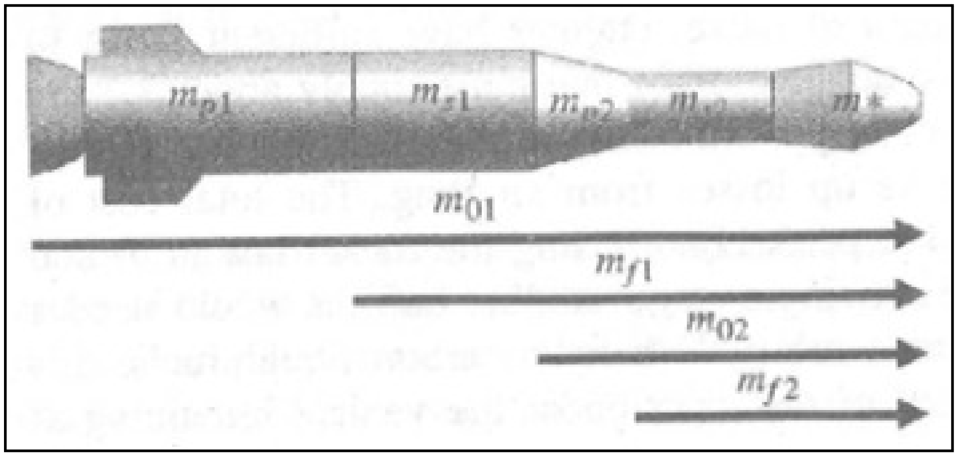
Multi-stage Configuration Design

Multi-stage Basic Concept

Consider the following **rocket configuration.**



Multi-stage Formulation

The **resulting** expressions for applicable **parameters** are as follows.

Lift-off Mass: $m_0 = \sum_{i=1}^{n} m_{pi} + \sum_{j=1}^{n} m_{sj} + m_*$

Stage-wise Starting Mass: $m_{0i} = m_{pi} + m_{si} + m_{0i+1}$

Stage-wise Ending Mass: $m_{fi} = m_{si} + m_{0i+1}$

Stage-wise Specific Impulse: I_{spi}

Multi-Stage Operation Strategy

For **i^{th} stage** operation of an **N-stage** rocket, the **sum** of masses of all the **stages** from 'i+1' to 'N' and the **final** payload mass is **treated** as its **payload**.

Inert mass of i^{th} stage is **separated** after its burnout but **before** the operation of the $i + 1^{th}$ **stage** is begun.

Multi-stage Design Variables

Three **staging** parameters are then **defined** as follows.

$$\pi_i = \frac{m_{0i+1}}{m_{0i}} \rightarrow \text{ Stage-wise Payload Ratio}$$

$$\varepsilon_i = \frac{m_{si}}{m_{si} + m_{pi}} \rightarrow \text{Stage-wise Structural Ratio}$$

$$I_{spi} \rightarrow \text{ Stage-wise Propellant Specific Impulse}$$

While, $\boldsymbol{\varepsilon_i}$ denotes the status of **structural** technologies and I_{spi} **captures** the status of **propulsion** technologies, π_i's indicate how $\mathbf{m_0}$ is distributed between **stages.**

Multi-stage Problem Analysis

Formulation starts by **defining** mission payload **ratio**, $\boldsymbol{\pi_*}$**,** in terms of **stage** payload ratios, $\boldsymbol{\pi_i}$, as follows.

$$\text{Payload Ratio: } \pi_* = \frac{m_*}{m_0} = \frac{m_*}{m_{0n}} \times \frac{m_{0n}}{m_{0(n-1)}} \times \cdots \times \frac{m_{02}}{m_{01}} = \prod_{i=1}^{n} \pi_i; \quad m_{01} = m_0$$

$$\text{In case of strap-on stage: } \pi_0 = \frac{m_{01}}{m_0}$$

In this **contex**t, generally, $\mathbf{I_{spi}}$ and $\boldsymbol{\varepsilon_i}$ are available as a set of **discrete** values, based on technological **options**. Thus, the design **solution** involves only the $\boldsymbol{\pi_i}$**'s** as unknowns.

Staging Problem Definition

Staging **problem** definition starts with **specifications** in terms of $\mathbf{m}_*$, and ideal burnout **velocity**.

This is **adequate** as all other trajectory **parameters** are related to it, and hence it can **represent** the requirements.

Multi-stage Formulation

Ideal **burnout velocity** for a multi-stage **rocket** is nothing but **sum** of ideal **velocity** increments as **expressed** below.

$$\text{Ideal Burnout Velocity: } V_0 = \sum_{i=1}^{n} \Delta V_i = -\sum_{i=1}^{n} g_0 I_{spi} \ln \frac{m_{0i} - m_{pi}}{m_{0i}}$$

As **masses** can be expressed in terms of $\boldsymbol{\pi_i}$**'s,** $\boldsymbol{\varepsilon_i}$**'s**, we can obtain the **velocity** also in terms of **these**, as shown next.

Multi-stage Formulation

V_0, in terms of I_{spi}, ε_i & π_i, can be **written** as follows.

$$\frac{m_{0i}-m_{pi}}{m_{oi}}=\frac{m_{si}+m_{0i+1}+m_{pi}-m_{pi}}{m_{oi}}=\frac{m_{0i+1}}{m_{0i}}+\frac{m_{si}}{m_{0i}}$$

$$=\pi_i+\left(\frac{m_{si}}{m_{si}+m_{pi}}\right)\times\left(\frac{m_{0i}-m_{0i+1}}{m_{0i}}\right)$$

$$=\pi_i+\varepsilon_i\times(1-\pi_i)=\varepsilon_i+\pi_i\times(1-\varepsilon_i)$$

$$V_0=-g_0\sum_{i=0}^{n}I_{spi}\ln\left(\pi_i+\varepsilon_i\times(1-\pi_i)\right)$$

$$=-g_0\sum_{i=0}^{n}I_{spi}\ln\left(\varepsilon_i+\pi_i\times(1-\varepsilon_i)\right)$$

Summary

Thus, to **summarize**, multi-stage configuration **design** is a simple algebraic **approach** that makes use of the **basic** staging philosophy.

We also **note** that by defining stage **payload** and structural ratios as the **design** variables, it is possible to create a **feasible** design methodology.