Flight Mechanics/Dynamics

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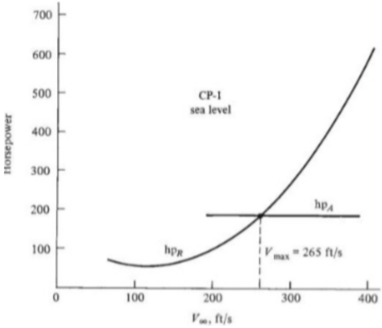
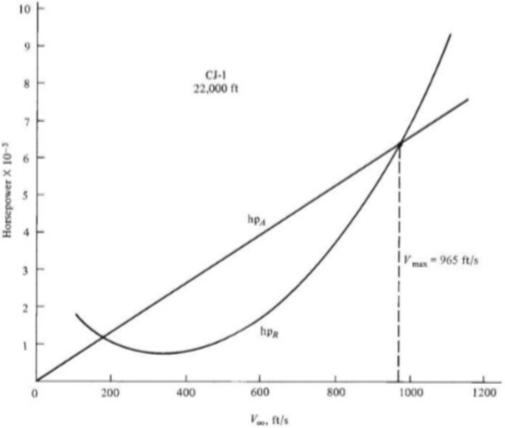


CP-1
sea level
Horsepower
700
600
500
400
300
200
100
0
100
200
300
400
hp_A
hp_R
V_{max} = 265 ft/s
V_∞, ft/s



CJ-1
22,000 ft
Horsepower × 10^{-3}
10
9
8
7
6
5
4
3
2
1
0
200
400
600
800
1000
1200
hp_A
hp_R
V_{max} = 965 ft/s
V_∞, ft/s

- How to compute required power at different altitudes, as densities at those altitudes are different?
- Assume that at sea-level,

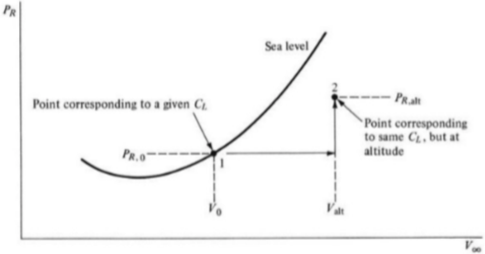
$$V_0 = \sqrt{\frac{2W}{\rho_0 S C_L}}, \quad P_{R,0} = \sqrt{\frac{2W^3 C_D^2}{\rho_0 S C_L^3}}$$

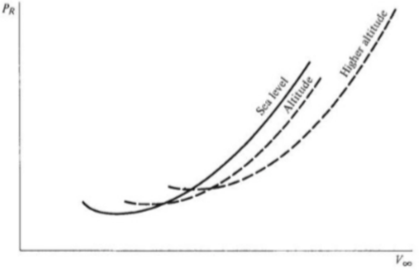
- At an altitude h with density ρ,

$$V_h = \sqrt{\frac{2W}{\rho S C_L}}, \quad P_{R,h} = \sqrt{\frac{2W^3 C_D^2}{\rho S C_L^3}}$$

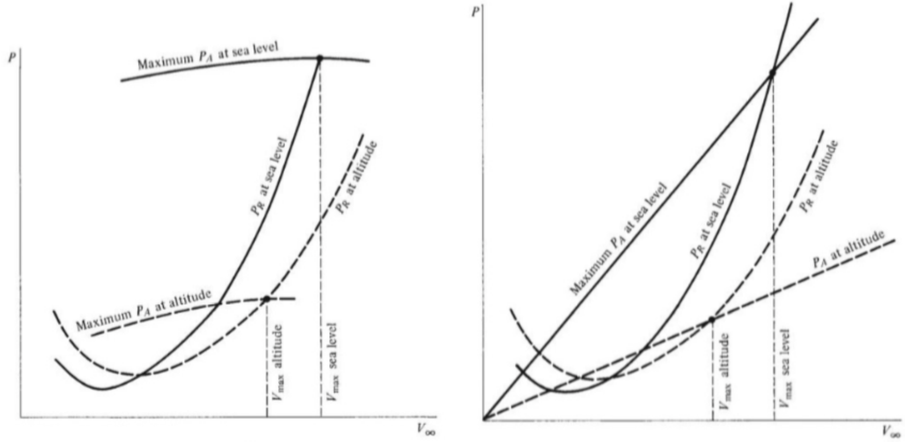
- Assuming C_L and thus C_D is fixed,

$$\boxed{V_h = V_0\sqrt{\frac{\rho_0}{\rho}}, \quad P_{R,h} = P_{R,0}\sqrt{\frac{\rho_0}{\rho}}}$$



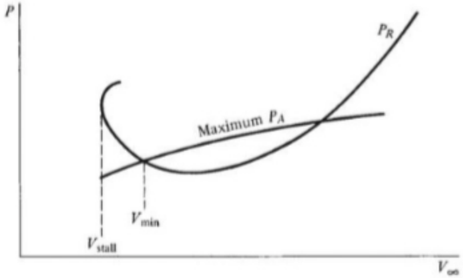


Upward and rightward translation as well as a slight clockwise rotation



Assumptions: $P_A \propto \rho,\ T_A \propto \rho$

- How is the low speed of aircraft determined?
- Low speed limit is dictated by stalling phenomenon.
- Is it the same for high altitude?
- At high altitude, low-speed limit may instead be determined by maximum P_A.
- At velocities just above stalling, P_R exceeds P_A.
- Stalling speed cannot be reached in level, steady flight, at high altitude.
- Minimum speed is governed by stalling or low speed intersection of power curves.



- What design aspects of the airplane dictate the maximum velocity?
- Thrust required

$$
\begin{aligned}
T = q_\infty S C_D &= q_\infty S \left(C_{D,0} + \frac{C_L^2}{\pi e \mathsf{AR}} \right) \\
&= q_\infty S \left(C_{D,0} + \frac{1}{\pi e \mathsf{AR}} \frac{W^2}{q_\infty^2 S^2} \right) = q_\infty S C_{D,0} + \frac{1}{\pi e \mathsf{AR}} \frac{W^2}{q_\infty S}
\end{aligned}
$$

- Quadratic equation in q_∞

$$
q_\infty^2 S C_{D,0} - q_\infty T + \frac{W^2}{S \pi e \mathsf{AR}} = 0 \implies q_\infty = \frac{T \pm \sqrt{T^2 - \frac{4 W^2 C_{D,0}}{\pi e AR}}}{2 S C_{D,0}}
$$

- Maximum available thrust, at full throttle, is $(T_A)_{\max}$.

- Maximum velocity

$$\boxed{V_{\max} = \left[\left(\frac{W}{S}\right)\left\{\left(\frac{T_A}{W}\right)_{\max} + \sqrt{\left(\frac{T_A}{W}\right)^2_{\max} - \frac{4C_{D,0}}{\pi e \mathsf{AR}}}\right\}\right]^{1/2} \bigg/ \sqrt{\rho_\infty C_{D,0}}}$$

- Maximum thrust-to-weight ratio: $\left(\dfrac{T_A}{W}\right)_{\max}$
- Wing loading: $\dfrac{W}{S}$
- Maximization of velocity
  - $\Rightarrow$ Increasing maximum thrust-to-weight ratio
  - $\Rightarrow$ Increasing wing loading
  - $\Rightarrow$ Decreasing zero-lift drag coefficient
- $\pi e \mathrm{AR} = 4C_{D,0}(L/D)^2_{\max} \implies (L/D)_{\max}$ is also important here.
- Imagine decreasing wind loading by reducing S

Example

A flying wing with an area of 27.75 m^2 has a NACA 2412 airfoil section. The mass of flying wing is 2270.663 kg and AR is 6. For level flight at an altitude of 1500 m and a velocity of 160 km/h, determine the angle of attack, induced drag coefficient, and the drag. Assume $e = 0.95$. For NACA 2412, $a_0 = 0.104/\text{deg}$ $C_{D0} = 0.0060, \rho = 1.0584$.

- Speed of aircraft: 160 kmph = 44.44 m/s, AR =6.
- We know that

$$a = \frac{a_0}{1 + \frac{57.3 a_o}{\pi A R e}} = 0.079/\text{deg}$$

- Lift coefficient

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} = \frac{W}{\frac{1}{2}\rho V^2 S} = \frac{2270.663 \times 9.81}{\frac{1}{2} \times 1.0584 \times 44.44^2 \times 27.75} = 0.768$$

- Angle of attack

$$\alpha = \frac{C_L}{a} = \frac{0.768}{0.079} = 9.7203 \text{ deg}$$

- Induced drag coefficient

$$C_{Di} = \frac{C_L^2}{\pi ARe} = \frac{0.768^2}{\pi \times 6 \times 0.95} = 0.03294$$

- Total drag coefficient

$$C_D = C_{D0} + C_{Di} = 0.006 + 0.03294 = 0.03894$$

- Drag acting on aircraft

$$D = \frac{1}{2}\rho V^2 S C_D = \frac{1}{2} \times 1.0584 \times 44.44^2 \times 27.75 \times 0.03894 = 1129.3456$$

- High aspect ratio implies a lower $C_{D,i}$ and thus higher $(L/D)_{\max}$.
- For an airplane in steady, level flight, which design parameters dictate the induced drag itself (not just $C_{D,i}$)?
- Induced drag

$$\boxed{D_i = q_\infty S \frac{C_L^2}{\pi e \mathsf{AR}} = q_\infty S \left[\frac{W}{q_\infty S}\right]^2 \frac{S}{\pi e b^2} = \frac{1}{\pi e q_\infty}\left(\frac{W}{b}\right)^2}$$

- Induced drag $\propto (\text{Span loading})^2$.
- Induced drag $\downarrow$ as wing span $b\uparrow$, Why?
- Span loading, wing loading, and AR are related as

$$\boxed{\underbrace{\frac{W}{b}}_{\text{Span loading}} = \underbrace{\left[\frac{W}{S}\right]}_{\text{Wing loading}} \frac{b}{\mathsf{AR}}}$$

- Zero-lift drag $D_0 = q_\infty S C_{D,0}$
- Ratio of induced drag to zero-lift drag

$$\frac{D_i}{D_0} = \left[\frac{1}{\pi e q_\infty}\left(\frac{W}{b}\right)^2\right]\frac{1}{q_\infty S C_{D,0}}$$

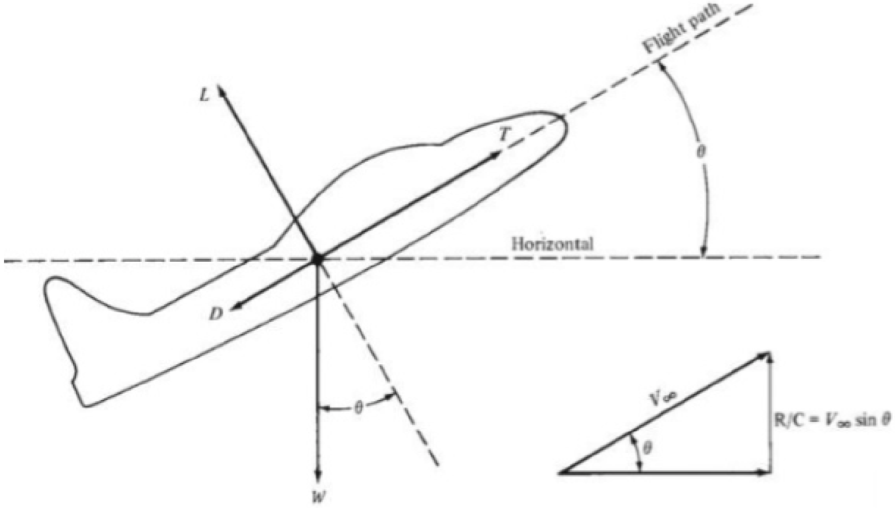
- We can rewrite

$$\frac{(W/b)^2}{S} = \frac{(W/S)^2}{(b^2/S)} = \frac{(W/S)^2}{\mathsf{AR}}$$

- Ratio of induced drag to zero-lift drag reduces to

$$\boxed{\frac{D_i}{D_0} = \frac{1}{\pi e q_\infty^2 C_{D,0}}\frac{(W/S)^2}{\mathsf{AR}}}$$

- $\uparrow$ in AR will $\downarrow$ D_i, relative to D_0.
- AR predominantly controls D_i/D_0, whereas span loading controls D_i.



Flight path
L
T
θ
Horizontal
D
θ
W
V_∞
θ
R/C = $V_\infty \sin\theta$

- Equation of motion for steady, climbing flight

$$T = D + W \sin\theta, \;\; L = W \cos\theta$$

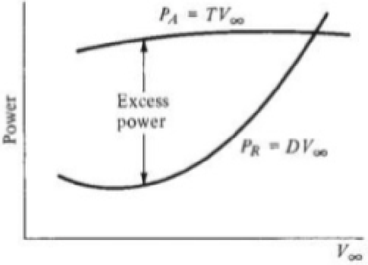
- For such flight, vertical velocity (rate of climb R/C),

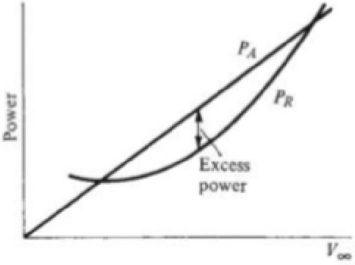
$$\boxed{R/C = V_\infty \sin\theta = \frac{TV_\infty - DV_\infty}{W}}$$

- Power required is not only DV_∞ because power is required to compensate for weight as well.
- Excess power

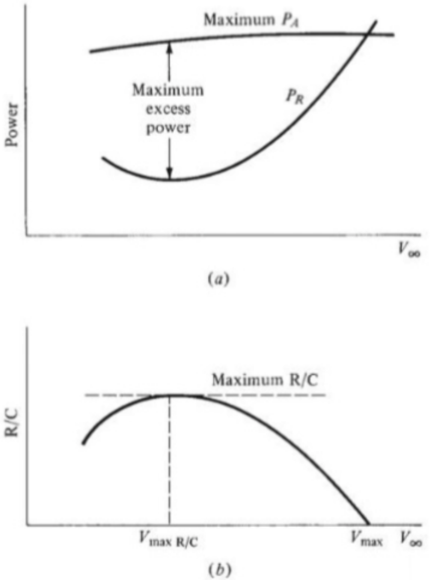
$$\text{Excess power} = TV_\infty - DV_\infty$$

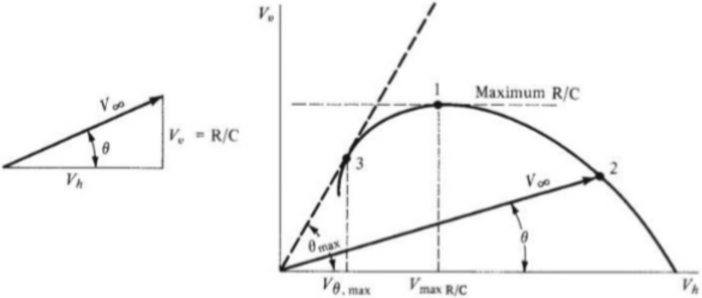
- Rate of climb $\boxed{R/C = \dfrac{\text{Excess power}}{W}}$





- Drag is smaller for climbing flight than level flight at same speed. Why?
- Lift and thus induced drag are small.
- For a piston engine-propeller combination, large excess powers are available at low V_∞ just above the stall. Any benefit?
- During landing, this gives a comfortable margin of safety.
- For jet aircraft at low V_∞, R/C is small.


Maximum P_A
Maximum excess power
P_R
Power
V_∞
(a)
Maximum R/C
R/C
$V_{\max R/C}$
$V_{\max}$
V_∞
(b)



- A horizontal tangent defines the point of maximum R/C.
- As line joining point 2 and origin is rotated counterclockwise, R/C first increases, then goes through its maximum, and finally decreases.
- Tangent line gives the maximum climb angle for which the airplane can maintain steady flight.
- Maximum R/C does not occur at maximum climb angle.

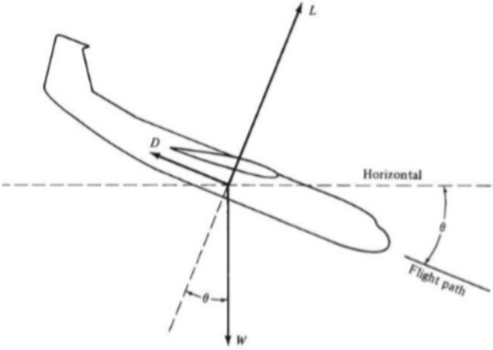
- For such flight, $T = 0$.

$$D = W \sin\theta, \quad L = W \cos\theta$$

- Glide angle

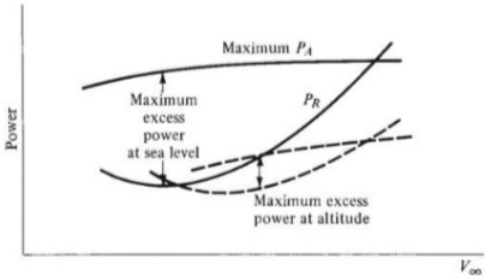
$$\theta = \tan^{-1}\left(\frac{1}{L/D}\right)$$

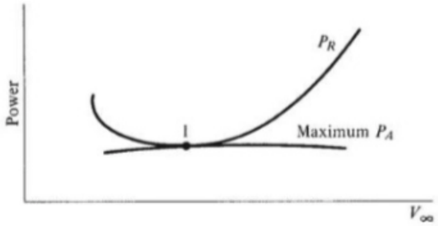
- Glide angle is strictly a function of L/D ratio.
- What would be the L/D for maximizing range?

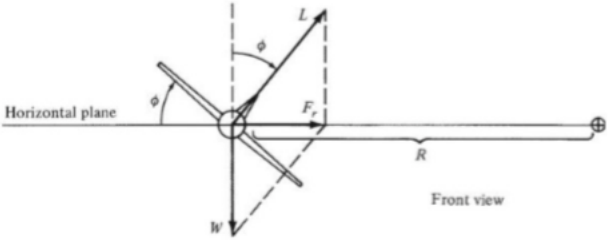


Higher L/D ensures the shallower glide angle, and thus maximum range.

- How does R/C vary with altitude?
- Maximum excess power decreases with altitude.
- Maximum R/C also decreases.
- **Absolute ceiling**: Altitude at which maximum $R/C = 0$.
- **Service ceiling**: Altitude where maximum R/C =100 ft/min.







- Resultant force $F_r = \sqrt{L^2 - W^2} = L \sin\phi$
- Define load factor $n = L/W$
- Using Newton's law

$$F_r = m\frac{V_\infty^2}{R} = \frac{WV_\infty^2}{gR} \implies \boxed{R = \frac{V_\infty^2}{g\sqrt{n^2-1}}}$$

- Turn rate

$$\boxed{\omega = \frac{V_\infty}{R} = \frac{g\sqrt{n^2-1}}{V_\infty}}$$

- Resultant force
  $F_r = L \mp W = (n \mp 1)W$
- For pullup flight

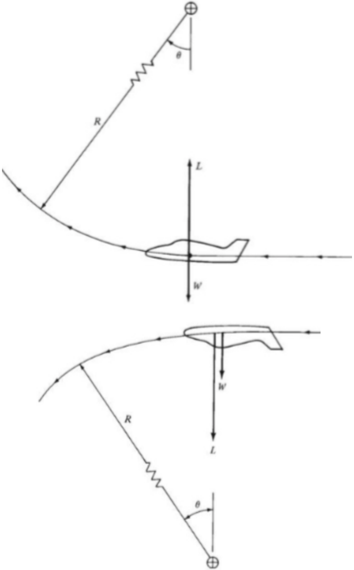
$$R = \frac{V_\infty^2}{g(n-1)}, \quad \omega = \frac{g(n-1)}{V_\infty}$$

- For pulldown flight

$$R = \frac{V_\infty^2}{g(n+1)}, \quad \omega = \frac{g(n+1)}{V_\infty}$$

- For large n,

$$\boxed{R = \frac{V_\infty^2}{gn}, \quad \omega = \frac{gn}{V_\infty}}$$

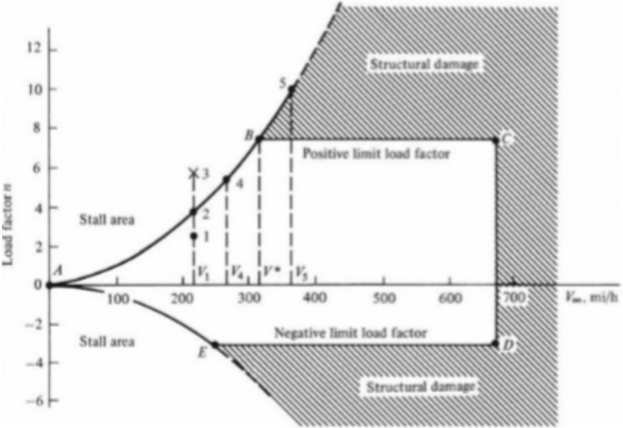


- Turn radius and turn rate, with $V_\infty^2 = \frac{2L}{\rho_\infty S C_L}$, are

$$R = \frac{2(W/S)}{\rho_\infty C_L g}, \quad \omega = g\sqrt{\frac{\rho_\infty C_L n}{2(W/S)}}$$

- Wing loading: W/S
- Airplanes with lower wing loadings will have smaller turn radii and larger turn rates, everything else being equal.
- For an airplane with a given wing loading, under what conditions will R be minimum and ω maximum?
- This will happen if both C_L and n are maximum.
- Best performance will occur at sea level, with maximum density.
- Maximum load factor depends on $C_{L,\max}$ at lower speed and structural design at higher speed.

$$\boxed{n_{\max} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{L,\max}}{W/S}}$$



Maneuver point: Point of highest possible C_L and $n \implies$ both $\omega_{\max}, R_{\min}$.

$$\boxed{V^\star = \sqrt{\frac{2n_{\max}}{\rho_\infty C_{L,\max}}\frac{W}{S}}}$$

Reference

1. John Anderson Jr., *Introduction to Flight*, McGraw-Hill Education, Sixth Edition, 2017.

Thank you for your attention !!!