to its original angle of attack.
- **Control**: Use of a longitudinal control (elevator) to change the equilibrium value of the angle of attack.
- Analysis is limited to angle of attack disturbance

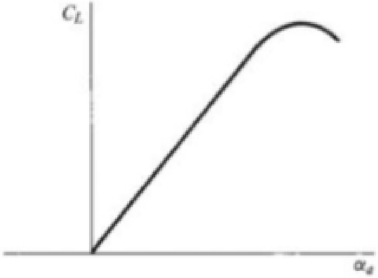

V_∞
Zero-lift line
$\alpha_{L=0}$
Chord line

(a)


α_a
$\alpha_{L=0}$
Zero-lift line
Chord line
α
V_∞

(b)


C_L
α
$\alpha_{L=0}$


C_L
α_a

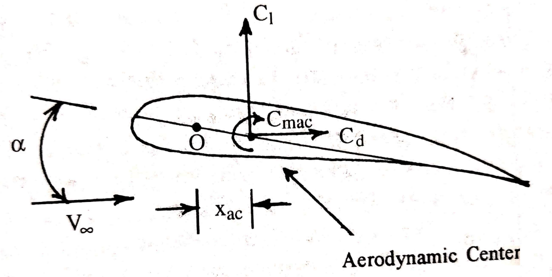
- For linear range of angle of attack α, wing pitching moment is

$$C_m = C_{m0} + \zeta C_l$$

where C_{m0} is pitching moment coefficient at zero-lift coefficient, ζ is empirical constant, depending on location of moment reference point (MRP).

- At zero-lift condition, lifts on lower and upper surface are equal and opposite.
- For symmetric airfoil, these forces act through same point $\implies C_{m0} = 0$.
- For cambered airfoil, it results in a pure couple, C_{m0}, independent of MRP.
- Sign of ζ: depend on location of MRP.
- For MRP near leading edge: $\zeta < 0$.
- For MRP near trailing edge: $\zeta > 0$.
- There is a point where $\zeta = 0$, called aerodynamic center (ac).

- Pitching moment about this aerodynamic center as MRP is $C_{mac} = C_{m0}$.
- What would be the value of C_{mac} in case of symmetric airfoil? Zero
- Can we obtain location of ac from a given lift, drag and pitching moment coefficients?
- Let O be the MRP about which we know these coefficients, c_m, and c_l, c_d,. x_{ac} be the distance from ac from O.
- Let C_{mac} be the pitching moment coefficient about ac, invariant of α.



- Pitching moment about given MRP

$$M = M_{ac} - x_{ac} L \implies \boxed{C_m = C_{m_{ac}} - \bar{x}_{ac} C_l}, \quad \bar{x}_{ac} = x_{ac}/c$$

- On differentiating w.r.t. C_l, one may get

$$\frac{dC_m}{dc_l} = -\bar{x}_{ac} \implies \boxed{\bar{x}_{ac} = -\frac{dC_m}{dc_l}}$$

- $\bar{x}_{ac}$ depends on slope of pitching moment.
- If $\frac{dC_m}{dc_l} > 0$, ac is located ahead MRP.
- If $\frac{dC_m}{dc_l} < 0$, ac is located behind MRP.
- If lift and moment coefficient vary nonlinearly with α then $\frac{dC_m}{dc_l}$ is variable and the concept of ac is not valid.

Example

A two-dimensional wing was tested in a wind tunnel, and the pitching moment coefficient about the leading edge at zero-lift was found to be -0.02. At $\alpha = 8°$, $C_l = 0.7, C_d = 0.04, C_{m,le} = -0.20$. Determine the location of aerodynamic center.

- We know that

$$C_m = C_{mac} - \bar{x}_{ac} C_l, \quad \bar{x}_{ac} = x_{ac}/c$$

- $C_{mac} = C_{m0} = -0.02, C_l = 0.7, C_m = -0.20$ at $\alpha = 8°$.
- Location of ac

$$\bar{x}_{ac} = \frac{C_{m0} - C_m}{C_l} = \frac{-0.02 - (-0.20)}{0.7} = 0.2571$$

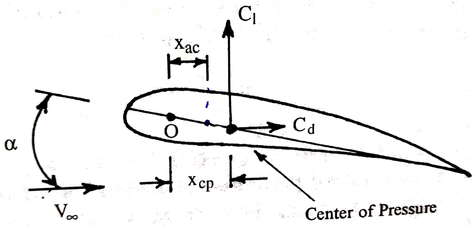
- The ac is location at 25.71$\%$ chord from the leading edge.

- For linear range of α, with $x_{cp} = \bar{x}_{cp}/c$,

$$C_m = C_{m0} + \frac{dC_m}{dc_l} C_l$$
$$= -\bar{x}_{cp} C_l$$

- We can write

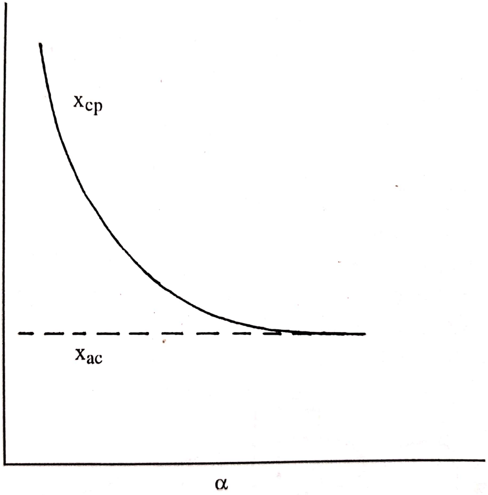
$$\bar{x}_{cp} = -\frac{C_{m0}}{C_l} - \frac{dC_m}{dc_l}$$



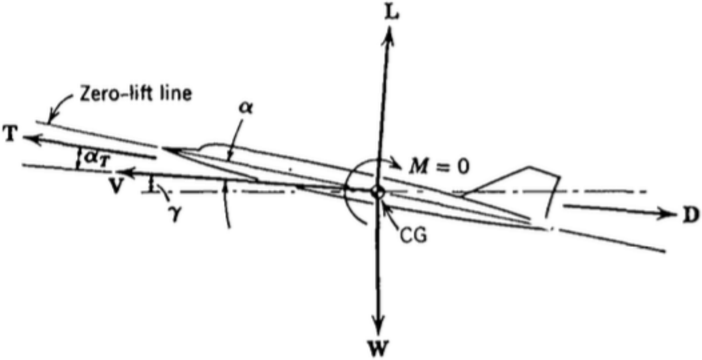
Relation between centre of pressure (cp) and aerodynamic center.

$$\boxed{\bar{x}_{cp} = \bar{x}_{ac} - \frac{C_{m0}}{C_l}}$$

- Location of ac w.r.t. cp: sign of C_{m0}
- For positive cambered airfoil, $C_{m0} < 0 \implies \bar{x}_{cp} > \bar{x}_{ac}$
- For high α, cop moves towards ac.

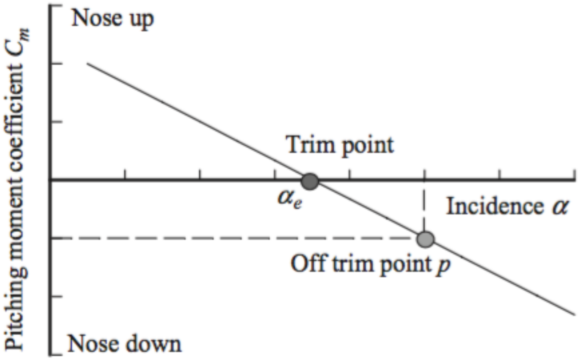


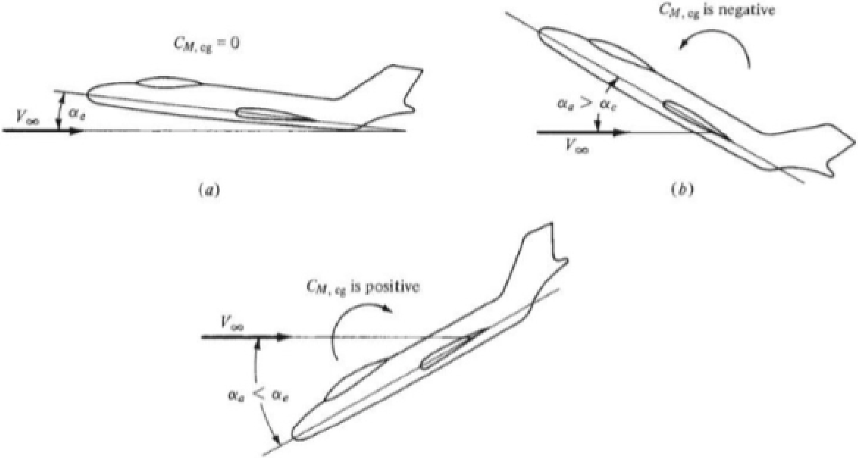
- Symmetric steady flight:

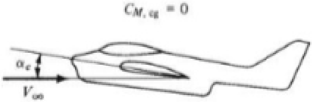


- Idealizations
  - Constant thrust: T independent of V
  - Constant power: TV independent of V

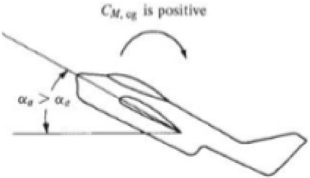
- An airplane can continue in steady unaccelerated flight only when the resultant external force and moment about the CG both vanish.
- Longitudinal balance: Zero pitching moment
- Nonzero pitching moment $\Longrightarrow$ Rotation in direction of unbalanced moment



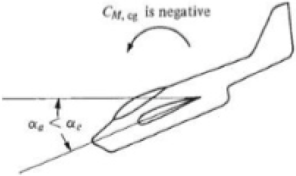

$C_{M,cg} = 0$
V_∞
α_e
(a)
$C_{M,cg}$ is negative
$\alpha_a > \alpha_e$
V_∞
(b)
$C_{M,cg}$ is positive
V_∞
$\alpha_a < \alpha_e$


$C_{M,cg} = 0$
α_e
V_∞

(a)


$C_{M,cg}$ is positive
$\alpha_a > \alpha_e$

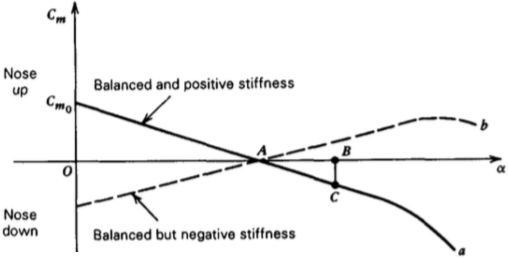
(b)


$C_{M,cg}$ is negative
$\alpha_a < \alpha_e$

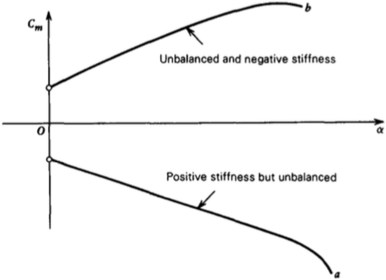
- Suppose that airplane is disturbed from its equilibrium attitude, the angle of attack being increased while its speed remains unaltered.
- A negative moment, whose magnitude corresponds to off trim point P.
- This moment tends to reduce the angle of attack to its equilibrium value, and hence is a restoring moment.
- Airplane has positive pitch stiffness: a desirable characteristic.
- Unstable: If moment acting when disturbed would be positive, or nose-up, and would tend to rotate airplane still farther from its equilibrium attitude.
- Pitch stiffness is determined by the sign and magnitude of the slope $\partial C_m/\partial\alpha$.
- For positive pitch stiffness at equilibrium α,

$$C_m = 0,\ C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} < 0$$

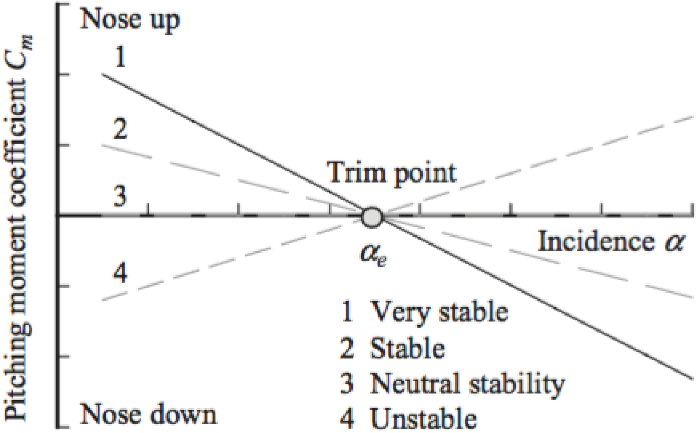
- What if one of these is not satisfied?



C_m
Nose up
C_{m_0}
Balanced and positive stiffness
b
A
B
O
α
C
Nose down
Balanced but negative stiffness
a

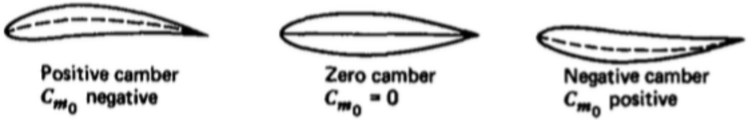


C_m
b
Unbalanced and negative stiffness
O
α
Positive stiffness but unbalanced
a



Nose up
1
2
3
4
Trim point
α_e
Incidence α
Pitching moment coefficient C_m
Nose down
1 Very stable
2 Stable
3 Neutral stability
4 Unstable

- What are the possible configurations for statically stable airplanes?
- $\partial C_m/\partial\alpha$ can be made negative for virtually any combination of lifting surfaces and bodies by placing the center of gravity far enough forward.
- What is the problem in ensuring static stability then?
- Airplane must be simultaneously balanced and have positive pitch stiffness.
- Any configuration with a positive C_{m_0} can satisfy the (limited) conditions for balanced and stable flight.



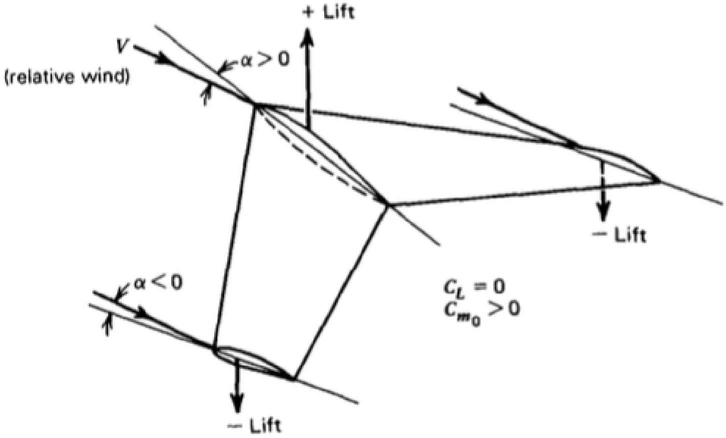
- In which of these cases, stable flights are possible with straight flying wing airplanes?
- Reasons for not using negative camber airplane?
  Lift, drag, dynamic characteristics, permissible CG range

How most of airplanes are using positive camber airfoil?

Any other possible configurations?

What about angle of attack for both configurations?

- Benefits of using canard
  - ⇒ The flow past canard is relatively free from wing or engine interference.
  - ⇒ Requirement of less lift to be produced by wing. Why is it so?
  - ⇒ Less induced drag generated from wing
- Issues
  - ⇒ As the wing is located relatively aft, the c.g. of the airplane moves aft and consequently the moment arm for the vertical tail is small.
  - ⇒ Contribution of canard to C_{m_α} is positive i.e. destabilizing.



+ Lift
V
(relative wind)
α > 0
− Lift
α < 0
− Lift
$C_L = 0$
$C_{m_0} > 0$

Reference

1. John Anderson Jr., *Introduction to Flight*, McGraw-Hill Education, Sixth Edition, 2017.
2. Bernard Etkin and Llyod Duff Reid, *Dynamics of Flight Stability and Control*, John Wiley and Sons, Third Edition, 1996.

Thank you for your attention !!!