Fields & Waves II

Project V - BVP 1 - Infinite Trough

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Fields & Waves II - ECEN-3623

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Contents

1 Introduction

A waveguide is a structure that guides waves, such as high-frequency, high-power electromagnetic waves, with minimal loss of energy by restricting the transmission of energy in one direction. For Project 5 we are given a structure considered an infinite trough. The trough is bounded on the y axis from 0 to 1m. The x axis extends from 0 to infinity. At 0 on the x axis and all along the y axis is a voltage potential of 10 V. We are tasked to use MATLAB to effectively plot $\phi(x, y)$. Then change the discrete mode value to [1, 2, 5, 10, 100, 1000]. We are also tasked to plot $E(x, y)$, E_x, E_y, and $|E|$.

2 Part A

First we plot $\phi(x, y)$ with $\Phi = 10$ and $b = 1m$. This is completed using the equation:

$$\sum_{k=1}^{N} \frac{4\Phi}{m\pi} \exp \frac{-m\pi x}{b} \sin \frac{m\pi y}{b} \tag{1}$$

Where $m = 2k - 1$

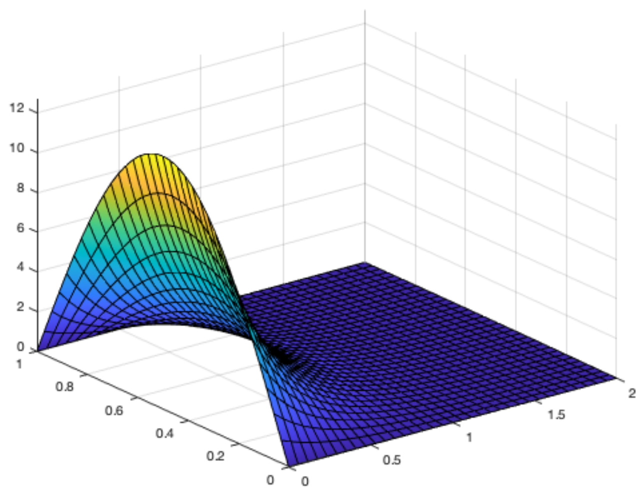

Figure 1: ϕ @ N = 1

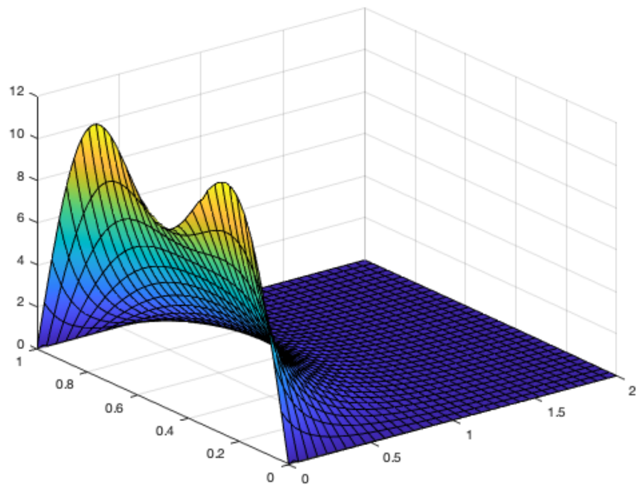

Figure 2: ϕ @ N = 2

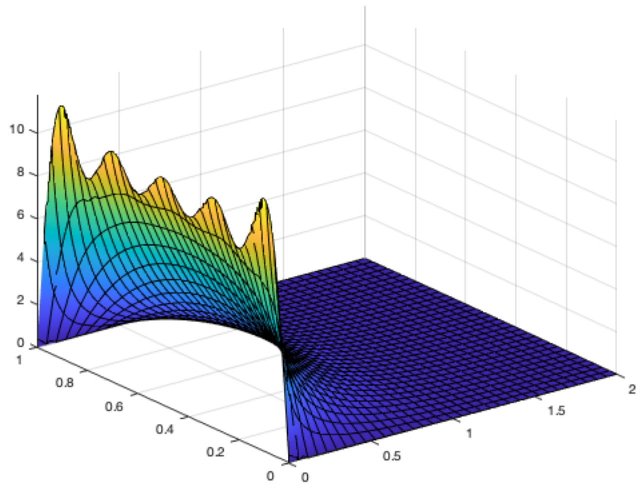

Figure 3: ϕ @ N = 5

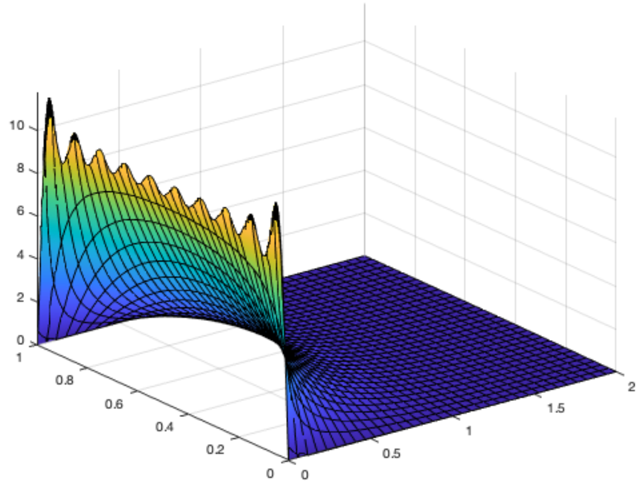

Figure 4: ϕ @ N = 10

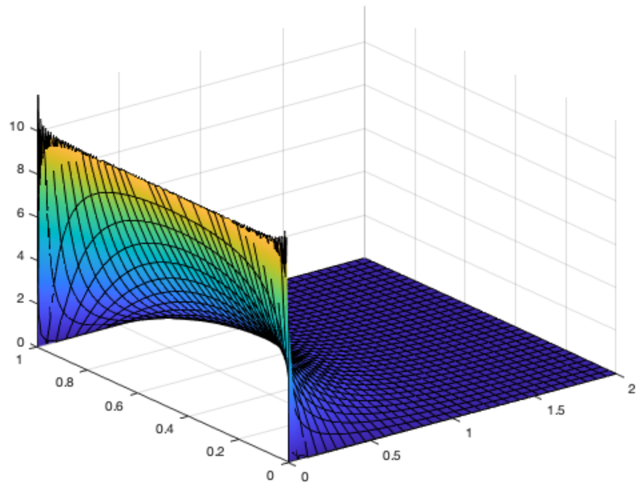

Figure 5: ϕ @ N = 100

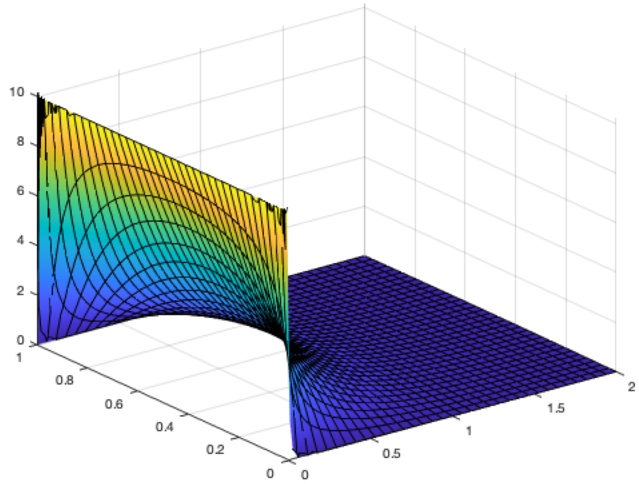

Figure 6: ϕ @ N = 1000

Observing the graphs we see a half sinusoid relating to N (ie., if $N = 1$ then we have one half a sinusoid). Continuing, we see boundary condition is continuous along x = 0 relating to the waves bouncing off the walls of the wave guide causing standing waves to exist inside the wave-guide. As N increases the boundary problem is similar to a Fourier series by the increasing summation adds up to a "rectangle." As you increase through the x axis you can see the boundary condition is no longer needed and the wave falls to zero. The waves also range on the z axis from $8 - 10V$ the waves peak at certain points due to making a tangential for the boundary crossover.

3 Part B

For part 2 we were required to plot $E(x, y)$, E_x, E_y, and $|E|$. This was done by first determining $E(x, y)$ using equation:

$$E(x, y) = \nabla\phi(x, y) \tag{2}$$

3.1 $E(x,y)$

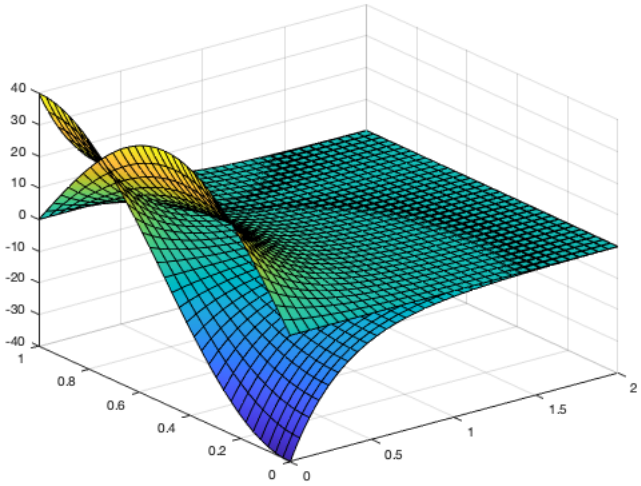

Figure 7: $E(x,y)$ @ N $=1$

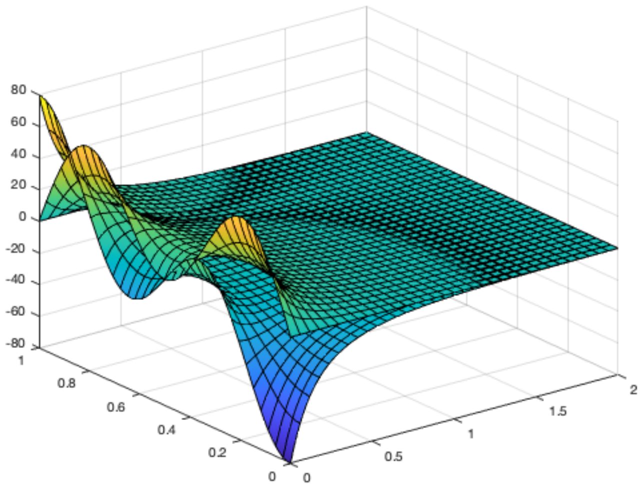

Figure 8: $E(x,y)$ @ N $=2$

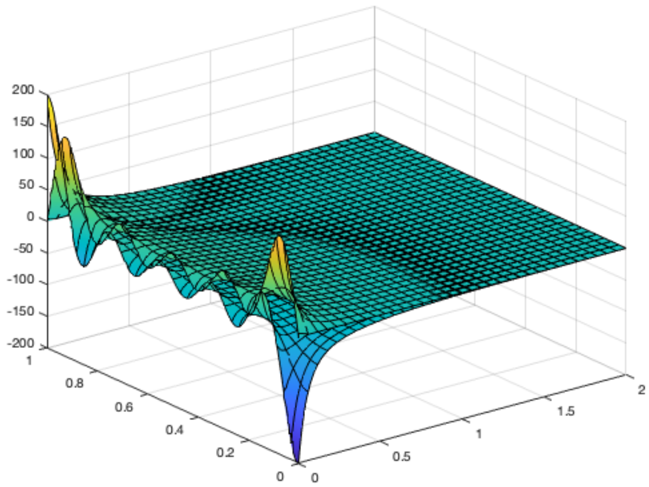

Figure 9: $E(x, y)$ @ N $= 5$

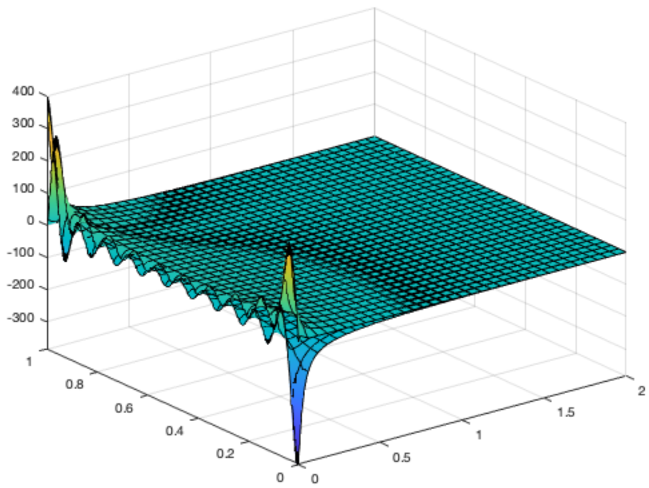

Figure 10: $E(x, y)$ @ N $= 10$

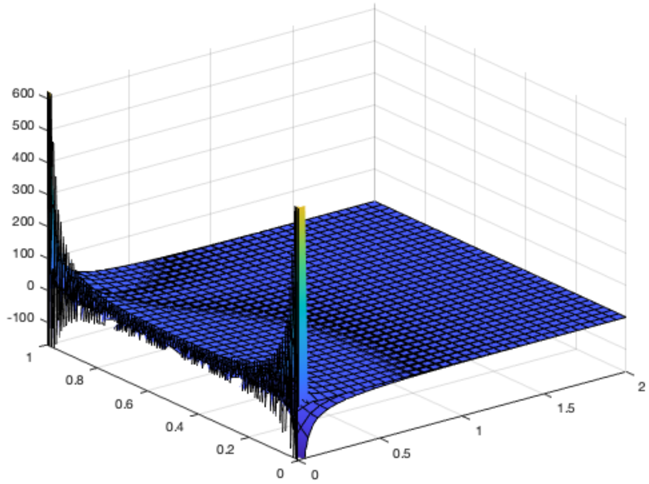

Figure 11: $E(x, y)$ @ N $= 100$

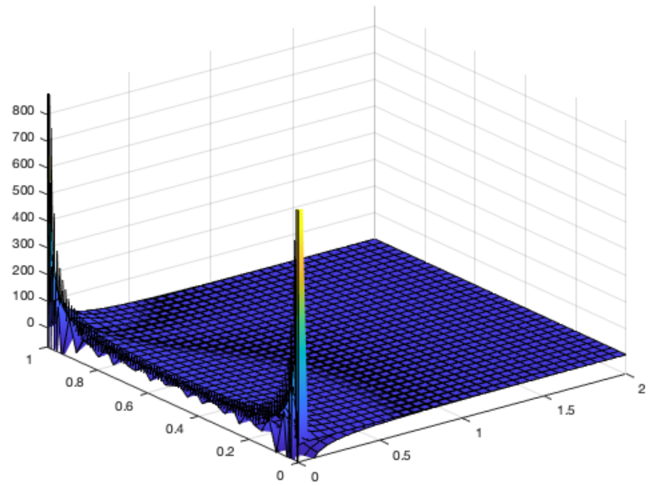

Figure 12: $E(x, y)$ @ N $= 1000$

Given these graphs we are able to visualize the different modes for a transverse electric field inside of the wave-guide. We know it is transverse electric because the electric field propagates on the z axis, not the x axis/ propagation axis. We can see the modes: TE_{10}, TE_{20}, TE_{50}, TE_{100}, TE_{1000}, and TE_{10000}. The intensity of the electric field is described by these graphs and explains where the electric field is most prominent. In future, I will utilize contour and quiver function as they're probably easier to read, but I ran out of time and figured these sufficed as they're very telling of the electric field activity. We see the electric field peaks at the mode peaks and the y axis extremities/ outer bounds (0 & 1).

3.2 E_x

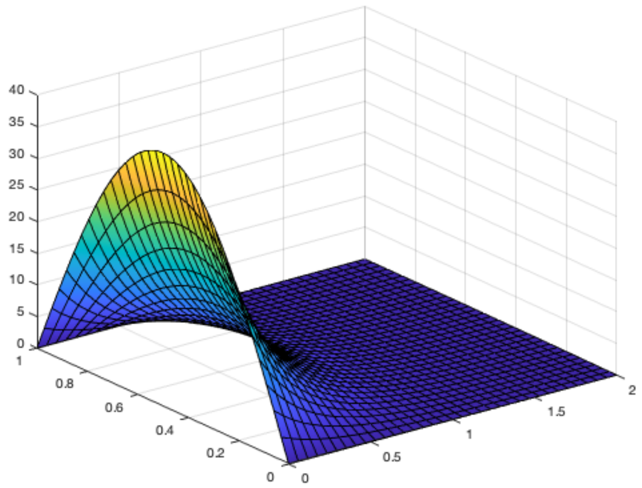

Figure 13: E_x @ N = 1

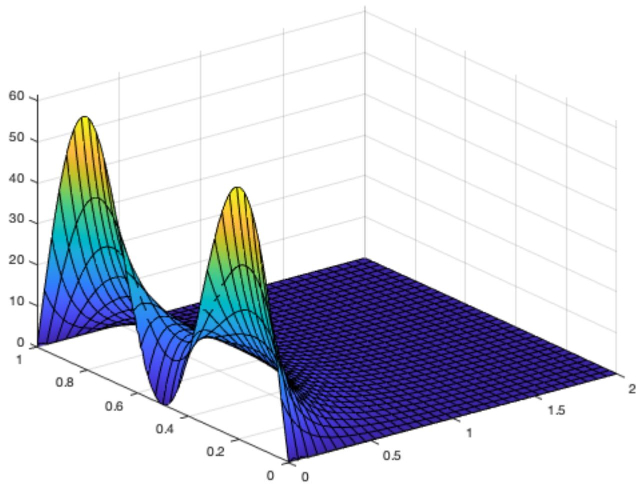

Figure 14: E_x @ N = 2

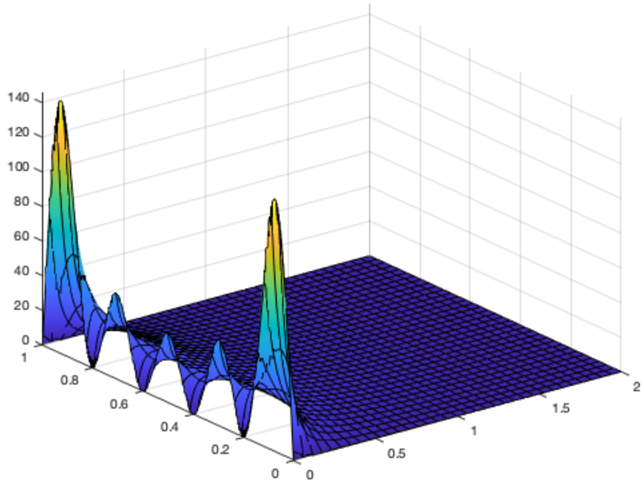

Figure 15: E_x @ N $= 5$

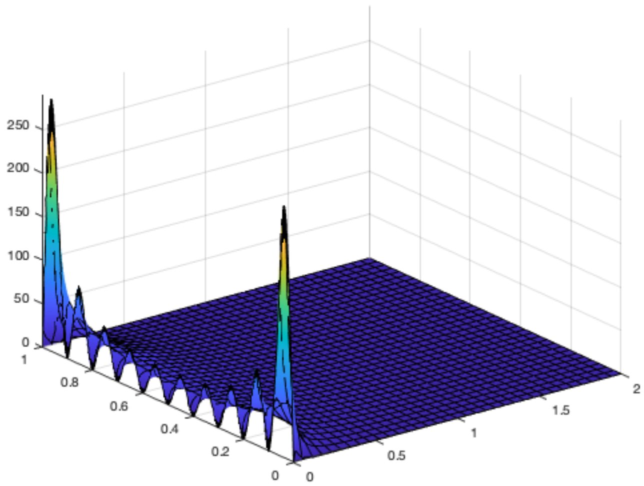

Figure 16: E_x @ N $= 10$

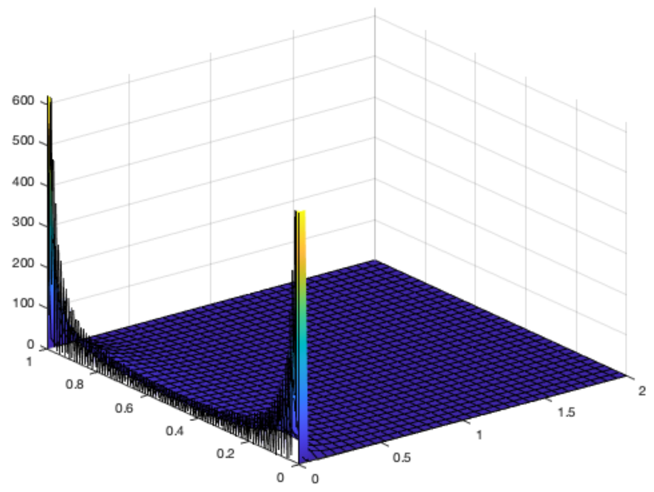

Figure 17: E_x @ N = 100

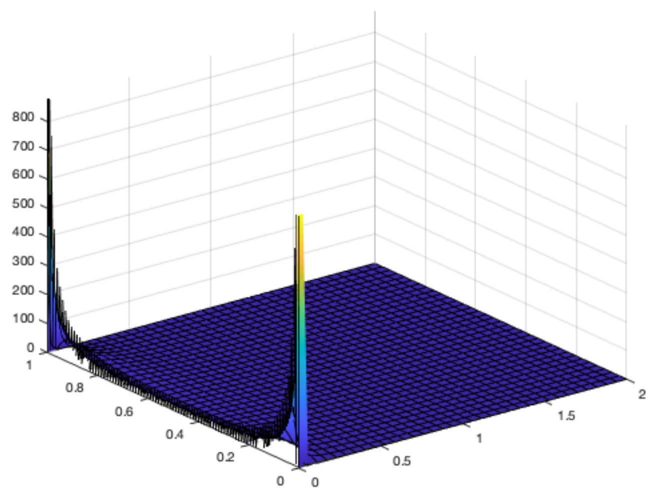

Figure 18: E_x @ N = 1000

3.3 E_y

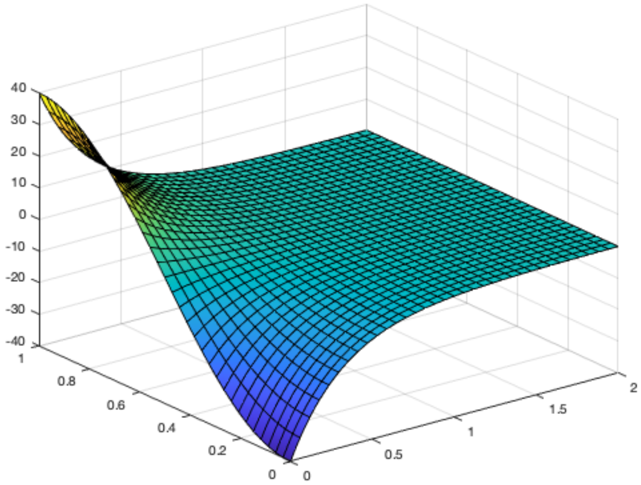

Figure 19: E_y @ N = 1

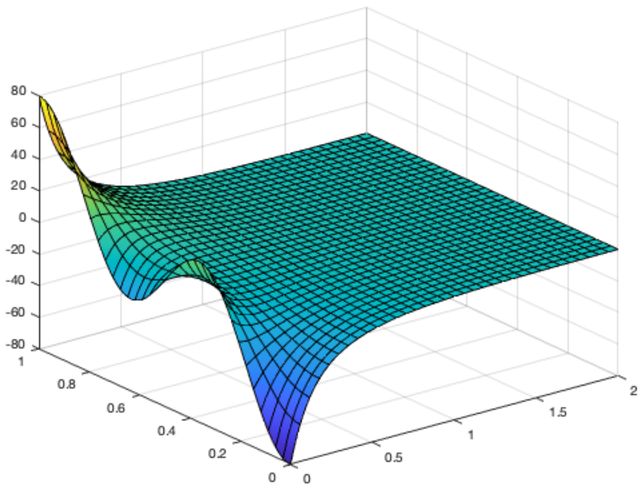

Figure 20: E_y @ N = 2

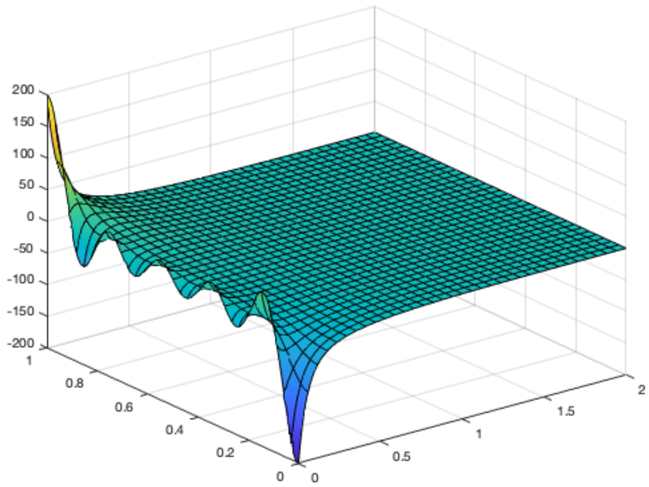

Figure 21: E_y @ N = 5

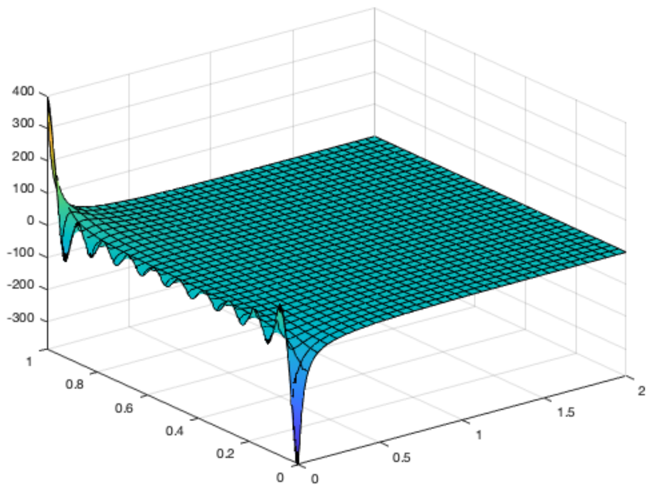

Figure 22: E_y @ N = 10

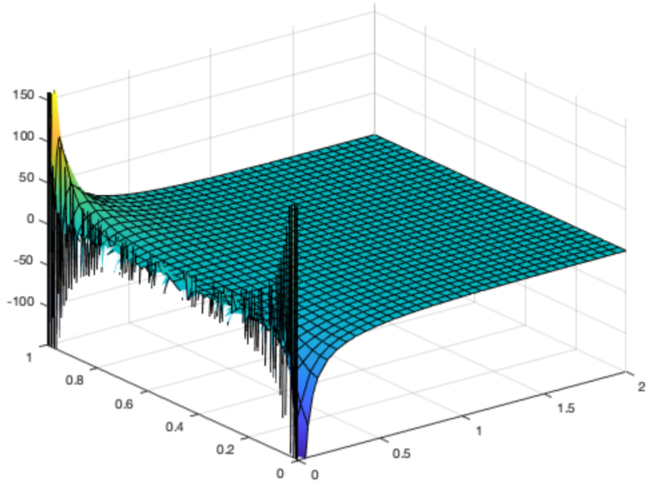

Figure 23: E_y @ N $= 100$

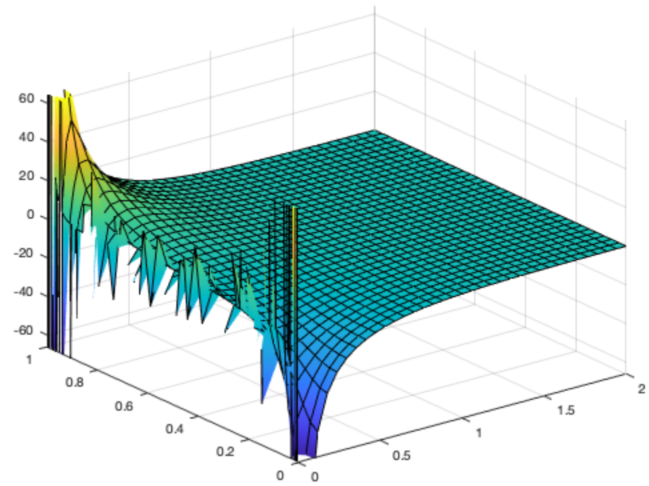

Figure 24: E_y @ N $= 1000$

3.4 $|E|$

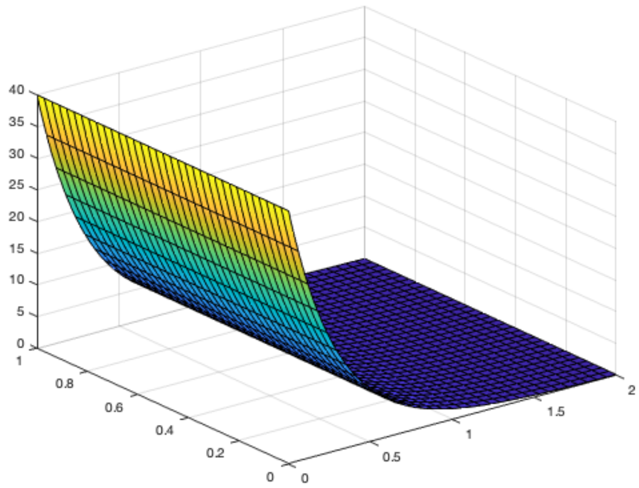

Figure 25: $|E|$ @ N $= 1$

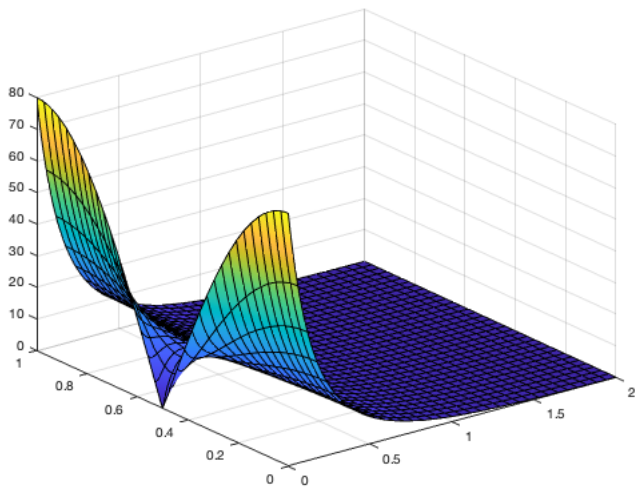

Figure 26: $|E|$ @ N $= 2$

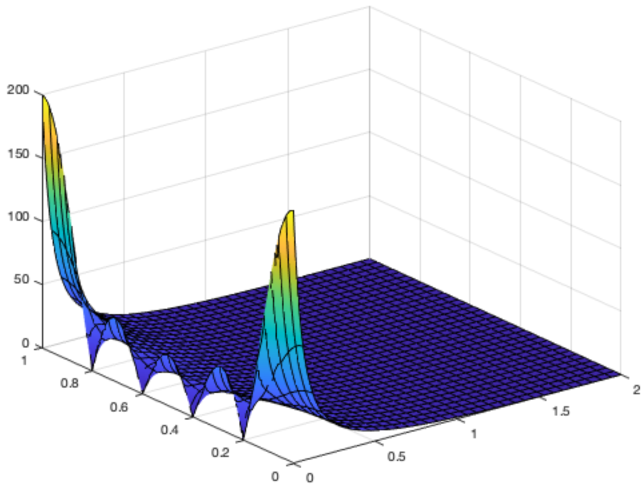

Figure 27: $|E|$ @ N = 5

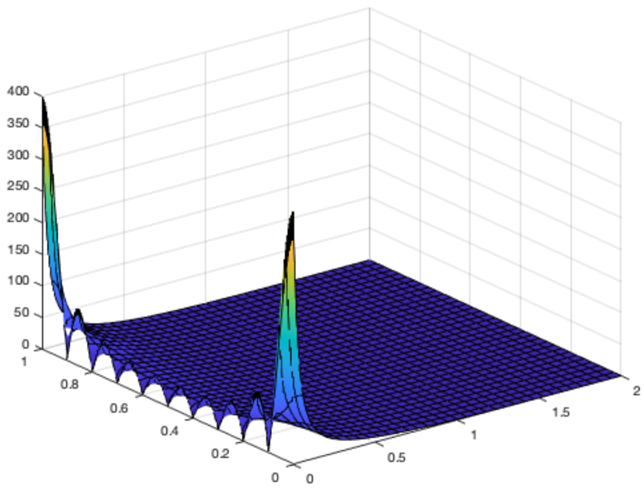

Figure 28: $|E|$ @ N = 10

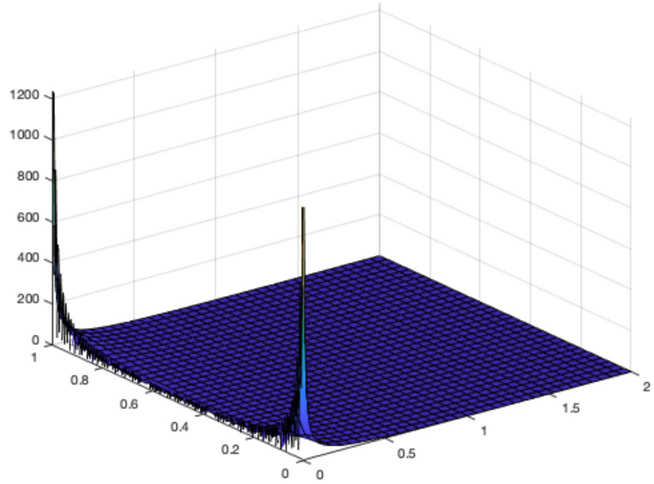

Figure 29: $|E|$ @ N = 100

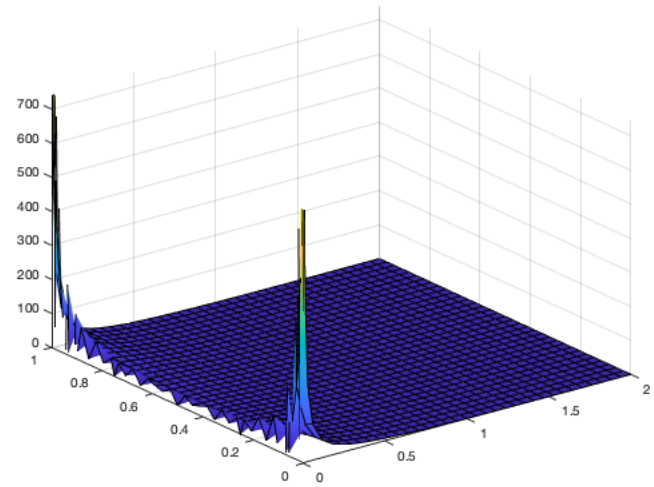

Figure 30: $|E|$ @ N = 1000

The above graphs further describe the electric field function. I am not sure if we needed to include them, but I found them fascinating. Using the graphs we can determine the intensity of the electric field at certain areas of the wave-guide. The intensities were separated by variable x, y, and absolute value/ magnitude. The magnitude is very telling of the activity of the electric field and how it peaks towards the extremities of the wave-guide y bounds.

4 Appendix

```
%Aaron Rosen - A20198898 - March242022
%Fields & Waves II - Project 5 - BVP 1 - Infinite Trough
```

```

syms y k x

Io = 10; %Voltage Potential (V)
b = 1; %trough height (m)


begin = "Would you like to begin? ";
beg = input(begin);
while beg == 1






prompt = "Set N ---> ";
N = input(prompt);
func1 = (4.*Io)./((2.*k-1).*pi);
func2 = exp((-(2.*k-1).*pi.*x)./b);
func3 = sin(((2.*k-1).*pi.*y)./b);
WG = symsum((func1.*func2.*func3), k, 1, N);
%WG = symsum((((4*Io)/((2*k-1)*pi))*exp(-((2*k-1)*pi*x)/b)*sin(((2*k-1)*pi*y
    )/b)), k, 1, N);
%figure
%fsurf(WG, [0 2 0 1]);


WGX = -1.*gradient(WG)
WGXofX = -1.*gradient(WG, x)
WGXofY = -1.*gradient(WG, y)
figure
fsurf(WGX, [0 2 0 1]);
%figure
%fsurf(WGXofX, [0 2 0 1]);
%figure
%fsurf(WGXofY, [0 2 0 1]);
%figure
%fsurf(norm(WGX), [0 2 0 1]);


continue2 = "Would you like to continue? ";
```

```
cxnt = input(continue2);
    if cxnt == 1
else
    break
    end


end
```