Fields & Waves II

Project VII - Final Project

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Fields & Waves II - ECEN-3623

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Contents

8.1

In free space, a wave propagating radially away from an antenna at the origin has

$$\boldsymbol{H_s} = \frac{-I_s}{r} cos^2(\theta) \boldsymbol{a_\theta} \tag{1}$$

where the driving current phasor $I_s = I_o e^{j\alpha}$. Determine

a) $\mathbf{E_s}$

b) **P**(r, θ, ϕ)

c) R_{rad}

Solving for a)

$$\mathbf{E_o s} = -\eta_o \mathbf{a_r} \times \mathbf{H_{os}}$$
$$= \frac{-\eta_o I_o}{r} e^{j\alpha} cos^2(\theta) \boldsymbol{\phi}$$

Solving for b)

$$\mathbf{P}(r, \theta, \phi) = \frac{1}{2} Re[\mathbf{E_s} \times \mathbf{H_s}]$$
$$= \frac{1}{2} \eta_o \frac{I_o^2}{r^2} cos^4 \mathbf{r}$$

Solving for c)

$$\mathbf{P}_{rad} = \oint \mathbf{P}(r, \theta, \phi)\, d\mathbf{S}$$
$$= \frac{1}{2} \eta_o I_o^2 \oint \frac{1}{r^2} cos^4 \mathbf{r} * r^2 sin(\theta) d\phi d\theta \mathbf{r}$$
$$= \frac{1}{2} \eta_o I_o^2 \int_0^\pi cos^4(\theta) sin\theta d\theta \int_0^2 \pi d\phi$$
$$\frac{1}{2} \eta_o I_o^2 (\frac{2}{5})(2\pi) = \mathbf{P}_{rad} = \frac{1}{2} I_o^2 R_{rad}$$
$$\eta_o = 120\pi$$
$$R_{rad} = 96\pi^2 \Omega = 947.48\Omega$$

8.5

You are given the following normalized radiation intensity:

$$P_n(\theta,\phi) = \begin{cases} sin^2\theta sin^3\phi & 0 \le \phi \le \pi \\ 0 & otherwise \end{cases} \tag{2}$$

Determine the beamwidth, pattern solid angle, and directivity.

Beamwidth

$$P_n(\theta,\phi) = sin^2\theta sin^3\phi$$

$$P_n(\theta,\phi)_{iso} = P_n(\theta,\phi)_{max} = 1$$

$$P_{BW} = 1/2$$

$$\frac{1}{2} = sin^2\theta$$

$$\theta = sin^{-1}(\frac{1}{\sqrt{2}}) = 45°$$

$$\frac{1}{2} = sin^3\phi$$

$$\phi = sin^{-1}(\frac{1}{\sqrt[3]{2}}) = 52.53°$$

$$BW_\theta = 180° - 45° - 45° = 90°$$

$$BW_\phi = 180° - 52.53° - 52.53° = 74.93°$$

$$BW = \frac{1}{2}(BW_\theta + BW_\phi)$$

$$= \frac{1}{2}(90° + 74.93°)$$

$$= 82.47°$$

Pattern Solid Angle

$$\Omega_\text{p} = \int\int P_n d\Omega$$

$$= \int\int sin^2\theta sin^3\phi sin\theta d\theta d\phi$$

$$= \int_0^\pi sin^3\theta d\theta \int_0^\pi sin^3\phi d\theta d\phi$$

$$= \frac{16}{9} = 1.78$$

Directivity

$$D_{max} = \frac{4\pi}{\Omega_\text{p}} = \frac{4\pi}{1.78} = 7.069$$

8.13

Suppose in the far field for an antenna at the origin,

$$\mathbf{H_{os}} = \frac{\beta I_s}{4\pi}\frac{e^{-j\beta r}}{r} sin\theta cos\phi \boldsymbol{\phi} \tag{3}$$

where $\mathbf{I_s} = I_o e^{j\alpha}$. What is the radiation resistance of this antenna at 100MHz?

$$\mathbf{E_{os}} = -\eta_o \boldsymbol{r} \times \mathbf{H_{os}}$$

$$= \eta_o \frac{\beta I_s}{4\pi}\frac{e^{-j\beta r}}{r} sin\theta cos\phi \boldsymbol{\theta}$$

$$\mathbf{P}(r,\theta,\phi) = \frac{1}{2} Re[\mathbf{E_s} \times \mathbf{H_s}]$$

$$= \frac{\beta^2 I_o^2 \eta_o}{32\pi^2 r^2} sin^2\theta cos^2\phi$$

$$R_{rad} = \oint \mathbf{P}(r,\theta,\phi) d\boldsymbol{S}$$

$$= \frac{\beta^2 I_o^2 \eta_o}{32\pi^2 r^2} \int\int\int \frac{1}{r^2} sin^2\theta cos^2\phi r^2 sin\theta d\theta d\phi dr \boldsymbol{r}$$

$$= \frac{\beta^2 I_o^2 \eta_o}{32\pi^2 r^2} \int_0^{\pi} sin^3\theta d\theta \int_0^{2\pi} cos^2\phi d\phi$$

$$= \frac{\beta^2 I_o^2 \eta_o}{32\pi^2 r^2} \frac{4}{3}(\pi)$$

$$\beta = \frac{2\pi}{\lambda}$$

$$P_{rad} = \frac{4\pi^2 I_o^2 \eta_o}{32\pi^2 \lambda^2} (\frac{4}{3})(\pi)$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

$$= 43.86\Omega$$

8.17

How long is a 1.5λ-long dipole antenna at 1.0 GHz? Suppose this antenna is constructed using AWG#20 (0.406mm radius) copper wire. Determine R_{diss}, e, G_{max}.

Length

$$\lambda = \frac{c}{f} = 0.3m$$

$$1.5\lambda = (0.3m)(1.5) = 0.45m$$

R_{diss}

$$\sigma = 5.8 \times 10^7, \mu_o = 4\pi \times 10^{-7}$$

$$\delta_{Cu} = \frac{1}{\sqrt{\pi f \mu_o \sigma}}$$

$$= 2.09 \times 10^{-6} m$$

$$S = 2\pi r \delta_{Cu}$$

$$= 5.33 \times 10^{-9}$$

$$R_{diss} = \frac{1}{\sigma}\frac{l}{S}$$

$$= 1.455\Omega$$

e

Using MATLAB: $F_{max} = 1.3661, \Omega_p = 8.0867$

$$R_{rad} = \frac{30}{\pi} F(\theta)_{max} \Omega_p$$

$$= \frac{30}{\pi}(1.3661)(8.0867)$$

$$= 105.493\Omega$$

$$e = \frac{R_{rad} R_{rad} + R_{diss}}{}$$

$$= 0.986 = 98.6\%$$

G_{max}

Using MATLAB: $D_{max} = 1.5540$

$$G_{max} = eD_{max}$$

$$G_{max} = 1.533$$

8.19

A 2.45GHz $\lambda/2$ dipole antenna is driven by a 2.0A amplitude current source. Find the maximum power density at a distance of 1.0 km.

$$P_{max} = \frac{15I_o^2}{\pi r^2}$$

$$= 0.0000191 = 19.1\frac{\mu W}{m^2}$$

8.21

MATLAB: Modify MATLAB 8.4 to calculate directivity and radiation resistance for an arbitrary length dipole antenna. Evaluate these properties for a 0.75λ dipole antenna.

Using MATLAB:

$$D_{max} = 1.8821$$

$$R_{rad} = 185.8086\Omega$$

Appendix

```
%Aaron Rosen - F&WII - 4/20/2022
% ProjIV - Final Project
%MATLAB 8.4
%Variables
%L          dipole length (in wavelengths)
%bL2        phase constant * length/2
%N          number of theta points
%th, thr    angle theta in degrees, radians
%dth        differential theta
%num, den   temporary variables
%F          un-normalized power function
%Fmax       max power function (W/m^2)
%omegaP     beam solid angle (sr)
%Dmax       Directivit

clc
clear

L = 0.75;
bL2 = pi*L;
N = 90;

i = 1:1:N;
dth = pi/N;
th(i) = i*pi/N;
num(i) = cos(bL2.*cos(th(i)))-cos(bL2);
den(i) = sin(th(i));
F(i) = ((num(i)).^2)./den(i);
Fmax = max(F);
Pn =F./Fmax;
omegaP = 2*pi*dth*sum(Pn)
Dmax = 4*pi/omegaP
Fmax

%Addition for 8.21
Rrad = (30/pi)*Fmax*omegaP
```