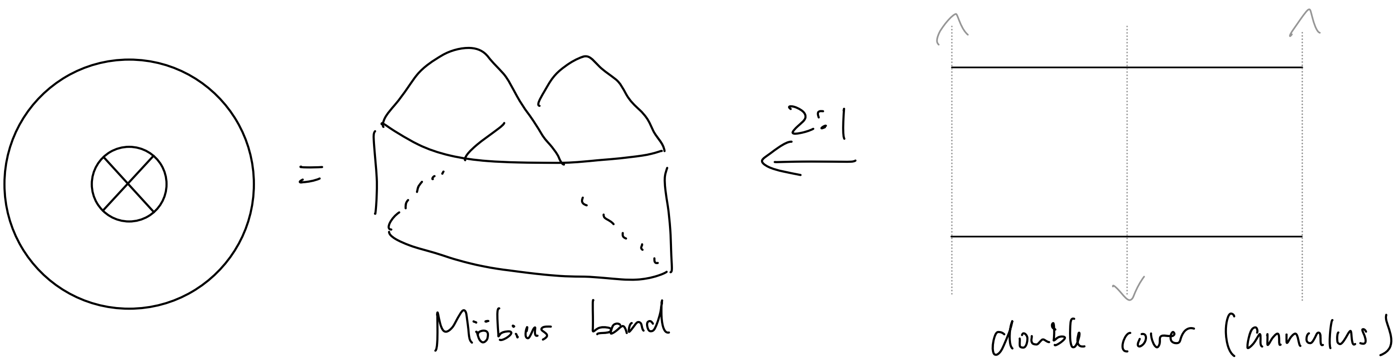
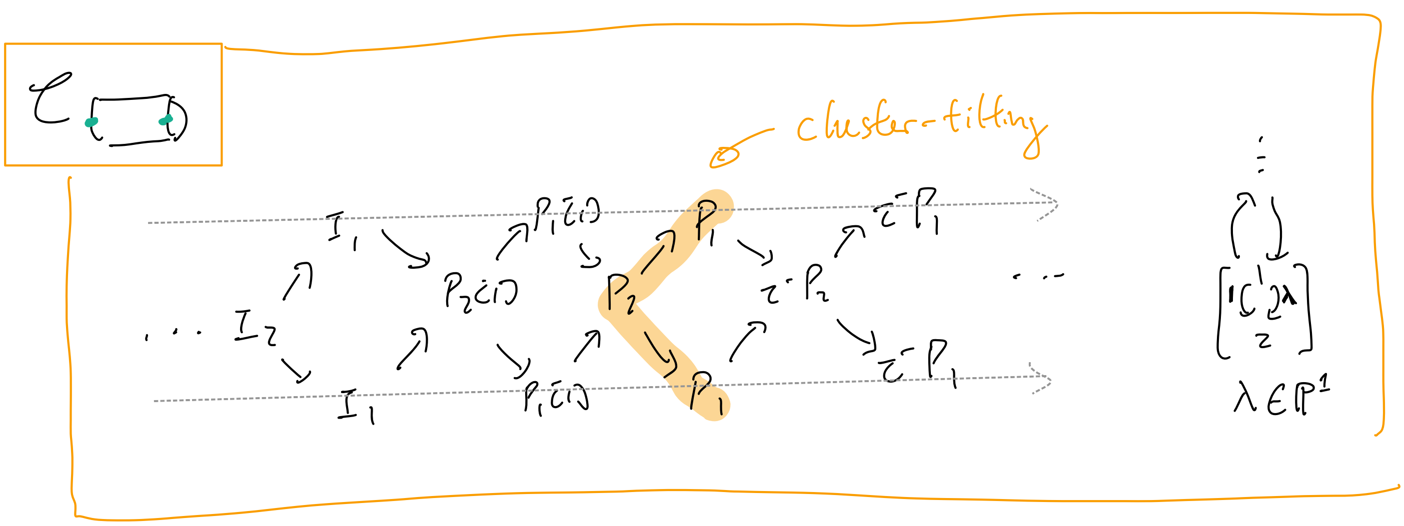
Categorification of unpunctured non-orientable marked surfaces

work in progress, jt. with V. Bazier-Matte, K. Wright





★ Green texts = extra info

Summary

(Known):

	Topology		Category
		I	
	Orientable surface w/ triangulation	$\xleftrightarrow{1:1}$	gentle Jacobian algebra
			$\rightsquigarrow$
			cluster category
			$\rightsquigarrow$
		II	
$\rightsquigarrow$	curves $(+\cdots)$	$\xleftrightarrow{1:1}$	indec. (families of) obj's
		III	
	triangulations	$\xleftrightarrow{1:1}$	cluster-tilting obj.'s
		$\rightsquigarrow$ IV	
	Categorification of mutation (= flip) and associated cluster algebra [Fomin-Shapiro-Thurston, Fock-Goncharov]		

Question : Non-orientable surfaces (NoS) ?

Idea : $\exists$ orientable double cover $\tilde{S}$ for an NoS S.

i.e., $S = \tilde{S}/\sigma$ $\qquad$ σ : orientation-reversing auto.

$\sigma^2 = 1$

Today : Goal I, II, "III"

§§1 Surface topology

Setup

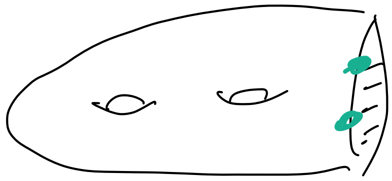
$\partial S \neq \phi$

S: Surface = compact 2-dim^l/$\mathbb{R}$ w/ non-empty boundary

$\mathbb{M}$: finite set of marked points in ∂S (∴ unpunctured)

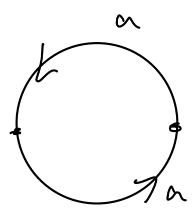
s.t. each boundary component contains ≥ 1 marked pt.

and $(S, \mathbb{M}) \neq 1, 2, 3$-gon

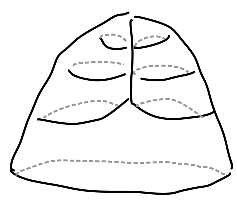


$|\mathbb{M}| = 2$.

§1.1 Working with non-orientable surfaces (NoS)

- $\mathbb{R}\mathbb{P}^2$ = 

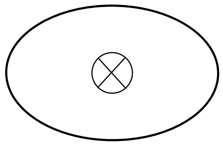
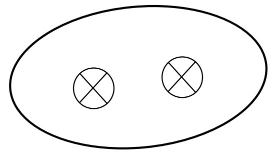
Youtube
"The cross-cap"

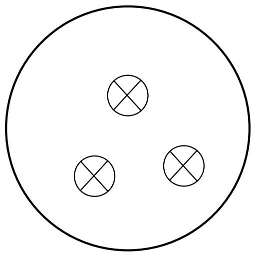
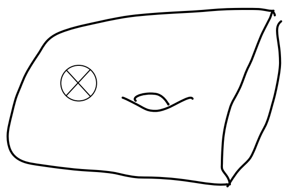
- crosscap $:= \mathbb{R}\mathbb{P}^2 \setminus$ disc  $\overset{\text{homeo}}{\simeq}$ Möbius strip,

In practice, represent by the symbol $\otimes$ (or $\circledast$, $\circledast$, etc.)

Rmk: Some literature call $\mathbb{R}\mathbb{P}^2$ the crosscap instead.

E.g.

 = Möbius strip ,  = Klein bottle $\setminus$ disc ,

 =  = Dyck's surface $\setminus$ disc.

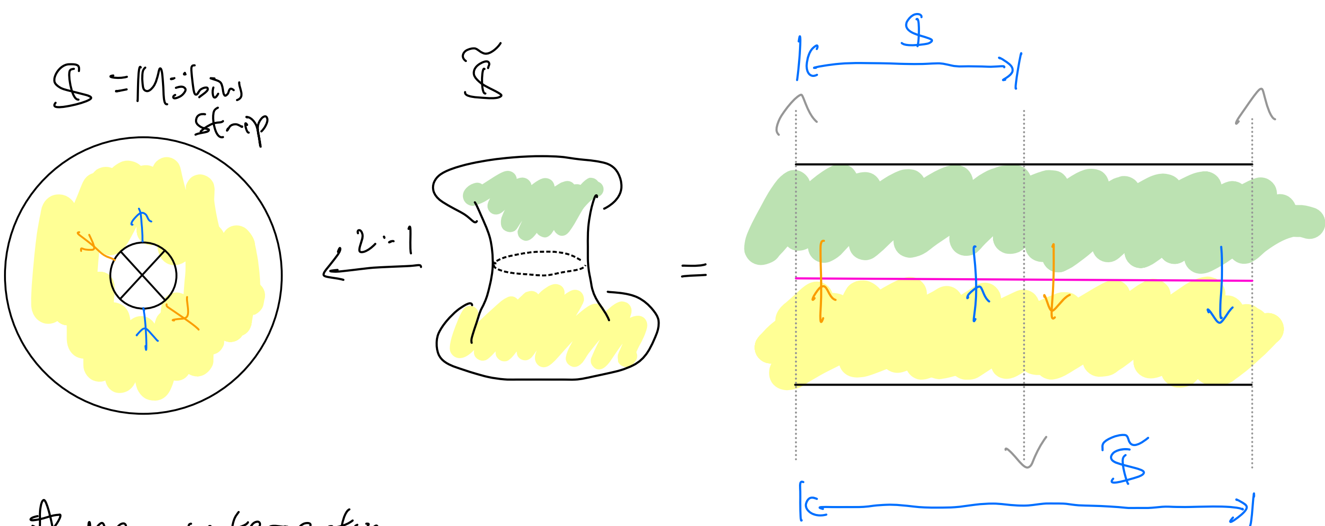
§1.2 Double cover

$$\boxed{\text{(N)OS } (\mathbb{S}, \mathbb{M}) \longleftrightarrow (\tilde{\mathbb{S}}, \mathbb{M})/\sigma}$$

where

$(\tilde{\mathbb{S}}, \mathbb{M})$:= Orientable double cover of a marked nOS. $(\mathbb{S}, \mathbb{M})$

σ : orientation-reversing auto. of order 2.

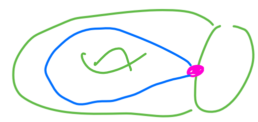


※ no intersection between the 2 curves shown!

§1.2 Objects of interest

γ : **Curve** on $(\mathbb{S}, \mathbb{M})$ $:\Leftrightarrow$ either $\begin{cases} \textbf{closed} & \gamma \simeq S^1 \\ \text{s.t.} & \gamma \cap \mathbb{M} = \emptyset \\ & \text{non-contractible} \end{cases}$

or $\begin{cases} \textbf{non-closed} & \gamma : [0,1] \to \mathbb{S} \\ & \gamma(0), \gamma(1) \in \mathbb{M} \\ & \gamma(0,1) \subset \mathbb{S} \setminus \partial \mathbb{S} \end{cases}$

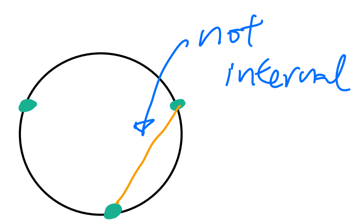
 $\in$

* Always considered up to isotopies that fix $\partial\mathbb{S}$ pointwise

Non-crossing (set of) curves $:\Leftrightarrow$ no intersection except possibly at endpoints.
$\shortparallel$
NC

- **Arc** $:\Leftrightarrow$ NC non-closed curves

- **Internal arc** $:\Leftrightarrow$ arc $\not\simeq$ boundary interval

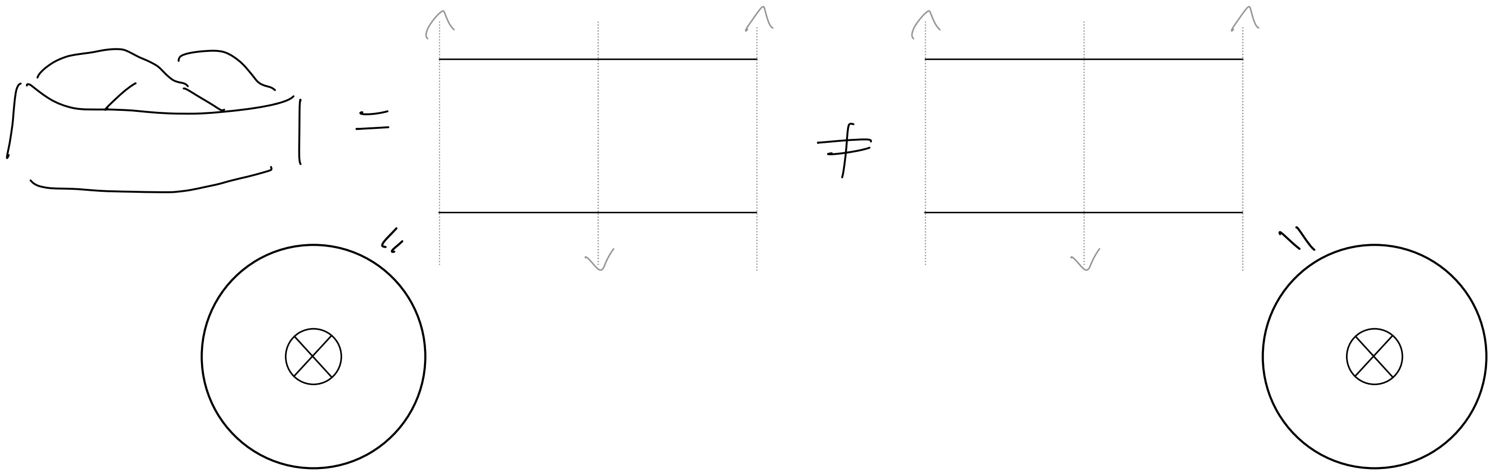


- **1-sided** closed curve $:\Leftrightarrow$ non-orientable closed curve

  (If simple, then equiv. to $\exists$ regular nbhd $\underset{\text{homeo}}{\cong}$ Möbius strip)

- **2-sided** closed curve $:\Leftrightarrow$ not 1-sided

  (If simple, then equiv. to $\exists$ regular nbhd $\underset{\text{homeo}}{\cong}$ annulus)



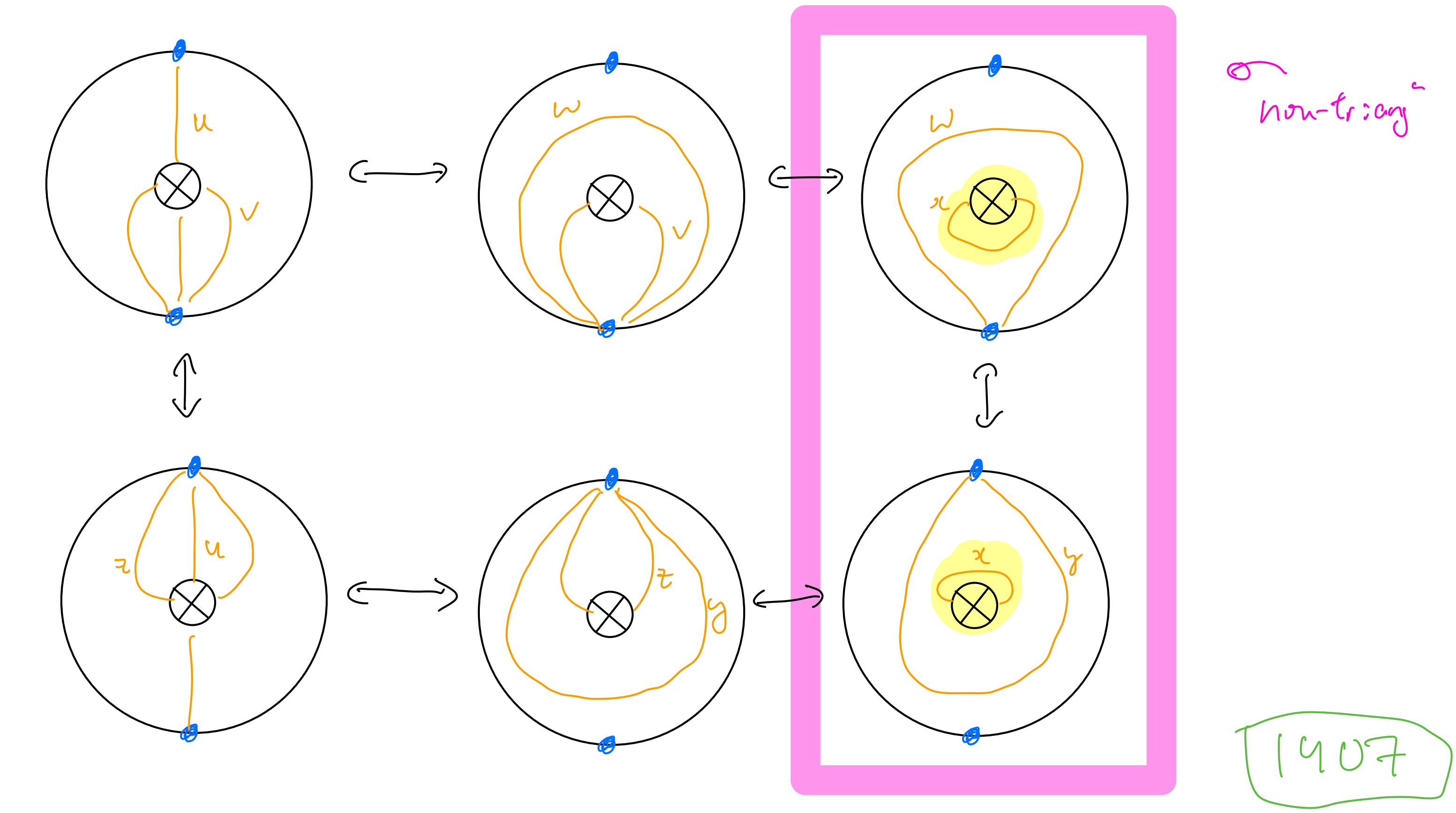
- **Quasi-arc** :⟺ either internal arc,

  or 1-sided simple closed curve

  (simple: no self-intersection)

- **(quasi-)triangulation** :⟺ maximal NC set of (quasi-)arcs

E.g. M_2 (:= Möbius strip w/ 2 marked pt's) has 6 quasi-triang^n's.



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§§2) Triangulation vs QP

2.1) Orientable case $(\widetilde{S}, \widetilde{M})$, $\widetilde{T}$

$\rightsquigarrow$ (Q, W) : QP (Quiver with Potential)

where
$$\begin{cases} Q_0 = \{\text{internal arcs of } \widetilde{T}\} \\ Q_1 = \text{CW oriented angles of triangles of } \widetilde{T} \\ W = \sum\limits_{\substack{\text{internal} \\ \text{triangles}}} \end{cases}$$

$\rightsquigarrow$ Jacobian algebra $\underset{\substack{\| \\ J_{\widetilde{T}}}}{}$ $\mathrm{Jac}(Q, W) := \mathbb{k}Q / (\partial_a W) = \mathbb{k}Q / (\text{length 2 paths in int. } \Delta\text{'s})$

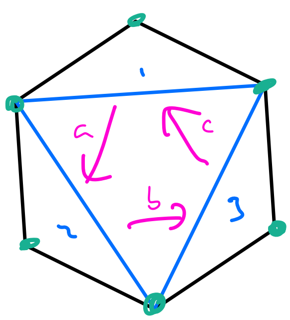
$$J_{\widetilde{T}} = \mathrm{Jac}(Q, W)$$

N.B. $J_{\widetilde{T}}$ is f.d. and a gentle algebra (cf. next talk)

Convention/Definition QP is gentle if induced by a triang^n.

E.g.

$(\widetilde{S}, \widetilde{M})$, $\widetilde{T}$:



$$Q = \begin{tikzcd} & 1 \arrow[dr, "a"] & \\ 3 \arrow[ur, "c"] & & 2 \arrow[ll, "b"] \end{tikzcd}, \quad W = abc$$

$$J_{\widetilde{T}} = \begin{tikzcd} & 1 \arrow[dr] & \\ 3 \arrow[ur] & & 2 \arrow[ll] \end{tikzcd} \Big/ (ab, bc, ca)$$

Thm [Assem, Brüstle, Charbonneau-Jodoin, Plamondon]

$$\left\{ (\tilde{S}, \tilde{M}; \tilde{T}) : \begin{array}{l} \text{triangulated} \\ \text{ori. surfaces} \end{array} \right\} \overset{1:1}{\longleftrightarrow} \{ (Q, W) : \text{gentle QP} \}$$

2.2) NoS case

$$(S, M) \xleftarrow{2:1} (\tilde{S}, \tilde{M}) \circlearrowleft \sigma$$

Def ① Q : quiver

An involution $\sigma : Q \to Q$

In [Derksen-Weyman] language: (Q, σ) is a symmetric quiver

$:\Leftrightarrow \quad \sigma = (\sigma_0 : Q_0 \to Q_0, \ \sigma_1 : Q_1 \to Q_1)$

s.t. $\begin{cases} \sigma^2 = 1 \\ \sigma(v \xrightarrow{\alpha} w) = \left(\sigma(v) \xleftarrow{\sigma(\alpha)} \sigma(w) \right) \end{cases}$

N.B. Specifying involution $\sigma \Leftrightarrow$ Specifying $\Bbbk Q \xrightarrow[\sim]{\text{alg}} \Bbbk Q^{op}$

② An involution on a QP (Q, W).

$\sigma W = \pm W$ OK

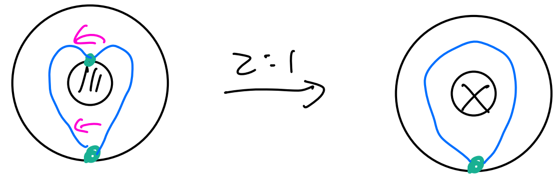
$:\Leftrightarrow$ an involution σ on Q s.t. $\sigma W = W$

$\Leftrightarrow$ —— " —— s.t. $\sigma(\partial_a W) = (\partial_a W)$

Eg. ① $\overset{1}{\bullet} \xrightarrow{\alpha} \overset{2}{\bullet}$ $\quad \sigma : \begin{array}{l} 1 \longleftrightarrow 2 \\ \alpha \longleftrightarrow \alpha \end{array}$

② $\overset{1}{\bullet} \underset{\beta}{\overset{\alpha}{\rightrightarrows}} \overset{2}{\bullet}$ $\quad \sigma : \begin{array}{l} 1 \longleftrightarrow 2 \\ \alpha \longleftrightarrow \alpha \\ \beta \longleftrightarrow \beta \end{array}$

(3) $\overset{1}{\bullet} \underset{\beta}{\overset{\alpha}{\rightrightarrows}} \overset{2}{\bullet}$ σ ; $1 \longleftrightarrow 2$, $\alpha \longleftrightarrow \beta$



(4) Exercise: $A_{n,n}$-quiver

$$\begin{array}{c} 1 \nearrow 2 \to \cdots n \searrow \\ \qquad\qquad\qquad\quad n+1 \\ 1 \searrow 2n \to \cdots n+2 \nearrow \end{array}$$

has an involution

Obs: The setup $\sigma \circlearrowright (\widetilde{S, M}) \xrightarrow{2:1} (S, M)$

induces an involution σ on (Q, W)

Moreover, σ is fixed-point free (FPF)

$\sigma(v) \neq v \quad \forall v \in Q_0$

$\sigma(\alpha) \neq \alpha \quad \forall v \in Q_1$

Prop [BM-C-W] (Goal I)

$$\left\{ \begin{array}{c} \text{triangulated} \\ \text{NoS} \end{array} \right\} \overset{1:1}{\longleftrightarrow} \left\{ \begin{array}{c} (Q, W; \sigma) \text{ s.t. } (Q, W) \text{ is} \\ \text{gentle } \& \ \sigma \text{ is a FPF involution} \end{array} \right\}$$

$$(S, M; T) \longmapsto (Q, W; \sigma)$$

Rmk Same for locally gentle quivers version

1915

§3 Triangulation vs CTO (cluster-tilting obj)

Setup: From now on, $\boxed{\Bbbk = \text{complex numbers}}$

Orientable case

See [Brüstle-Zhang]

TOPOLOGY	CATEGORY
$(\tilde{S}, \tilde{M})$	cluster category $\mathcal{C} = \mathcal{C}_{(\tilde{S}, \tilde{M})} \circlearrowleft [1] \cong \tau$
non-closed curves	"string objects"
$\begin{pmatrix}\text{closed} \\ \text{curves}\end{pmatrix} \times \Bbbk^{\times}$	"band objects"

N.B. This is KS Hom-fin. 2-CY tri. cat.

all indec's of $\mathcal{C}_{(S,M)}$

Elements are (ω^n, λ), where ω: primitive,

i.e. $\omega \neq \rho^k$ in $\pi_1(\tilde{S})$ with $k > 1$

Moreover,

TOPOLOGY	CATEGORY
Crossing between curves (+ loc. sys.)	Canonical basis (+ local sys. hom.) of $\mathrm{Ext}^1_{\mathcal{C}}(-,-) := \mathrm{Hom}_{\mathcal{C}}(-,-[1])$
arcs	rigid objects $:=$ no self-ext^n $(\mathrm{Ext}^1 = 0)$
triangulation	cluster-tilting object (CTO) $:=$ maximal rigid object. (i.e. $M \oplus N$ rigid $\Rightarrow N \in \mathrm{add}\, M$)

Slogan: Crossings $=$ Non-split extensions

Prop [BM-C-W]

∃ exact $\underline{\underline{\text{contravariant}}}$ $\boxed{\text{duality}}$ functors ∇ i.e. $\nabla^2 \cong \mathrm{Id}$

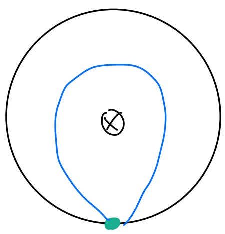
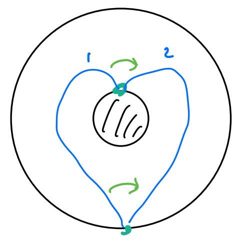
$$\begin{array}{ccc} \mathcal{C} & \xrightarrow[\sim]{\nabla} & \mathcal{C} \\ \downarrow & \circlearrowright & \downarrow \\ \mathrm{mod}\, J_T & \xrightarrow[\sim]{\nabla} & \mathrm{mod}\, J_T \end{array}$$

such that

① $\forall \gamma$: non-closed on $(\widetilde{S}, \widetilde{M})$, $\overbrace{\sigma(\gamma)}^{\text{Topology}} \longleftrightarrow \overbrace{\nabla(\gamma)}^{\text{Category}}$

② $\forall (\omega^n, \lambda) \in \left\{ \begin{array}{c} \text{c.c.'s} \\ \text{on } (\widetilde{S}, \widetilde{M}) \end{array} \right\} \times k^\times$, $(\sigma(\omega)^n, \lambda^{-1}) \longleftrightarrow \nabla(\omega^n, \lambda)$

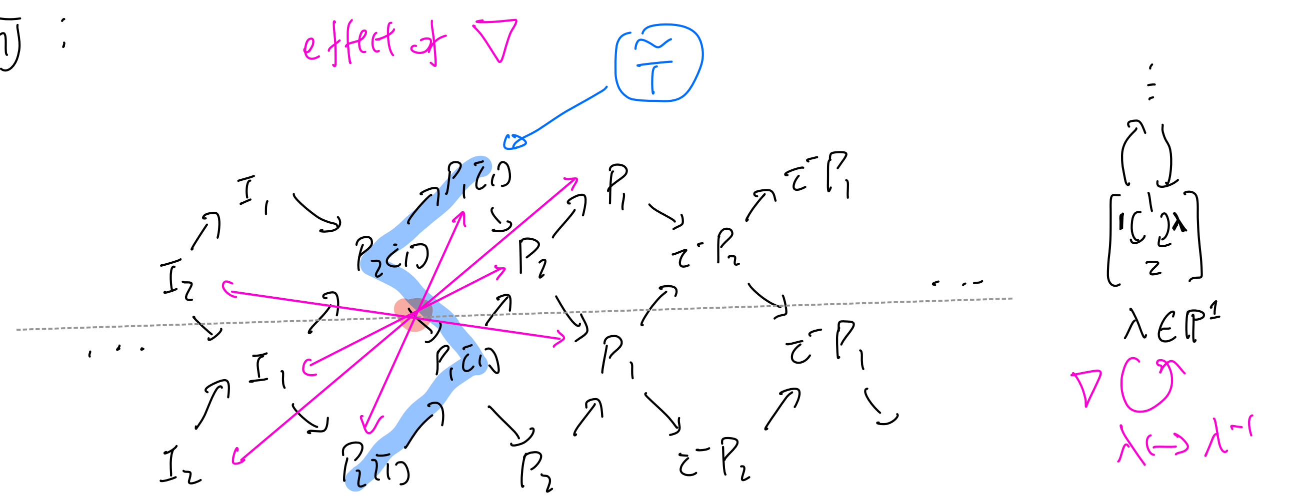
E.g.

$(S, M; T) =$  $\xleftarrow{2:1}$ $(\widetilde{S}, \widetilde{M}, \widetilde{T}) =$ 

$Q = \left(1 \overset{a}{\underset{b}{\rightrightarrows}} 2 \right)$, $W = 0$

σ : $1 \longleftrightarrow 2$, $a \longleftrightarrow b$

$\mathcal{C}_{(\widetilde{S}, \widetilde{M})}$:



Thm [BM-C-W] (Partial Goal II, III)

$\forall$ NoS $(\mathbb{S},\mathbb{M})$, have

$$\left\{ \begin{array}{l} \text{non-closed} \\ \quad \text{curves on } (\mathbb{S},\mathbb{M}) \end{array} \right\} \xleftrightarrow{1:1} \{ \nabla\gamma \oplus \gamma \mid \gamma : \text{string obj} \}$$

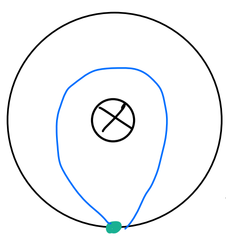
which induces

$$\left\{ \begin{array}{l} \text{triangulations} \\ \quad \text{of } (\mathbb{S},\mathbb{M}) \end{array} \right\} \xleftrightarrow{1:1} \left\{ \begin{array}{l} \text{basic CTO } M \in \mathcal{C} \\ \quad \text{s.t. } \underbrace{\nabla M \cong M}_{\Leftarrow \text{ self-dual}} \end{array} \right\}$$

Rmk

- This is mutation compatible (as long as it is possible.)
- Mutation send corresp. s.τ-tilting pair to incomparable s.τ-tilting pair.

E.g. $(\mathbb{S},\mathbb{M}) = M_1$

$\Rightarrow$ LHS $= \Big\{$  $\Big\}$, RHS $= \{$initial CTO$\}$

$\xleftrightarrow{1:1} \{ \Lambda[1] \} \subset s\tau\text{-tilt}(\Lambda)$

What about quasi-triangulations?

and their mutations?

1925

§§4 Symmetric representations ($= \varepsilon$-representations)

$\varepsilon = \{\pm 1\}$

[Derksen-Weyman]

orthogonal ($\varepsilon = +1$)

symplectic ($\varepsilon = -1$)

[Boos-Cerulli Irelli]

Throughout this section,

- $(A = \Bbbk Q/I, \sigma)$: σ involution on Q fixing I.
- modules = right f.d. modules

Def 1) ε-form $:\Leftrightarrow$ $\begin{cases} \cdot \text{ symmetric bilinear form if } \varepsilon = +1, \\ \cdot \text{ skew-symm. bilinear form if } \varepsilon = -1. \end{cases}$

2) An ε-representation over (A, σ)

$:\Leftrightarrow$ · M : oridinary rep^n.

· $\langle -, - \rangle : M \times M \to \Bbbk$ non-degen. ε-form

s.t. $\begin{cases} \cdot \ \langle m e_i, n e_j \rangle = 0 \quad \forall j \neq \sigma(i) \ ; \ e_i, e_j : \text{primitive idem's.} \\ \cdot \ \langle m\alpha, n \rangle + \langle m, n\sigma(\alpha) \rangle = 0 \\ \quad \text{(i.e. } \sigma(\alpha) \text{ is adjoint of } \alpha \text{)} \end{cases}$

☆ M : ε-rep^n $\Rightarrow \exists \psi_M : M \xrightarrow{\sim} \nabla M$ as A-module s.t. $\nabla(\psi_M) = \varepsilon \psi_M$

☆ Indecomposability makes sense for ε-repⁿ's.

"Slogan"

$\boxed{\varepsilon\text{-rep}^n \approx \text{anti-version of rep}^n \text{ over skew-group ring } \Lambda * \mathbb{Z}/2}$

Prop [D-W, B-CI] (Characterization of indec ε-repⁿ's)

M : indec. ε-repⁿ / (A, σ)

$\Rightarrow \exists\ \overline{M}$: indec. A-module

s.t. exactly one of the following holds.

a) $\nabla \overline{M} \not\cong \overline{M}$, $M = \overline{M} \oplus \nabla \overline{M}$ → call M split

b) $\nabla \overline{M} \cong \overline{M}$, $M = \overline{M} \oplus \nabla \overline{M}$ → call M ramified

c) $\nabla \overline{M} \cong \overline{M}$, $M = \overline{M}$ → call M ~~type I~~ 1-sided

<u>Notation</u> : ω: primitive cc, $\ell(\omega) := \# \omega \cap \tilde{\tau}$.

<u>Thm</u> [BM-C-W]

M: indec ε-rep^n / $Jac(Q,W)$, (Q,W): gentle FPF-Symm. QP

$\Rightarrow M \cong$ exactly one of the following.

split
- $M(\gamma) \oplus M(\sigma(\gamma))$ $\leftarrow$ all strings
- $M_\lambda(\omega^n) \oplus M_{\lambda^{-1}}(\sigma(\omega^n))$ s.t. $\begin{cases} \omega \neq \sigma(\omega) \text{ or } \lambda \notin \{\pm 1\}, \\ \text{any } n \geq 1. \end{cases}$

1-sided
- $M_\lambda(\omega)$ s.t. $\sigma(\omega) = \omega$ & $\varepsilon = (-1)^{\ell(\omega)/2} \lambda$
- $M_\lambda(\omega^2)$ s.t. $\sigma(\omega) = \omega$ & $\varepsilon = (-1)^{\frac{\ell(\omega)}{2}+1} \lambda$

ramified
- $M_\lambda(\omega^n)^{\oplus 2}$ s.t. all remaining cases

all band

<u>Hence</u>

$\underbrace{\text{"Indec" top. obj.}}$ $\underbrace{\text{Indec } \varepsilon\text{-rep}^n\text{'s}}$

$\{$1-sided c.c. ω on $(\mathbb{S}, M)\} \xleftrightarrow{1:1} \{$1-sided ε-indec $M_{\varepsilon_\omega}(\omega)\}$

$\updownarrow$ 1:1

$\{$2-sided c.c. ω on $(\mathbb{S}, M)\} \xleftrightarrow{1:1} \{$ramified ε-indec $M_{-\varepsilon_\omega}(\omega)^{\oplus 2}\}$

$\left\{ \begin{array}{c} \text{non-closed curve } \gamma \\ \text{on } (\mathbb{S}, M) \end{array} \right\} \setminus \tilde{\tau} \xleftrightarrow{1:1} \left\{ \begin{array}{l} \text{split } \varepsilon\text{-indec} \\ \text{"of strong type"} \end{array} M(\gamma) \oplus \nabla M(\gamma) \right\}$

$\rightarrow$ <u>Goal II</u> ✓

1935

§§5 ε-extension, ε-rigid

Go back to $\mathcal{C} = \mathcal{C}_{(\mathcal{S},\mathcal{M})}$.

<u>Fact</u>: $\tilde{T}$: triang. on $(\tilde{\mathcal{S}},\tilde{\mathcal{M}})$, $\Lambda := \mathrm{Jac}(Q,W)$

$\Rightarrow$ $C := \tilde{T}[-1]$ is a CTO

& $\pi := \mathrm{Hom}_{\mathcal{C}}(C,-) : \mathcal{C}/_{[\tilde{T}]} \xrightarrow{\sim} \mathrm{mod}\,\Lambda$

Fix $\varepsilon \in \{\pm 1\}$, T: triang^n.

<u>Def</u>:

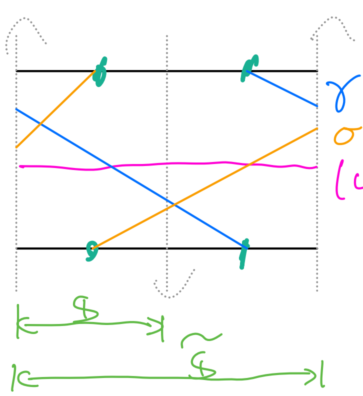
- $X \in \mathcal{C}$ is an indec. ε-object $\Leftrightarrow$ · πX an indec. ε-rep$^n/\Lambda$
  or
  · $X = \alpha \oplus \underset{\nabla(\alpha)}{\sigma(\alpha)}$, some $\alpha \in \tilde{T}$

- In this case,

  ① $\exists\ (\nabla \bar{X} \to X \to \bar{X} \to)$: Δ in $\mathcal{C}$ with $\bar{X}$: indec.

  we call any such $\bar{X}$ ε-factor. N.B. not unique!

e.g. $\mathcal{S} = M_2$:



$\sigma(\gamma) \to (\omega,\lambda) \to \gamma \to$

$\gamma = \overline{(\omega,\lambda)}$

② Isom. $X \xleftarrow[\varphi_X]{\sim} \nabla X$ s.t. $\nabla(\varphi_X) = \varepsilon\,\varphi_X$

- $X \in \mathcal{C}$ is an **$\mathcal{E}$-object** $:\Longleftrightarrow$ $X = \bigoplus$ indec $\mathcal{E}$-obj's.

- X, Y: $\mathcal{E}$-objects

$f : X \to \bar{Y}[1]$ s.t. $\begin{cases} \bar{Y} : \mathcal{E}\text{-factor of } Y \\ f \circ \psi_X \circ \nabla f = 0 \end{cases}$

$$\begin{tikzcd} (\nabla \bar{Y}[-1]) \arrow[d, "\nabla f"] & \\ X \arrow[r, "f"] & \bar{Y}[1] \end{tikzcd}$$

In this case, $\exists$ comm. dgm:

$$\begin{tikzcd}
 & & \nabla\bar{Y}[-1] \arrow[d, "\psi_X \nabla f"'] & \\
\bar{Y} \arrow[r] \arrow[d, equal] & C^f \arrow[r] \arrow[d] & X \arrow[r, "f"] \arrow[d] & \bar{Y}[1] \\
\bar{Y} \arrow[r] & E \arrow[r] \arrow[d] & C_{\psi_X \nabla f} \arrow[d] & \\
 & \nabla\bar{Y} \arrow[r, equal] & \nabla\bar{Y} &
\end{tikzcd}$$

s.t. all rows & all col^n's are Δ's of $\mathcal{C}$.

Fact: This is self-dual obj. in $\mathcal{C}$

We call this E an **$\mathcal{E}$-extension of $\begin{bmatrix} X \\ Y \end{bmatrix}$**.

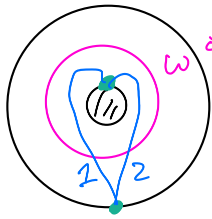
- An $\mathcal{E}$-extension **splits** $:\Longleftrightarrow$ $E \cong X \oplus \bar{Y} \oplus \nabla\bar{Y}$

- Write $\mathcal{E}\mathrm{Ext}^1(X, Y) = 0$ if all $\mathcal{E}$-ext^n of $\binom{X}{Y}$ splits

- X is **indec. $\mathcal{E}$-rigid** $:\Longleftrightarrow$ $\begin{cases} X \text{ indec } \mathcal{E}\text{-obj} \\ \mathcal{E}\mathrm{Ext}^1(X, X) = 0. \end{cases}$

- X is **$\mathcal{E}$-rigid** $:\Longleftrightarrow$ $\begin{cases} \cdot\ X = \bigoplus \text{ indec } \mathcal{E}\text{-rigid.} \\ \cdot\ \forall\ \mathcal{E}\text{-indec's } Y, Z \subset \oplus X, \\ \quad Y \not\cong Z \Rightarrow \mathrm{Ext}^1_{\mathcal{E}}(Y, Z) = 0. \end{cases}$

$\boxed{\text{Eg.}}$

$\mathbb{S} = M_1$, $\quad \tilde{\mathbb{S}} =$ 

← lift of the unique quasi-arc on $\mathbb{S}$

$\tilde{\gamma} \leftrightarrow (\alpha, w) = (1 \rightrightarrows 2, 0)$

$\Rightarrow \quad \pi\omega = 1 \overset{1}{\underset{2}{\circlearrowleft}} \lambda \qquad \varepsilon = -\lambda \in \{\pm 1\}$

$\pi\gamma = S_1$

$\pi\nabla\gamma = S_2$

$(\nabla\gamma)[-1] \to \omega \to \gamma[1]$

$\pi \circlearrowleft$

$\begin{smallmatrix}1\ 1\\ 2\ 2\ 2\end{smallmatrix} \xrightarrow{\nabla f} \begin{smallmatrix}1\\ 1\\ 2\end{smallmatrix} \xrightarrow{f} \begin{smallmatrix}2\ 2\ 2\\ 1\ 1\end{smallmatrix}$

If $f \neq 0$, this composition is non-zero.

$\Rightarrow$ only split extensions everywhere.

$\Rightarrow$ ε-rigid.

Goal III

Conj / "Thm" [IM-C-W] ("Baby" categorification of quasi-triang")

$\left\{ \begin{matrix}\text{indec}\\ \varepsilon\text{-rigid}\end{matrix} \right\} \xleftrightarrow{1:1} \{\text{quasi-arc of } (\mathbb{S}, M)\}$

which then induces

$\left\{ \begin{matrix}\text{maximal}\\ \varepsilon\text{-rigid obj}\end{matrix} \right\} \xleftrightarrow{1:1} \{\text{quasi-triangulations of } (\mathbb{S}, M)\}$

1950

Expectations

1) Define strictly $\mathbb{E}$-rigid obj X :$\Longleftrightarrow$ $\mathbb{E}\mathrm{Ext}^1(X,X)=0$

$\Rightarrow$ (maximal $\mathbb{E}$-rigid $\Longleftrightarrow$ maximal strictly $\mathbb{E}$-rigid)

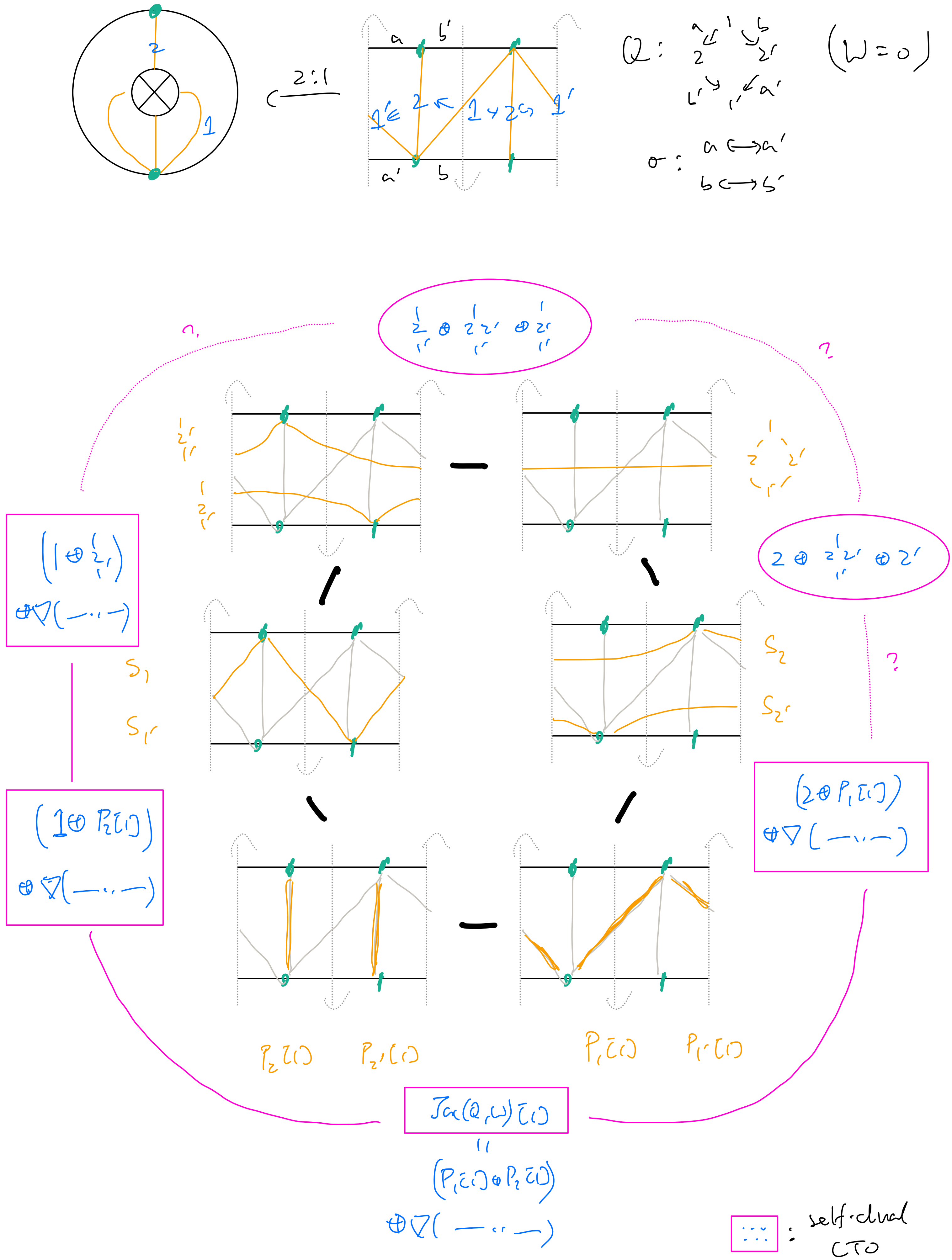
2) $\mathbb{E}$-ext^n's /equiv.?? count crossings on $(\mathbb{S},\mathbb{M})$

3) Exchange relation/Mutation of quasi-triangulation:

$$\alpha \in T \xleftrightarrow{\mu} (T\setminus\{\alpha\})\cup\{\alpha'\}.$$

is categorified by existence of some "special" $\mathbb{E}$-extension /equiv.??.

Part of Goal IV.



2:1
Q:
(W=0)
σ: a ⟷ a'
b ⟷ b'
?
S_1
S_1'
S_2
S_2'
(2 ⊕ P_1[1]) ⊕ ∇(—,—)
(1 ⊕ P_2[1]) ⊕ ∇(—,—)
P_2[1]
P_2'[1]
P_1[1]
P_1'[1]
Jac(Q,W)[1]
=
(P_1[1] ⊕ P_2[1]) ⊕ ∇(—,—)
: self-dual CTO