Topics in Mathematical Science VI
Autumn 2022

Course theme. Representations of finite groups and ordinary character theory

Preliminary lecture plan.

Week	Date	Topic
1	10-06	Representations of groups, group algebra
2	10-13	Jordan-Hölder theorem, Artin-Wedderbun theorem, Maschke's theorem
3	10-20	Krull-Schmidt property, tensor and dual representations
4	10-27	Self-duality, character, induction, restriction
5	11-03	(No lecture)
6	11-10	Permutation representation, class function, inner product
7	11-17	Inner product 2, character table
8	11-24	Induced and restricted characters
9	12-01	Frobenius algebra, symmetric algebra. Cartan matrix
10	12-08	p-group, cyclic group case.
11	12-15	Block decomposition, uniserial algebra, Brauer trees
12	12-22	Quiver algebras, Brauer tree algebras
13	12-29	(No lecture)
14	01-05	(No lecture)
15	01-12	Revision. Examples. Module diagrams
16	01-19	Tame and wild representation type
17	01-26	Tame and wild representation type

Time and venue. Thursday 13:00–14:30, Grad. School of Mathematics Room 409

Evaluation. There will be four sets of homework assignments, which will be distributed during the course. To pass the course, one must obtain at least 50% of the total score of the four assignments combined. The final grade is determined by the total score as follows:

$$A : 85 \sim 100\%, \quad B : 70 \sim 84\%, \quad C : 50 \sim 69\%, \quad F : 0 \sim 49\%.$$

Textbooks. Apart from Lecture 2 and 3 (in the plan above), the content of Lecture 1 to 8 will be based on Chapter 1 to 20 of

[JL] G. James and M. Liebeck: *Representations and characters of groups* 2nd ed, Cambridge University Press, 2001

The rest of the course will be based on Chapter 1 and some part of Chapter 3 and 4 from

[B] D. J. Benson: *Representations and Cohomology: Volume 1, Basic Representation Theory of Finite Groups and Associative Algebras*, Cambridge Studies in Advanced Mathematics 30, Cambridge University Press 1998

Benson's book is often too highly paced for beginners, one may find it much more helpful to consult other books. The following books are recommended for all material of the course, except for the character theory part.

[EH] K. Erdmann and T. Holm: *Algebras and representation theory.* Springer Undergraduate Mathematics Series, Springer International Publishing, 2018

[Z] A. Zimmermann: *Representation Theory: A Homological Algebra Point of View*, Algebra and Applications 19, Springer 2014

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Actual lecture content and remark (update weekly).

L1. • Content: Representations, trivial rep, sign rep, group algebra, regular rep, representations = modules, homomorphism, characteristic, simple modules (irreducible representation)
  • Reference: [JL, Chap. 1 to 7, 9], [EH, 1.1.2, 2.5.1]
  • Remark: In case reminder on basic module theory is needed, see, for example, [EH, Chapter 2]

L2. • Content: Jordan-Hölder, Jacobson radical, Artin-Wedderburn, Maschke's theorem
  • Reference: [JL, Chap. 8, 9, 10], [B, Chap. 1], [EH, Chap. 3 to 6], [Z, 1.4.2, 1.6, 1.2.2, 1.5]

L3. • Content: Krull-Schmidt property, tensor product, dual representation
  • Reference: [B, Ch. 1], [EH, Ch. 7], [Z, 1.4.1, 1.7], [JL, Chap. 19]

L4. • Content: Self-duality, character, induction, restriction
  • Reference: [JL, Chap. 20, 21], [EH, Chap. A], [Z, 1.7.2]

L5. • Content: Permutation representation, number of simples, class functions, inner product
  • Reference: [JL, Ch. 29, 13, 14, 15], [EH, 6.4], [Z, 1.7.2]

L6. • Content: Inner product 2, character table
  • Reference: [JL, Ch. 14, 16, 18]

L7. • Content: Induced and restricted characters
  • Reference: [JL, Ch. 20, 21]

L8. • Content: Forbenius, symmetric algebra. Cartan matrix
  • Reference: [B, Ch. 1], [Z, 1.10]

L9. • Content: Group algebra of p-group, cyclic group in characteristic p.
  • Reference: [Alperin - Local representation theory I.3, II.4, II.5]

L10. • Content: Block decomposition, uniserial algebras, Brauer trees
  • Reference: [B. 1.8][Z. 2.3], [B. 4.18], [Z. 5.10]

L11. • Content: Quiver algebras, Brauer tree algebras
  • Reference: [Z. 1.11][Z. 5.10]

L12. • Content: Revision. Examples. Module diagrams.
  • Reference: N/A

L13. • Content: Tame and wild representation type.
  • Reference: [B, 4.4]

L14. • Content: Tame and wild representation type continue, with Klein-4 group and $(\mathbb{Z}/2)^3$ as examples.
  • Reference: [B, 4.3]