Low Power 0/1 Coverage and Scheduling Techniques in Sensor Networks

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***Abstract* –** Distributed embedded systems have recently emerged as an economically attractive and technically challenging research direction. Among such systems, wireless ad-hoc sensor networks have a special place due to their numerous applications and potential to fill the interface gap between the Internet and the physical world. Wireless sensor networks are intrinsically energy constrained and therefore necessitate the design of new low power protocols.

Coverage, in its many forms, is a fundamental task in sensor networks. Our focus here is energy efficient operation strategies for sensor networks with sensor coverage as the primary objective. We present several ILP (Integer Linear Program) formulations and strategies to reduce overall energy consumption while maintaining guaranteed 0/1 coverage levels. We also demonstrate the practicality and effectiveness of these formulations using several examples and provide comparisons with alternative strategies.

I. INTRODUCTION

Recent technological advances in distributed embedded systems have prompted significant research efforts in both the industry and the academia. Among such systems, wireless ad-hoc sensor networks are particularly noteworthy due to their potentially numerous, economically attractive applications and their ability to bridge the interface between the user, the Internet, and the physical world. Unlike traditional embedded systems, the new wireless sensor network nodes have remarkable computational and storage capabilities. Consequently, implementing the necessary low power protocols to overcome the intrinsic energy constrains for common sensing and network tasks have become more tractable.

One of the fundamental metrics that can be used to quantify the quality of service of a sensor network is coverage. Due to the wide variety of sensing technologies and applications, coverage can have different meanings. In general, we use the term coverage to quantify the geographic extent in which sensors can observe and react to the environment. In this paper we present optimal and heuristic optimization algorithms using ILP (integer linear program) -based formulations, that are essential in implementing low-power operation strategies, with coverage as the main objective. In the next subsection, a simple motivational example provides insights into the nature of our approach and the goals at hand.

A. Motivational Example

Consider the shaded area covered by the sensor network in Figure 1. The sensors are labeled as s_1 through s_5. Let us assume that each sensor can observe a circular region as shown in the figure. The areas formed by the intersections of the circles are denoted using the label a and an appropriate index ranging from 1 to 13. As Figure 1 shows, the sensors cover the following area sets:

s_1: $\{a_1, a_2, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}\}$
s_2: $\{a_1, a_2, a_3, a_4, a_6, a_7, a_8, a_9\}$
s_3: $\{a_2, a_3, a_4, a_5, a_7, a_8, a_9, a_{11}, a_{12}, a_{13}\}$
s_4: $\{a_4, a_5, a_8, a_9, a_{12}, a_{13}\}$
s_5: $\{a_6, a_7, a_9, a_{10}, a_{11}, a_{12}, a_{13}\}$

It is easy to see that each region is often covered by several sensors. For example, a_1 is covered by two sensors (s_1 and s_2) and a_9 is covered by all sensors. As an enabling step in implementing complex power saving modes of operation, an interesting problem is to find a minimal set of sensors that cover the entire region shown in the figure. In this example, a valid solution to the minimum cover problem is the set $\{s_1, s_3\}$. As we will discuss in the following sections, another interesting question to answer is finding several different subsets of sensors that cover the entire region such as $\{s_2, s_3, s_5\}$ and $\{s_1, s_2, s_4\}$. Here, we transform the new coverage problem in sensor networks to the classical set-cover problem that has been the focus of intense study for decades.

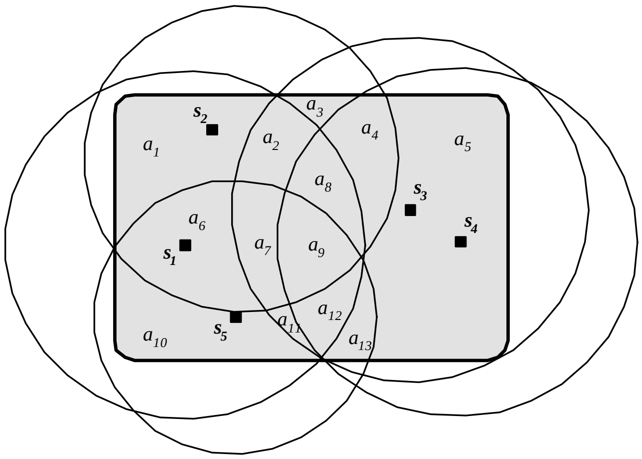

Figure 1. A Sensor Network Example

Although the field coverage is at maximum when all sensor nodes are active, it is clear from this example, that often, only a small subset of sensors are enough in providing adequate coverage. Having a limited set of sensor nodes active at any given time is the key in reducing the overall energy consumption of the network since unused sensors can be placed in special power-saving sleep modes.

B. Paper Organization

The remainder of this paper is organized as follows: In Section II, we discuss the related work. Section III provides a brief preliminary discussion and background that are essential in understanding the formulations and technical discussions in Section IV. Finally, in Section V we present and analyze several experimental results followed by the conclusion.

II. RELATED WORK

A. Low Power At System Level

Low power research has been a topic of prime importance since the early eighties. Although numerous approaches have been proposed and studied, references [Ben00a, Ben00b] have demonstrated that power can be optimized at the system level especially well.

For wireless designs, power consumption has been optimized at the circuits level [Abi00], architecture level [Rab00a], and the system level [Rab00b]. A comprehensive survey of power minimization techniques for state-of-the-art designs is presented in [Ver00]. A technique that addresses power minimization specific to sensor networks is presented in [Rab00b]. The approach focuses on optimizing power consumption of individual nodes in sensor networks. However, our goal is to minimize power consumption for a fundamental middleware task, coverage. It is interesting to note, that although the targeted problem is NP-complete, we are able to solve practically significant instances optimally.

B. Coverage Problems

The Art Gallery Problem deals with determining the number of observers necessary to cover an art gallery room such that every point is seen by at least one observer. It has found several applications in many domains such as the optimal antenna placement problems for wireless communication. The Art Gallery problem was solved optimally in 2D and was shown to be NP-hard in the 3D case. Reference [Mar96] proposes heuristics for solving the 3D case using Delaunay triangulations. Sensor coverage for detecting global ocean color where sensors observe the distribution and abundance of oceanic phytoplankton [Gre98] is approached by assembling and merging data from satellites at different orbits. Reference [Meg01] presents an efficient algorithm using computational geometry constructs to optimally characterize and calculate best- and worst-case coverage in sensor networks.

Coverage studies to maintain connectivity have also been the focus of study for many years. For example, [Mol99 and Lie98] calculate the optimum number of base stations required to achieve the system operator's service objectives in wireless networks. Previously, connectivity was achieved through mobile host attachments to a base station. However, the connectivity coverage is more important in the case of adhoc wireless networks since the connections are peer-to-peer. Reference [Has97] shows the improvement in the network coverage due to multi-hop routing features and optimizes coverage constrained with a limited path length.

C. The Set Cover Problem

There are three popular classes of combinatorial optimization problems that are known as set-covering, set-packing, and set-partitioning. Suppose $M=\{1,2,...,m\}$ is a finite set and $\{M_j\}$, where $j \in \{1,..,n\}$, is a given collection of subsets of M. We say that set of subsets F covers M if $\bigcup_j M_j \in F = M$.

The other relevant combinatorial problem is the packing problem, where the $M_j \cap M_k = \emptyset$ for all j and k $(j \neq k)$. If F is both covering and packing, then F is said to be a partition of *M*. In the set covering problem, c_j is the cost of M_j and we seek a minimum cost cover. In the set packing problem, however, c_j is the weight or value of M_j and we seek a maximum weight packing. An excellent reference on the set cover problems and their numerous applications is [Nem88].

D. ILP

Linear programming is an exceptionally efficient method for minimizing a linear form under linear inequality constraints [Nem88]. Since its invention [Dan54], it found a large number of applications in many areas [Dan63, Nem88]. Integer linear programming (ILP) is linear programming where additional constraints are imposed on the objective function in a form of a request that all solution variables be integers. While any linear programming instance can be solved in polynomial time, solving ILP is NP-hard. Nevertheless, numerous effective techniques and packages have been developed that solve fairly large instances in reasonably short run times. In particular, when problems have special structures such as set cover problems, the run-times can be drastically shortened. We use the public domain LP_SOLVE program for our experimentations.

III. PRELIMINARIES

A. Sensor Models

Sensing devices generally have widely different theoretical and physical characteristics. Numerous models of varying complexity can be constructed based on application needs and device features. Interestingly, most sensing device models share two facets in common:

1. Sensing ability diminishes as distance increases;
2. Due to diminishing effects of noise bursts in measurements, sensing ability can be improved as the allotted sensing time (exposure) increases.

Thus, for a sensor *s*, we express the general sensing model *S* at an arbitrary point *p* as:

$$S(s,p) = \frac{\lambda}{[d(s,p)]^K}$$

where *d(s,p)* is the Euclidean distance between the sensor *s* and the point *p*, and positive constants λ and *K* are sensor technology dependent parameters.

B. Average Sensor Field Intensity

Following the definition of the sensing model in the previous subsection, we now introduce the *Average Sensor Field Intensity* for a given region denoted by *a*.

Definition: ***Average Sensor Field Intensity I(s, a)*** for a region *a* due to sensor *s* is defined as:

$$I(s,a) = \left(\frac{1}{|a|}\right)\sum_{i=1}^{|a|} S(s, p_i)$$

In general, the region *a* and the intensity function are continuous. However, here we treat the region *a* as a set of discrete points. In order to simplify computations, without loss of generality and with minimal loss of optimality, we use a grid based approach to approximate the average sensor field intensity. Thus each point $p \in a$ actually represents a grid point and as the grid resolution increases, the result of the approximation approaches the optimum.

The overall average sensor field intensity for a region *a* due to *n* sensors $\{s_1, s_2, \ldots s_n\}$ is the sum of the field intensities from each sensor and can be stated as:

$$I(a) = \left(\frac{1}{|a|}\right)\sum_{j=1}^{n}\sum_{i=1}^{|a|} S(s_j, p_i)$$

IV. ILP FORMULATIONS

The general wireless sensor network domain encompasses countless applications, each with different requirements and objectives. Here, we use Integer Linear Programming formulations as a concise, intuitive, and clear method of representing several key optimization problems for sensor networks that provide the necessary framework for implementing complex operation strategies while providing guaranteed coverage levels. These results can be used as enabling steps for a wide range of objectives such as reducing overall power consumption, providing redundancy, fault tolerance, and security. We must note here that for ad-hoc sensor networks, the problem of scheduling sensor nodes is significantly more complex since maintaining network connectivity becomes an important factor. However, in cases where specific base-stations are used or when the radio transmission ranges are much larger than sensor sensing ranges then node scheduling based on sensor coverage, as presented, is more appropriate.

A. Minimum 0-1 Cover

The first problem that we consider is the task of finding the minimum set of sensors that cover the same regions as the complete set of sensors. The main goal here is to find the smallest number of sensors that must be active in order to guarantee that every observable point in the sensor field is observed by at least one active sensor. We formally state *minimum 0-1 cover problem in sensor networks* as follows:

Given: A sensor set $S=\{s_1, s_2, \ldots, s_{|S|}\}$; a sensor field A, where $\{a_1, a_2, \ldots a_{|A|}\}$ is a partitioning of A; an area coverage matrix $C_{|A|x|S|}$, where each element $C_{i,j}$ is 1 if sensor $s_j \in S$ covers area $a_i \in A$ and 0 otherwise.

Problem: Find the minimum set of sensors, represented by vector $X_{|S|\times 1}$, where X_i is 1 if sensor $s_i \in S$ is in the set and 0 otherwise, that cover A.

The ILP formulation that solves the minimum area cover problem for sensor networks can thus be expressed as:

$$\begin{aligned} &\textit{Minimize}: && \mathbf{1}_{1\times|S|} \cdot X_{|S|\times 1} \\ &\textit{Where}: && C_{|A|x|S|} \cdot X_{|S|\times 1} \geq \mathbf{1}_{|A|\times 1} \end{aligned}$$

The objective function being minimized is the number of sensors represented by the sum of the elements of vector X. The constraints to be satisfied are that each region a_i must be covered by at least one sensor as represented by the inequality.

B. Minimal Cover – Sensor Field Intensity

The sensor network minimal cover problem, as formulated above, is somewhat restrictive in that each region is either covered or not. In most situations, a sensor field intensity based approach may be more appropriate since for each area, closer sensors generally provide better coverage. The main goal here is to select a minimal set of sensors such that the average sensor field intensity I for each region a_i is above a user specified threshold level. We formally state the *sensor field intensity based minimal cover problem* as:

Given: A sensor set $S=\{s_1, s_2, \ldots, s_{|S|}\}$; a sensor field A, where $\{a_1, a_2, \ldots a_{|A|}\}$ is a partitioning of A; a real valued sensor field intensity coverage matrix $E_{|A|x|S|}$, where $E_{i,j}$ represents the average sensor field intensity $I(s_j, a_i)$ in region $a_i \in A$ due to sensor $s_j \in S$.

Problem: Find the minimum set of sensors, represented by vector $X_{|S|\times 1}$, where X_i is 1 if sensor $s_i \in S$ is in the set and 0 otherwise, such that for each $a_i \in A$, the average sensor field intensity $I(a_i) \geq E_{min}$.

The ILP formulation that solves the sensor field intensity based minimal cover problem can thus be expressed as:

$$\begin{aligned} &\textit{Minimize}: && \mathbf{1}_{1\times|S|} \cdot X_{|S|\times 1} \\ &\textit{Where}: && E_{|A|x|S|} \cdot X_{|S|\times 1} \geq \mathbf{1}_{|A|\times 1} \cdot E_{min} \end{aligned}$$

Although the objective function here is the same as the minimal-cover problem, the constraint inequalities contain the sensor field intensity coverage matrix E and the user specified minimum intensity value E_{min}. The inequalities guarantee that once a valid solution is found, each region $a_i \in A$ will have at least an average sensor field intensity equal to E_{min}. It is easy to see that this formulation can easily be modified to satisfy a variety of constraints by altering the semantics of the matrix E and the constraint E_{min}.

C. Balanced Operation Scheduling

In the previous subsections, we presented two generic ILP formulations that, when solved, produce a minimal subset of sensors that provide a certain level of coverage. However, it is sometimes desirable to have several, possibly disjoint set of such subsets. The benefit of having several (a maximal number of) subsets with guaranteed coverage levels is that each subset can be scheduled to be operational during a different time slice, thus increasing resource utilization, providing higher levels of redundancy, fault tolerance, and ease of maintenance. In the balanced operation scheduling task, our main goal is to produce a user specified number of sensor subsets such that each sensor is designated as active in as few sets as possible. This goal will ensure that no sensor is active for longer than necessary. We formally state the *balanced operation scheduling problem in sensor networks* as:

Given: A sensor set $S=\{s_1, s_2, \ldots, s_{|S|}\}$; a sensor field A, where $\{a_1, a_2, \ldots a_{|A|}\}$ is a partitioning of A; an area coverage matrix $C_{|A|x|S|}$, where each element $C_{i,j}$ is 1 if sensor $s_j \in S$ covers area $a_i \in A$ and 0 otherwise; scheduling parameter T.

Problem: Find the scheduling matrix $X_{|S|\times T}$, where each element $X_{i,j}$ is 1 if sensor $s_i \in S$ is active at time slice j and the regions covered by the active sensors during each time slice is equal to the area covered if all sensors were active, subject to the following requirements: *(1)* Minimize the total number of time slices where each sensor is active; *(2)* Minimize the number of active sensors in each time slice.

The ILP formulation for the balanced sensor node operation scheduling problem can be stated as:

$$\begin{aligned}
&\textit{Minimize}: && k \\
&\textit{Where}: && \mathbf{1}_{1\times|S|} \cdot X_{|S|\times T} \le \mathbf{1}_{1\times T} \cdot k \\
& && C_{|A|x|S|} \cdot X_{|S|\times T} \ge \mathbf{1}_{|A|\times T}
\end{aligned}$$

In this formulation, in addition to the *0-1*-integer variables represented by matrix X, we have introduced a new integer variable k. This variable represents the maximum number of time slices that any sensor is active. Note that solving this formulation would only satisfy requirement *(1)* of the problem statement. In order to satisfy requirement *(2)* and minimize the total number of active sensors in each time slice, we use the minimum cover formulation presented in Section IV.A.

V. Experimental results

A. Experimentation Platform

The experimentation platform we use to implement and solve the ILP formulations for low power optimization in sensor networks is comprised of the following four components:

1. Instance Generator
2. ILP Formulator
3. LP_SOLVE
4. Verifier

The Instance Generator creates a random sensor network based on user specified parameters. Node placements are performed according to the uniform distribution by using two uniform random variables X and Y to determine the (x,y) coordinate of each sensor node. The ILP Formulator is a preprocessor to the main solver and creates the requested ILP formulations pertaining to the network instance generated by the Instance Generator. We use LP_SOLVE, a non-commercial Mixed Integer Linear Program solver to obtain the solution for each instance. The Verifier then checks whether a valid solution was found and calculates several statistics, some of which are reported in the provided tables.

In order to establish a uniform experimentation domain, we opted to simulate a square sensor field 1200 meters wide and for minimum cover formulations, sensor radii ranging from 50 to 150 meters. In order to eliminate boundary effects, all calculations are performed on the 1000x1000 field centered in the sensor field.

B. Min-Cover Results

Table 1 presents a sampling of results for networks with the number of sensors ranging from 50 to 700 nodes. For each instance, the third column of the table contains the average number of sensors in the minimum cover solution for 100 randomly generated networks using the Instance Generator. For comparison, we also list the results obtained using a greedy-heuristic to find the minimal cover. In this greedy heuristic, we start with an empty set. Then we select an uncovered region and find a sensor that covers the selected region. We then mark all areas covered by the selected sensor as covered and repeat the process until all areas are covered. As expected, the ILP results always outperform the greedy approach, often by a large margin, and sometimes producing covers that require 40% or fewer sensors than the greedy method. Each case required less than one minute of run-time to execute on a 700Mhz Intel PIII-based workstation.

Figure 2 and 3 depict a graphical view of what is produced by solving the sensor network minimum-cover ILP formulation. For illustration purposes we only show the graphical result of the minimal 0-1 coverage problem due to its visually intuitive nature. Figure 2 shows a random network of 150 nodes and Figure 3 shows the 11 sensors that are required to cover the entire field.

In order to gain a deeper understanding of the benefits of using the ILP based approach to solve the minimum cover problem, we studied how the coverage provided by random sensor selections compare with the optimal solutions. Table 2 lists the outcomes of 100 attempts to cover each generated instance using a random selection method. In this method, for each instance, we randomly select n sensors, where n is the average number of sensors required to cover the field as determined by the min-cover ILP approach (Table 1). The results show that although a large portion of the field can generally be covered by random sensor selections, as the density of the sensor field increases, the performance of the random selection drops rapidly. Although the ILP method is fairly efficient, the benefits of using the method become more apparent as the density (and the total number) of sensors in the field increases.

Table 3 lists several results for the balanced operation scheduling problem in sensor networks. It is interesting to note here that as a result of our formulation, if any region is covered by only one sensor, in order to guarantee coverage, that sensor must be active in all subsets which renders the optimization algorithm ineffective. By a simple pre-processing step, we determine if such instances exist in each case. Table 3 only report results where all regions are covered by at least two sensors.

The third column in Table 3 shows the user specified scheduling parameter T which represents the total number of subsets (time slices). The fourth column shows the average k, the worst case number of subsets that each sensor is marked as active after solving the ILP formulation. As can be seen from the results, whenever each region is covered by at least two sensors, in the worst case on average, each sensor is marked active in only 50% of the subsets. This number improves as the minimum number of sensors covering each area is increased, i.e. as sensor density increases.

VI. CONCLUSION

Wireless ad-hoc sensor networks have recently emerged as an area of intense study. Although sensor networks are in many ways similar to traditional wireless networks, applications in this domain are intrinsically energy constrained. Making wireless sensor networks economically attractive mandates the design of new low power protocols to help in significantly reducing operation and maintenance costs.

In this paper we focused on power-aware operation strategies in sensor networks with coverage as the main criterion. We presented several ILP based formulations and strategies to reduce overall energy consumption while maintaining guaranteed sensor coverage levels. We also demonstrated the practicality and effectiveness of these formulations on a variety of examples and provided comparisons with several alternative strategies. It is interesting to note, that although the targeted problem is NP-complete, we are able to solve practically significant instances optimally. Overall, the technique performs well and, as our experiments have shown, can scale to large and dense networks with hundreds of sensor nodes.

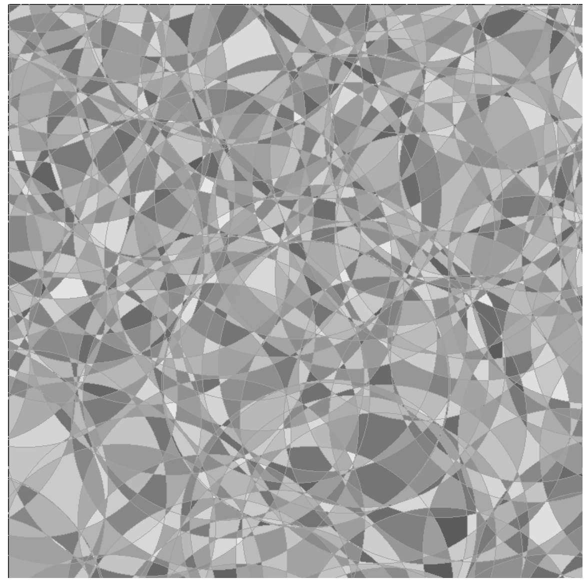

Figure 2. Example of a sensor network with 150 nodes with sensing radius set at 150m.

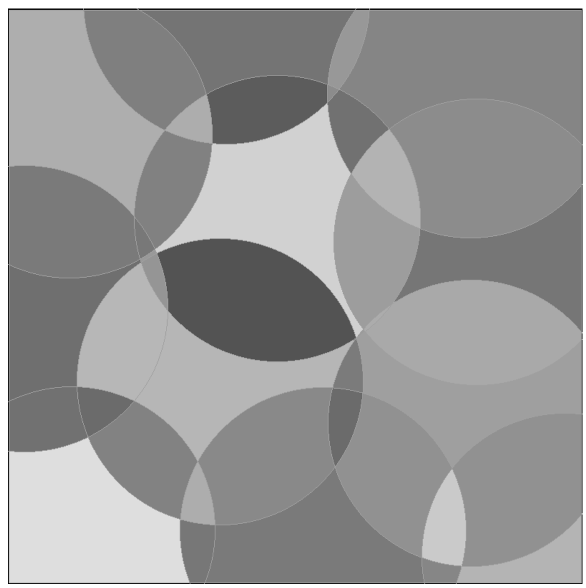

Figure 3. Minimum cover of the field shown in Figure 2 with 11 Sensors.

Table 1. Minimum number of sensors required to cover 100% of the field using the Min-Cover ILP approach and a greedy heuristic based approach.

Radius (m)	Sensor Count	Average # of Sensors	
		Min-Cover (ILP)	Greedy-Heuristic
50	700	269	356
50	500	262	309
50	300	211	220
100	250	68	104
100	200	74	102
100	150	74	93
150	150	30	51
150	100	32	49
150	50	31	38

Table 2. Percentage of regions covered by randomly selecting the number of sensors in an average sized minimum cover.

Radius (m)	Sensor Count	Sensors Selected	Random Cover Average # of Regions		
			Covered	Total	% Covered
50	700	269	6475	7605	85.1%
50	500	262	3678	4156	88.5%
50	300	211	1432	1571	91.2%
100	250	68	3483	4187	83.2%
100	200	74	2445	2834	86.3%
100	150	74	1434	1607	89.2%
150	150	30	2580	3182	81.1%
150	100	32	1259	1505	83.7%
150	50	31	376	413	91.0%

Table 3. Balanced operation scheduling results for sensor networks. Reported results for *k* are averages for 100 random instances.

Radius (m)	Sensor Count	Balanced Scheduling	
		Number of Subsets (T)	Max Active (k)
50	300	10	4.8
50	300	8	3.9
50	300	4	2.0
100	200	10	4.9
100	200	8	3.8
100	200	4	2.0
150	100	10	5.0
150	100	8	4.0
150	100	4	2.0

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