CENTER FOR RESEARCH IN HUMAN MOVEMENT VARIABLITY
NONLINEAR ANALYSIS CORE

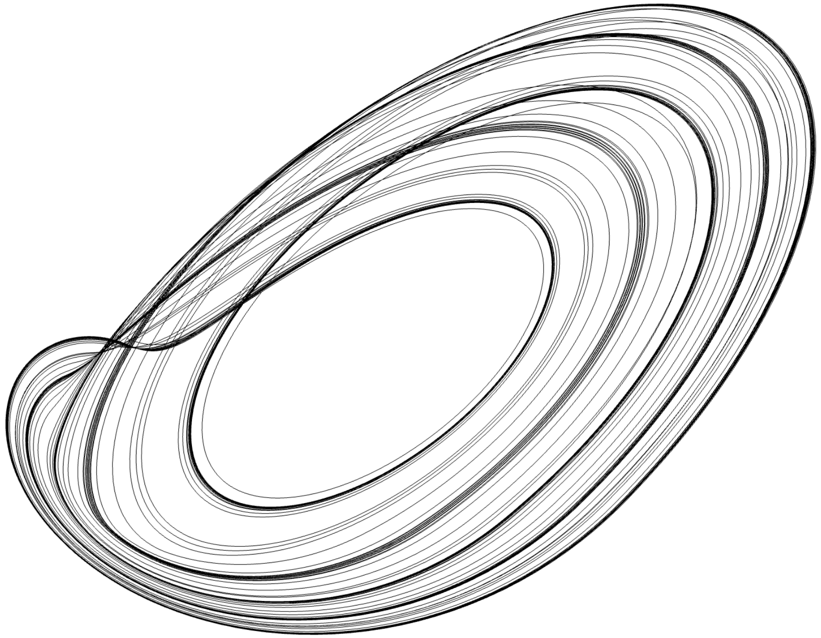
Recurrence Quantification Analysis

bmchnonan@unomaha.edu

University of Nebraska Omaha

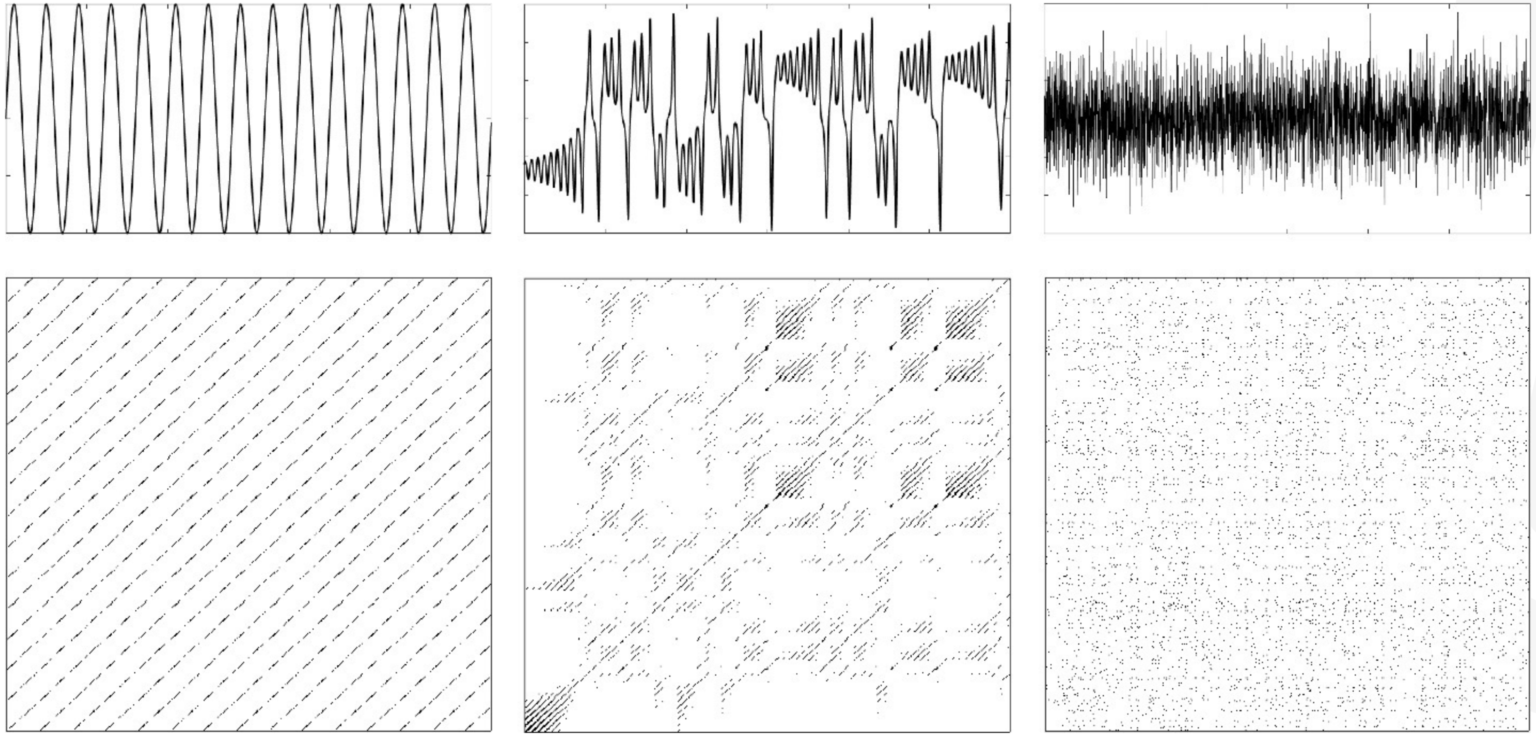
Phase Space Reconstruction

- What do we do if something is more than 3 dimensions?
- How can we visualize the system in higher dimensions?



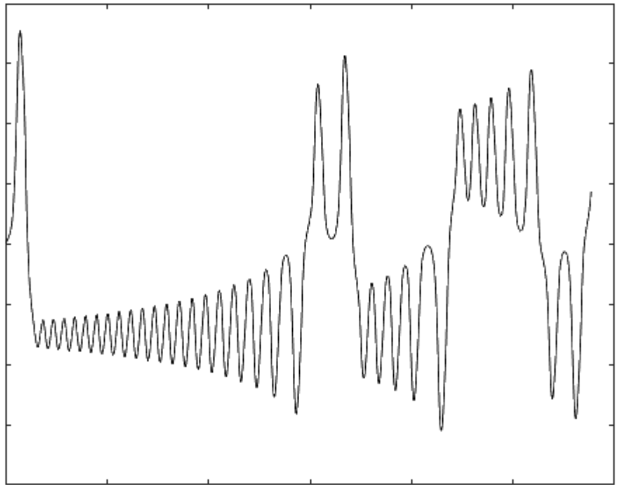
Recurrence Quantification Analysis (RQA)

1987: Jean Pierre Eckmann and colleagues sought to develop a visual tool for viewing the dynamics of higher dimensional systems



Eckmann et al., 1995

Recurrence Quantification Analysis (RQA)

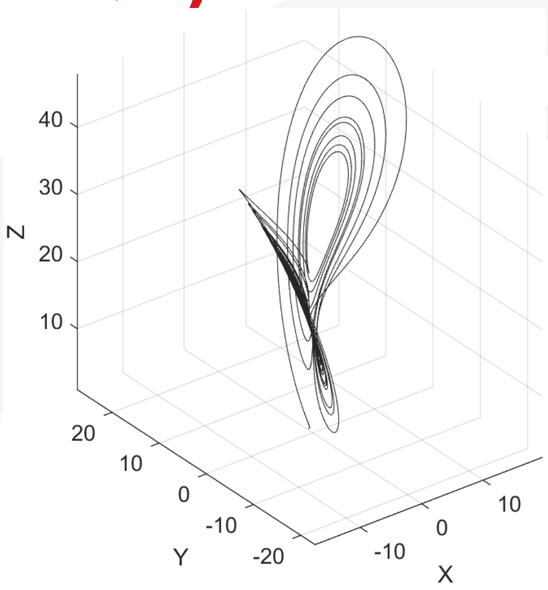


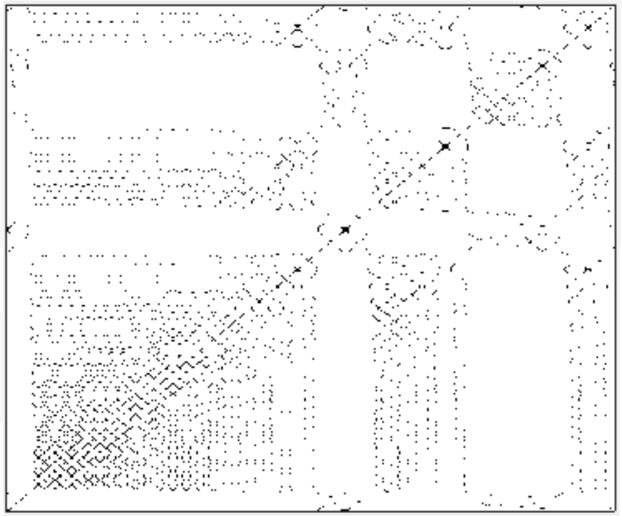
$$x = (x_1, x_2, x_3, \dots, x_n)$$

$$X_1 = (x_1, x_{1+\tau}, x_{1+2\tau}, \dots, x_{1+(D-1)\tau})$$

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_{n-(D-1)\tau} \end{pmatrix} = \begin{pmatrix} x_1 & x_{1+\tau} & \cdots & x_{1+(D-1)\tau} \\ x_2 & x_{2+\tau} & \cdots & x_{2+(D-1)\tau} \\ \vdots & \vdots & & \vdots \\ x_{n-(D-1)\tau} & x_{n-(D-2)\tau} & \cdots & x_n \end{pmatrix}$$

$$RP_{ij} = \Theta(T - (\|X_i - X_j\|)$$





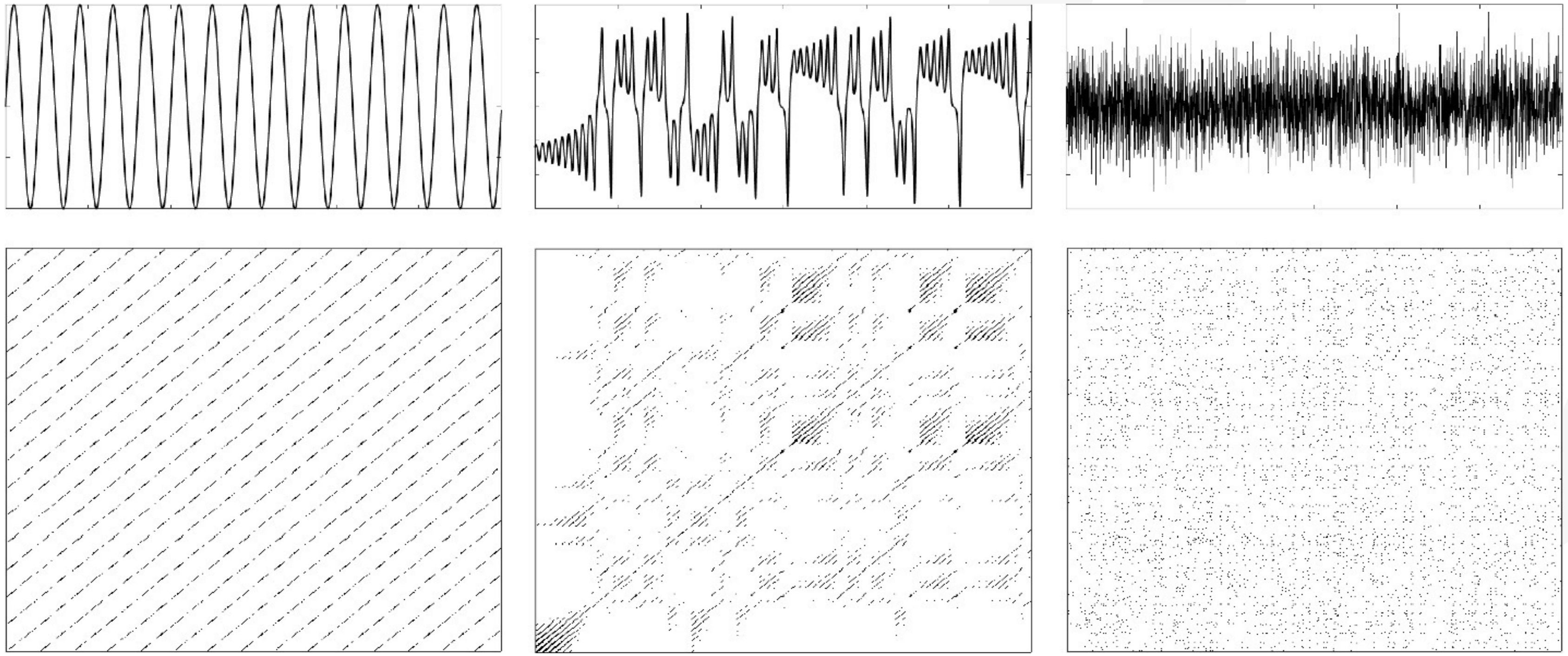
What is RQA?

- RQA is a nonlinear data analysis technique utilized for investigating the behavior of dynamical systems (i.e., irregular and nonstationary)
- It comprises a method to visualize systems of any dimension
- It focuses on the repetition of states within a system over time
- It is easily generalizable to bivariate and multivariate time series

Riley et al., 1999
Wallot et al., 2017

Recurrence Plot (RP)

The plot enables investigation into the *m*-dimensional phase space trajectory through a 2D representation of its recurrences



Riley et al., 2001

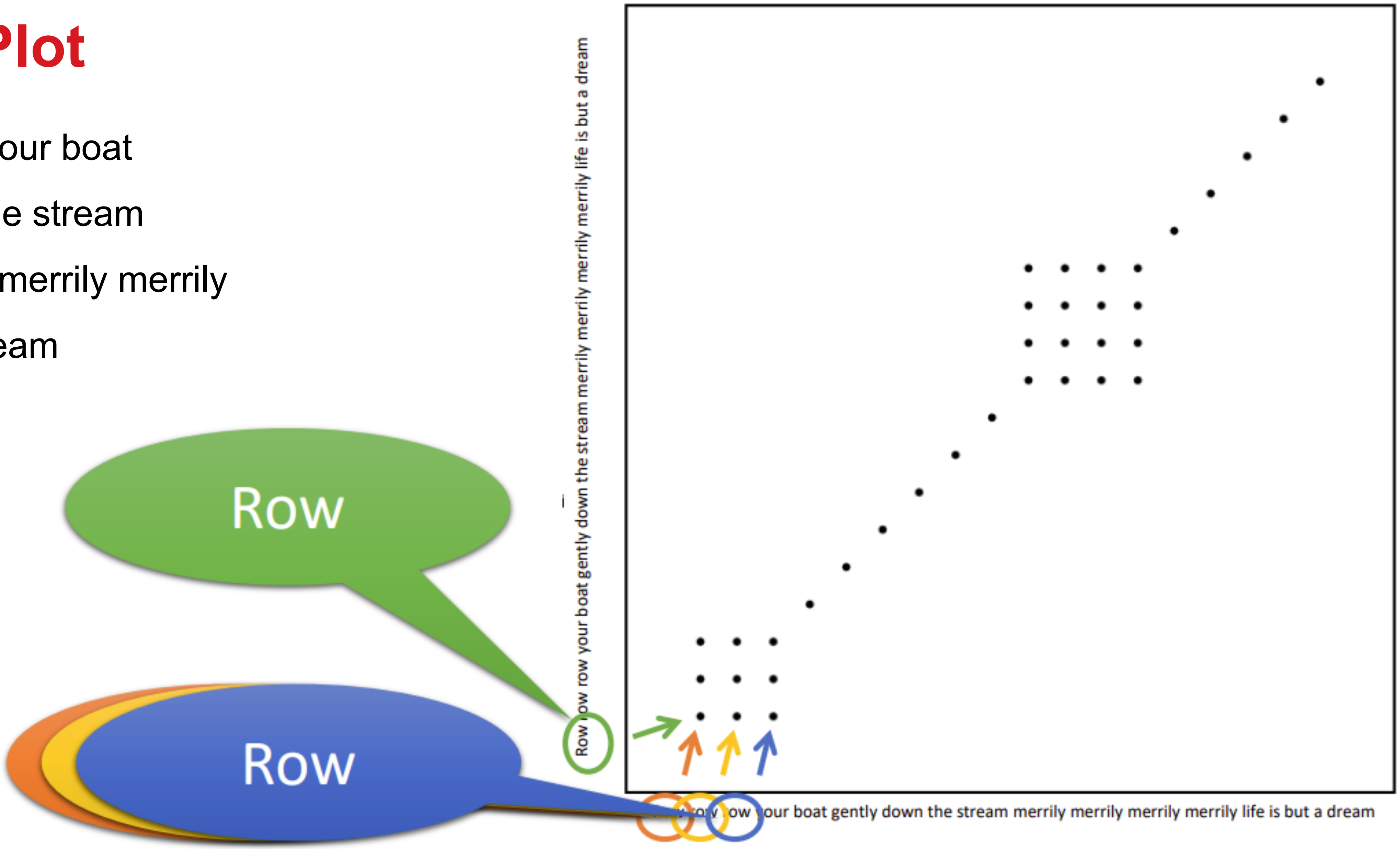
Example Plot

Row row row your boat

Gently down the stream

Merrily merrily merrily merrily

Life is but a dream



Example Plot

Rowrowrowyourboat

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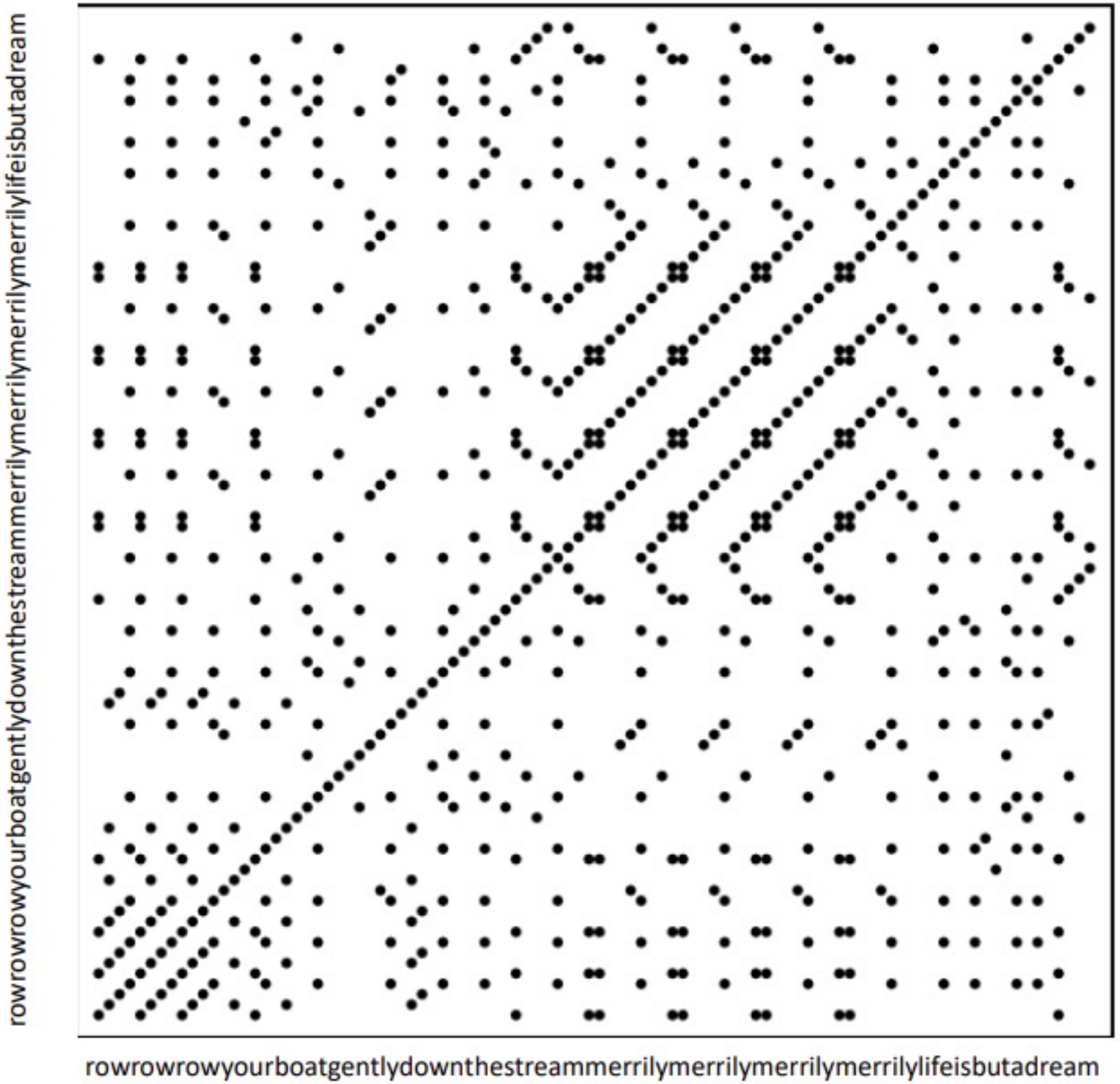
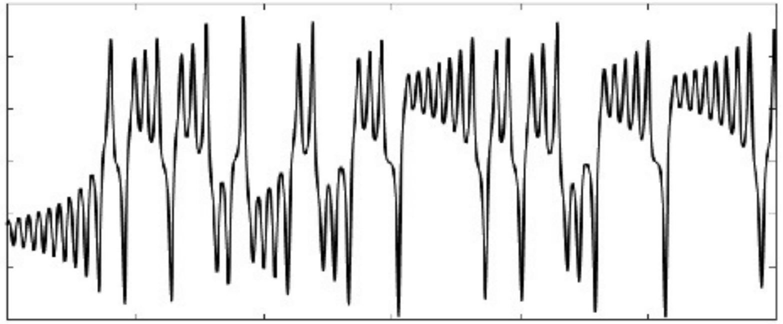
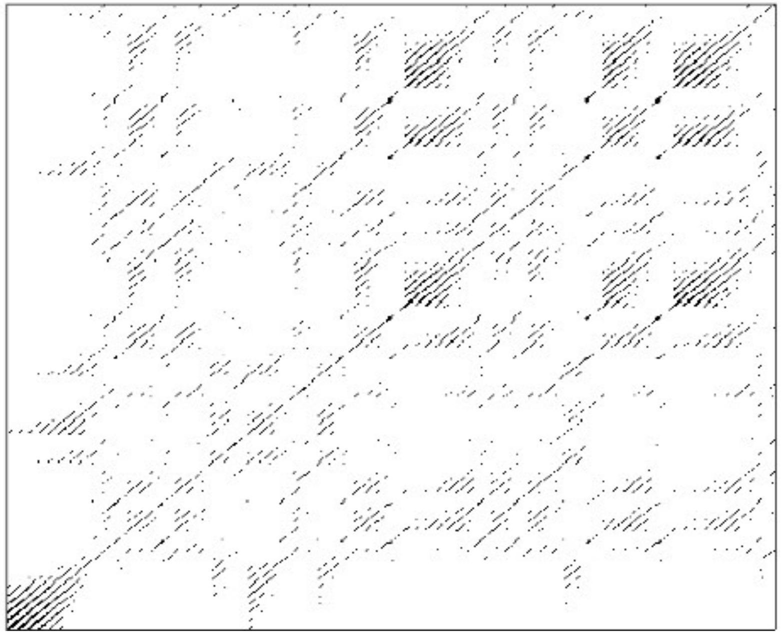
Lifeisbutadream



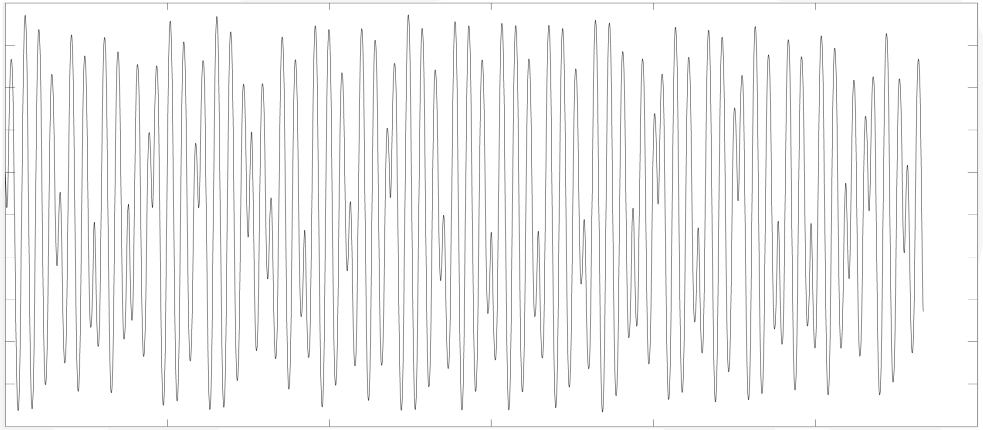
Recurrence Plots of Common Dynamical Systems

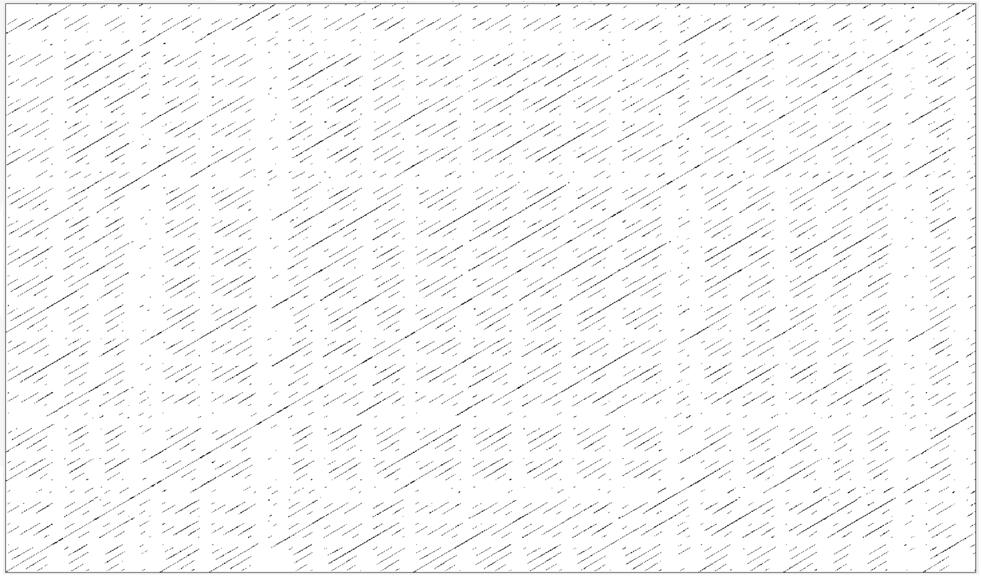




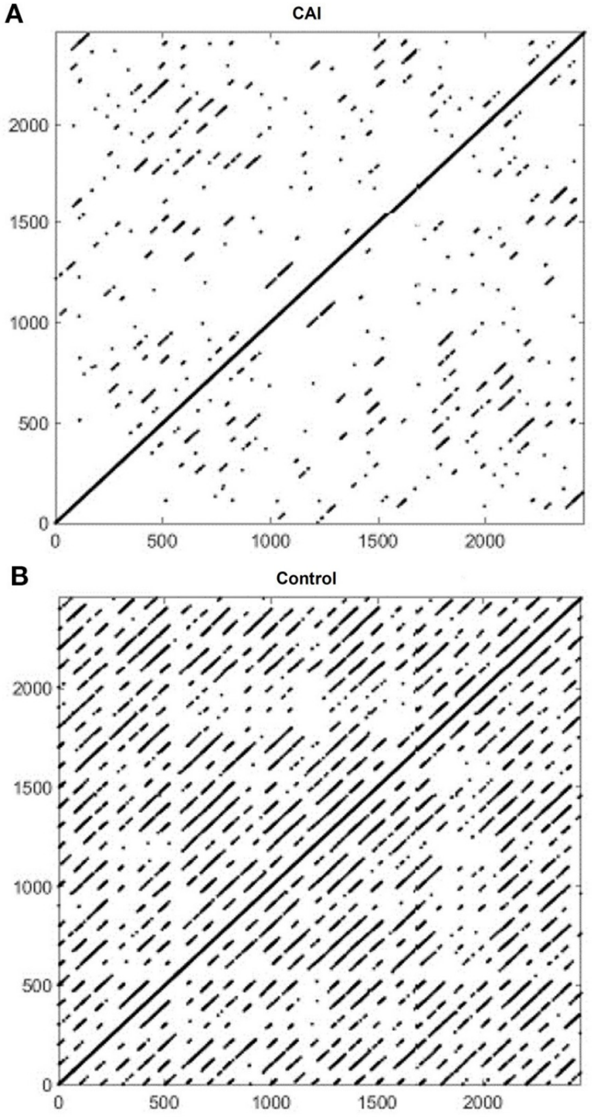
Lorenz

Duffing Van Der Pol





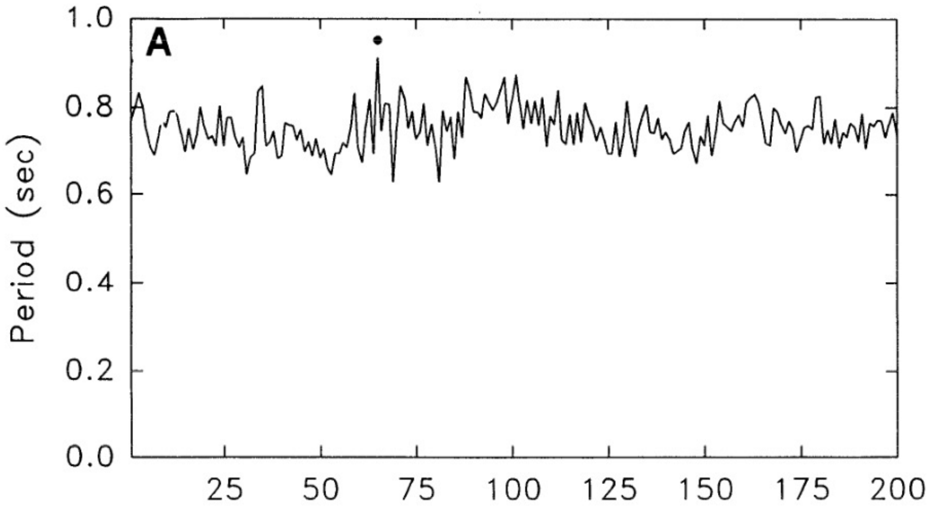
Recurrence Plots of Biological Data

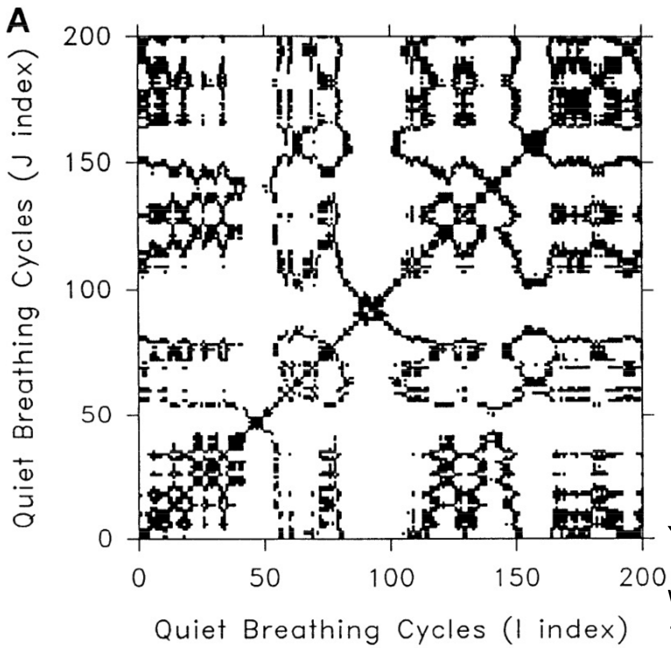


Quiet Breathing

Ankle Kinematics

Recurrence Plots





Yen et al. 2022

Webber et al. 1994

Building a Recurrence Plot

- Step 1:Time delay via *average mutual information (AMI)*
- Step 2: Embedding Dimension via *false nearest neighbors (FNN)*
- Step 3: Radius (i.e., threshold parameter to determine what points are recurrent)

Webber et al., 1994

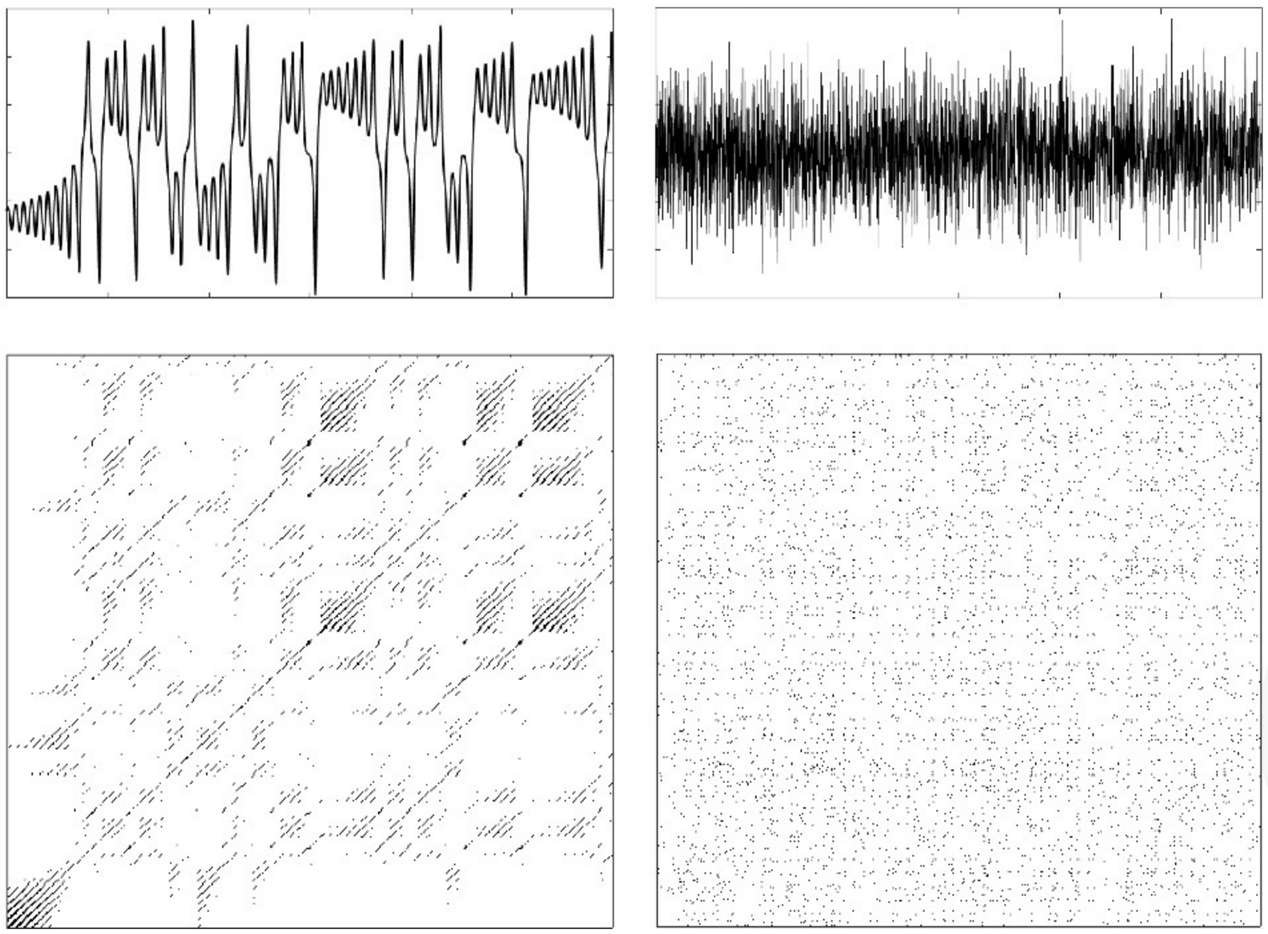
Choosing a proper radius

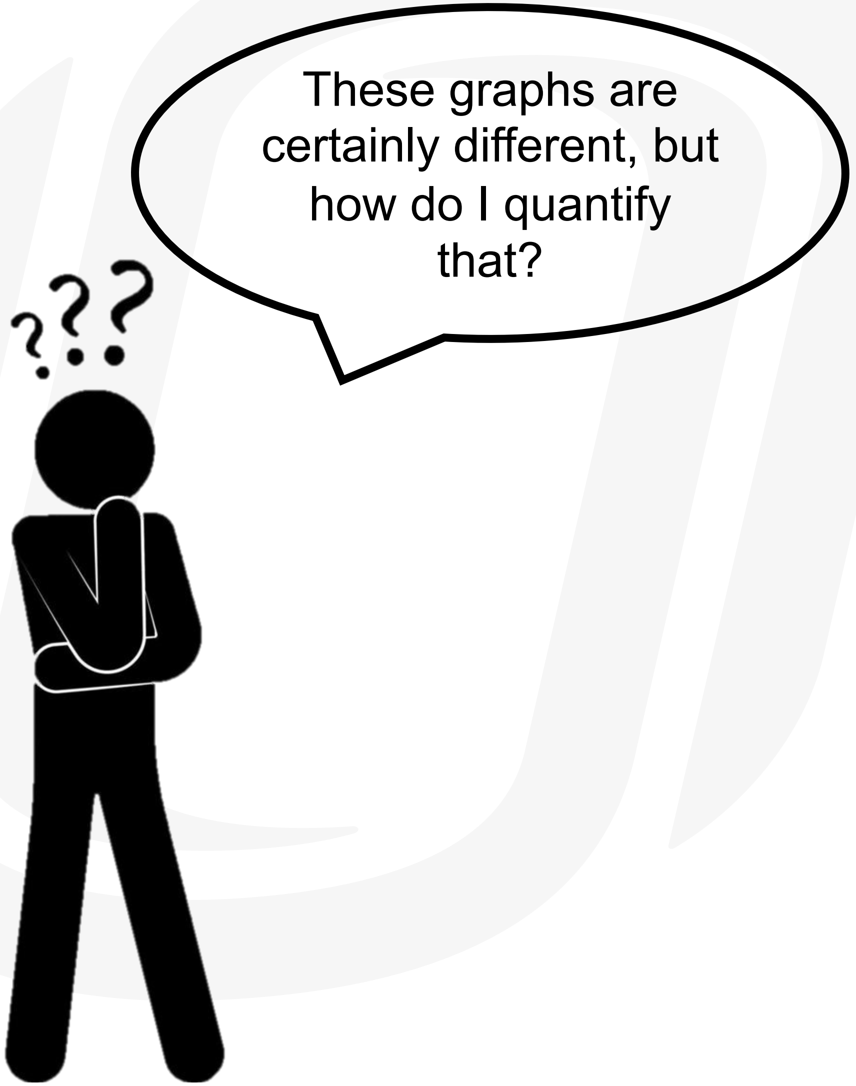
Several "Rules of Thumb" have been suggested in movement science:

- Choose a radius to achieve a % REC of 1-5%, 5-10%, or 2.5%
- Use a radius of 10% of the maximum diameter of the phase space
- Select a predefined radius of 1/16 or 0.2 of the phase space
- Set a radius of 40% of the maximum diameter of the phase space
- Use a radius equal to 10% of the mean distance between data points in the reconstructed space

- Most typical is choosing a radius to achieve a %REC of 2.5%

Recurrence Plots Provide More Information!

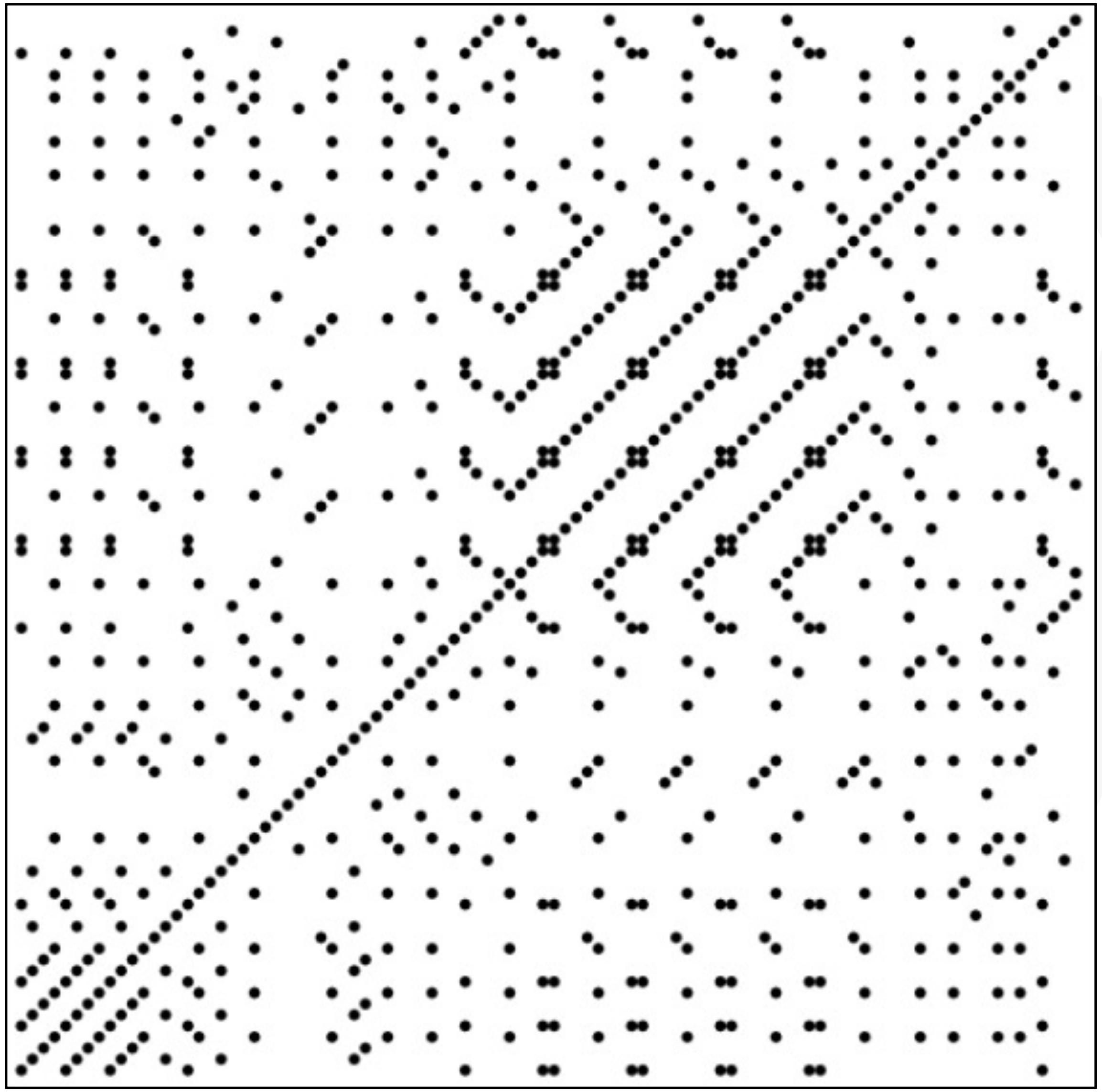




RP Metrics

- **Percent Recurrence (%REC):** percentage of the plot occupied by points
- **Percent Determinism (%DET):** percentage of points that fall on a diagonal line

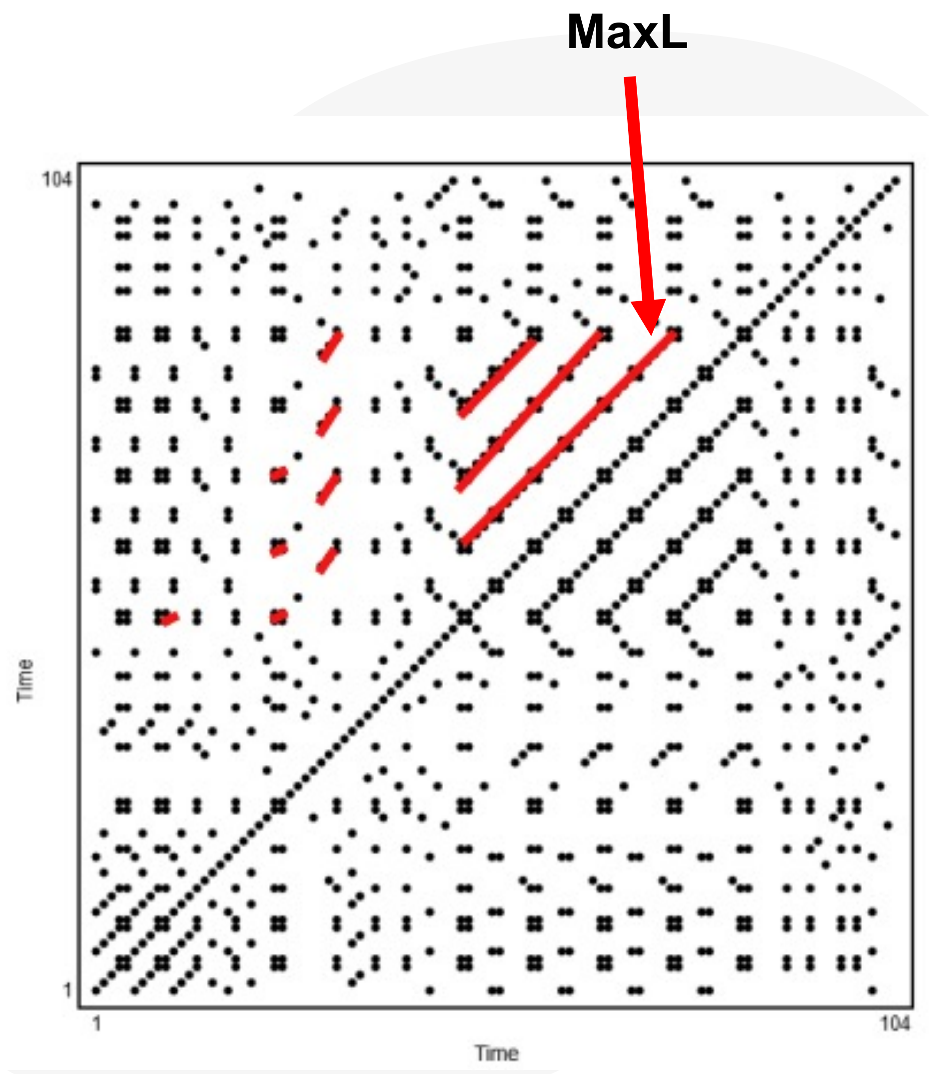
Row Row Row
Your Boat



RP Metrics

- **Average Line Length (MeanL):** average length of time the system repeats itself
- **Maximum Line Length (MaxL):** the longest diagonal line; not including the main diagonal (i.e., line of identity)
- **Entropy (EntrL):** Shannon entropy based on the distribution of the line lengths, not the time series.

Row Row Row
Your Boat

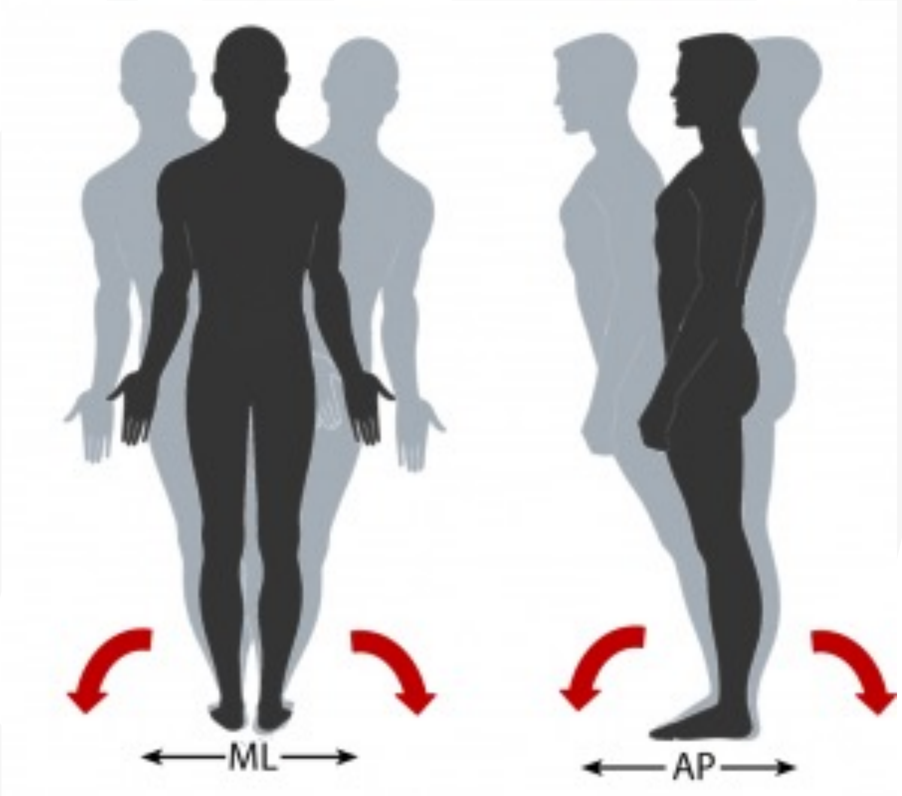


Analyzing RQA Metrics

Names and definitions of four RQA measures of temporal structure.		
Variable Name	**Definition**	**Interpretation**
Percent Recurrence (%REC)	Sum of recurrence points in RP/size of RP.	Repetitiveness of the trajectories in the time series.
Percent Determinism (%DET)	Sum of diagonally adjacent recurrent points/sum of recurrent points in RP.	How many of the individual repetitions occur in connected trajectories. The larger the %DET, the more predictable (i.e., deterministic).
Average Diagonal Line Length (MeanL)	Average diagonal lines in RP.	How long the average repeating trajectory is. The longer the line, the more deterministic the pattern is.
Maximum Diagonal Line Length (MaxL)	Length of longest diagonal line in RP.	How long the longest repeating trajectory is. Periodic signals (sine waves) will give very long diagonal lines.
Entropy (EntrL)	Shannon information entropy of the distribution of all diagonal line lengths.	A measure of signal complexity. Lower entropy indicates deterministic behavior is present.
Percent Laminarity %(LAM)	Sum of vertically adjacent recurrent points/sum of recurrent points in RP.	How constant the system appears over time. High values correspond to slow predictable changes.
Average Vertical Line Length (MeanV)	The average length of vertical lines.	How long the system state does not change or changes slowly over time. A higher value suggests the system remains in specific state for a longer period of time.
Maximum Vertical Line Length (MaxV)	Length of longest vertical line in RP.	How long the system state does not change. A higher value indicates greater stability.
Entropy (EntrV)	Shannon information entropy of the distribution of all vertical line lengths.	A measure of signal complexity. Higher entropy indicates chaotic behavior is present.

Examples of RQA in Human Movement Variability

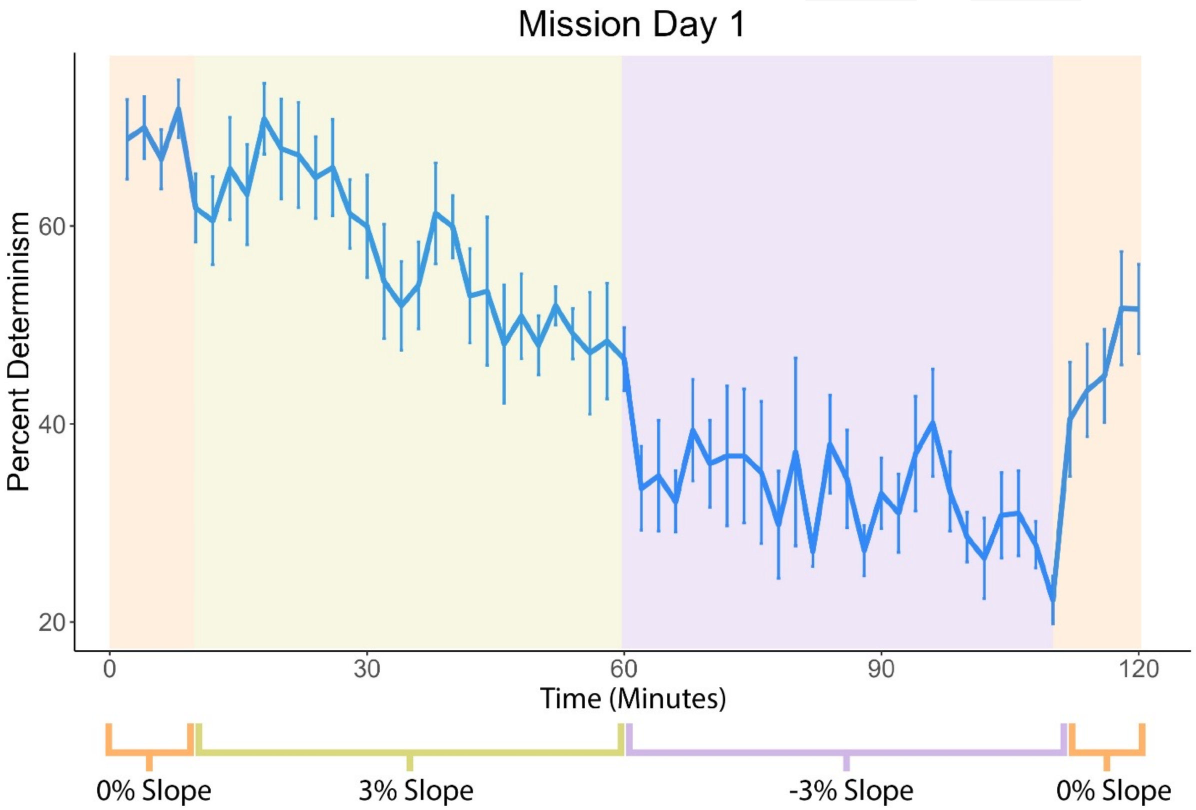
- Center of pressure displacement (COP)
  - Analyses have revealed that COP fluctuations are a blend of deterministic and random dynamics
  - RQA measures from COP tend to increase (i.e., become more stable) in the AP direction, as sensory input became unavailable or altered
  - The temporal structure of postural sway provides a window into the functional organization of the postural control system



Riley et al., 2003

Examples of RQA in Human Activity Monitoring

- Muscular fatigue detection
  - It is possible to detect neuromuscular fatigue based on changes in EMG signals from loaded biceps (Webber et al., 1994)



O2.4.6: A novel biomarker for detecting fatigue in soldiers during loaded walks

Webber et al., 1994

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Cross Recurrence Quantification Analysis

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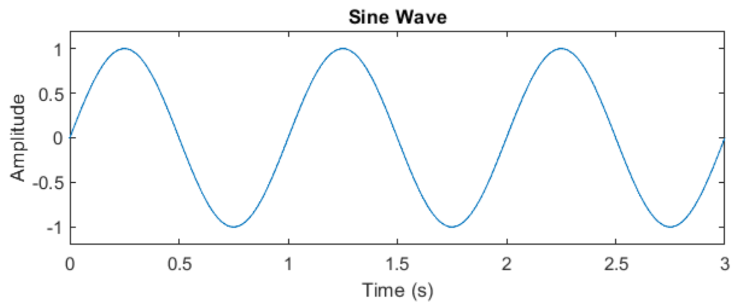
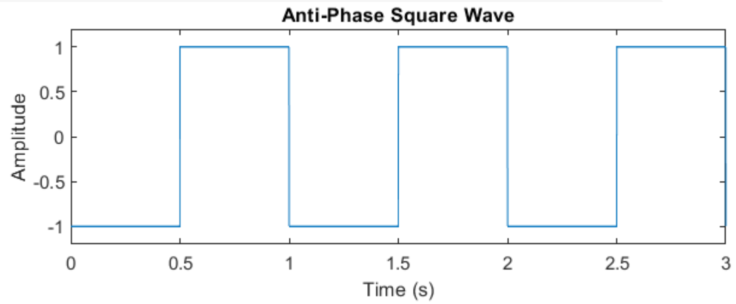
Cross Recurrence Quantification Analysis (CRQA)

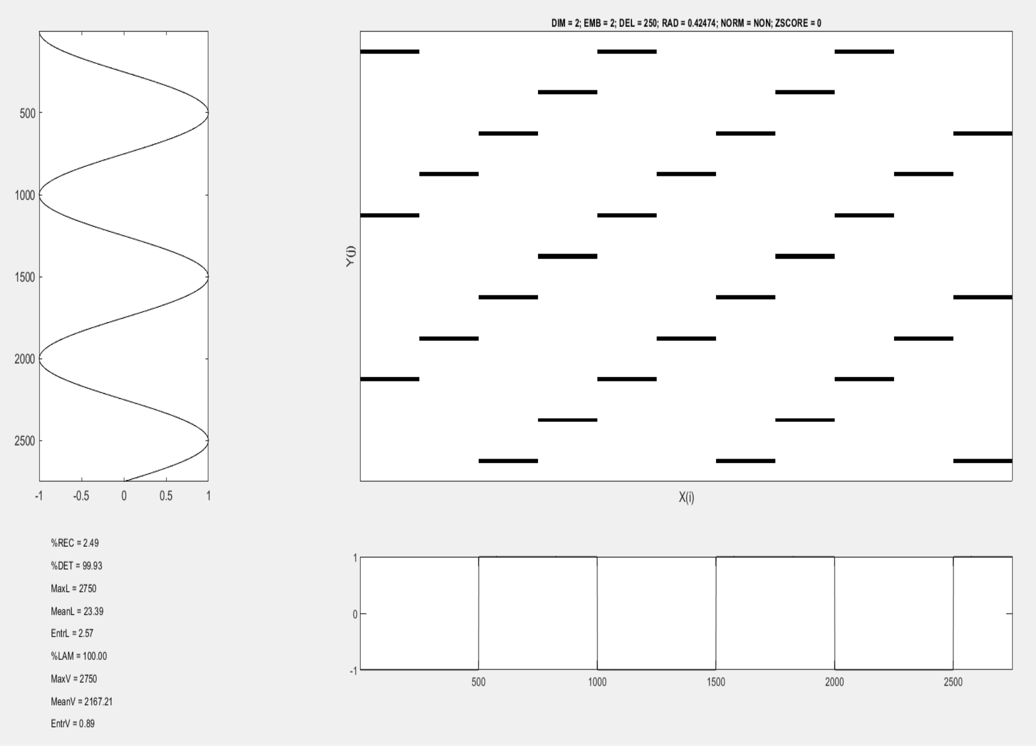
- Introduced by Zbilut, Giuliani, and Webber in 1998
- Deals with cross recurrences between two series
- Calculates the distance between points in one series with points in another series

$$CR_{i,j}^{x,y} = \theta\left(T - \left\|X_i - Y_j\right\|\right),$$

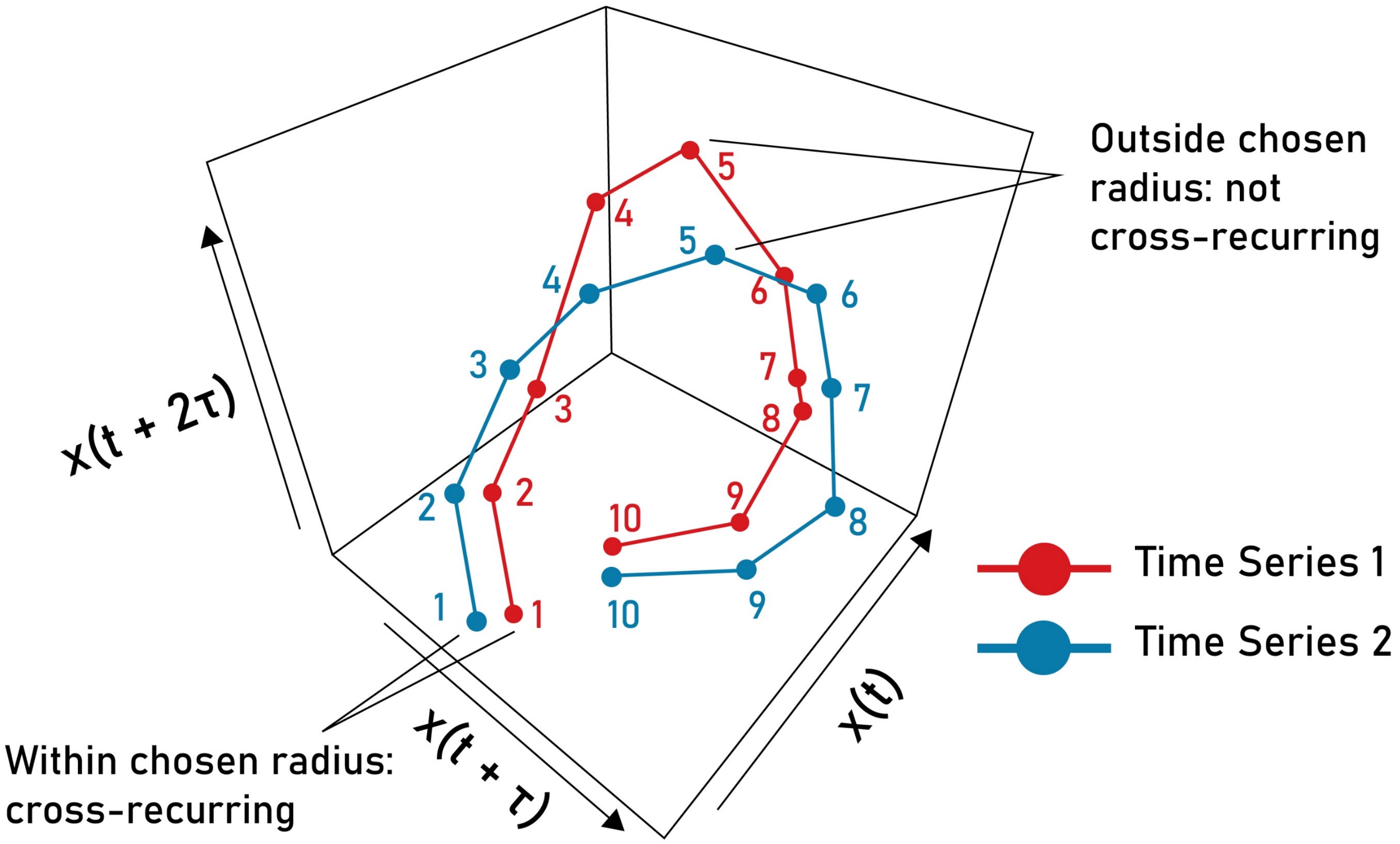
$$i = 1, \ldots N, j = 1, \ldots, M$$

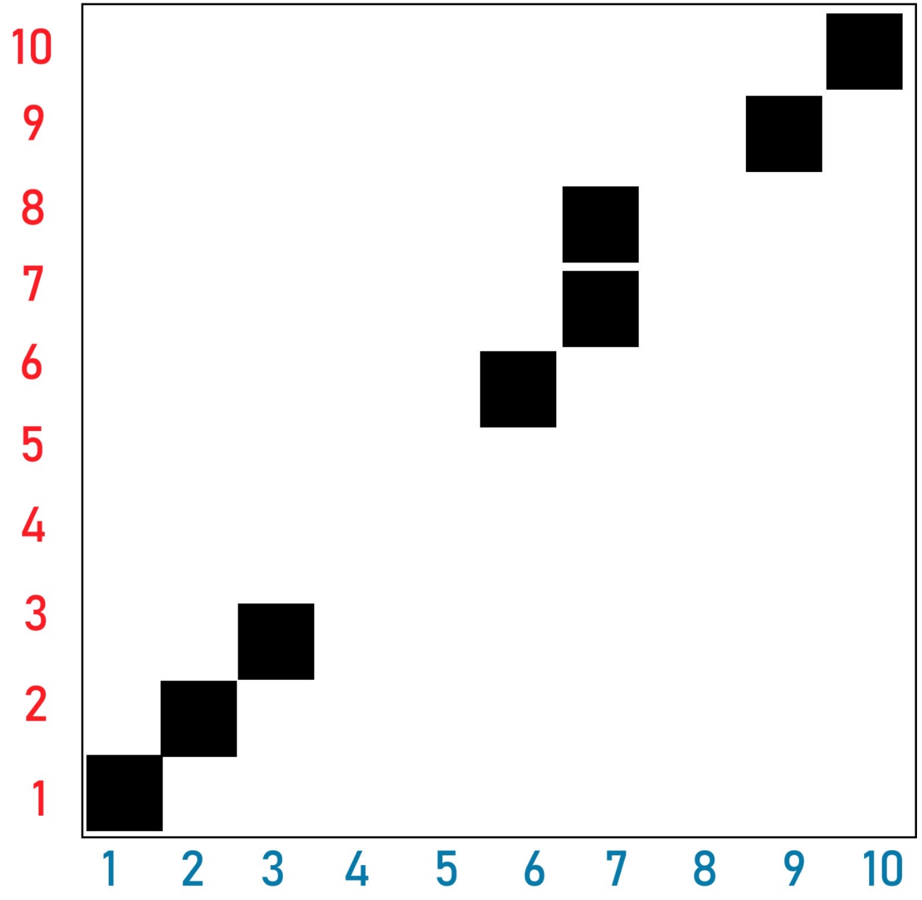






Cross Recurrence Quantification Analysis (CRQA)

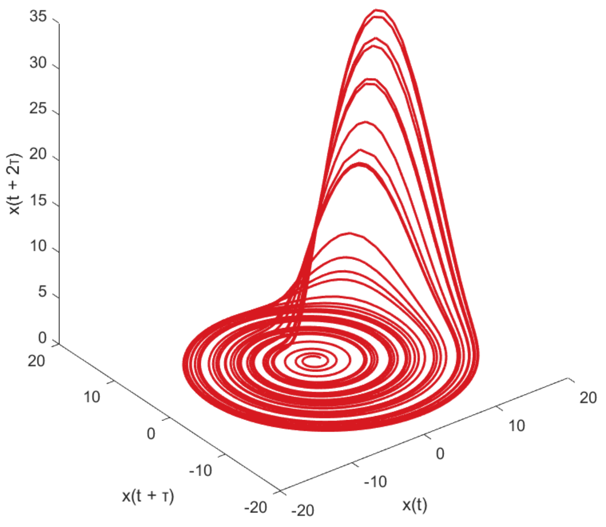




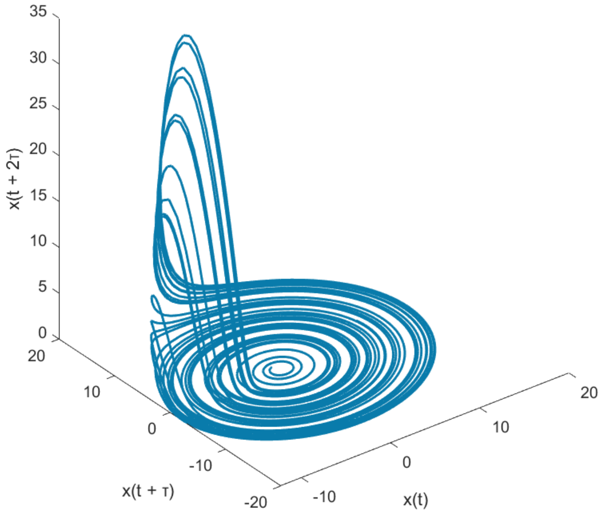
Another Example of CRQA

Coupled Rossler Systems

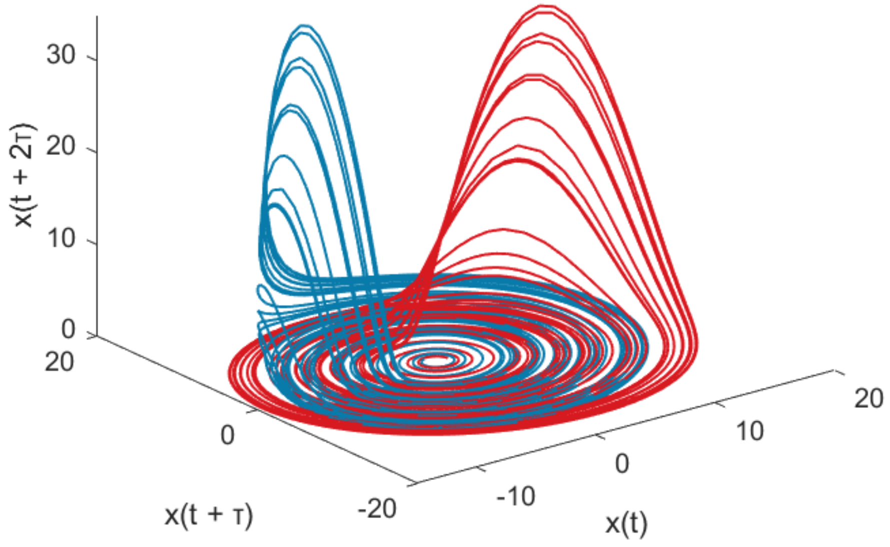
Rossler System



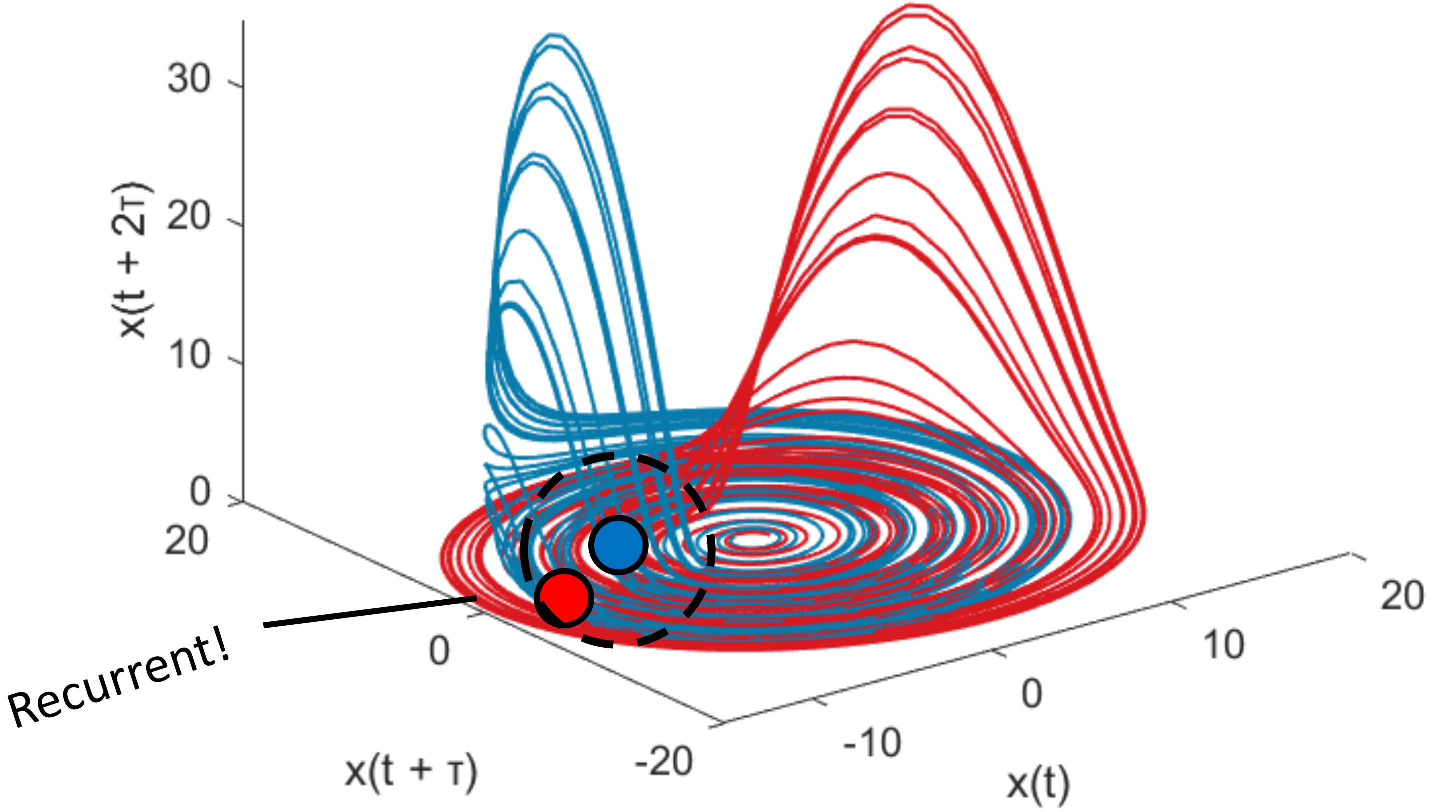
Rotated Rossler System



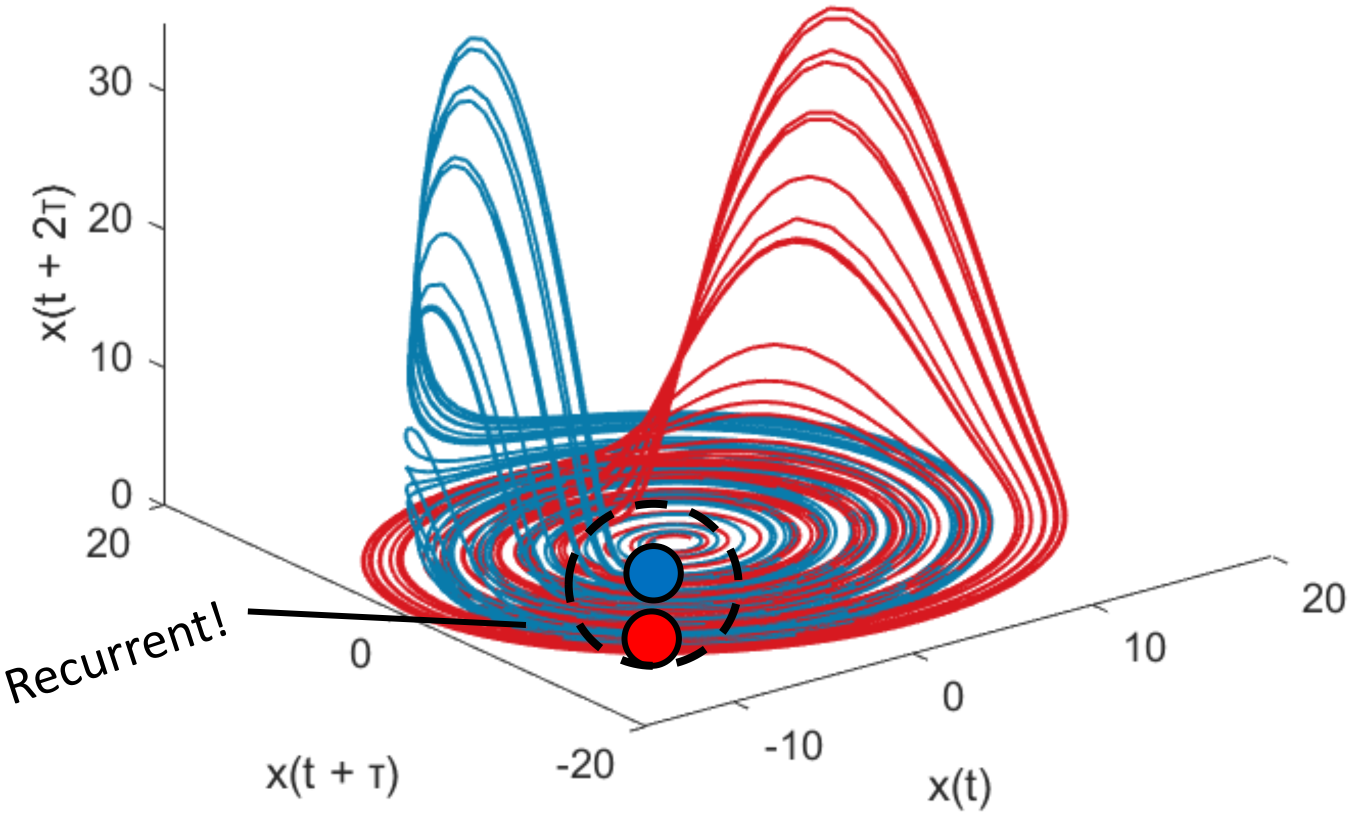
Shared State Space of Rossler Systems



CRQA: Looks at when recurrences occur between the systems in the phase space



CRQA: Looks at when recurrences occur between the systems in the phase space



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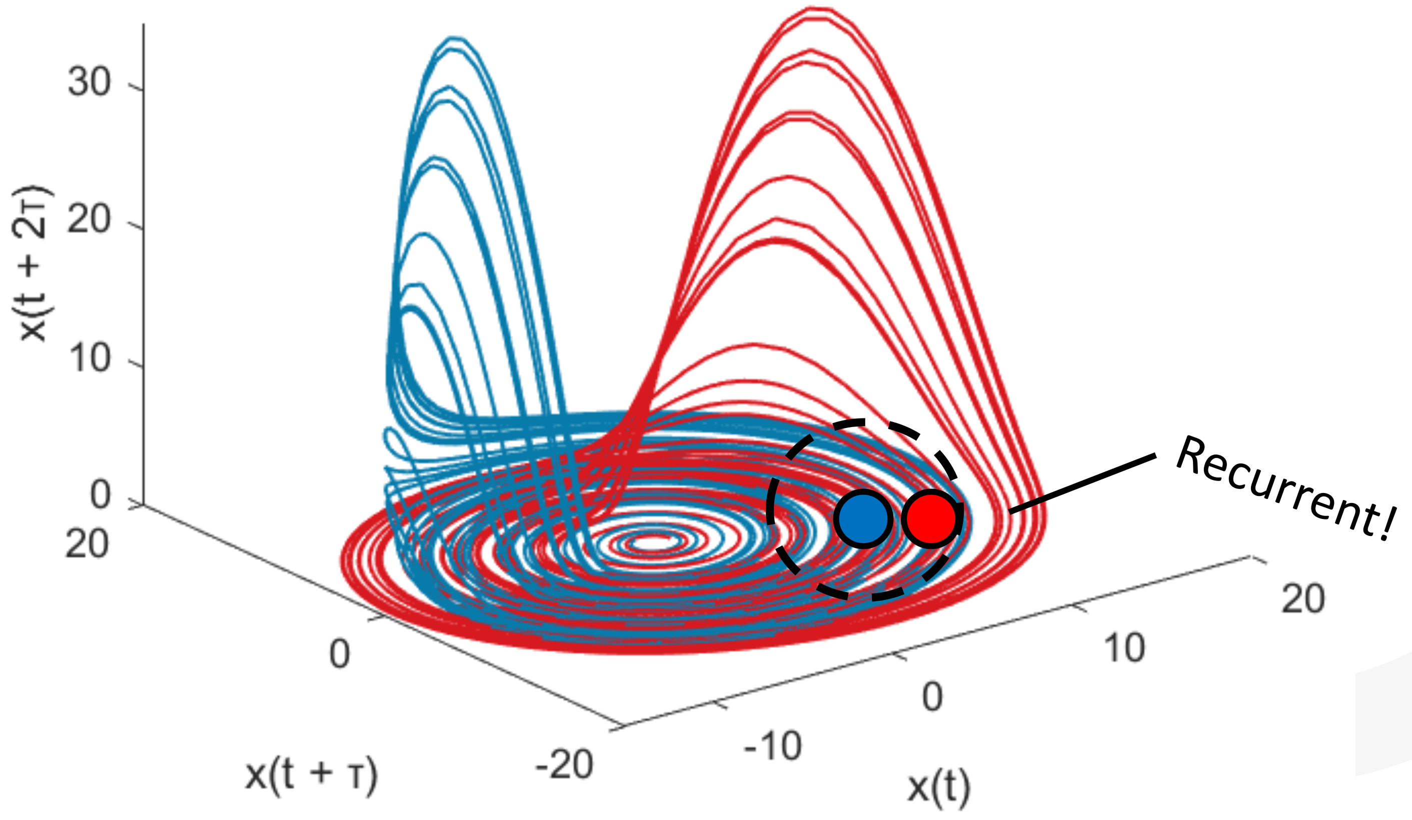
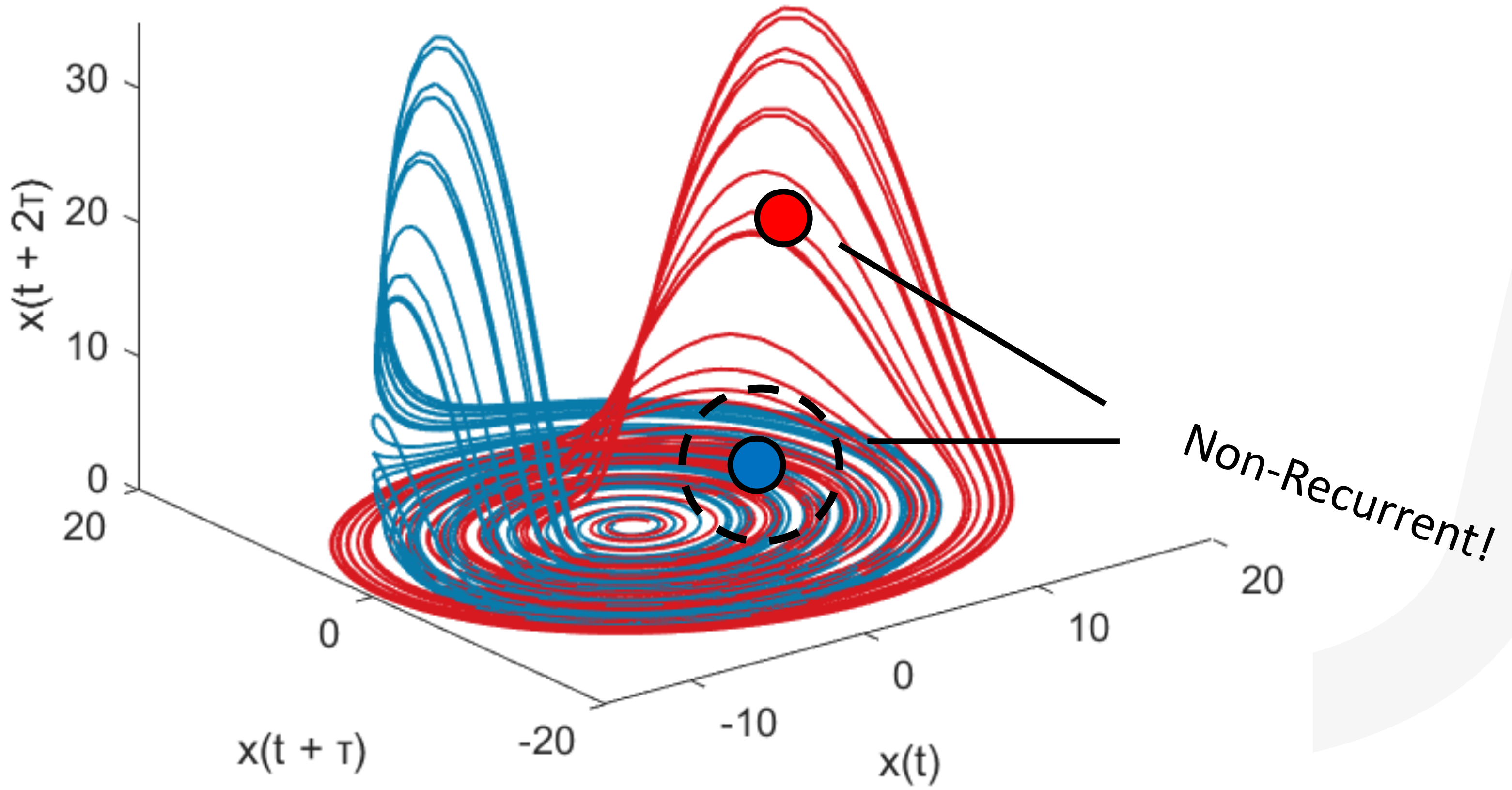
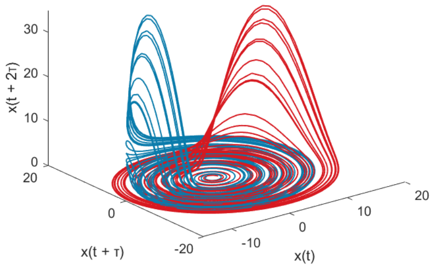
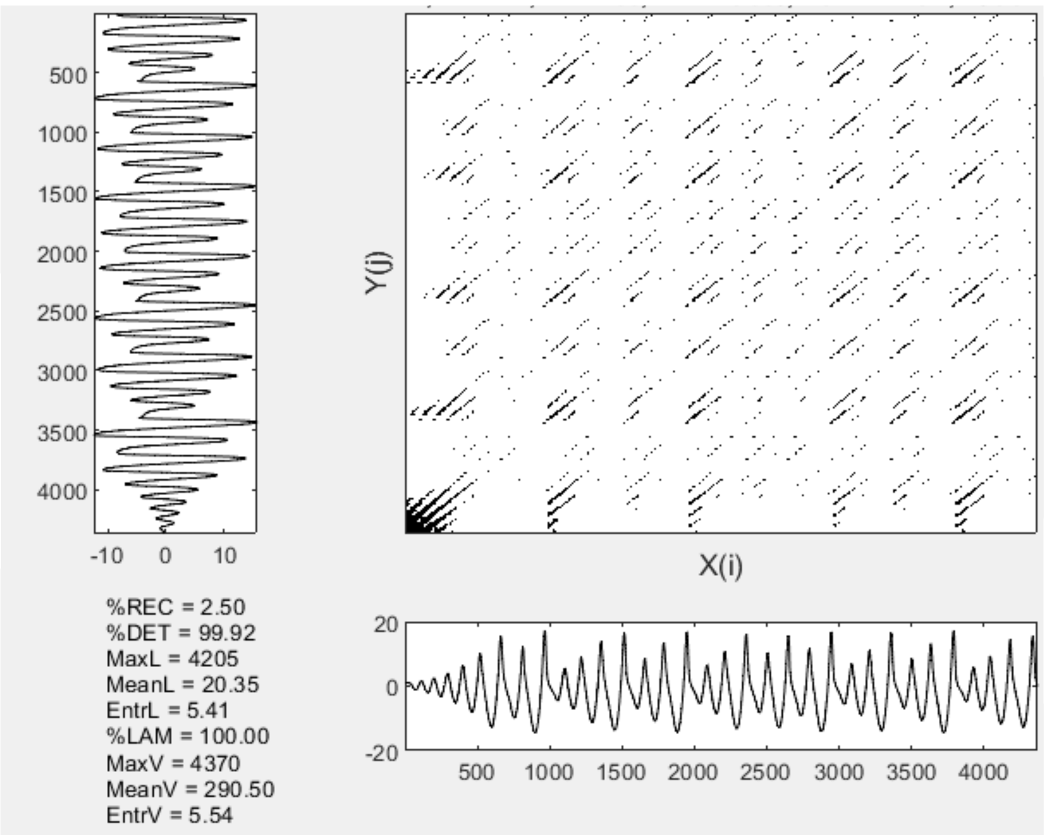


CRQA: Looks at when recurrences occur between the systems in the phase space



Cross Recurrence Quantification Analysis (CRQA)





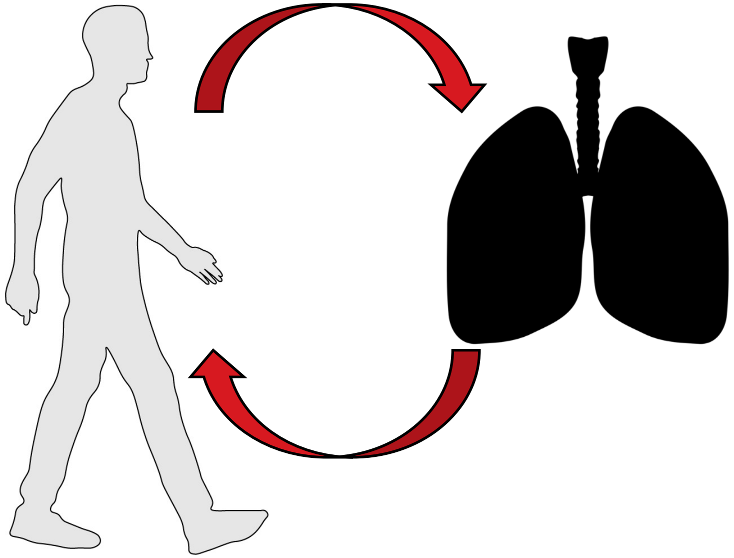
- Compares recurrence patterns over different time periods
- Useful for studying the coordination between two systems over time

Parameter Selection

- For cRQA calculation involving two time series, both series must be embedded in a common phase space
- Selection of τ and *d* practices:
  - Option 1: Use the average of time lags and maximum value of embedding dimensions from both series as inputs for cRQA calculation
  - Option 2: Use average values (rounded to the next integer) for each parameter
  - **Option 3: Use the highest values for each parameter**

A Few Examples of CRQA in Movement Science

Locomotor and Respiratory Coupling

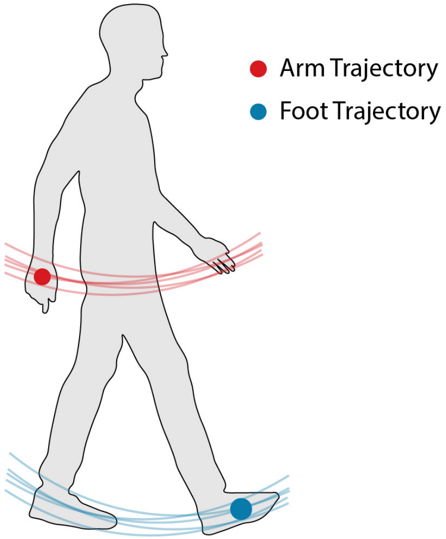


- Older adults exhibited less locomotor-respiratory coupling compared to younger adults
- cRQA entropy is able to discriminate between healthy subjects and COPD patients
- Increased energy expenditure was associated with stronger and less complex coupling between gait and respiration

Denton et al., 2017
McCamley et al., 2017
Goldberger et al., 2019

A Few Examples of CRQA in Movement Science

Inter/Intra-Limb Coordination



Interlimb Coordination:

- Observing the dynamical coordination between the thumb and index finger during precision grip tasks
- The coupling of different joints within the leg

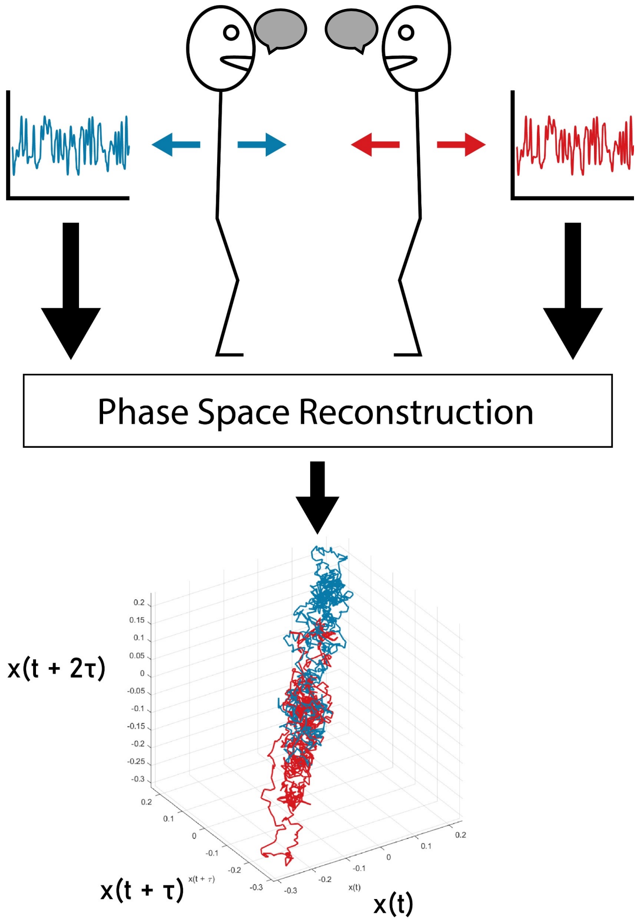
Intralimb Coordination:

- How the coupling between legs changes during gait
- The coupling between upper and lower limbs during gait
- The coupling between both hands during coordinated pendulum swings

Sado et al., 2022
Stephenson et al., 2009
Richardson et al., 2007
Bonnette et al., 2023

A Few Examples of CRQA in Movement Science

Interpersonal Postural Dynamics



- Pairs of participants exhibited higher inter-personal postural recurrence when engaged in conversation to solve a puzzle task together compared to conversations with others
- The observations provide objective measures of interpersonal postural coordination
- CRQA was found to be a more sensitive and appropriate method for quantifying shared postural activity compared to linear methods

Shockley (2005)

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Joint Recurrence Quantification Analysis

bmchnonan@unomaha.edu

University of Nebraska Omaha

Joint Recurrence Quantification Analysis (JRQA)

- A **Joint Recurrence Plot (JRP)** is a graphical representation that illustrates the simultaneous occurrences of recurrences in two **or more** dynamical systems
- This approach allows for the preservation of each system's respective phase space
- Is a valuable tool for detecting coordination phenomena and determining the strength of coupling between distinct systems
- **Joint Recurrence Quantification Analysis (JRQA)** quantifies the tendency of systems to have recurrences at corresponding positions in their respective recurrence matrices

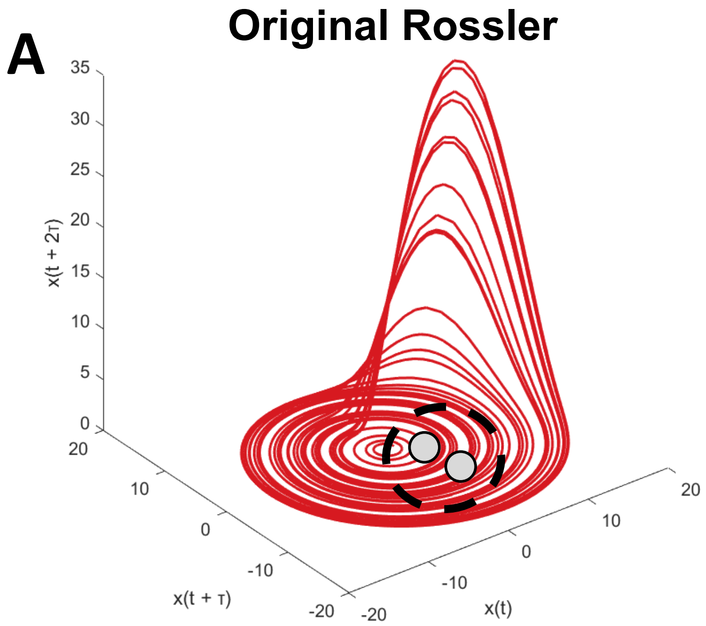
Joint Recurrence Quantification Analysis (JRQA)

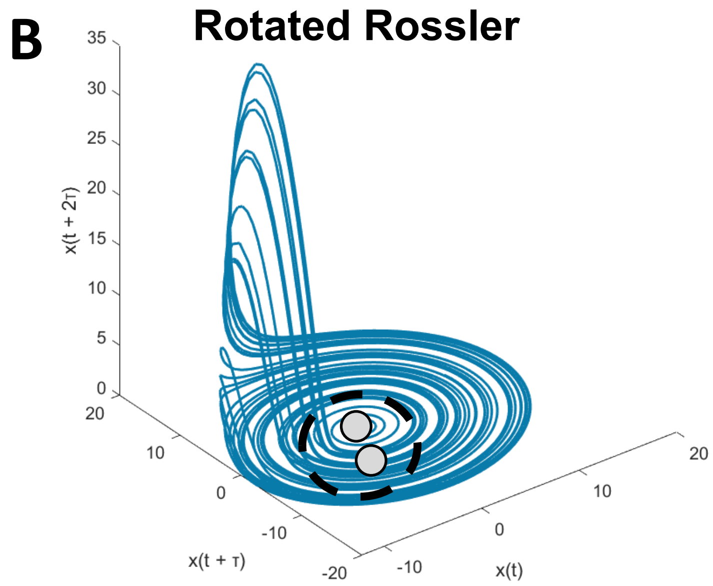
- The joint recurrence matrix for two systems x and y is then the element wise product of the single RPs

$$JR_{i,j}^{x,y} = \theta\left(T^X - \left\|X_i - X_j\right\|\right)\theta\left(T^Y - \left\|Y_i - Y_j\right\|\right),$$

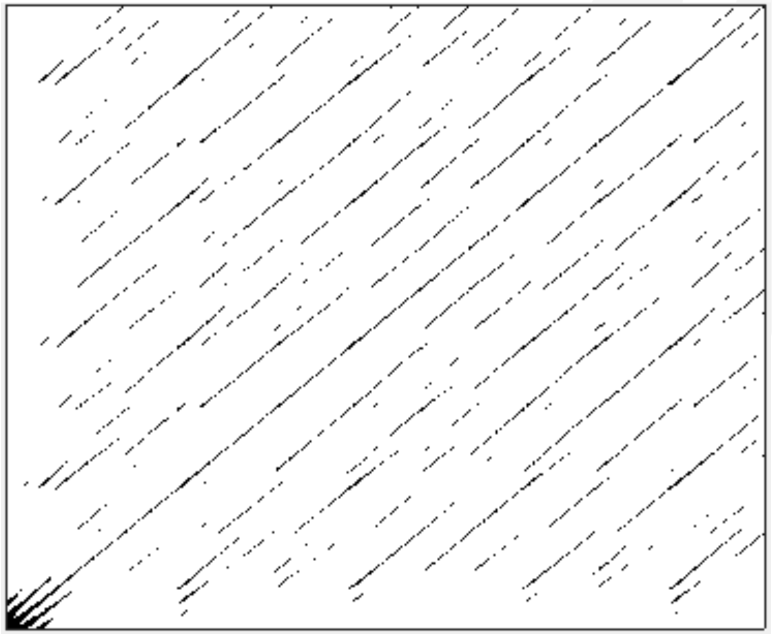
$$i,j = 1, \ldots, N$$

System X Recurrence Matrix		System Y Recurrence Matrix		Joint Recurrence Matrix (Hadamard Product)
$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$	$\bullet$	$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$	$=$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$



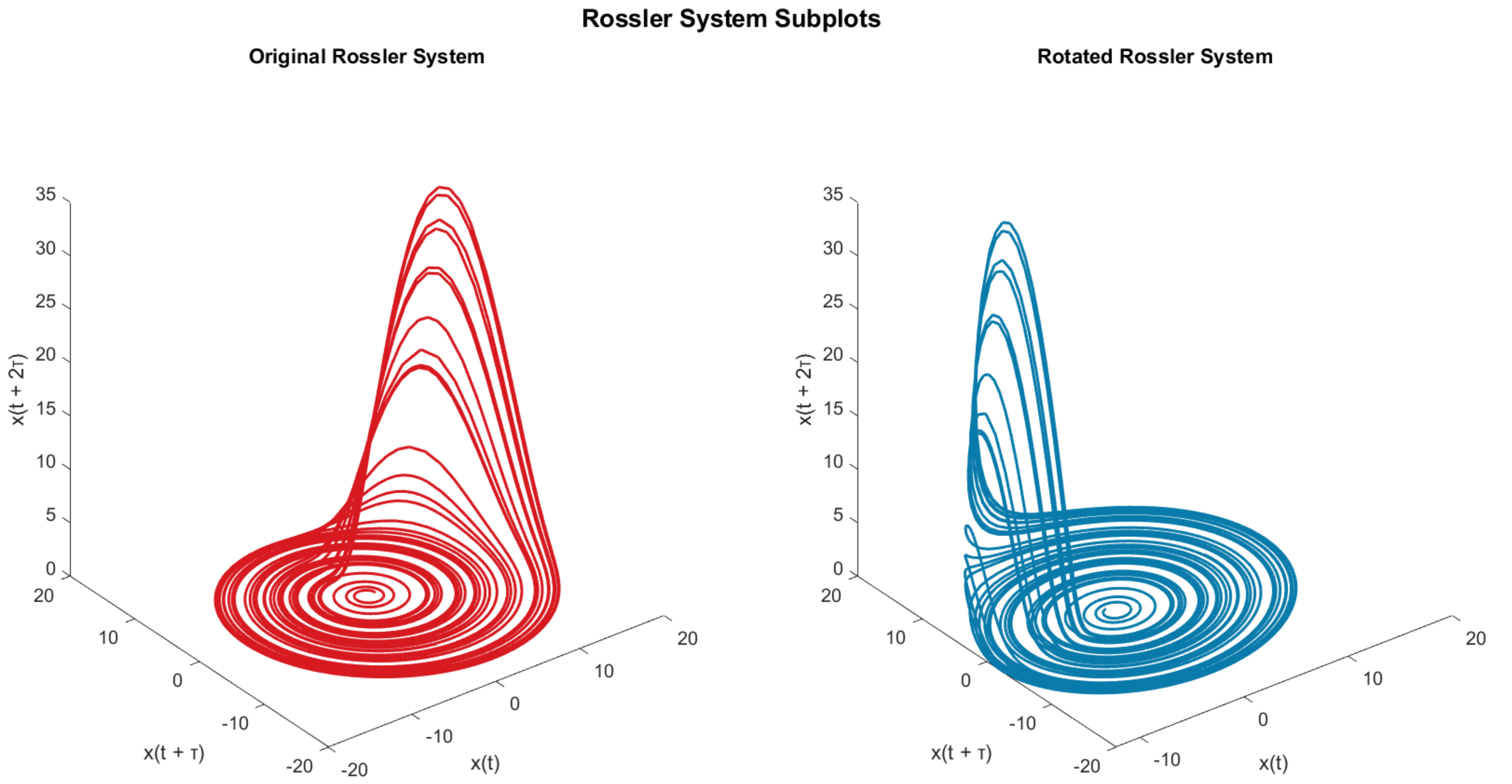


Joint Recurrence Plot



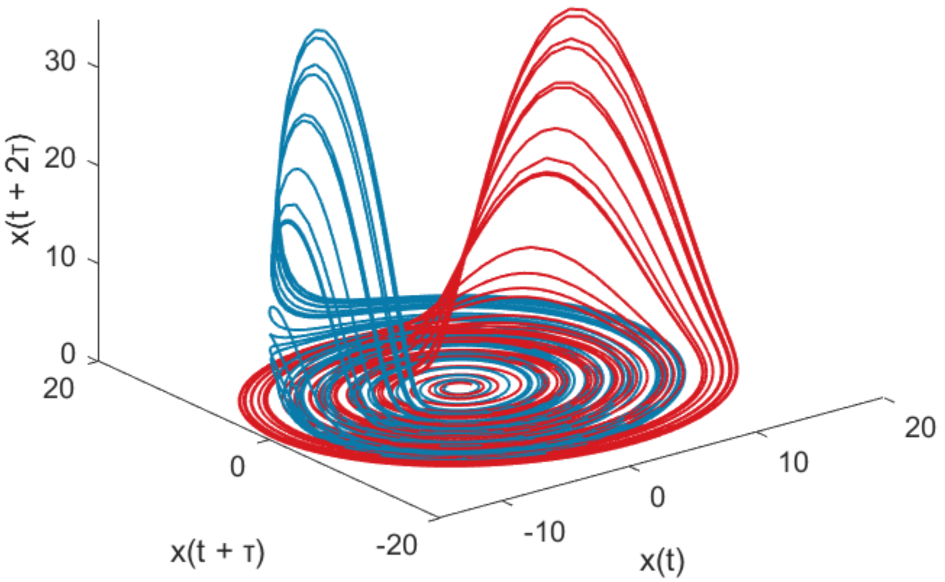
- **If** two phase space vectors of the second Rössler system at time *i* and *j* are neighbors [two dots in (B)]…
- **And if** two phase space vectors of the first Rössler system at same time *i* and *j* are also neighbors [two dots in (A)]…
- **Then** a black point in the JRP at the location *(i, j)* will occur

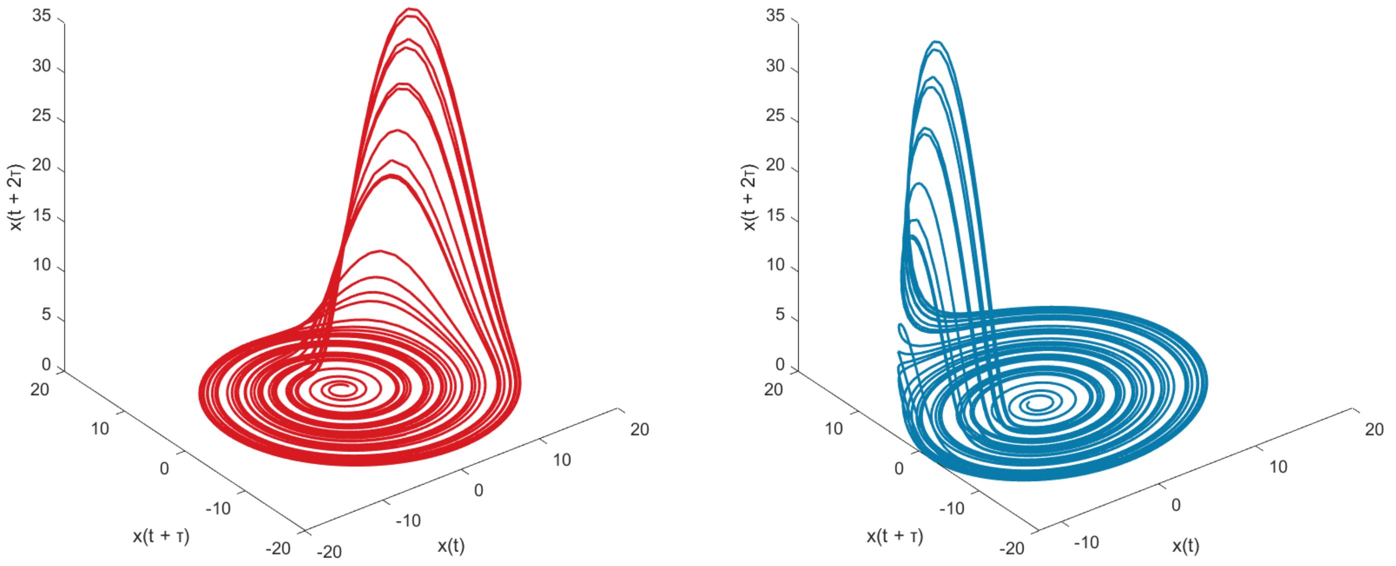
Comparing JRQA to CRQA



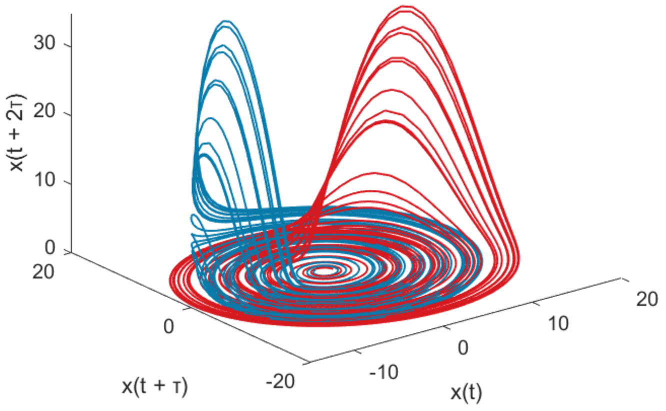
CRQA: Single phase space of both systems

JRQA: Both systems have their own respective phase space

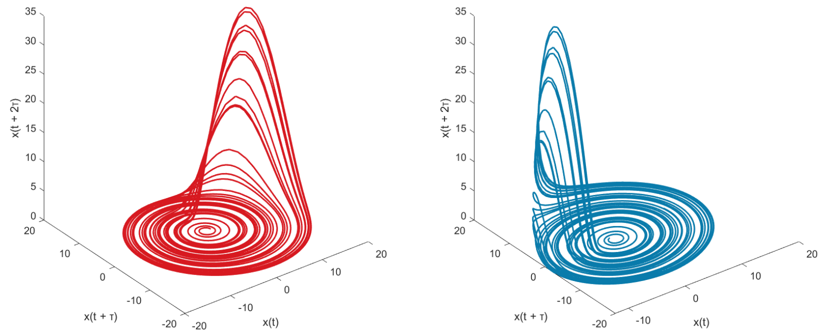




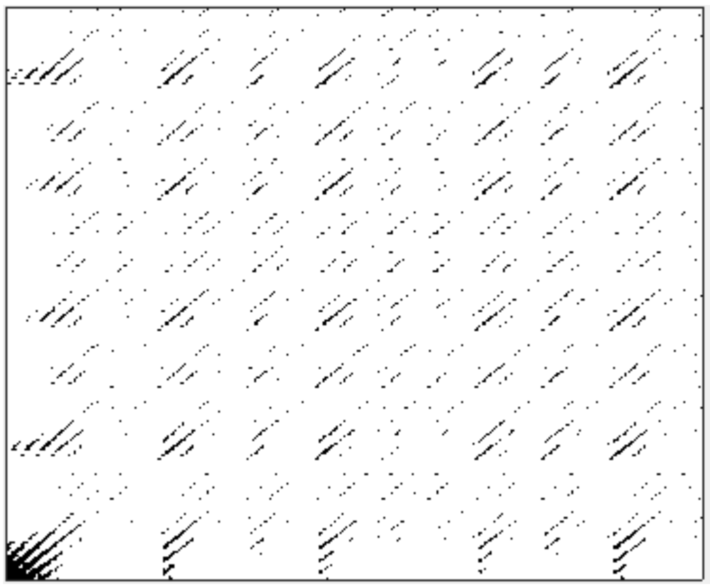
CRQA Phase Space (both systems in one)



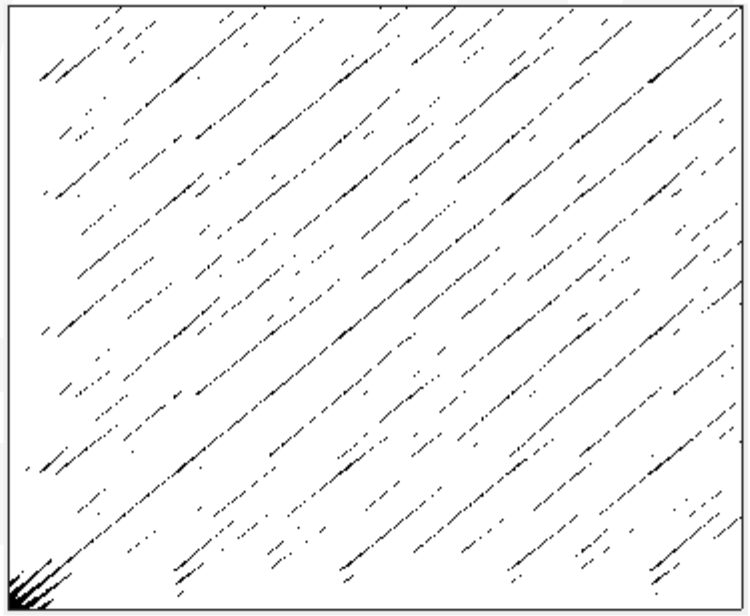
JRQA Phase Space (systems in their respective phase space)

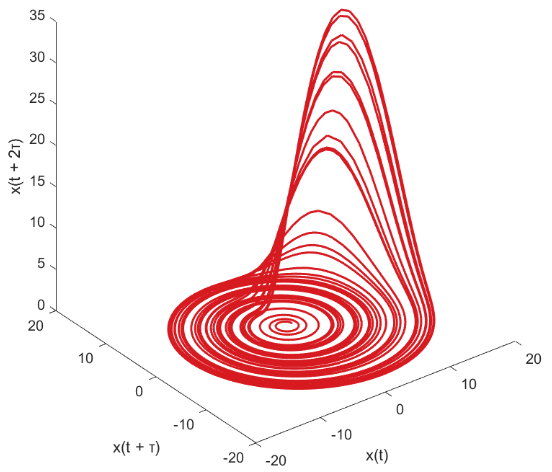


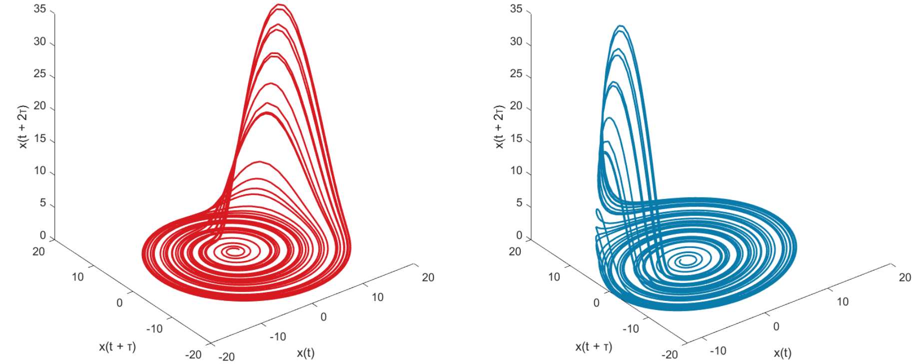
Cross Recurrence Plot of both systems



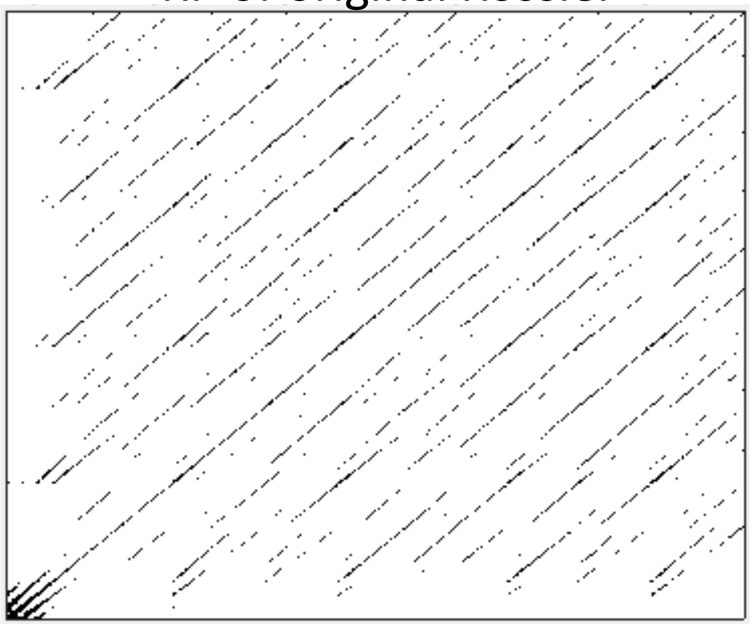
Joint Recurrence Plot of both systems



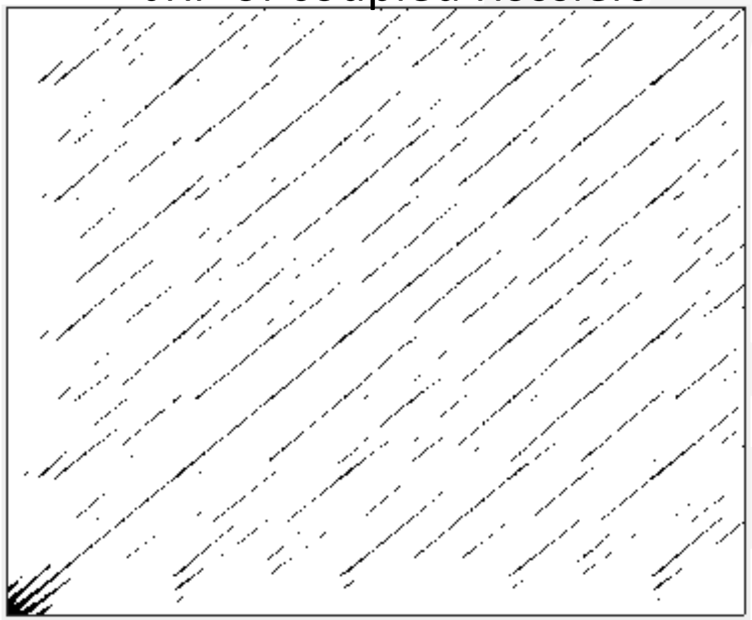




RP of original Rossler



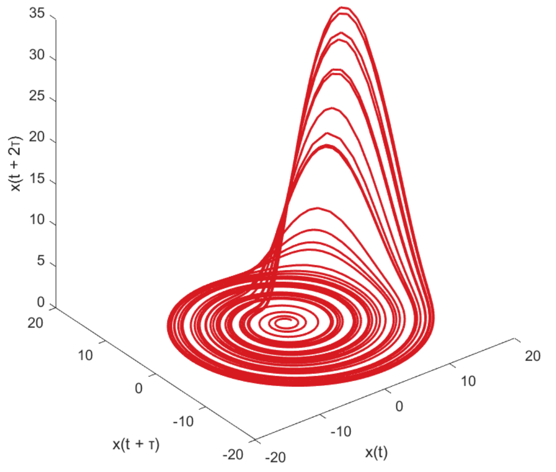
JRP of coupled Rosslers

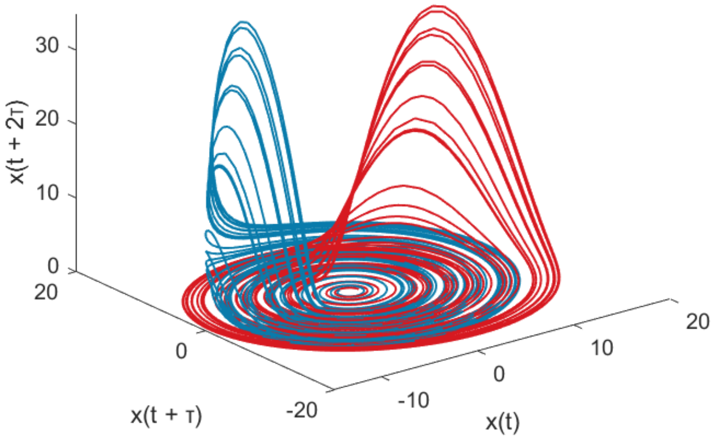


Recurrence plots are identical

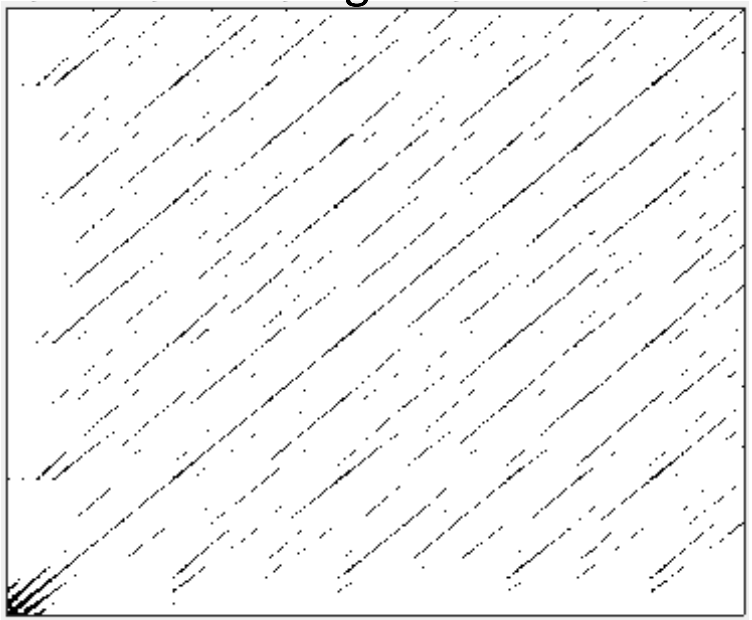
This is because the JRP considers joint recurrences

- Recurrences which occur simultaneously in both systems, and they are invariant under affine transformations

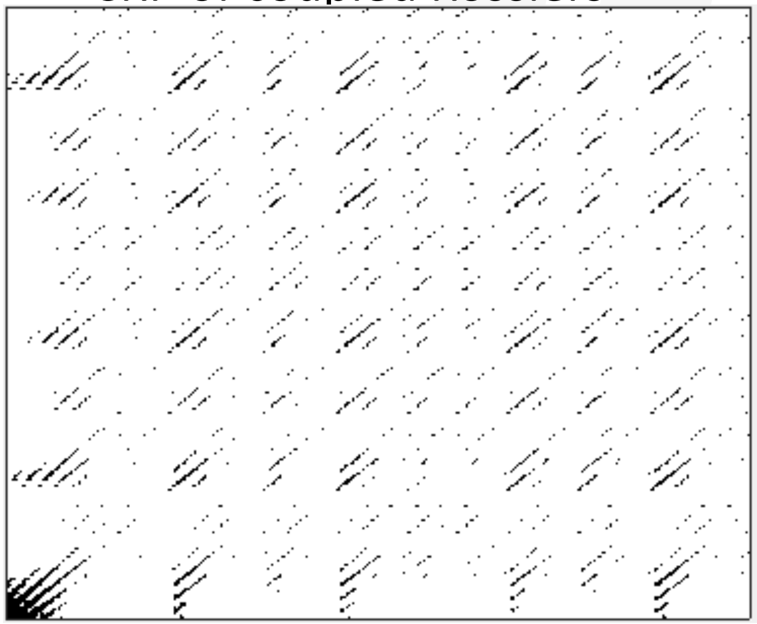




RP of original Rossler

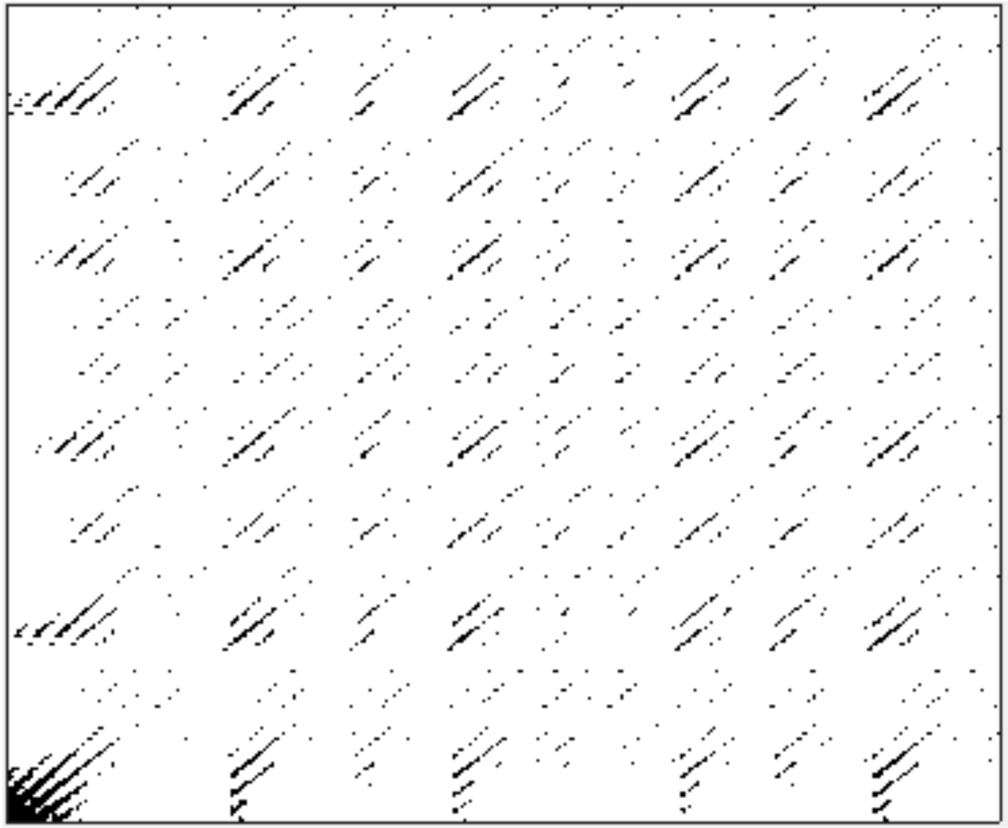


CRP of coupled Rosslers

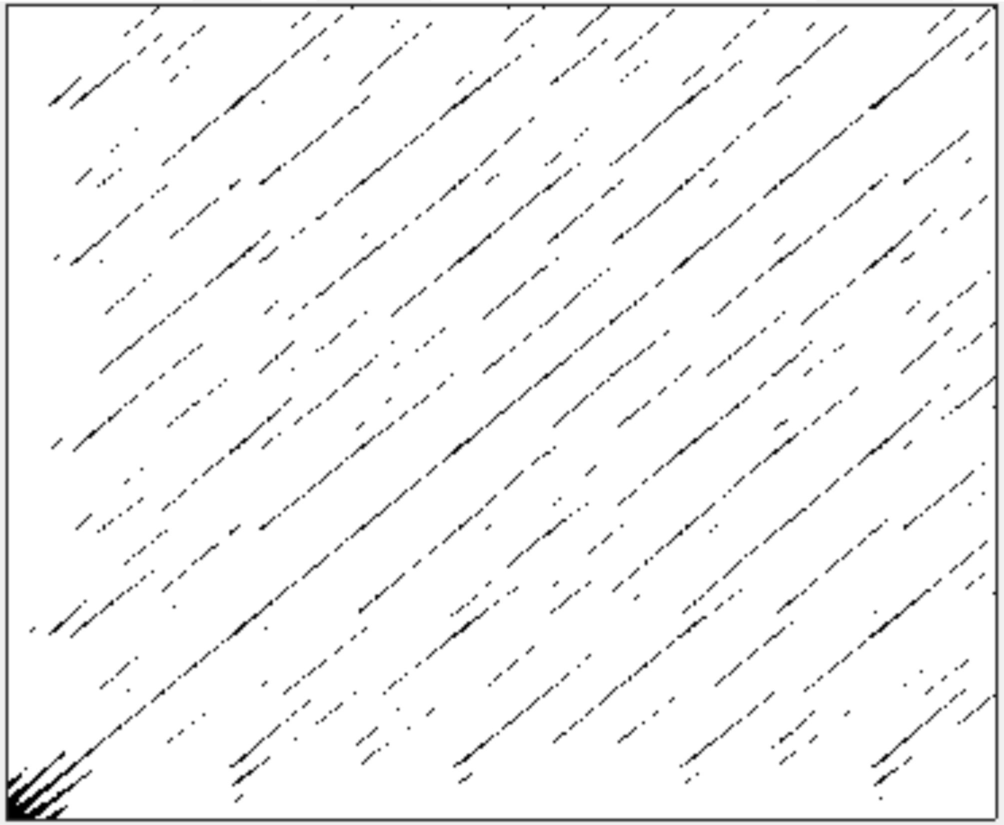


- Recurrence plots are ***not*** identical
- A rotation of the reference system of one trajectory changes the CRP
- Therefore, the CRP cannot detect that both trajectories are identical up to a rotation

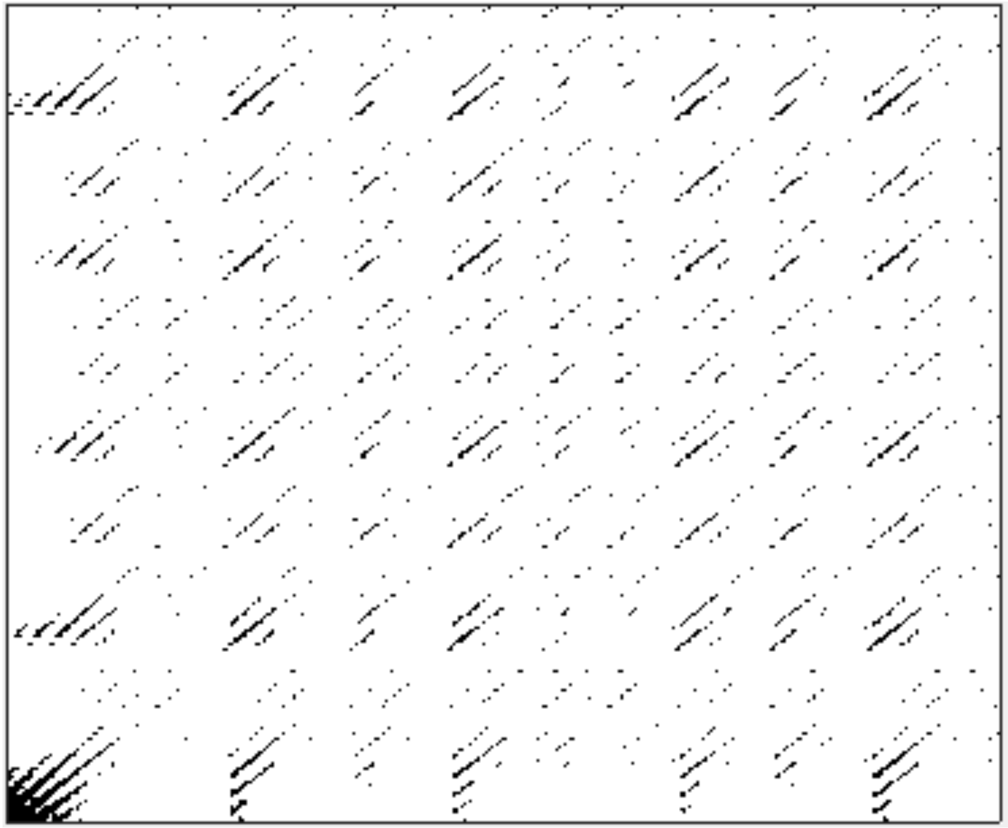
Cross Recurrence Plot



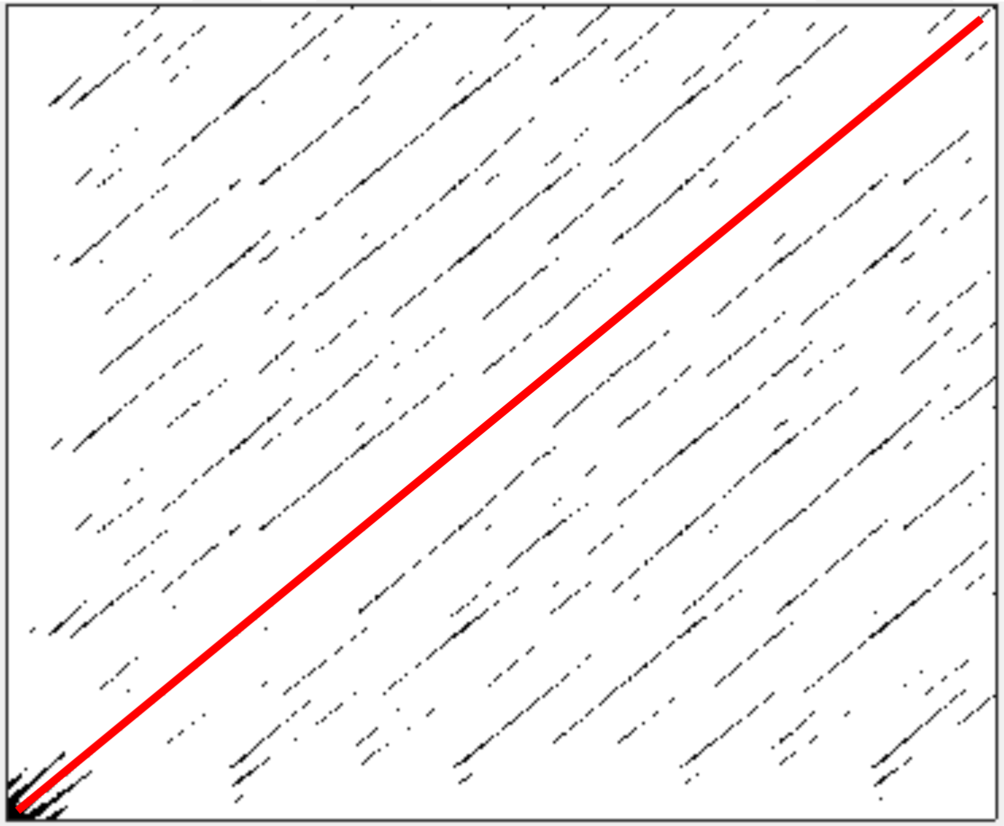
Joint Recurrence Plot



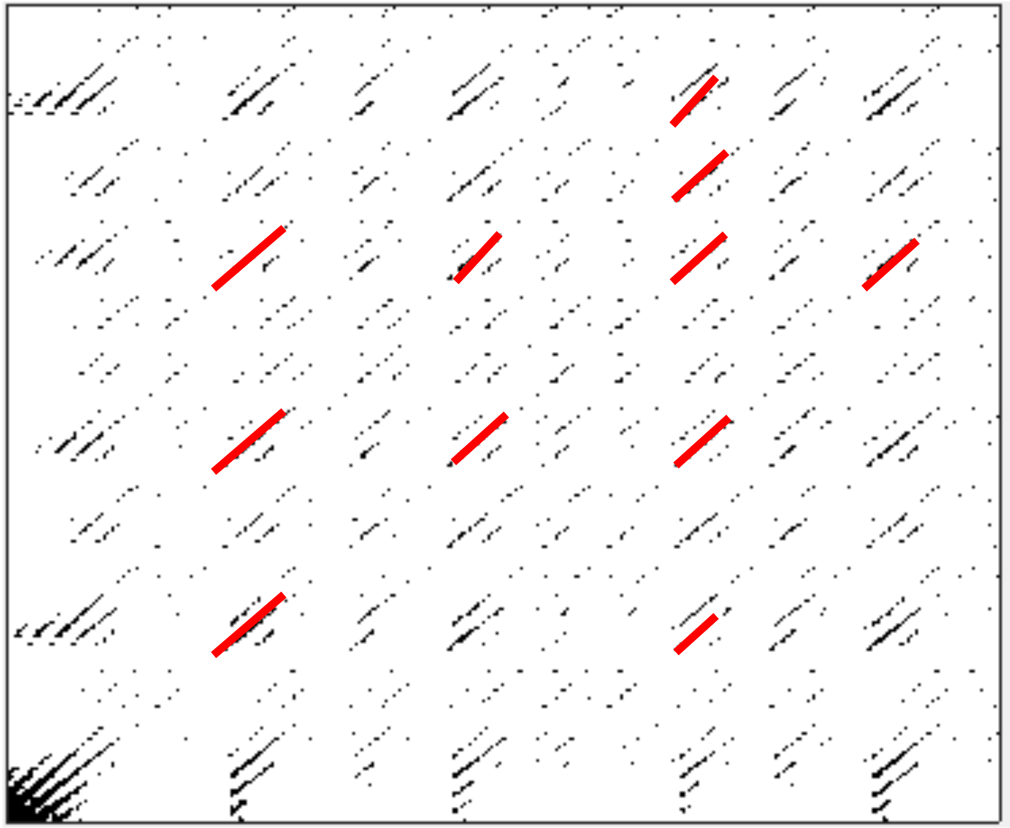
Cross Recurrence Plot



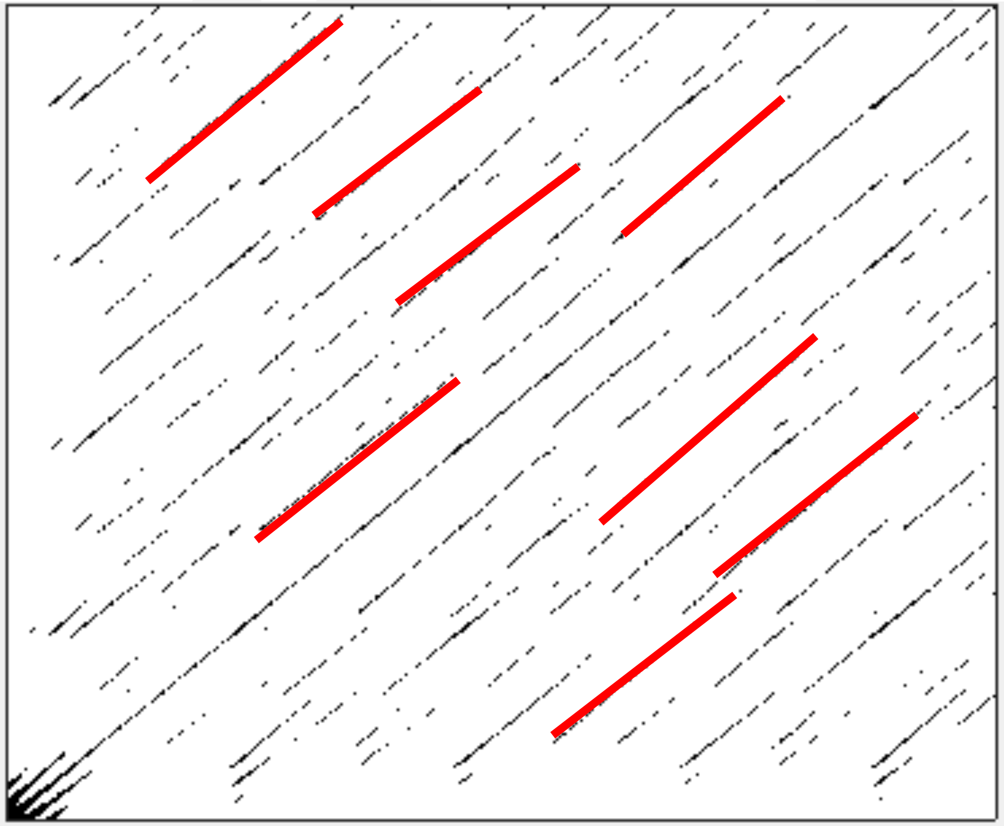
Joint Recurrence Plot



Cross Recurrence Plot



Joint Recurrence Plot

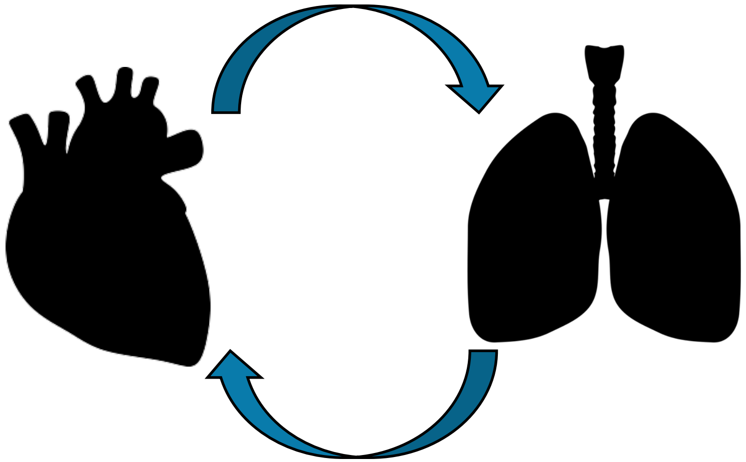


Parameter Selection

- By means of using jRQA, individual phase space systems are created for both of the given series
- jRQA allows for both systems to have different embedding dimensions
- Two different thresholds for each system may be considered
- The criteria for choosing the threshold can be applied to both systems separately, respecting the natural measure of both systems

Uses of JRQA

Heart Rate Variability and Respiratory Flow

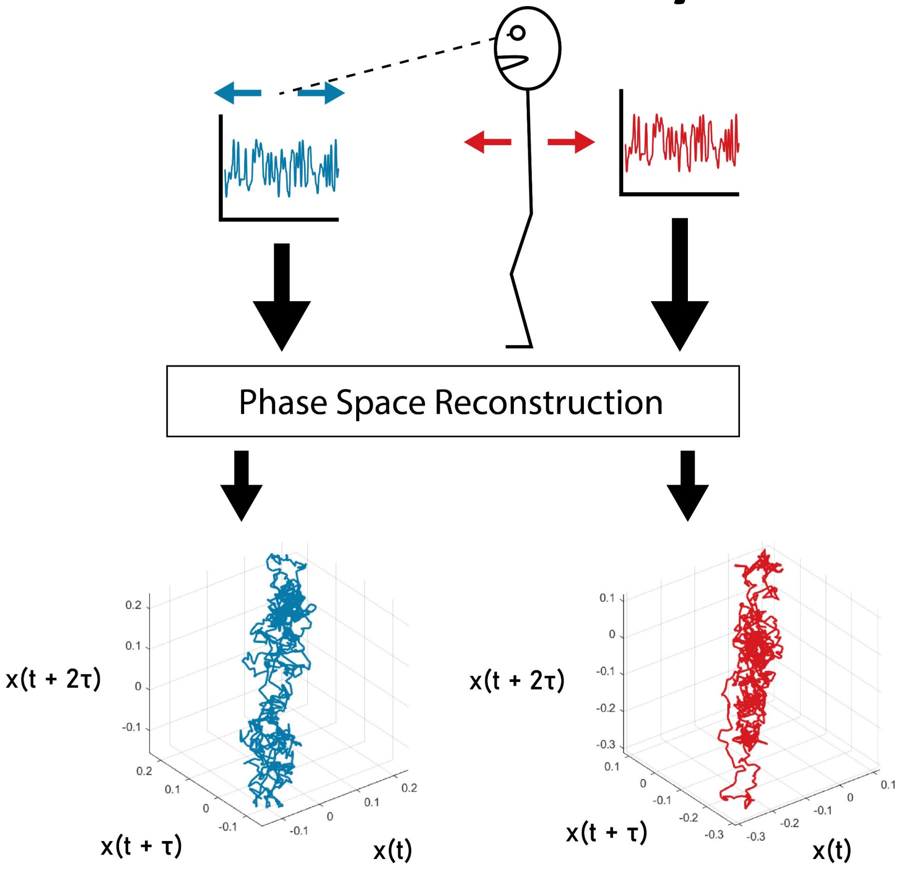


- JRQA has been used to compare patients that had successful weaning from mechanical ventilation to patients that failed to maintain spontaneous breathing
- All JRQA parameters were higher in the group with successful weaning than in the group that failed to maintain spontaneous breathing
- JRQA has shown to be a promising tool for characterizing patients who necessitate mechanical ventilation

Arcentales et al,. (2005)

Uses of JRQA

Gaze and Postural Dynamics

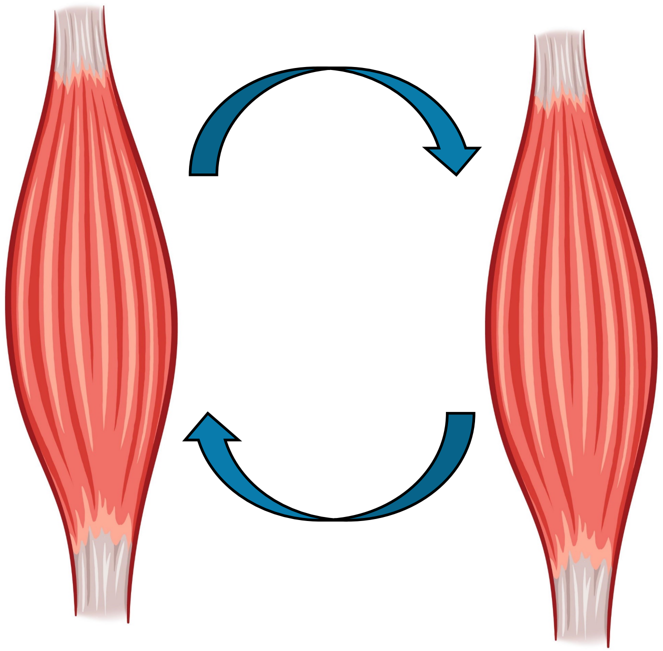


- JRQA was implemented to observe how asymmetric restraints on movement impact postural dynamics, gaze dynamics and other multivariate behavioral signals
- JRQA was able to detect coupling between systems and changes in coupling due to asymmetric restraints

Tolston et al., (2020)

Uses of JRQA

Muscle Synergies



- Previous studies have investigated how multiple muscles coordinate at different submaximal voluntary contractions during power grip with JRQA
- JRQA parameters in both right and left hands increased with the force level
- This demonstrated the utility of JRQA to explore the coordination mechanism of multiple muscles under neurological control

Zhang et al., (2018)

CENTER FOR RESEARCH IN HUMAN MOVEMENT VARIABLITY
NONLINEAR ANALYSIS CORE

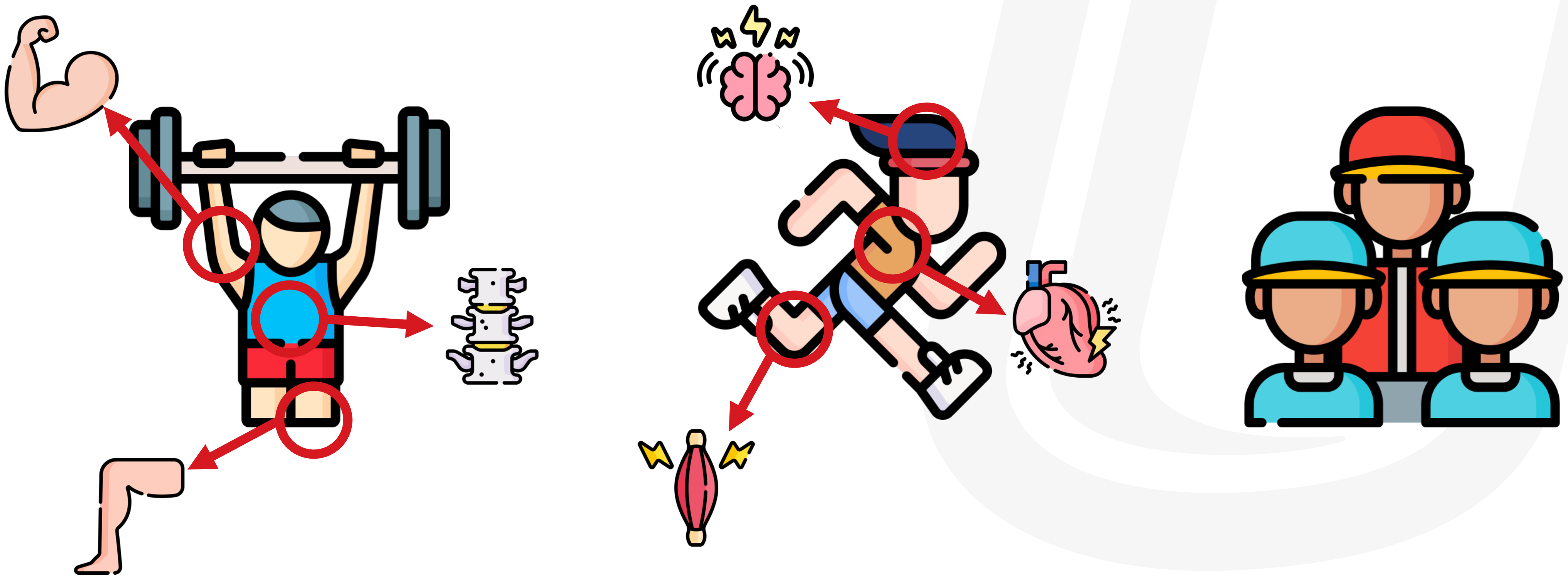
Multidimensional Recurrence Quantification Analysis

bmchnonan@unomaha.edu

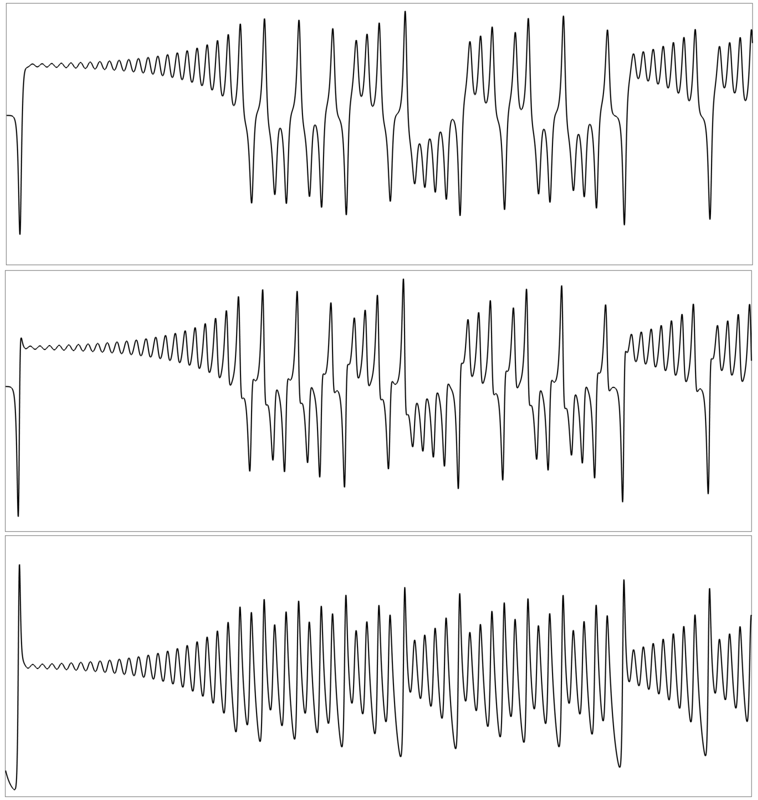
University of Nebraska Omaha

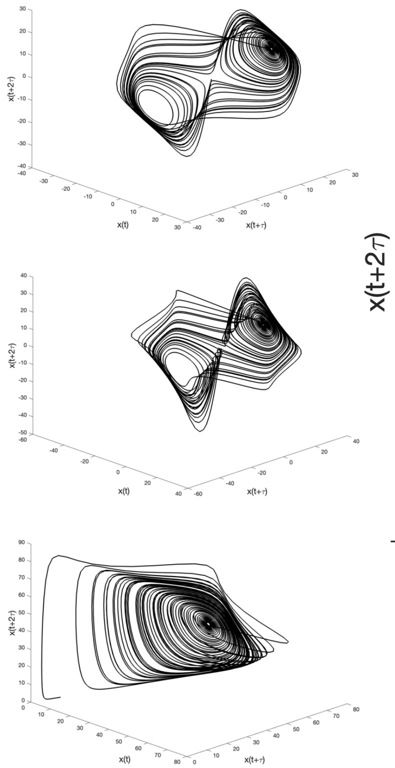
Multidimensional Recurrence Quantification Analysis (MdRQA)

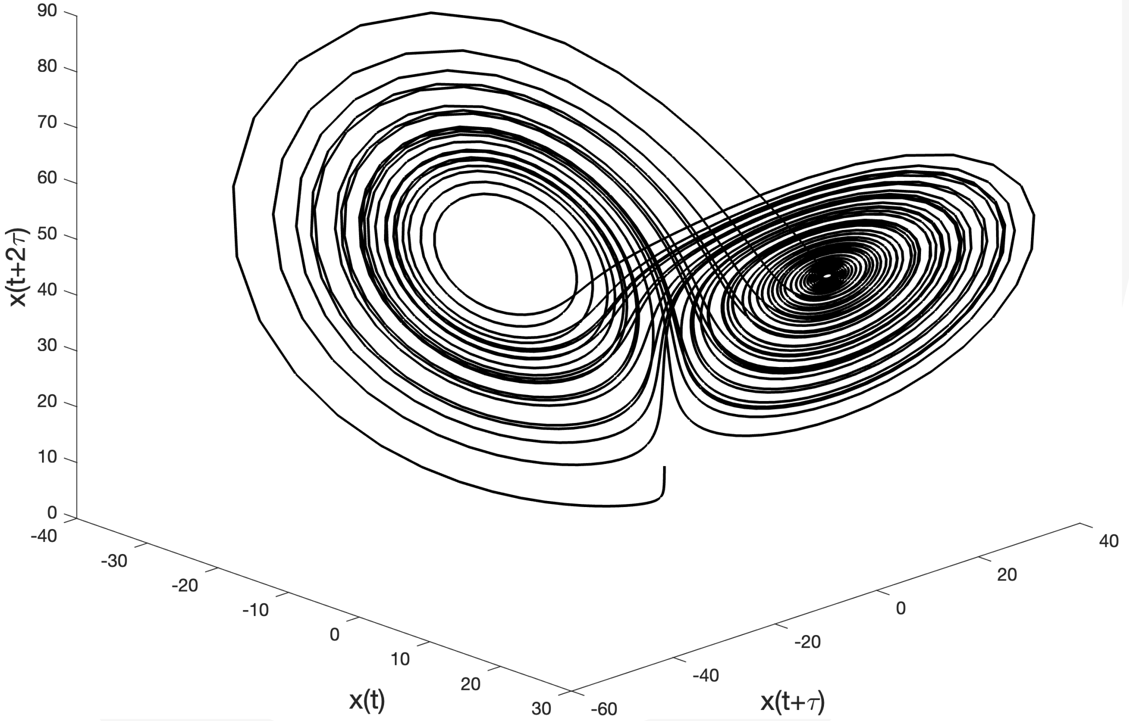
A lot of phenomena that we deal with are the products of the **interaction among numerous components / subsystems**



Multidimensional Recurrence Quantification Analysis (MdRQA)







Multidimensional Recurrence Quantification Analysis (MdRQA)

- Measuring all relevant variables allows us to apply RQA directly to the multivariate time series without time delayed embedding

➔ **Multidimensional Recurrence Quantification Analysis (MdRQA)**

- Is useful to examine multidimensional systems / multiscale properties of systems

Multidimensional Recurrence Quantification Analysis (MdRQA)

$$x = (x_1, x_2, x_3, \dots \dots, x_n)$$

$$X_1 = (x_1, x_{1+\tau}, x_{1+2\tau}, \dots \dots, x_{1+(D-1)\tau})$$

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_{n-(D-1)\tau} \end{pmatrix} = \begin{pmatrix} x_1 & x_{1+\tau} & \cdots & x_{1+(D-1)\tau} \\ x_2 & x_{2+\tau} & \cdots & x_{2+(D-1)\tau} \\ \vdots & \vdots & & \vdots \\ x_{n-(D-1)\tau} & x_{n-(D-2)\tau} & \cdots & x_n \end{pmatrix}$$

$$RP_{ij} = \Theta(T - \|X_i - X_j\|)$$

$$x_i = (x_{i,1}, x_{i,2}, x_{i,3}, \dots \dots, x_{i,n})$$

$$W_n = (x_{1,n}, x_{2,n}, x_{3,n}, \dots \dots, x_{N,n})$$

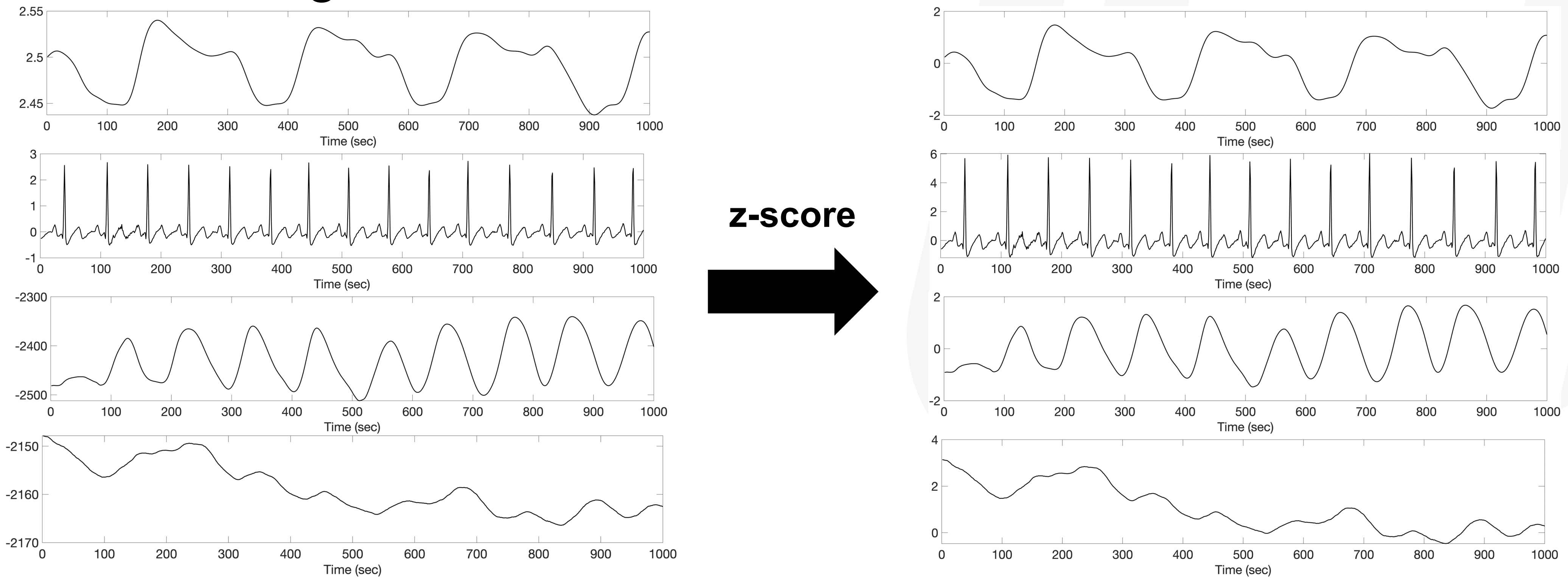
$$W = \begin{pmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{pmatrix} = \begin{pmatrix} x_{1,1} & x_{2,1} & \cdots & x_{N,1} \\ x_{1,2} & x_{2,2} & \cdots & x_{N,2} \\ \vdots & \vdots & & \vdots \\ x_{1,n} & x_{2,n} & \cdots & x_{N,n} \end{pmatrix}$$

$$MdRP_{ij} = \Theta(T - \|W_i - W_j\|)$$

Wallot et al 2016

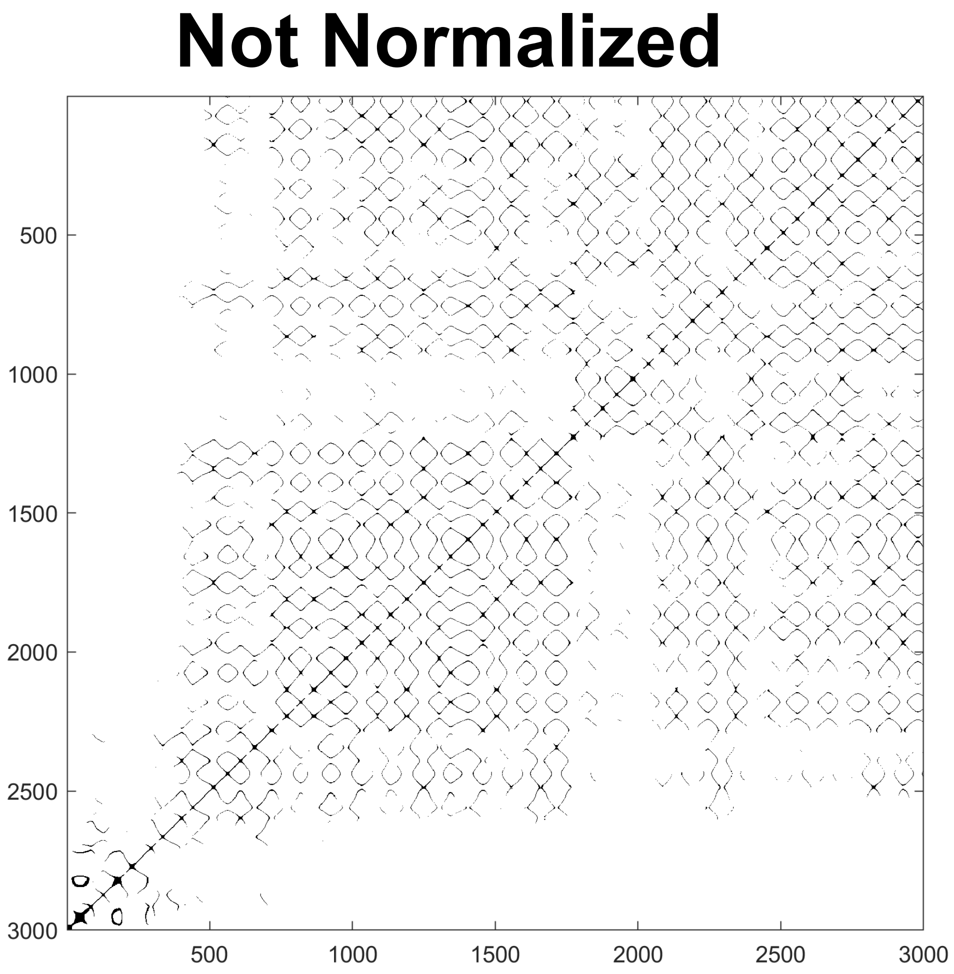
Considerations for MdRQA – Normalization

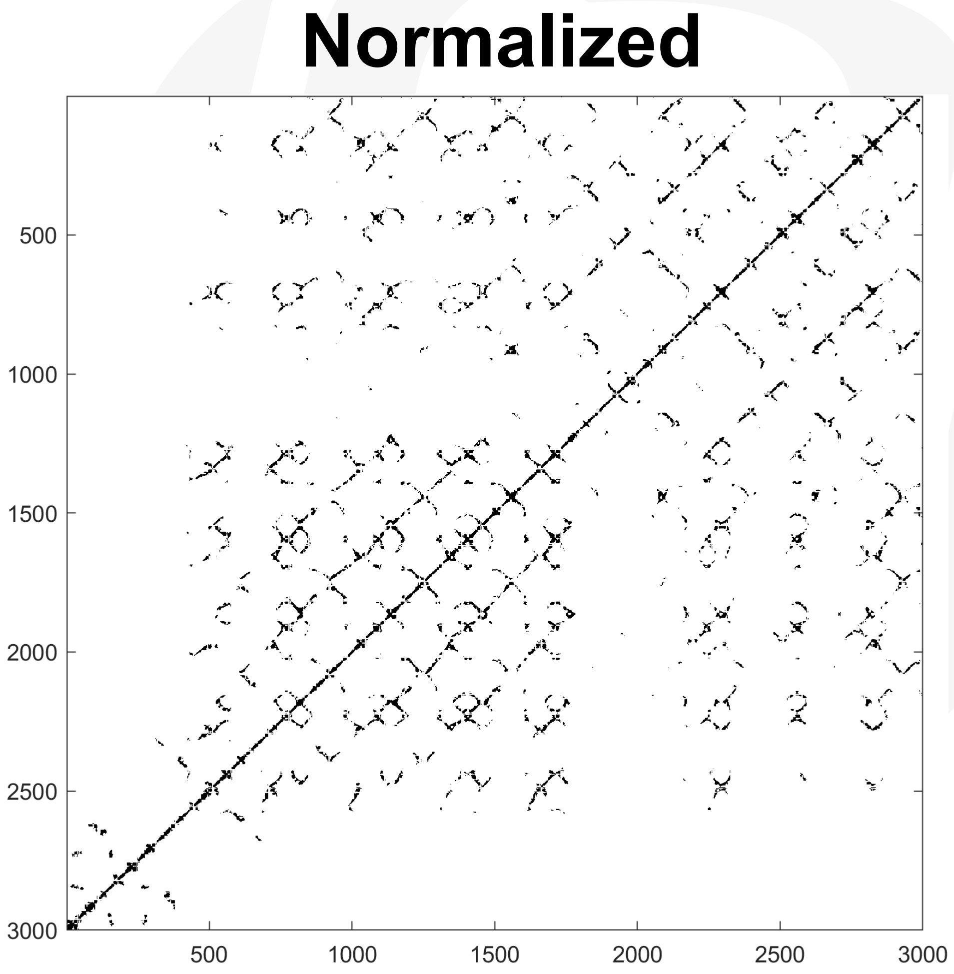
- If differences among the magnitudes of signals are significant, consider rescaling signals onto comparable scale while maintaining the distribution



Likens et al 2023

Considerations for MdRQA – Normalization





Likens et al 2016

Considerations for MdRQA – Parameter Selection

- Option 1 – If all relevant variables are measured
  - No need for time delayed embedding
  - Treat each variable as each dimension of the embedded state space
- Option 2 – If all relevant variables aren't measured
  - Need to perform time delayed embedding
  - Estimating embedding parameters based on the estimated embedding parameters from each individual signal
  - May not capture the embedding properties of the actual multivariate data

Considerations for MdRQA – Parameter Selection

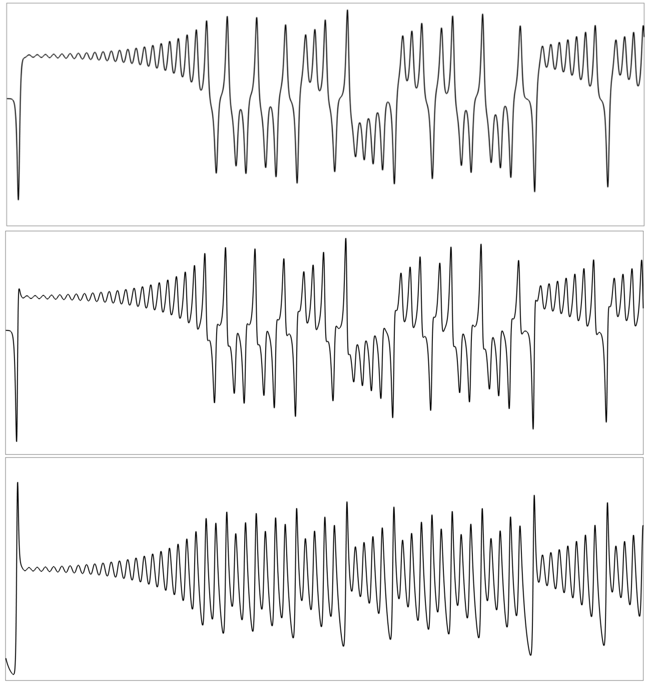
Time delay (Wallot & Mønster, 2018)

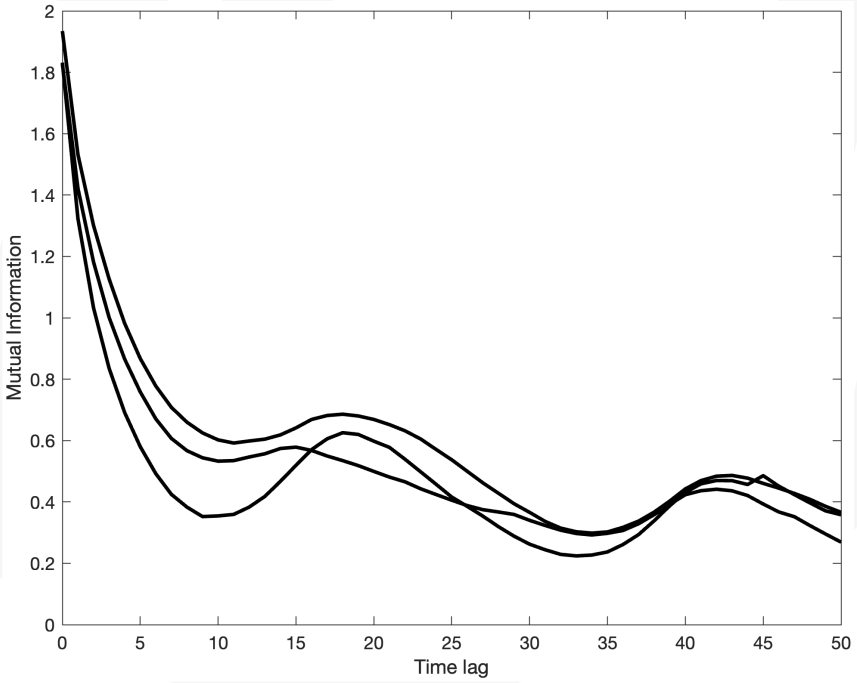
1) Compute Average Mutual Information (AMI) of each dimension / variable
2) Check AMI functions of each dimension / variable
3) If AMI functions are not significantly different, take the mean of the τ values where AMI functions attain first local minimums
4) If there are any significantly different AMI functions, consider…
   a) Embedding dimensions using different time delays
   b) Re-sampling some of the dimensions

Considerations for MdRQA – Parameter Selection

Time delay (Wallot & Mønster, 2018)

1) Average Mutual Information (AMI) on each dimension / variable

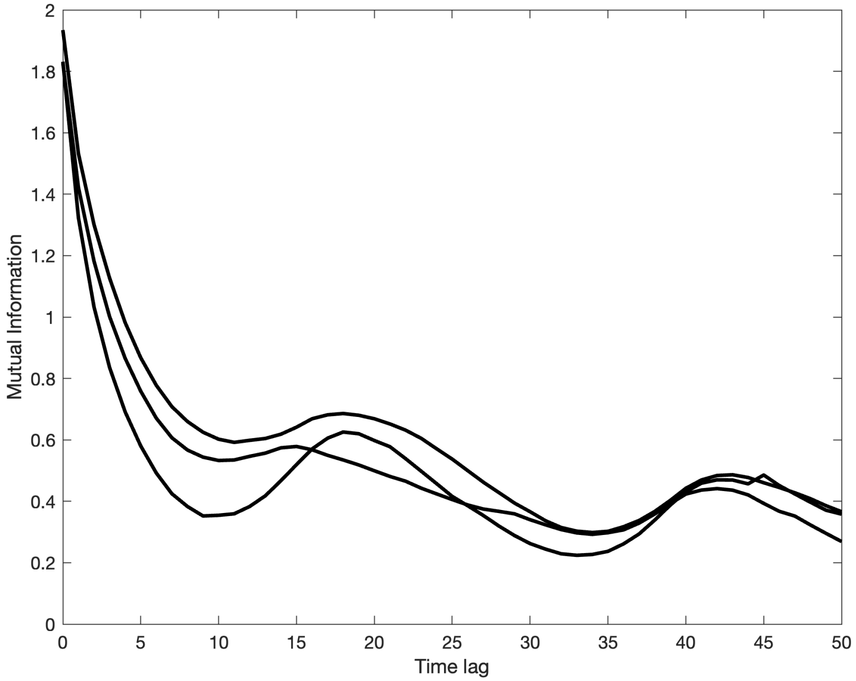




Considerations for MdRQA – Parameter Selection

Time delay (Wallot & Mønster, 2018)

2) Check AMI functions of each dimension / variable

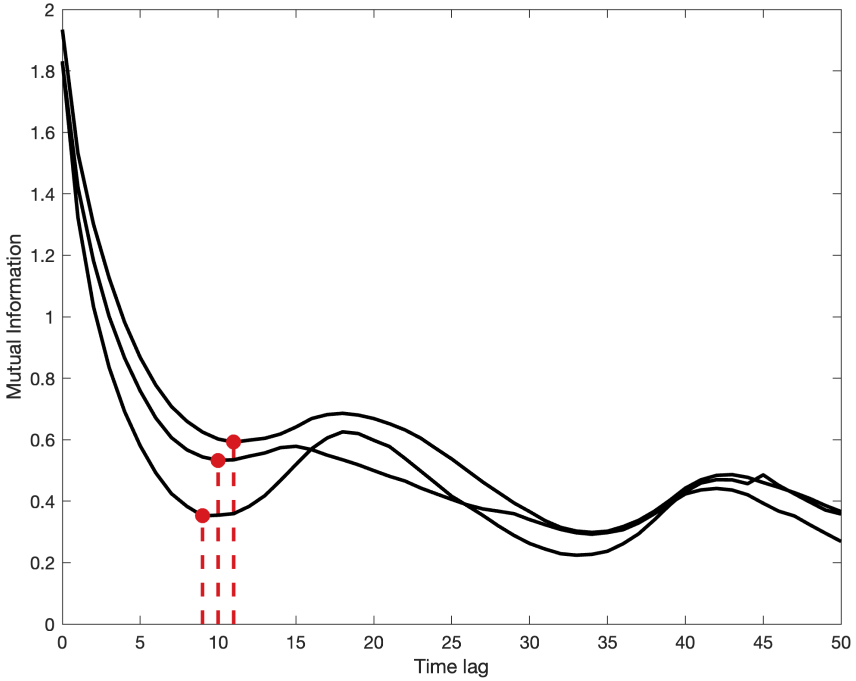


No deviating AMI functions

Considerations for MdRQA – Parameter Selection

Time delay (Wallot & Mønster, 2018)

3) If AMI functions are not significantly different, take the mean of the τ values where AMI functions attain first local minimums



$\tau = 9, 10, 11$

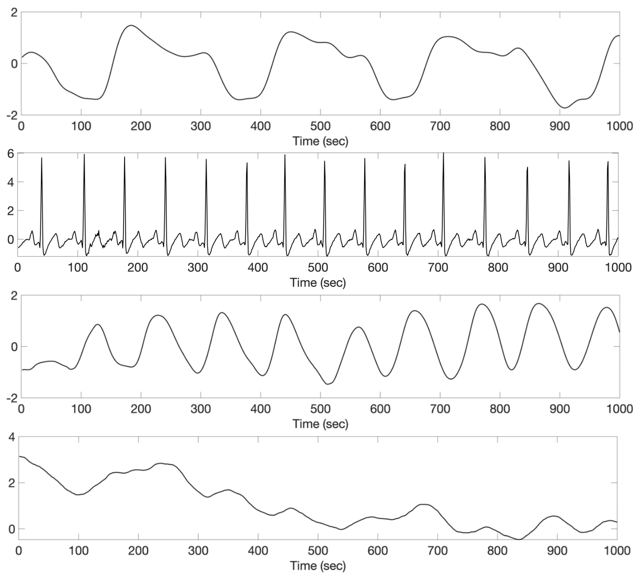
$mean\ \tau \approx 10$

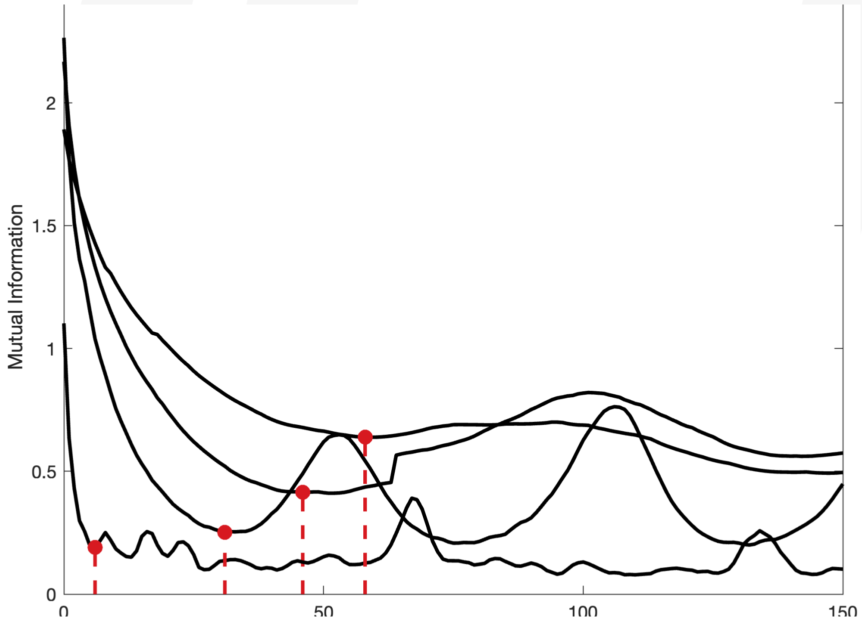
If decimal, round to the closest integer

Considerations for MdRQA – Parameter Selection

Time delay (Wallot & Mønster, 2018)

4) If there are any significantly different AMI functions, consider…
   a) Embedding dimensions using different time delays
   b) Re-sampling some of the dimensions





Likens et al 2023

Considerations for MdRQA – Parameter Selection

Embedding Dimension (Wallot & Mønster, 2018)

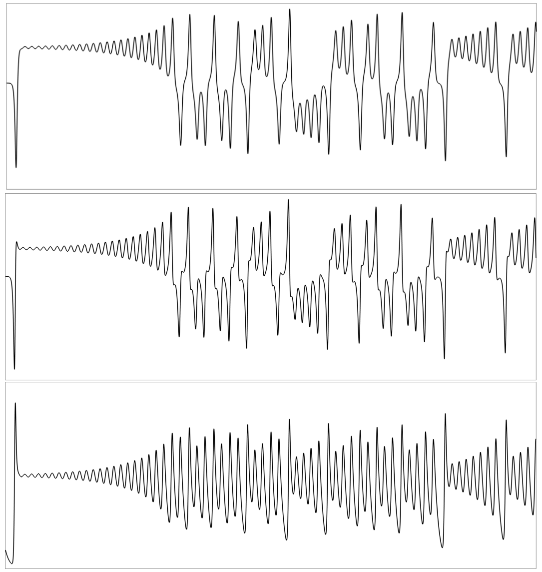
1) False Nearest Neighbors (FNN) Algorithm on the multivariate time series
2) Count the number of FNNs by increasing the embedding dimension D by the number of dimension of given time series
3) Find D where the number of FNN drops to 0 or plateaus

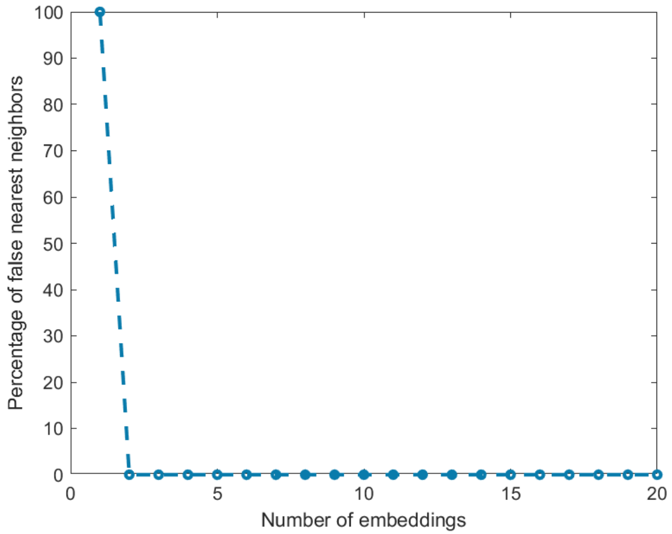
Considerations for MdRQA – Parameter Selection

Embedding Dimension (Wallot & Mønster, 2018)

1) False Nearest Neighbors (FNN) Algorithm on the multivariate time series

2) Count the number of FNNs by increasing the embedding dimension, D, by the number of dimension of given time series

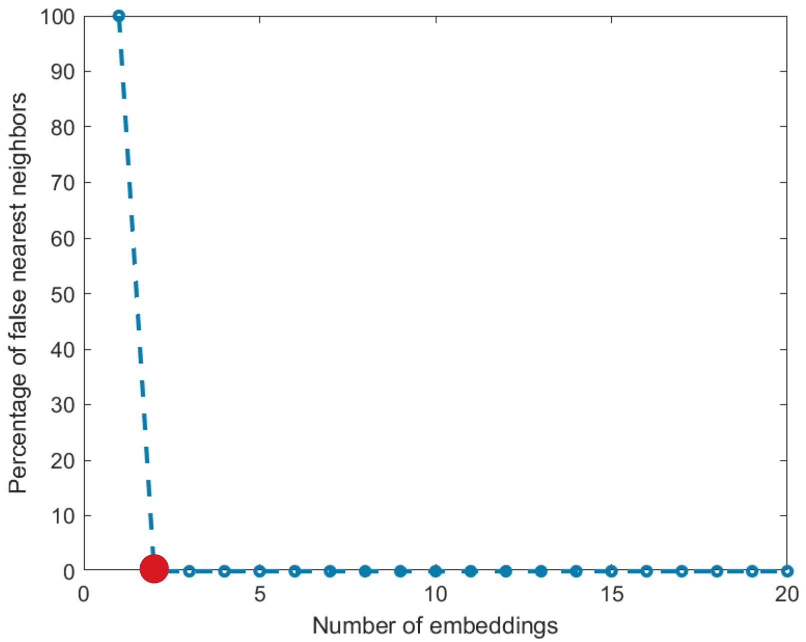




Considerations for MdRQA – Parameter Selection

Embedding Dimension (Wallot & Mønster, 2018)

3) Find D where FNN drops to 0 or plateaus

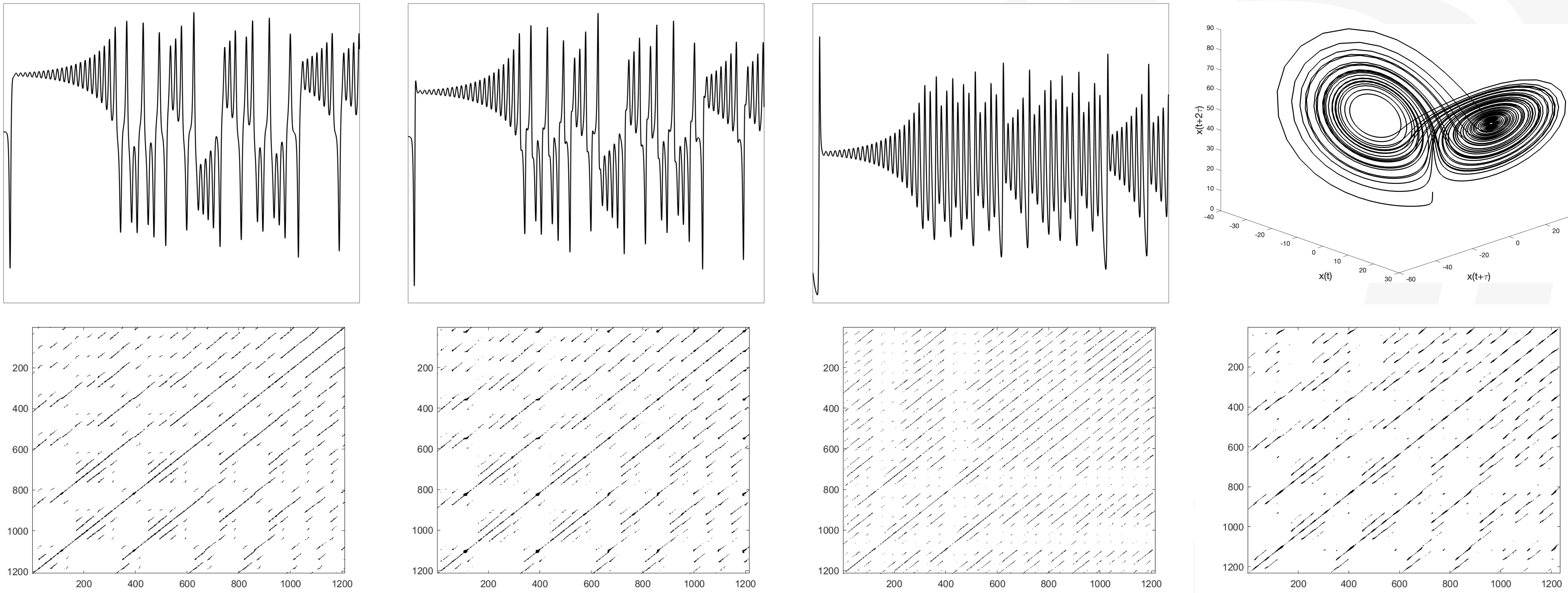


FNN drops to 0 at $D = 2$

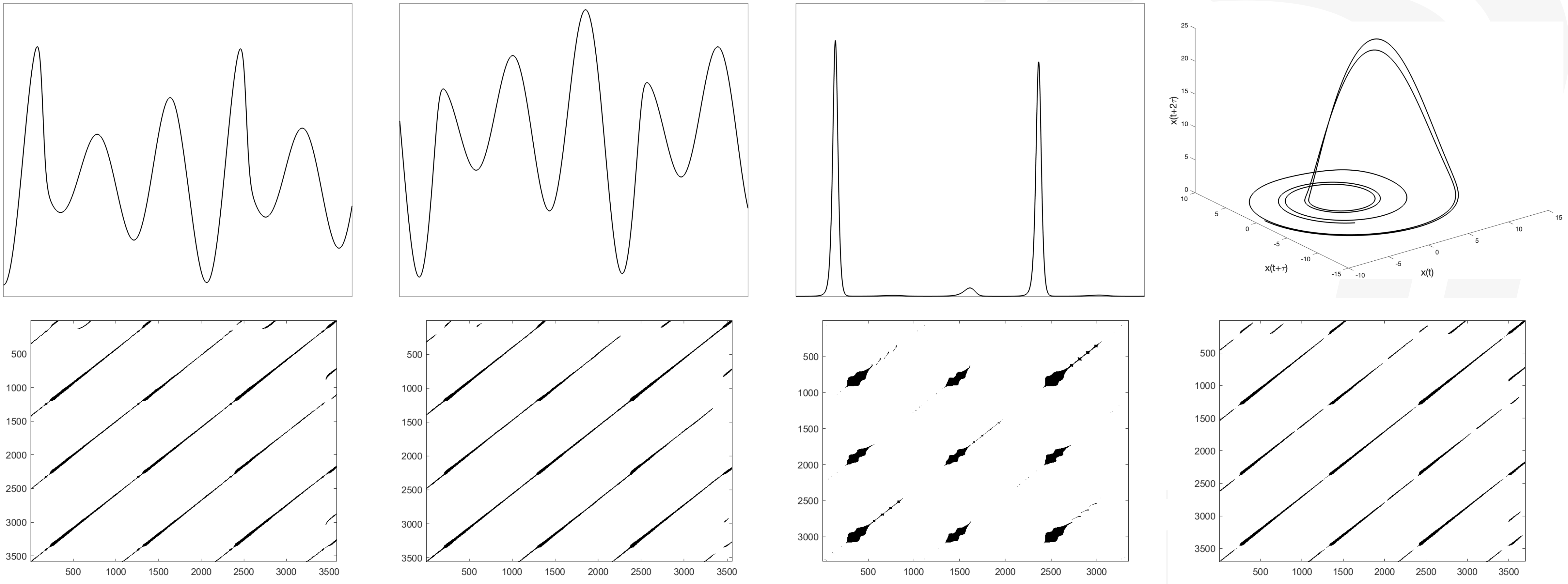
➔ Embed each time series twice

➔ Final dimension $= \ 6$

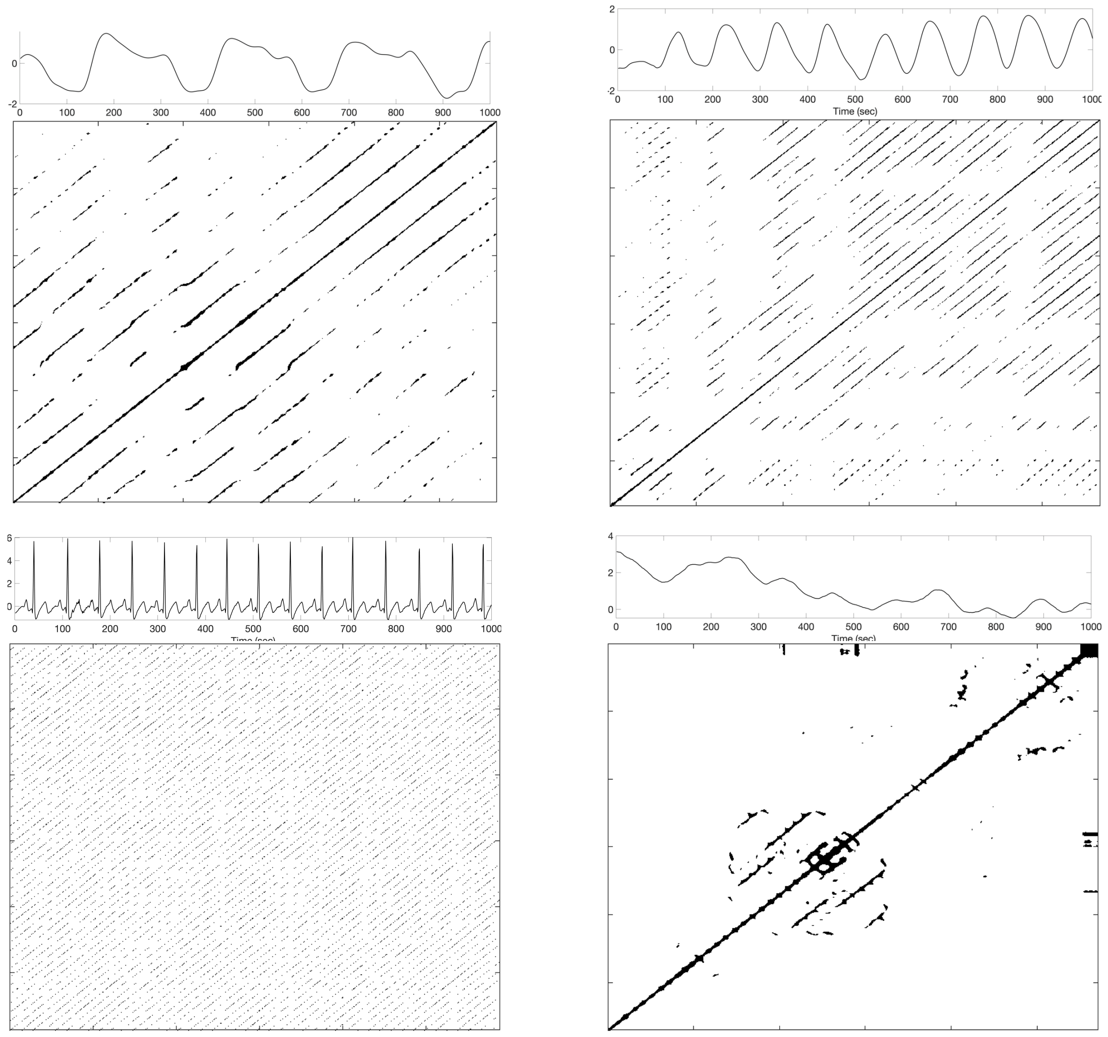
Comparing RQA and MdRQA

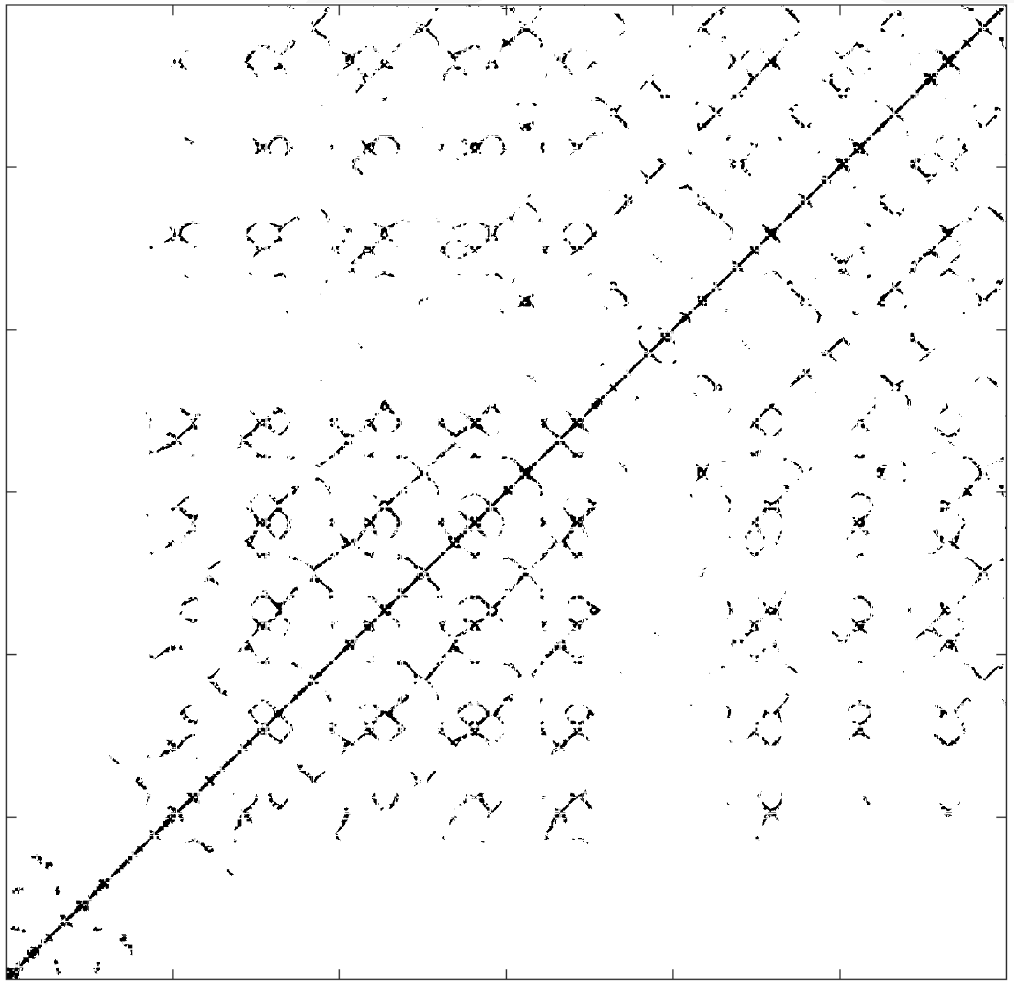


Comparing RQA and MdRQA

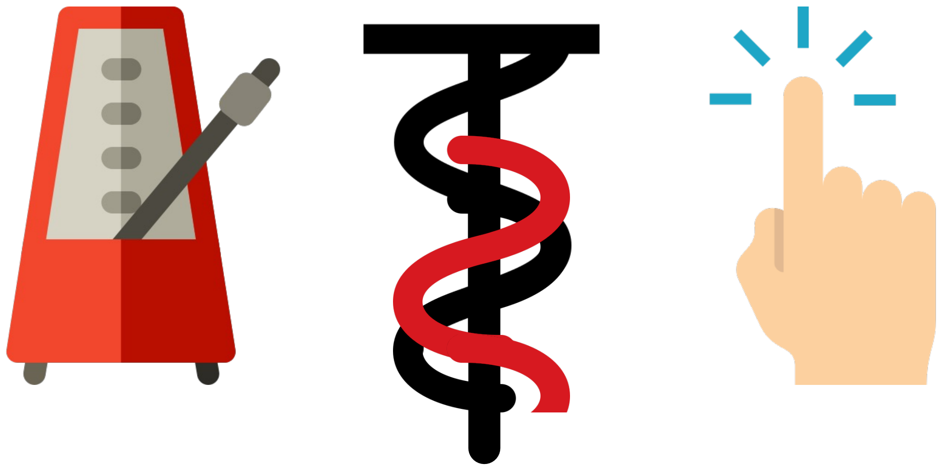


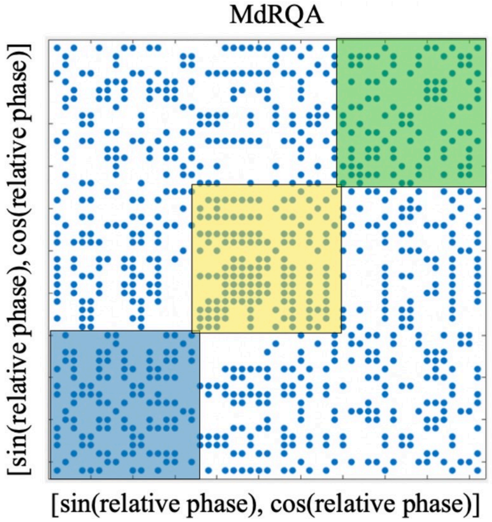
Comparing RQA and MdRQA

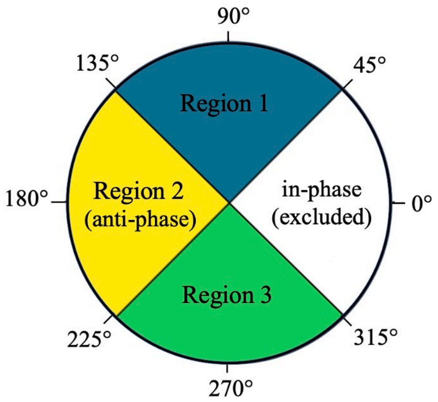




Examples of MDRQA



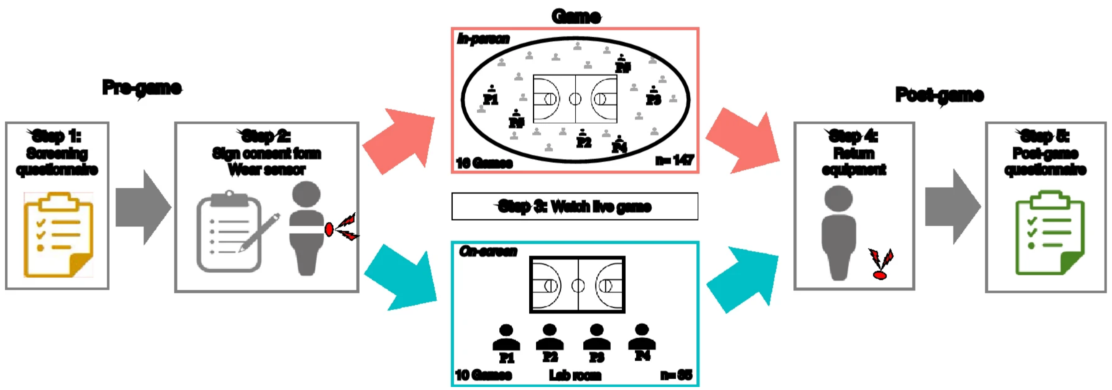


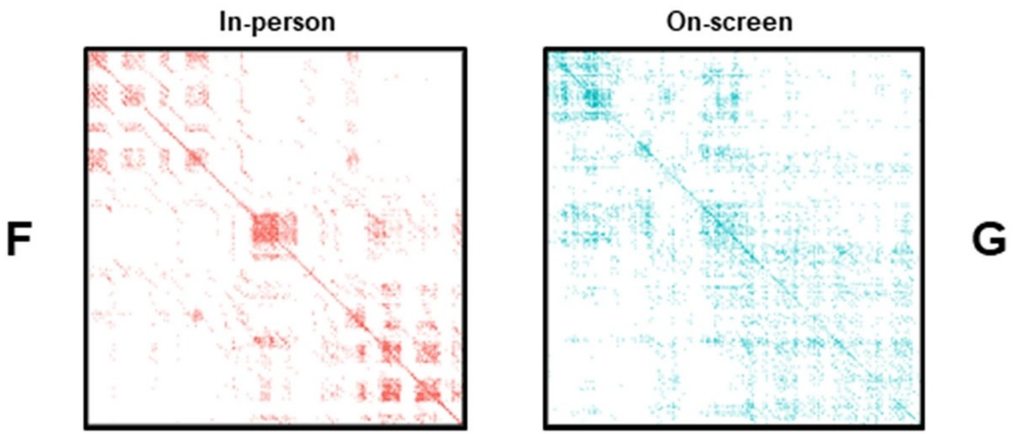


- MdRQA has been used to examine how sensorimotor synchronization is impacted by individual and situational factor
- MdRQA revealed greater attractor strength at faster tempi
- MdRQA revealed weakest coupling with metronome when transitioning from anti-phase to in-phase
- MdRQA has shown to be able to quantify sensorimotor synchronization

Hall et al., (2023)

Examples of MDRQA





- MdRQA has been used to quantify physiological synchrony while watching basketball game
- MdRQA metrics were higher in the in-person crowd compared to on-screen crowd
- MdRQA was able to detect physiological synchrony due to shared experience

Baranowski-Pinto et al., (2022)

Summary

- RQA has been successfully used for describing dynamic systems that are too complex to be characterized adequately by standard methods in time series analysis
- It can be applied to short or long chronological data series and the data do not need to be stationary
- Recurrence is a basic feature of many deterministic systems
- RQA quantifies the structural variability of a time series by counting recurrent patterns in the underlying dynamical system

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