# Project II

Impedance Matching

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Fields & Waves II - ECEN3623

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#### 1 Introduction

The purpose of this lab was to further understanding of quarter wave transformers and matching load impedance. The instructions directed the design of a quarter wave transformer to match a load impedance of  $25\,\Omega$  at a frequency of 2.4GHz and an assumption of a  $50\,\Omega$  impedance transmission line. The permittivity of the line (2.7 F/m) was also provided. Given this information, the instruction described creating a sketch of the transmission line solution, plotting magnitude and phase of the input impedance, and plotting the scattering parameters. The Bonus section of the project required the plotting of scattering parameters for an identical transmission line and quarter wave transformer and an adjusted load impedance of 25 -  $j5\Omega$ .

## 2 Calculations

Angular Velocity

$$\omega = 2*\pi*f$$

Propagation Velocity (c = 3e8, Speed of Light):

$$u_p = \frac{c}{\sqrt{\epsilon_{\rm r}}}$$

Phase Constant:

$$\beta = \frac{\omega}{u_p}$$

Lambda (Wavelength):

$$\lambda = \frac{2 * \pi}{\beta}$$

Length (for quarter wave transformer):

$$l = \frac{\lambda}{4}$$

Input Impedance:

$$Z_{in} = Z_o * \frac{R_l + j * Z_o * tan(\beta * l)}{Z_o + j * R_l * tan(\beta * l)}$$

Quarter Wave Transformer Impedance:

$$Z_t = \sqrt{Z_o * R_l}$$

Scattering Parameter:

$$S11 = \frac{(Z_{inQWT} - Z_o)}{(Z_{inQWT} + Z_o)}$$

$$S21 = \sqrt{1 - |S11|^2}$$

## 3 Part I

Design a quarter wave transformer to match a load impedance of 25 Ohms at a frequency of 2.4 GHz. Assume a 50 Ohm transmission line with a permittivity of 2.7.

## 3.1 1.1

Neatly sketch the transmission line solution (provide all lengths in m).

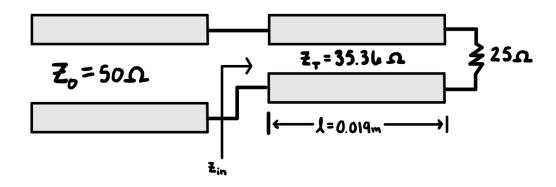


Figure 1: Quarter Wave Transmission Line Solution

## 3.2 1.2

Plot  $Z_{in}$  (magnitude and phase) from DC to 5 X the design frequency.

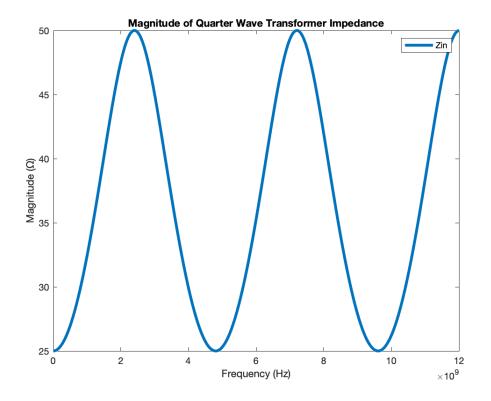


Figure 2: Input Impedance Magnitude vs. Frequency Sweep

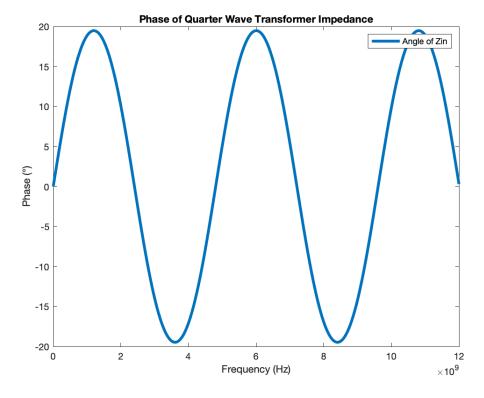


Figure 3: Input Impedance Phase vs. Frequency Sweep

## 3.3 1.3

Plot the s-parameters: S11 and S21 in dB from DC to 5 X the design frequency.

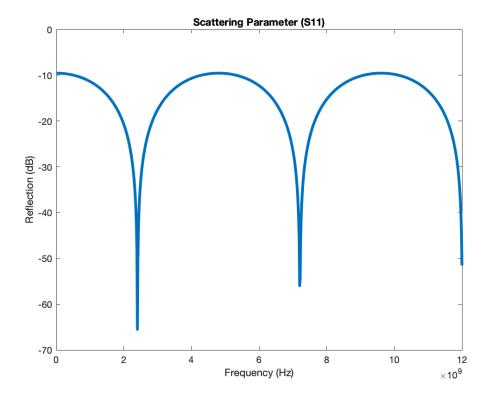


Figure 4: Quarter Wave Transmission Line Scattering Parameter (Reflection vs. Input)

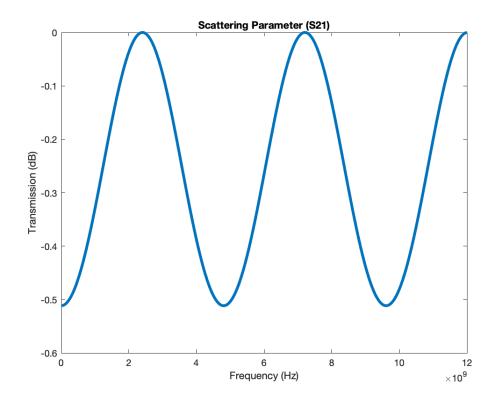


Figure 5: Quarter Wave Transmission Line Scattering Parameter (Transmission vs.Input)

#### 3.4 1.4

Discuss the quarter-wave transformer to match the purely real load across the frequency band of step 1.2 above. How do you know it's matched?

The impedance can be described as matching for multiple reasons. First, the impedance at frequency of zero shows the input impedance magnitude as  $25\,\Omega$  this is as expected because  $\frac{Z_t*R_l}{Z_t}=R_l=25\Omega$ . The input impedance for a wavelength of zero should be equal to the load impedance. Next, the input impedance magnitude at 2.4 GHz is  $100\,\Omega$ . This is also expected because the tangent (imaginary portion) of the impedance goes to infinity and is cancelled (due to L'Hopital's Rule) leaving  $\frac{Z_t*Z_t}{R_l}=\frac{35.3553^2}{25}=50\,\Omega$ . The impedance is also shown to be matched due to Figure 4, or S11. S11 can be seen going to  $-\inf$  at 2.4GHz. This explains reflections are minimized (or equal to zero), which is achieved by making the load impedance equal to the source impedance. Another quality of the matched impedance means complete power is being transmitted, which can be observed at 2.4GHz in Figure 5: S21 where the transmission loss is zero. I utilize 2.4GHz as the marker for observation, but it should be noted that any frequency  $\beta*l=\pi/2$  can also be utilized. The bandwidth, determined using Figure 4: S11 and Figure 5: S21, is roughly 1.3GHz.

## 4 Part II

Design a quarter wave transformer to match a load impedance of 25-j5 Ohms at a frequency of 2.4 GHz. Assume a 50 Ohm transmission line with a permittivity of 2.7.

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## 4.1 2.1

Neatly sketch the transmission line solution.

#### 4.2 2.2

Plot S11 and S21 in dB from DC to  $5\ X$  the design frequency.

#### 4.3 2.3

Discuss the quarter-wave transformer to match the complex load across the frequency band of step 2.2 above.

## 5 Appendix

#### 5.1 Part I - MATLAB Code

```
1 %Aaron Rosen — Fields & Waves II — Project II: Impedance Matching
 %Part I
  clear all; clc; close all;
 f = 2.4e9; %frequency
 w = 2*pi*f; %angular frequency
  c = 3e8; %speed of light
  Zo = 50; %Line Impedance
  RI = 25; %Load Impedance
  Er = 2.7; %permittivity
  fsweep = 0:15000000: f*5; %frequency sweep
15
  up = c/sqrt(Er); %propagation velocity
  B = w/up; %phase constant
  %lambda = up/f; %lambda
  %I = lambda/4; %length of Quarter Wave transformer
  I = 0.019;
22
  B2 = (2*pi*fsweep)/up;
  lambda = up./fsweep; %lambda
25
  Zt = sqrt(Zo*RI); %matched impedance for quarter wave transformer
27
28
  Zin2 = Zt*((RI+1j.*Zt*tan(B2*I))./(Zt + 1j.*RI*tan(B2*I))); %input impedance
      qwt
30
  figure; plot(fsweep, abs(Zin2), 'Linewidth', 3), title('Magnitude of
     Quarter Wave Transformer Impedance'), xlabel('Frequency (Hz)'), ylabel(['
     Magnitude ('char(0x2126)')']), legend('Zin'); %plotting magnitude
 figure; plot(fsweep, rad2deg(angle(Zin2)), 'Linewidth', 3), title('Phase of
     Quarter Wave Transformer Impedance'), xlabel('Frequency (Hz)'), ylabel(['
     Phase ('char(176)')']), legend('Angle of Zin'); %plotting angle
```

```
s11 = (Zin2-Zo)./(Zin2+Zo);
  ab = abs(s11);
  ab = ab.^2;
  s21 = (sqrt(1-ab));
  dbs21 = mag2db(s21);
40
41
42
43
44
45
 figure; plot((fsweep), (20.*log10(s11)), 'LineWidth',3), title('Scattering
     Parameter (S11)'), xlabel('Frequency (Hz)'), ylabel('Reflection (dB)'); %
     plotting s11
   figure; plot( (fsweep), (20*log10(s21)), 'LineWidth',3), title('
      Scattering Parameter (S21)'), xlabel('Frequency (Hz)'), ylabel('
      Transmission (dB)'); %plotting s21
```

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#### 5.2 Part II - MATLAB Code