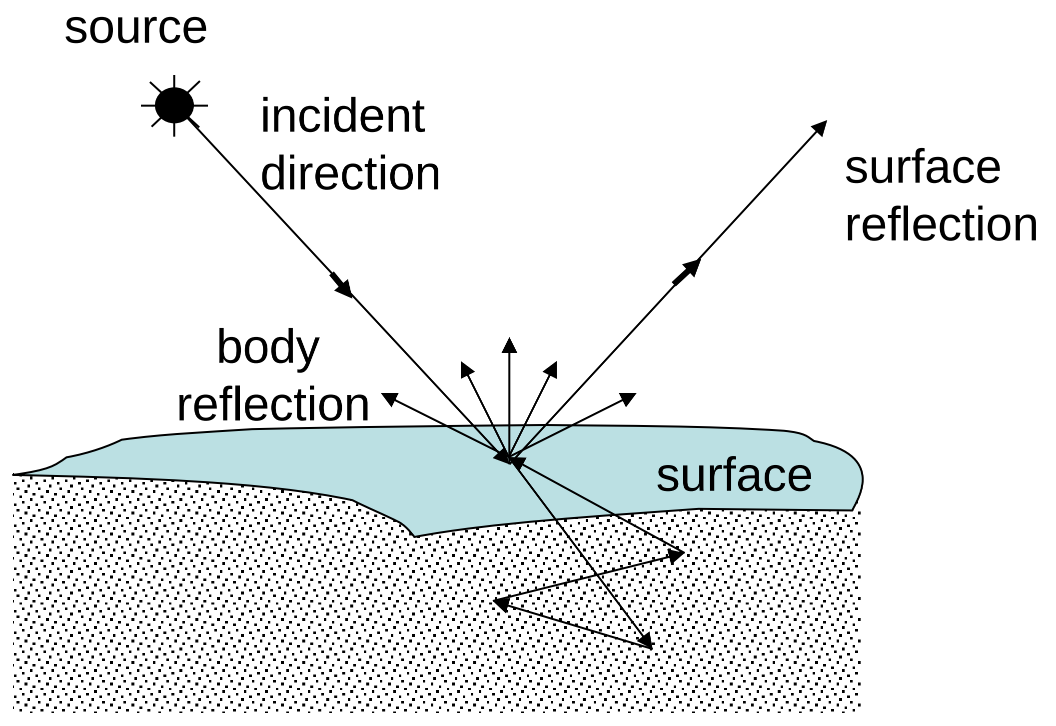
Photometric Stereo

Slide set courtesy: S. Narasimhan

Mechanisms of Reflection



- Body Reflection:

  Diffuse Reflection
  Matte Appearance
  Non-Homogeneous Medium
  Clay, paper, etc

- Surface Reflection:

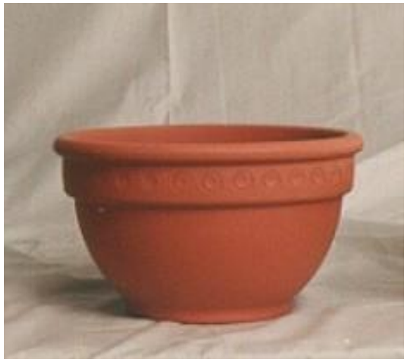
  Specular Reflection
  Glossy Appearance
  Highlights
  Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection

Example Surfaces

Body Reflection:

- Diffuse Reflection
- Matte Appearance
- Non-Homogeneous Medium
- Clay, paper, etc

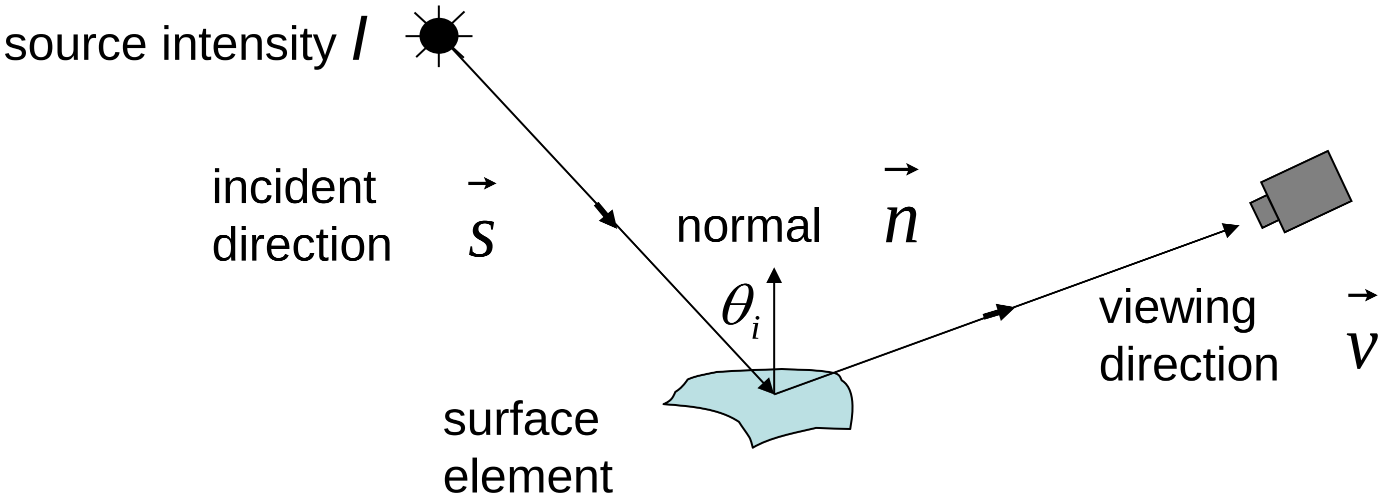


Surface Reflection:

- Specular Reflection
- Glossy Appearance
- Highlights
- Dominant for Metals

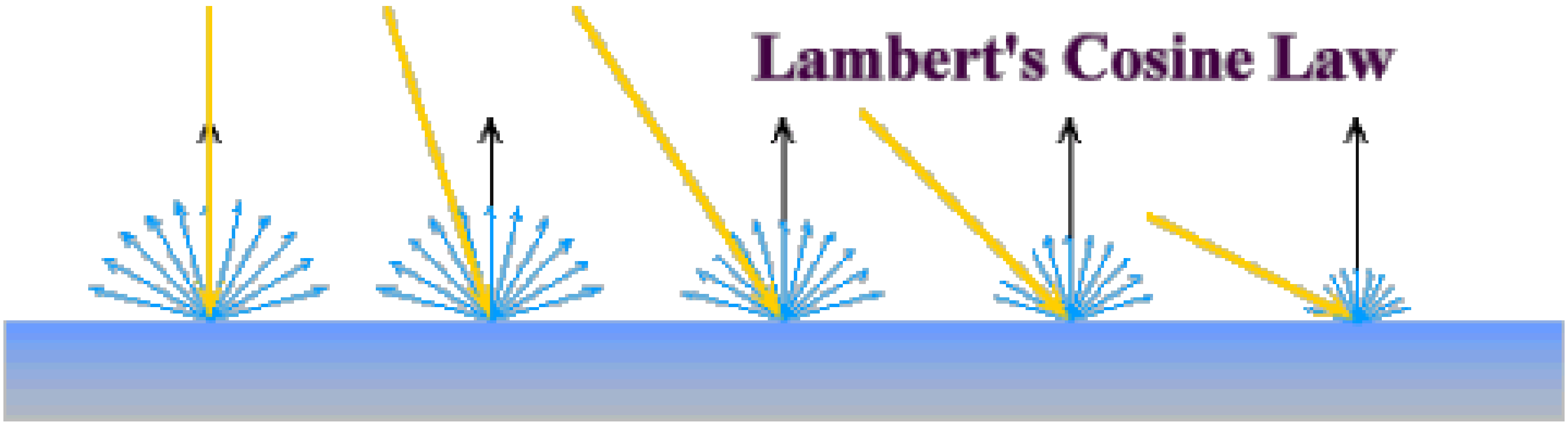
Many materials exhibit both Reflections:

Diffuse Reflection and Lambertian BRDF



- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ albedo
- Surface Radiance : $L = \frac{\rho_d}{\pi} I \cos\theta_i = \frac{\rho_d}{\pi} I \; \vec{n} \cdot \vec{s}$ source intensity
- Commonly used in Vision and Graphics!

Diffuse Reflection and Lambertian BRDF



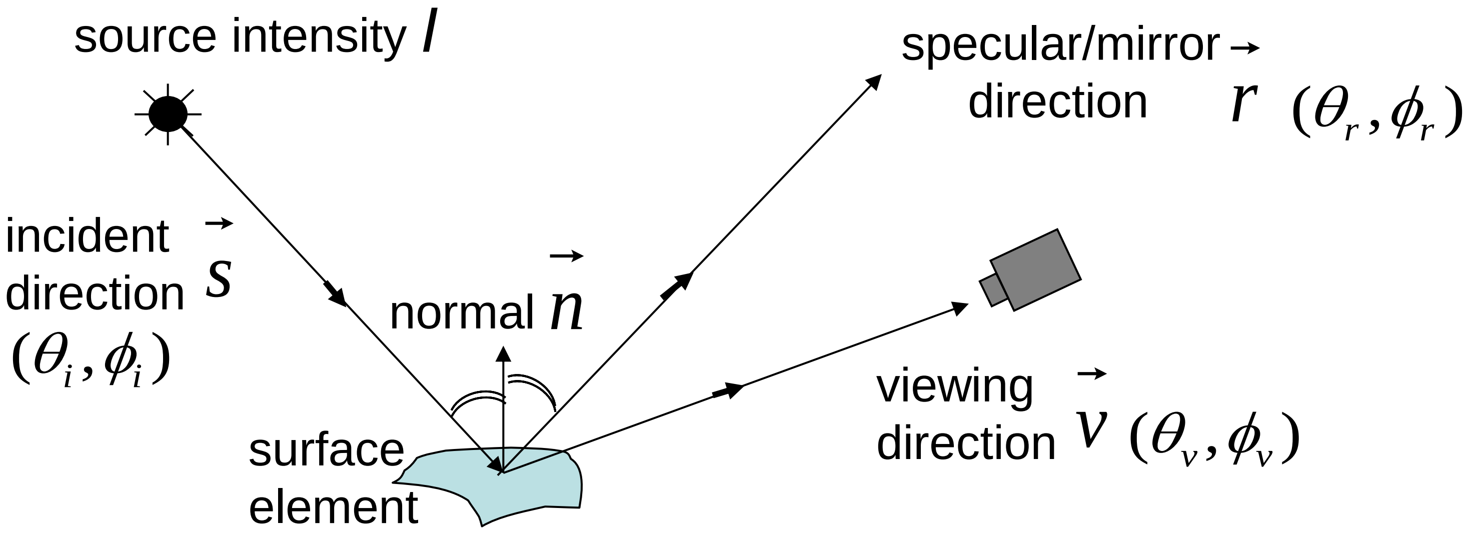
White-out: Snow and Overcast Skies

CAN'T perceive the shape of the snow covered terrain!

CAN perceive shape in regions lit by the street lamp!!

WHY?

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).
- Mirror BRDF is simply a double-delta function :

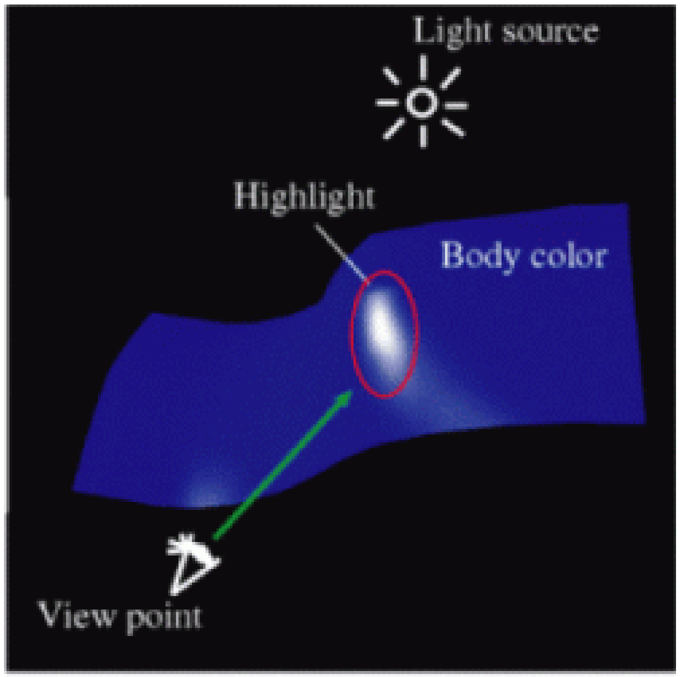
specular albedo

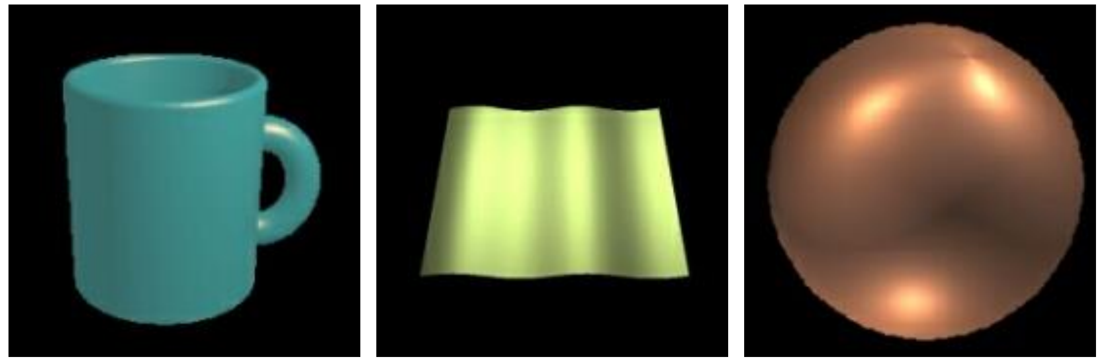
$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \; \delta(\theta_i - \theta_v) \; \delta(\phi_i + \pi - \phi_v)$$

- Surface Radiance : $L = I \; \rho_s \; \delta(\theta_i - \theta_v) \; \delta(\phi_i + \pi - \phi_v)$

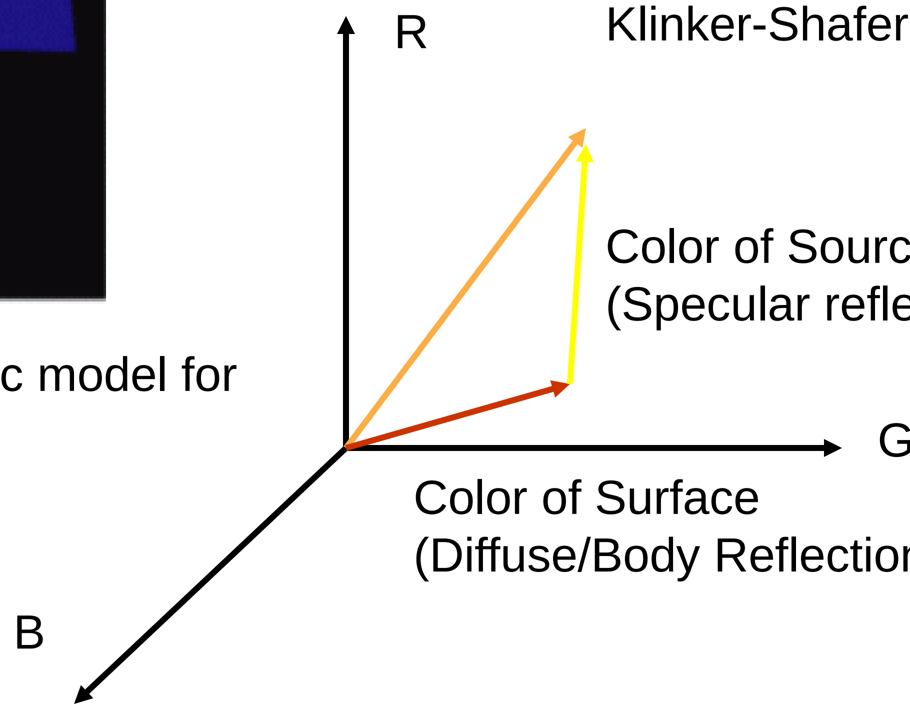
Combing Specular and Diffuse: Dichromatic Reflection

Observed Image Color = a x Body Color + b x Specular Reflection Color



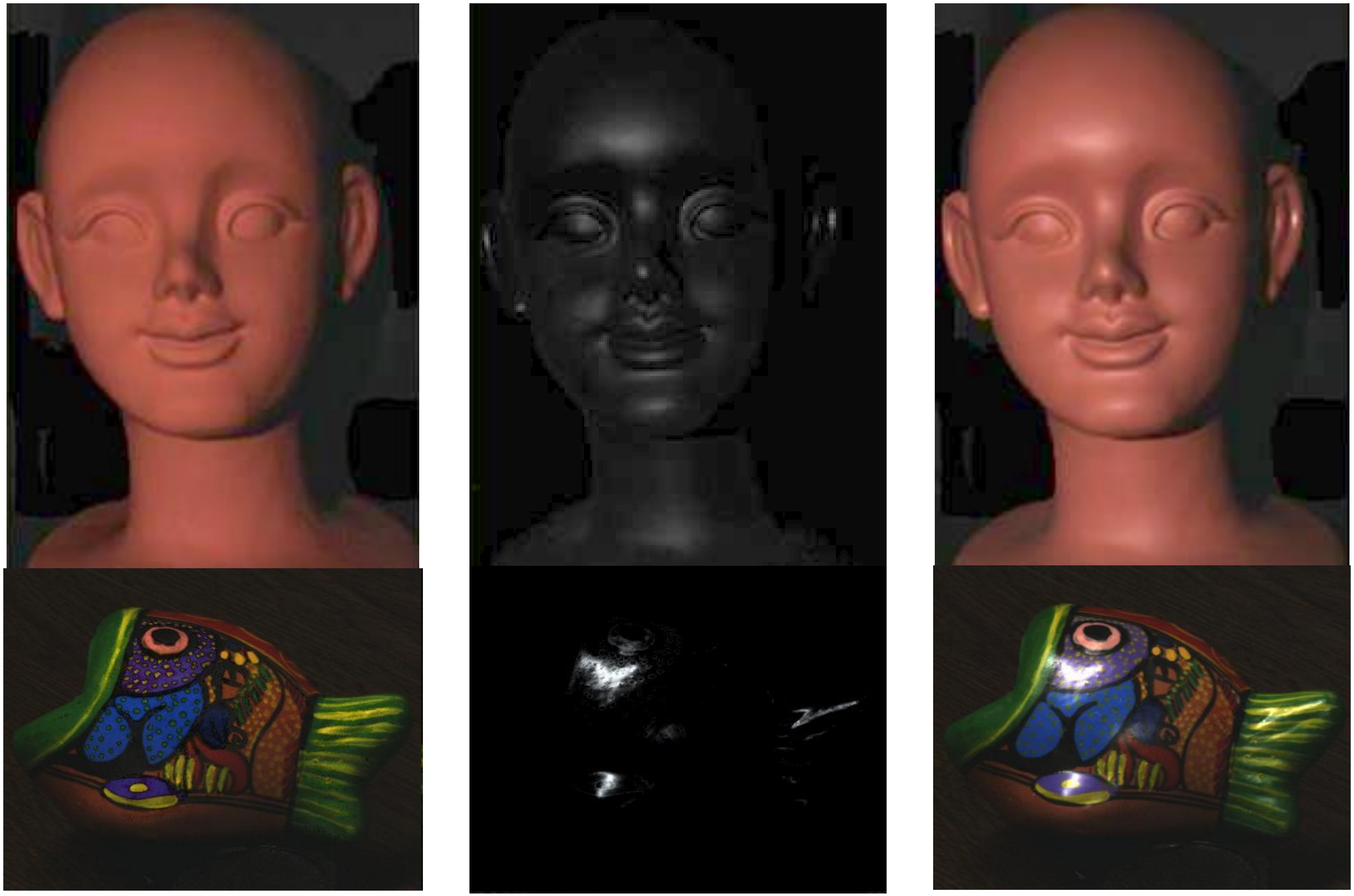


Klinker-Shafer-Kanade 1988



Does not specify any specific model for
Diffuse/specular reflection

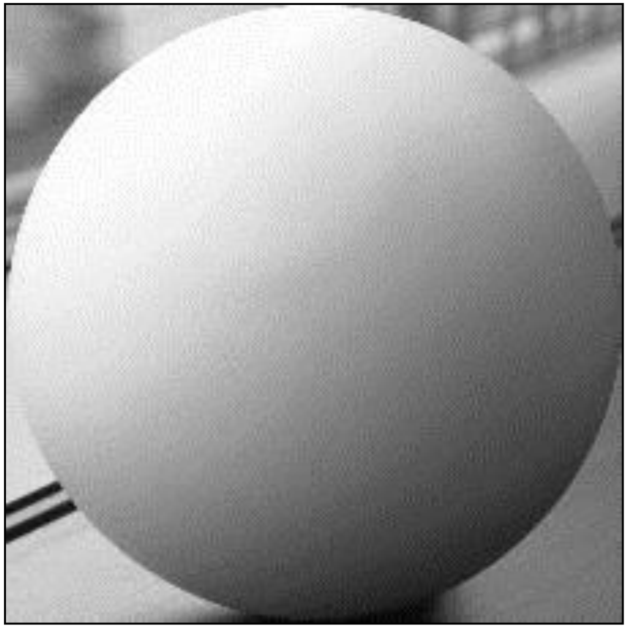
Diffuse and Specular Reflection



diffuse specular diffuse+specular

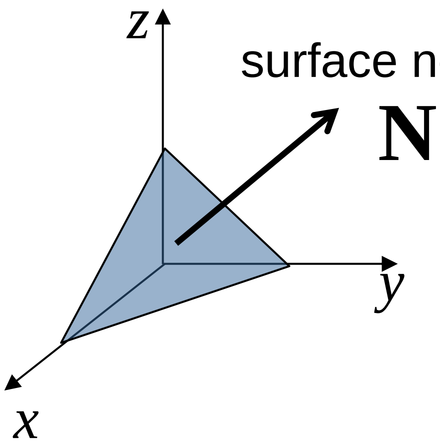
Photometric Stereo

Image Intensity and 3D Geometry



- Shading as a cue for shape reconstruction
- What is the relation between intensity and shape?
  - Reflectance Map

Surface Normal



Equation of plane $$Ax + By + Cz + D = 0$$

or $$\frac{A}{C}x + \frac{B}{C}y + z + \frac{D}{C} = 0$$

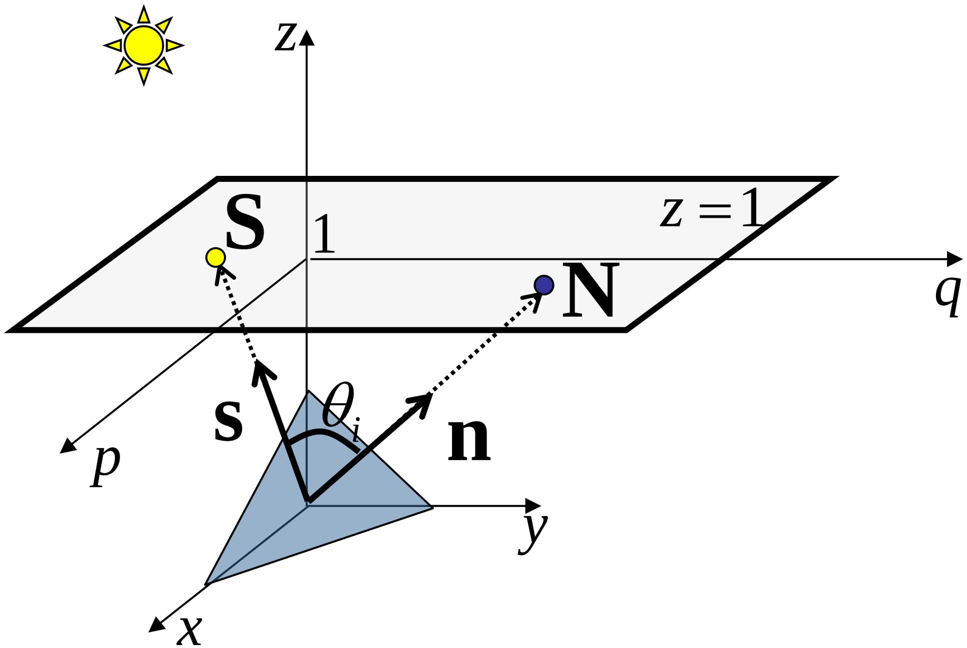
Let

$$-\frac{\partial z}{\partial x} = \frac{A}{C} = p \qquad -\frac{\partial z}{\partial y} = \frac{B}{C} = q$$

Surface normal

$$\boxed{\mathbf{N} = \left(\frac{A}{C}, \frac{B}{C}, 1\right) = (p, q, 1)}$$

Gradient Space



Normal vector

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = \frac{(p, q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

Source vector

$$\mathbf{s} = \frac{\mathbf{S}}{|\mathbf{S}|} = \frac{(p_S, q_S, 1)}{\sqrt{p_S^2 + q_S^2 + 1}}$$

$$\cos\theta_i = \mathbf{n} \cdot \mathbf{s} = \frac{(pp_S + qq_S + 1)}{\sqrt{p^2 + q^2 + 1}\sqrt{p_S^2 + q_S^2 + 1}}$$

$z = 1$ plane is called the Gradient Space (*pq* plane)

- Every point on it corresponds to a particular surface orientation

Reflectance Map

- Relates image irradiance *I(x,y)* to surface orientation *(p,q)* for given source direction and surface reflectance
- Lambertian case:

k : source brightness

ρ : surface albedo (reflectance)

c : constant (optical system)

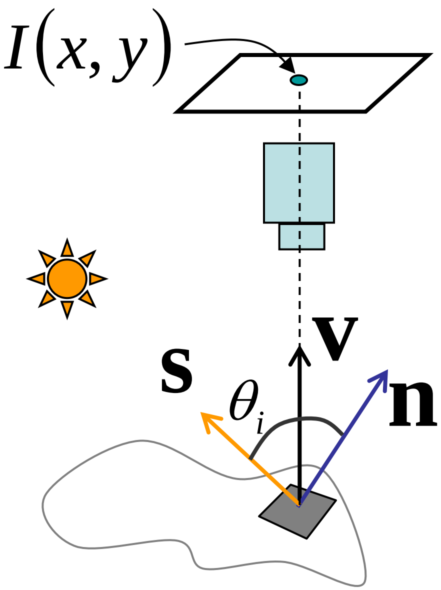


Image irradiance:

$$I = \frac{\rho}{\pi} k c \cos\theta_i = \frac{\rho}{\pi} k c \, \mathbf{n} \cdot \mathbf{s}$$

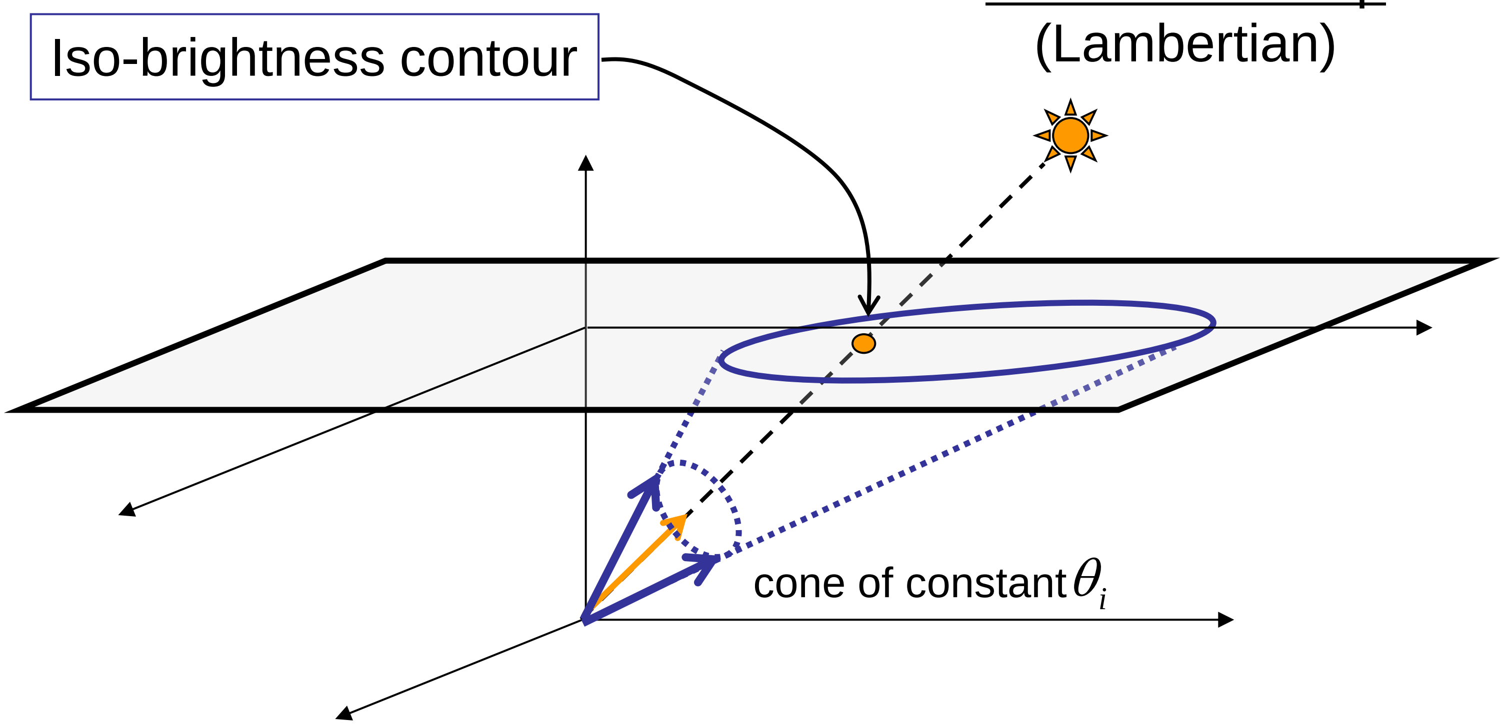
Let $\frac{\rho}{\pi} k c = 1$ then $\boxed{I = \cos\theta_i = \mathbf{n} \cdot \mathbf{s}}$

Reflectance Map

- Lambertian case

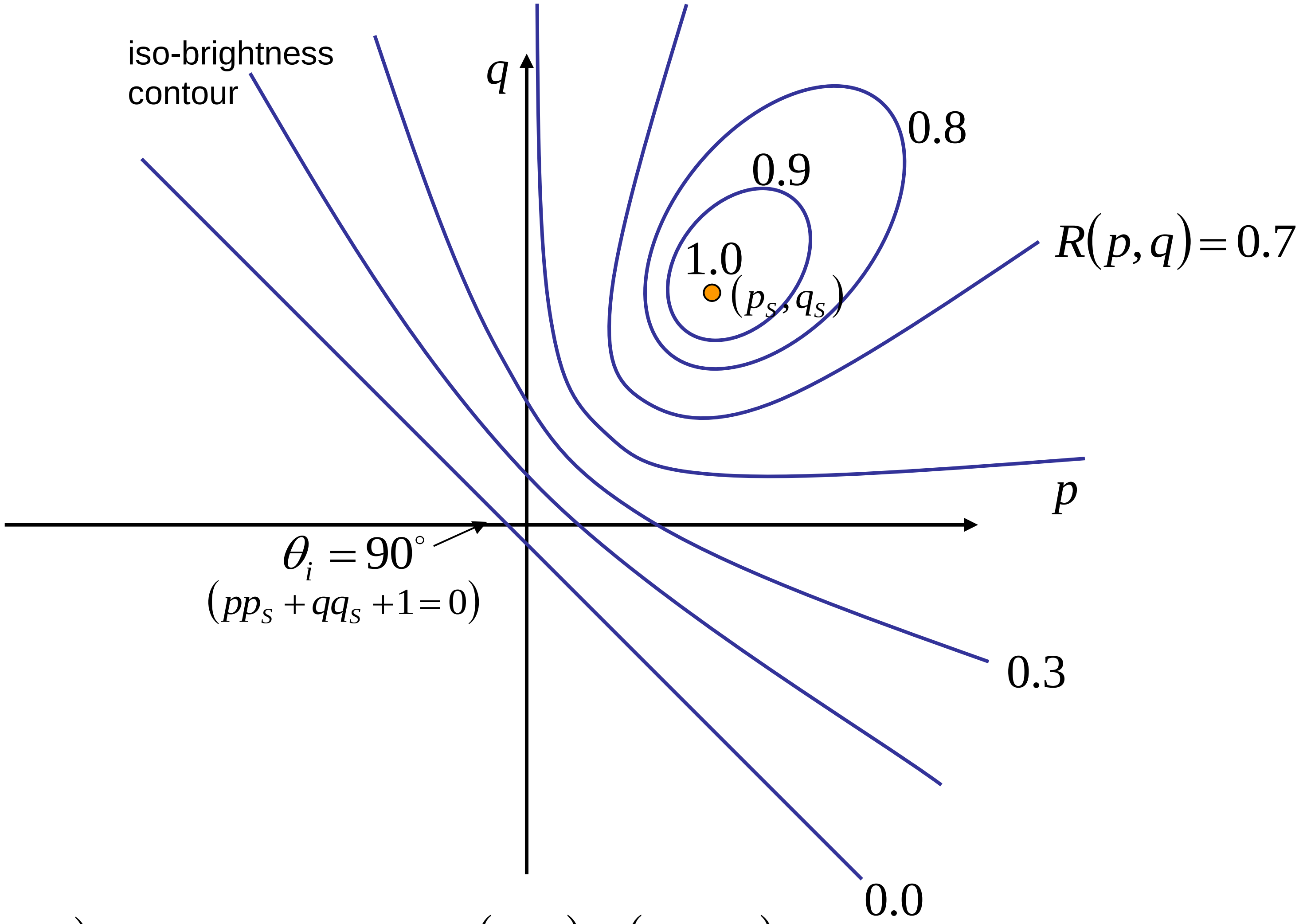
$$I = \cos\theta_i = \mathbf{n}\cdot\mathbf{s} = \frac{(pp_s + qq_s + 1)}{\sqrt{p^2+q^2+1}\sqrt{p_S^2+q_S^2+1}} = \boxed{R(p,q)}$$

Reflectance Map
(Lambertian)



Reflectance Map

- Lambertian case



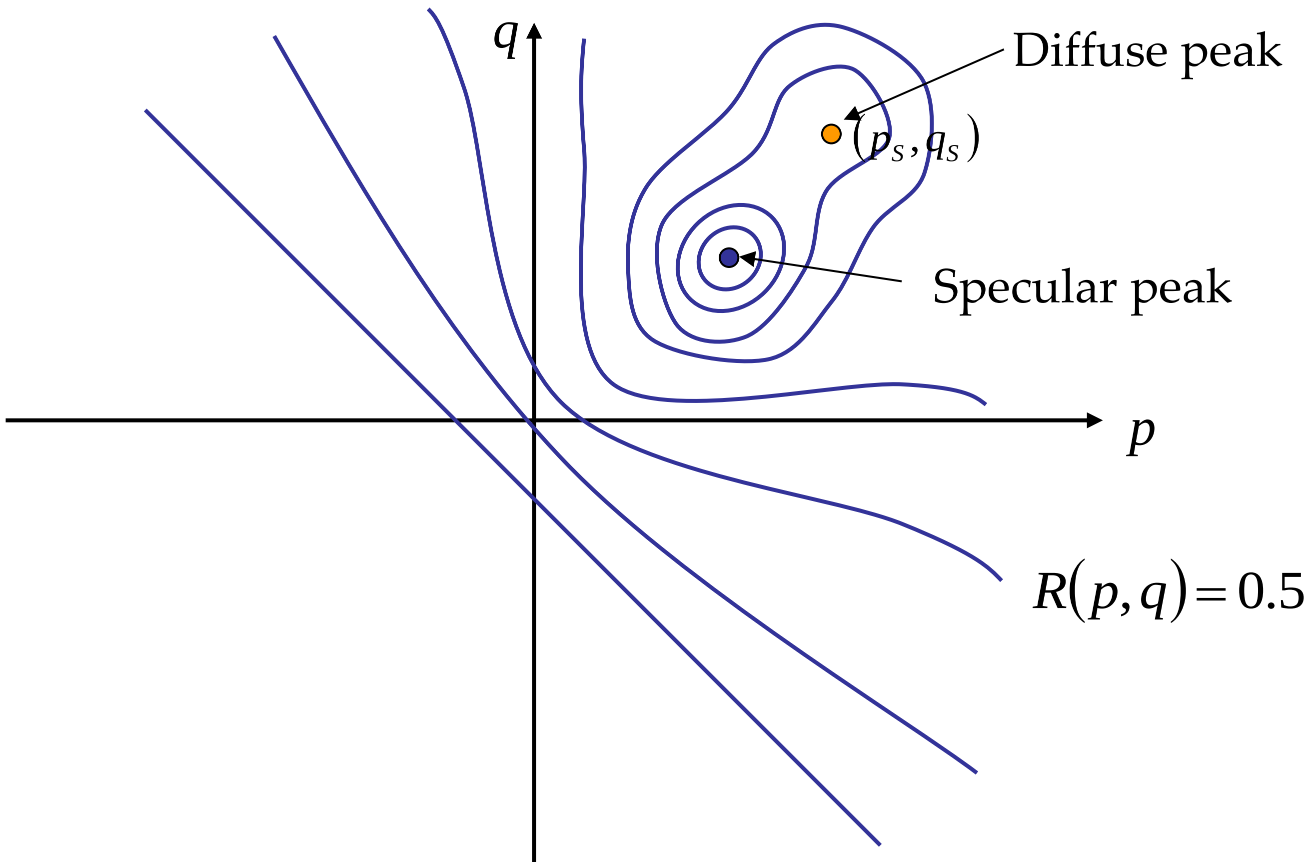
Note: $R(p,q)$ is maximum when $(p,q) = (p_S, q_S)$

Reflectance Map

- Glossy surfaces (Torrance-Sparrow reflectance model)

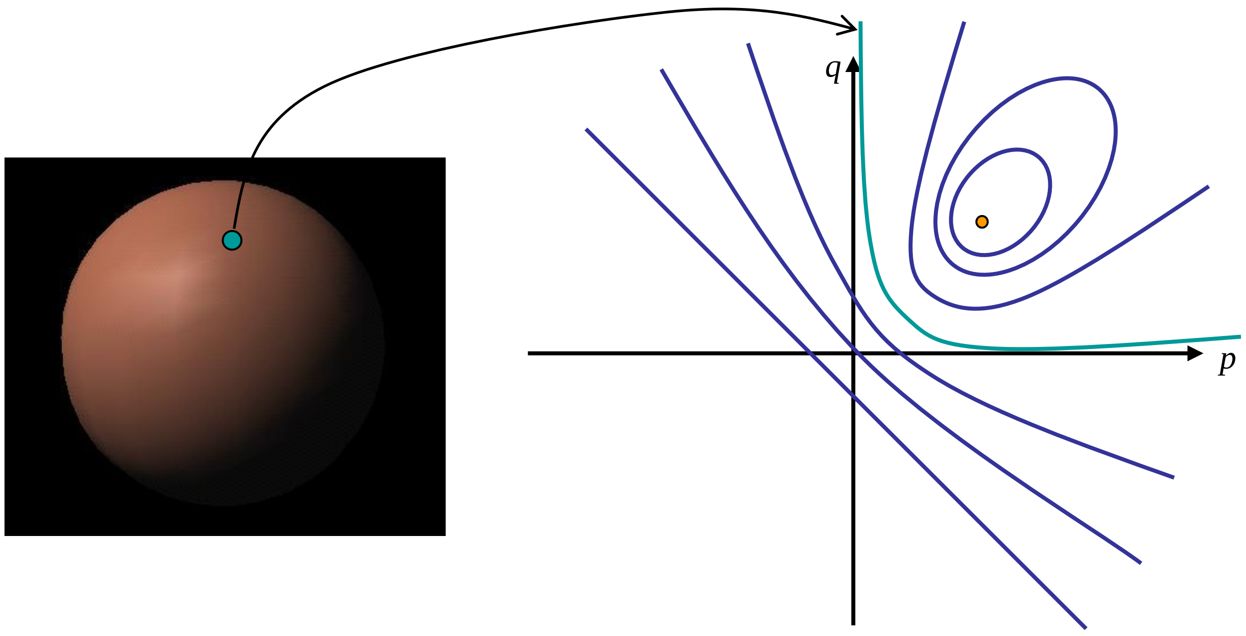
$$I = \boxed{\frac{\rho_d}{\pi} kc \cos\theta_i} + \boxed{\frac{\rho_s kc}{\cos\theta_r} p(\beta) G} = R(p,q)$$

diffuse term specular term



Shape from a Single Image?

- Given a single image of an object with known surface reflectance taken under a known light source, can we recover the shape of the object?
- Given *R(p,q)* ((p_S,q_S) and surface reflectance) can we determine *(p,q)* uniquely for each image point?



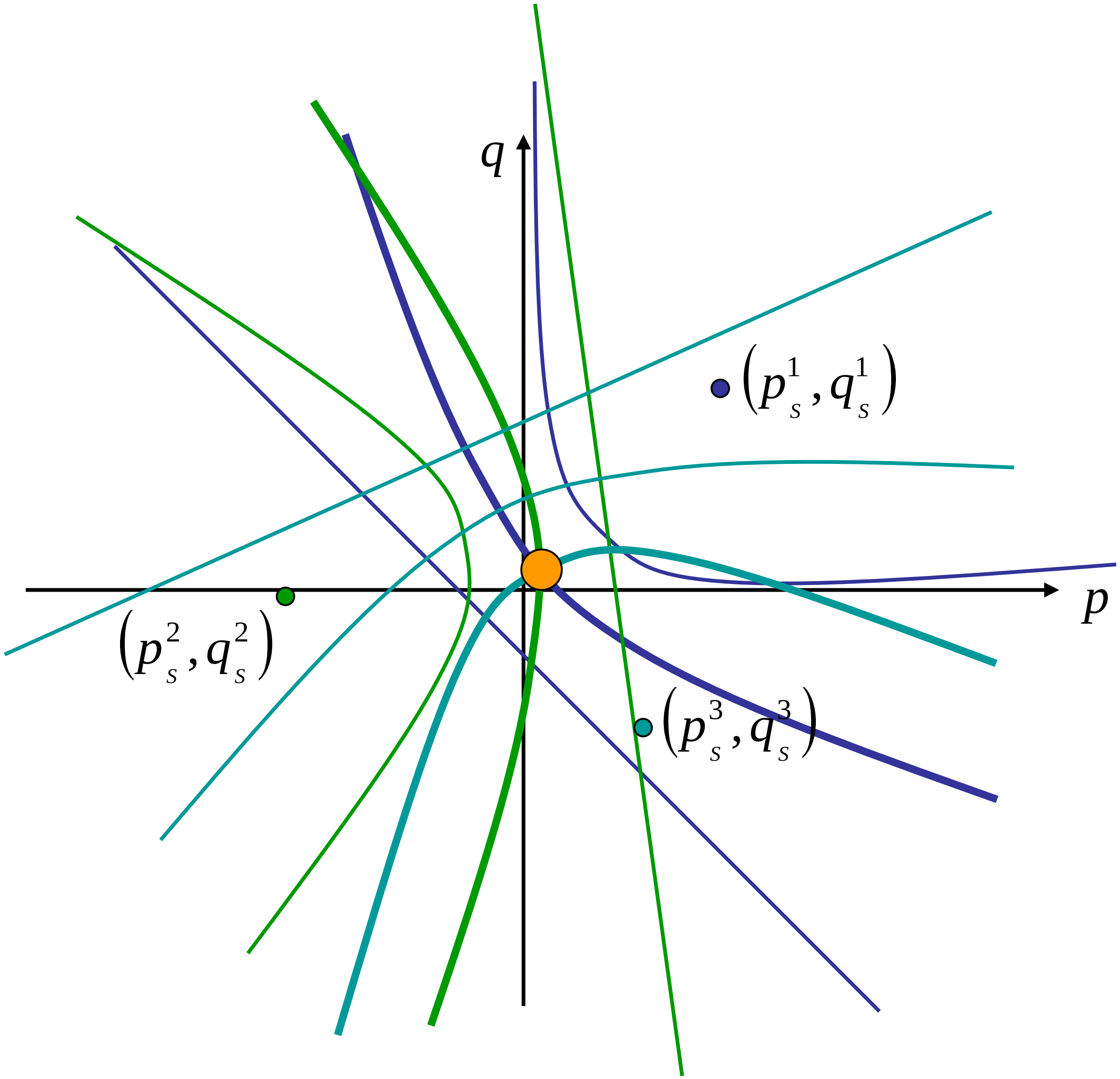
NO

Solution

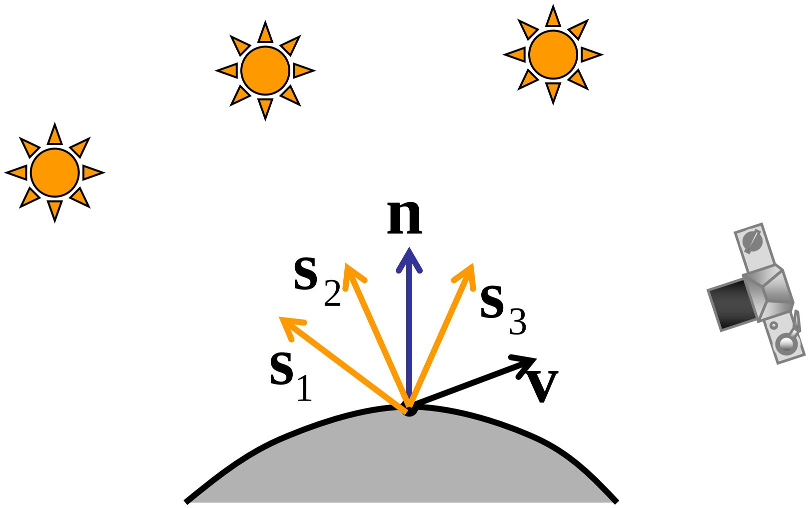
- Take more images
  - Photometric stereo

- Add more constraints
  - Shape-from-shading (next class)

Photometric Stereo



Photometric Stereo



Lambertian case:

$$I = \frac{\rho}{\pi} kc \cos\theta_i = \rho \mathbf{n} \cdot \mathbf{s} \qquad \left(\frac{kc}{\pi} = 1 \right)$$

Image irradiance:

$$I_1 = \rho \mathbf{n} \cdot \mathbf{s}_1$$
$$I_2 = \rho \mathbf{n} \cdot \mathbf{s}_2$$
$$I_3 = \rho \mathbf{n} \cdot \mathbf{s}_3$$

- We can write this in matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_2 \end{bmatrix} = \rho \begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix} \mathbf{n}$$

Solving the Equations

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_2 \end{bmatrix}}_{\substack{\mathbf{I} \\ 3\times 1}} = \underbrace{\begin{bmatrix} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \mathbf{s}_3^T \end{bmatrix}}_{\substack{\mathbf{S} \\ 3\times 3}} \underbrace{\rho \mathbf{n}}_{\substack{\tilde{\mathbf{n}} \\ 3\times 1}}$$

$$\tilde{\mathbf{n}} = \mathbf{S}^{-1}\mathbf{I}$$

inverse

$$\rho = \left|\tilde{\mathbf{n}}\right|$$

$$\mathbf{n} = \frac{\tilde{\mathbf{n}}}{\left|\tilde{\mathbf{n}}\right|} = \frac{\tilde{\mathbf{n}}}{\rho}$$

More than Three Light Sources

- Get better results by using more lights

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1^T \\ \vdots \\ \mathbf{s}_N^T \end{bmatrix} \rho \mathbf{n}$$

- Least squares solution:

$$\mathbf{I} = \mathbf{S}\tilde{\mathbf{n}} \qquad \longleftarrow \qquad N \times 1 = \underline{(N \times 3)}(3 \times 1)$$

$$\mathbf{S}^T\mathbf{I} = \mathbf{S}^T\mathbf{S}\tilde{\mathbf{n}}$$

$$\tilde{\mathbf{n}} = \boxed{\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T}\mathbf{I}$$

Moore-Penrose pseudo inverse

- Solve for $\rho, \mathbf{n}$ as before

Color Images

- The case of RGB images
  - get three sets of equations, one per color channel:

$$\mathbf{I}_R = \rho_R \mathbf{S}\,\mathbf{n}$$
$$\mathbf{I}_G = \rho_G \mathbf{S}\,\mathbf{n}$$
$$\mathbf{I}_B = \rho_B \mathbf{S}\,\mathbf{n}$$

  - Simple solution: first solve for $\mathbf{n}$ using one channel
  - Then substitute known $\mathbf{n}$ into above equations to get

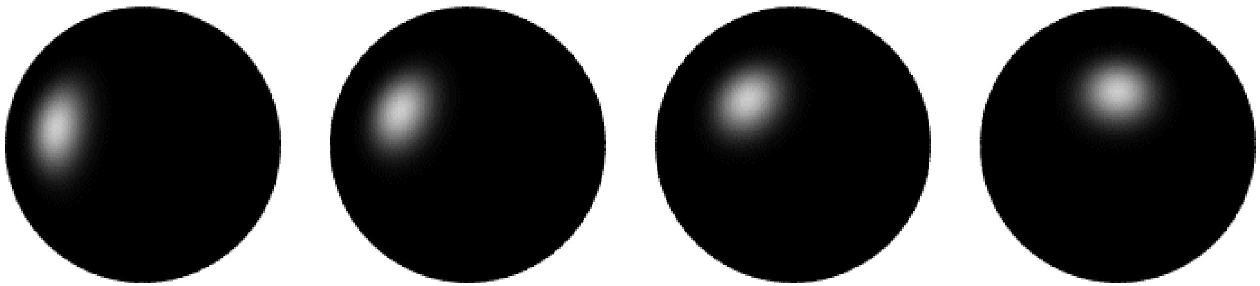
$$(\rho_R, \rho_G, \rho_B)$$

  - Or combine three channels and solve for $\mathbf{n}$

$$\mathbf{I} = \sqrt{\mathbf{I}_R^2 + \mathbf{I}_G^2 + \mathbf{I}_B^2} = \rho \mathbf{S}\,\mathbf{n}$$

Computing light source directions

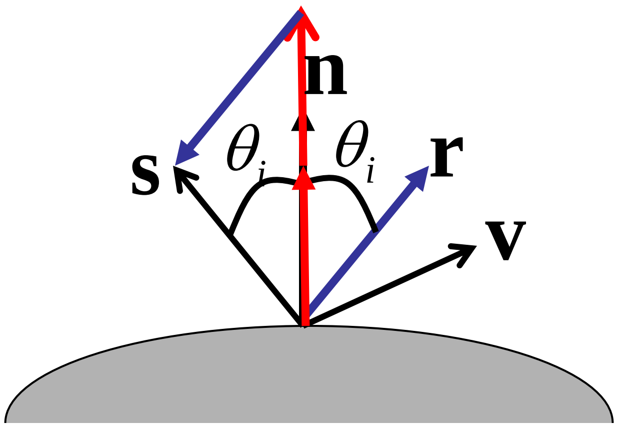
- Trick: place a chrome sphere in the scene



  - the location of the highlight tells you the source direction

Specular Reflection - Recap

- For a perfect mirror, light is reflected about **N**



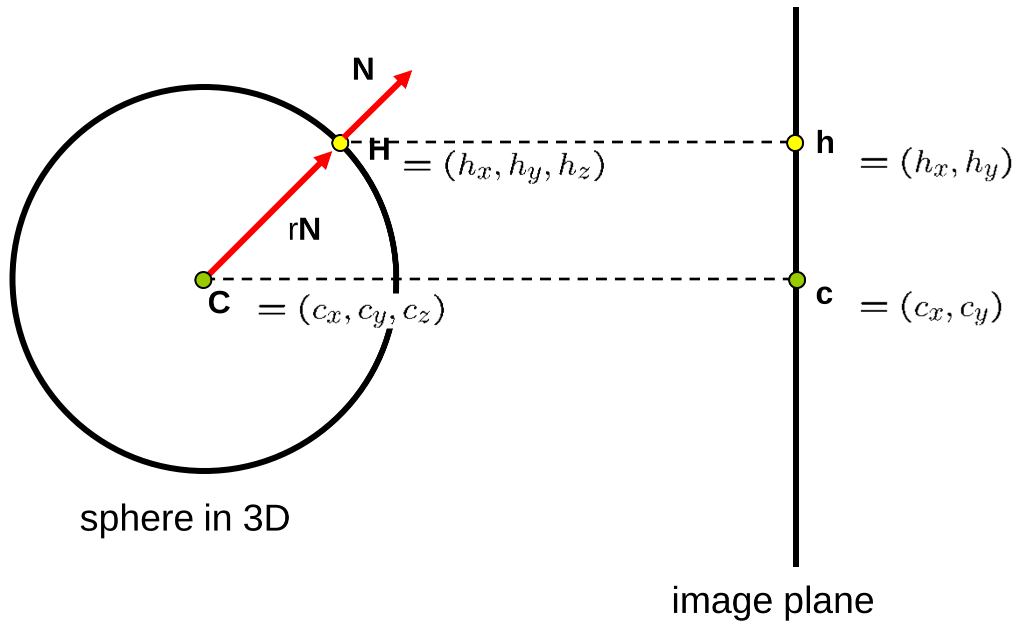
$$R_e = \begin{cases} R_i & \text{if } \mathbf{v} = \mathbf{r} \\ 0 & \text{otherwise} \end{cases}$$

- We see a highlight when $\mathbf{v} = \mathbf{r}$
- Then $\mathbf{s}$ is given as follows:

$$\mathbf{s} = 2(\mathbf{n} \cdot \mathbf{r})\mathbf{n} - \mathbf{r}$$

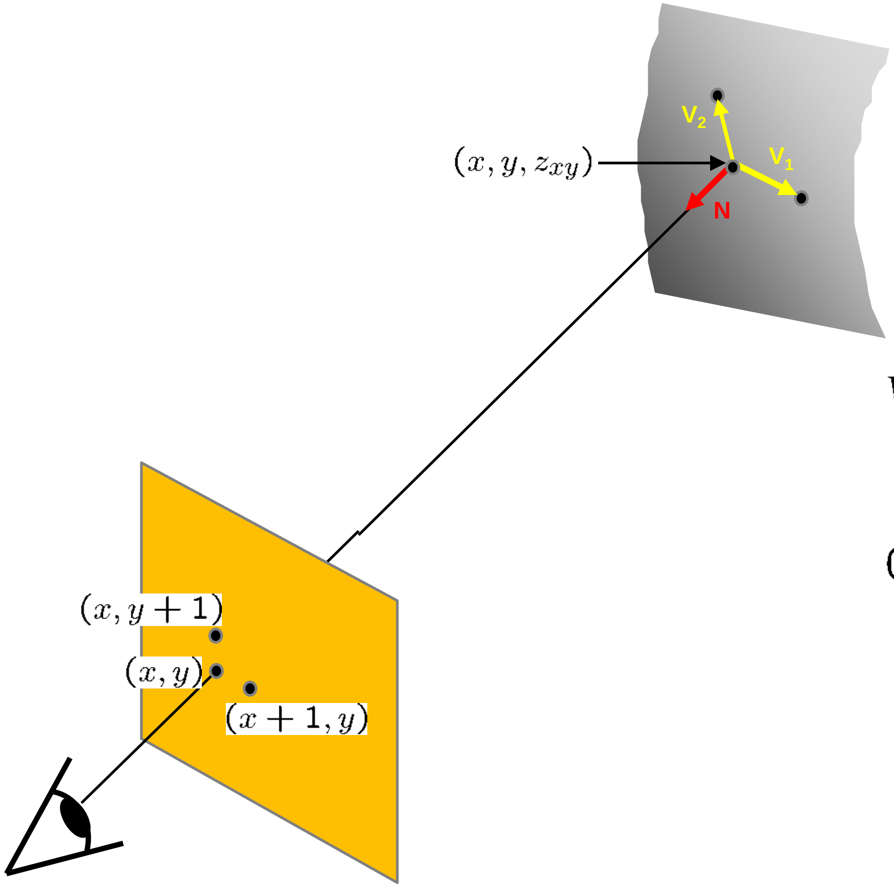
Computing the Light Source Direction

Chrome sphere that has a highlight at position **h** in the image



- Can compute **N** by studying this figure
  - Hints:
    - use this equation: $||H - C|| = r$
    - can measure **c**, **h**, and r in the image

Depth from Normals



$$
\begin{aligned}
V_1 &= (x+1, y, z_{x+1,y}) - (x, y, z_{xy}) \\
&= (1, 0, z_{x+1,y} - z_{xy})
\end{aligned}
$$

$$
\begin{aligned}
0 &= N \cdot V_1 \\
&= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\
&= n_x + n_z(z_{x+1,y} - z_{xy})
\end{aligned}
$$

- Get a similar equation for $\mathbf{V_2}$
  - Each normal gives us two linear constraints on z
  - compute z values by solving a matrix equation

Limitations

- Big problems
  - Doesn’t work for shiny things, semi-translucent things
  - Shadows, inter-reflections
- Smaller problems
  - Camera and lights have to be distant
  - Calibration requirements
    - measure light source directions, intensities
    - camera response function

Trick for Handling Shadows

- Weight each equation by the pixel brightness:

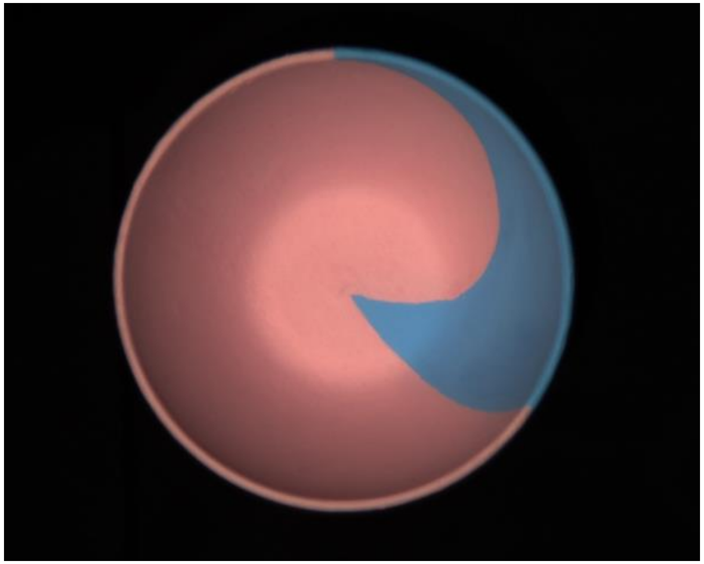
$$I_i(I_i) = I_i(\rho \mathbf{n} \cdot \mathbf{s}_i)$$

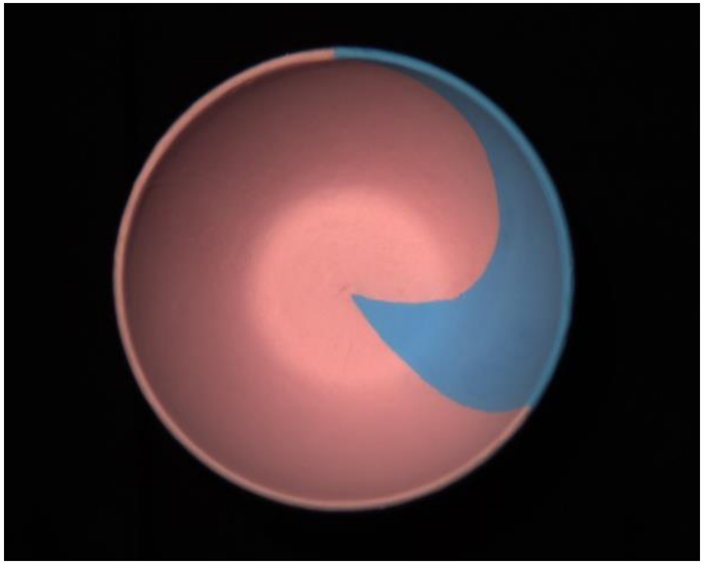
- Gives weighted least-squares matrix equation:

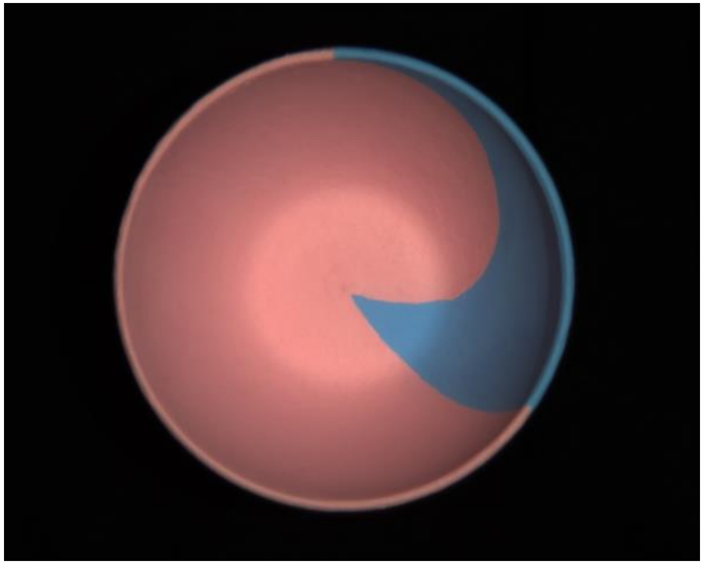
$$\begin{bmatrix} I_1^2 \\ \vdots \\ I_N^2 \end{bmatrix} = \begin{bmatrix} I_1 \mathbf{s}_1^T \\ \vdots \\ I_N \mathbf{s}_N^T \end{bmatrix} \rho \mathbf{n}$$

- Solve for $\rho, \mathbf{n}$ as before

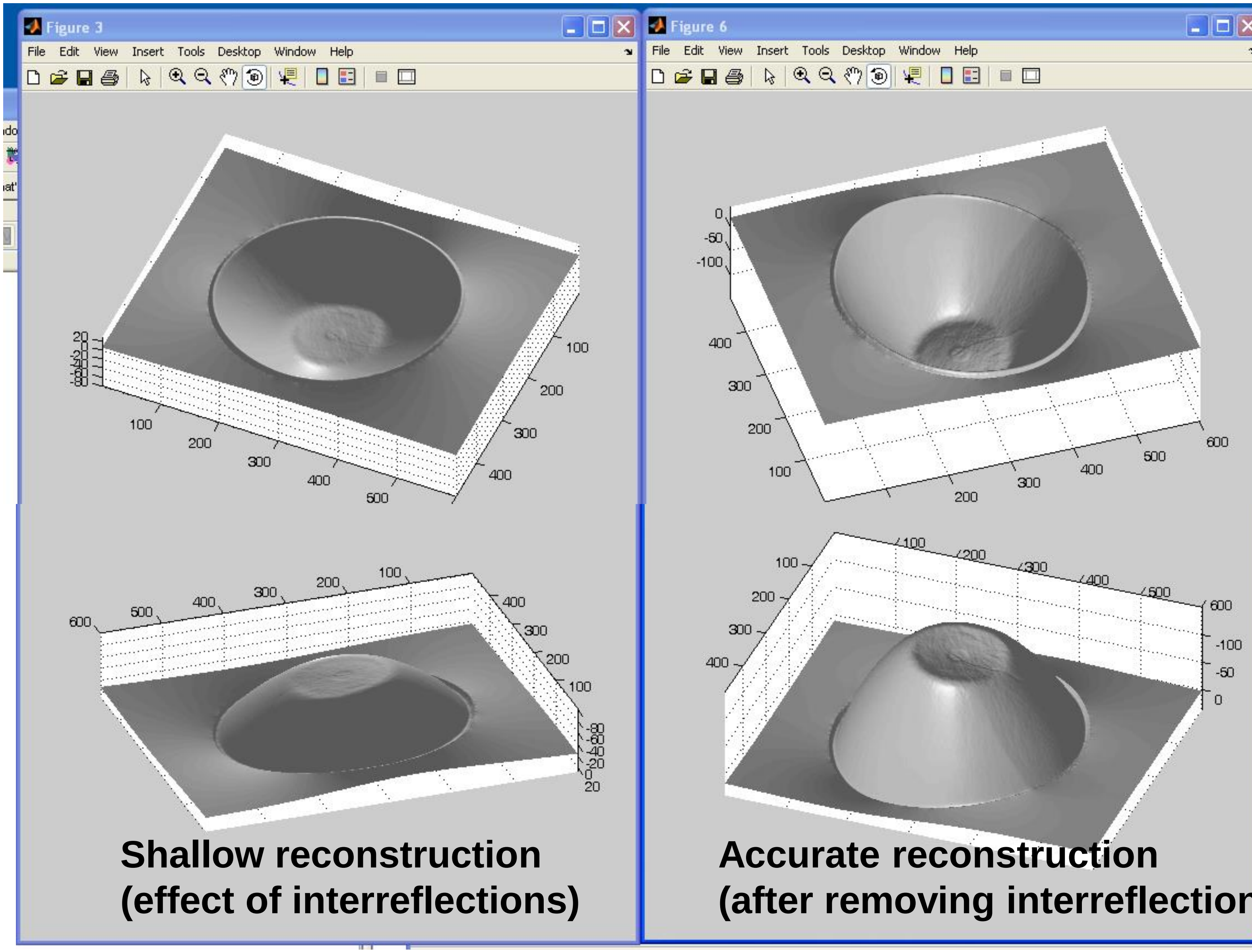
Original Images







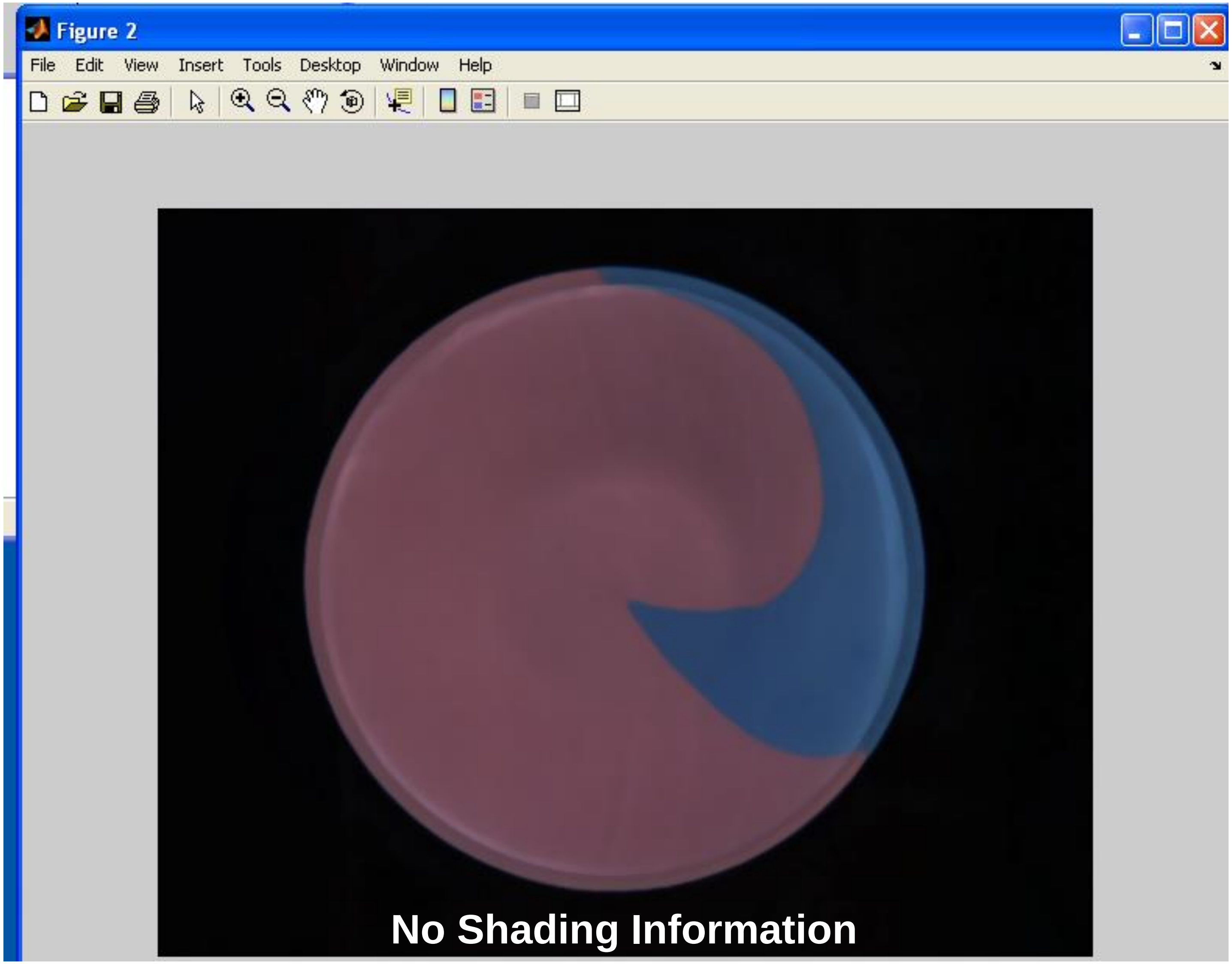
Results - Shape



Shallow reconstruction (effect of interreflections)

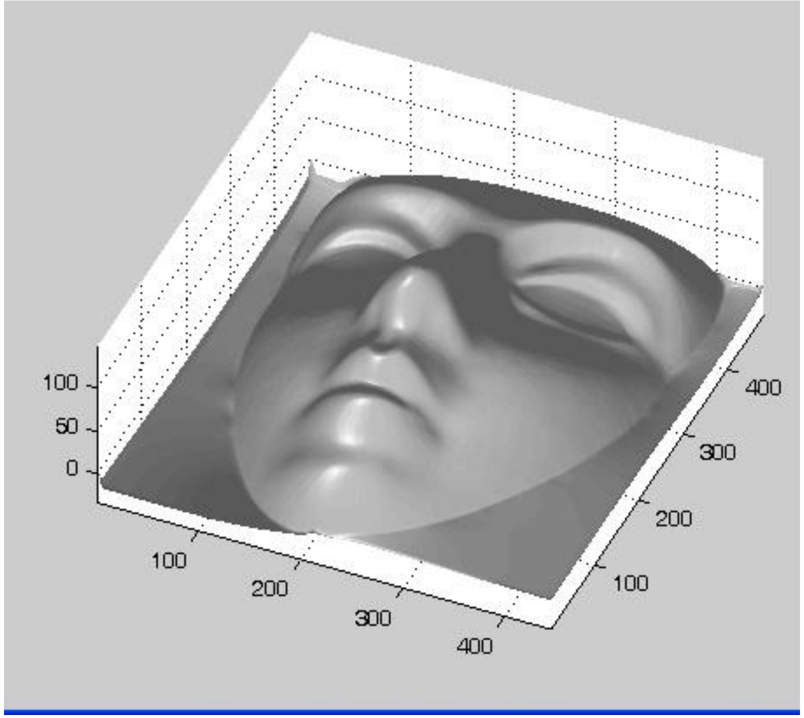
Accurate reconstruction (after removing interreflections)

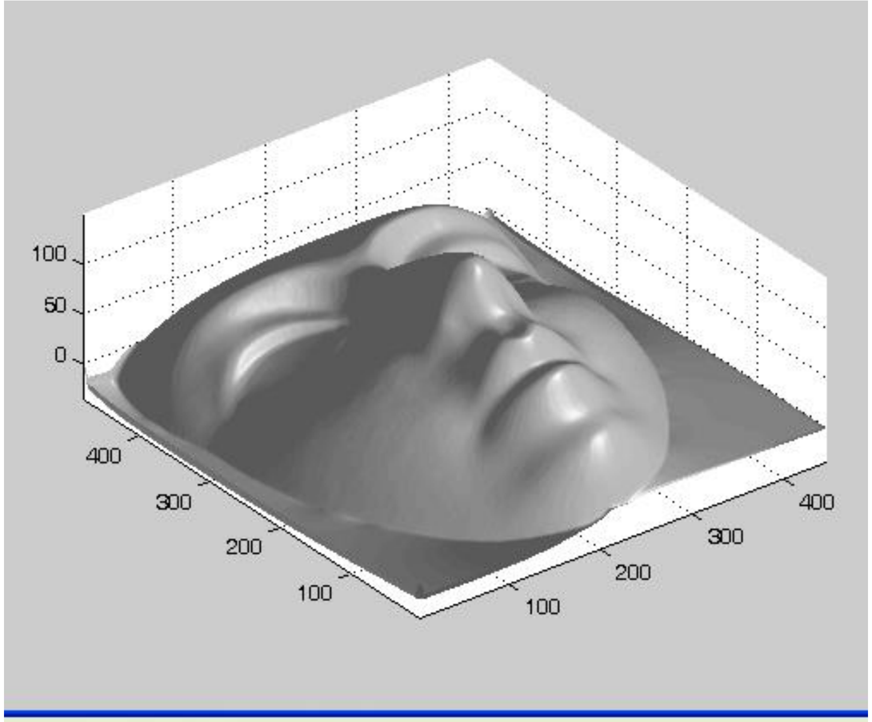
Results - Albedo



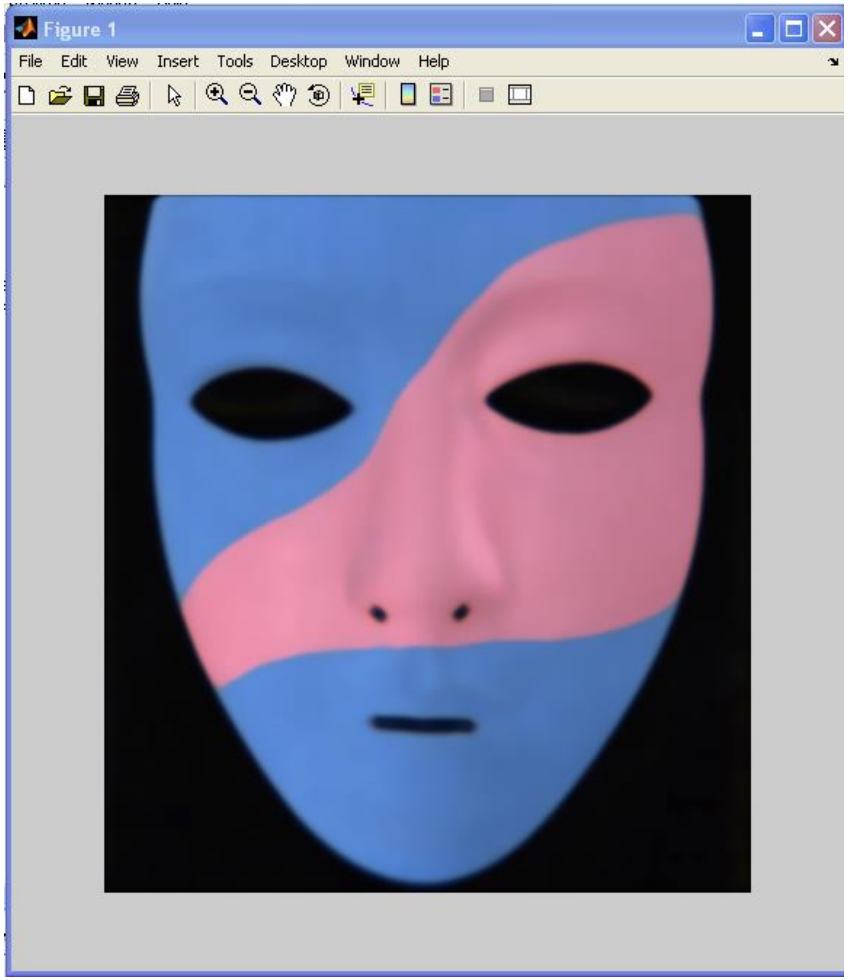
Original Images

Results - Shape





Results - Albedo



Results

1. Estimate light source directions
2. Compute surface normals
3. Compute albedo values
4. Estimate depth from surface normals
5. Relight the object (with original texture and uniform albedo)