Flight Mechanics/Dynamics

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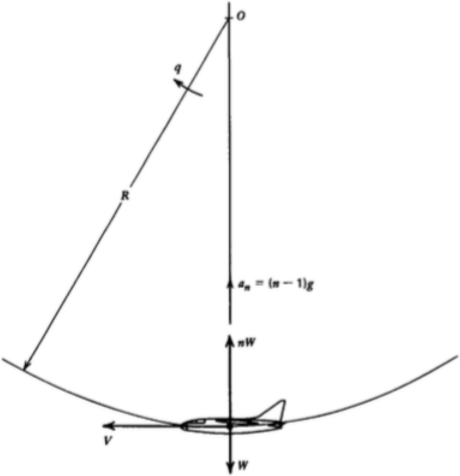
- Consider a pull up maneuver of airplane with load factor n.
- Horizontal flight path
- Net normal force $L - W = (n-1)W$
- Normal acceleration $a_n = (n-1)g$
- Assume that elevator angle and control force, when airplane is in straight horizontal flight, be denoted as δ_e and P, respectively.
- In pull-up flight, they are changed to $\delta_e + \Delta\delta_e$ and $P + \Delta P$, respectively.



- Measure of maneuverability
  - $\Rightarrow$ Elevator angle per g
  - $\Rightarrow$ Control force per g
- Elevator angle per $g = \dfrac{\Delta\delta_e}{(n-1)}$
- Control force per $g = \dfrac{\Delta P}{(n-1)}$
- Angular velocity

$$q = \frac{V}{R} = \frac{a_n}{V} = \frac{(n-1)g}{V}$$

- Field of relative air flow past airplane is curved, due to angular velocity.
- Curvature of flow field alters pressure distribution, and thus aerodynamic forces from their values in translational flight.

- Assuming q, $\Delta\alpha$ and $\Delta\delta_e$ small, the increments in lift and moment

$$\Delta C_L = C_{L_\alpha}\Delta\alpha + C_{L_q}\hat{q} + C_{L_{\delta_e}}\Delta\delta_e$$
$$\Delta C_m = C_{m_\alpha}\Delta\alpha + C_{m_q}\hat{q} + C_{m_{\delta_e}}\Delta\delta_e,$$

where $\hat{q} = \dfrac{q\bar{c}}{2V}$, $C_{L_q} = \dfrac{\partial C_L}{\partial \hat{q}}$, and $C_{m_q} = \dfrac{\partial C_m}{\partial \hat{q}}$.

- Angular velocity $\hat{q}$

$$\hat{q} = (n-1)\frac{g\bar{c}}{2V^2} = (n-1)\frac{C_W}{2\mu},$$

where weight coefficient $C_W = \dfrac{W}{(1/2)\rho V^2 S}$, mass ratio $\mu = \dfrac{2m}{\rho S\bar{c}}$

- As curved flight is assumed to be steady, $\Delta C_m = 0$.
- Increment in lift coefficient

$$\Delta C_L = \frac{(n-1)W}{(1/2)\rho V^2 S} = (n-1)C_W$$

- Assuming q, $\Delta\alpha$ and $\Delta\delta_e$ small, increment in lift and moment

$$C_{L_\alpha}\Delta\alpha + C_{L_q}\hat{q} + C_{L_{\delta_e}}\Delta\delta_e = (n-1)C_W$$
$$C_{m_\alpha}\Delta\alpha + C_{m_q}\hat{q} + C_{m_{\delta_e}}\Delta\delta_e = 0$$

- On substituting for $\hat{q}$,

$$C_{L_\alpha}\Delta\alpha + C_{L_{\delta_e}}\Delta\delta_e = (n-1)C_W\left[1 - \frac{C_{L_q}}{2\mu}\right]$$
$$C_{m_\alpha}\Delta\alpha + C_{m_{\delta_e}}\Delta\delta_e = -(n-1)C_{m_q}\frac{C_W}{2\mu}$$

- On solving these equations,

$$\frac{\Delta\delta_e}{n-1} = -\frac{C_W}{\text{det}}\left[C_{m_\alpha} - \frac{1}{2\mu}(C_{L_q}C_{m_\alpha} - C_{L_\alpha}C_{m_q})\right]$$
$$\frac{\Delta\alpha}{n-1} = \frac{1}{C_{L_\alpha}}\left[C_W\left(1 - \frac{C_{L_q}}{2\mu}\right) - C_{L_{\delta_e}}\frac{\Delta\delta_e}{n-1}\right]$$

- As determinant is independent of CG position, variation of $\Delta\delta_e/(n-1)$ w.r.t. h depends on numerator only.

- Using $C_{m_\alpha} = C_{L_\alpha}(h - h_n)$,

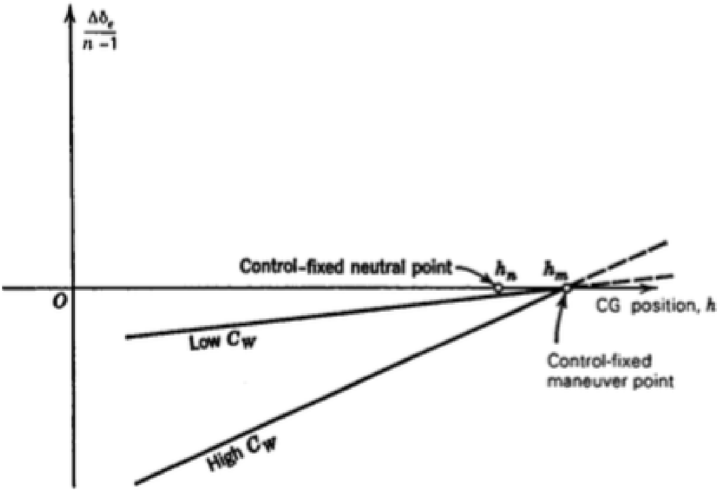
$$\frac{\Delta\delta_e}{n-1} = -\frac{C_W C_{L_\alpha}(2\mu - C_{L_q})}{2\mu\,\mathrm{det}}\left[h - h_n + \frac{C_{m_q}}{2\mu - C_{L_q}}\right]$$

- Both C_{L_q} and C_{m_q} vary with h, linearly and quadratically, respectively.
- Variation of $\Delta\delta_e/(n-1)$ is not linear in h.
- For tailed aircraft, $C_{L_q} \ll 2\mu$, $dC_{m_q} \approx 0$, and thus relation is linear.
- For tailless aircraft, variation has more curvature.
- **Control-fixed maneuver point** (h_m): point where $\Delta\delta_e/(n-1) = 0$

$$\boxed{h_m = h_n - \frac{C_{m_q}(h_m)}{2\mu - C_{L_q}(h_m)}}$$

- If C_{L_q} and C_{m_q} are independent of h then

$$\frac{\Delta\delta_e}{n-1} = -\frac{C_W C_{L_\alpha}(2\mu - C_{L_q})}{2\mu\,\mathrm{det}}(h - h_m)$$



Control-fixed maneuver margin: $h_m - h$

- Incremental control force

$$\Delta P = G S_e \bar{c}_e \frac{1}{2}\rho V^2 \Delta C_{h_e}$$

- Increment in C_{he}, with $\Delta\delta_t = 0$

$$\Delta C_{he} = C_{he}\Delta\alpha + C_{he_q}\hat{q} + b_2\Delta\delta_e$$

- On substituting $\Delta\alpha$ and $\hat{q}$,

$$\begin{aligned}
\frac{\Delta C_{he}}{n-1} =& C_{he_\alpha}\frac{\Delta\alpha}{n-1} + C_{he_q}\frac{\hat{q}}{n-1} + b_2\frac{\Delta\delta_e}{n-1} \\
=& C_{he_\alpha}\frac{\Delta\alpha}{n-1} + C_{he_q}\frac{C_W}{2\mu} + b_2\frac{\Delta\delta_e}{n-1} \\
=& \frac{C_{he_\alpha}}{C_{L_\alpha}}\left(C_W - C_{L_q}\frac{C_W}{2\mu} - C_{L_{\delta_e}}\frac{\Delta\delta_e}{n-1}\right) + C_{he_q}\frac{C_W}{2\mu} + b_2\frac{\Delta\delta_e}{n-1} \\
=& \frac{C_W}{2\mu C_{L_\alpha}}\left[C_{he_\alpha}(2\mu - C_{L_q}) + C_{he_q}C_{L_\alpha}\right] + \frac{\Delta\delta_e}{n-1}\left[b_2 - \frac{C_{L_{\delta_e}}C_{he_\alpha}}{C_{L_\alpha}}\right]
\end{aligned}$$

- On substituting for $\Delta\delta_e$, we get

$$
\begin{aligned}
\frac{\Delta C_{he}}{n-1} =& \frac{C_W}{2\mu C_{L_\alpha}}\left[C_{he_\alpha}(2\mu - C_{L_q}) + C_{he_q}C_{L_\alpha}\right] \\
& - \left[b_2 - \frac{C_{L_{\delta_e}}C_{he_\alpha}}{C_{L_\alpha}}\right]\left[\frac{C_W C_{L_\alpha}(2\mu - C_{L_q})}{2\mu \text{ det}}(h - h_m)\right] \\
=& -\frac{C_W}{2\mu}\frac{a'b_2}{\text{det}}(2\mu - C_{L_q})(h - h_m')
\end{aligned}
$$

where control-free maneuver point, h_m', is given by

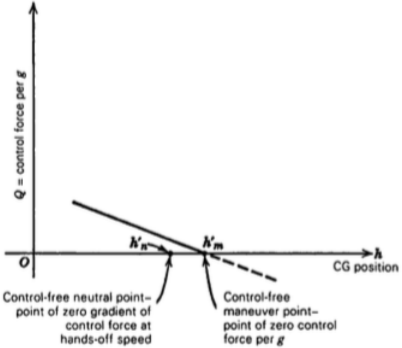
$$
\boxed{h_m' = h_m + \frac{\Delta}{a'b_2}\left(\frac{C_{he_\alpha}}{C_{L_\alpha}} + \frac{C_{he_q}}{2\mu - C_{L_q}}\right)}
$$

- Control-free maneuver margin: $h_m' - h$.
- Wing loading $w = \dfrac{W}{S} = C_W q_\infty$

- Control force per g

$$\boxed{Q = \frac{\Delta P}{n-1} = -GS_e\bar{c}_e \frac{w}{2\mu}\frac{a' b_2}{\text{det}}(2\mu - C_{L_q})(h - h'_m)}$$

- Result applicable for both tailed and tailless aircraft.



- What is the effect of high lift devices on stiffness?
- Several kinds of high lift devices such as flap, slots, boundary layer control, etc. are used.
- "Configuration-type" devices (flaps): Specific changes can always be incorporated with the appropriate changes to $h_{n_{wb}}$, $C_{m_{ac_{wb}}}$, $C_{L_{wb}}$.
- What are the effect of trailing edge flap?

  $\Rightarrow$ Distortion of shape of span-wise distribution of lift on wing, increasing vorticity behind the flap tips,

  $\Rightarrow$ Same effect locally as an increase in wing-section camber, i.e, a negative increment in $C_{m_{ac}}$, and a positive increment in $C_{L_{wb}}$.

  $\Rightarrow$ Increased downwash at the tail, both ϵ_0, and $\partial\epsilon/\partial\alpha$ will change.

- Change in wing-body moment coefficient

$$\Delta C_{m_{wb}} = \Delta C_{m_{ac_{wb}}} + \Delta C_{L_{wb}}(h - h_{n_{wb}})$$

- Change in airplane lift coefficient

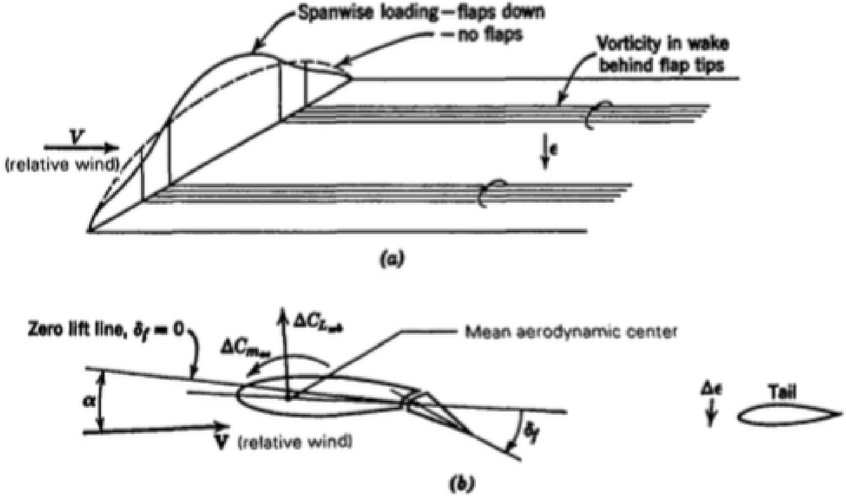
$$\Delta C_L = \Delta C_{L_{wb}} - a_t \frac{S_t}{S} \Delta \epsilon$$

- Change in tail pitching moment

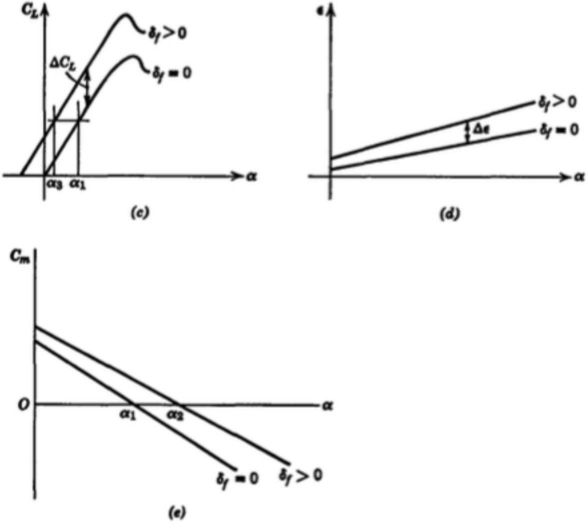
$$\Delta C_{m_t} = a_t V_H \Delta \epsilon$$

- Changes in lift-slope and stiffness, with $\Delta C_{m_{wb}}, \Delta C_{L_{wb}}$ constant w.r.t. α,

$$\Delta a = \Delta C_{L_\alpha} = -a_t \frac{S_t}{S} \Delta \frac{\partial \epsilon}{\partial \alpha}, \quad \Delta C_{m_\alpha} = (h - h_{n_{wb}})\Delta a + a_t \bar{V}_H \Delta \frac{\partial \epsilon}{\partial \alpha}$$

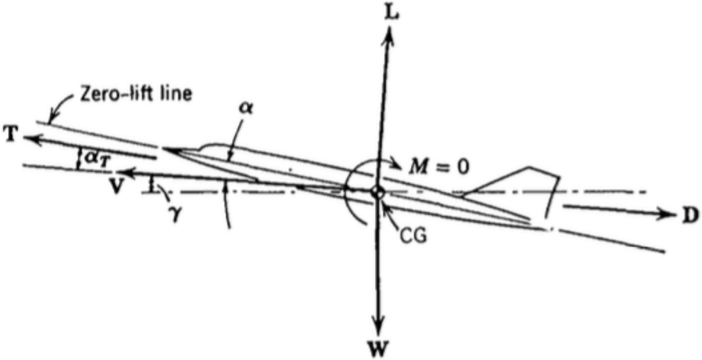


What would be the effect on trim condition?



Can we maintain trim speed, even with a flap operation?

- What is the effect of propulsion system on trim and stability?
- Types of propulsive units
  - $\Rightarrow$ Reciprocating-engine-driven propellers
  - $\Rightarrow$ Turbojets
  - $\Rightarrow$ Turboprops
  - $\Rightarrow$ Rockets
- Sufficient theoretical or empirical information do not exist to enable reliable predictions to be made under all conditions.
- Appropriate direct effect on C_{m_p} and $\dfrac{\partial C_{m_p}}{\partial \alpha}$, with indirect effect on wing-body and tail coefficients
- While calculating trim curves, thrust must be the one required to maintain equilibrium at condition of speed and angle of climb being investigated.



- Using force balance, with $\alpha_T \ll 1$

$$C_T = C_D + C_W \sin\gamma,\ C_W \cos\gamma = C_L + C_T \alpha_T$$

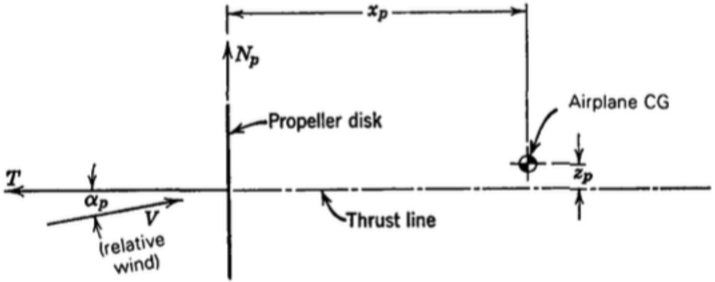
- Can we solve for C_T in terms of C_L, C_D?

- On solving for C_T

$$\boxed{C_T = \frac{C_D + C_L \tan\gamma}{1 - \alpha_T \tan\gamma}}$$

- For small angle assumption of α_T, $\alpha_T \tan\gamma \ll 1$, and

$$\boxed{C_T = C_D + C_L \tan\gamma}$$



- Let the thrust line be offset by a distance z_p from CG.
- Moment coefficient due to thrust

$$C_{m_p} = C_T \frac{z_p}{c} = (C_D + C_L \tan\gamma)\frac{z_p}{c}$$

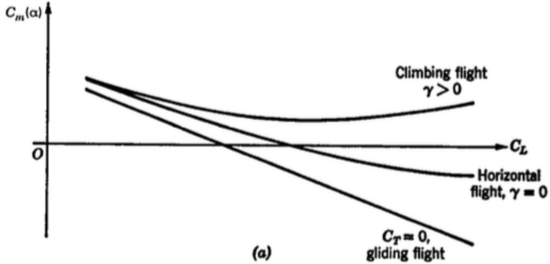
- Using equation for drag polar

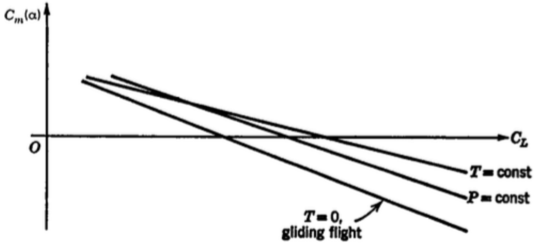
$$C_{m_p} = (C_{D_{\min}} + KC_L^2 + C_L \tan\gamma)\frac{z_p}{c}$$

- Thrust moment Tz_p is independent of α,

$$\boxed{\frac{\partial C_{m_p}}{\partial \alpha} = 0}$$

- No change in the NP from that for unpowered flight.


$C_m(\alpha)$
Climbing flight
$\gamma > 0$
O
C_L
Horizontal
flight, $\gamma = 0$
$C_T = 0$,
gliding flight
(a)


$C_m(\alpha)$
O
C_L
$T =$ const
$P =$ const
$T = 0$,
gliding flight

- If full throttle thrust does not change with speed then

$$C_{m_p} = \frac{T}{W} C_L \frac{z_p}{\bar{c}} \Rightarrow \boxed{\frac{dC_{m_p}}{dC_L} = \frac{T}{W}\frac{z_p}{\bar{c}}}$$

- In constant thrust case, C_m-C_L graph simply has its slope changed by the addition of thrust.
- If power is invariant then $T = P/V$ and

$$C_{m_p} = \frac{P}{VW}\frac{z_p}{\bar{c}} C_L = \frac{P}{W}\frac{z_p}{\bar{c}}\sqrt{\frac{\rho}{2w}} C_L^{3/2}$$

- What would be the slope $\frac{dC_{m_p}}{dC_L}$?

$$\boxed{\frac{dC_{m_p}}{dC_L} = \frac{3P}{2W}\frac{z_p}{\bar{c}}\sqrt{\frac{\rho}{2w}} C_L^{1/2}}$$

- In constant power case, the shape is also changed.

- Resultant force on propeller: T along axis, and N_p in plane of propeller.
- Change in moment due to N_p

$$\Delta C_m = C_{N_p} \frac{x_p}{\bar{c}} \frac{S_p}{S}, \quad C_{N_p} = \frac{N_p}{(1/2)\rho V^2 S_p}$$

- To get total moment for several propellers, increments must be calculated for each and summed.
- For small angles, $C_{N_p} \propto \alpha_p$.

$$\boxed{\frac{\partial C_{m_p}}{\partial \alpha} = \frac{x_p}{\bar{c}} \frac{S_p}{S} \frac{\partial C_{N_p}}{\partial \alpha_p} \frac{\partial \alpha_p}{\partial \alpha}}$$

- If the propellers were situated far from flow field of wing, $\dfrac{\partial \alpha_p}{\partial \alpha} = 1$.

- For wing-mounted propellers with propeller plane close to wing, there is a strong upwash ϵ_p, at propeller.

$$\alpha_p = \alpha + \epsilon_p + \text{constant}, \quad \boxed{\frac{\partial \alpha_p}{\partial \alpha} = 1 + \frac{\partial \epsilon_p}{\partial \alpha}}$$

where the constant is angle of attack of propeller axis relative to airplane zero-lift line.

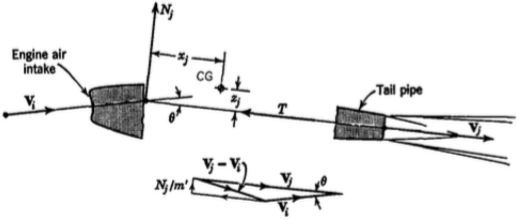
- Now, $\dfrac{\partial C_{m_p}}{\partial \alpha}$ reduces to

$$\boxed{\frac{\partial C_{m_p}}{\partial \alpha} = \frac{x_p}{\bar{c}} \frac{S_p}{S} \left(1 + \frac{\partial \epsilon_p}{\partial \alpha}\right) \frac{\partial C_{N_p}}{\partial \alpha_p}}$$

- Air that passes through a propulsive duct experiences changes in both direction and magnitude of its velocity.
- Change in magnitude is the principal source of thrust, and direction change entails a force $\perp$ thrust line.
- Using momentum principle, reaction of air flow on the airplane

$$\boldsymbol{F} = -m'(\boldsymbol{V}_j - \boldsymbol{V}_i) + \boldsymbol{F}',$$

where $\boldsymbol{F}'$ is resultant of pressure forces across inlet and outlet areas.



- It is assumed that V_i has that direction which the flow would take in the absence of the engine.
- What would be the component of $\boldsymbol{F} \perp$ thrust line?

$$N_j = m' V_i \sin\theta \approx m' V_i \theta$$

- Angle θ equals the angle of attack of the thrust line plus upwash angle due to wing induction.

$$\boxed{\theta = \alpha_j + \epsilon_j}$$

- Speed V_i, with inlet area A_i and air density ρ_i, is obtained as

$$\boxed{V_i = \frac{m'}{A_i \rho_i}}$$

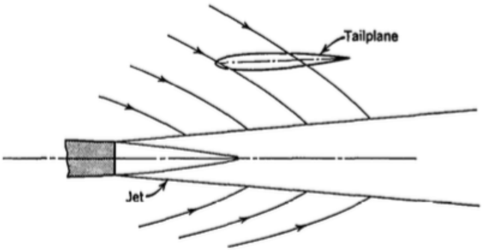
- Pitching moment coefficient

$$N_j = \frac{m'^2}{A_i \rho_i}(\alpha_j + \epsilon_j) \implies \boxed{\Delta C_m = \frac{m'^2}{A_i \rho_i} \frac{x_j(\alpha_j + \epsilon_j)}{(1/2)\rho V^2 S\bar{c}}}$$

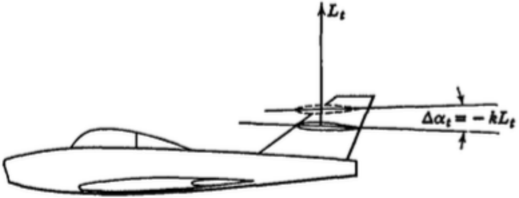
- As pitching moment varies wth α, change in C_{m_α}

$$\boxed{\Delta C_{m_\alpha} = \frac{m'^2}{A_i \rho_i} \frac{1}{(1/2)\rho V^2 S\bar{c}} \left[x_j \left(1 + \frac{\partial \epsilon_j}{\partial \alpha}\right) + \theta \frac{\partial x_j}{\partial \alpha} \right]}$$

- Quantities m' and ρ_i can be determined from the engine performance data.
- For subsonic flow, $\dfrac{\partial \epsilon_j}{\partial \alpha} = \dfrac{\partial \epsilon_p}{\partial \alpha}$.
- $\dfrac{\partial x_j}{\partial \alpha}$ can be calculated from the geometry.



- Consider the influence of fuselage flexibility on stiffness and control.



- Assume that L_t bends fuselage so that tail rotates through angle $\Delta\alpha_t = -kL_t$ while wing angle of attack remains unaltered.
- Net angle of attack of tail

$$\boxed{\alpha_t = \alpha_{wb} - \epsilon - i_t - kL_t}$$

- Tail lift coefficient with $\delta_e = 0$

$$C_{L_t} = a_t\alpha_t = a_t(\alpha_{wb} - \epsilon - i_t - kL_t) = a_t\left(\alpha_{wb} - \epsilon - i_t - k\frac{1}{2}\rho V^2 S_t C_{L_t}\right)$$

- On solving for tail lift coefficient

$$\boxed{C_{L_t} = \frac{a_t(\alpha_{wb} - \epsilon - i_t)}{1 + (ka_t\rho V^2 S_t)/2}}$$

- Tail effectiveness has been reduced by the factor $\boxed{\dfrac{1}{1 + (ka_t\rho V^2 S_t)/2}}$.
- Reduction is largest at high speed.
- Location of NP

$$h_n = h_{n_{wb}} + \frac{a_t}{a}\bar{V}_H\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right) - \frac{1}{a}\frac{\partial C_{m_p}}{\partial\alpha}$$

- Forward shift in NP

$$\Delta h_n = \frac{\Delta a_t \bar{V}_H}{a}\left(1 - \frac{\partial\epsilon}{\partial\alpha}\right), \quad \Delta a_t = a_t\left(\frac{1}{1 + (ka_t\rho V^2 S_t)/2} - 1\right)$$

- What would happen to the elevator effectiveness?
- Elevator effectiveness is also reduced by same factor. How?
- For nonzero δ_e, we have tail lift coefficient

$$C_{L_t} = a_t\left(\alpha_{wb} - \epsilon - i_t - k\frac{1}{2}\rho V^2 S_t C_{L_t}\right) + a_e\delta_e$$

- On simplifying, we get

$$\boxed{C_{L_t} = \frac{a_t\left(\alpha_{wb} - \epsilon - i_t\right)}{1 + (k a_t \rho V^2 S_t)/2} + \frac{a_e\delta_e}{1 + (k a_t \rho V^2 S_t)/2}}$$

- Presence of ground modifies air flow significantly, affecting trim and stability.
- Take-off and landing: governing design criteria of airplanes
- Reduced downwash due to ground effect
  - $\Rightarrow$ A reduction in downwash angle at the tail, ϵ.
  - $\Rightarrow$ An increase in wing-body lift slope, a_{wb}.
  - $\Rightarrow$ An increase in tail lift slope, a_t.
- Most important items to be determined are the elevator angle and control force required to maintain $C_{L,\max}$ in level flight close to ground.
- Ratio a_t/a decreases, leading to forward movement of NP. However, the reduction in $\partial\epsilon/\partial\alpha$ results in a net effect of large rearward shift of NP.
- Since C_{m_0} is only slightly affected, $\delta_{e_{\text{trim}}}$ at $C_{L\,\max}$ is much larger than in flight remote from ground.

Reference

1. Bernard Etkin and Llyod Duff Reid, *Dynamics of Flight Stability and Control*, John Wiley and Sons, Third Edition, 1996.

Thank you for your attention !!!