# Force Analysis for Active Cams (SLCD)

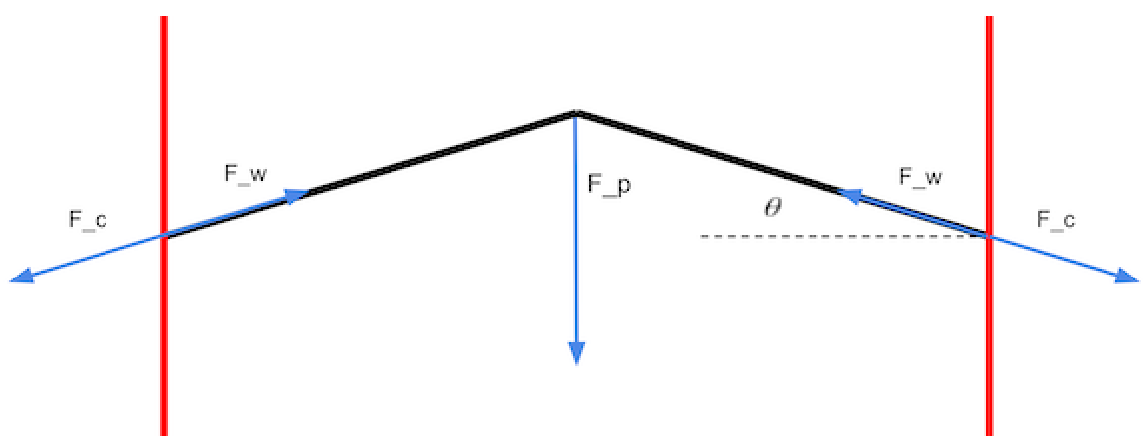
## Summary of analysis

A simplified analysis of the camming forces for an active Spring Loaded Caming Device (SLCD), focusing on the relationship between the applied downward force $F_p$, the camming angle $\theta$ (which measures the angle from the pivot point to the point of contact on the surface), and the resulting outward force $F_{c,x}$, assuming no slipping. It was found that as the camming angle $\theta$ decreases, the outward force $F_{c,x}$ increases significantly, and thus should increase frictional resistance against the rock surface. The outward expansive force (on each surface of contact) is given by:

$$F_{c,x} = \frac{F_p}{2 \cdot \tan(\theta)}$$

This assumes a cam in a symmetric parallel sided placement. The total expansive force is would be $2$ times this, and for a four lobed cam, each lobe would generate $1/2$ of $F_{c,x}$ or $1/4$ of the total expansive force. The outward force varies proportionately to the downward force $F_p$ and inversly proportionally to $\tan(\theta)$. It seems that generally cams are designed with a logrithmic spiral shape to maintain a consistent camming angle throughout range of retraction. For Black Diamond cams this angle is $14.5^\circ$ (see *UKC: Black Diamond C4 review*). Considerations of when slipping will occur based on friction are considered at the close.

The question: *What is the force on the rock if we pull down with 2 kn?* can be answered theoretically as $3.87$ kn (on each side of the surface) or $7.73$ kn of total outward force for a Black Diamond C4 Cam.




## Definitions

- $F_p$ is the downward pulling force at the central pivot point.
- $\theta$ is the angle of the camming arm relative to horizontal.

- $F_c$ is the distributed force along cam arm.
- $F_w$ is the balancing force exerted by the wall or surface on each cam arm.

## Assumptions

- The placement surfaces are parallel and immovable; they cannot translate or rotate. (In reality, the possibility of surface (rock or other material) failure or deformation exists.)
- A simplified cam arm (straight line) is used to model a cam lobe. This should be a decent approximation given a rigid cam lobe.
- The cam rotation axis is assumed to be at the center where the force is applied. For offset double axle cams, the force distribution is offset, however assuming a rigid structure, this force will translate to the cam pivot point and the analysis will be unchanged (if we measure the camming angle from cam axle to where the cam lobe meets rock surface).
- There is no slipping at the contact points between the cam arms and the walls; the contact points are effectively fixed pivots. This is equivalent to perfect or infinite friction.
- The cam arms are assumed to make point contact with the rock. In reality there will be a surface zone of contact and pressure forces over that surface will be distributed in potentially complex ways.
- We begin by studying a simplified two lobe system. Extending to four lobes is considered is straightforward and added in the following section.
- The cam arms (and other cam components) are rigid and incompressible, meaning their length remains constant and they cannot deform.
- The system is symmetric, so the forces on the left and right cam arms are equal in magnitude.
- The cam arms remain attached at the central point where the force $F_p$ is applied.
- The force $F_p$ is applied directly downward at the central point where the cam arms meet.

### Horizontal (Outward) Force Balance

The horizontal component of the force exerted by the rock surface on each cam arm is:

$$F_{w,x} = F_c \cdot \cos(\theta)$$

For static equilibrium, the horizontal components on both sides must be equal in magnitude:

$$F_{w,x,\text{left}} = F_{w,x,\text{right}}$$

### Vertical (Downward) Force Balance

The vertical component of the force exerted by the rock surface on each cam arm is:

$$F_{w,y} = F_c \cdot \sin(\theta)$$

For static equilibrium, the sum of the vertical components must balance the downward force $F_p$:

$$2 \cdot F_{w,y} = F_p$$

Substituting $F_{w,y} = F_c \cdot \sin(\theta)$:

$$2 \cdot F_c \cdot \sin(\theta) = F_p$$

Thus, the force along the cam arm is:

$$F_c = \frac{F_p}{2 \cdot \sin(\theta)}$$

## Decomposition of the force $F_c$

The force $F_c$ along the cam arm can be decomposed into horizontal (outward) and vertical (downward) components:

- **Outward Component** $F_{c,x}$:

  The outward component of $F_c$ is given by:

  $$F_{c,x} = F_c \cdot \cos(\theta)$$

- **Downward Component** $F_{c,y}$:

  The upward component of $F_w$ must be such that it balances $F_p$, as already seen. $F_w$ and $F_c$ are equal in magnitude and opposite in direction so:

  $$F_{c,y} = F_{w,y} = \frac{F_p}{2}$$

To find the outward component in terms of $F_p$, substitute the expression for $F_c$ from above:

$$F_c = \frac{F_p}{2 \cdot \sin(\theta)}$$

giving:

- **Outward Component**:

$$F_{c,x} = \frac{F_p}{2 \cdot \sin(\theta)} \cdot \cos(\theta) = \frac{F_p \cdot \cos(\theta)}{2 \cdot \sin(\theta)} = \frac{F_p}{2 \cdot \tan(\theta)}$$

Thus, the force $F_c$ is decomposed into:

- Outward component: $F_{c,x} = \frac{F_p}{2 \cdot \tan(\theta)}$
- Downward component: $F_{c,y} = \frac{F_p}{2}$

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## Outward force for 2 kn pull on Black Diamond C4 Cam

In [ ]:

```
import numpy as np

theta_degrees = 14.5
theta = theta = np.radians(theta_degrees) # set theta to 15 degrees.
F_p = 2 # set the force to 2kn
F_cx = F_p / (2 * np.tan(theta)) # calculate F_cx on the rock
F_lobe = F_cx / 2
F_x_total = 2 * F_cx

print(f"F_x_total = {F_x_total:.2f} kn")
print(f"F_cx = {F_cx:.2f} kn")
print(f"F_lobe = {F_lobe:.2f} kn")
```

```
F_x_total = 7.73 kn
F_cx = 3.87 kn
F_lobe = 1.93 kn
```

For $F_p = 2.0\ \mathrm{kn}$, with a cam angle of $\theta = 14.5^\circ$ (BD C4 Cam)

The force on each side of the surface is predicted to be

$$F_{c,x} = 3.87\ \mathrm{kn} \ \ (1.93\ \mathrm{kn\ per\ lobe})$$

The total expansive force would be

$$F_{\mathrm{total}} = 7.73\ \mathrm{kn}$$

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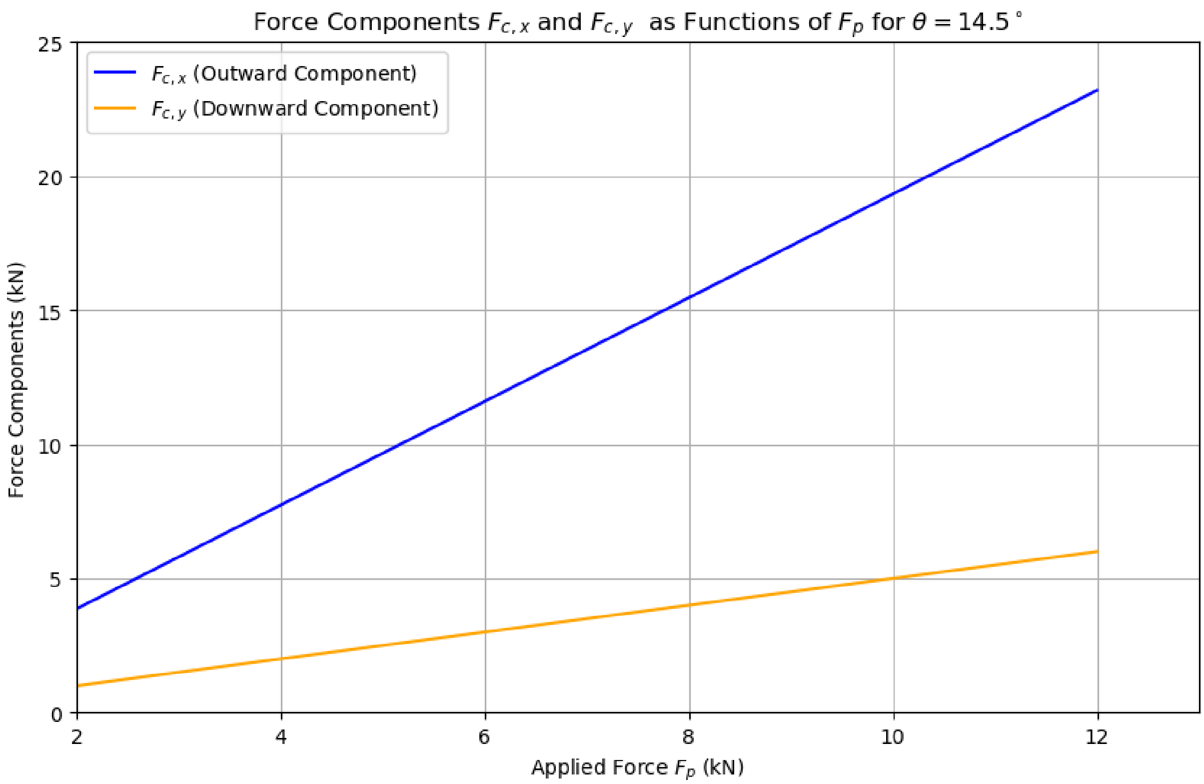
## $F_{c,x}$ and $F_{c,y}$ as functions of $F_p$, with $\theta = 14.5^\circ$

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt

# Constants
theta = np.radians(14.5)  # Example angle in degrees, convert to radians

# Force range (F_p) from 1 kN to 12 kN
F_p = np.linspace(2e3, 12e3, 100)  # Force in Newtons

# Calculate F_c,x and F_c,y
F_c_x = F_p / (2 * np.tan(theta))  # F_c,y as a function of F_p
F_c_y = F_p / 2  # F_c,x as a function of F_p

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(F_p / 1e3, F_c_x / 1e3, label=r'$F_{c,x}$ (Outward Component)', col
```

```
plt.plot(F_p / 1e3, F_c_y / 1e3, label=r'$F_{c,y}$ (Downward Component)', co
plt.xlim(2, 13)  # Set x-axis limits
plt.ylim(0, 25)  # Set y-axis limits
plt.xlabel('Applied Force $F_p$ (kN)')
plt.ylabel('Force Components (kN)')
plt.title(r'Force Components $F_{c,x}$ and $F_{c,y}$  as Functions of $F_p$
plt.legend()
plt.grid(True)
plt.show()
```

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## $F_{c,x}$ and $F_{c,y}$ as functions of $\theta$, with $F_p = 2$ kn

In [ ]:

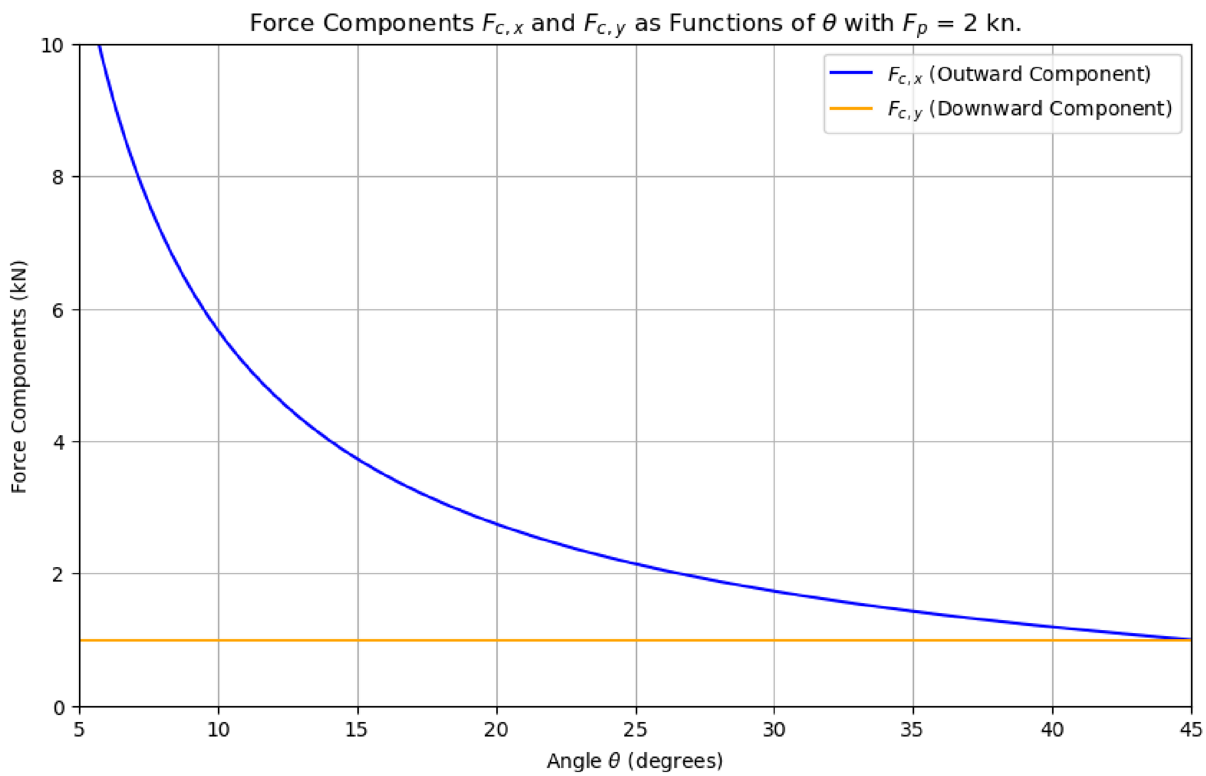
```
# Constants
F_p = 2e3  # Applied force fixed at 5 kN (5000 N)
theta = np.radians(np.linspace(5, 45, 100))  # Theta ranging from 5 degrees

# Calculate F_c,x and F_c,y
F_c_x = F_p / (2 * np.tan(theta))  # F_c,x as a function of theta
F_c_y = np.linspace(F_p / 2, F_p /2, 100) * theta / theta # F_c,y as a funct

# Convert theta to degrees for plotting
theta_degrees = np.degrees(theta)

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(theta_degrees, F_c_x / 1e3, label=r'$F_{c,x}$ (Outward Component)',
plt.plot(theta_degrees, F_c_y / 1e3, label=r'$F_{c,y}$ (Downward Component)'
plt.xlim(5, 45)  # Set x-axis limits
plt.ylim(0, 10)  # Set y-axis limits
```

```
plt.xlabel(r'Angle $\theta$ (degrees)')
plt.ylabel('Force Components (kN)')
plt.title(r'Force Components $F_{c,x}$ and $F_{c,y}$ as Functions of $\theta
plt.legend()
plt.grid(True)
plt.show()
```

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## Force Multiplier: Ratio of $2F_{c,y}$ to $F_p$

> *Thanks to @Tim Parkin*

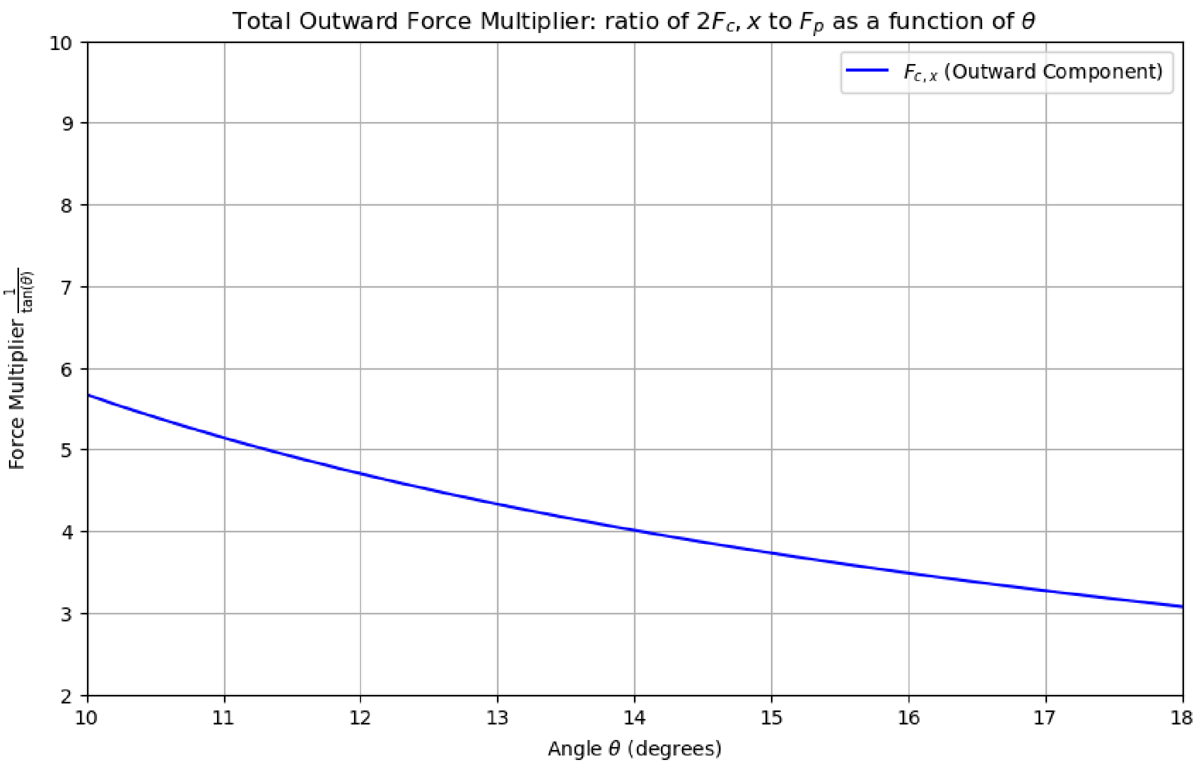
In [ ]:
```
# Constants
theta = np.radians(np.linspace(5, 45, 100))  # Theta ranging from 5 degrees

# Calculate F_c,x and F_c,y
F_c_multiple = 1 / (np.tan(theta))  # F_c,x as a function of theta

# Convert theta to degrees for plotting
theta_degrees = np.degrees(theta)

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(theta_degrees, F_c_multiple, label=r'$F_{c,x}$ (Outward Component)'
plt.xlim(10, 18)  # Set x-axis limits
plt.ylim(2, 10)  # Set y-axis limits
plt.xlabel(r'Angle $\theta$ (degrees)')
plt.ylabel(r'Force Multiplier $\frac{1}{\tan(\theta)}$')
plt.title(r'Total Outward Force Multiplier: ratio of $2F_c,x$ to $F_p$ as a
```

```
plt.legend()
plt.grid(True)
plt.show()
```

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## Force multiplier for $13.75^{\circ}$

In [ ]:
```
# calculate the force multiplier for 13.75 degree cam angle
theta = np.radians(13.75)
print(f"{1/np.tan(theta):.2}")
```

```
4.1
```

The force multiplier for $\theta = 13.75^{\circ}$ is 4.1

This implies that the total outward force will be about 4 times the downward force.

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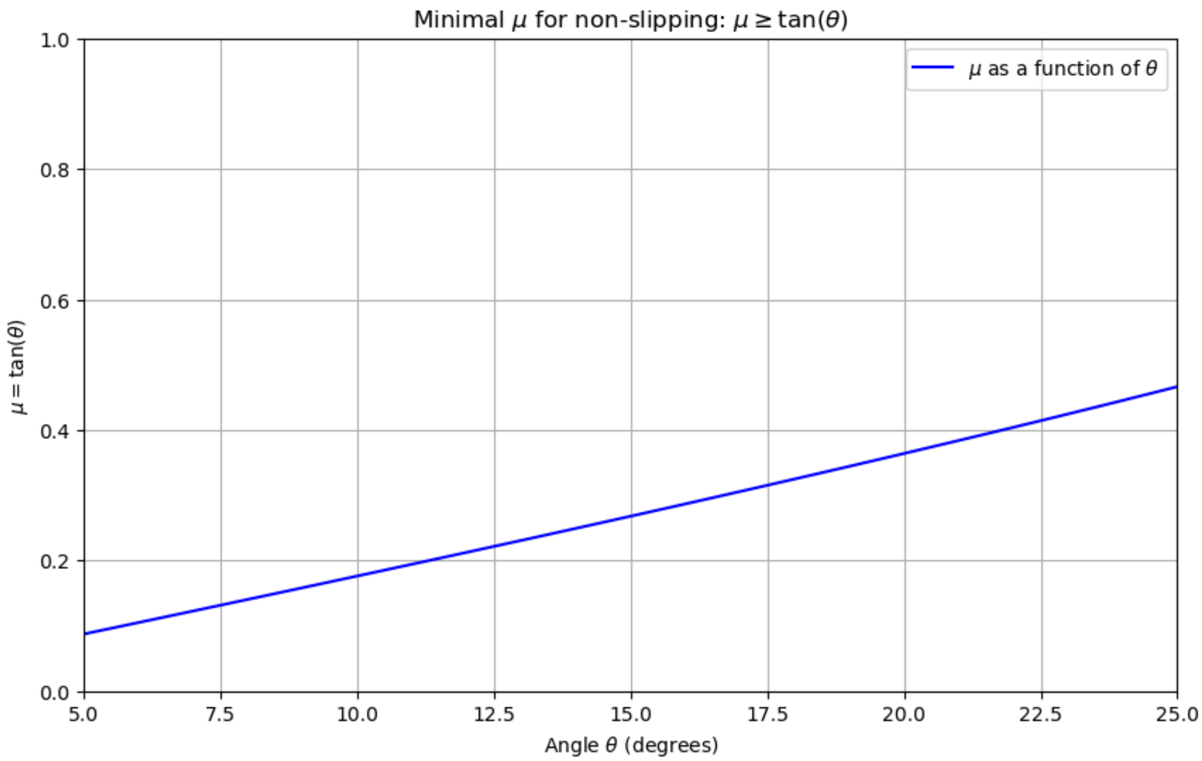
## Plot minimal coefficient of friction $\mu$ for nonslipping

In [ ]:
```
# What is the minimal coefficient of friction for non slipping given a cam a
# Constants
theta = np.radians(np.linspace(5, 45, 100))  # Theta ranging from 5 degrees

# Calculate minimal mu base on mu = tan(theta)
mu = np.tan(theta)  # mu as a function of theta

# Convert theta to degrees for plotting
theta_degrees = np.degrees(theta)
```

```python
# Plotting
plt.figure(figsize=(10, 6))
plt.plot(theta_degrees, mu, label=r'$\mu$ as a function of $\theta$', color=
plt.xlim(5, 25)  # Set x-axis limits
plt.ylim(0, 1)  # Set y-axis limits
plt.xlabel(r'Angle $\theta$ (degrees)')
plt.ylabel(r'$\mu =  \tan(\theta)$')
plt.title(r'Minimal $\mu$ for non-slipping: $\mu \geq \tan(\theta$)')
plt.legend()
plt.grid(True)
plt.show()
```

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## Observations and Commentary

### Force Distribution and Balance

- There is a simple relationship between the applied force $F_p$, the angle $\theta$ of the cam arms, and the resulting forces $F_{c,x}$ and $F_{c,y}$.
- The vertical components of the forces $F_{c,y}$ must balance the downward force $F_p$ to ensure static equilibrium.
- The horizontal components $F_{c,x}$, which exert outward pressure against the rock, are sensitive to the angle $\theta$. As $\theta$ decreases, the outward force increases significantly. (Note that the camming angle is *NOT* the same as the percentage of camming, e.g. from fully open to fully cammed).
- The amount of camming (between open and fully cammed) only affects the system if it changes the camming angle $\theta$, that is the angle from the pivot point to the point

where the lobe contacts the surface it is cammed against.

## Behavior as $\theta$ Approaches Extremes

- When $\theta$ approaches zero, the system exhibits a dramatic increase in the outward force $F_{c,x}$. This suggests that a cam with too small a camming angle may create too great an outward force (either deforming/destroying cam or deforming/breaking rock surface, potentially causing failure of placement).
- Conversely, as $\theta$ increases (steeper camming angle), the outward force diminishes, this in turn would reduce the available friction force to maintain cam placement.
- This suggest that there may be an ideal camming angle. From a cursory look at modern Black Diamond C4 cams with a double axle, it seems clear that they are designed with a logrithmic spiral such that the contact point with the surface remains approximately constant throughout the camming range due to the cam lobe shape. It appears that the choice of camming angle by designers is a trade off over expansive range of the cam and amount of force mulitplication which the cam offers to increase frictional forces and counteract downward pull.

## Considering Four Lobes

- The initial analysis focused on 2 lobes to capture the basic mechanics. In a four-lobed cam, the applied force $F_p$ is distributed among all four lobes. In the case of a Metolius TCU, a tri-lobed cam, the factor might be more complex, as the central wider cam must generate 1/2 the outward force load, while each of the two side lobes will generate 1/4 of the outward force load (to maintain the static equilibrium condition in the $x$ plane). This may be why Metolius makes the central cam wider!

- For 4 lobes: Assuming a symmetric setup, the force $F_p$ would be divided equally among the four lobes, meaning that each lobe would bear a force of $\frac{F_p}{4}$.

- This redistribution of force changes the factors of 2 in our previous equations to factors of 4. Specifically: The force along each cam arm $F_{\text{lobe}}$ would be:

$$F_{\text{lobe}} = \frac{F_p}{4 \cdot \sin(\theta)}$$

  Consequently, the horizontal and vertical components of the force would also be adjusted:

$$F_{\text{lobe},x} = \frac{F_p}{4 \cdot \tan(\theta)}$$

  And

$$F_{\text{lobe},y} = \frac{F_p}{4}$$

- This shows the efficiency of the four-lobed design in distributing forces evenly across multiple contact points, reducing the load on each individual lobe and likely increasing the overall stability.

### Considering Friction Forces

- In the initial analysis, it was assumed that there was no slipping at the contact points between the cam lobes and the rock surface, that they acted like fixed pivot points. However, in reality, it is friction that plays the crucial role in preventing a cam from slipping. For example, imagine a frictionless surface--the cam will simply slide out (assuming parallel sided placement).
- The outward force $F_{c,x}$ exerted by the cam lobes against the rock surface generates a normal force and hence a frictional force that resists slipping. The maximum frictional force that can be achieved (using the simple normal force and coefficient model) is:

  $$F_{\text{friction}} = \mu \cdot F_{c,x}$$

  where $\mu$ is the coefficient of friction between the cam lobes and the rock surface. It's important to note that if we are dealing with significantly large forces, on the order of 10 kn, this may imply some deformation of materials and other more complex factors than can be captured by this equation. It is however signicant to note that things like water and ice are likely to drastically reduce $\mu$ (all else being equal)!
- For a cam to remain securely in place, the frictional force on each lobe must be greater than or equal to the fraction of the force $F_p$ that each lobe is counteracting. Once slipping begins, it may lead to further failure, however it may also lead to the cam reaching a more stable position. The complexities of cam placement!
- This highlights the importance of both the outward force $F_{c,x}$ and the coefficient of friction $\mu$ in ensuring the cam's effectiveness. As the camming angle $\theta$ decreases, the outward force, and therefore frictional force, increases. Understanding the frictional properties of the contact surface and the dependence on camming angle $\theta$ to multiplicatively increase normal force, are critical to maximizing the cam's holding power.
- It is important to note that the degree of cam contraction (open to fully cammed) is not modelled here, and given that a cams are typically designed to maintain a fairly constant contact angle, the degree of cam contraction (e.g. not cammed to fully cammed) seems to mostly affect the security of the placement, or the amount of reserve camming action that is available.

### Cam Design Implications

- This analysis reinforces the importance of cam angle $\theta$ based on the desired balance between vertical load balance and lateral force exertion to generate friction.

## Visualization

- The plots of $F_{c,x}$ and $F_{c,y}$ as functions of both $F_p$ and $\theta$ provided a visual of how these forces behave under different conditions.
- Its interesting to note that both the outward and downward forces behave linearly for fixed camming arm angle, $\theta$.
- The non-linear nature of the force distribution in terms of $\theta$ is evident, and particularly the rapid increase in $F_{c,x}$ as the cam angle decreases.

## Remarks and Further Investigation

- This analysis is sufficient to give a baseline prediction of the outward force of a cam, if the camming angle is known and we can assume no slippage.
- Understanding how the percentage of camming affects the cam behavior remains an interesting question.
- Modeling friction directly to calculate the slipping point of a lobe would be interesting, however the coefficient of friction could be quite complex and could be effectively infinite in the case where there is anything that blocks the cam from moving, such as a pocket or constriction, or sharp protrusion of rock (a crystal) wedged into the metal rib or "tooth" of the cam. Working with uniform surfaces (as much as possible) having known coefficients of friction would be a good way to get started with collecting data. The ribs or "teeth" in cams is definitely something to take into consideration!
- The effects of surface and cam structure deformation under load would be quite interesting though expected to be complex.
- Understanding how the surface of the lobes interact with the surface of the rock would be interesting, vs. just using a point model: where the "teeth" come in, and other factors.
- Understanding why a more retracted cam is considered stronger than a less retracted (tipped out, or under-cammed) cam would be interesting, as this is not evident in this model. (Is it actually stronger?)
- Comparing smaller and larger cams of course would be interesting.