CENTER FOR RESEARCH IN HUMAN MOVEMENT VARIABILITY

NONLINEAR ANALYSIS CORE

Univariate and Bivariate Fractal Methods for Movement Science

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Outline

Part I: Monofractal Analysis

i. Monofractal introduction
ii. Detrended Fluctuations Analysis step by step
iii. MATLAB Tutorial

Part II: Multifractal Analysis

i. From monofractal to multifractal analysis
ii. Multifractal Detrended Fluctuations Analysis step by step
iii. MATLAB Tutorial

Part III: Fractal Regression Analysis

i. From multifractal to multivariate fractal regression analysis
ii. Fractal regression analysis step by step
iii. MATLAB Tutorial

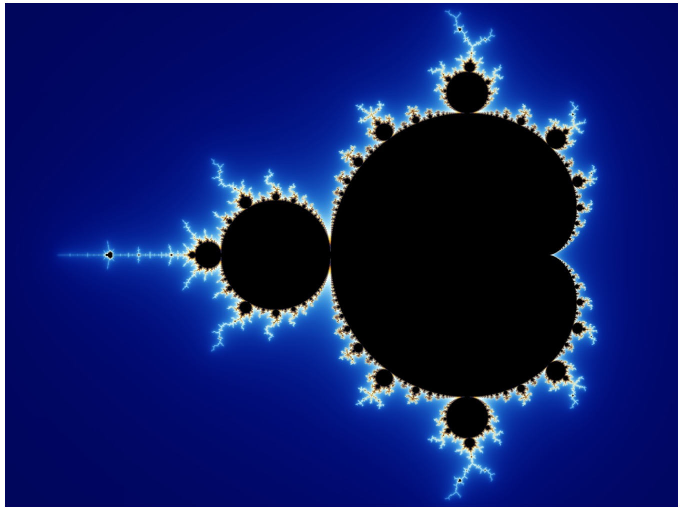
Part I: MONOFRACTAL ANALYSIS

Tyler M. Wiles

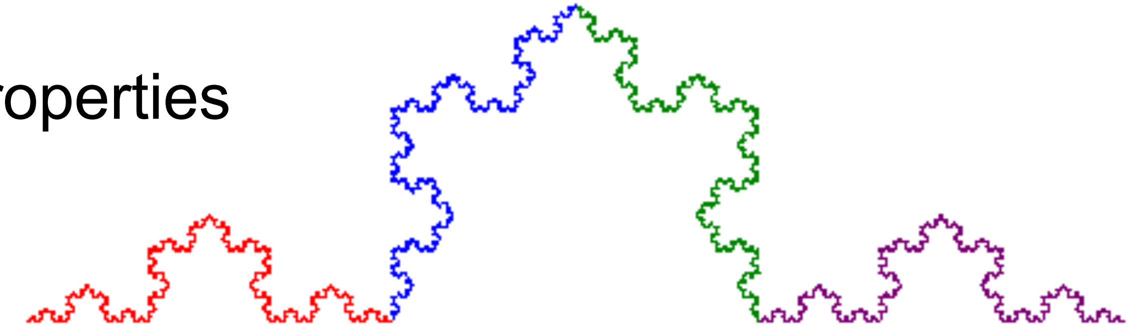
OUTLINE

- Four properties of fractals
  - Self similarity
  - Scaling
  - Dimension
  - Statistical properties
- How, what, when, where are fractals?
- Fractal analysis step by step
- Best practices

BACKGROUND- Fractals Introduction

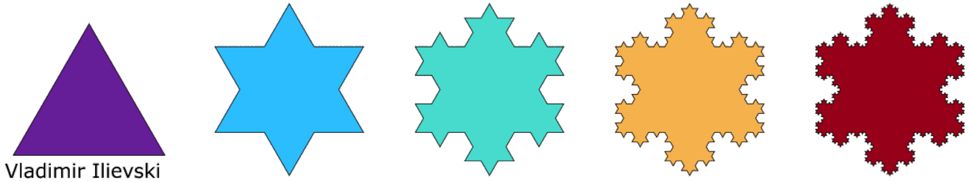


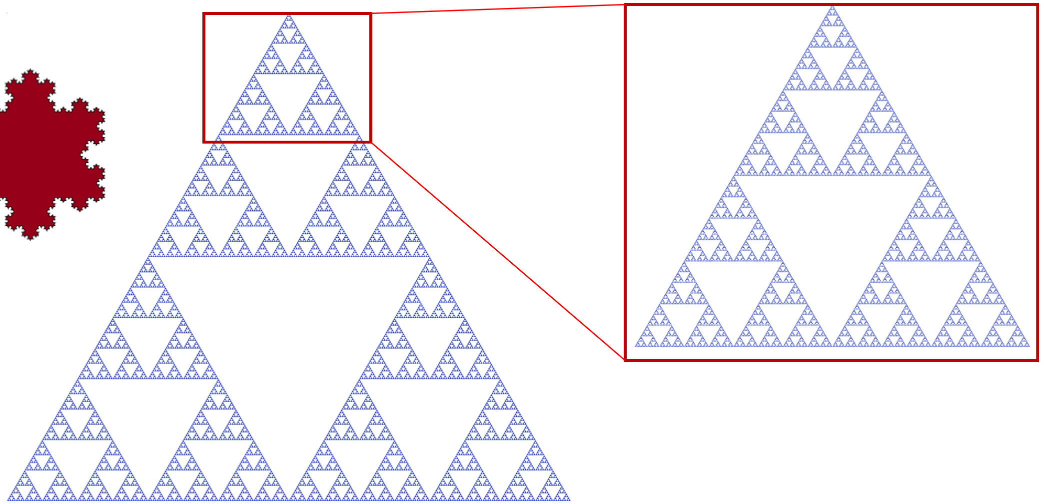
- **Fractal** objects when magnified:
  - Reveal finer and finer features;
  - Special case: look similar to larger features
- Four Properties:
  - Self-Similarity (Affinity)
  - Scaling
  - Dimensions
  - Statistical Properties

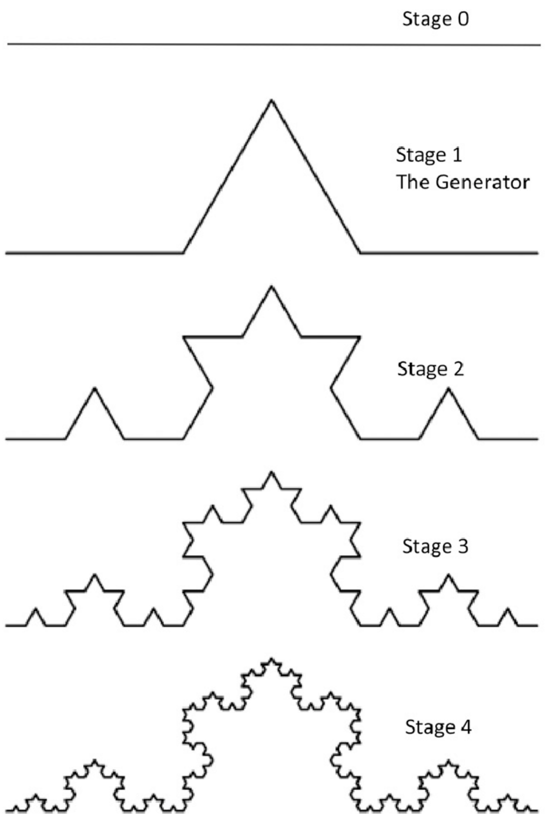


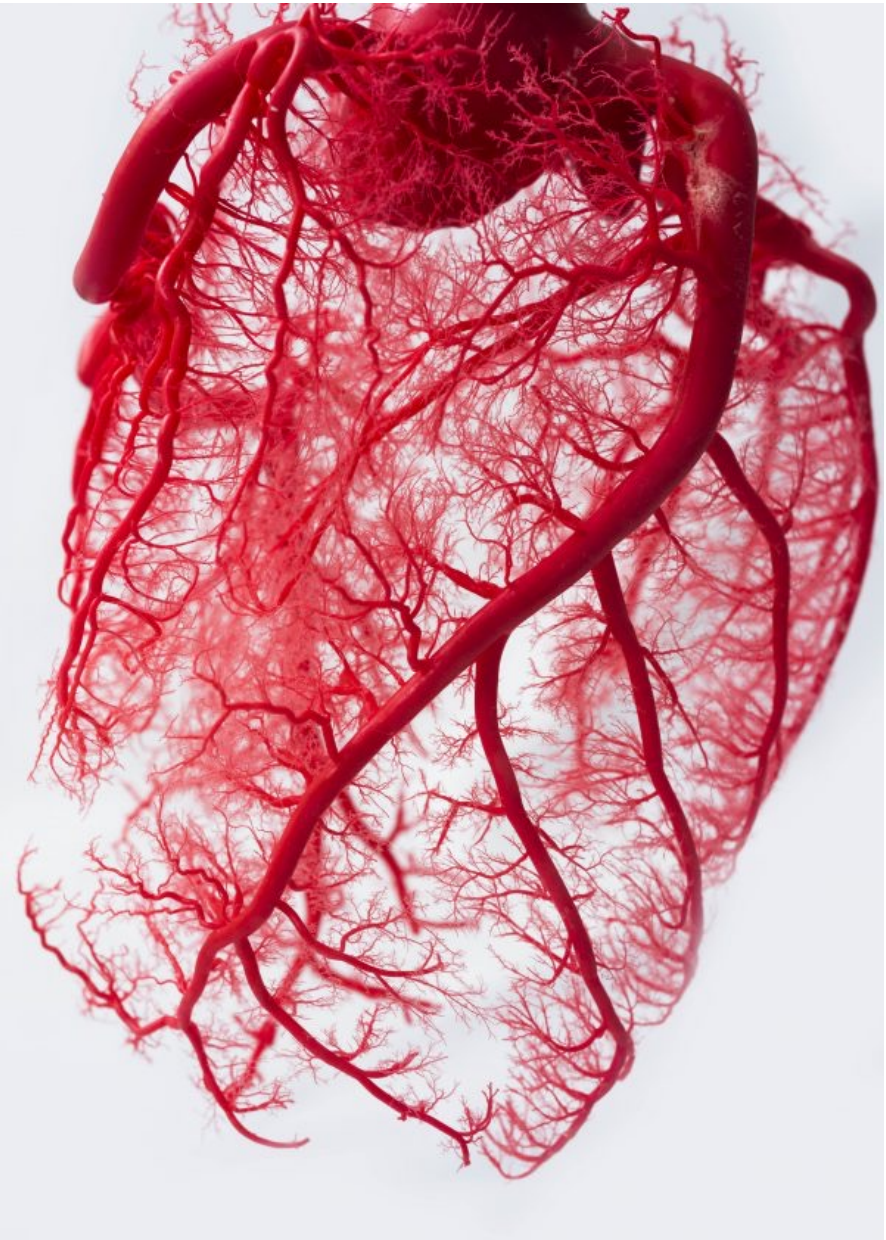
BACKGROUND- Self-Similarity

- **A geometrical interpretation** is that small parts of an object (exactly) resemble the shape formed by the entire object
  - Scale-free
  - Inherent roughness


by Vladimir Ilievski

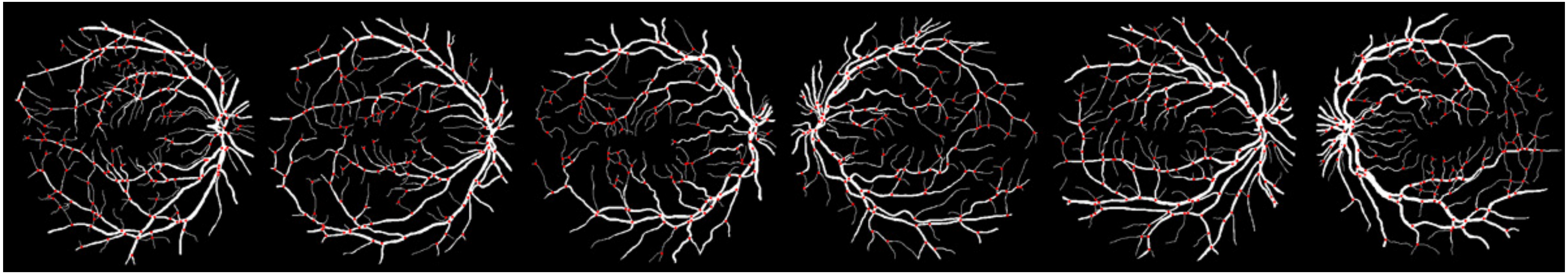






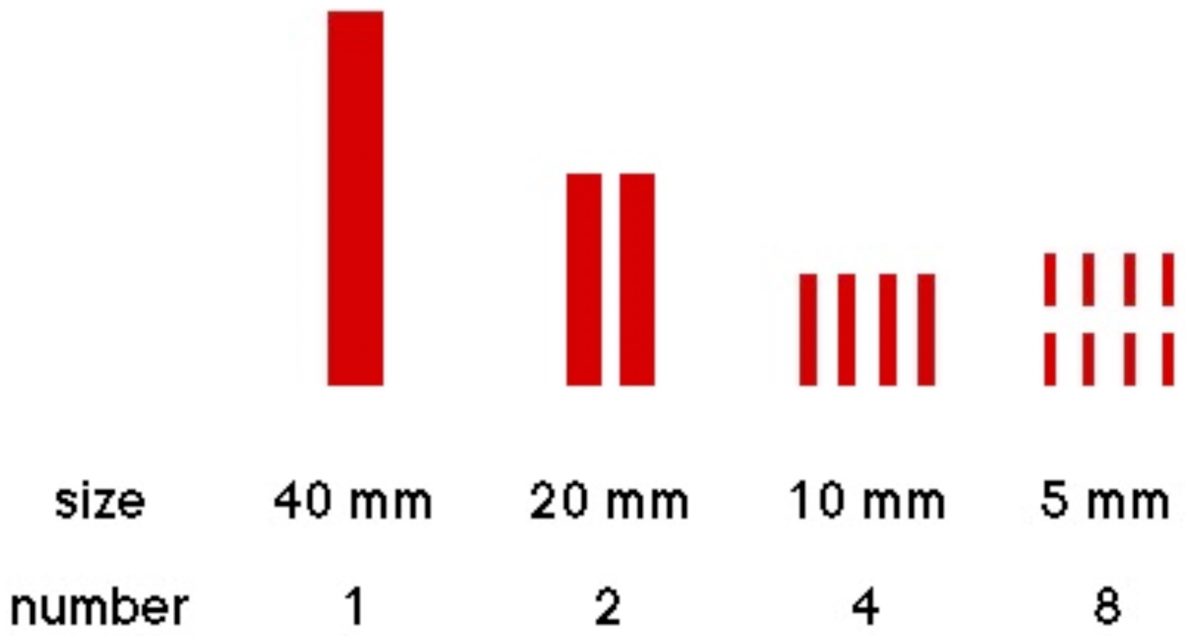
BACKGROUND- Self-Similarity

- **A statistical interpretation** is that spatial limitations may prevent infinite expansion, therefore geometric self similarity may not exist, but statistical self similarity exists.
- It may not always **LOOK** fractal but it may **BE** fractal

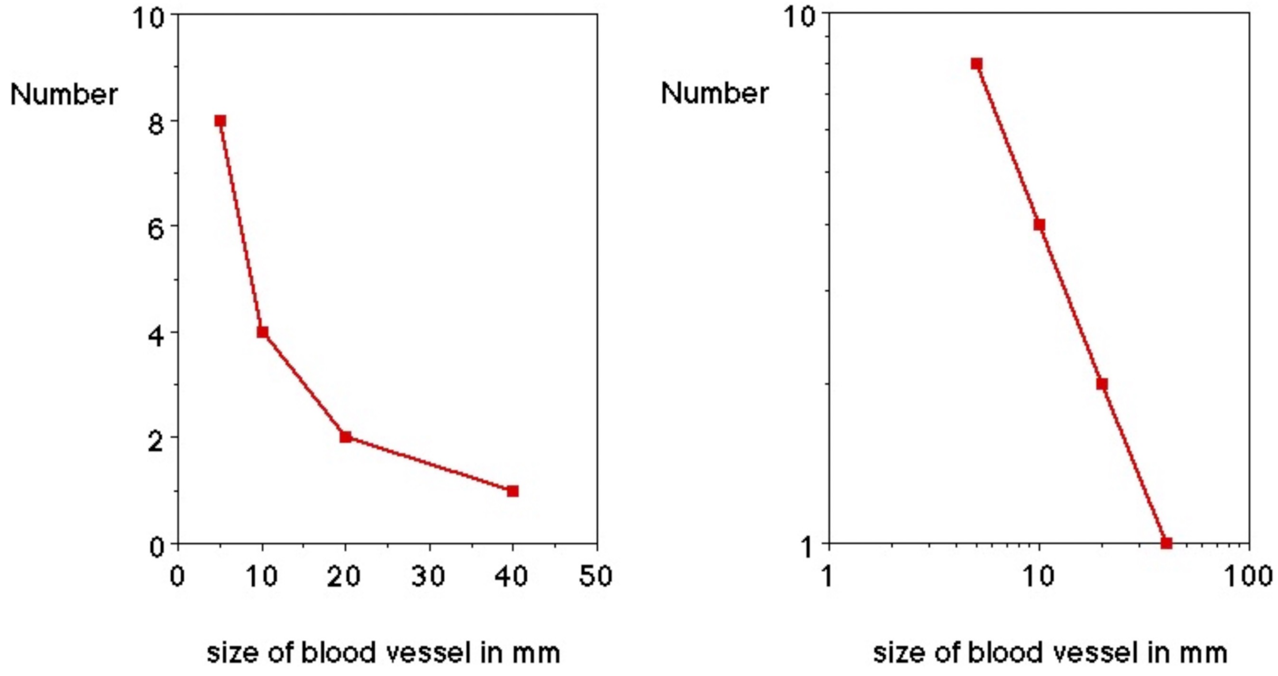


BACKGROUND- Self-Similarity

Retinal Blood Vessels

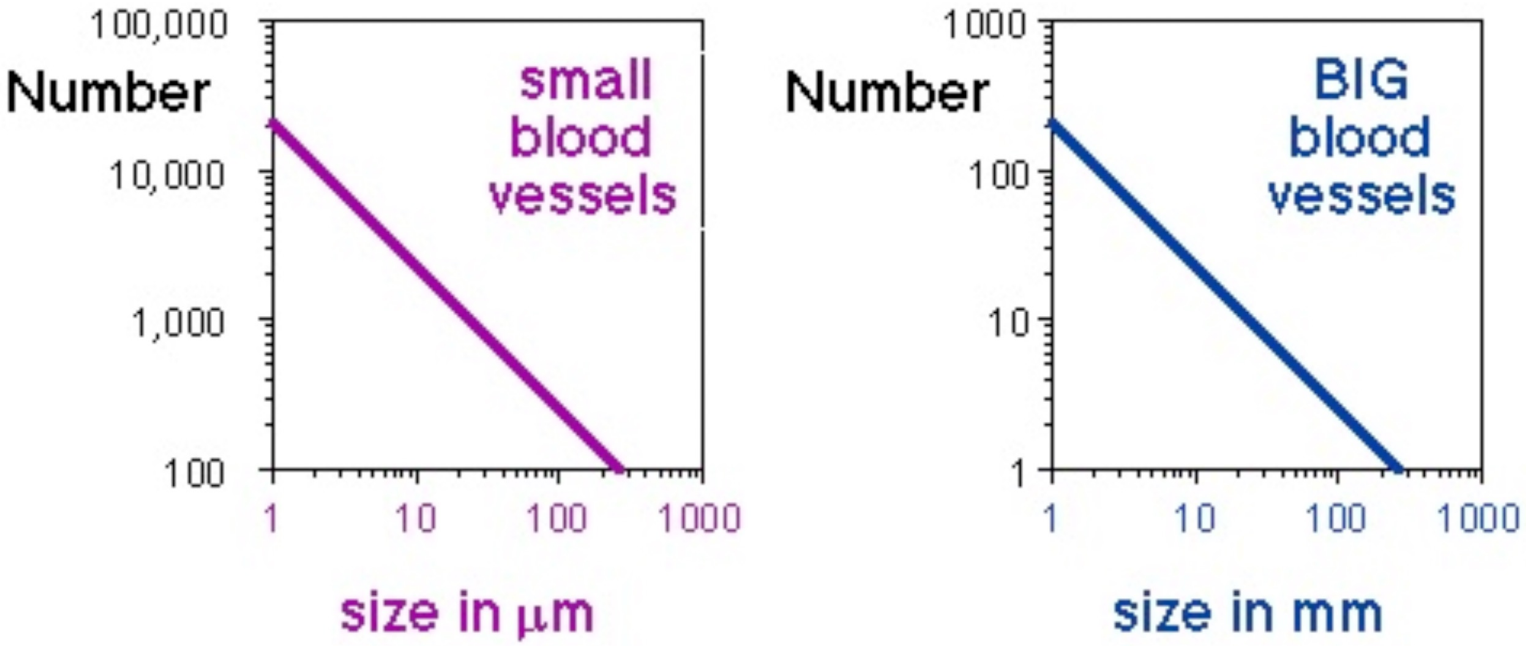


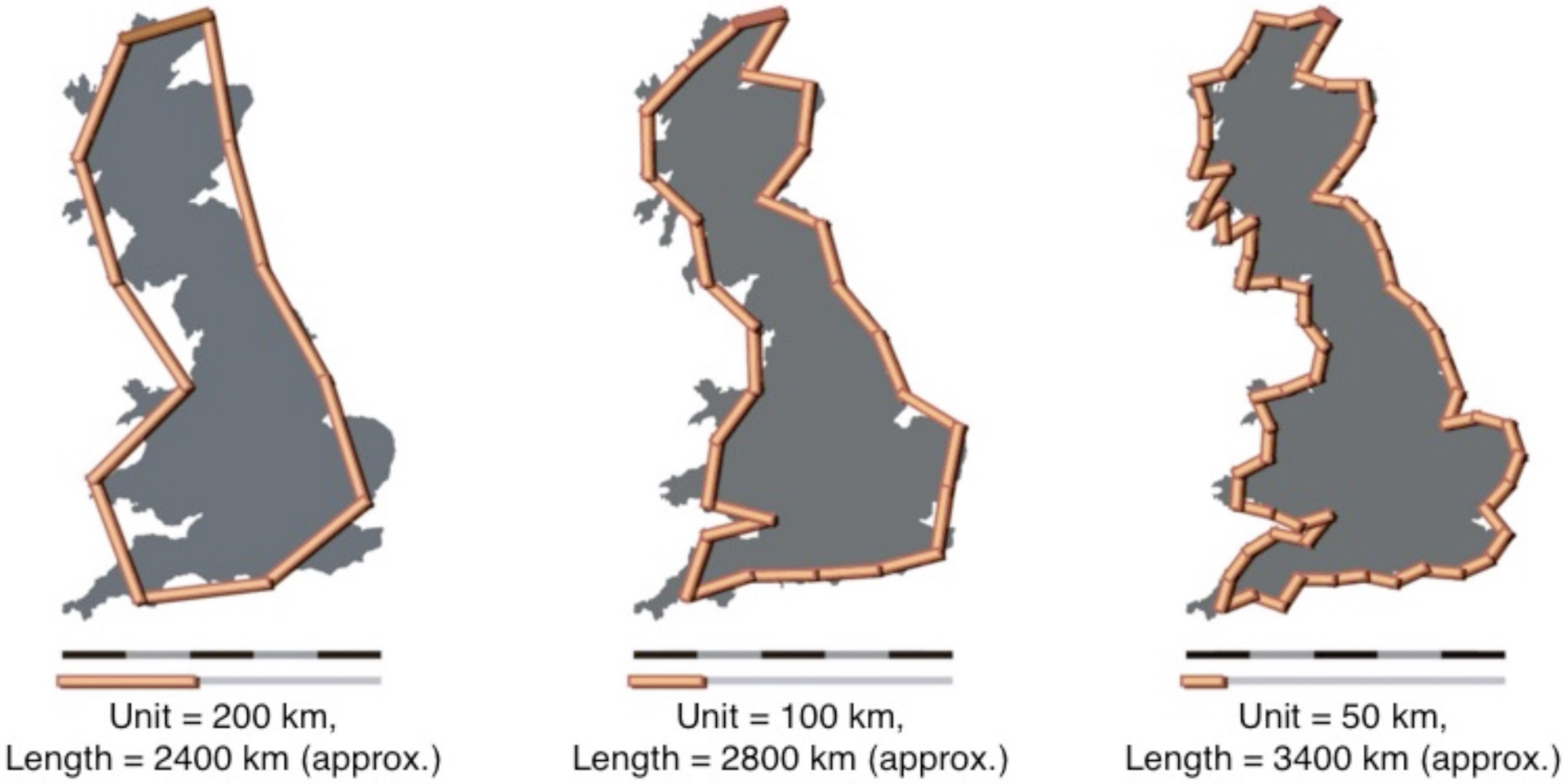
Probability Density Function (PDF)



BACKGROUND- Self-Similarity/Scaling

- Meaningful information is lost using a single resolution
- Our analysis is on this scaling relationship!





How long is the coast of Britain?
Mandelbrot, 1967, Science

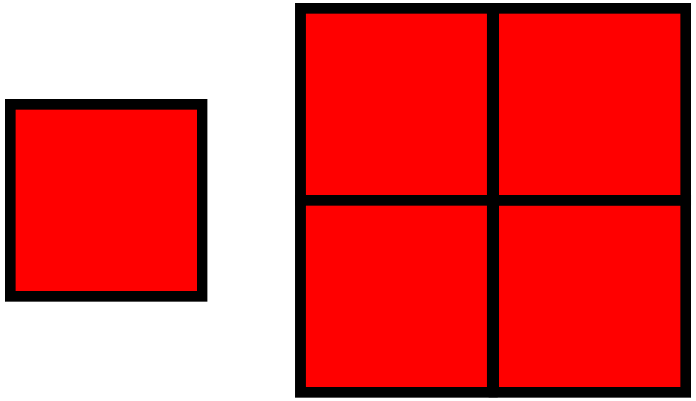
BACKGROUND- Dimension

- The **fractal dimension** provides a quantitative measurement of self-similarity and scaling.
- Characterizes how the fractal object fills space

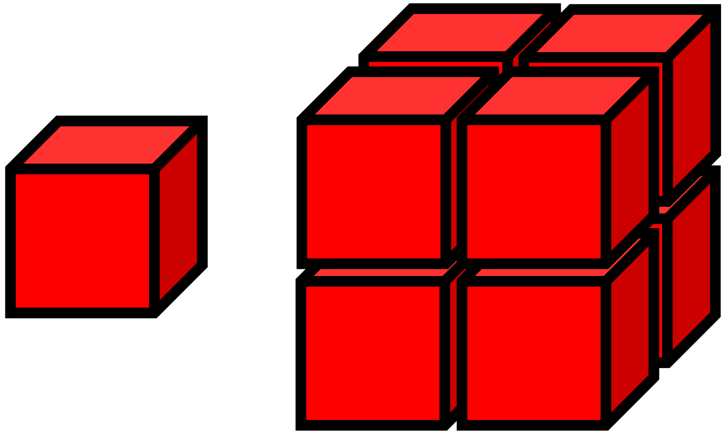
$$\text{Fractal Dimension (D)} = \frac{\ln N}{\ln R} = \frac{N\ Number\ of\ figures}{R\ times\ larger}$$

BACKGROUND- Dimension

$$\text{Fractal Dimension (D)} = \frac{\ln N}{\ln R} = \frac{N\ Number\ of\ figures}{R\ times\ larger}$$



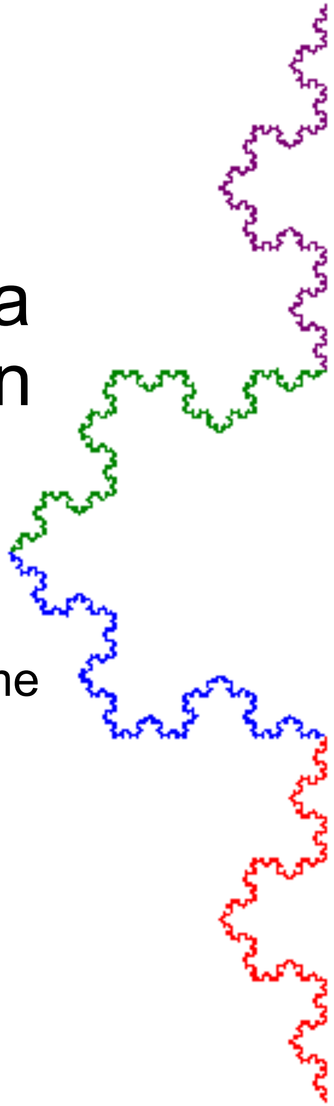
$$D = \frac{\ln 4}{\ln 2} = \frac{2}{1} = 2$$



$$D = \frac{\ln 8}{\ln 2} = \frac{3}{1} = 3$$

BACKGROUND- Dimension

- A **fractal** is an object in space or process in time that has a fractal dimension (D) greater than its topological dimension
  - Example: D = 1.26
    - Since D > 1, it covers more than a 1-D line, but less than a 2-D area
  - Topological dimension is 1
    - This dimension tells us what the object is, such as an edge, surface, or volume
  - When fractal dimension > topological dimension
    - The edge, surface, or volume has more finer pieces than we would have expected of an object with its topological dimension



BACKGROUND- Statistical Properties

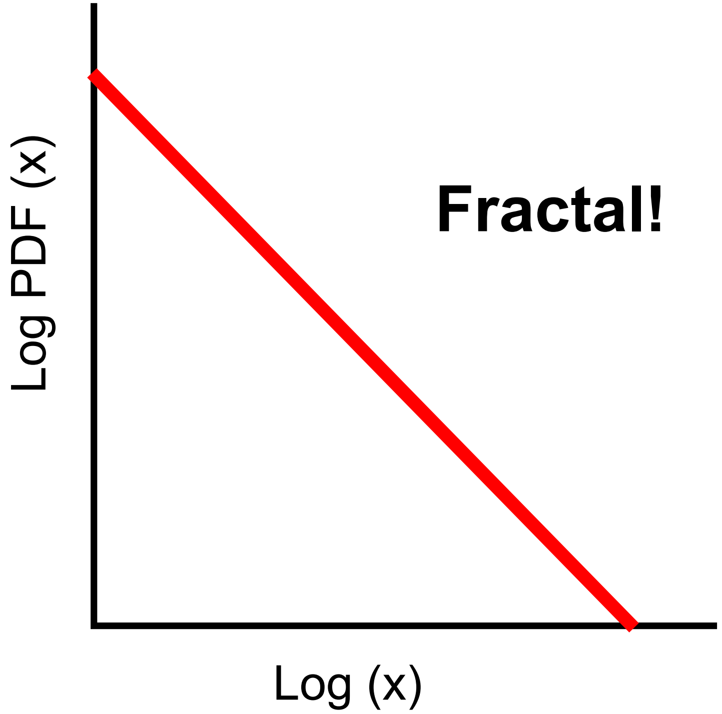
Fractal or Not Fractal?

How can you tell?

BACKGROUND- Statistical Properties

Fractal or Not Fractal?

How can you tell?



BACKGROUND- Statistical Properties

Fractal or Not Fractal?

How can you tell?

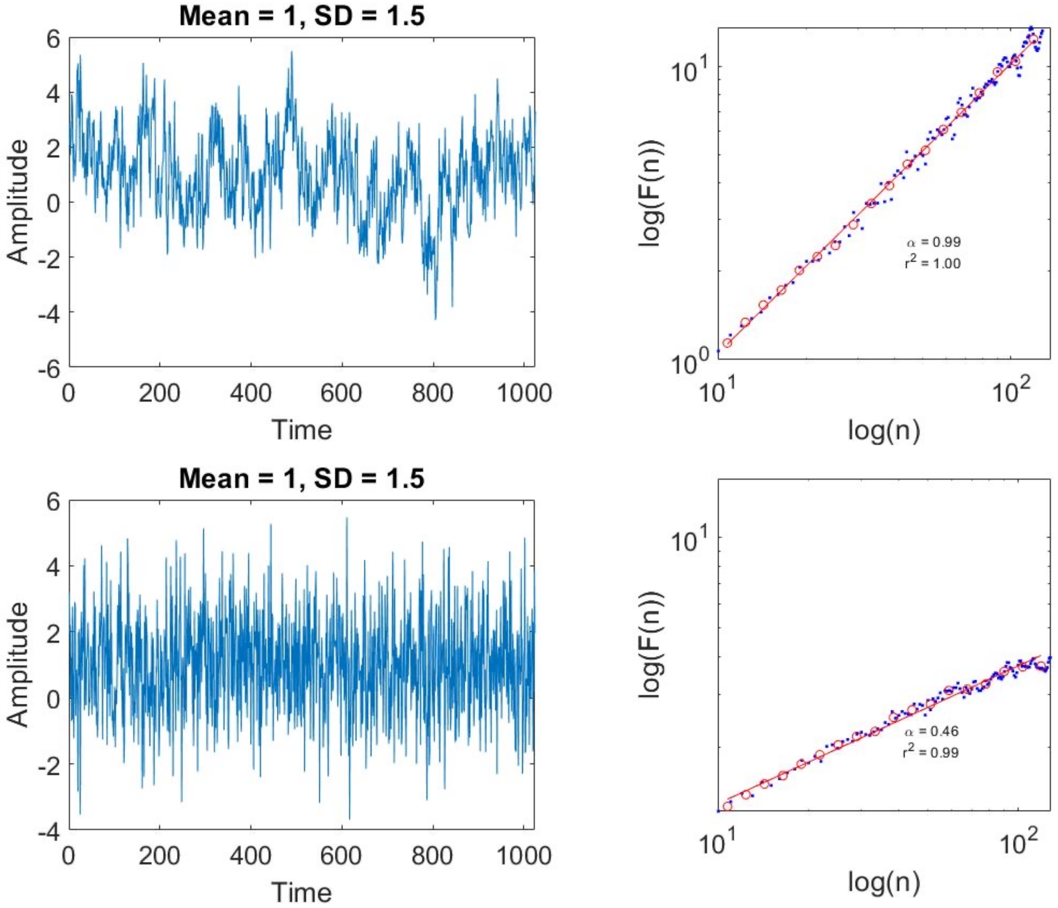
Where can it be found and what does it tell us?

- Fractal patterns/behavior exists in:
  - DNA nucleotides
  - Neural patterns
  - Cardiovascular system
  - Heart rate
  - Inter-step intervals
  - Inter-organism communication
  - Geographical features
  - Weather patterns

- Fractal patterns/behavior can distinguish between healthy and unhealthy patients in:
  - **Center of Pressure**
  - **Gait**
  - Heart rate
  - Huntington's
  - Parkinson's
  - Alzheimers
  - ADHD

What does this complex structure tell us?

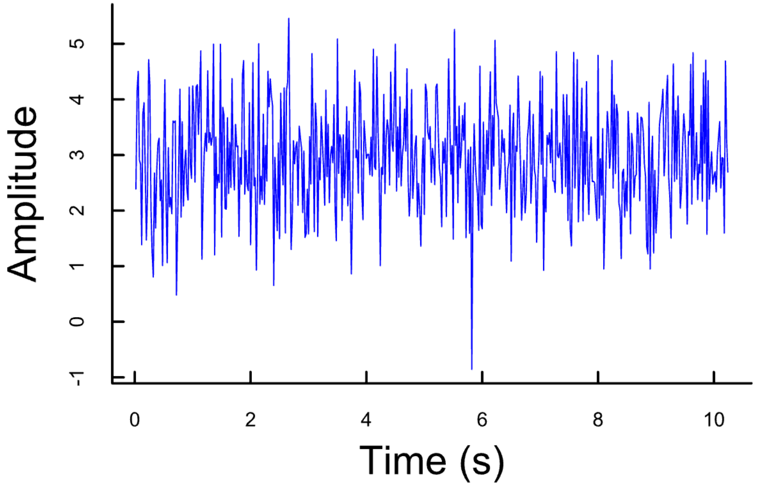
- Characteristics that may not be present from traditional perspectives (i.e. Mean/SD)
- We can look at changes overtime in a system at different time-scales
- **Fractal analysis is measuring how some measure of variability changes as a function of scale**

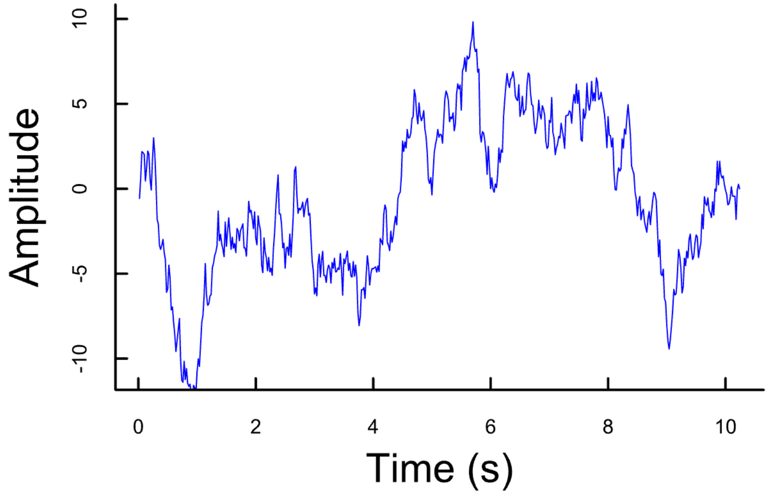


How can we capture this complex structure?

- Fractal analysis techniques
  - **Monofractal Detrended Fluctuation Analysis (DFA)** (Peng et al 1994)
    - **A method to determine the statistical self-similarity of long time-series that may have memory**
  - Multifractal Detrended Fluctuation Analysis (MFDFA)
  - Fractal Regression
  - And many more!

DFA ANALYSIS STEP BY STEP

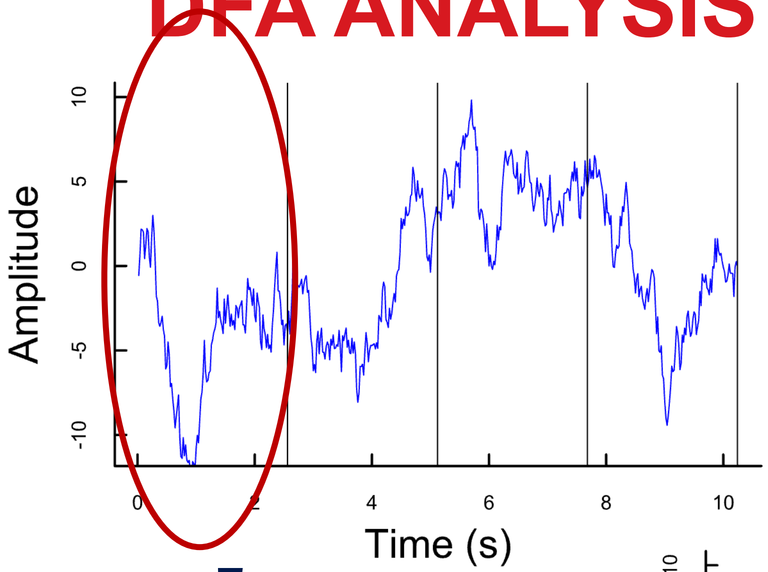


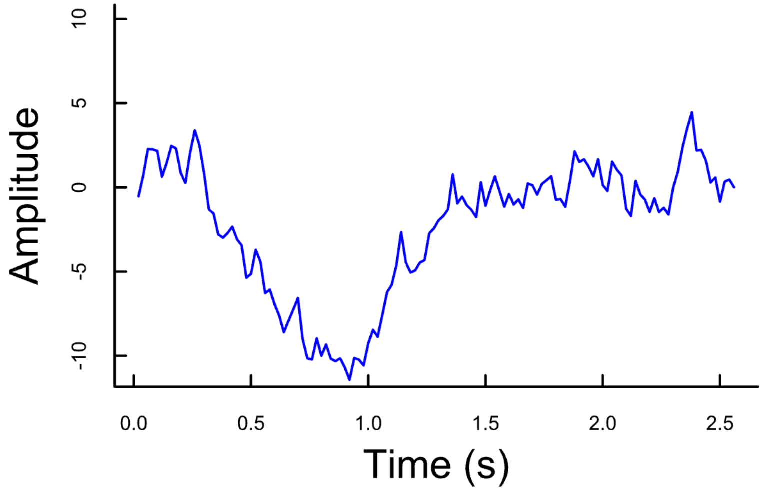


Step 1

- Create a profile of a time series:
  - subtracting its mean from each data point
  - Integrate the time series
- This step allows the conversion of the time series to random walk-like process that meets the theoretical assumptions of DFA

DFA ANALYSIS STEP BY STEP



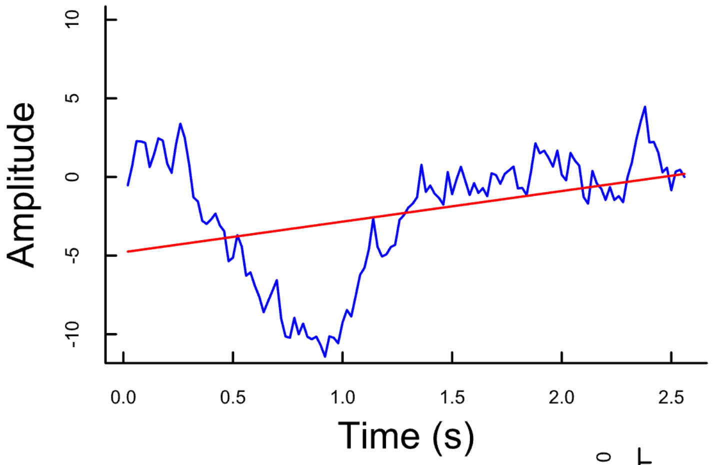


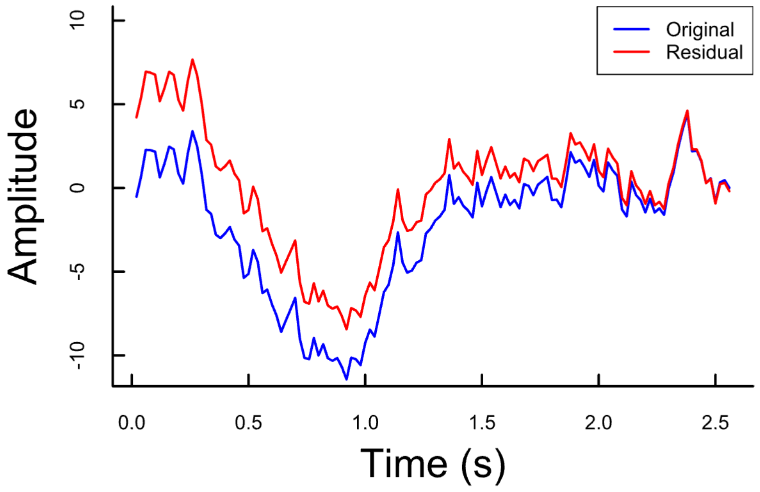
<u>Step 2</u>

- Divide the series into a sequence of non-overlapping windows.
- In this example, the time series was separated into 4 windows (Top).

→ Zooming in

DFA ANALYSIS STEP BY STEP





<u>Step 3</u>

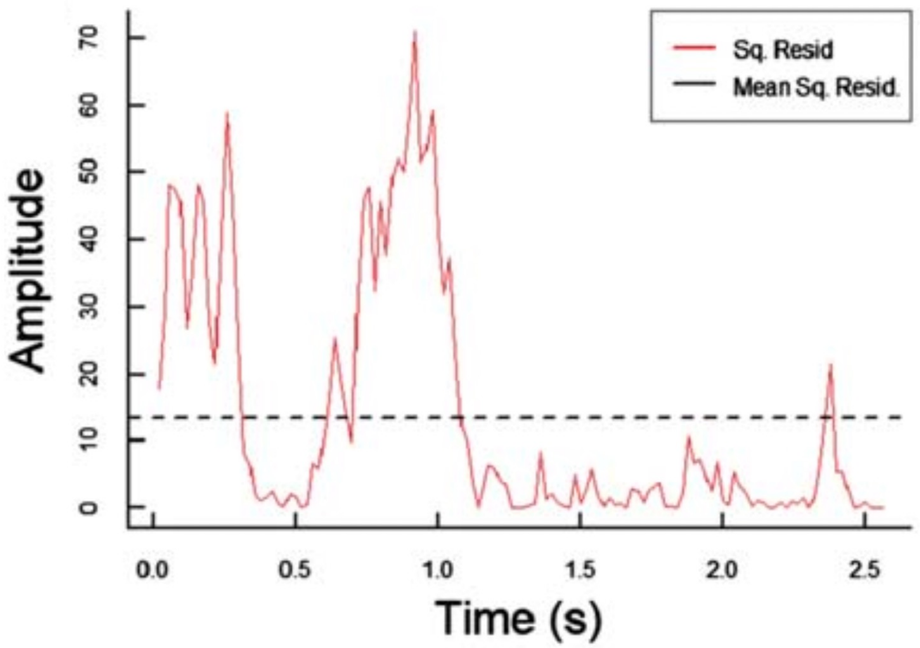
Detrending stage:

- This means fitting a regression line within each of the windows (Top)
- Then, we subtract the fitted trend line from the data in each window (Bottom)

DFA ANALYSIS STEP BY STEP

Step 4

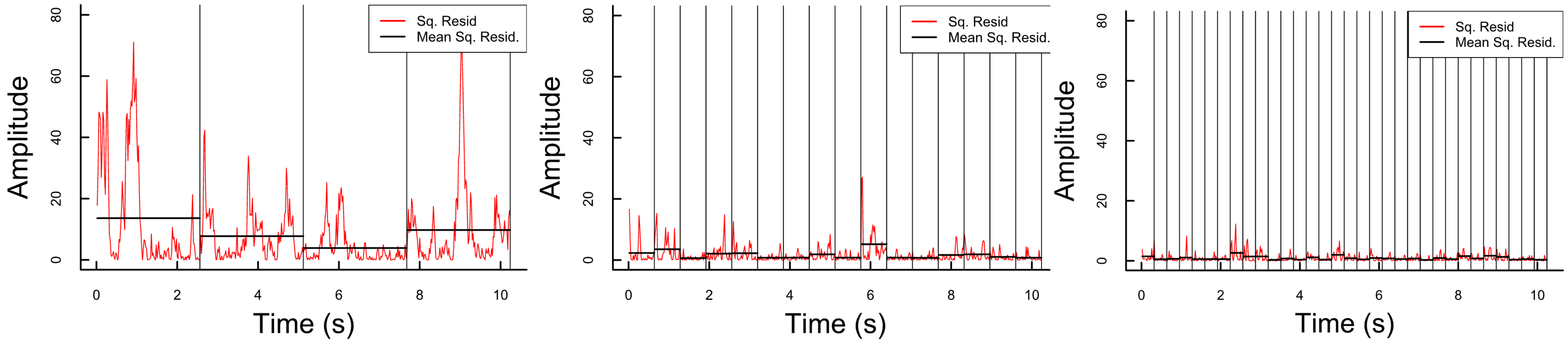
- Find the average fluctuation F(s) by taking the root mean square of the local residuals within each scale.



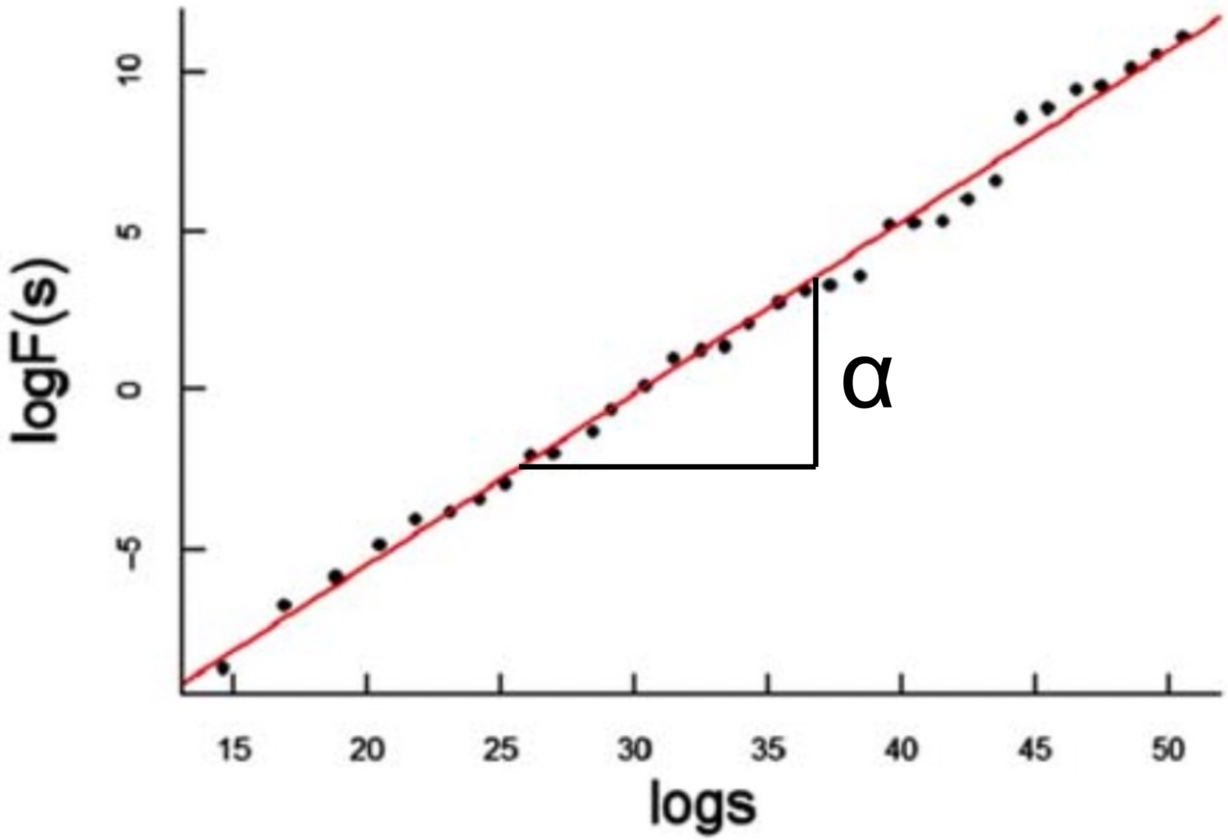
DFA ANALYSIS STEP BY STEP

Step 4

- Find the average fluctuation F(s) by taking the root mean square of the local residuals within each scale.



DFA ANALYSIS STEP BY STEP



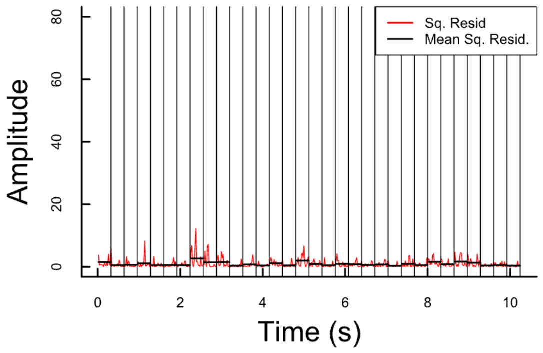
Step 5

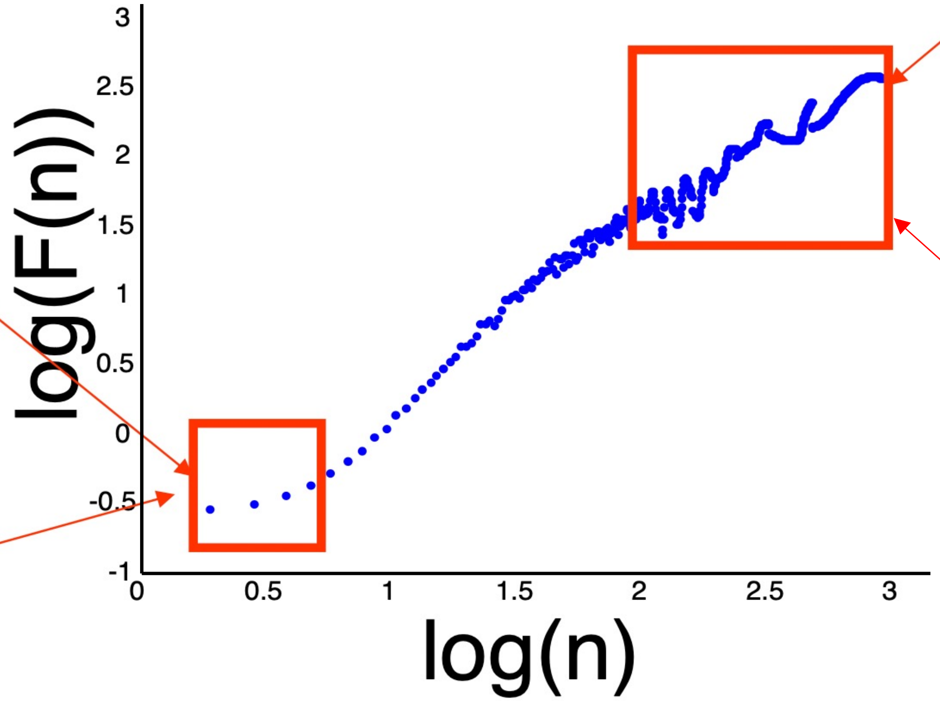
- Perform a regression to estimate the scaling exponent α
- α is a measure of how fast the standard deviation changes as a function of timescale

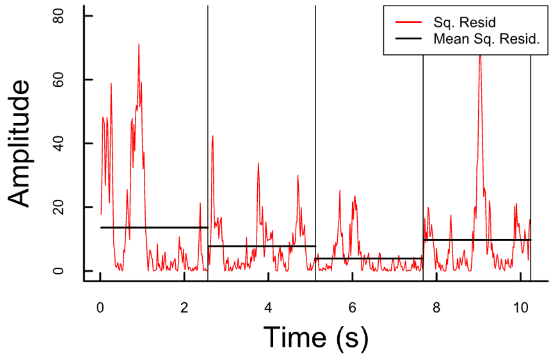
DFA ANALYSIS Interpretation

- Many small windows
- SD's computed from only a few points
- Statistically less reliable

- Few large windows
- Fewer windows to compute average fluctuations
- More variable

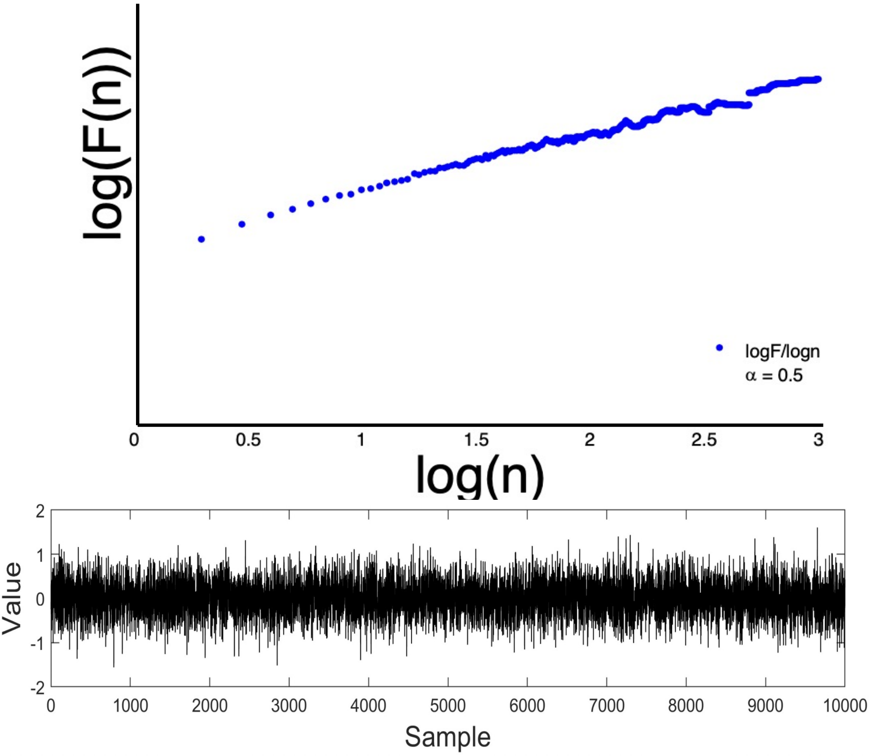


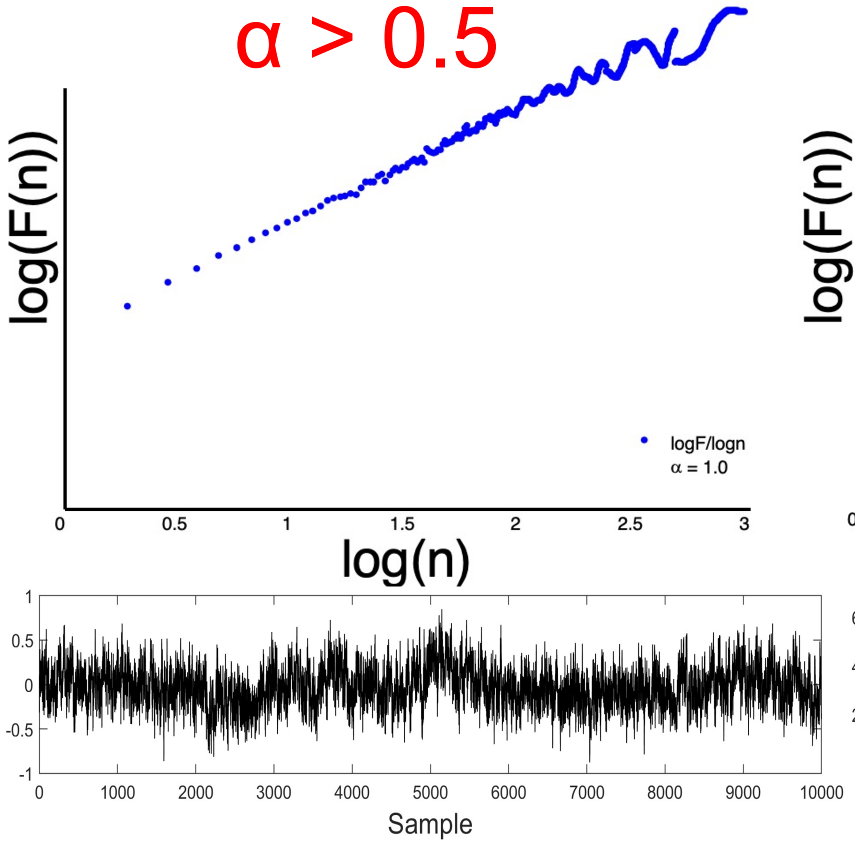


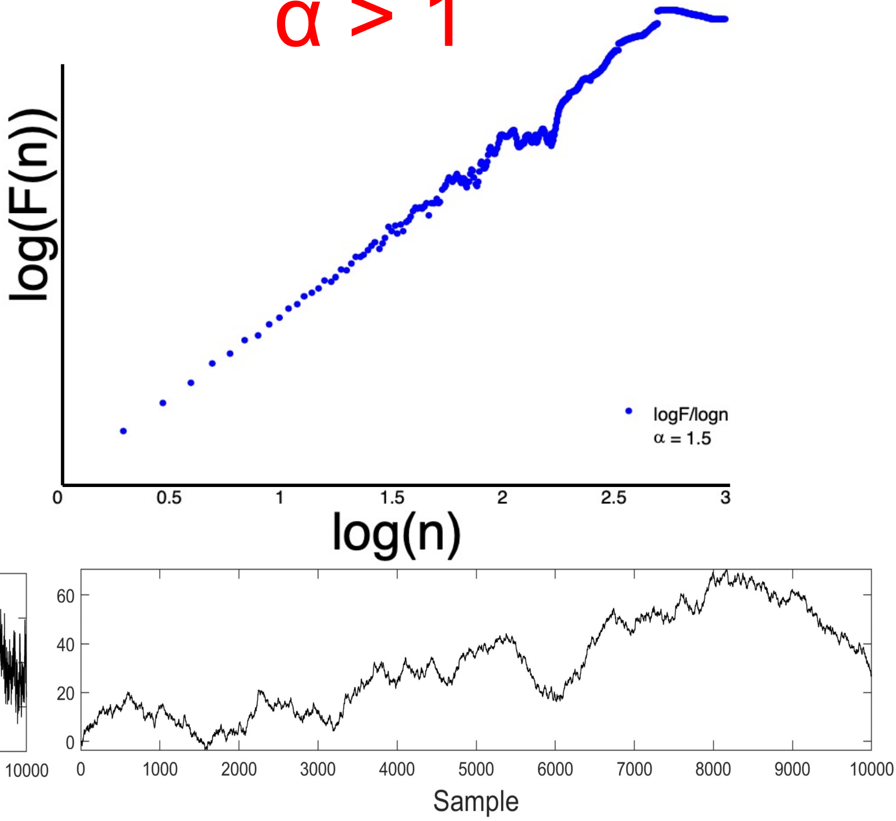


DFA ANALYSIS Interpretation

Negatively Correlated	Positively Correlated	Nonstationary/Unbounded
α < 0.5	α > 0.5	α > 1

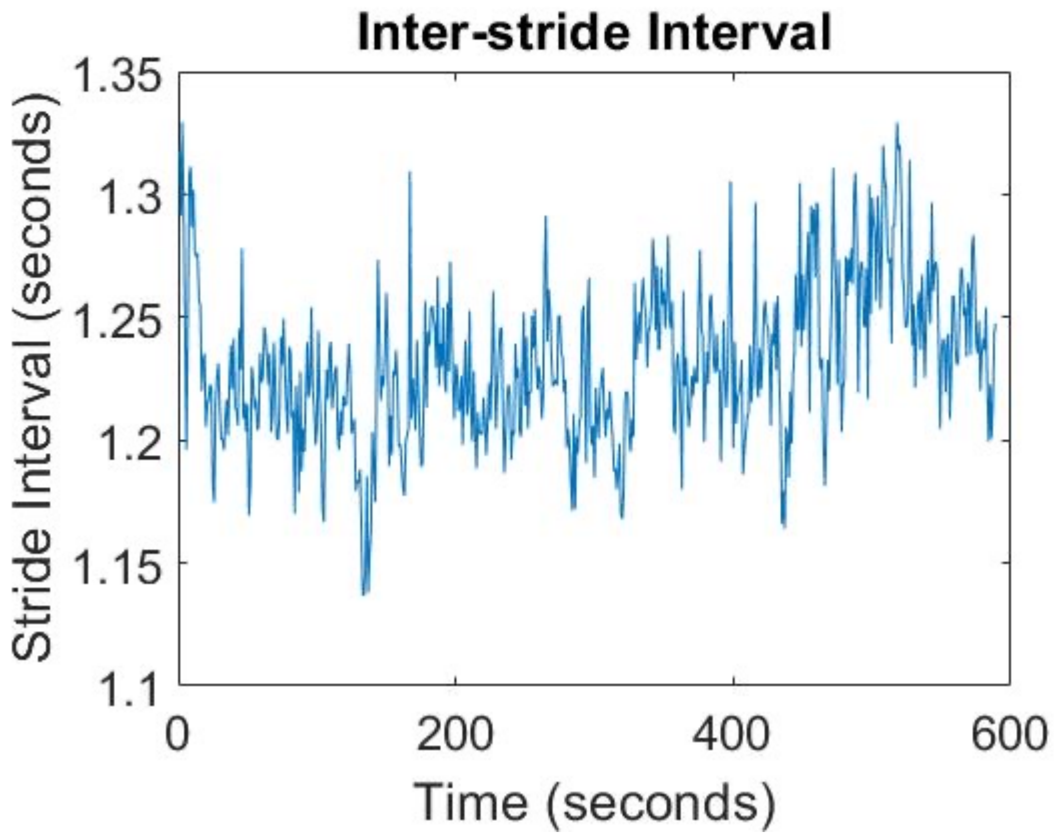






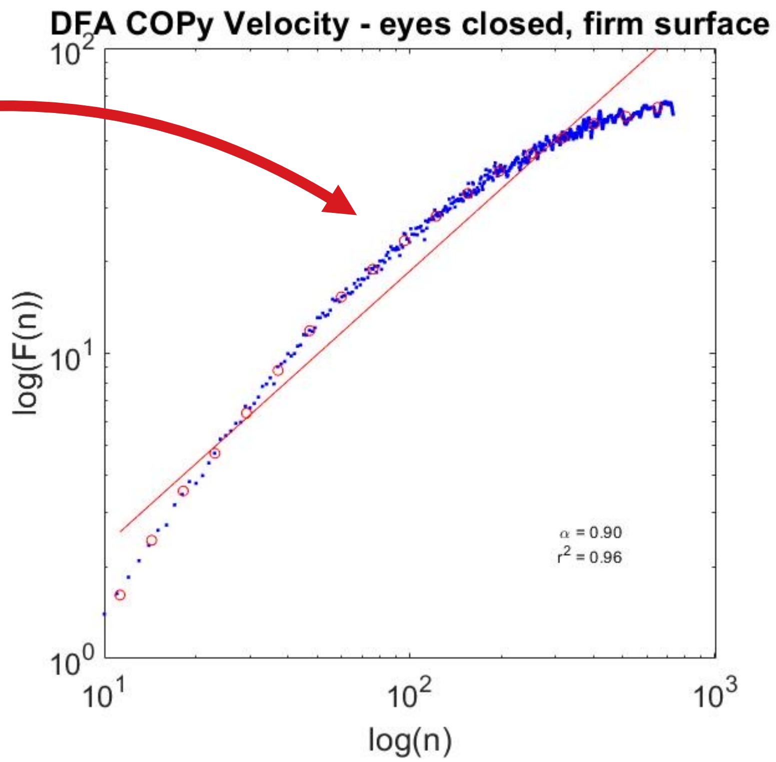
DFA Best Practices

- Plot your data
  - Visually inspect for:
    - ↑ or ↓ trends
    - Structure or roughness
- Time series length
  - More is almost always better
  - ≥ 500 datapoints for stable α estimates
    - i.e. 500 steps, taps, beats
    - Options exist for shorter time series



DFA Best Practices

- Investigating crossovers
  - Crossovers of short/long-term scaling behaviors
  - Begin DFA at lowest order
    - Inspect logF(n) x log(n) plot for crossover
      - Repeat DFA with higher order if crossover exists
- Timescales
  - Minimum = 16 data points
  - Maximum = $\frac{Length\ of\ series}{9}$



*******5 min Break*******

Github link: https://github.com/aaronlikens/ISPGR-2022.git

MATLAB VERSION

There are no known incompatibilities using MATLAB version R2019a or later.

MATLAB Toolboxes Required:

Statistics and Machine Learning Toolbox

Signal Processing Toolbox

Image Processing Toolbox

Images

- By Created by Wolfgang Beyer with the program Ultra Fractal 3. - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=321973
- https://phys.org/news/2017-03-fractal-patterns-nature-art-aesthetically.html
- https://larryriddle.agnesscott.org/ifs/kcurve/kcurve.htm
- By Beojan Stanislaus, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=8862246
- https://www.researchgate.net/figure/The-first-stages-of-the-construction-of-von-Koch-curve-and-a-random-version-of-the-curve_fig1_329695536
- https://isquared.digital/visualizations/2020-06-15-koch-curve/
- https://iternal.us/what-is-a-fractal/
- https://webvision.med.utah.edu/tag/retinal-vasculature/
- By Ivar Leidus - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=97556224

Papers on polynomial order

- Kantelhardt, J. W., Koscielny-Bunde, E., Rego, H. H. A., Havlin, S., & Bunde, A. (2001). Detecting long-range correlations with detrended fluctuation analysis. Physica A: Statistical Mechanics and Its Applications, 295(3), 441454. Available from https://doi.org/10.1016/S0378-4371(01)00144-3.
- Likens, A. D., Fine, J. M., Amazeen, E. L., & Amazeen, P. G. (2015). Experimental control of scal- ing behavior: What is not fractal? Experimental Brain Research, 233(10), 28132821.

Peng paper on DFA

- Peng, C.-K., S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, and A. L. Goldberger. “Mosaic Organization of DNA Nucleotides.” *Physical Review E* 49, no. 2 (February 1, 1994): 1685–89. https://doi.org/10.1103/PhysRevE.49.1685.

Papers on time series length

- Delignieres, D., Ramdani, S., Lemoine, L., Torre, K., Fortes, M., & Ninot, G. (2006). Fractal analyses for ‘short’ time series: A re-assessment of classical methods. Journal of Mathematical Psychology, 50(6), 525544. Available from https://doi.org/10.1016/j.jmp.2006.07.004.
- Stroe-Kunold, E., Stadnytska, T., Werner, J., & Braun, S. (2009). Estimating long-range dependence in time series: An evaluation of estimators implemented in R. Behavior Research Methods, 41(3), 909923. Available from https://doi.org/10.3758/BRM.41.3.909.
- Marmelat, V., & Meidinger, R. L. (2019). Fractal analysis of gait in people with Parkinson’s disease: Three minutes is not enough. Gait & Posture, 70, 229234. Available from https://doi.org/10.1016/j.gaitpost.2019.02.023.
- Yuan, Q., Gu, C., Weng, T., & Yang, H. (2018). Unbiased detrended fluctuation analysis: Long-range correlations in very short time series. Physica A: Statistical Mechanics and Its Applications, 505, 179189. Available from https://doi.org/10.1016/j.physa.2018.03.043.

Papers on time scales

- Almurad, Z. M. H., & Delignières, D. (2016). Evenly spacing in detrended fluctuation analysis. Physica A: Statistical Mechanics and Its Applications, 451, 6369. Available from https://doi.org/10.1016/j.physa.2015.12.155.
- Likens, A. D., Fine, J. M., Amazeen, E. L., & Amazeen, P. G. (2015). Experimental control of scaling behavior: What is not fractal? Experimental Brain Research, 233(10), 28132821.
- Yuan, Q., Gu, C., Weng, T., & Yang, H. (2018). Unbiased detrended fluctuation analysis: Long-range correlations in very short time series. Physica A: Statistical Mechanics and Its Applications, 505, 179189. Available from https://doi.org/10.1016/j.physa.2018.03.043.

Part II: MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS

Anaelle E. Charles

Outline:

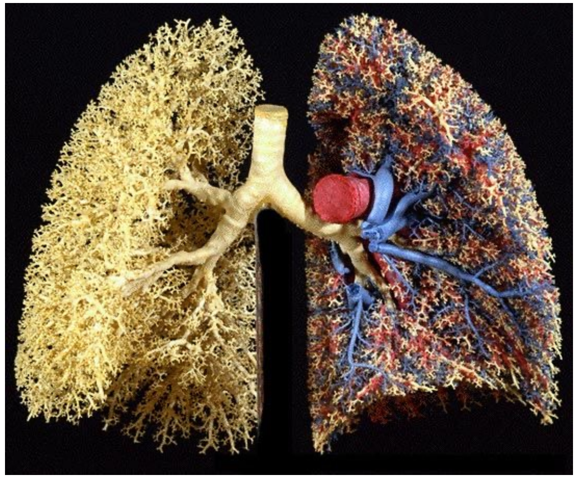
Part II: Multifractal Analysis

i. From monofractal to multifractal analysis
ii. Multifractal Detrended Fluctuations Analysis step by step
iii. MATLAB Tutorial

Brief Recap on Fractals

Aloe Vera plant

Fractals are ubiquitous in nature



Many natural structures exhibit ***self-similarity***, where their structure repeat itself over many scales

Romanesco broccoli

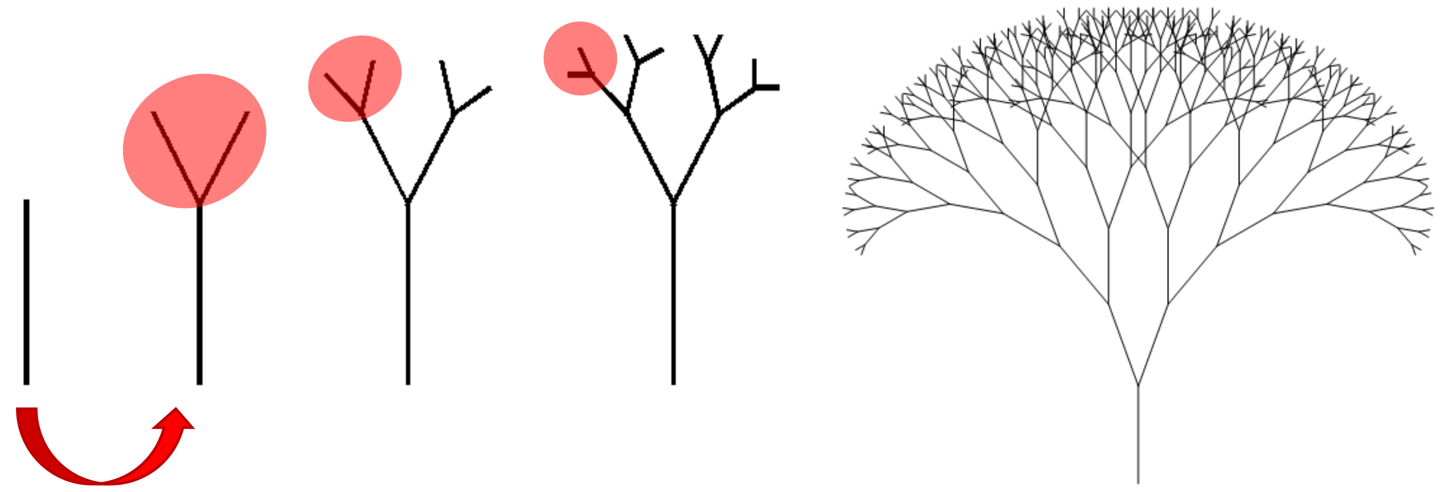
Brief Recap on Fractals

In fact, ***self-similarity*** can be easily illustrated through very simple mathematical rules.

RULES:

1. **Create a vertical line**
2. **Break that line in half and add it on top**
3. **Repeat**

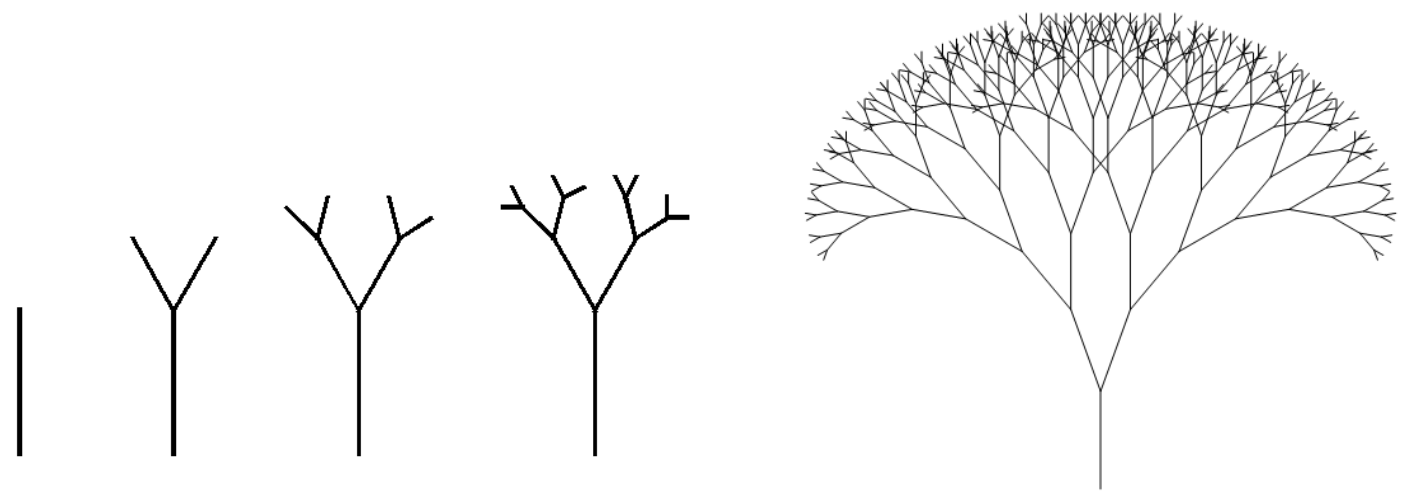
From a very simple set of rules, you get a complex structure



Fractal properties: *self-similarity*, *scale invariance*, *roughness*.

Brief Recap on Fractals

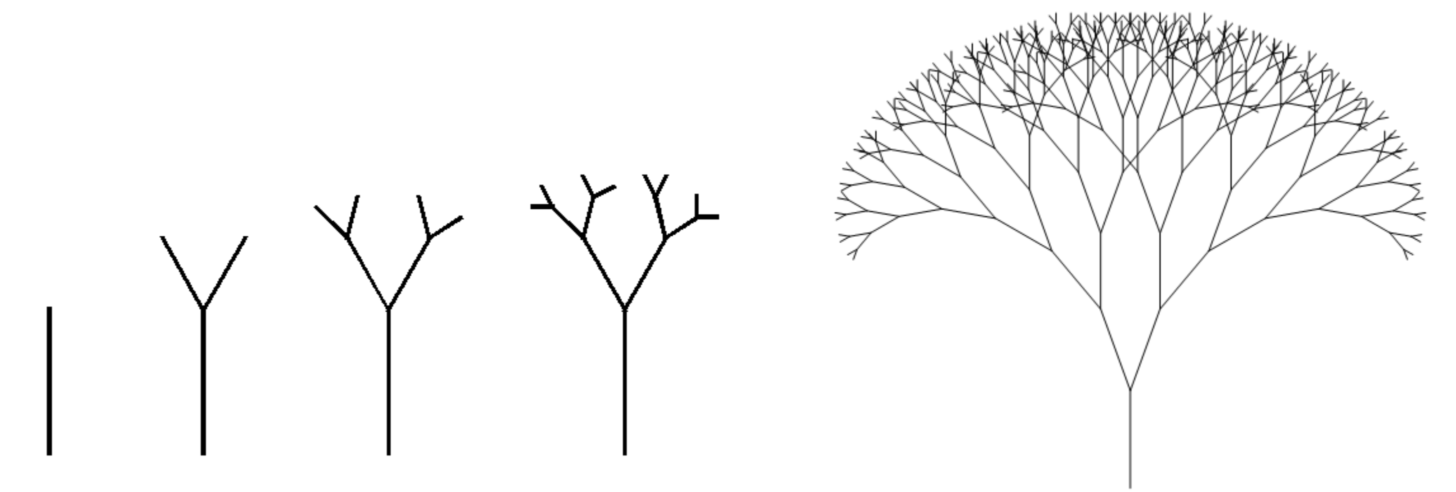
What are differences that you observe between those two pictures?



?

Brief Recap on Fractals

What are differences that you observe between those two pictures?

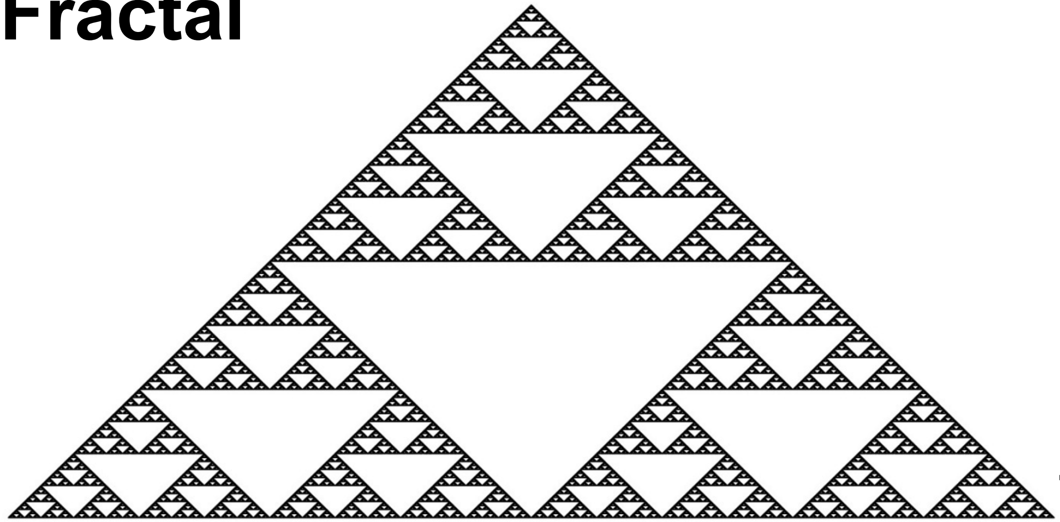


- There are some similarity, but nature is not quite perfect
- There is a physiological limit: trees don't grow forever

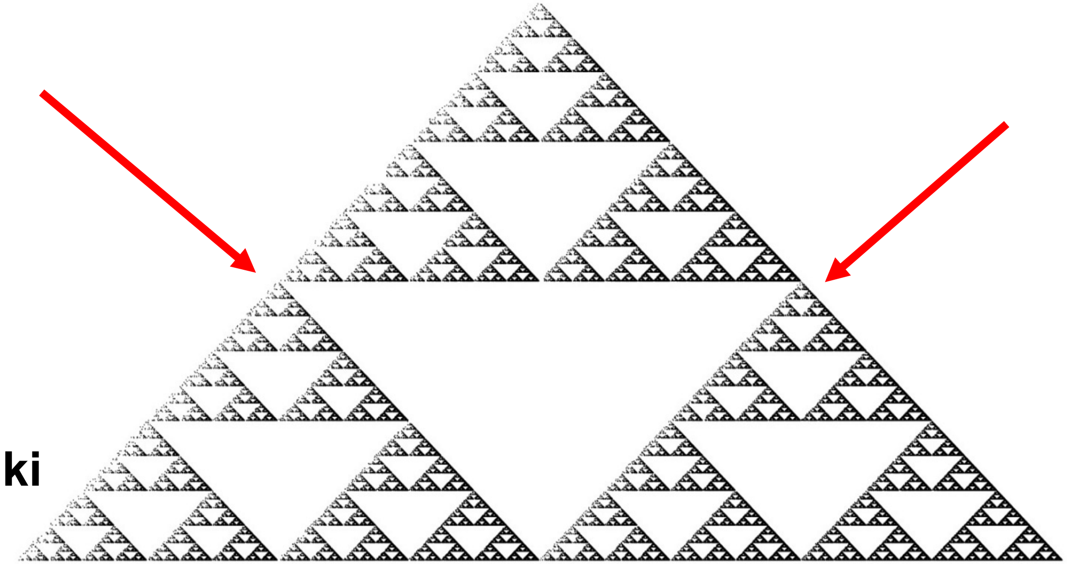
MULTIFRACTALS ARE SELF-AFFINE*

Here the scaling behavior is the same everywhere
→ **Fractal**

Here the scaling behavior depends on where you look
→ **Multifractal**



Sierpiński triangle



Multifractals have more than one scaling relationship, more than one scaling exponent

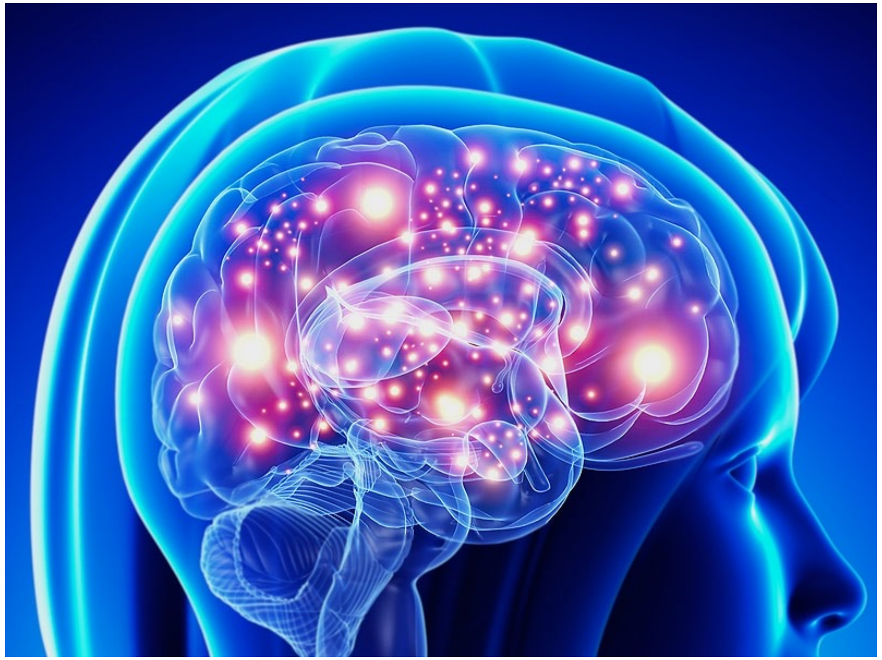
JUST LIKE THE EXAMPLE ABOVE OUR MOVEMENT IS NEVER PERFECTLY SIMILAR…

The surface you walk on is not always flat and stable

Your movement pattern constantly adapts to the environment

Movement patterns also vary from task to task (i.e., walk, run, …)

So…Does it really makes sense to only have a single strategy?



Monofractals are overly restrictive!!!

MULTIFRACTALS

- Multifractals are **<u>important</u>** because they characterize many aspects of human variability:

  - Entail coordination of multiple time scales
  - Reflect the time varying demands and adaptability

  - **Human gait**
  - **Upright posture**
  - Heart rate
  - Neural activity
  - Haptic perception
  - Reaction times
  - Speech production
  - Time estimation
  - Visual perception

Cavanaugh et al., 2017; Harrison & Stergiou, 2015; Ihlen (2013); Ihlen & Vereijken (2010); Ivanov et al., (2001); Likens et al., 2014; Palatinus et al., 2014; West & Scafetta (2003); Scafetta, Griffin, & West, (2003)

DFA vs. MFDFA

DFA

DFA is used to measure how **variance (2nd Statistical moment)** changes as a function of different time scales

MFDFA

Multifractal Detrended Fluctuation Analysis (**MFDFA**) is used to measure how **variance** and **other statistical moments** change as a function of different time scales

*Before we go further into the analysis, let's refresh our memory on a few earlier statistical concepts…

Who remembers the four main statistical moments from introductory statistics?

QUICK RECALL

Just like there are higher derivative orders:

$y = f(x)$	Position (Displacement)
$\frac{dy}{dx} = y' = f'(x)$	?
$\frac{d^2y}{dx^2} = y'' = f''(x)$	
$\frac{d^3y}{dx^3} = y''' = f'''(x)$	
$\frac{d^4y}{dx^4} = y^{(4)} = f^{(4)}(x)$	
$\frac{d^5y}{dx^5} = y^{(5)} = f^{(5)}(x)$	
$\frac{d^6y}{dx^6} = y^{(6)} = f^{(6)}(x)$	

Etc.

There are higher statistical moments:

1. ?
2.
3.
4.
5.
6. …

QUICK RECALL

Just like there are higher derivative orders:

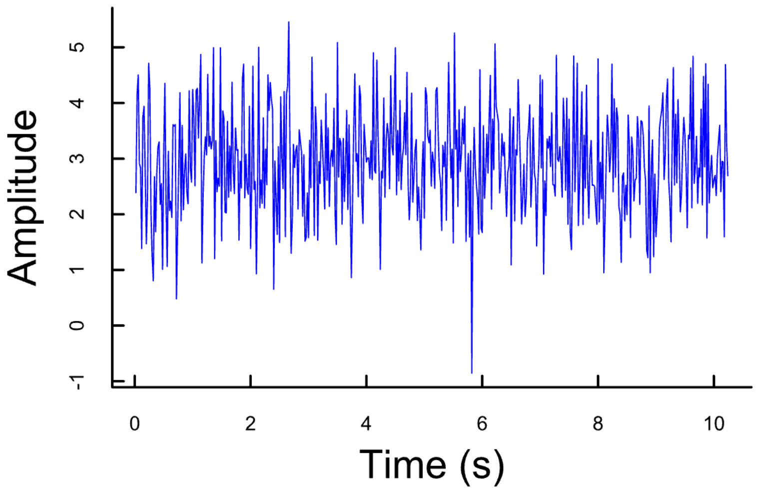
$y = f(x)$	Position (Displacement)
$\frac{dy}{dx} = y' = f'(x)$	Velocity
$\frac{d^2y}{dx^2} = y'' = f''(x)$	Acceleration
$\frac{d^3y}{dx^3} = y''' = f'''(x)$	Jerk
$\frac{d^4y}{dx^4} = y^{(4)} = f^{(4)}(x)$	Snap (Jounce)
$\frac{d^5y}{dx^5} = y^{(5)} = f^{(5)}(x)$	Crackle (Flounce)
$\frac{d^6y}{dx^6} = y^{(6)} = f^{(6)}(x)$	Pop (Pounce)

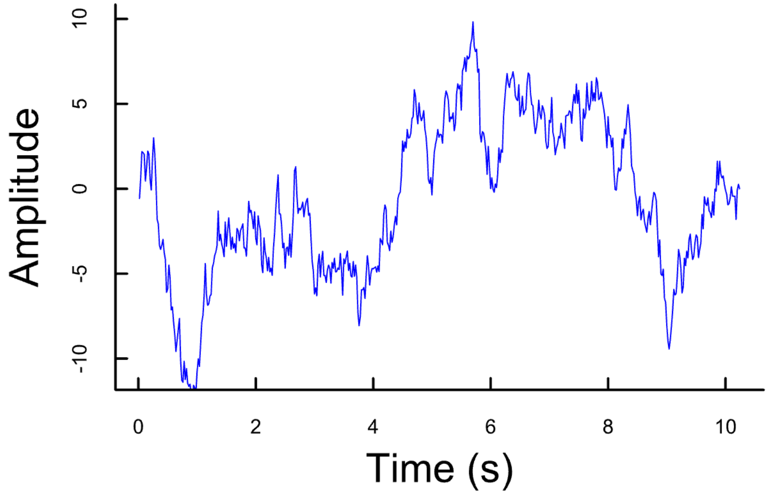
Etc.

There are higher statistical moments:

1. Mean
2. Variance
3. Skewness
4. Kurtosis
5. …
6. …

MFDFA ANALYSIS STEP BY STEP

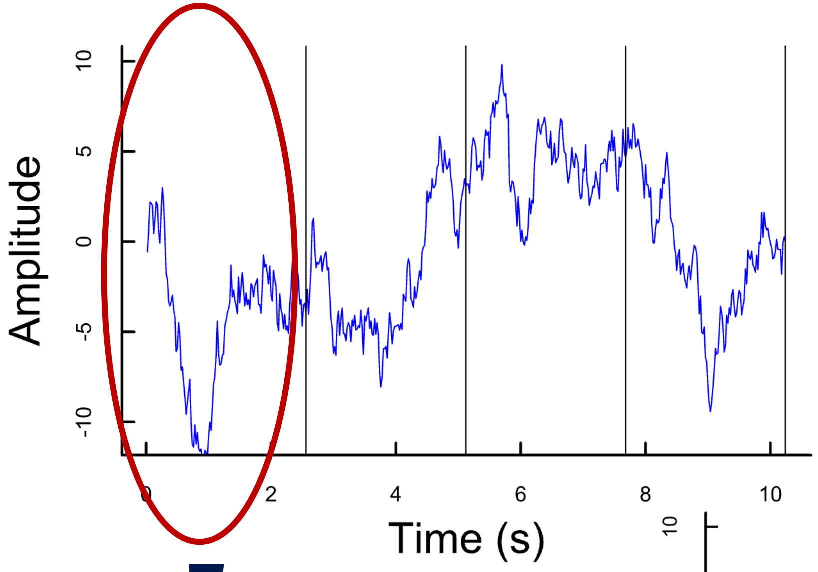


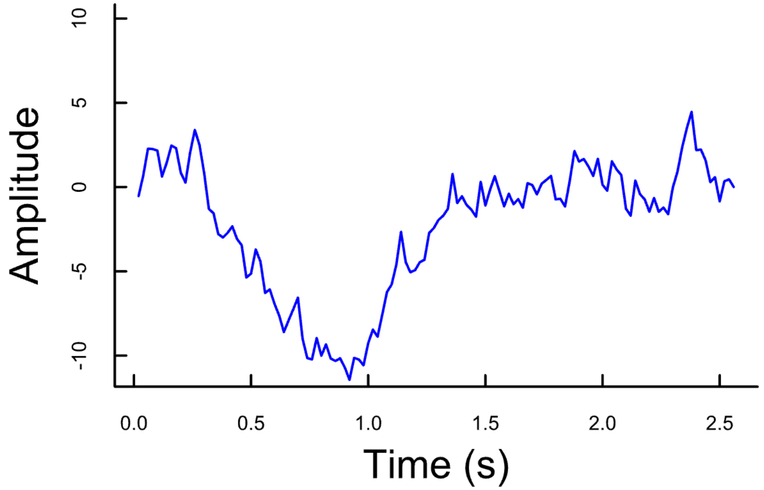


Step 1

- Create a profile of a time series:
  - subtracting its mean from each data point
  - Integrate the time series
- This step allows the conversion of the time series to random walk-like process that meets the theoretical assumptions of DFA

MFDFA ANALYSIS STEP BY STEP





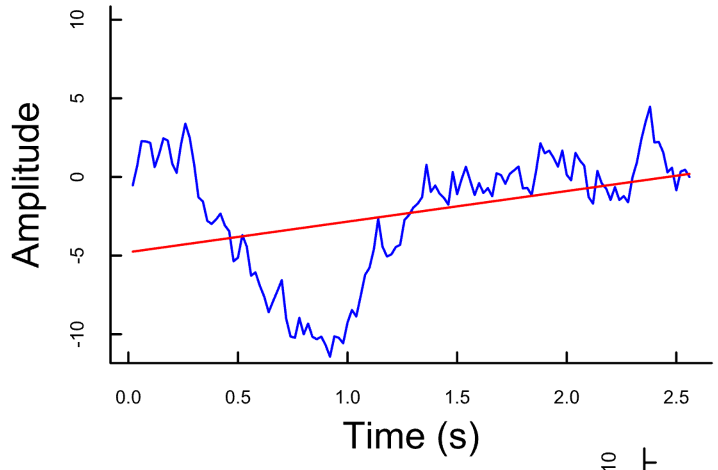
Step 2

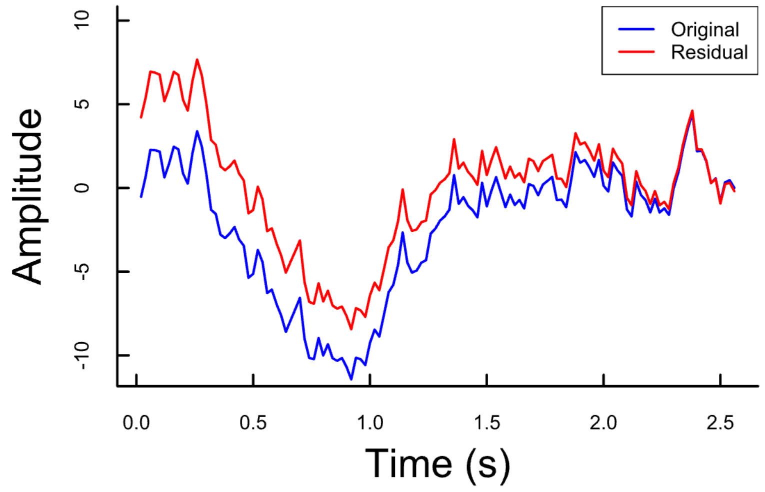
- Divide the series into a sequence of nonoverlapping windows.
- In this example, the time series was separated into 4 windows* (Top).

→ Zooming in

*In practice, 4 windows is not enough to obtain unbiased estimates of fluctuation but for the sake of this tutorial it will do

MFDFA ANALYSIS STEP BY STEP





Step 3

Detrending stage:

- This means fitting a regression line within each of the windows (Top).
- Then, we subtract the fitted trend line from the data in each window (Bottom).

MFDFA ANALYSIS STEP BY STEP

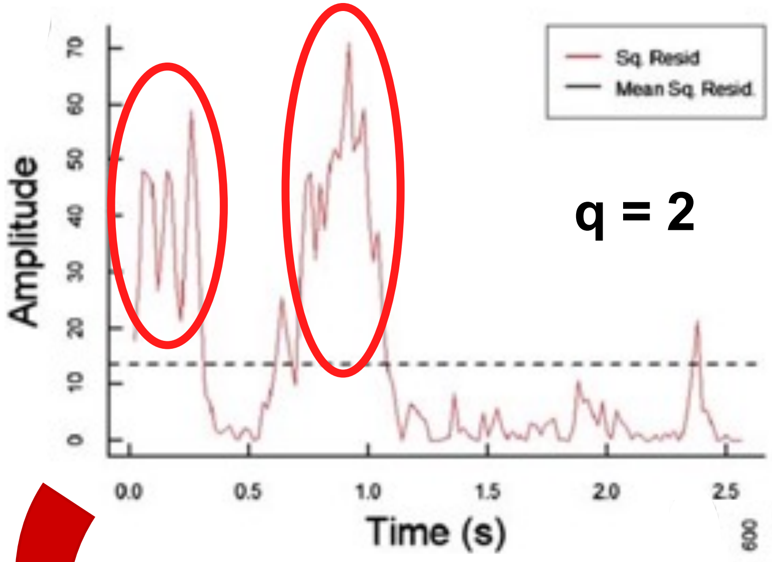
Step 4

- Multifractal analysis involves analyzing several statistical moments (q-order)
- MFDFA focuses on analyzing the scaling behavior of these statistical moments.
- ***Monofractality*** (single power law) vs. ***Multifractality*** (multiple power law).

Squared the residuals $q = 2$

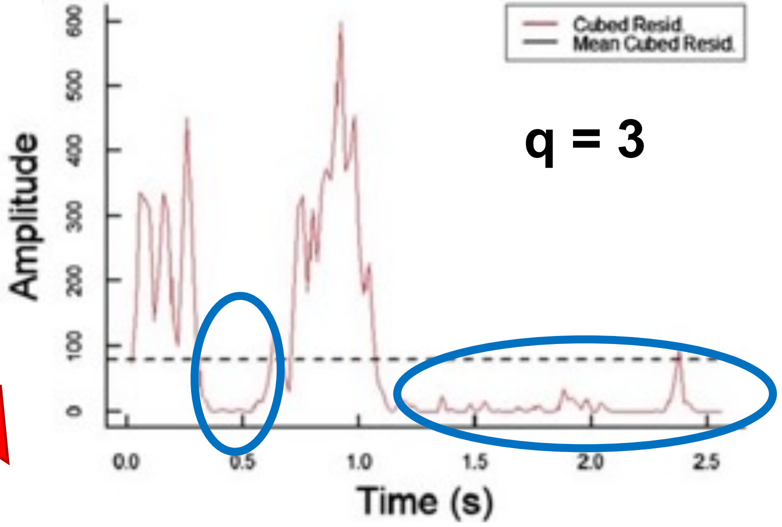
Multiple q-order

MFDFA ANALYSIS STEP BY STEP



→ Represents the squared residuals obtained in the 4th DFA step within a single window

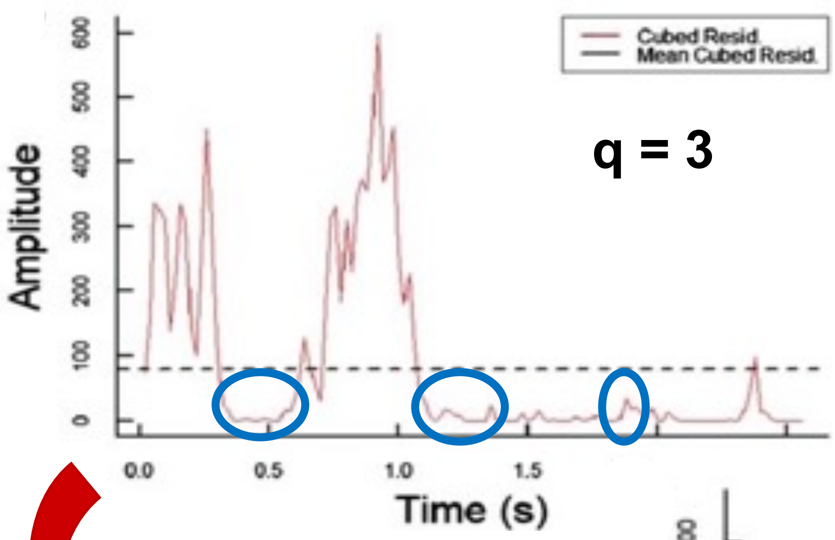
Note: Range is significantly larger



<u>Step 4</u> (continued)

→ In MFDFA we cubed the residuals instead of squaring them. This **magnifies large fluctuations** like those between 0 and 1.0s, while **minimizing smaller fluctuations**.

MFDFA ANALYSIS STEP BY STEP

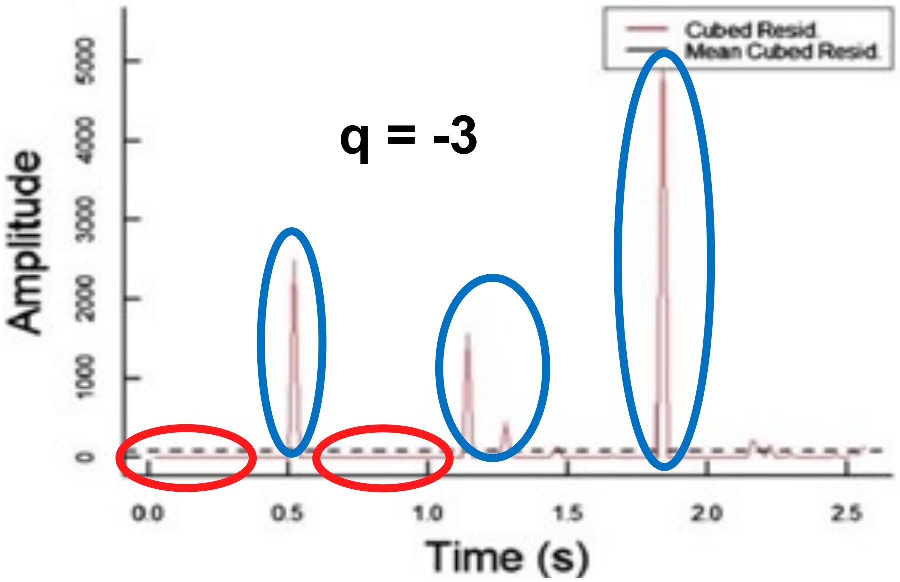


→ Represents the cubed residuals, raised to the **3rd power**

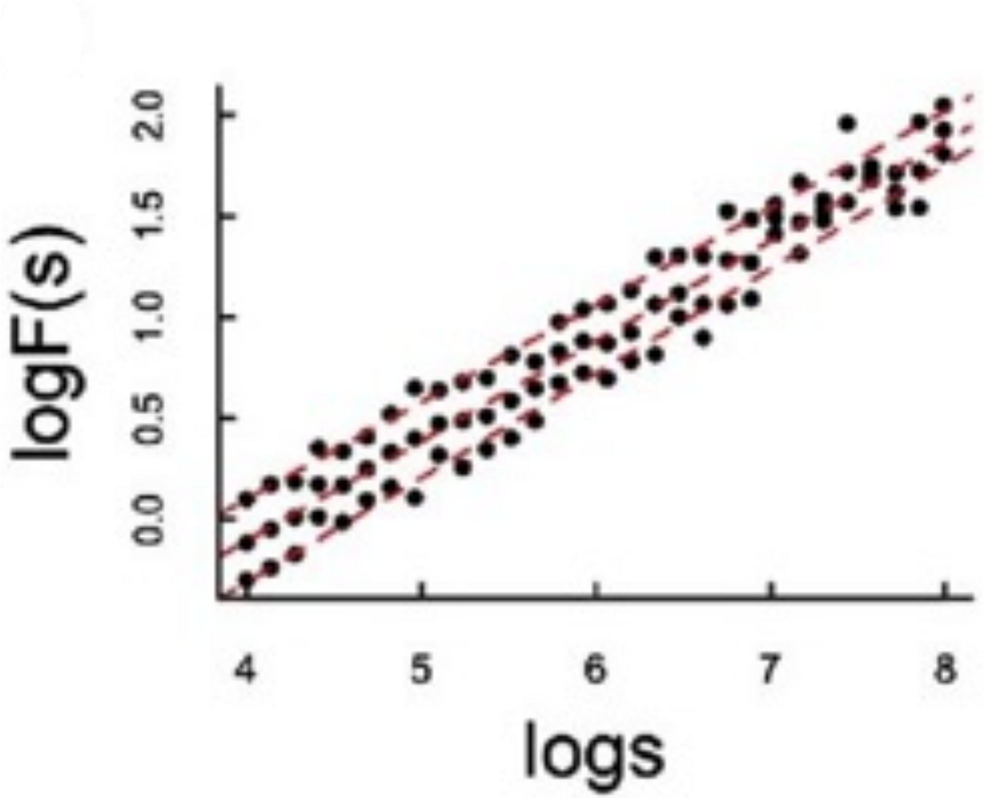
Step 4 (*continued*)

→ Residuals raised to a negative value, **-3rd power, emphasize small fluctuations** and **minimize larger fluctuations**.

Note: Range is significantly larger



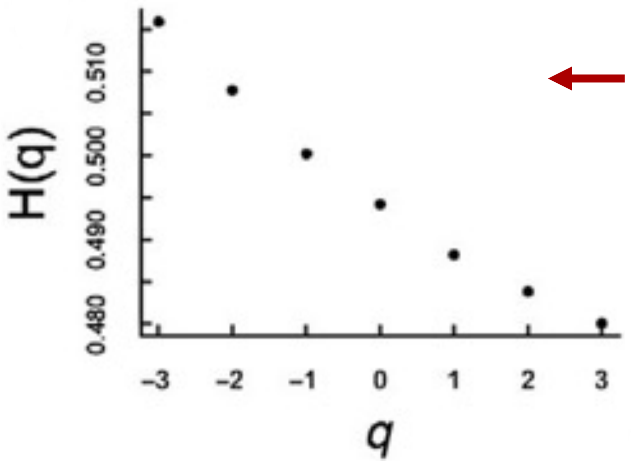
MFDFA ANALYSIS STEP BY STEP



Step 5

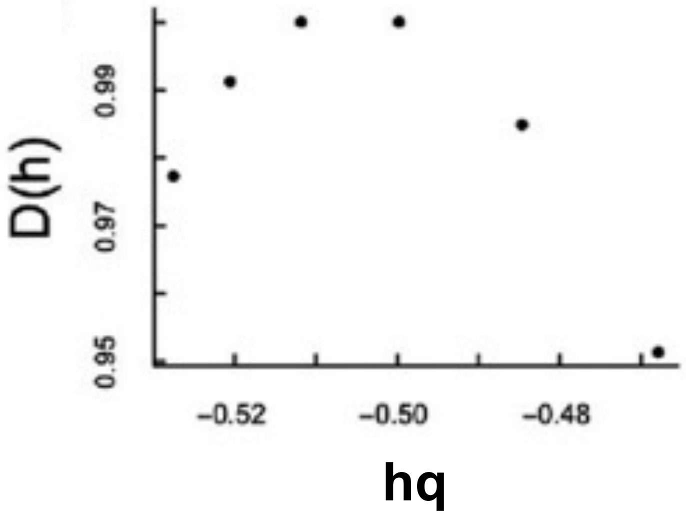
Now we can measure how $F^{(q)}(s)$ varies as a function of scale, ‘s’, via ordinary least squares regression.

MFDFA ANALYSIS STEP BY STEP



Spectrum generalized Hurst exponents, H(q)

Alternative representation of the spectrum– Normalized probability distribution obtained with Legendre Transform



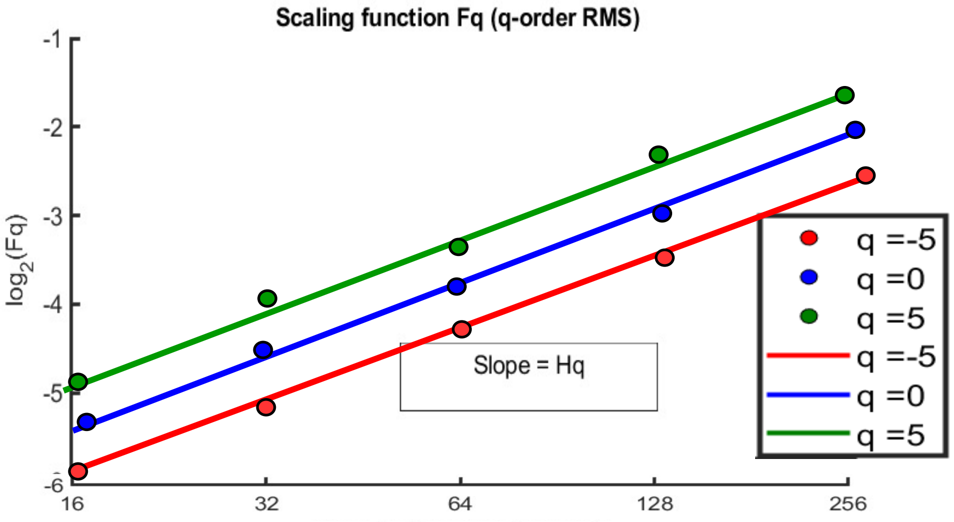
Step 5 (continued)

- The multifractal exponent provides a summary of the time-varying scaling exponents present in the data known as the ***generalized exponent***, H(q).

Monofractals VS. Multifractals

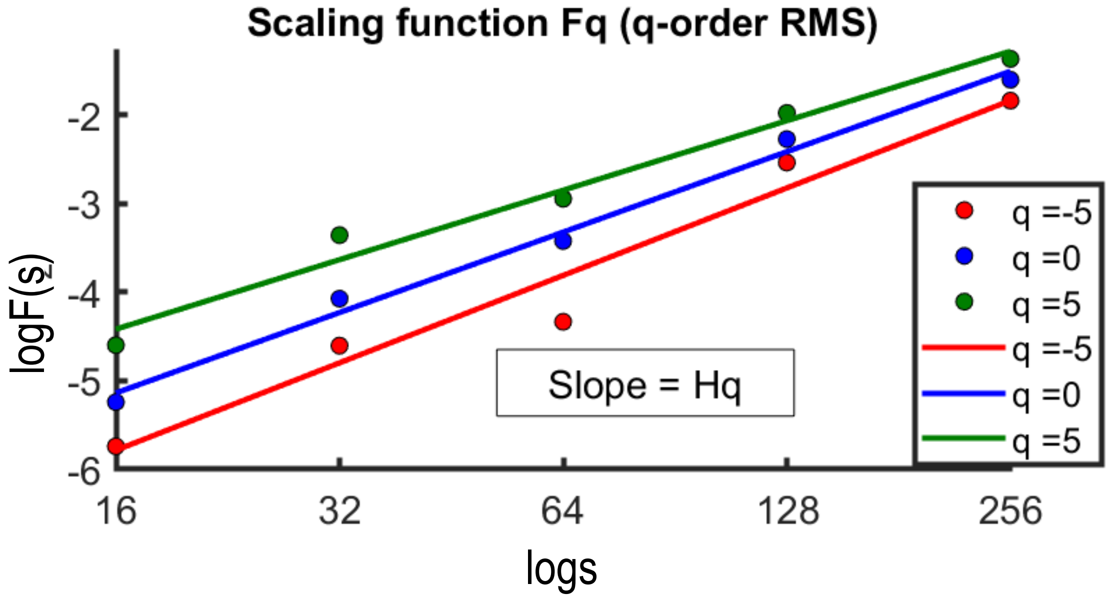
Monofractals

- All regression lines are:
  - Parallel, single exponent
  - Constant (relatively)



Multifractals

- q-order regression lines have different slopes, reflecting differing scaling behavior for different statistical moments



MATLAB TUTORIAL PREVIEW

- MFDFA Functions quick run through:

[Hq, tq, hq, Dq, Fq] = MFDFA (signal, scale, q, m, Fig)

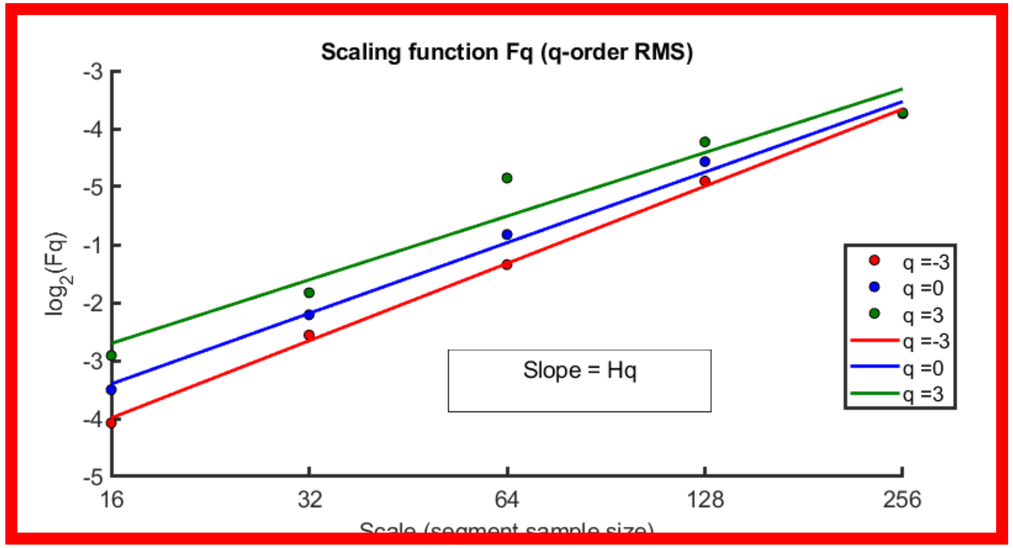
OUTPUT VARIABLES:

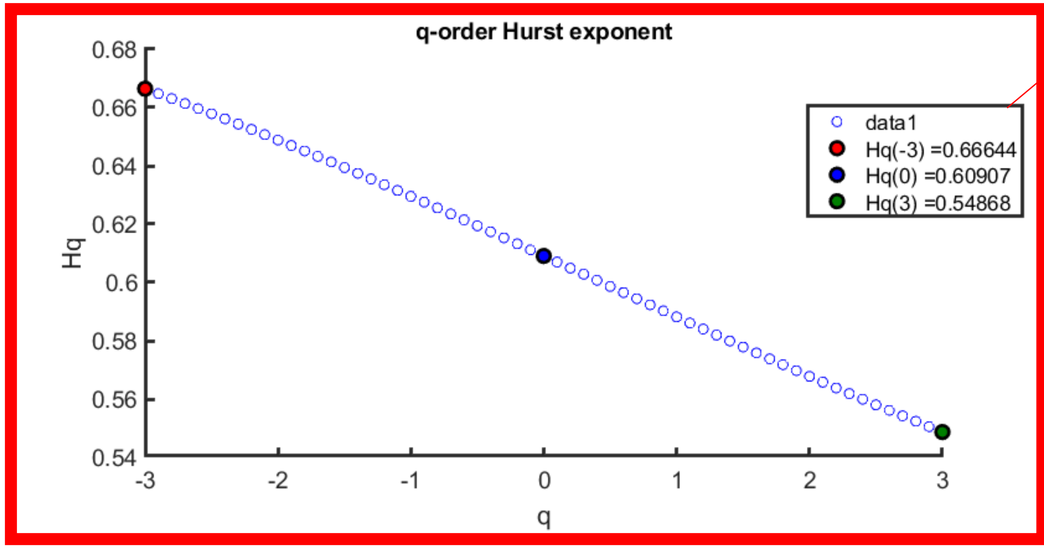
- ***Hq:*** q-order Hurst exponent
- ***tq:*** q-order mass exponent
- ***hq:*** q-order singularity exponent
- ***Dq:*** q-order dimension
- ***Fq:*** q-order scaling function

INPUT PARAMETERS:

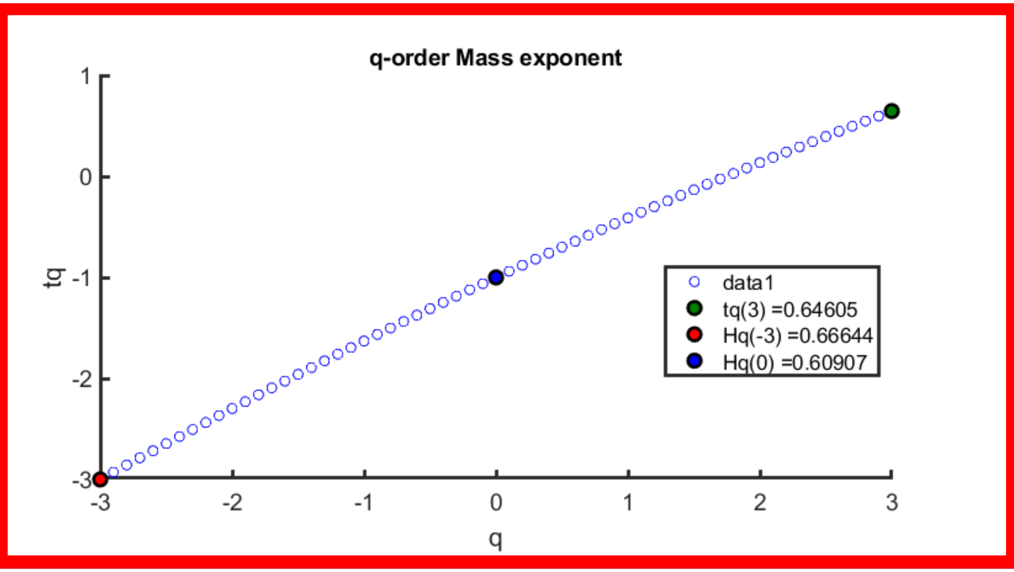
- ***signal***: input signal
- ***scale***: vector of scales
- ***q-order:*** weights the local variations
- ***m***: polynomial order for the detrending
- ***Fig***: 1 or 0 binary for output plot

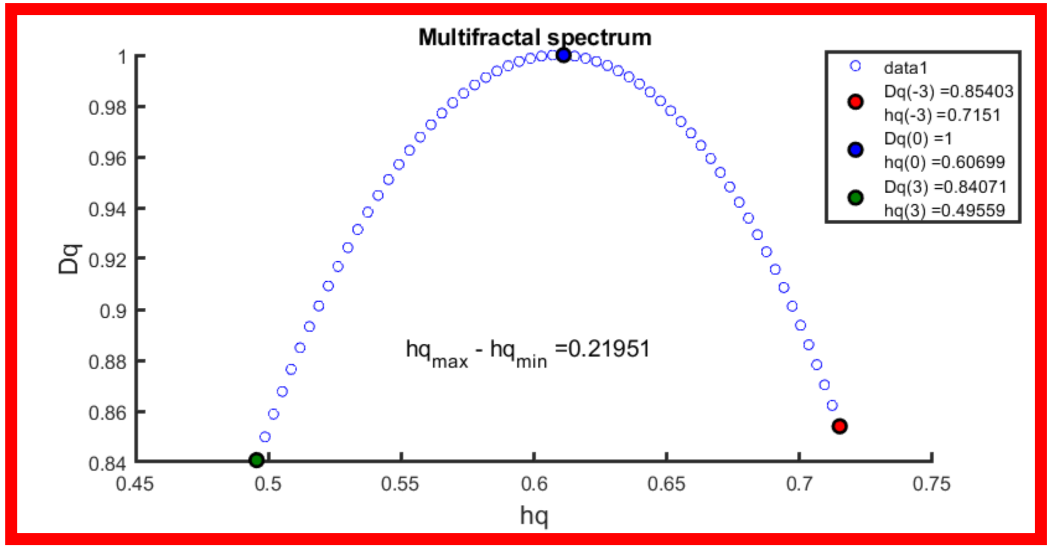
ANALYSIS OUTPUT- What to look for?



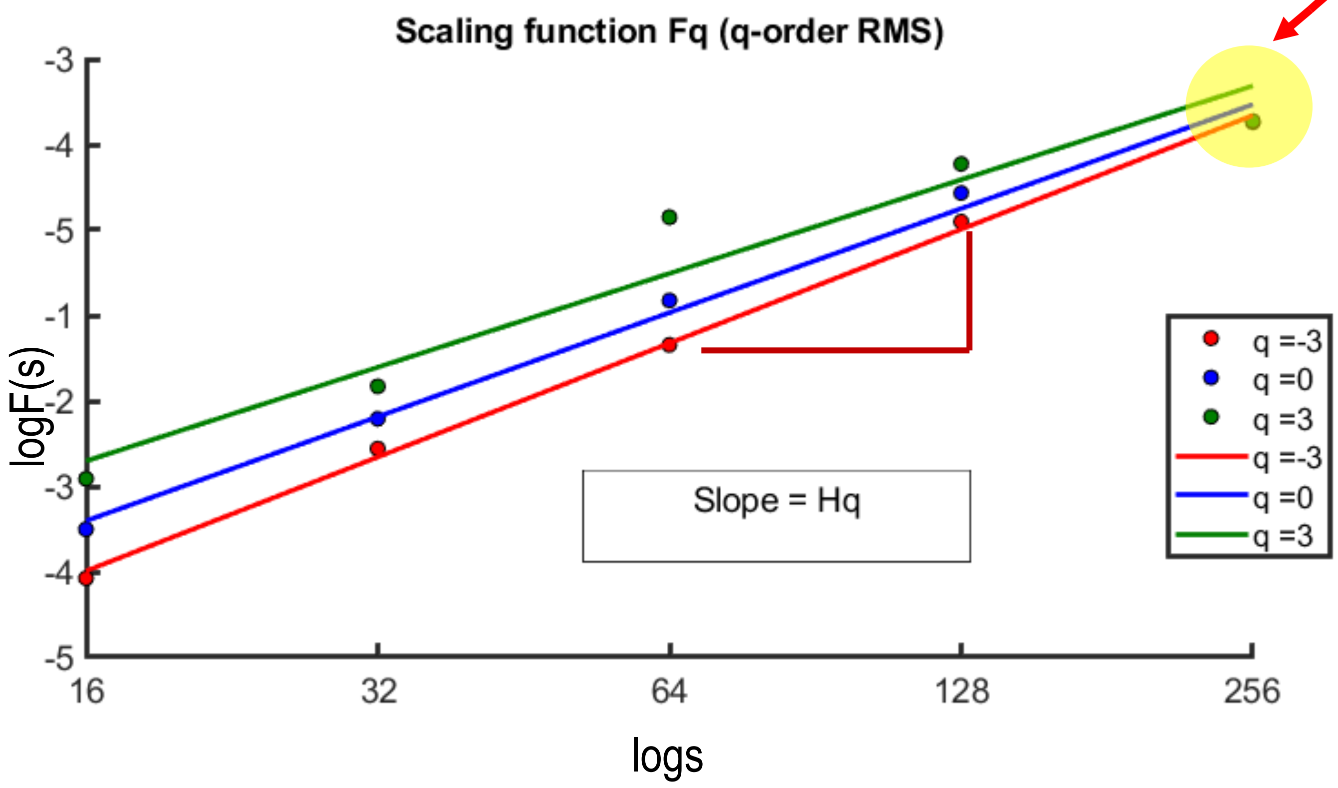


Hq





ANALYSIS OUTPUT- What to look for?

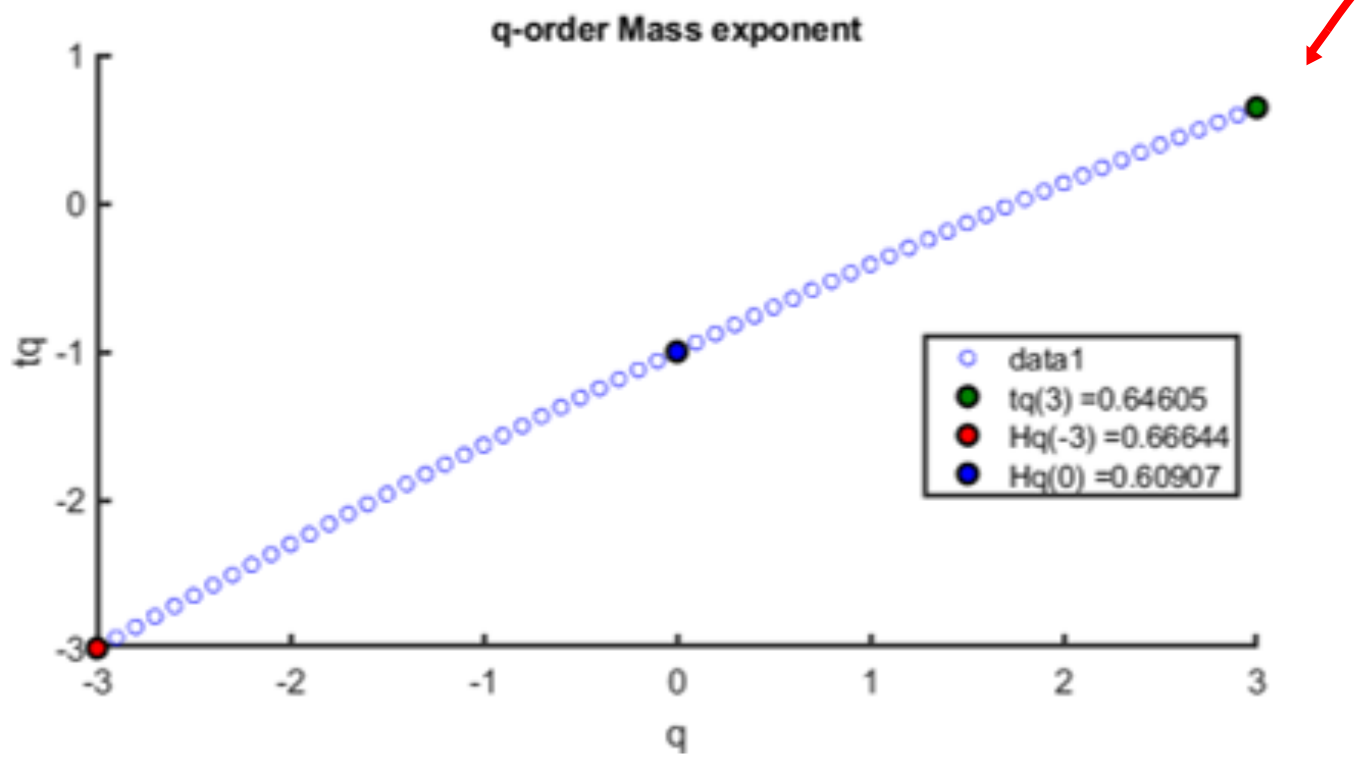


What should I look for?

You should have a linear relationship.

- **Monofractal** = All regression lines are parallel and do not **converge**.
- **Multifractal** = q-order regression lines have different slopes, reflecting differing scaling behavior for different statistical moments.

ANALYSIS OUTPUT- What to look for?



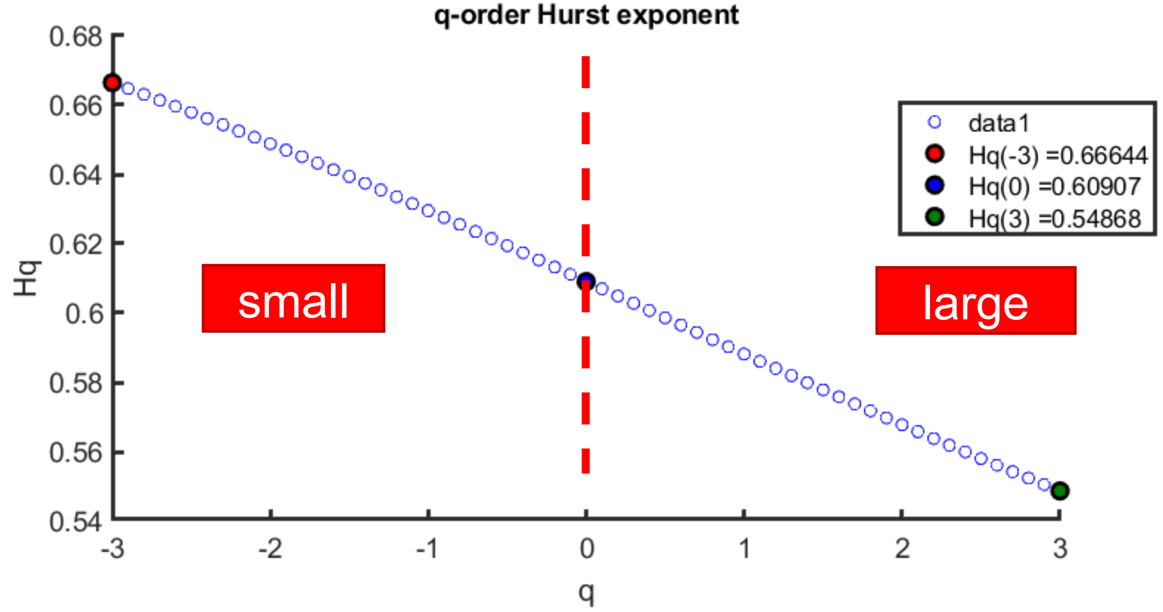
This graph represents a different scaling exponent.

What should I look for?

- Nonlinearity: different slope on either side of q = 0
- Increasing function

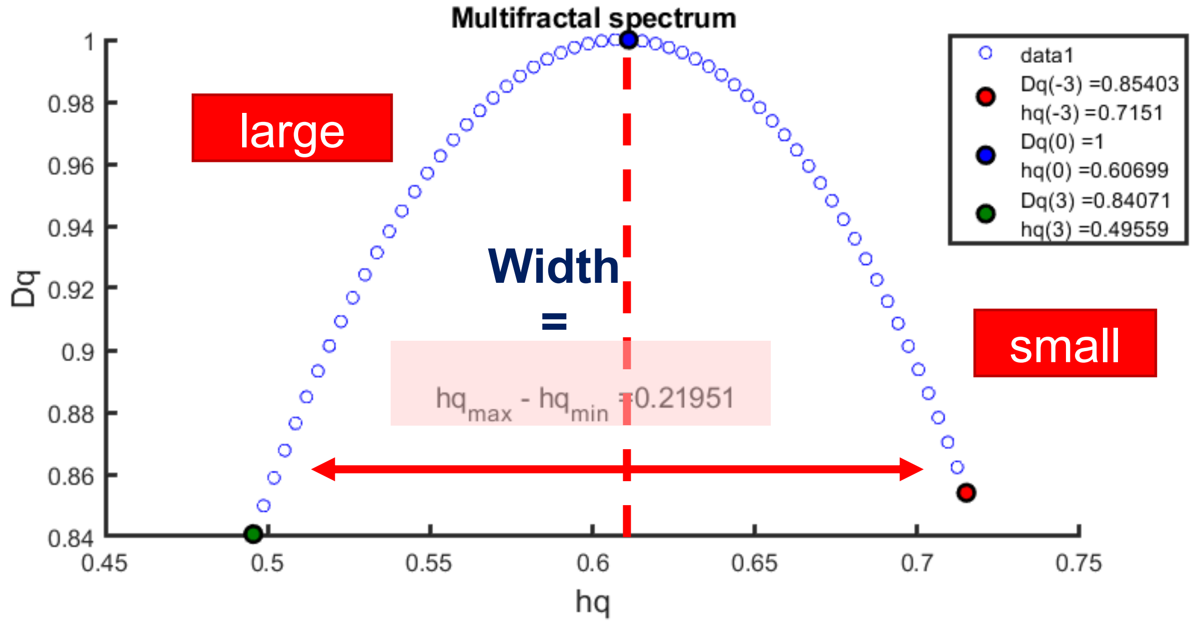
ANALYSIS OUTPUT- What to look for?

Spectrum generalized Hurst exponents, H(q)



- Should be a **monotonically decreasing function** (negative slope and no change of direction)
- Values **< q** correspond to **smaller** fluctuations
- Values **> q** correspond to **larger** fluctuations

Alternative representation of the spectrum



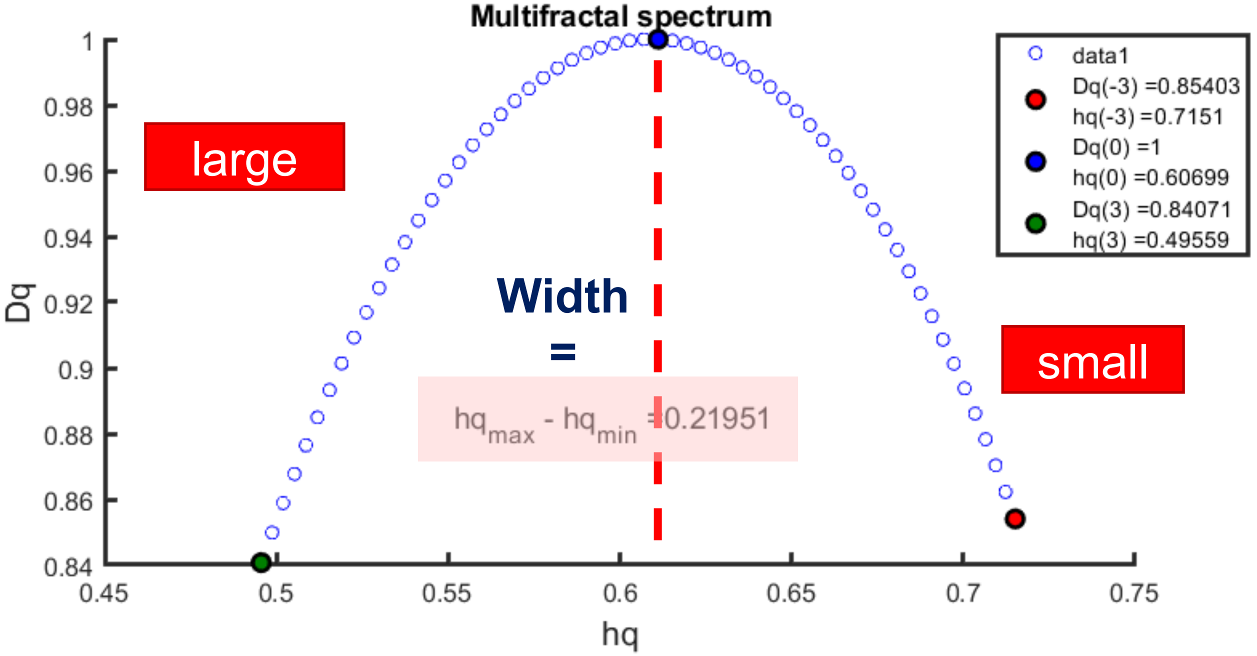
- **Inverted “U”**
- **hq- x-axis:** large values correspond to small fluctuations; small values correspond to large fluctuations
- **Width** will increase as the q-order increases
- **Look for asymmetry**

How do I interpret the multifractal width?

The ***width*** and ***shape*** of the multifractal spectrum reflect the temporal variation of the local *Hurst exponent****,*** to put it simply***,*** it **reflects the change in scaling exponents overtime**.

Larger width can be interpreted as greater number of patterns (i.e., more behavioral strategies) in a time series.

Alternative representation of the spectrum

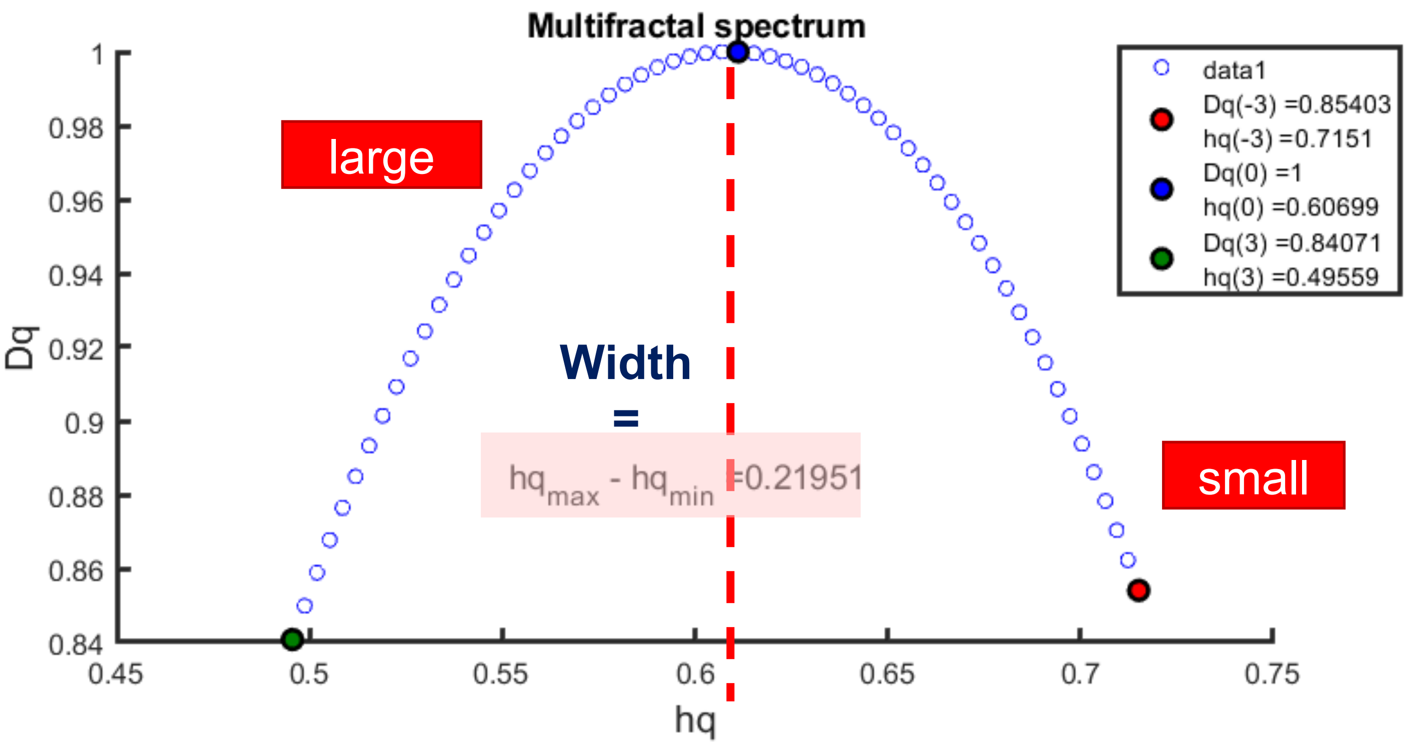


INTERPRETATION

What does the inverted "U" mean if it is asymmetric?

Indicates more variability in the larger asymmetric side. For instance, let's say that the left side (**large fluctuation**) is larger than the right side. It would mean that for this trial and task the participant exhibited more variability in the large fluctuations compared to the small fluctuations.

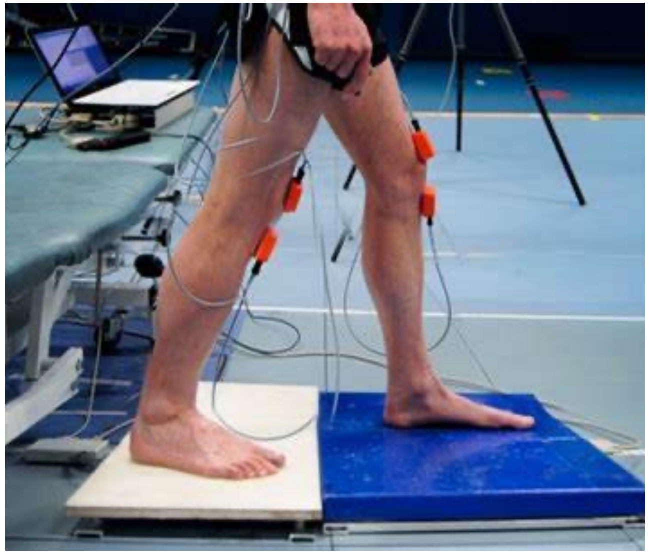
Alternative representation of the spectrum



MFDFA Best Practices

What should my data look like to perform MFDFA?

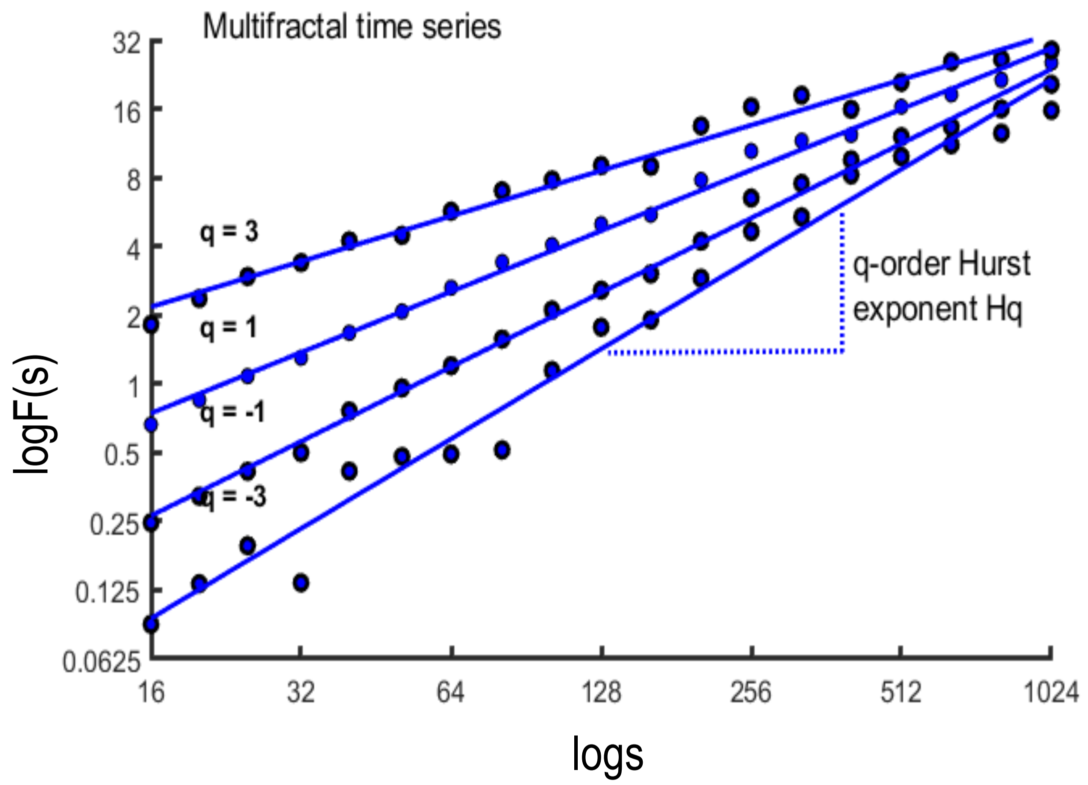
1. Structurally your data should be a ***time-series similar to noise***, meaning your time series should NOT be smooth. *Examples of noisy data:* stride intervals, velocity,… If your time series is smooth (i.e., COP), raw data will require some transformation in order to run this analysis.
2. Local fluctuations within the time series ***cannot be close to zero.***
3. Time series should be ***scale-invariant*** within the predefined range of scales



Ilhen, 2012

MFDFA Best Practices

- **Check for scale-invariance -** *examine the logscale by log(fq) plot for linearity*
- **Series Length -** 500 time points minimum
- **Scales -** [16 < scale < N/9]
- **q range/magnitude -** more extreme values of q require longer data sets. Recommend q = -3:3
- **Detrending order, m -** linear detrending has been used in movement science
  - Selecting a min scale is recommended to avoid overfitting
  - See Kantelhardt et al. (2001)



MATLAB TUTORIAL

TIME TO PRACTICE WHAT WE HAVE LEARNED!!!!!!

You can use your own data or you can practice on the data set provided in **Github**: https://github.com/aaronlikens/ISPGR-2022.git

DOWNLOAD MFDFA FOLDER

This folder contains the following folders: **ANALYSIS OUTPUT** (figures, summary tables), **DATA** (Stride Intervals for each trials and participants), **FUNCTION** (MFDFA)

OPEN MATLAB

Open the script called 'MFDFA_gait'

********5 min Break********

Github link: https://github.com/aaronlikens/ISPGR-2022.git

MATLAB VERSION:

There are no known incompatibilities using MATLAB version R2019a or later.

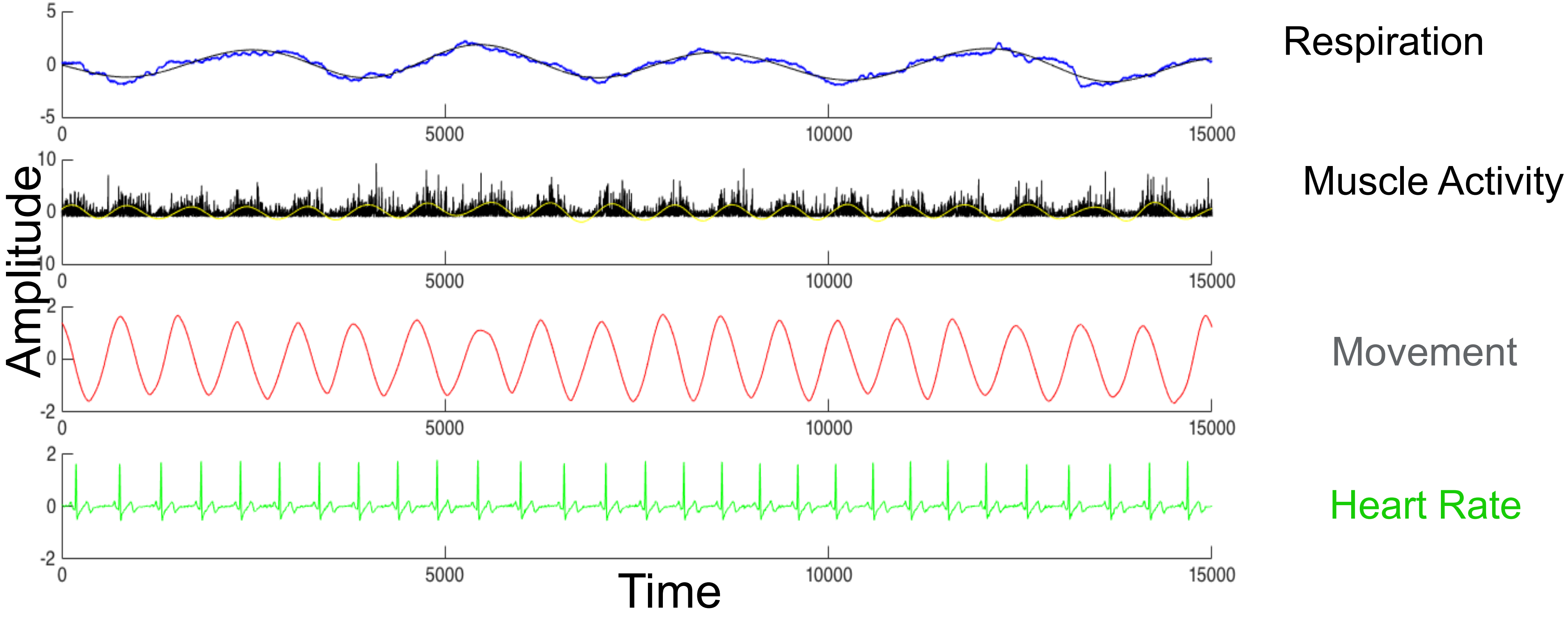
MATLAB Toolboxes Required:

Statistics and Machine Learning Toolbox
Signal Processing Toolbox
Image Processing Toolbox

Part III: Fractal Regression

Aaron D. Likens

Movement involves coordination of multiple nested concurrent processes



Movement requires coordination of many nested processes

Respiration

Muscle Activity

Movement

Heart Rate

What is the scale and time course of influence across coordinating processes?

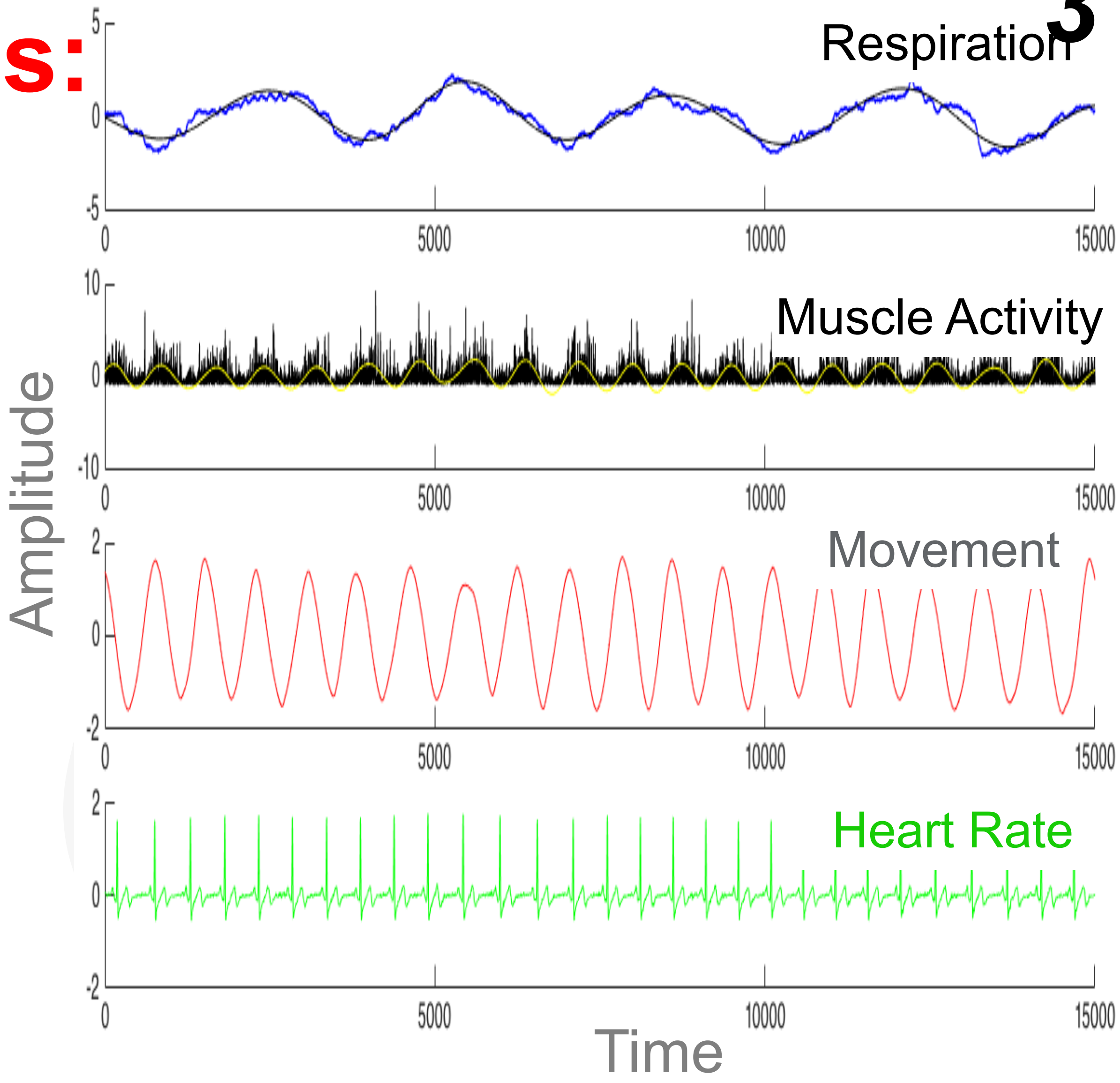
Two Classes of Questions:

Scale dependency:
are coordination effects
temporary or **long-lasting?**

- scale

Time dependency:
are coordination effects
immediate or **delayed?**

- lag



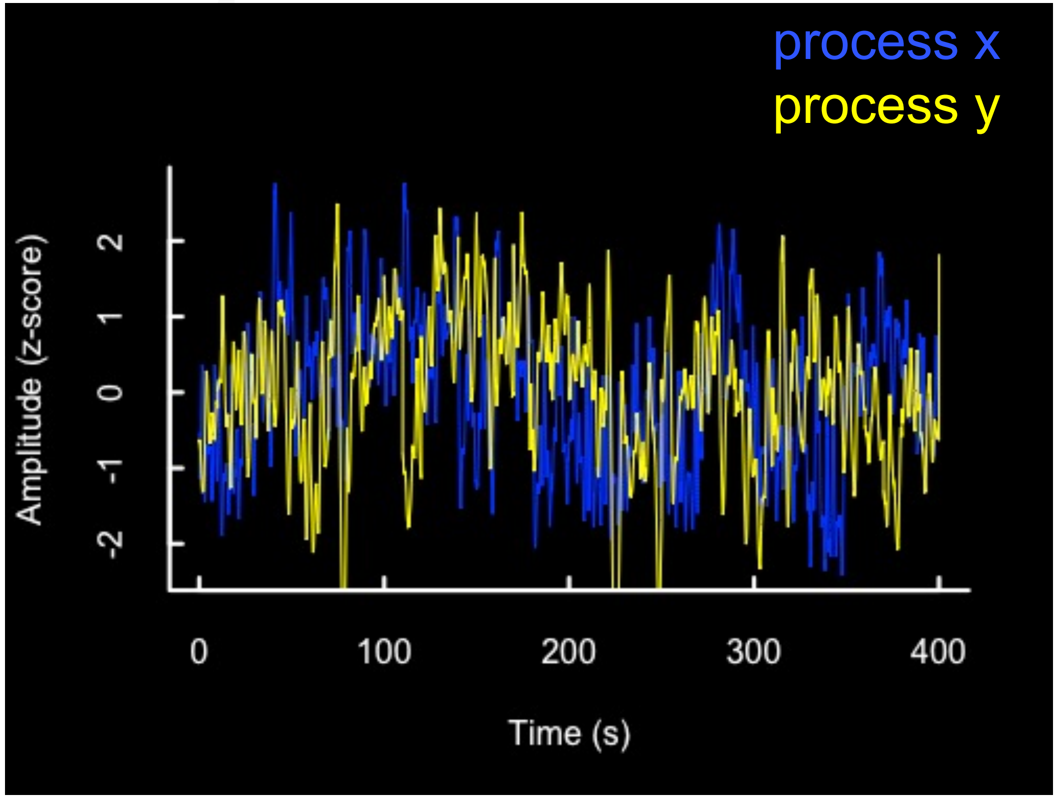
Previous Methods
capture multiscale influence

Scale Dependency / Multiscale Influence:

- Fractal analysis (e.g., Eke et al., 2002; Kantelhardt et al., 2002)
- Spectral and Wavelet Coherence (e.g., Grinsted et al., 2004; Mandel & Wolf, 1976)
- Fractal regression and correlation (e.g., Kristoufek, 2015; Podobnik & Stanley, 2008; Zebende, 2011)

MRA combines
Detrended Fluctuation Analysis
(DFA, fractal analysis) and
ordinary least squares regression
(OLS) to capture coordination of
two processes across scale *(s)*
i.e. scale dependency

We extend Kristoufek (2015), by developing a statistical test for the method he introduced



Ordinary Least Squares (OLS) Regression

- regressing one variable on another

$$Y_t = \beta_0 + \beta_1 X_t + e_t$$

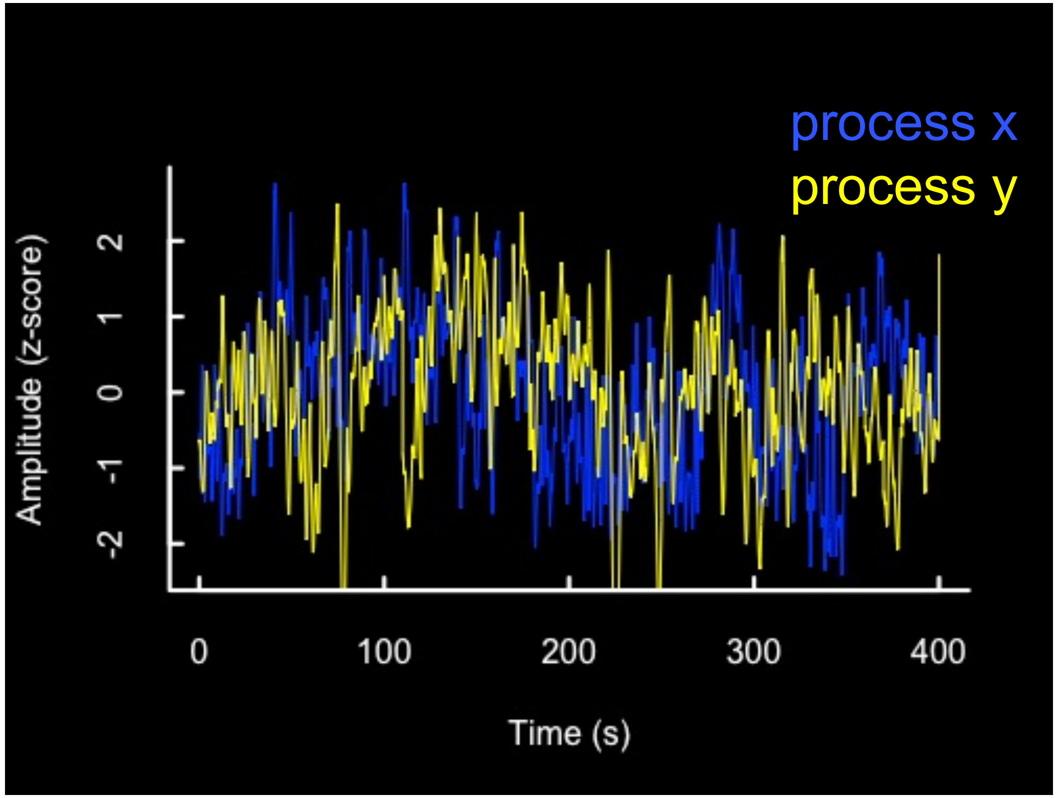
Regression coefficient for that simple regression:

- covariance / variance

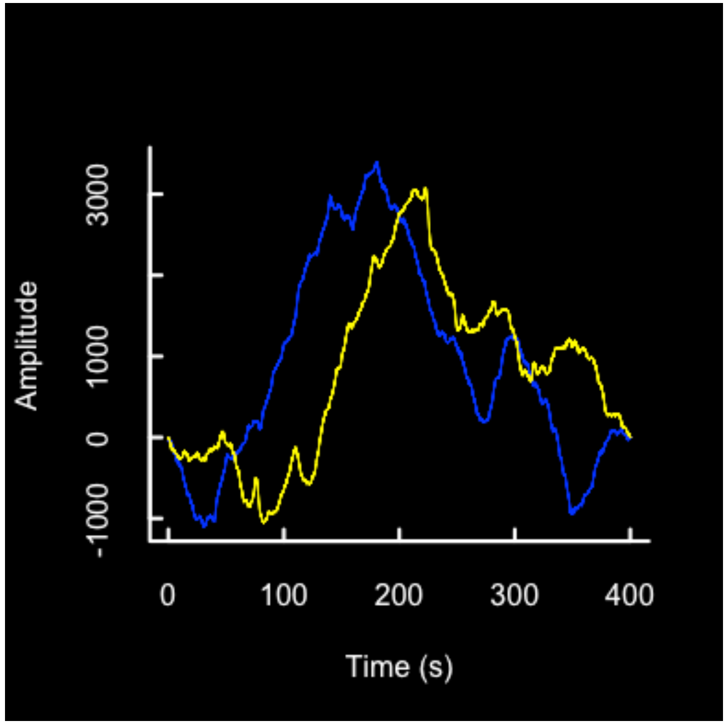
$$\hat{\beta}_1 = \frac{\sum_{t=1}^{T} (x_t - \overline{x})(y_t - \overline{y})}{\sum_{t=1}^{T} (x_t - \overline{x})^2}$$

How do 2 variables relate to one another at different scales of analysis?

For each series, start as you would for DFA:

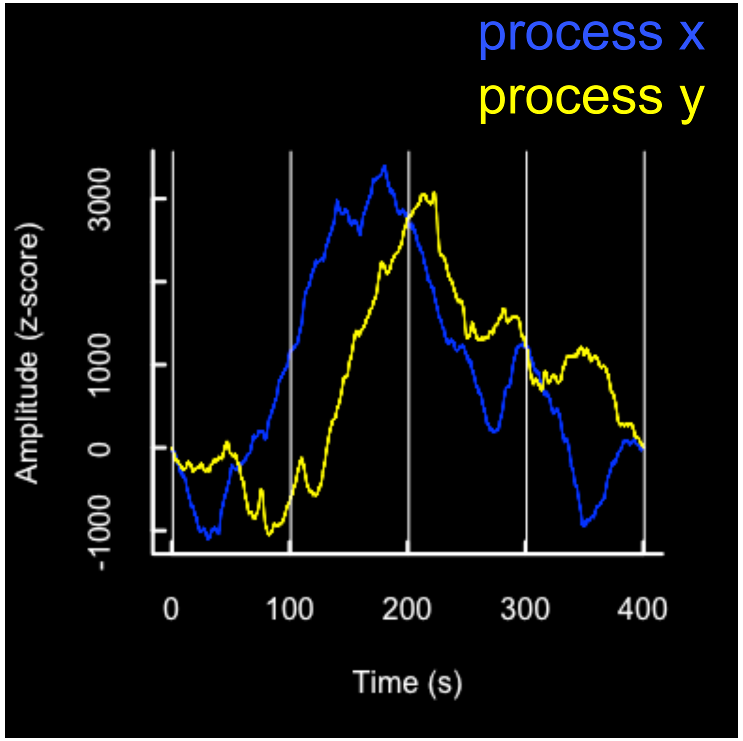


1. center about the mean and take the cumulative sum

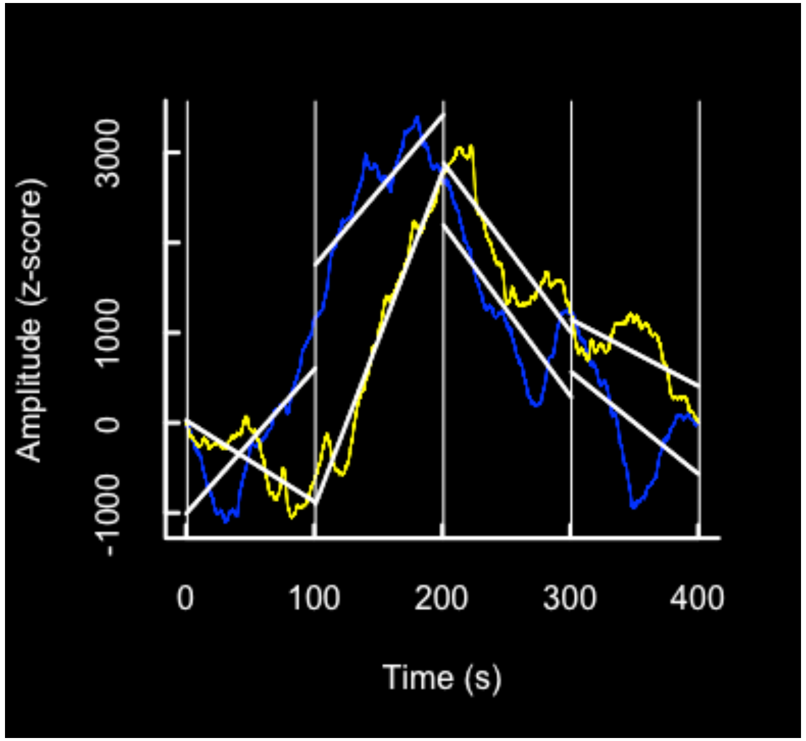


How do 2 variables relate to one another at different scales of analysis?

2. divide each series into equal-sized bins



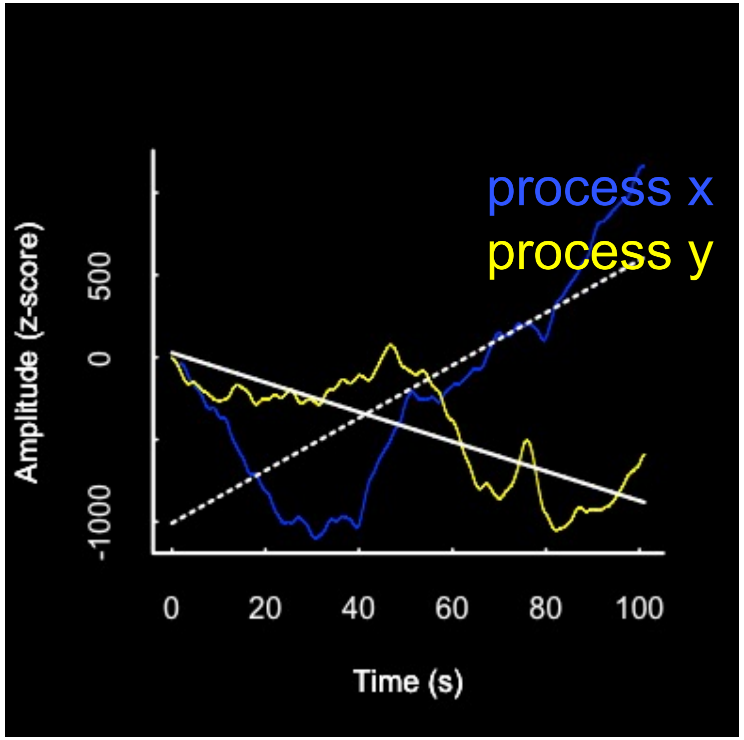
3. estimate a regression line* within each bin



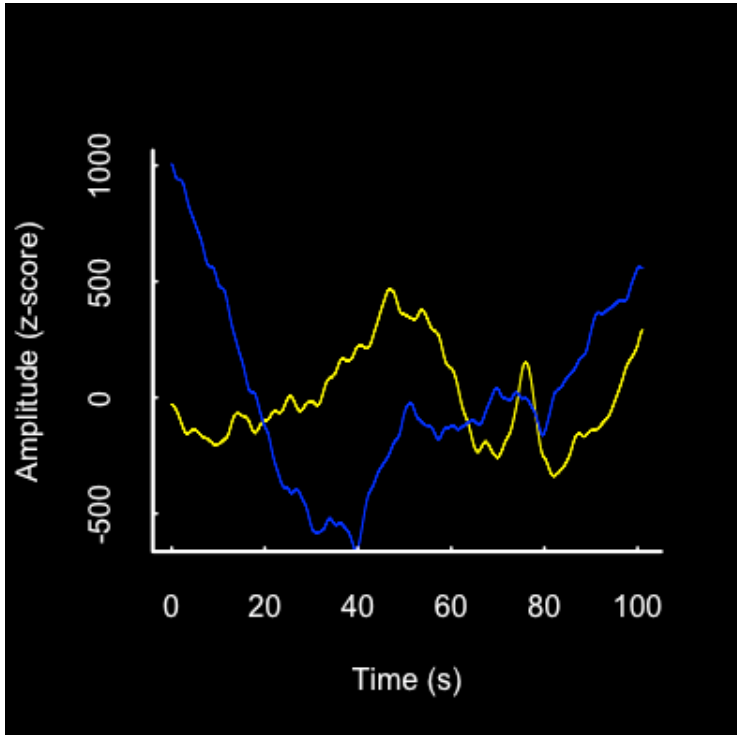
* linear regression is pictured, but you can improve fit by using other fitting functions

How do 2 variables relate to one another at different scales of analysis?

Within a bin,
there are different regression lines for each series



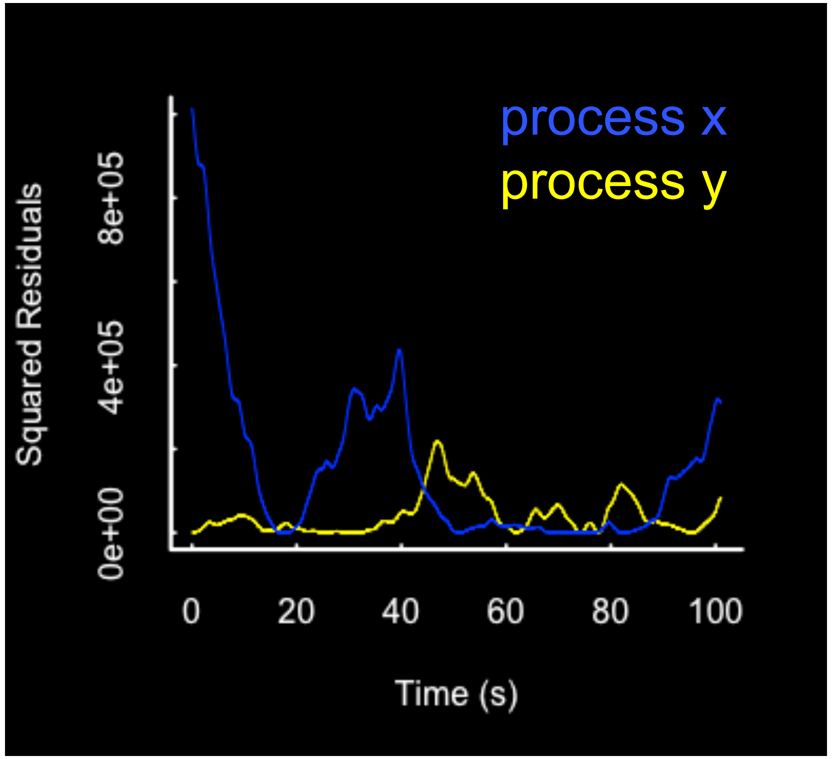
... and residuals for each series:



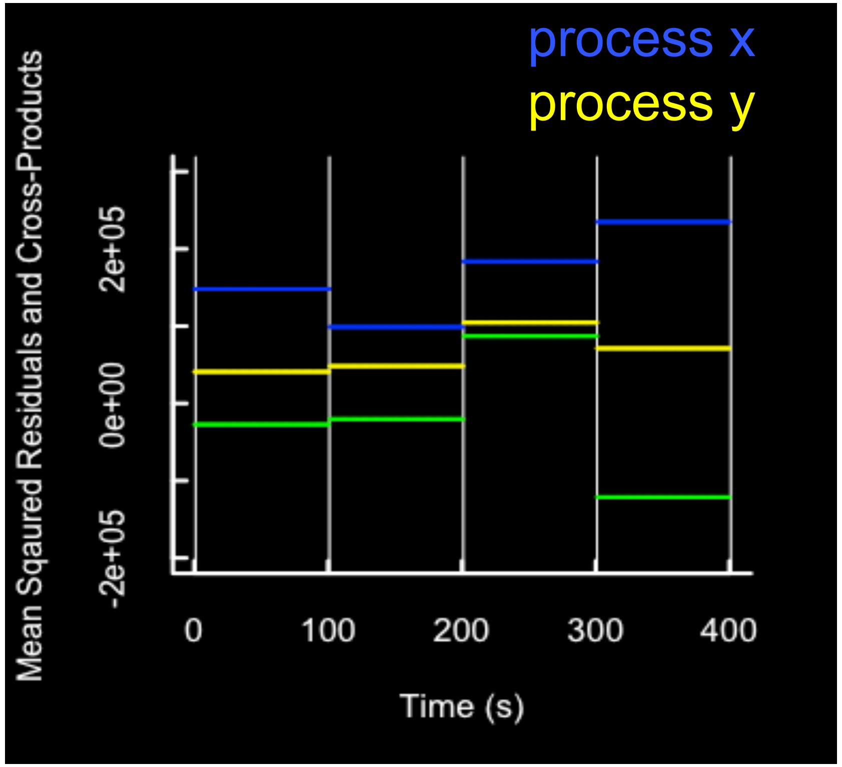
How do 2 variables relate to one another at different scales of analysis?

4. Square those residuals to compute variance for each series

... and multiply those residuals to compute the covariance across series



How do 2 variables relate to one another at different scales of analysis?



... now you have variance per process and covariance across the two processes for all bins at that scale size.

5. Average those bin-wise estimates to obtain scale-wise estimates of covariance and variance

6. Scale-wise regression coefficients are β_{scale} = covariance/variance

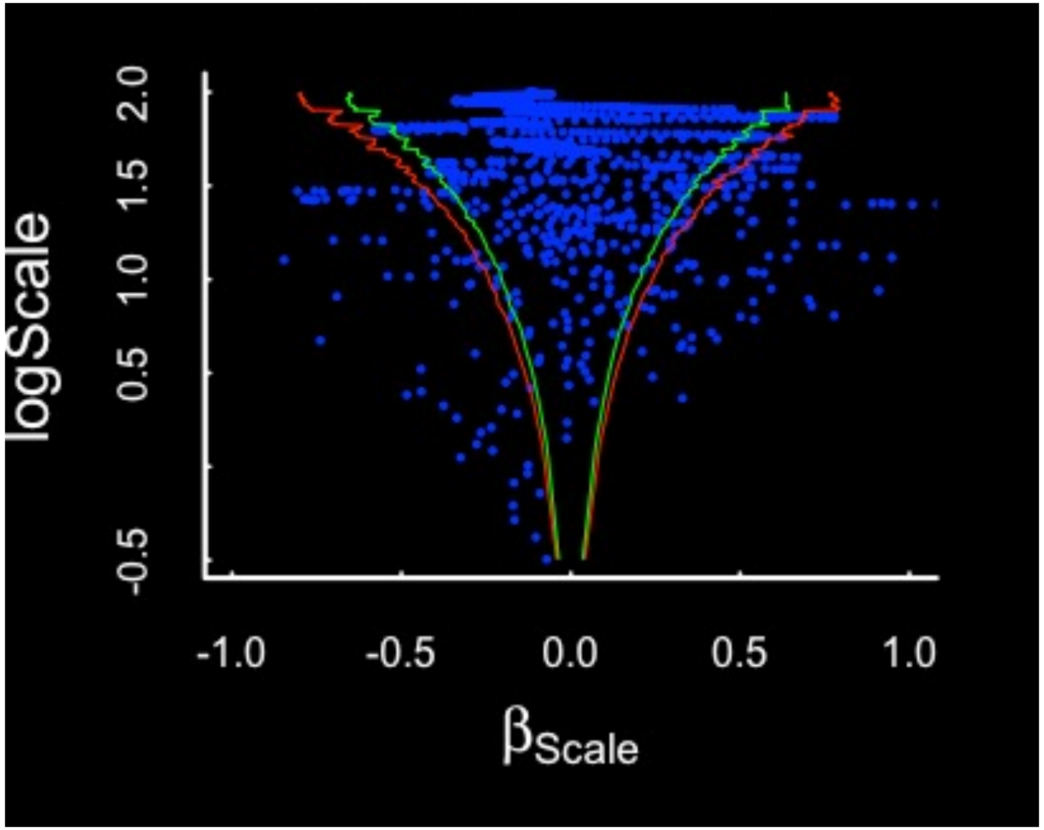
$$\hat{\beta}_1(s) = \frac{F_{XY}^2(s)}{F_X^2(s)}$$

How do 2 variables relate to one another at different scales of analysis?

7. **Repeat** over many scales to generate multiscale influence at zero lag (Kristoufek, 2015)

Multiscale Regression Analysis

8. Generate **confidence intervals** to test for statistical significance (Likens, Amazeen, West, & Gibbons, 2019)
   - surrogates
   - simulations



Ordinary Least Squares (OLS) Regression

- regressing one variable on another

$$Y_t = \beta_0 + \beta_1 X_t + e_t$$

Regression coefficient for that simple regression:

- covariance / variance

$$\hat{\beta}_1 = \frac{\sum_{t=1}^{T}(x_t - \bar{x})(y_t - \bar{y})}{\sum_{t=1}^{T}(x_t - \bar{x})^2}$$

Kristoufek (2015) estimated covariance and variance at each scale (s)

$$\hat{\beta}_1(s) = \frac{F_{XY}^2(s)}{F_X^2(s)}$$

Method Development

Investigate how Multiscale Regression Analysis behaves for different data sets:

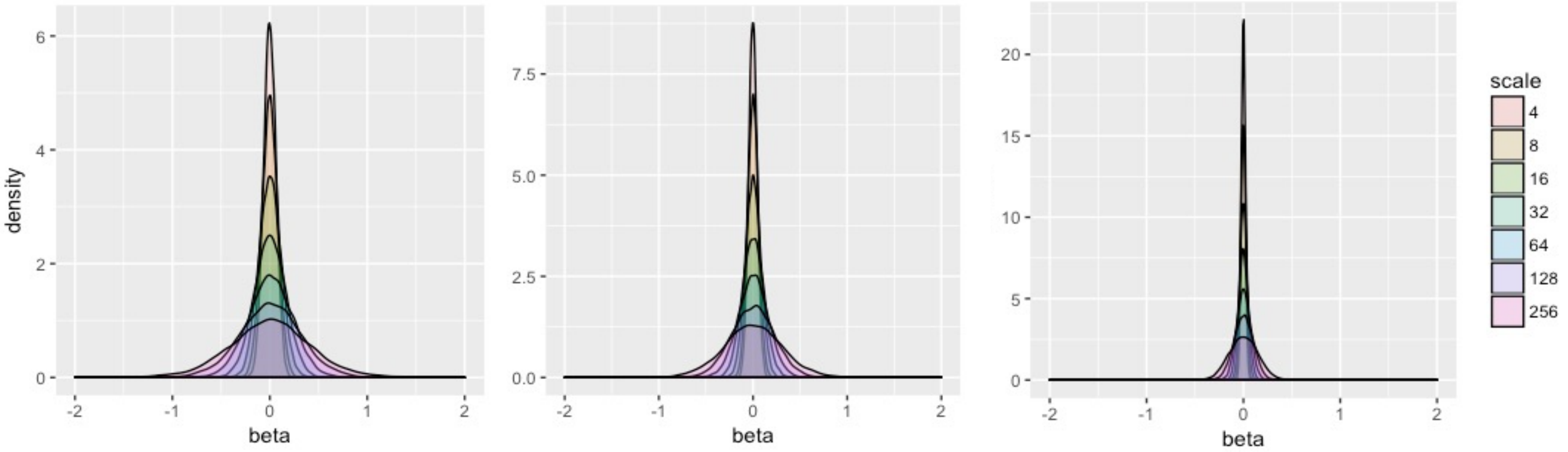
- how do confidence intervals change as a function of **series length**?
    - T = 512, 1024, 4096
    - relatedly, s = 4... 2^n... 256
- how robust is MRA for different **series types**?
    - uncorrelated (Gaussian) noise / autocorrelated signals
    - stationary / nonstationary series

Multiscale Regression Analysis (MRA)

How do confidence intervals change as a function of **series length?**

- calculated **beta** for 10,000 simulated series of Gaussian (uncorrelated) noise

T = 512 T = 1024 T = 4096



Multiscale Regression Analysis (MRA)

How do confidence intervals change as a function of **series length?**

T = 512

scale	0.1	1	2.5	97.5	99	99.9
4	-0.1889316	-0.1493981	-0.124541	0.1262811	0.1496859	0.1973205
8	-0.2504436	-0.1861518	-0.1589527	0.1600898	0.192147	0.2618819
16	-0.3536967	-0.2641565	-0.220866	0.2225776	0.2694265	0.354777
32	-0.5262445	-0.3765088	-0.311902	0.3145898	0.374583	0.4920462
64	-0.7299614	-0.5251441	-0.4347303	0.449918	0.5391599	0.7759531
128	-1.0068969	-0.7291901	-0.6141151	0.6259065	0.7316962	1.0718401
256	-1.3409292	-0.9856829	-0.8103346	0.8505748	1.0482059	1.4787476

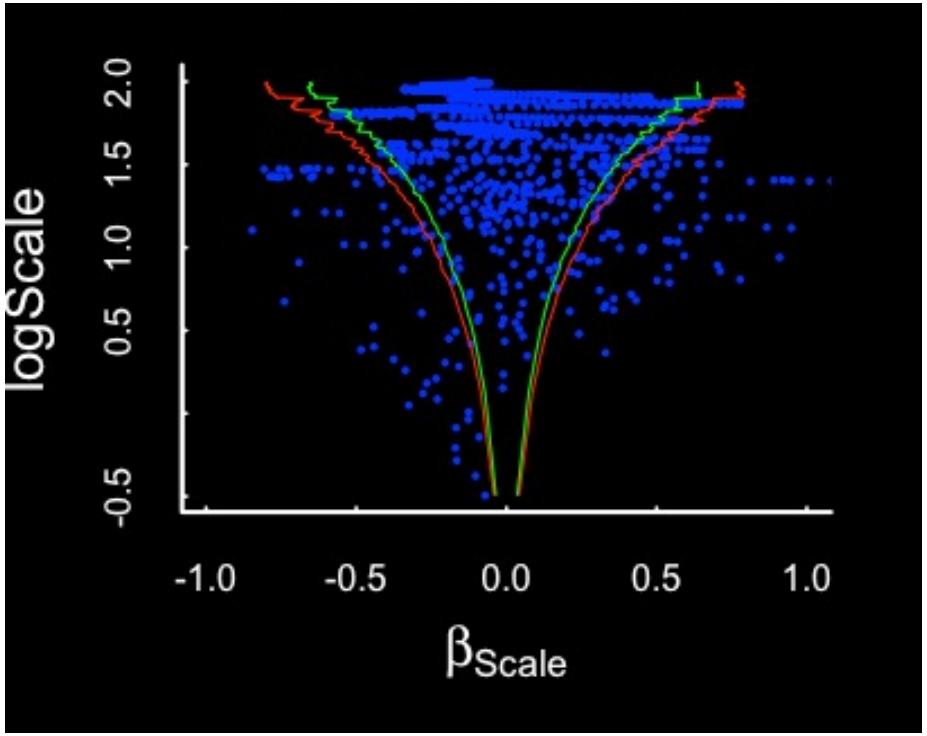
T = 1024

scale	0.01	1	2.5	97.5	99	99.9
4	-0.1387652	-0.1043384	-0.0868504	0.09085313	0.10669765	0.14447768
8	-0.1806002	-0.1350455	-0.1140263	0.1150365	0.13748864	0.17898405
16	-0.234342	-0.1875568	-0.1573156	0.15664268	0.18415054	0.24592646
32	-0.3443485	-0.2600243	-0.219513	0.218135	0.25809487	0.33627318
64	-0.4830369	-0.3766349	-0.3149791	0.30915307	0.36880947	0.47622003
128	-0.7222387	-0.5327281	-0.4372649	0.44137776	0.5268568	0.69231233
256	-1.0393115	-0.7379413	-0.6087975	0.61311657	0.76122092	1.01065238

Lesson: Larger coefficients are required to reach conventional statistical significance as series length decreases, scale (bin size) increases, and alpha-level increases.

Multiscale Regression Analysis (MRA)

how do confidence intervals change as a function of **series length?**



Lesson: Larger coefficients are required to reach conventional statistical significance as series length decreases, scale (bin size) increases, and alpha-level increases

Method Development
guided by regression first principles

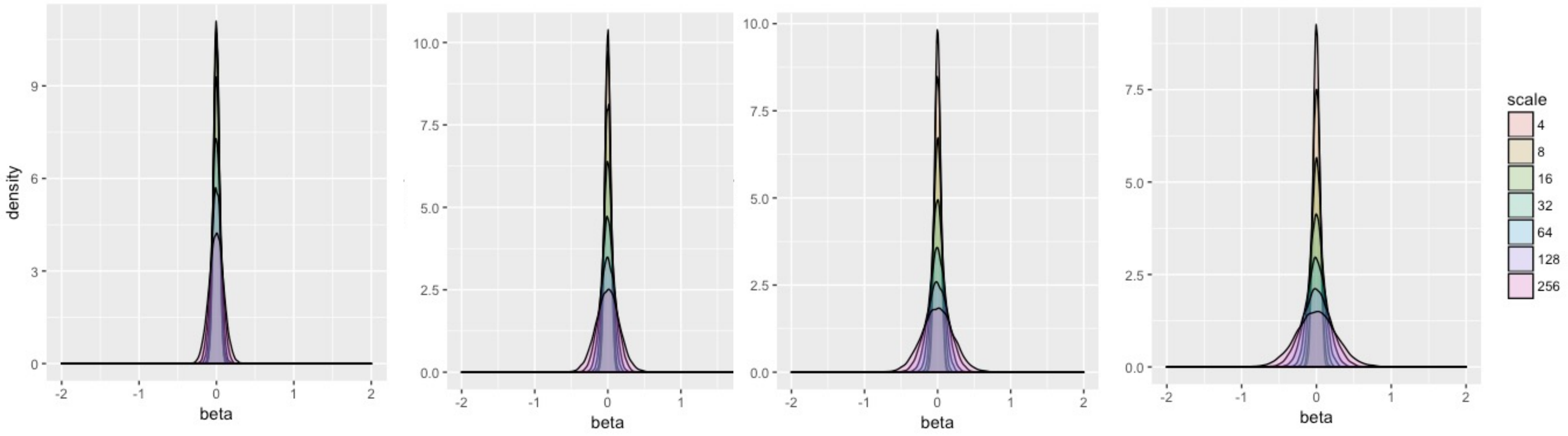
Investigate how Multiscale Regression Analysis behaves for different data sets:

- how do confidence intervals change as a function of **series length**?
- how robust is MRA for different **series types**?
    - uncorrelated (Gaussian) noise / autocorrelated signals
    - stationary / nonstationary series

Method Development guided by regression first principles

- how robust is MRA for different **series types**?
  - autocorrelated signals: anti-persistent time series

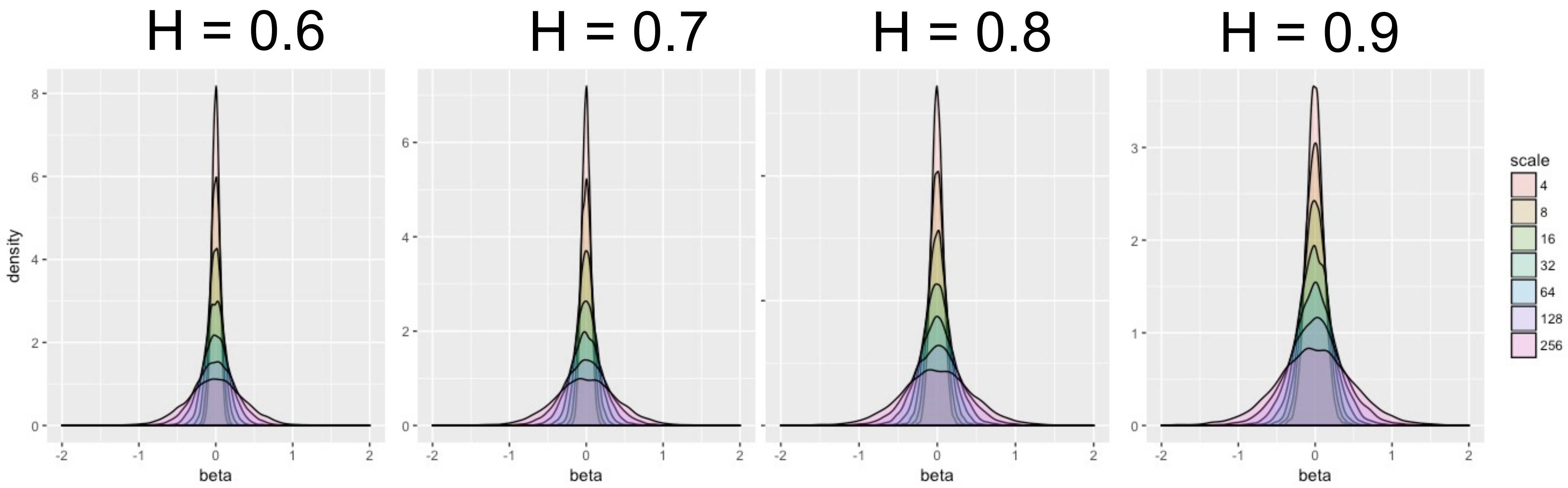
H = 0.1 H = 0.2 H = 0.3 H = 0.4



Lesson: Confidence intervals spread (larger coefficients are required) with increasing H

Method Development guided by regression first principles

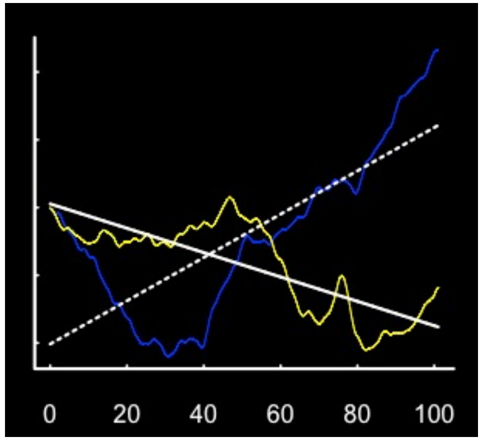
- how robust is MRA for different **series types**?
  - autocorrelated signals: persistent time series

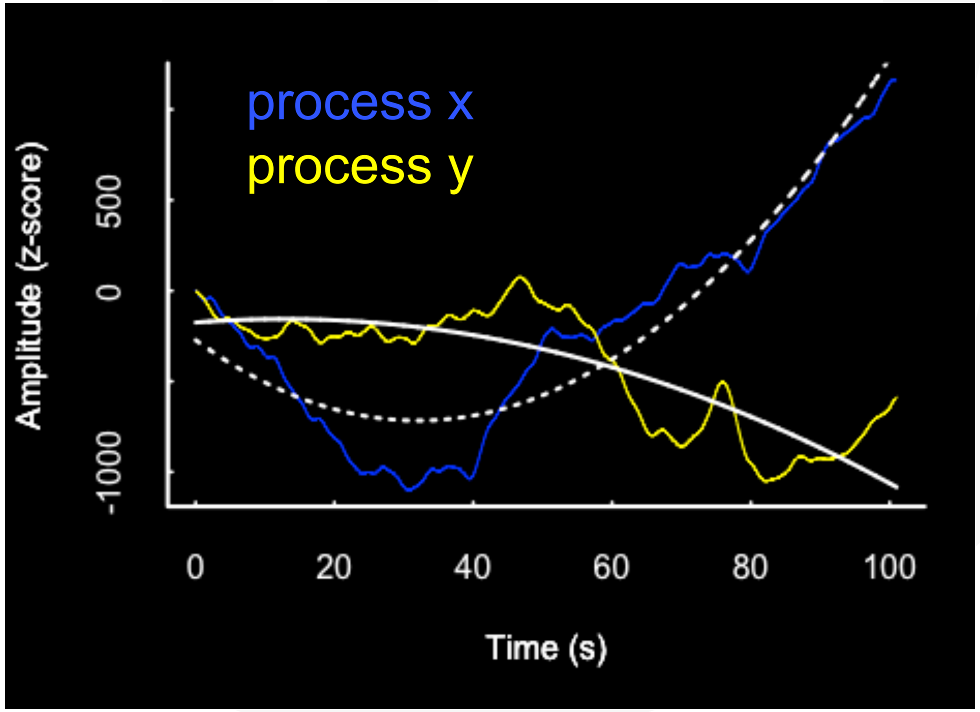


Lesson: Confidence intervals spread (larger coefficients are required) as H increases; higher-order detrending functions will take care of this.

Multiscale Regression Analysis

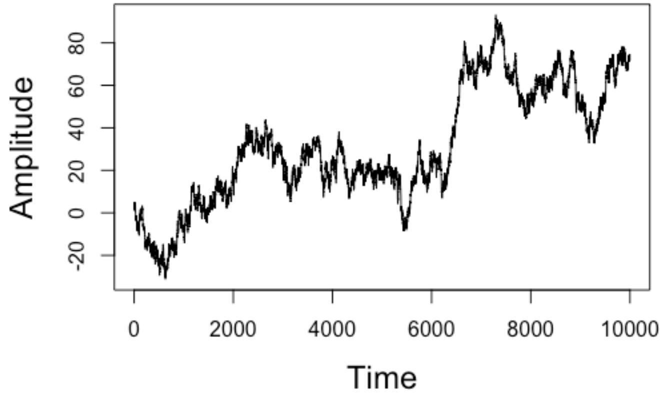
- understand the effects of polynomial detrending
  - linear is not always most appropriate



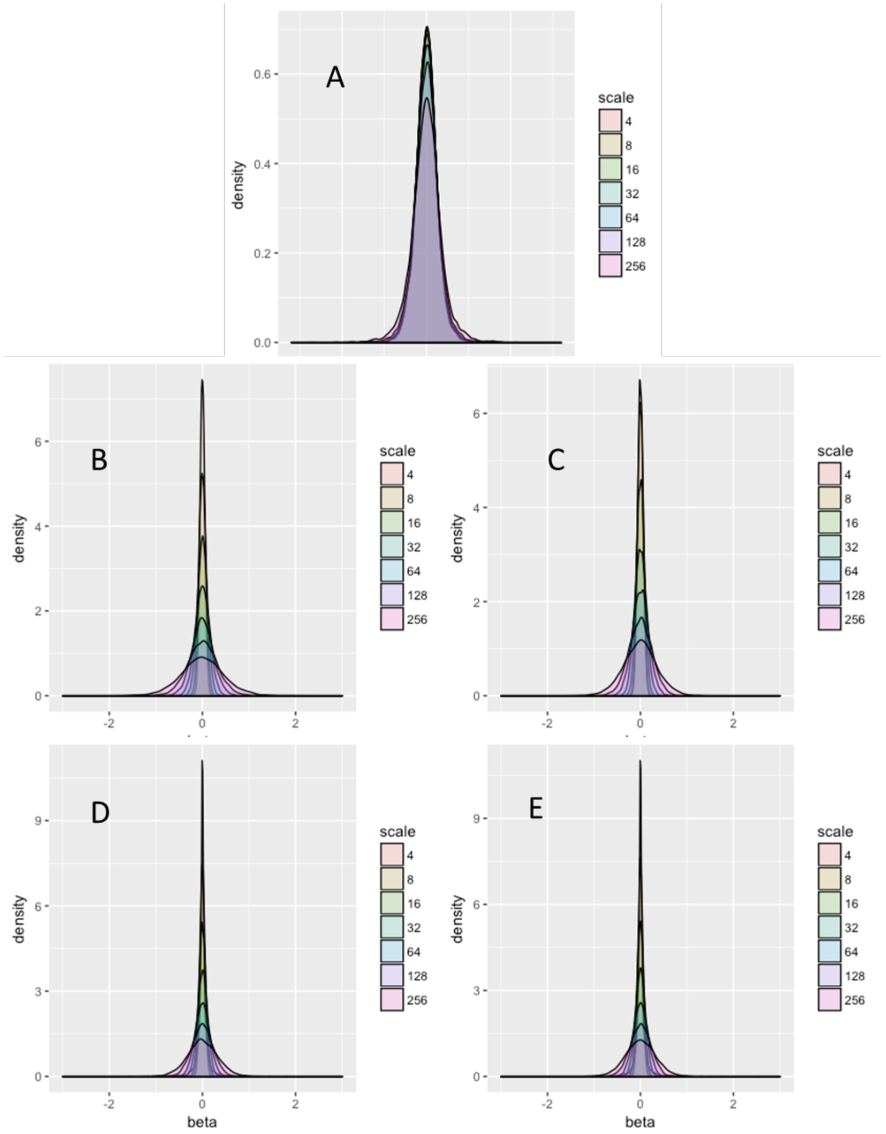


nonstationary signals (e.g., Brownian motion; Gaussian superimposed with linear / quadratic trends).... may need to apply higher-order detrending.

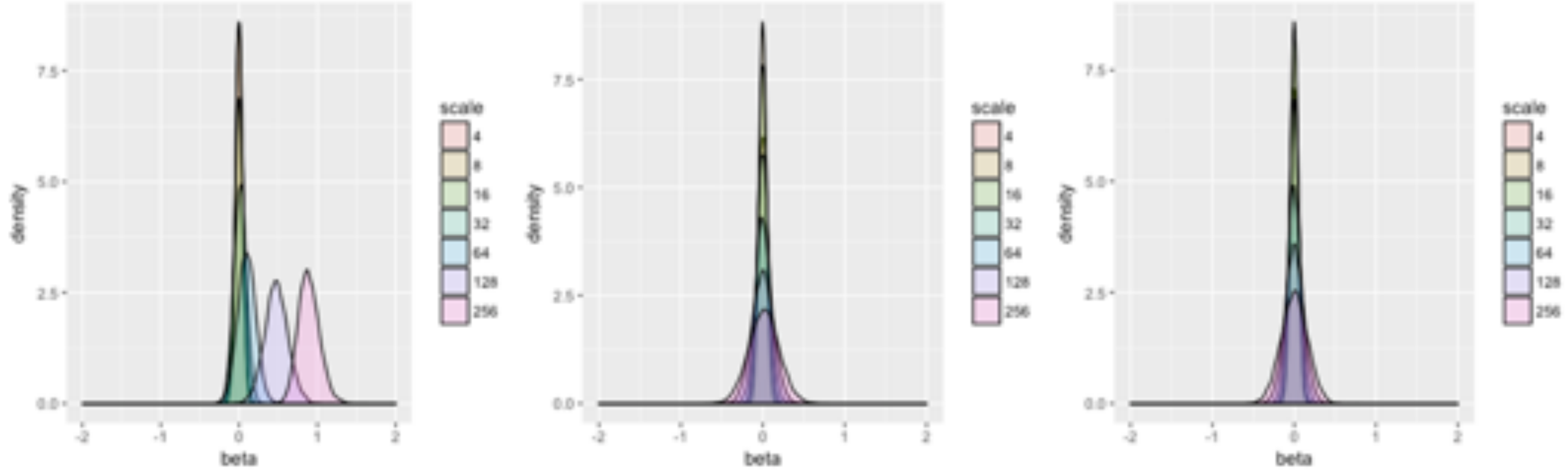
Brownian nonstationarity



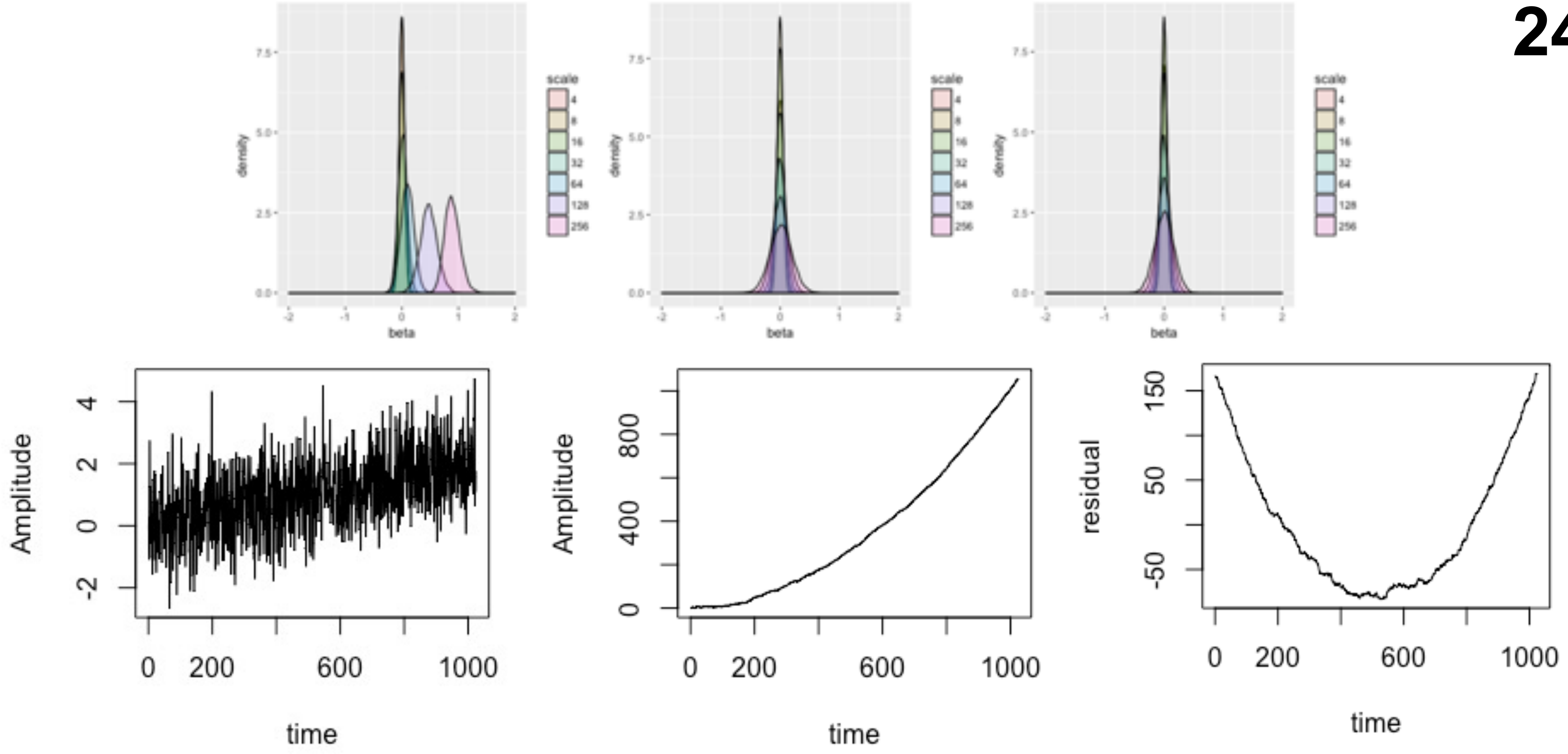
- Biased only without detrending
- Higher order polynomials further attenuate variance inflation



Linear time trends (quadratics behave similarly)



- Growth trends over entire series introduce considerable bias
- Bias is attenuated by increasing the detrending order in MRA


density
beta
scale
4
8
16
32
64
128
256
Amplitude
time
0
200
600
1000
-2
0
2
4
Amplitude
0
400
800
residual
-50
50
150

Real data

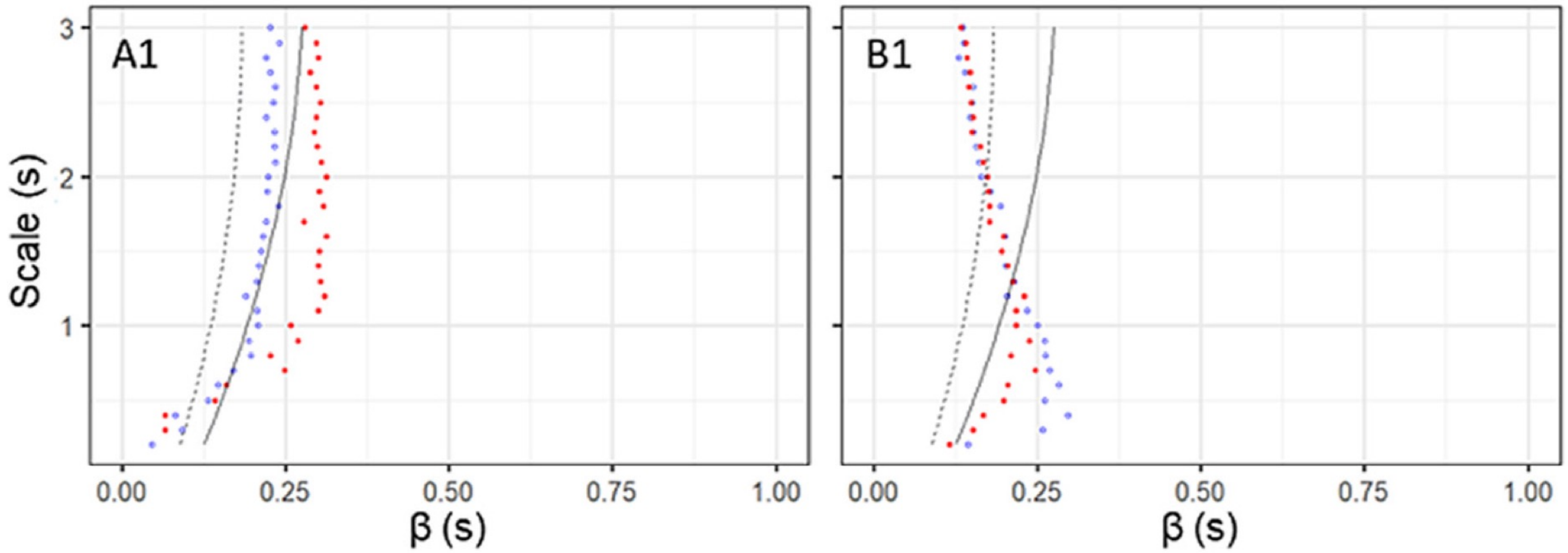


Figure 7. Averaged MRA results for (a) ankle-shoulder and (b) ankle-hip

Previous Methods capture multiscale or historical influence but not both

Scale Dependency / Multiscale Influence:

- Fractal analysis (e.g., Eke et al., 2002; Kantelhardt et al., 2002)
- Spectral and Wavelet Coherence (e.g., Grinsted et al., 2004; Mandel & Wolf, 1976)
- Fractal regression and correlation (e.g., Kristoufek, 2015; Podobnik & Stanley, 2008; Zebende, 2011)

Time Dependency / Historical Influence:

- Vector Autoregressive (VAR) models (e.g., Hamilton, 1994; Sims, 1980)
- Recurrence Quantification Analysis (RQA) techniques (e.g., Eckmann et al., 1987; Marwan et al., 2007; Weber & Zbilut, 1994)

Thank you for attending our workshop!!! Annual workshop (in-person/virtual) on July 25-29, 2022

https://www.unomaha.edu/college-of-education-health-and-human-sciences/biomechanics-core-facility/community-engagement/nonlinear-workshop.php

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