Sphere of Influence Concept

Planetary Limit Solution

Non-Keplerian solution provides a **framework** to set up and understand the **inter-planetary** travel.

In this context, **equilibrium** solutions (Lagrange points) play an **important** role in providing **approximate** locations to place an object for the **next** stage.

Planetary Limit Solution

Lagrange points represent the planetary **limits,** beyond which, the **object** would be on an **inter-planetary** path**.**

Sphere of Influence / Activity (**SOI**) is a re-statement of the above result, in a **simplified** format**.**

Concept of Sphere of Activity

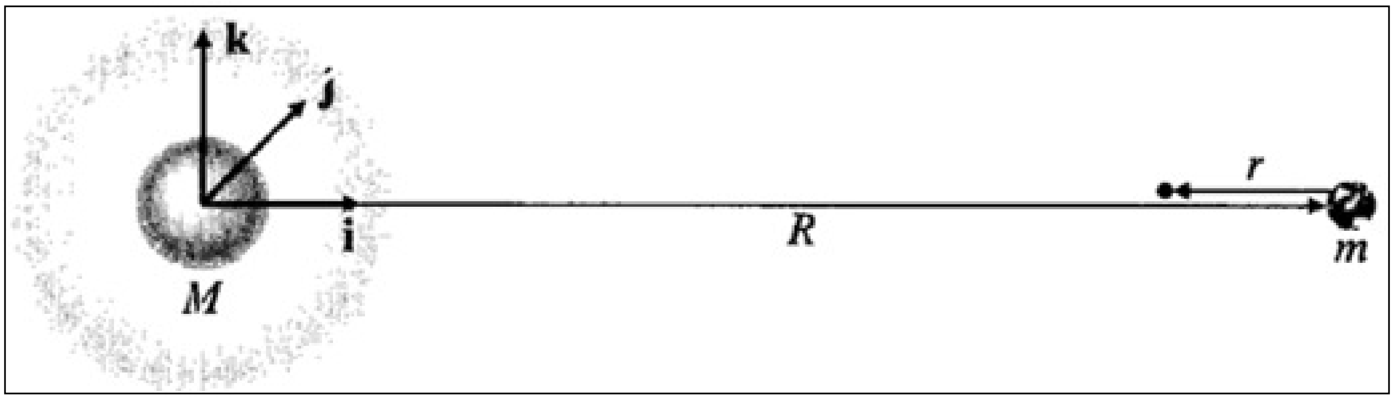
As per the **non-Keplerian** formulation, objects at **great** distances from **earth** are influenced by **sun/**other planets**.**

SOI defines the **distance** at which the **space** object is **considered** to have become **helio-centric.**

In reality, there is **no** such **sphere**, but it is still **possible** to arrive at a reasonabl**e approximation** for ‘far enough’, through the **steady-state** non-Keplerian formulation.

Sphere of Activity Formulation

Consider following **schematic** representing **sun (**‘M’**) &** **earth (**‘m’**),** at a distance of **‘R’** from each other, and a **test mass** (spacecraft) at a distance of **‘r’** from earth.



Further, both ‘**m**’ & **test** mass are **assumed** to be in **circular** orbit around ‘**M**’, in the same **plane**.

Sphere of Activity Formulation

The **forces** acting on the **test** mass and the **equilibrium** condition, are as follows**.**

Gravitational acceleration on test mass: $\bar{a}_g = -\dfrac{GM\hat{i}}{(R-r)^2} + \dfrac{Gm\hat{i}}{r^2}$

Angular velocity of planet (Circular orbit): $\bar{\omega} = \left(\dfrac{GM}{R^3}\right)^{1/2} \hat{k}$

Centripetal acceleration of test mass: $\bar{a} = \bar{\omega} \times \left(\bar{\omega} \times (R-r)\hat{i}\right)$

$$= -\frac{GM}{R^3}(\bar{R} - \bar{r})\hat{i}$$

Under equilibrium: $\bar{a} = \bar{a}_g \rightarrow -\dfrac{GM}{R^3}(R-r) = -\dfrac{GM}{(R-r)^2} + \dfrac{Gm}{r^2}$

Sphere of Activity Features

Under the **above** equilibrium, influence of the **sun** and **earth** are such that a **circular** orbit at a distance **'r'** from earth is an **equilibrium** solution**.**

However, it is also **seen** that this equilibrium is **unstable** in the sense that if the **object** is perturbed on either **side**, it will be either **earth** bound or sun **bound.**

Therefore, this distance '**r**' is treated as the **distance** at which the influence of **earth** ends and the influence of **sun** begins**.** (Lagrange Point L_1?)

Sphere of Activity Solution

We can write the **above** equation in terms of the distance ratio, **(r/R)** and mass ratio, **(m/M),** as follows.

$$\left(\frac{r}{R}\right)^5 - 3\left(\frac{r}{R}\right)^4 + 3\left(\frac{r}{R}\right)^3 - \lambda\left(\frac{r}{R}\right)^2 + 2\lambda\left(\frac{r}{R}\right) - \lambda = 0$$

Sphere of Activity Solution

Here, 'λ', is the mass ratio, (**m/M**), as employed in the restricted **3-body** solution and (r/R) is the **normalized** distance**.**

We note that there are **only** 3 real **roots** of the equation **seen** earlier**.**

Sphere of Activity Solution

As we aim to seek **solutions** close to earth (**L_1 or L_2**), (r/R) is a small **quantity** and thus, we can simplify the **equation** through binomial **expansion**, as shown below.

$$-\left(1-\frac{r}{R}\right)=-\frac{1}{\left(1-\frac{r}{R}\right)^2}+\frac{\lambda}{\left(\frac{r}{R}\right)^2}=-\left(1+2\frac{r}{R}\right)+\frac{\lambda}{\left(\frac{r}{R}\right)^2}$$

$$-\left(1-\frac{r}{R}\right)\left(\frac{r}{R}\right)^2=-\left(1+2\frac{r}{R}\right)\left(\frac{r}{R}\right)^2+\lambda;\quad 3\left(\frac{r}{R}\right)^3=\lambda\rightarrow\frac{r}{R}\approx\left(\frac{\lambda}{3}\right)^{1/3}$$

Typical SOIs for Solar System

Typical **values** of ‘r’ for different **planets** in our solar **system** are given in table **along side.**

Planet	AU	Meters	Nautical miles
Mercury	0.000747	1.117×10^8	60,710
Venus	0.00411	6.163×10^8	334,900
Earth	0.00618	9.245×10^8	502,500
Mars	0.00386	5.781×10^8	314,200
Jupiter	0.3222	4.820×10^{10}	26,200,000
Saturn	0.3761	5.627×10^{10}	30,580,000
Uranus	0.3457	5.172×10^{10}	28,110,000
Neptune	0.5792	8.664×10^{10}	47,090,000

Inter-planetary Travel Basics

Interplanetary Travel Concept

Interplanetary travel is concerned with motion of manmade objects when these **travel** through outer space, passing many **planets** in the process.

Such motions need **clear** understanding of the changing **nature** of forces as well as **strength** of the gravitational **field** of the planets involved.

While, we **need** general N-body **solutions**, as a 1st step, we employ **2-body** and restricted **3-body** solutions, in order to **understand** the basic nature of **motion.**

Interplanetary Travel Concept

It is **found** that good **solutions** to trajectories are **possible** by considering **these** as a sequence of multiple **2-body** segments, **'joined'** together at a suitable **common** point**.**

Thus, the **motion** of a spacecraft starting from **earth** and going to **Moon**, Mars, Jupiter etc. can be **captured** by solving successively, a number of **2-body** problems**.**

Concept of '**patched conics**', is used for a **smooth** transition between **segments**, while, SOI represents the **common** point**.**

Patched Conic Hypothesis

Patched conic hypothesis can be **investigated** as follows.

A **spacecraft**, escaping from a planet on a **hyperbolic** path**,** reaches the edge of **SOI** of that planet.

At this point, the **spacecraft** becomes **heliocentric**, which represents a point of **patching** between the two trajectories (**Departure**).

Patched Conic Hypothesis

Next, when the **spacecraft** reaches the **SOI** of the target **planet**, it again becomes **planeto-centric** and this point now **represents** the second patch **(Arrival).**

At **each** patch point, **velocity** is the '**patch**' parameter.

Summary

To **conclude**, sphere of influence, though a **conceptual** sphere, provides an important **reference** point for setting up **inter-planetary** missions.

We also **note** that interplanetary missions can be **adequately** modelled through the 2-body **problem** solutions, using the patched conic **hypothesis**.