Perigee Argument Changing Manoeuvre

Perigee Argument Changing Concept

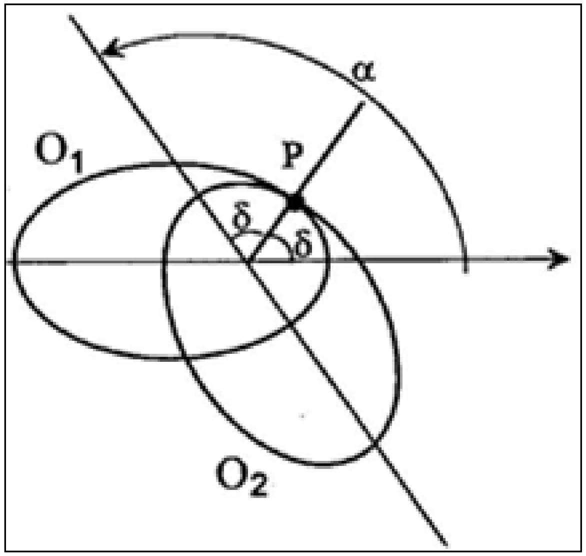
We know that ‘ω’ is determined by the **line** of nodes and **eccentricity** vectors at the time of **injection**, which also locates the **perigee** in the **geographical** context.

However, there are **many** cases where we wish for a **different** location for perigee (e.g. higher/ lower latitude) and hence we **need** to change it.

Above **manoeuvre** is equivalent to **rotating** orbit about its **angular** momentum vector, which **preserves** orbital **plane**, though it still **requires** spending ‘ΔV’**.**

Perigee Argument Changing Concept

Consider the **schematic** given below.



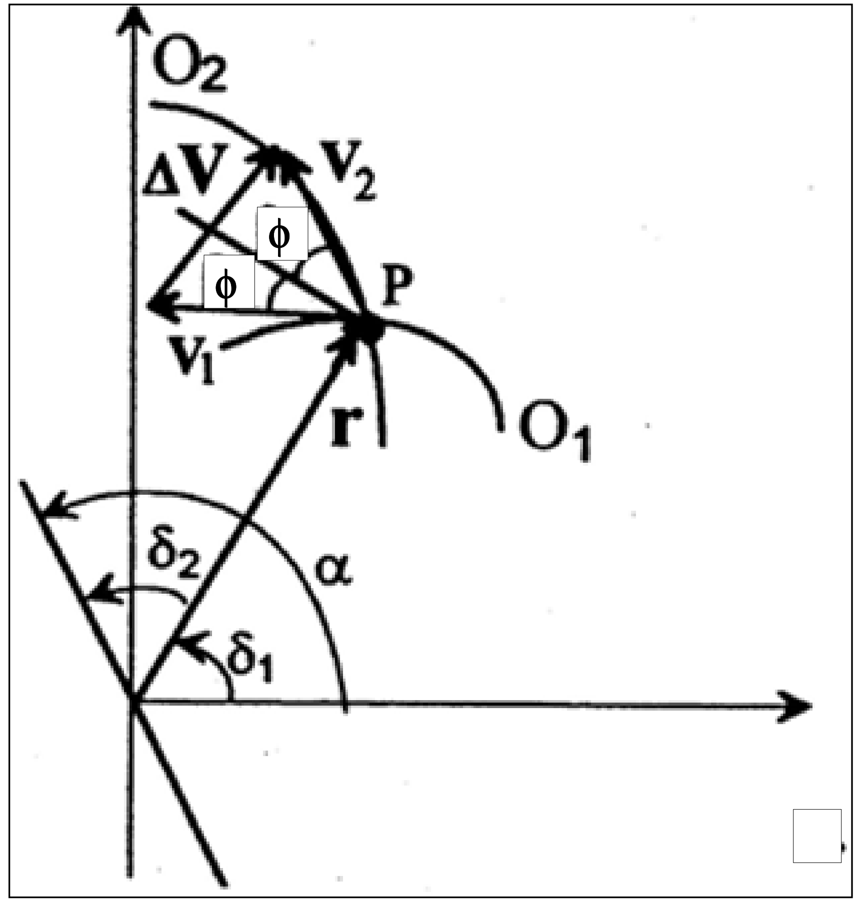
Here, '**P**' is the point at which the two **ellipses** intersect and 'α' is the **angle** by which the **'ω'** needs to be changed.

Perigee Argument Changing Concept

As **line** drawn from **centre** to ‘P’ represents common **radius** vector, ‘δ’ is true anomaly.

Figure **alongside** shows applicable **velocity** triangle.

We see that as **velocity** triangle is **isosceles**, the local **horizon** at ‘P’ is **normal** to the ‘Δv’.



Argument Change Formulation

Since both orbits are **same**, following relation for **'ΔV',** to be applied along **local** velocity, can be obtained.

$$r_1 = r_2 = r, \quad p_1 = p_2 = p, \quad V_1 = V_2 = V, \quad \delta_1 = \delta_2 = \delta = \frac{\alpha}{2}$$

$$p = a\left(1 - e^2\right) = \frac{V^2 r^2 \cos^2\phi}{\mu}; \quad r = \frac{p}{1 + e\cos\delta}; \quad V^2 = \frac{2\mu}{r} - \frac{\mu}{a}$$

Argument Change Solution

The velocity **impuls**e expression is **obtained** as follows.

$$V^2 = \frac{2\mu}{r} - \frac{\mu}{a} = \frac{2\mu}{p}(1+e\cos\delta) - \frac{\mu}{a} = \frac{\mu}{a}\frac{\left[1+e^2+2e\cos\delta\right]}{1-e^2}$$

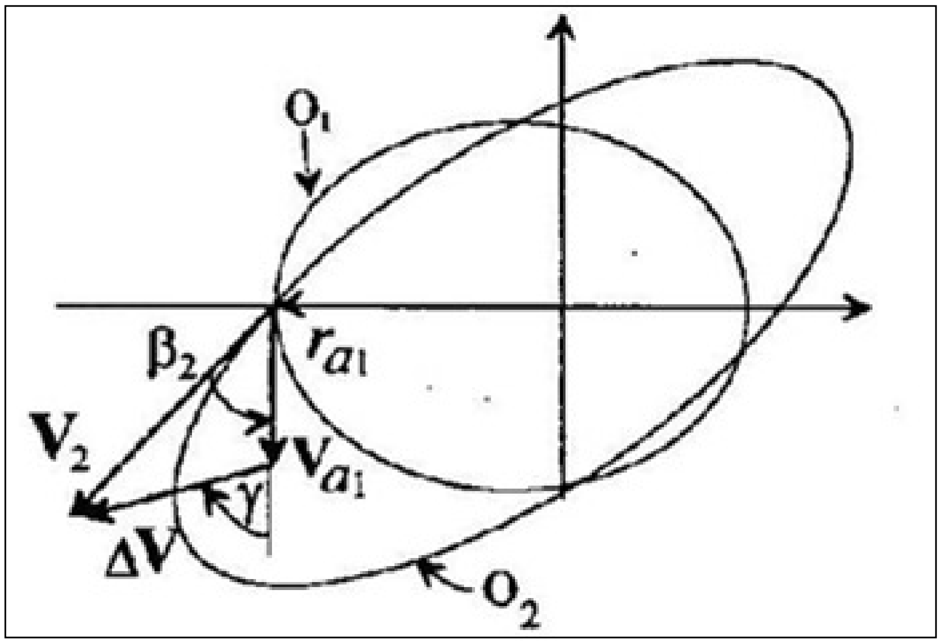
$$\cos^2\phi = \frac{\mu a\left(1-e^2\right)}{V^2 r^2} = \frac{(1+e\cos\delta)^2}{1+e^2+2e\cos\delta}; \quad \sin^2\phi = \frac{e^2\sin^2\delta}{1+e^2+2e\cos\delta}$$

$$\Delta V = 2V\sin\phi = 2\sqrt{\frac{\mu}{a(1-e^2)}}\cdot\left(e\sin\frac{\alpha}{2}\right); \quad \Delta V = 0 \text{ for circular orbit.}$$

General Single Impulse Manoeuvre

Changing both Perigee & Apogee

Both perigee & apo**gee** can be **modified** together with single 'ΔV', at **apogee** of O_1, as shown below.



Single Impulse Change Solution

The **applicable expressions** are as given below.

$$\frac{V_{a1}^2}{2} - \frac{\mu}{r_{a1}} = -\frac{\mu}{2a_1} \rightarrow V_{a1}^2 = 2\mu\left[\frac{1}{r_{a1}} - \frac{1}{2a_1}\right] = \mu\frac{(1-e_1)}{a_1(1+e_1)}$$

$$V_2^2 = 2\mu\left[\frac{1}{r_2} - \frac{1}{2a_2}\right] = 2\mu\left[\frac{1}{r_{a1}} - \frac{1}{2a_2}\right] = 2\mu\left[\frac{1}{a_1(1+e_1)} - \frac{1}{2a_2}\right]$$

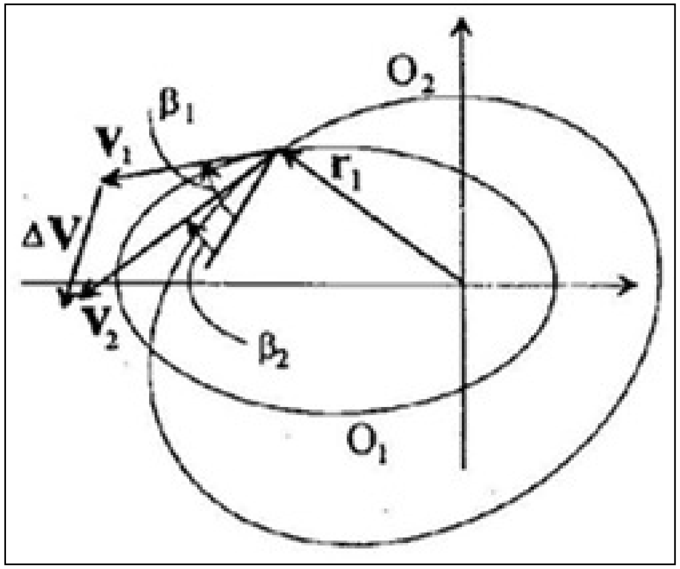
$$p_2 = a_2\left(1-e_2^2\right) = \frac{h_2^2}{\mu}; \quad h_2 = r_{a1}V_2\cos\beta_2; \quad \cos^2\beta_2 = \frac{\mu a_2(1-e_2^2)}{V_2^2 a_1^2(1+e_1)^2}$$

$$\Delta V^2 = V_{1a}^2 + V_2^2 - 2V_{1a}V_2\cos\beta_2 = \left(V_2\cos\beta_2 - V_{1a}\right)^2 + V_2^2 - V_2^2\cos^2\beta_2$$

$$\Delta V = \sqrt{\left[V_2\cos\beta_2 - V_{a1}\right]^2 + V_2^2\sin^2\beta_2}; \quad \sin\gamma = \sin\left(\beta_2\right)\cdot V_2 / \Delta V$$

Single Impulse Change Restrictions

Not all orbit **changes** are possible using a **single** velocity impu**lse**. Consider the **figure** given below.



$O_1 : a_1; \quad O_2 : a_2; \quad r_1 = r_2$

$$\frac{V_2^2}{2} = \frac{\mu}{r_1} - \frac{\mu}{2a_2} \geq 0 \rightarrow 2a_2 \geq r_1$$

$$h_2 = \mu a_2 \left(1 - e_2^2\right) = \left(v_2 r_1 \cos \beta_2\right)^2$$

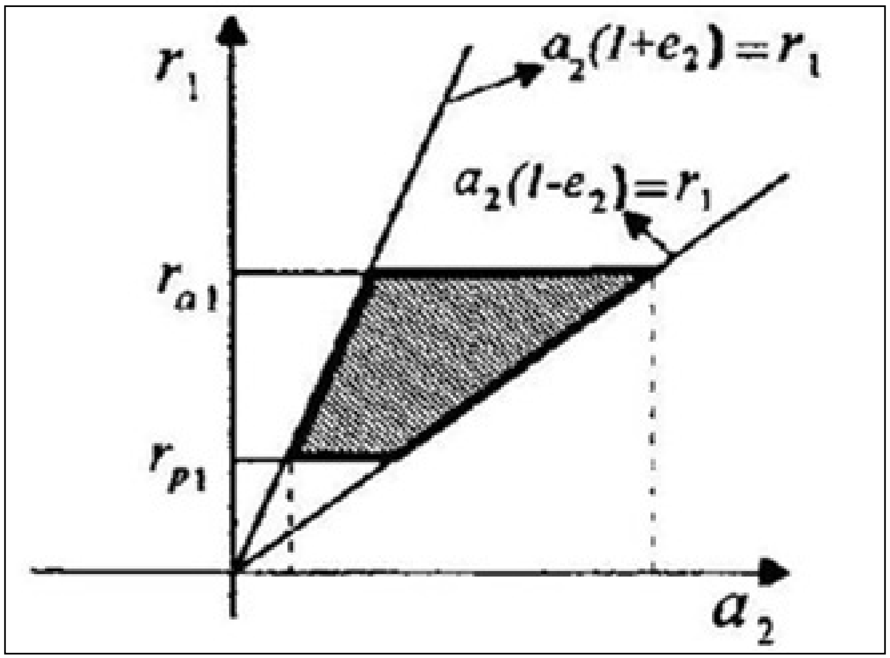
$$\cos^2 \beta_2 = \frac{\mu a_2 \left(1 - e_2^2\right)}{r_1^2 v_2^2} = \frac{\mu a_2 \left(1 - e_2^2\right)}{r_1^2 \left(\dfrac{2\mu}{r_1} - \dfrac{\mu}{a_2}\right)} \leq 1$$

$$r_1^2 - 2a_2 r_1 + a_2^2 \left(1 - e_2^2\right) \leq 0$$

$$r_{p2} = a_2 \left(1 - e_2\right) < r_1 < r_{a2} = a_2 \left(1 + e_2\right)$$

Single Impulse Change Restrictions

Figure below **shows** the available design **space** under single **impulse** context.



Single Impulse Change Restrictions

Table below **summarizes** these **restrictions** for all orbits.

Element changed	Fixed elements	Restrictions
a	e, ω	Impossible
a, e, ω	None	$1+D>\frac{a_1}{a_2}>1-D$
e, ω	a	$\left(\frac{e_1}{e_2}\right)^2+1-2\left(\frac{e_1}{e_2}\right)\cos(\Delta\omega)>0$
a, ω	e	$1+D'>\frac{a_1}{a_2}>1-D'$
a, e	ω	$1\pm\left(\frac{a_1}{a_2}e_1-e_2\right)\geq\frac{a_1}{a_2}\geq 1\pm\left(e_2-\frac{a_1}{a_2}e_1\right)$
e	a , ω	None
ω	a, e	None

$$D^2=\left(\frac{a_1}{a_2}\right)^2 e_1^2+e_2^2-2\left(\frac{a_1}{a_2}\right)e_1e_2\cos(\Delta\omega);\ D'=e\left[\left(\frac{a_1}{a_2}\right)^2+1-2\left(\frac{a_1}{a_2}\right)e_1e_2\cos(\Delta\omega)\right]^{1/2}$$

Summary

We **see** that an in-plane rotation of **orbit** requires expenditure of **energy**.

We further **note** that while single impulse **orbital** manoeuvres are simple, there are **constraints** that need to be observed while **setting** these up.