Ex 1. Write down all Brauer tree with 5 edges and trivial exceptional multiplicity.

Ex 2. Let $(T, v_0, m_0 = 1)$ be the multiplicity-free Brauer star with 3 edges.

(1) Write down quiver Q_{T,v_0,m_0} and the admissible ideal I_{T,v_0,m_0}.

(2) Write down all 9 strings of $(Q_{T,v_0,m_0}, I_{T,v_0,m_0})$; you can write the associated module diagram instead if you prefer.

Ex 3. Repeat the previous question for the multiplicity-free Brauer line with 3 edges.

Ex 4. Consider an arrow $(y|x)_v \in Q_T$ for a Brauer tree (T, v_0, m_0). Let B be the associated Brauer tree algebra. Define a module $H_{x,v} := B(y|x)_v$. Note that when x is a leaf attached to valency 1 vertex v, then $H_{x,v}$ is the simple module S_x.

(1) Show that the module $H_{x,v}$ is uniserial and find the corresponding string.

(2) There is a surjective homomorphism $P_x \twoheadrightarrow H_{x,v}$ with kernel $H_{y,u}$. What is y and u?

Ex 5. Recall the construction of trivial extension algebra $\Lambda \ltimes D\Lambda$ from Lecture 8. Let Λ be the lower triangular matrix ring

$$\begin{pmatrix} K & 0 \\ K & K \end{pmatrix} \cong K(1 \xrightarrow{\alpha} 2).$$

Note that $D\Lambda \cong \left(\begin{smallmatrix} K & K \\ 0 & K \end{smallmatrix}\right)$ as (left or right or bi-) module. Show that the induced trivial extension algebra is isomorphic to the multiplicity-free Brauer tree algebra with 2 edges

$$B(T, v_0, m_0 = 1) = K(\begin{tikzcd} 1 \arrow[r, bend left, "\alpha"] & 2 \arrow[l, bend left, "\alpha^*"] \end{tikzcd})/(\alpha\alpha^*\alpha, \alpha^*\alpha\alpha^*).$$

Deadline: 30th January, 2023