

Q.1 A two-stage rocket is required to achieve a  $V^*$  of 10000 m/s. Further, I and II stage use  $\epsilon$  of 0.1 and 0.2 respectively. However, ' $I_{sp}$ ' of both the stages is 350s. Determine  $\pi_i$ 's of each stage, using the approximate constrained optimization method for maximizing  $\pi^*$  and also obtain its value. (Hint: use constraint to express  $\pi_1$  in terms of  $\pi_2$ .  $g_0 = 9.81 \text{ m/s}^2$ ). (4)

Q.2 Consider a two-stage rocket with the following configuration.

$$\frac{m_{p1}}{m_{p1} + m_{s1}} = \frac{m_{p2}}{m_{p2} + m_{s2}} = 0.92; \quad m_{s1} = 4450 \text{ kg}; \quad I_{sp1} = 253 \text{ s}; \quad m_{s2} = 451 \text{ kg}; \quad I_{sp2} = 280 \text{ s}; \quad m_* = 2065 \text{ kg}$$

If the second stage fuel  $I_{sp}$  is changed to 350s, how much additional payload can be carried, assuming ideal burnout velocity to be constant? (Hint: all stage-wise mass values remain the same. Further, solutions, based on  $I_{sp}$  related derivative or through any other consistent methodology, in the context of question, are also admissible, as long as solution is close to the correct value.). (3)

Q.3 Ariane rockets are capable of directly launching a small satellite in GPS constellation (i.e. circular orbit at 20,000 km altitude above Earth's surface). In a particular mission, however, the rocket underperforms by 5% on its desired injection velocity, though the injection altitude and inclinations are as desired. What kind of orbit can be expected for the satellite under these conditions? Give orbit parameters and comment on the apogee and perigee altitudes. ( $R_E = 6,378 \text{ km}$ ;  $\mu = 3.986 \times 10^{14} \text{ m}^3/\text{s}^2$ ,  $g_0 = 9.81 \text{ m/s}^2$ ). (3)

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