Aircraft Mechanics II

(Course Code: AE 305/305M/717)

Department of Aerospace Engineering
Indian Institute of Technology Bombay

Instructor: Shashi Ranjan Kumar

March 17, 2021

Tutorial 4

TA: Jinay Patel

1. Consider a vector $P(1,0)$ lying on the x -axis of Frame $\mathcal{A}$. Rotate $\mathcal{A}$ by 15° to frame $\mathcal{B}$ and then rotate frame $\mathcal{B}$ by 30° to Frame $\mathcal{C}$.
   (a) What are the new coordinates of the vector $\vec{P}$ in frame $\mathcal{C}$?
   (b) What are the new coordinates of the vector $\vec{P}$ if the sequence of rotation is reversed?

2. Consider a rotation of vector using quaternion about an axis defined by vector (1,0,1) through an angle of $2\pi/3$.
   (a) Obtain the quaternion $[Q]$ to perform this rotation.
   (b) Compute the effect of rotation on the basis vector $\boldsymbol{k} = (0, 0, 1)$.
   (c) Find out the conjugate $[Q]^\star$ and inverse $[Q]^{-1}$ of the quaternion $[Q]$.
   (d) Find the coordinates of above vector in new frame if we rotate the coordinate frame itself about the same axis and angle while keeping the vector constant?

3. If $\mathbf{R}$ is a general rotation matrix

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{33} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

   which represents rotation of an aircraft along three principal axes, such as about x with angle ϕ, about y with angle θ, and about z with angle ψ respectively.
   (a) Find the values of Euler angles (ϕ, θ, ψ) in terms of elements of $\mathbf{R}$.
   (b) If $\mathbf{R} = \begin{bmatrix} 0.5 & -0.1464 & 0.8536 \\ 0.5 & 0.8536 & -0.1464 \\ -0.7071 & 0.5 & 0.5 \end{bmatrix}$, find the values of roll, pitch and yaw angles.

4. Recall that the quaternion operator with unit quaternion $[Q]$ acts on a vector $\boldsymbol{v}$ as

$$L_Q(\boldsymbol{v}) = [Q]\boldsymbol{v}[Q]^\star = \left(q_0^2 - \|\boldsymbol{q}\|^2\right)\boldsymbol{v} + 2(\boldsymbol{q}\cdot\boldsymbol{v})\boldsymbol{q} + 2q_0(\boldsymbol{q}\times\boldsymbol{v})$$

   (a) Show that length of the vector $\boldsymbol{v}$ is invariant under the operation.
   (b) Show that direction of the vector $\boldsymbol{v}$ remains unchanged under the operation.
   (c) Show that the operation is a linear map over $\mathbb{R}^3$.