1 Expand summation values

$$
\begin{aligned}
p(u,v) &= \sum_{i=0}^{3}\sum_{j=0}^{3} B_i^3(u)B_j^3(v)\mathbf{k}_{ij} \\
&= \sum_{i=0}^{3}[B_i^3(u)B_0^3(v)\mathbf{k}_{i0} + B_i^3(u)B_1^3(v)\mathbf{k}_{i1} + B_i^3(u)B_2^3(v)\mathbf{k}_{i2} + B_i^3(u)B_3^3(v)\mathbf{k}_{i3}] \\
&= [B_0^3(u)B_0^3(v)\mathbf{k}_{00} + B_0^3(u)B_1^3(v)\mathbf{k}_{01} + B_0^3(u)B_2^3(v)\mathbf{k}_{02} + B_0^3(u)B_3^3(v)\mathbf{k}_{03}] \\
&+ [B_1^3(u)B_0^3(v)\mathbf{k}_{10} + B_1^3(u)B_1^3(v)\mathbf{k}_{11} + B_1^3(u)B_2^3(v)\mathbf{k}_{12} + B_1^3(u)B_3^3(v)\mathbf{k}_{13}] \\
&+ [B_2^3(u)B_0^3(v)\mathbf{k}_{20} + B_2^3(u)B_1^3(v)\mathbf{k}_{21} + B_2^3(u)B_2^3(v)\mathbf{k}_{22} + B_2^3(u)B_3^3(v)\mathbf{k}_{23}] \\
&+ [B_3^3(u)B_0^3(v)\mathbf{k}_{30} + B_3^3(u)B_1^3(v)\mathbf{k}_{31} + B_3^3(u)B_2^3(v)\mathbf{k}_{32} + B_3^3(u)B_3^3(v)\mathbf{k}_{33}]
\end{aligned}
$$

2 Convert summation values into matrices

2.1 Rearrange summation values

$$
\begin{aligned}
p(u,v) &= [B_0^3(u)B_0^3(v)\mathbf{k}_{00} + B_1^3(u)B_0^3(v)\mathbf{k}_{10} + B_2^3(u)B_0^3(v)\mathbf{k}_{20} + B_3^3(u)B_0^3(v)\mathbf{k}_{30}] \\
&+ [B_0^3(u)B_1^3(v)\mathbf{k}_{01} + B_1^3(u)B_1^3(v)\mathbf{k}_{11} + B_2^3(u)B_1^3(v)\mathbf{k}_{21} + B_3^3(u)B_1^3(v)\mathbf{k}_{31}] \\
&+ [B_0^3(u)B_2^3(v)\mathbf{k}_{02} + B_1^3(u)B_2^3(v)\mathbf{k}_{12} + B_2^3(u)B_2^3(v)\mathbf{k}_{22} + B_3^3(u)B_2^3(v)\mathbf{k}_{32}] \\
&+ [B_0^3(u)B_3^3(v)\mathbf{k}_{03} + B_1^3(u)B_3^3(v)\mathbf{k}_{13} + B_2^3(u)B_3^3(v)\mathbf{k}_{23} + B_3^3(u)B_3^3(v)\mathbf{k}_{33}] \\
\\
&= [B_0^3(v)(B_0^3(u)\mathbf{k}_{00} + B_1^3(u)\mathbf{k}_{10} + B_2^3(u)\mathbf{k}_{20} + B_3^3(u)\mathbf{k}_{30})] \\
&+ [B_1^3(v)(B_0^3(u)\mathbf{k}_{01} + B_1^3(u)\mathbf{k}_{11} + B_2^3(u)\mathbf{k}_{21} + B_3^3(u)\mathbf{k}_{31})] \\
&+ [B_2^3(v)(B_0^3(u)\mathbf{k}_{02} + B_1^3(u)\mathbf{k}_{12} + B_2^3(u)\mathbf{k}_{22} + B_3^3(u)\mathbf{k}_{32})] \\
&+ [B_3^3(v)(B_0^3(u)\mathbf{k}_{03} + B_1^3(u)\mathbf{k}_{13} + B_2^3(u)\mathbf{k}_{23} + B_3^3(u)\mathbf{k}_{33})] \\
\\
&= [B_0^3(v)\alpha] \\
&+ [B_1^3(v)\beta] \\
&+ [B_2^3(v)\gamma] \\
&+ [B_3^3(v)\delta]
\end{aligned}
$$

2.2 Convert to Matrix Format

2.2.1 Factor out Bernstein polynomials for v values

$$p(u,v) = \begin{bmatrix} \alpha & \beta & \gamma & \delta \end{bmatrix} \begin{bmatrix} B_0^3(v) \\ B_1^3(v) \\ B_2^3(v) \\ B_3^3(v) \end{bmatrix}$$

2.2.2 Factor out Bernstein polynomials for u values

$$p(u,v) = \begin{bmatrix} B_0^3(u) & B_1^3(u) & B_2^3(u) & B_3^3(u) \end{bmatrix} \begin{bmatrix} \mathbf{k}_{00} & \mathbf{k}_{01} & \mathbf{k}_{02} & \mathbf{k}_{03} \\ \mathbf{k}_{10} & \mathbf{k}_{11} & \mathbf{k}_{12} & \mathbf{k}_{13} \\ \mathbf{k}_{20} & \mathbf{k}_{21} & \mathbf{k}_{22} & \mathbf{k}_{23} \\ \mathbf{k}_{30} & \mathbf{k}_{31} & \mathbf{k}_{32} & \mathbf{k}_{33} \end{bmatrix} \begin{bmatrix} B_0^3(v) \\ B_1^3(v) \\ B_2^3(v) \\ B_3^3(v) \end{bmatrix}$$

2.2.3 Expand Bernstein polynomial matrix

$$\begin{aligned}
p(u,v) &= \begin{bmatrix} (1-u)^3 & 3u(1-u)^2 & 3u^2(1-u) & u^3 \end{bmatrix} \begin{bmatrix} \mathbf{k}_{00} & \mathbf{k}_{01} & \mathbf{k}_{02} & \mathbf{k}_{03} \\ \mathbf{k}_{10} & \mathbf{k}_{11} & \mathbf{k}_{12} & \mathbf{k}_{13} \\ \mathbf{k}_{20} & \mathbf{k}_{21} & \mathbf{k}_{22} & \mathbf{k}_{23} \\ \mathbf{k}_{30} & \mathbf{k}_{31} & \mathbf{k}_{32} & \mathbf{k}_{33} \end{bmatrix} \begin{bmatrix} (1-v)^3 \\ 3v(1-v)^2 \\ 3v^2(1-v) \\ v^3 \end{bmatrix} \\
&= \begin{bmatrix} 1-3u+3u^2-u^3 & 3u-6u^2+3u^3 & 3u^2-3u^3 & u^3 \end{bmatrix} \begin{bmatrix} \mathbf{k}_{00} & \mathbf{k}_{01} & \mathbf{k}_{02} & \mathbf{k}_{03} \\ \mathbf{k}_{10} & \mathbf{k}_{11} & \mathbf{k}_{12} & \mathbf{k}_{13} \\ \mathbf{k}_{20} & \mathbf{k}_{21} & \mathbf{k}_{22} & \mathbf{k}_{23} \\ \mathbf{k}_{30} & \mathbf{k}_{31} & \mathbf{k}_{32} & \mathbf{k}_{33} \end{bmatrix} \begin{bmatrix} 1-3v+3v^2-v^3 \\ 3v-6v^2+3v^3 \\ 3v^2-3v^3 \\ v^3 \end{bmatrix} \\
&= \begin{bmatrix} 1 & u & u^2 & u^3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{k}_{00} & \mathbf{k}_{01} & \mathbf{k}_{02} & \mathbf{k}_{03} \\ \mathbf{k}_{10} & \mathbf{k}_{11} & \mathbf{k}_{12} & \mathbf{k}_{13} \\ \mathbf{k}_{20} & \mathbf{k}_{21} & \mathbf{k}_{22} & \mathbf{k}_{23} \\ \mathbf{k}_{30} & \mathbf{k}_{31} & \mathbf{k}_{32} & \mathbf{k}_{33} \end{bmatrix} \begin{bmatrix} 1 & -3 & 3 & -1 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ v \\ v^2 \\ v^3 \end{bmatrix}
\end{aligned}$$