Fields & Waves II

Project VI - BVP 2 - Trough with a top

Aaron Rosen

Oklahoma State University

Fields & Waves II - ECEN-3623

31 March 2022

A20198898

Contents

1 Introduction

A wave guide is a structure that guides waves, such as high-frequency, high-power electromagnetic waves, with minimal loss of energy by restricting the transmission of energy in one direction. For Project 6 we are given a trough structure with a top. For my project and MATLAB computing time, the trough is bounded on the y axis from 0 to 1m and x axis from 0 to 1m. At 1 on the y axis and all along the x axis is a voltage potential of 10 V. We are tasked to obtain $\Phi(x,y)$ through derivation, utilize MATLAB to plot $\Phi(x,y)$, **E**(x,y), and $\rho_s(x,y)$, and using Excel, plot the finite difference solution for $\Phi(x,y)$.

2 Part A

First we must derive $\Phi(x,y)$

Boundary Conditions:

1. Left: $\phi(0,y) = 0$
2. Right: $\phi(a,y) = 0$
3. Bottom: $\phi(x,0) = 0$
4. Top: $\phi(x,b) = \Phi_o$

$$V(x,y) = X(x)Y(y) \tag{1}$$

$$X''(x)Y(y) + Y''(y)X(x) = 0 \tag{2}$$

$$\frac{X''(x)}{X(x)} = \frac{-Y''(y)}{Y(y)} = \lambda \tag{3}$$

Giving us two Equations:
Equation (1):

$$X''(x) + \lambda X(x) = 0 \tag{4}$$

Equation (2):

$$Y''(y) - \lambda Y(y) = 0 \tag{5}$$

Where $\lambda = k^2$ Solution to Equation (1):

$$X(x) = C_1 cos(kx) + C_2 sin(kx) \tag{6}$$

Using Boundary Values (Left & Right):

$$BV(Left) = \phi(0,y) = 0 => x(0) = 0 = C_1 \tag{7}$$

$$BV(Right) = \phi(a,y) = 0 => x(a) = 0 = C_2 sin(kx) \tag{8}$$

This gives:

$$X_n(x) = g_n sin(\frac{n\pi}{a}x) \tag{9}$$

Solution to Equation (2):

$$Y(y) = h_0 cosh(\frac{n\pi}{a}y) + h_1 sin(\frac{n\pi}{a}y) \tag{10}$$

Using Boundary Value (Bottom):

$$BV(Bottom) = \phi(x,0) = 0 => Y(0) = 0 = h_0 \tag{11}$$

Resulting in:

$$Y_n(y) = h_n sinh(\frac{n\pi}{a}y) \tag{12}$$

Using the two solutions:

$$V(x,y) = \sum_{n=1}^{\infty} C_n sin(\frac{n\pi}{a}x) sinh(\frac{n\pi}{a}y) \tag{13}$$

Using Boundary Condition (Top):

$$\phi(x,b) = \Phi \tag{14}$$

$$\Phi = \sum_{n=1}^{\infty} C_n sin(\frac{n\pi}{a}x) sinh(\frac{n\pi}{a}b) \tag{15}$$

Multiplying both sides by $sin(\frac{m\pi}{a}x)$ and integrating from [0-9]:

$$\int_0^9 \Phi_o sin(\frac{m\pi}{a}x)dx => \tag{16}$$

$$\int_{n=1}^{\infty} C_n sinh(\frac{n\pi}{a}b) \int_0^a sin(\frac{m\pi}{a}x) sin(\frac{n\pi}{a}x)dx \tag{17}$$

Finally resulting in:

$$\phi(x,y) = \frac{4\Phi_o}{\pi} \sum_{n=1,3,5...}^{\infty} \frac{sin(\frac{n\pi}{a}x) sinh(\frac{n\pi}{a}y)}{n sinh(\frac{n\pi}{a}b)} \tag{18}$$

3 Part B: Plot $\phi(x,y)$

For part B we were required to plot the resulting $\phi(x,y)$ we derived [Equation 18 above]. This was done replicating MATLAB code for Project V - BVP using the different equation obtained. The result was:

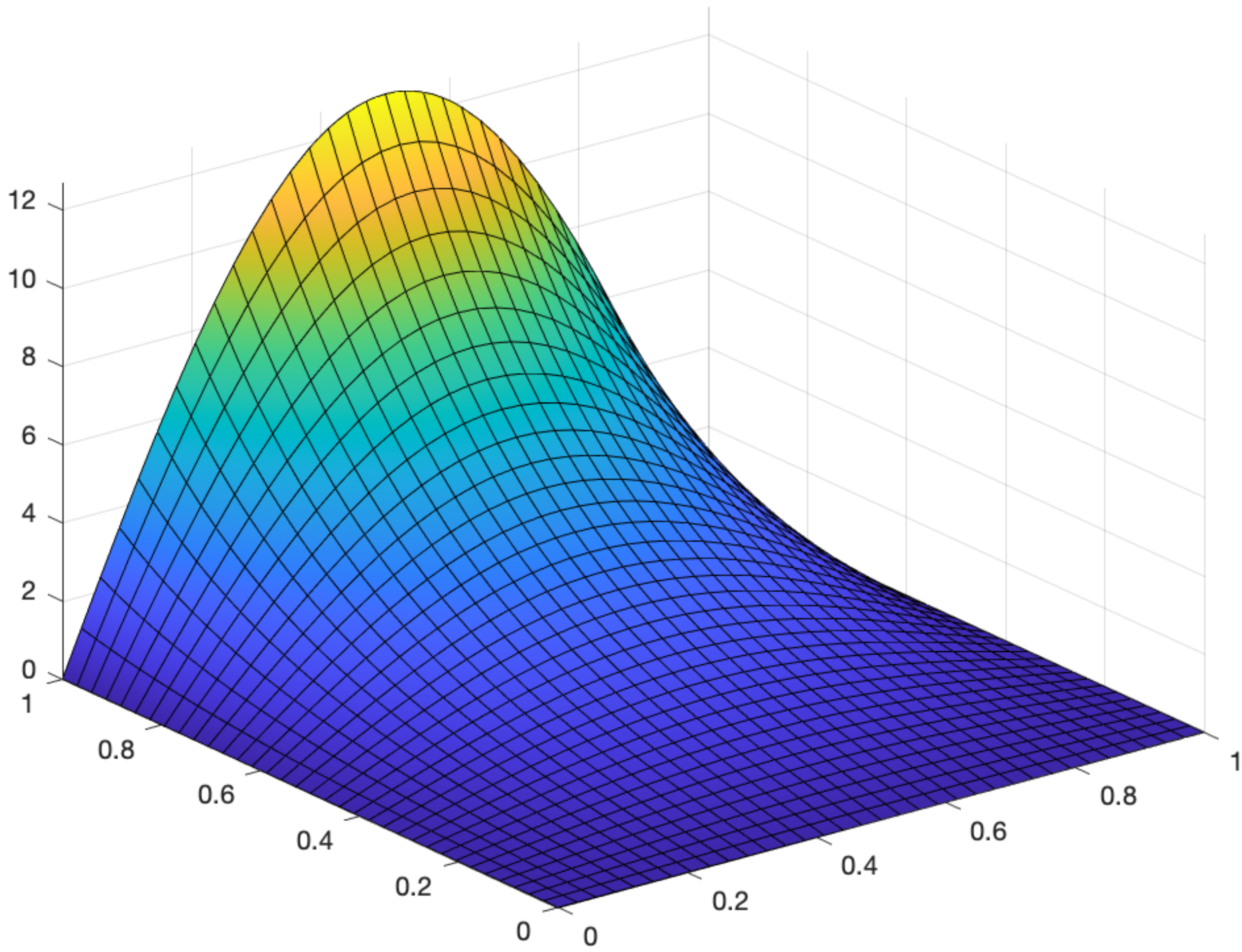

Figure 1: $\phi(x, y)$ for N = 1

The figure above demonstrates the Boundary Values clearly. We can see the peak going over the 10 V boundary due to the summing of the sinusoids. The Boundary Values also stop at the expected points and peak at expected points. As we increase N, due to the Fourier Series like behavior the sinusoids will sum to 10 rather than 12. This will be tested by plotting N = 10:

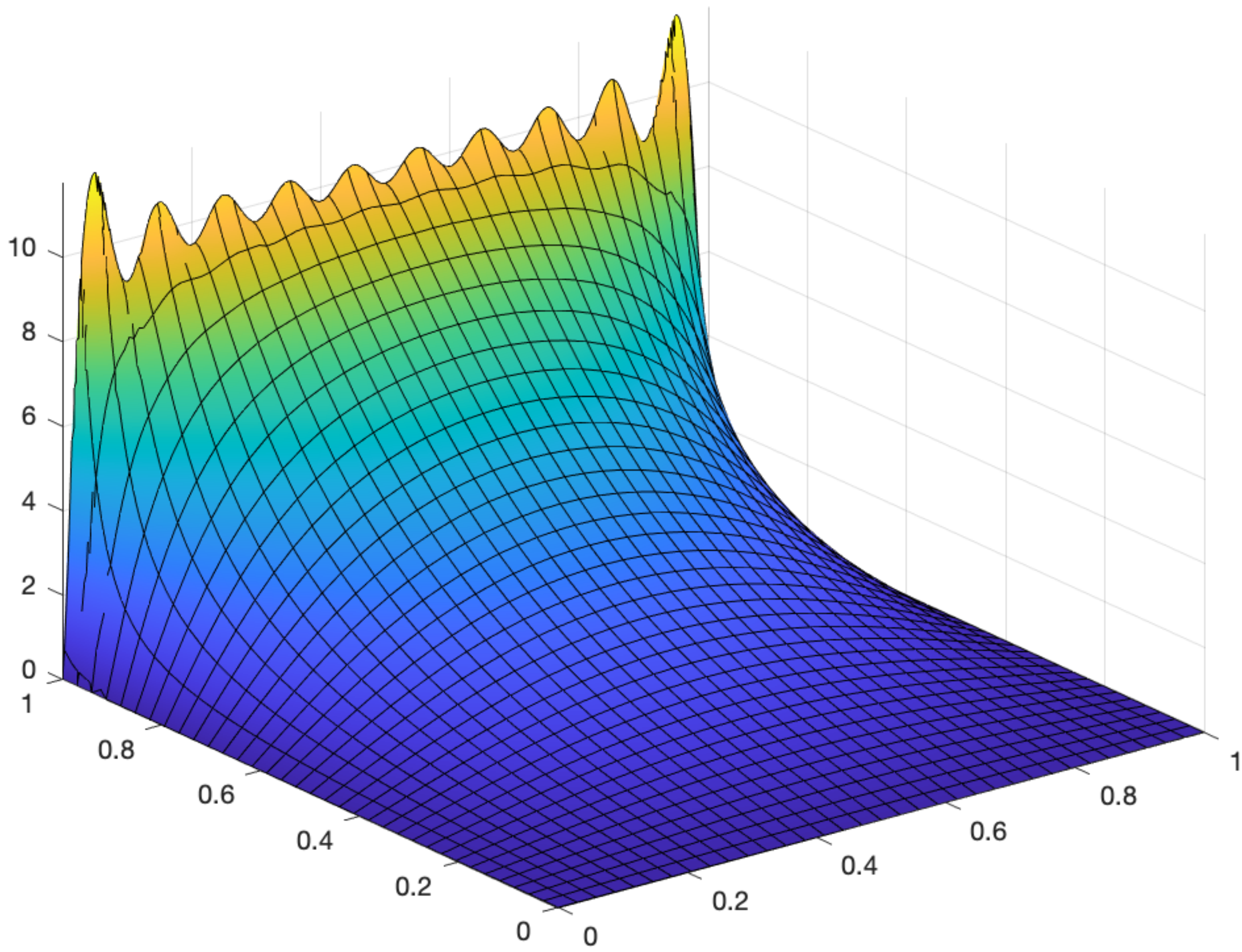

Figure 2: $\phi(x, y)$ for N = 10

Utilizing this graph, we are able to see $\phi(x, y)$ approach 10 V rather than 12 V. With more increasing of N the graph will eventually level out to 10 V.

3.1 Part C: Plot E(x, y)

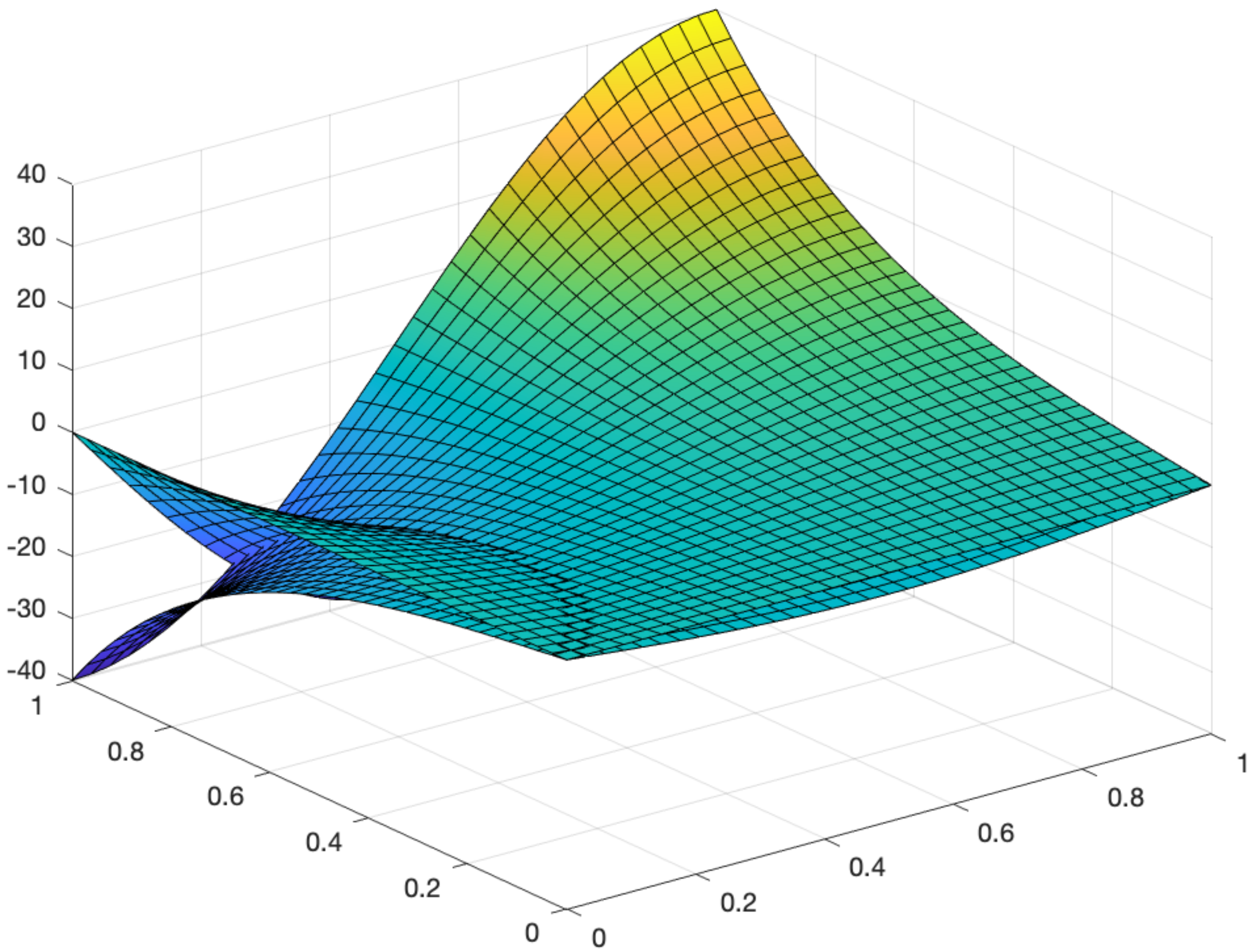

Figure 3: Plot of $E(x, y)$ for N $= 1$

This figure doesn't show much so I chose to break. The electric field into X and Y components for a better understanding.

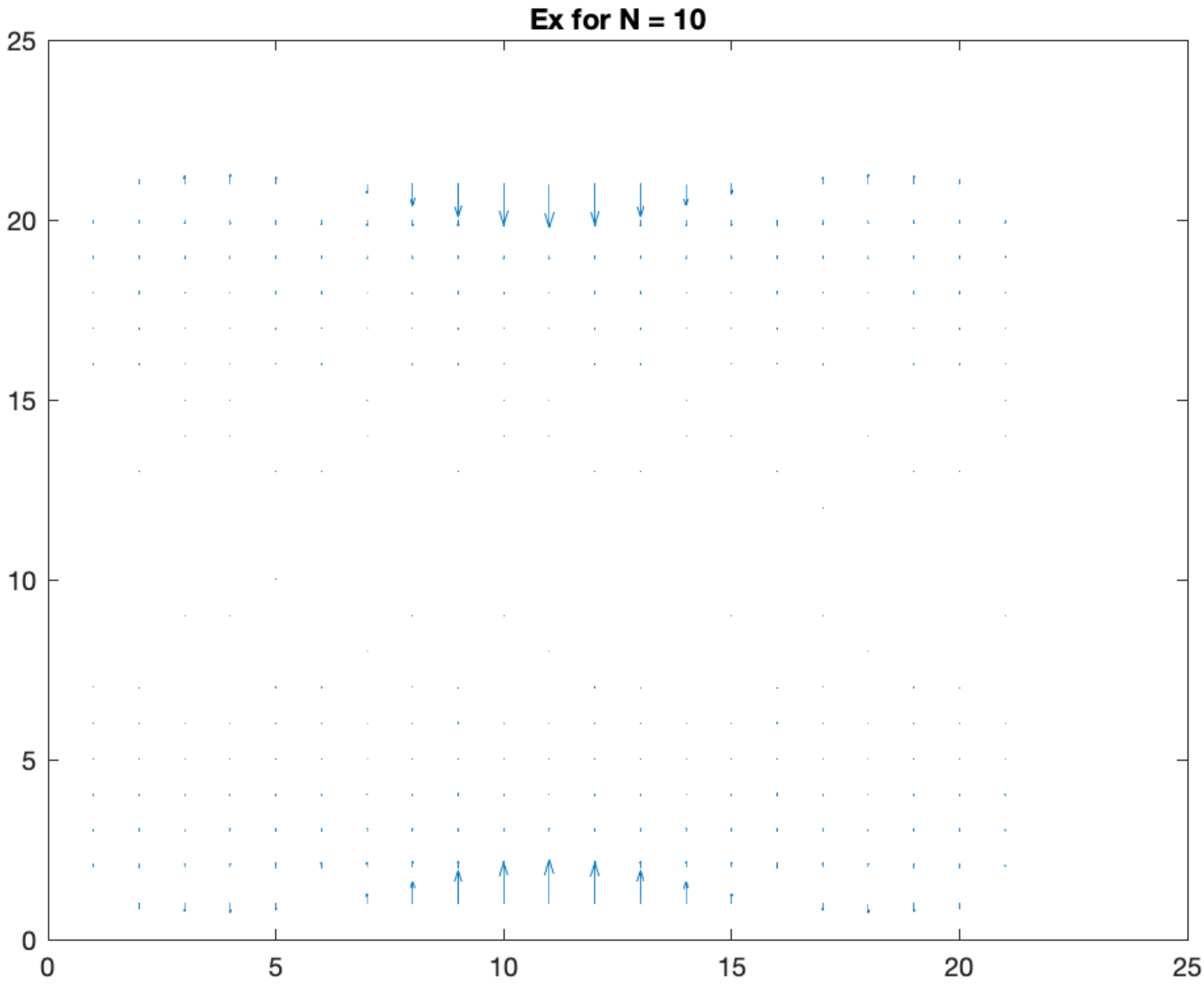

Figure 4: $E(x)$ for N $= 10$

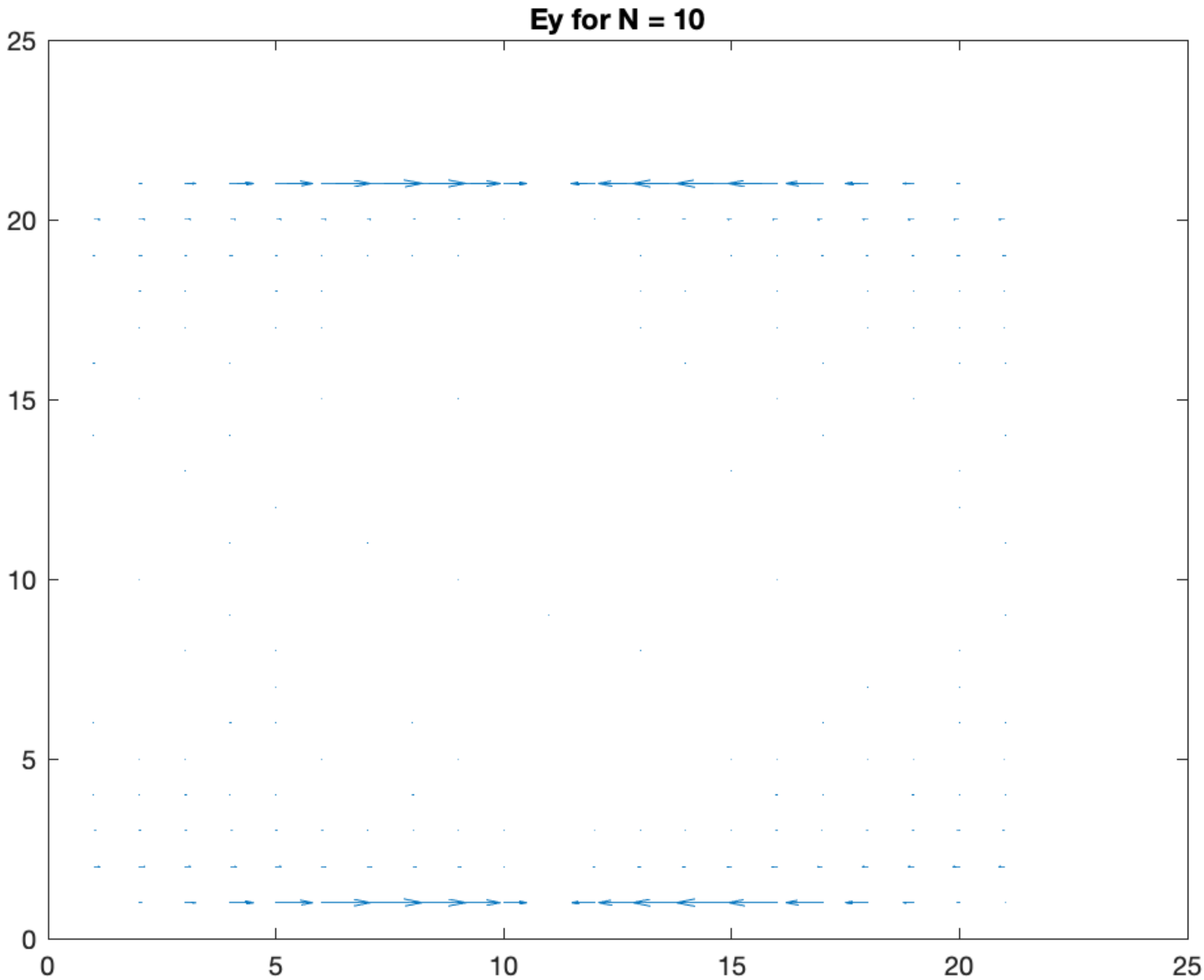

Figure 5: $E(y)$ for N $= 10$

Given these graphs we are able to visualize tangential behavior of the electric field at the boundaries. The graphs represent the tangential behavior we discussed in class. The intensity of the electric field is described by these figures and explains where the electric field is most prominent.

4 Part D: Plot $\rho_s(x, y)$

For this section the Surface Density ρ was plotted. This was completed by taking the negative Laplacian times Epsilon. Where $\epsilon = 8.854 * 10^{-12}$:

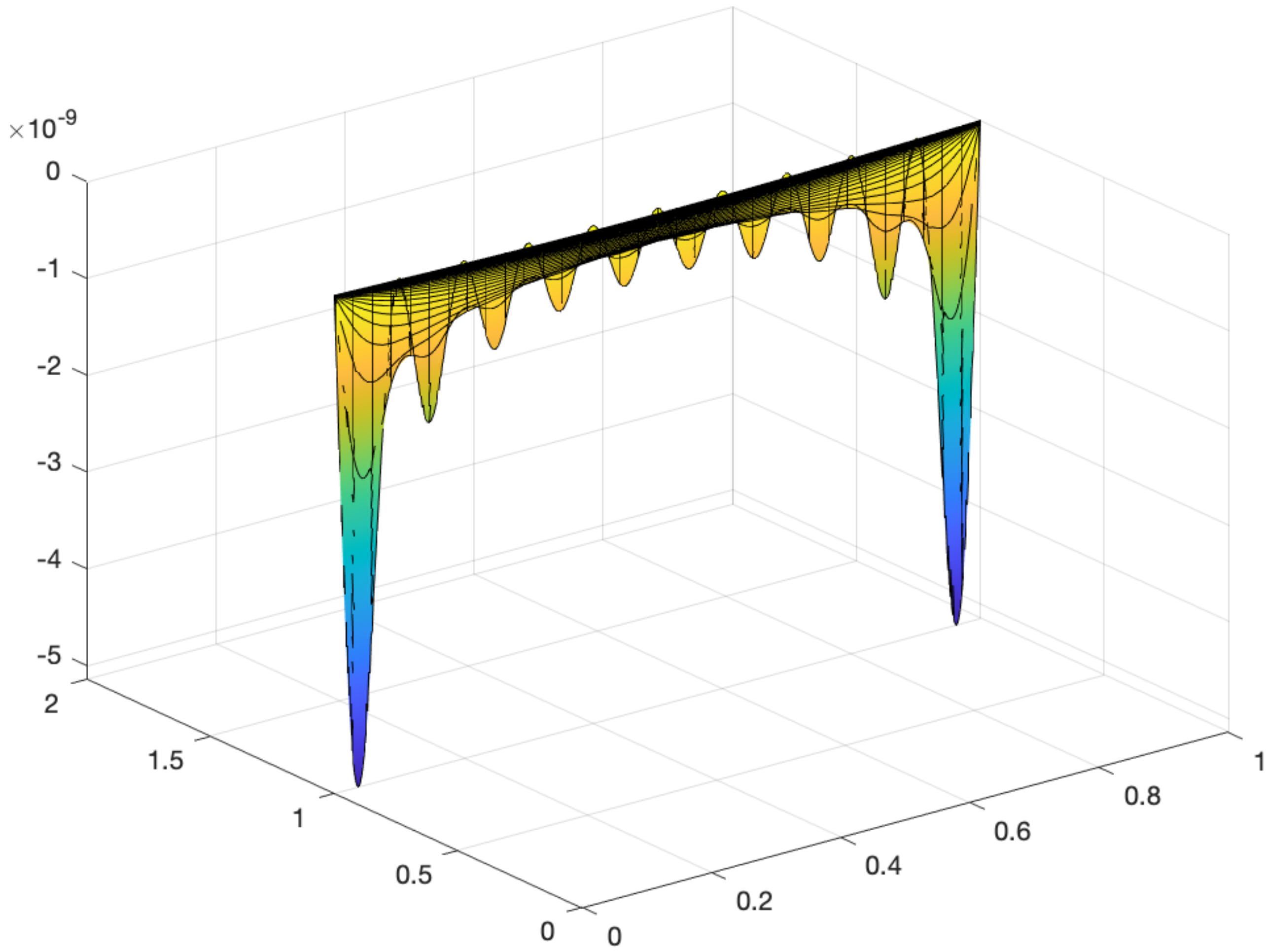

Figure 6: ρ_s for N = 10

This figure demonstrates the surface charge density on the top of the trough. Using this figure we can see where the most charge is stored throughout the trough interface. These stored charges can lead to a capacitance within the wave guide. When increasing N we are able to see differences of magnitude along the sides describing fluxuation within the surface charges.

5 Part E: Excel Laplacian

Utilizing the video provided, the following Excel table was constructed:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1		10	10	10	10	10	10	10	10	10	10	10	10	10	10	
2	0	4.95415374	6.88330376	7.76233966	8.21562154	8.4660685	8.60280537	8.66385249	8.66378884	8.60262274	8.4657913	8.21528845	7.76200084	6.8830171	4.95398049	0
3	0	2.93319581	4.81652802	5.95017215	6.63376245	7.04549241	7.28092318	7.3884328	7.38830826	7.28056585	7.04495003	6.63311071	5.94950919	4.81596712	2.93285682	0
4	0	1.96190451	3.49911709	4.58762652	5.32324494	5.80063488	6.08634656	6.22002441	6.21984255	6.08582479	5.7998429	5.32229329	4.58665849	3.49829808	1.96140953	0
5	0	1.41503182	2.62996482	3.57738047	4.27024735	4.74666402	5.04296579	5.18462902	5.1843942	5.04229207	4.7456414	4.26901855	3.57613052	2.62890729	1.41439268	0
6	0	1.06791446	2.02777424	2.82094558	3.4328176	3.87182335	4.15318211	4.29008023	4.28979751	4.15237096	3.87059212	3.43133813	2.81944063	2.02650096	1.06714492	0
7	0	0.82844527	1.59161679	2.24494147	2.76721623	3.15347226	3.40663577	3.53147791	3.53115298	3.40570353	3.15205725	2.76551592	2.24321187	1.59015344	0.82756085	0
8	0	0.65378831	1.26456407	1.79900493	2.23646055	2.56690633	2.78702963	2.89664965	2.89628879	2.7859943	2.56533482	2.23457219	1.79708403	1.26293886	0.65280606	0
9	0	0.52163609	1.01303138	1.44897598	1.81142871	2.08923029	2.27641394	2.37027704	2.36988699	2.27529486	2.08753165	1.80938759	1.44689968	1.01127468	0.52057437	0
10	0	0.41917993	0.81607676	1.17128583	1.46967275	1.70064078	1.85750219	1.93652824	1.93611613	1.85631979	1.69884603	1.46751613	1.16909203	0.81422064	0.41805811	0
11	0	0.33843505	0.65989527	0.94921017	1.19389596	1.38455536	1.51473465	1.58051366	1.58008687	1.51351013	1.38269668	1.19166251	0.94693821	0.65797301	0.33727325	0
12	0	0.27407627	0.53491873	0.7705228	0.97066701	1.12730487	1.23463174	1.28895662	1.28852269	1.23338674	1.1254151	0.9683962	0.76821283	0.53296429	0.27289502	0
13	0	0.22235602	0.43423097	0.62604315	0.789453	0.91770641	1.00578115	1.0503965	1.049963	1.00453738	0.91581849	0.7871844	0.62373542	0.43227842	0.2211759	0
14	0	0.1805253	0.35266365	0.50872402	0.64191697	0.74664263	0.81865661	0.85514019	0.85471457	0.81743545	0.74478902	0.6396896	0.50645822	0.35074656	0.17936662	0
15	0	0.14650395	0.28625533	0.41306211	0.52140802	0.6066896	0.66537499	0.69509472	0.69468418	0.66419712	0.60490171	0.5192596	0.41087661	0.28440618	0.14538631	0
16	0	0.11868124	0.23191152	0.33470286	0.42258568	0.49180186	0.53944583	0.56355632	0.5631677	0.53833085	0.49010941	0.42055195	0.33263403	0.23016107	0.11762325	0
17	0	0.09578841	0.18718001	0.2701651	0.3411375	0.39705065	0.43553781	0.45499574	0.45463537	0.43450385	0.39548119	0.33925155	0.2682466	0.18555675	0.09480729	0
18	0	0.07681265	0.15009521	0.21664169	0.27356218	0.31840813	0.34927182	0.36485856	0.36453212	0.34833522	0.31698646	0.27185383	0.21490384	0.1486248	0.07592391	0
19	0	0.06093608	0.11906544	0.1718502	0.21699964	0.25256946	0.27704233	0.28938764	0.2891001	0.27621733	0.2513172	0.21549484	0.17031941	0.11777023	0.06015323	0
20	0	0.0474907	0.09278817	0.13391765	0.16909537	0.19680578	0.2158651	0.22546916	0.22522465	0.21516357	0.19574092	0.16781577	0.13261595	0.09168678	0.04682501	0
21	0	0.03592371	0.07018406	0.10128903	0.12789036	0.14884189	0.16324799	0.17050032	0.17030208	0.16267921	0.14797854	0.1268529	0.10023365	0.06929109	0.03538398	0
22	0	0.02576995	0.05034408	0.07265299	0.09173011	0.10675351	0.11708082	0.12227583	0.12212615	0.11665137	0.10610165	0.09094679	0.07185614	0.04966985	0.02536243	0
23	0	0.01662934	0.03248587	0.04687988	0.0591879	0.0688797	0.07554081	0.07888987	0.07879007	0.07525446	0.06844505	0.0586656	0.04634856	0.03203631	0.01635761	0
24	0	0.00814771	0.01591655	0.02296866	0.02899864	0.03374669	0.03700977	0.03865001	0.03860042	0.03686749	0.03353073	0.02873911	0.02270466	0.01569317	0.0080127	0
25		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 7: Excel Chart for Laplacian

Using this chart, the following graph was created:

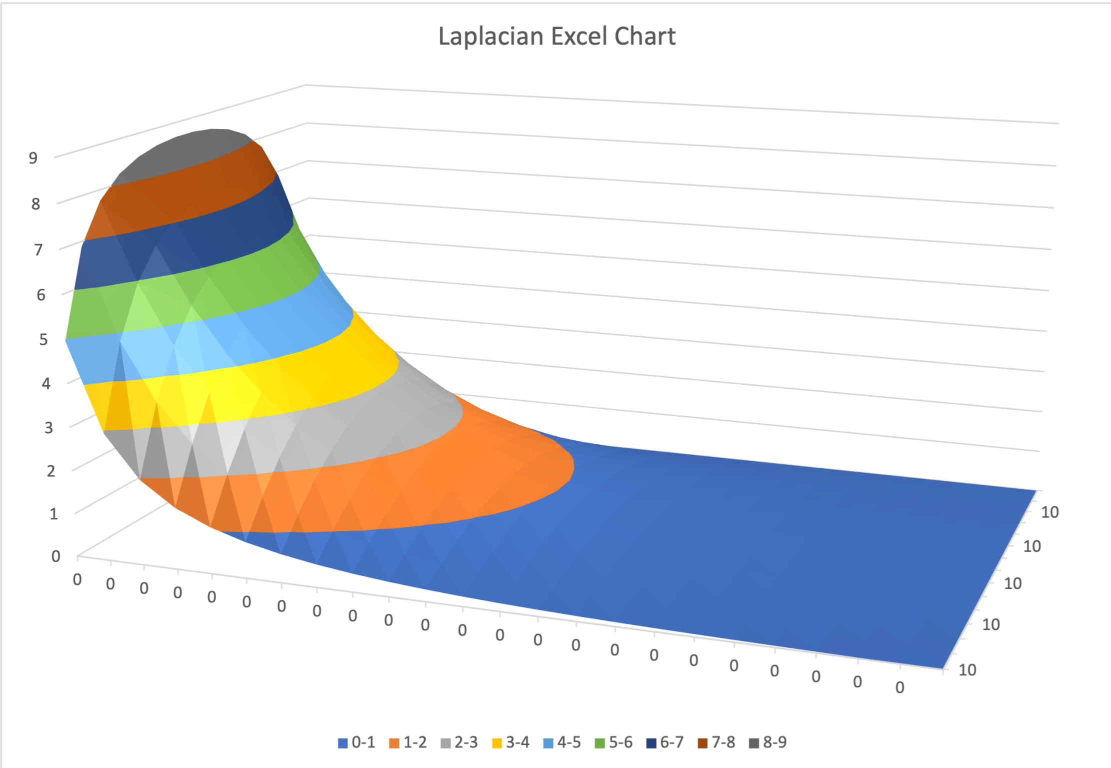

Figure 8: Laplacian Graph from Excel

The figure above vastly resembles our $\phi(x, y)$ for N = 1. This chart provides a secondary measure to ensure the calculations were carried out correctly in Part B. The only variation between the two that I notice is the peak at 12 V for Part B and a peak at 10 V for the Excel chart.

6 Appendix

```
%Aaron Rosen - A20198898 - March242022
%Fields & Waves II - Project 5 - BVP 1 - Infinite Trough

syms y k x z m

Io = 10; %Voltage Potential (V)
b = 1; %trough height (m)
a = 1; %trough height (m)
epsilon = 8.854E-12;

begin = "Would you like to begin? ";
beg = input(begin);
while beg == 1




prompt = "Set N ---> ";
N = input(prompt);

func1 = ((4*Io)/pi);
func2 = sin(((2*k-1)*pi*x)/a)*sinh((((2*k-1)*pi*y)/b));
func3 = 1/((2*k-1)*sinh(((2*k-1)*pi*b)/a));
WG = symsum((func1*func2*func3), k, 1, N);
%WG = symsum((((4*Io)/((2*k-1)*pi))*exp(-((2*k-1)*pi*x)/b)*sin(((2*k-1)*pi*y
    )/b)), k, 1, N);
figure
fsurf(WG, [0 a 0 b]);


WGX = -1.*gradient(WG)
WGXofX = -1.*gradient(WG, x)
WGXofY = -1.*gradient(WG, y)

[X, Y] = meshgrid(-0.1:0.01:0.1, -1:.1:1);
G1 = subs(WGX(1),[x y],{X,Y});
G2 = subs(WGX(2),[x y],{X,Y});

%X Plot
figure
quiver(X, G1);
title("Ex for N = 10");
```

```
%Y Plot
figure
quiver(G2,Y);
title("Ey for N = 10");

figure
fsurf(WGX, [0 a 0 b]);
figure
fsurf(WGXofX, [0 a 0 b]);
figure
fsurf(WGXofY, [0 a 0 b]);
figure
fsurf(norm(WGX), [0 a 0 b]);
rho = WGX(1) * 2* epsilon;
rho2 = WGX(2) * 2 * epsilon;

figure
y = b;
fsurf(x, y, rho, [0 a 0 b])

figure
y = b;
fsurf(x, y, rho2, [0 a 0 b])
%fsurf(rho2, [0 a 0 b])




continue2 = "Would you like to continue? ";
cxnt = input(continue2);
    if cxnt == 1
else
    break
    end


end
```