Quiz No. 01 Model Solutions

Question No. 01

It is required to launch a payload with an ideal burnout velocity of 9 km/s. The existing technological limitations restrict the I_{sp} to 300s. If the maximum rocket mass permissible on launch pad is 300 Tons, what is the maximum permissible inert mass of the rocket, if a 5 Ton payload is to be launched? Also, what is the resulting propellant fraction? ($g_0 = 9.81$ m/s^2).

Solution No. 01

Following is the applicable solution.

$$\frac{dV}{dt} = -\frac{\dot{m}}{m} g_0 I_{sp} \rightarrow \frac{dm}{m} = -\frac{1}{g_0 I_{sp}} dV \rightarrow \ln m = -\frac{V}{g_0 I_{sp}} \rightarrow \frac{m_b}{m_0} = e^{-\frac{\Delta V}{g_0 I_{sp}}}$$

$$\frac{m_b}{m_0} = e^{-\frac{9000}{9.81 \times 300}} = 0.047; \quad m_b = 0.047 \times 300 = 14.09T; \quad m_{inert} = 9.09T; \quad \frac{m_p}{m_0} = 0.953$$

Question No. 02

A rocket of lift-off mass m_0 and carrying propellant mass of m_p, having specific impulse, I_{sp}, is executing a rectilinear vertical motion under the assumptions of constant sea-level gravity, vacuum and has a propellant burn rate of β_0 at the start. However, due to fault in the engine, this burn rate increases slowly with a small constant rate of increment ($d\beta/dt$). Derive the expression for burnout time, t_b, (both exact and approximate, but correct up to first iteration), mass profile, m(t), and burnout velocity, V_b, using the approximate expression of the burnout time.

Solution No. 02

The **solution** is as follows.

$$\frac{dV}{dt} = -\frac{\dot{m}}{m} g_0 I_{sp} - g_0; \quad m = m_0 - \beta t; \quad \beta = \beta_0 + \dot{\beta} t$$

$$m = m_0 - \beta_0 t - \dot{\beta} t^2; \quad V(t) = g_0 I_{sp} \ln\frac{m_0}{m} - g_0 t; \quad V_b = g_0 I_{sp} \ln\frac{m_0}{m_b} - g_0 t_b$$

$$m_p = \beta_0 t_b + \dot{\beta} t_b^2 \rightarrow \dot{\beta} t_b^2 + \beta_0 t_b - m_p = 0; \quad t_b = \frac{\sqrt{\beta_0^2 + 4\dot{\beta} m_p} - \beta_0}{2\dot{\beta}}$$

However, as $\dot{\beta} \rightarrow 0$, equation is nearly of first order. Thus, we can employ the constant solution to extract approximate first iterative solution.

$$t_{b1} = \frac{m_p}{\beta_0}; \quad t_{b2} \approx t_{b1} - \dot{\beta} t_{b1}^2 = \frac{m_p}{\beta_0}\left(1 - \frac{\dot{\beta} m_p}{\beta_0^2}\right); \quad V_b = g_0 I_{sp} \ln\frac{m_0}{m_b} - g_0 \times \frac{m_p}{\beta_0}\left(1 - \frac{\dot{\beta} m_p}{\beta_0^2}\right)$$

$$m_b = m_0 - \beta_0 t_b - \dot{\beta} t_b^2 = m_0 - t_b\left(\beta_0 - \dot{\beta} t_b\right) = m_0 - \frac{m_p}{\beta_0}\left(1 - \frac{\dot{\beta} m_p}{\beta_0^2}\right) \times \left[\beta_0 - \dot{\beta}\frac{m_p}{\beta_0}\left(1 - \frac{\dot{\beta} m_p}{\beta_0^2}\right)\right]$$

Question No. 03

A rocket with m_0 = 100 Tons, m_p = 80 Tons (I_{sp} = 300 s), moves vertically under constant sea-level gravity (9.81 m/s^2) and constant burn rate β = 0.8 Tons / sec. Give the applicable expressions for velocity and altitude as functions of burn time, 't' and predict approximate dynamic pressure peak and corresponding altitude. Also, what is the magnitude of the average drag acceleration, as per the rectangular model of drag energy. (Hint: Neglect loss due to drag. Use atmospheric table given below for applicable density values. Solve for V, m & h for 20s, 30s, 40s & 50s. C_D = 1.0, S = 1 m^2).

Solution No. 03

The **formulation** of altitude is as given **below**.

$$V = g_0 I_{sp} \ln\frac{m_0}{m_0 - m_p} - g_0 t = 9.81\times\left[300\times\ln\left(\frac{100}{100-0.8\times t}\right)-t\right]$$

$$h = \frac{m_0 g_0 I_{sp}}{\beta}\left[(1-\Lambda)\ln(1-\Lambda)+\Lambda\right]-\frac{1}{2}\tilde{g}t^2$$

$$= \frac{100\times 9.81\times 300}{0.8}\left[(1-0.8t)\ln(1-0.8t)+0.8t\right]-\frac{1}{2}\times 9.81\times t^2$$

Solution No. 03

The **dynamic** pressure table is as **follows**.

t, sec	V, m/sec	h, km	ρ, kg/m^3	Q, N/m^2	m, kg
20	317.0	3.01	0.909	45,672	84
30	503.5	7.15	0.579	73,304	76
40	742.6	13.4	0.251	69,207	68
50	1,013	22.1	0.068	34,890	60

Solution No. 03

The **final** solution is as follows.

Dynamic pressure peak is approximately at the intersection of two lines, one line between t = 20s & 30s and the other line between t = 40s & 50s.

$$73304 + 6674.4(h - 7.15) - 3944(13.4 - h) = 69207 \rightarrow h = 9.08; \quad Q = 86{,}185$$

Further, the above **altitude** occurs approximately at t **= 33.1s**, at which instant, the propellant **consumed** is 26.47T. This results in the rocket mass, ‘m’ as **73.53T.**

Thus, the **peak** acceleration is 1.172 m/s^2 and the **average** acceleration is 0.586 m/s^2.