CENTER FOR RESEARCH IN HUMAN MOVEMENT VARIABILITY

NONLINEAR ANALYSIS CORE

Multifractal Methods for Movement Science

Presenters:

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Outline

Part I: Monofractal Analysis

i. Monofractal introduction
ii. Detrended Fluctuations Analysis step by step
iii. MATLAB Tutorial

Part II: Multifractal Analysis

i. From monofractal to multifractal analysis
ii. Multifractal Detrended Fluctuations Analysis step by step
iii. MATLAB Tutorial

Part III: Surrogate Analysis

i. Surrogate analysis
ii. Surrogate analysis step by step
iii. MATLAB Tutorial

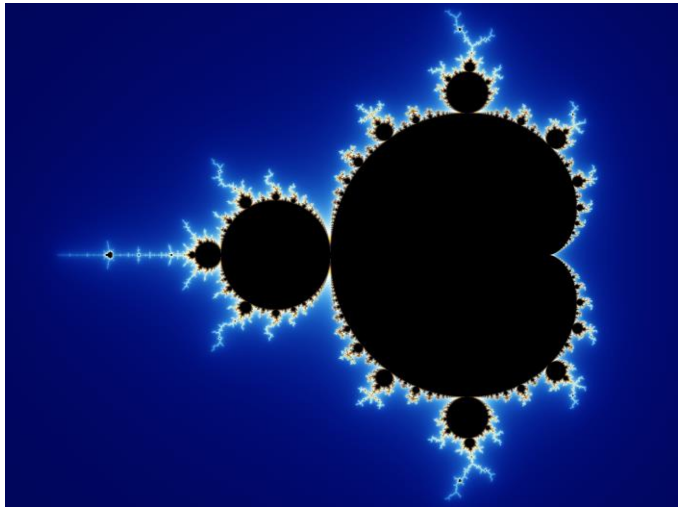
Part I: MONOFRACTAL ANALYSIS

Aaron Likens

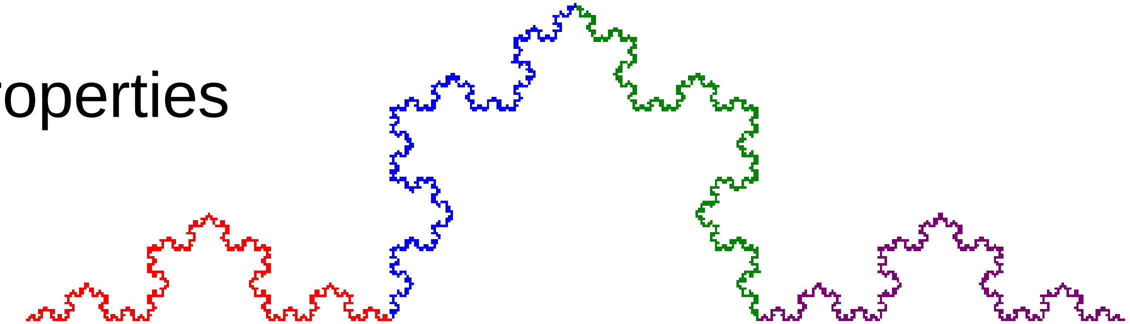
OUTLINE

- Four properties of fractals
  - Self similarity
  - Scaling
  - Dimension
  - Statistical properties
- How, what, when, where are fractals?
- Fractal analysis step by step
- Best practices

BACKGROUND- Fractals Introduction

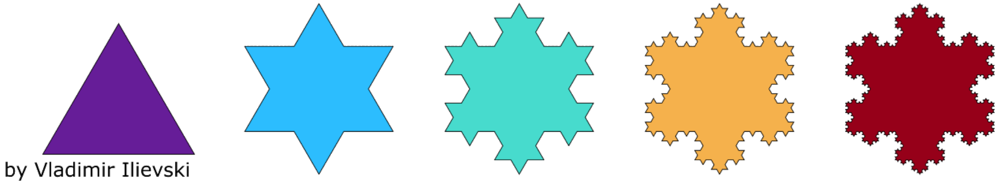


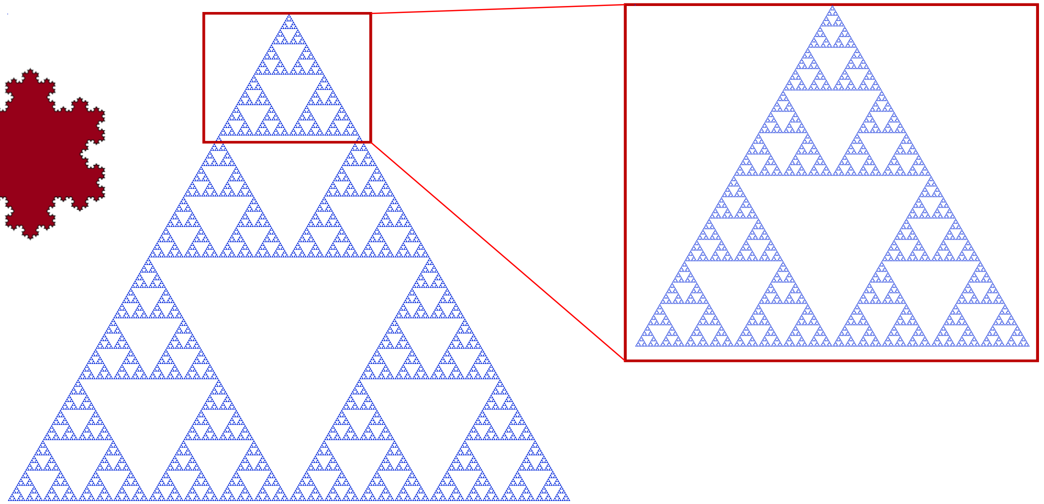
- **Fractal** objects when magnified:
  - Reveal finer and finer features;
  - Special case: look similar to larger features
- Four Properties:
  - Self-Similarity (Affinity)
  - Scaling
  - Dimensions
  - Statistical Properties

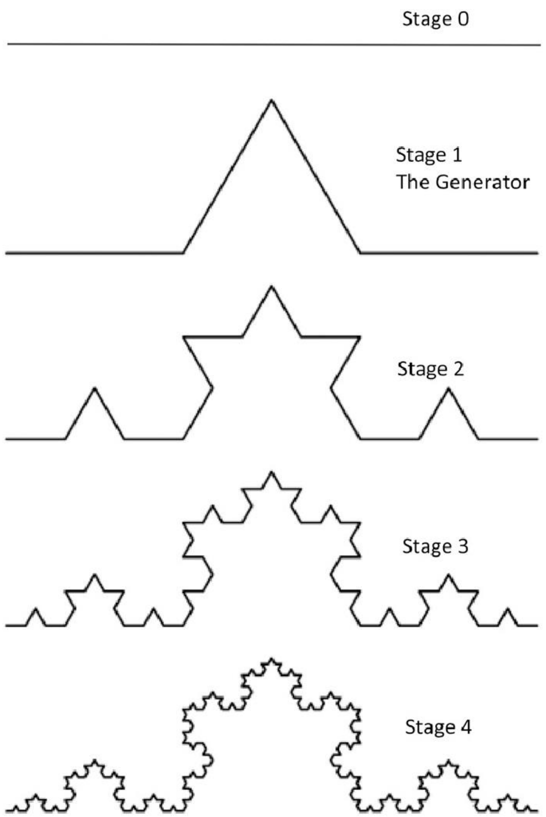


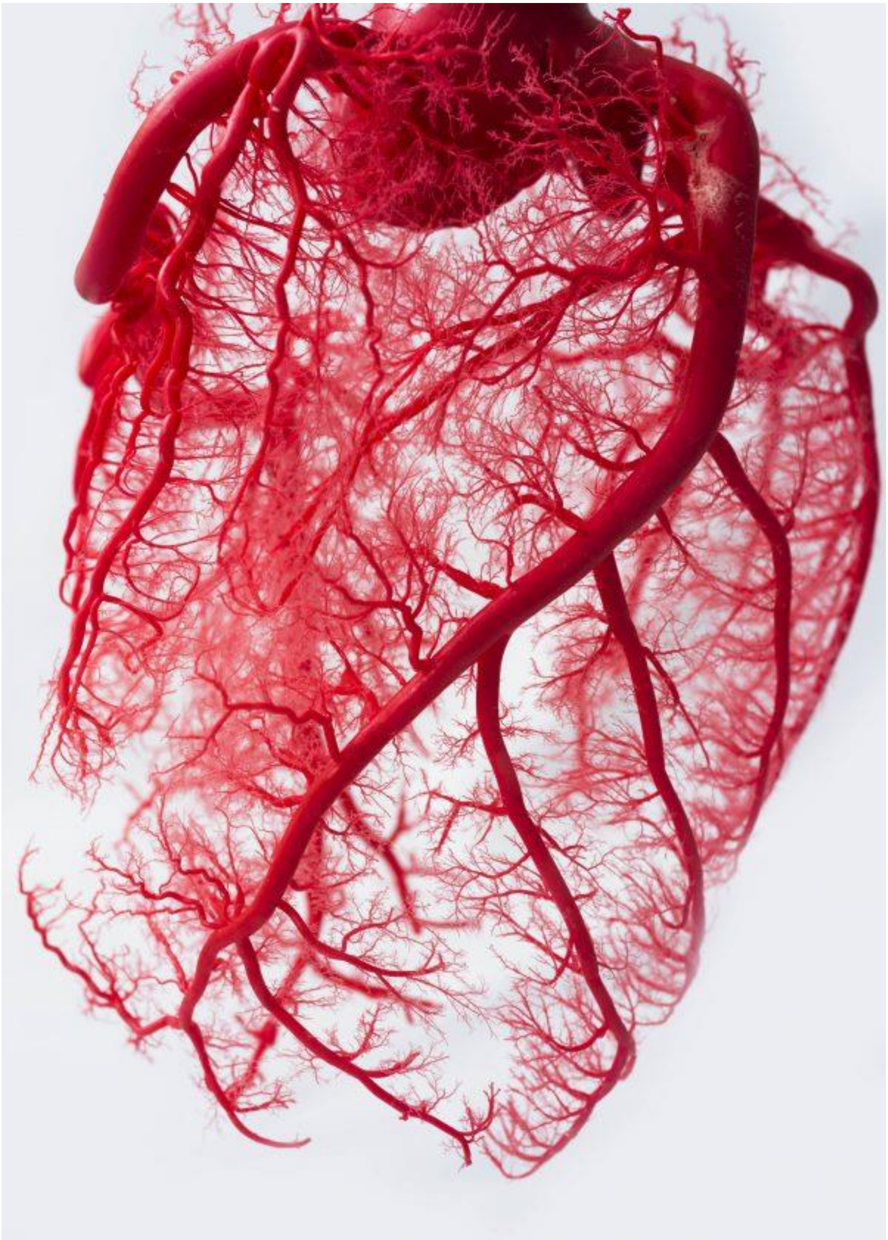
BACKGROUND- Self-Similarity

- **A geometrical interpretation** is that small parts of an object (exactly) resemble the shape formed by the entire object
  - Scale-free
  - Inherent roughness



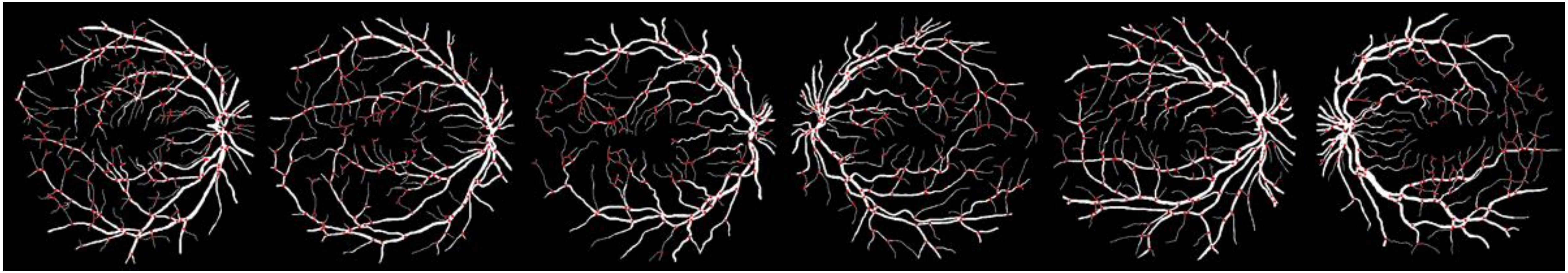






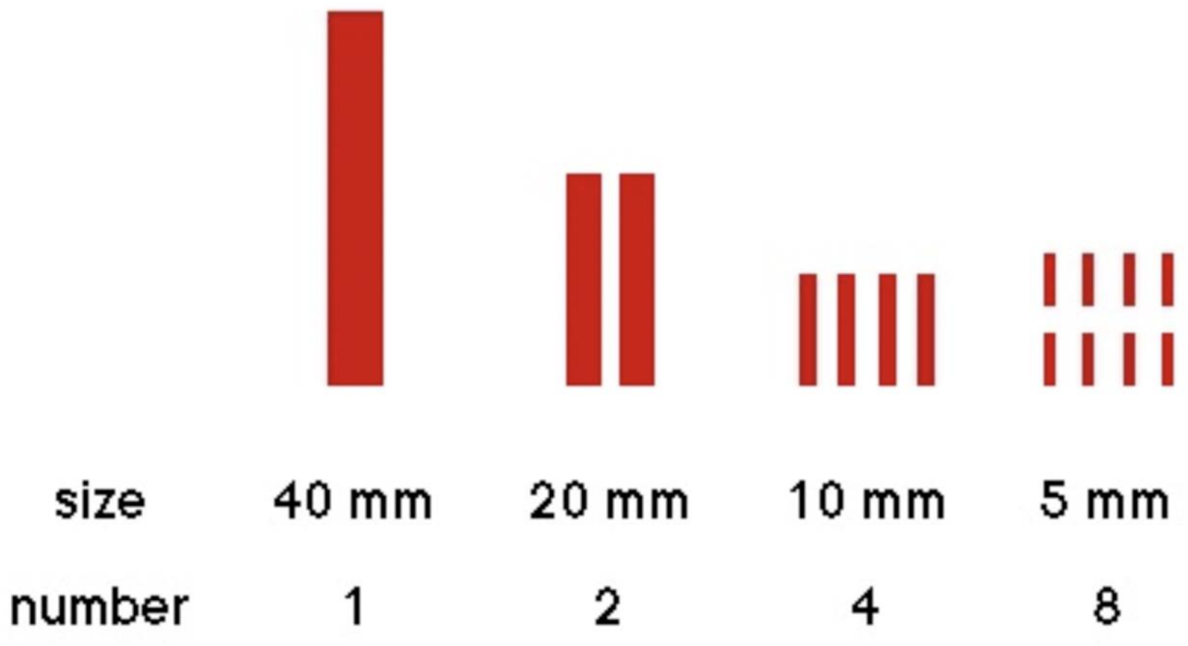
BACKGROUND- Self-Similarity

- **A statistical interpretation** is that spatial limitations may prevent infinite expansion, therefore geometric self similarity may not exist, but statistical self similarity exists.
- It may not always **LOOK** fractal but it may **BE** fractal

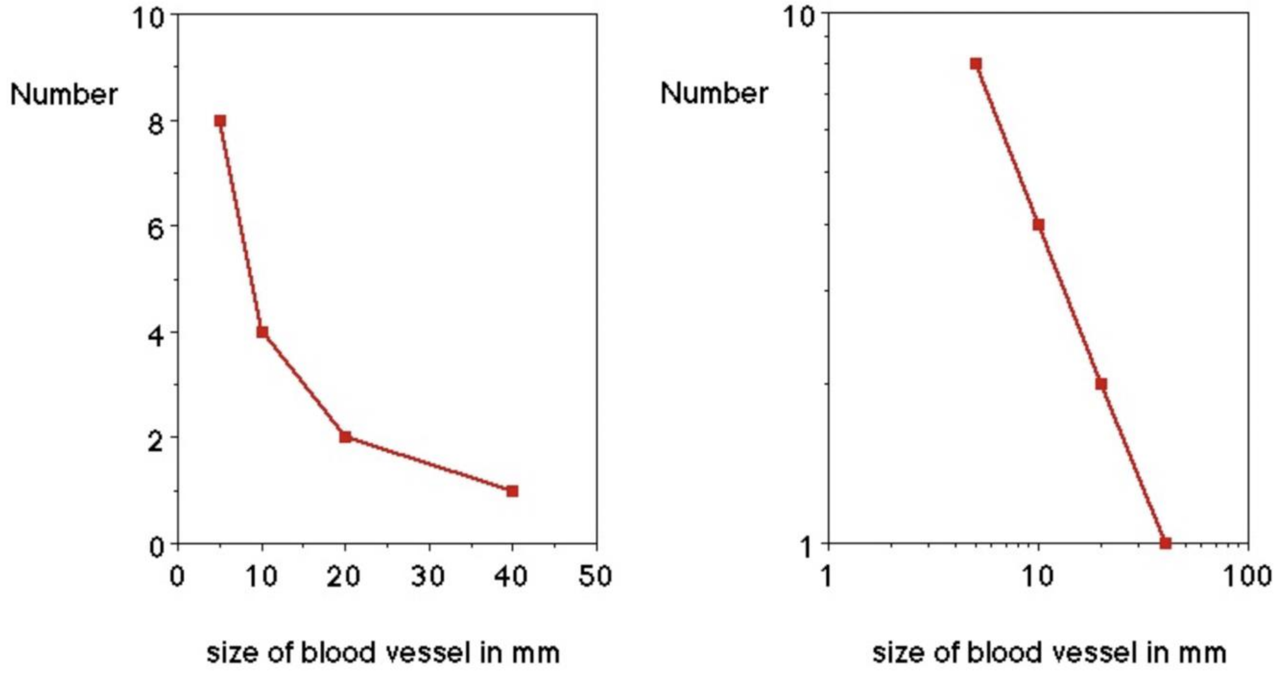


BACKGROUND- Self-Similarity

Retinal Blood Vessels

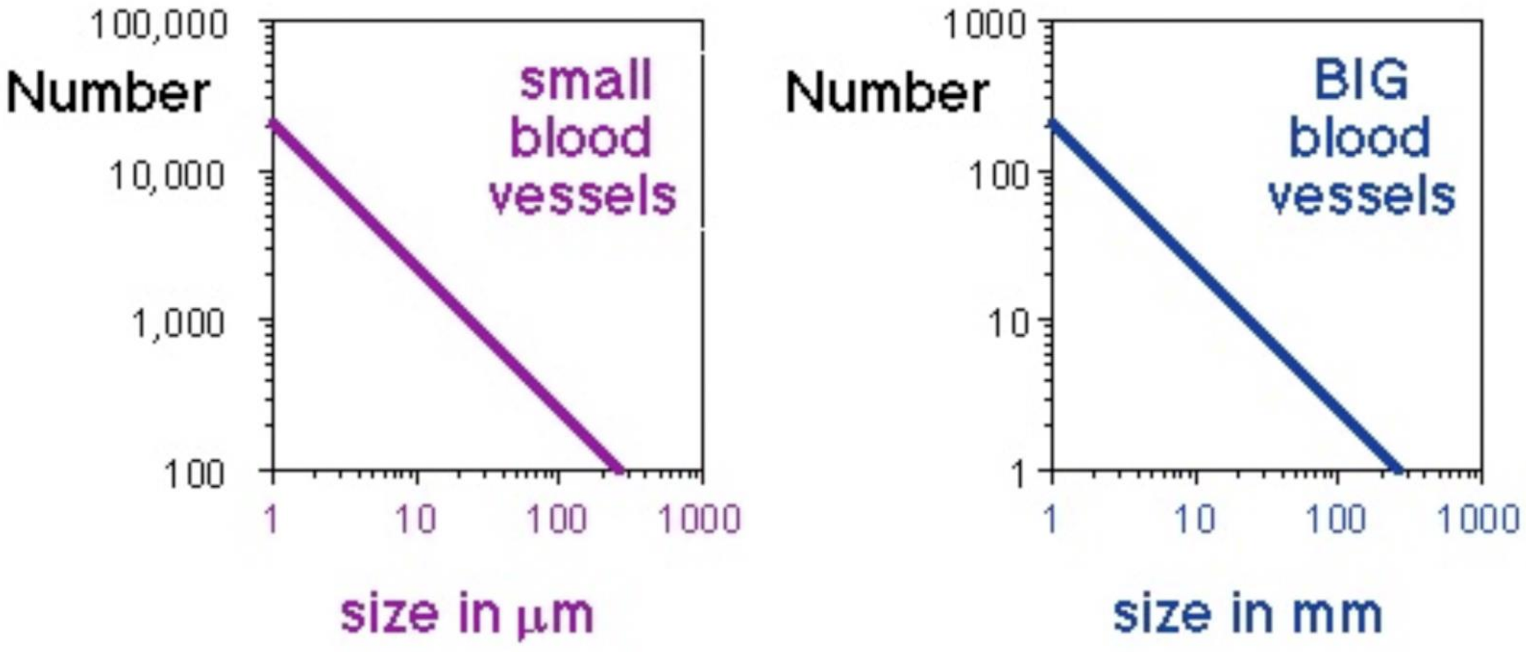


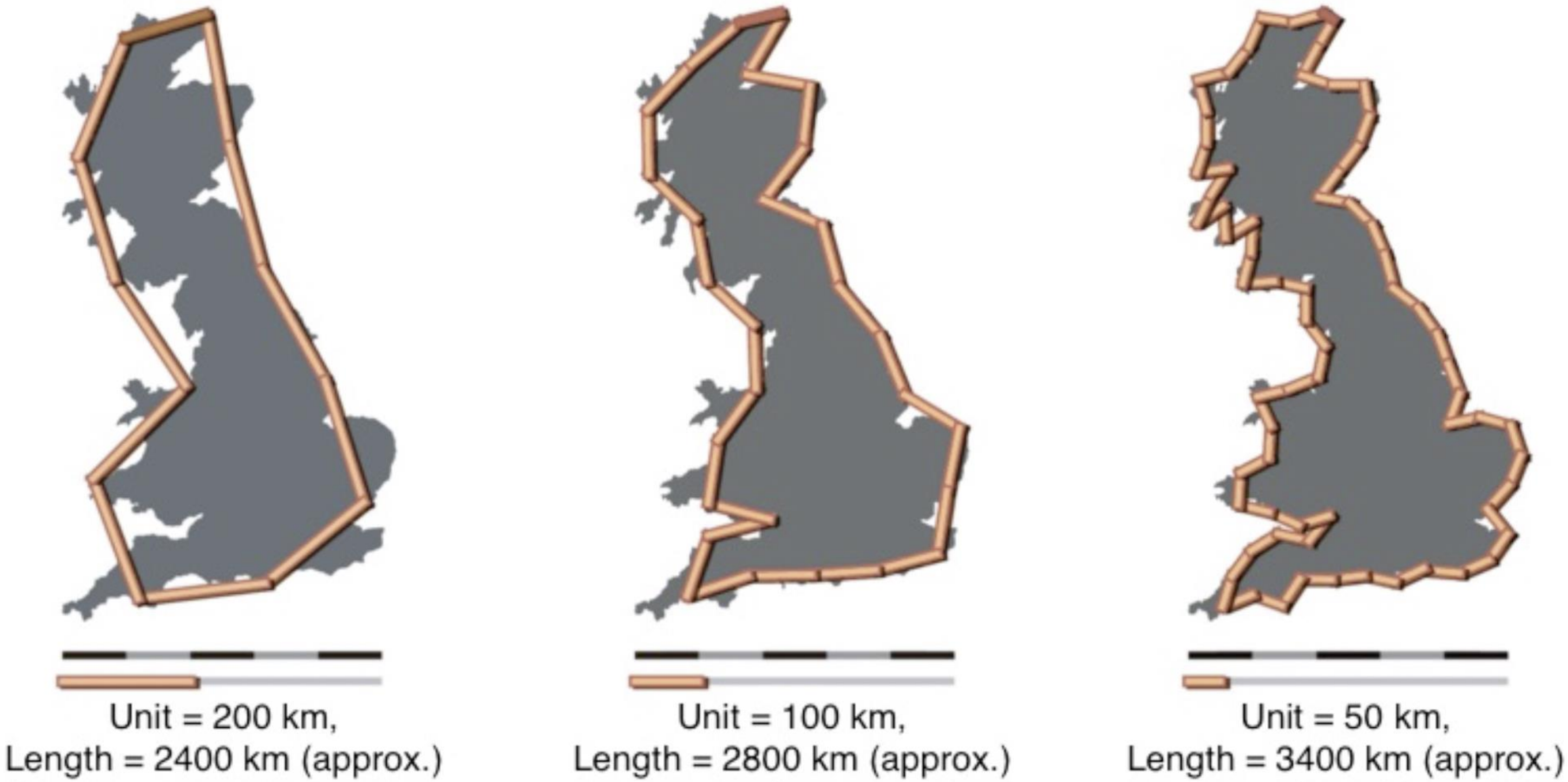
Probability Density Function (PDF)



BACKGROUND- Self-Similarity/Scaling

- Meaningful information is lost using a single resolution
- Our analysis is on this scaling relationship!





How long is the coast of Britain?
Mandelbrot, 1967, Science

BACKGROUND- Dimension

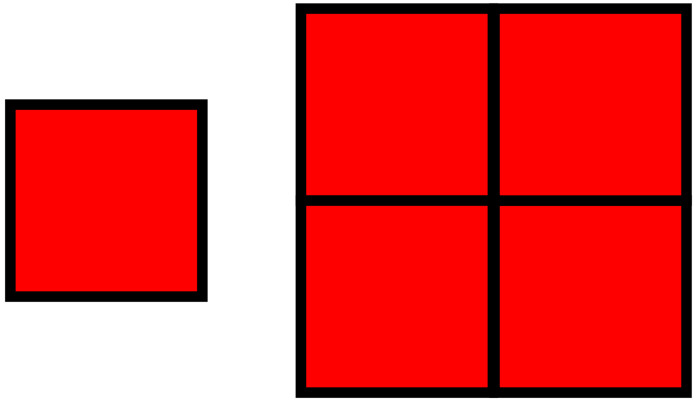
D = 0 D = 1 D = 2 D = 3

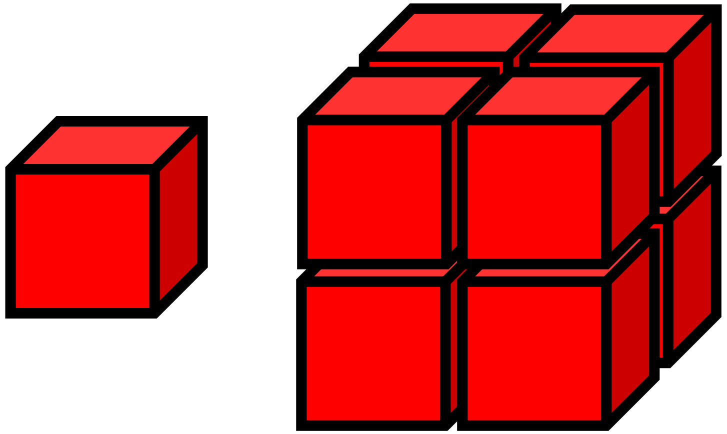
- The **fractal dimension** provides a quantitative measurement of self-similarity and scaling.
- Characterizes how the fractal object fills space

$$\text{Fractal Dimension (D)} = \frac{\ln N}{\ln R} = \frac{N\ Number\ of\ figures}{R\ times\ larger}$$

BACKGROUND- Dimension

$$\text{Fractal Dimension (D)} = \frac{\ln N}{\ln R} = \frac{N\ Number\ of\ figures}{R\ times\ larger}$$



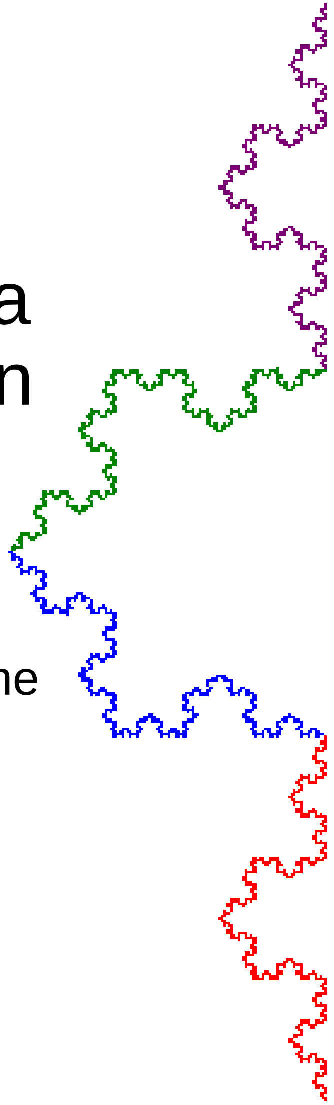


$$D = \frac{\ln 4}{\ln 2} = \frac{2}{1} = 2$$

$$D = \frac{\ln 8}{\ln 2} = \frac{3}{1} = 3$$

BACKGROUND- Dimension

- A **fractal** is an object in space or process in time that has a fractal dimension (D) greater than its topological dimension
  - Example: D = 1.26
    - Since D > 1, it covers more than a 1-D line, but less than a 2-D area
  - Topological dimension is 1
    - This dimension tells us what the object is, such as an edge, surface, or volume
  - When fractal dimension > topological dimension
    - The edge, surface, or volume has more finer pieces than we would have expected of an object with its topological dimension



BACKGROUND- Statistical Properties

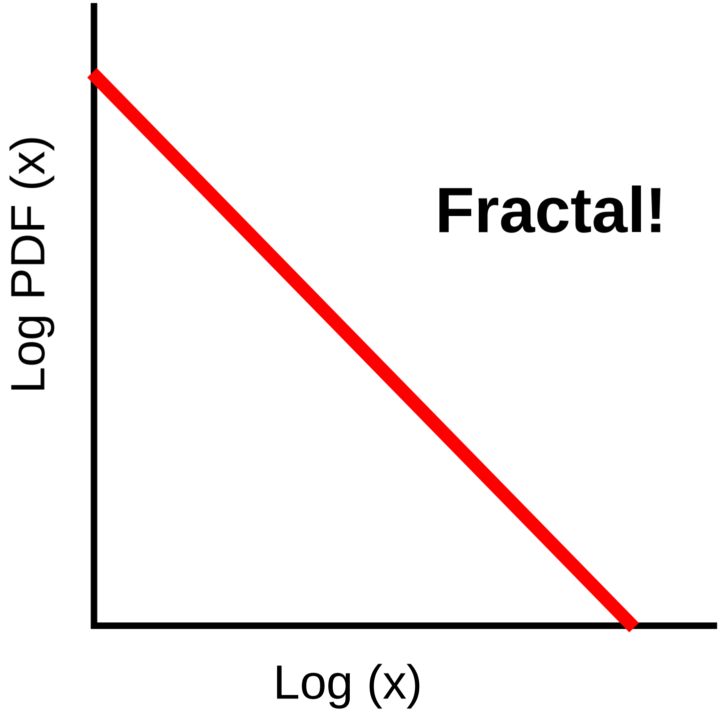
Fractal or Not Fractal?

How can you tell?

BACKGROUND- Statistical Properties

Fractal or Not Fractal?

How can you tell?



BACKGROUND- Statistical Properties

Fractal or Not Fractal?

How can you tell?

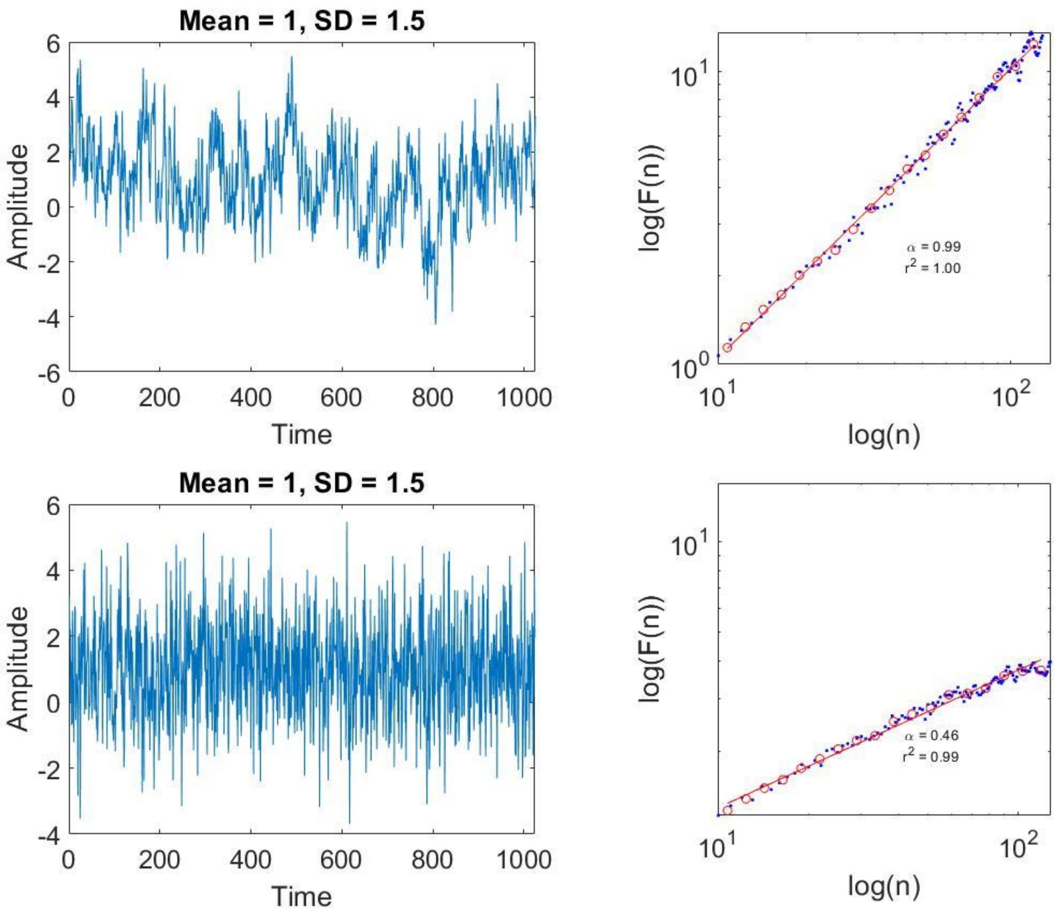
Where can it be found and what does it tell us?

- Fractal patterns/behavior exists in:
  - DNA nucleotides
  - Neural patterns
  - Cardiovascular system
  - Heart rate
  - Inter-step intervals
  - Inter-organism communication
  - Geographical features
  - Weather patterns

- Fractal patterns/behavior can distinguish between healthy and unhealthy patients in:
  - **Center of Pressure**
  - **Gait**
  - Heart rate
  - Huntington's
  - Parkinson's
  - Alzheimers
  - ADHD

What does this complex structure tell us?

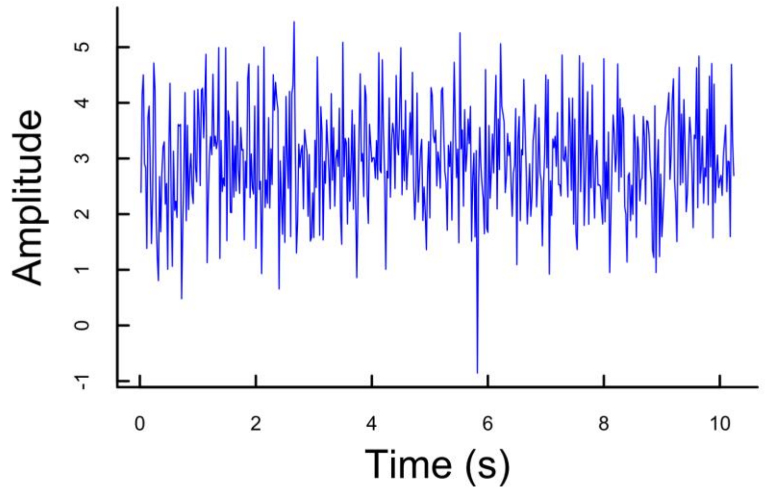
- Characteristics that may not be present from traditional perspectives (i.e. Mean/SD)
- We can look at changes overtime in a system at different time-scales
- **Fractal analysis is measuring how some measure of variability changes as a function of scale**

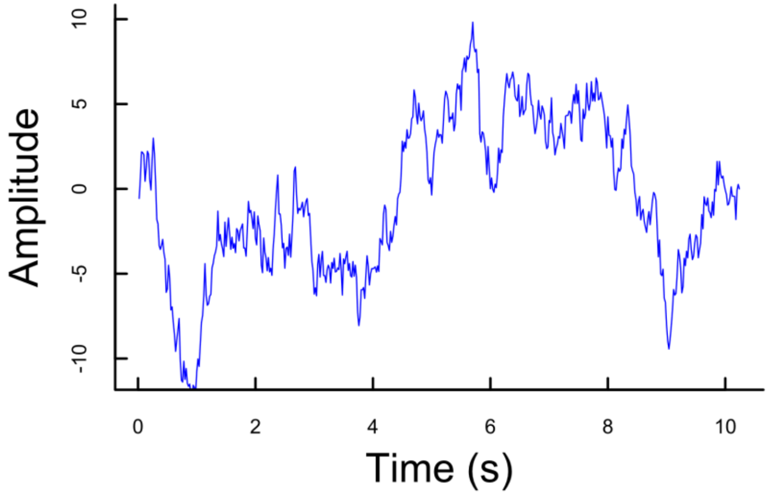


How can we capture this complex structure?

- Fractal analysis techniques
  - **Monofractal Detrended Fluctuation Analysis (DFA)** (Peng et al 1994)
    - **A method to determine the statistical self-similarity of long time-series that may have memory**
  - Multifractal Detrended Fluctuation Analysis (MFDFA)
  - Fractal Regression
  - And many more!

DFA ANALYSIS STEP BY STEP

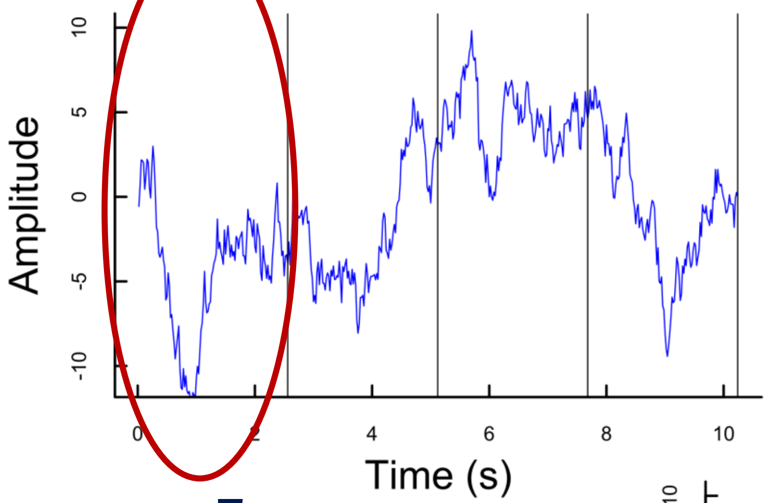


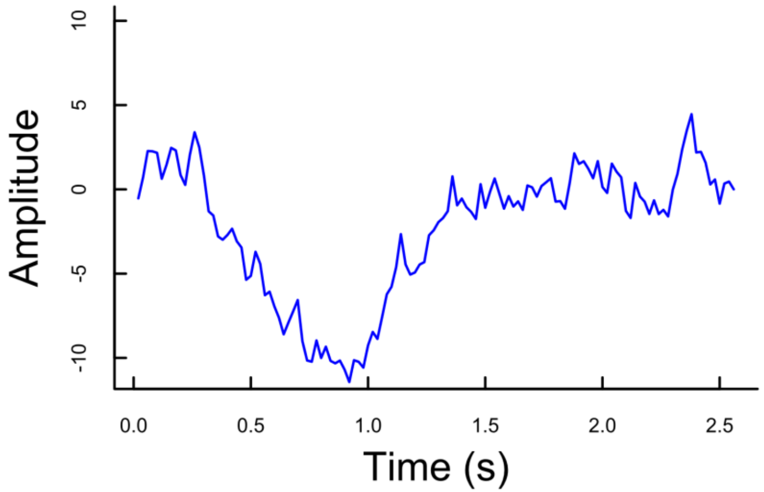


<u>Step 1</u>

- Create a profile of a time series:
  - subtracting its mean from each data point
  - Integrate the time series
- This step allows the conversion of the time series to random walk-like process that meets the theoretical assumptions of DFA

DFA ANALYSIS STEP BY STEP



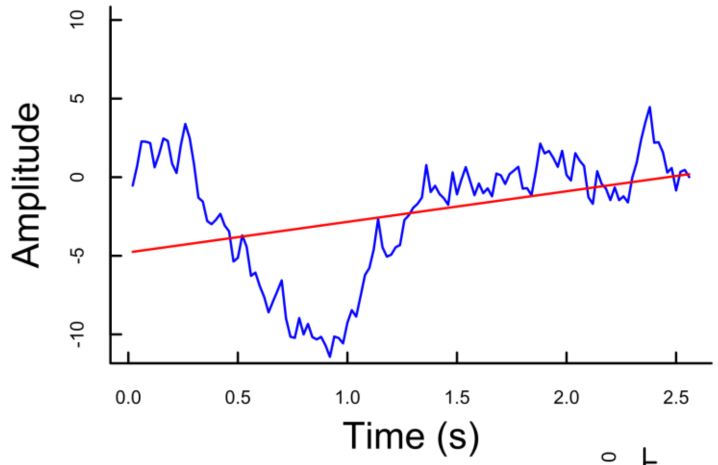


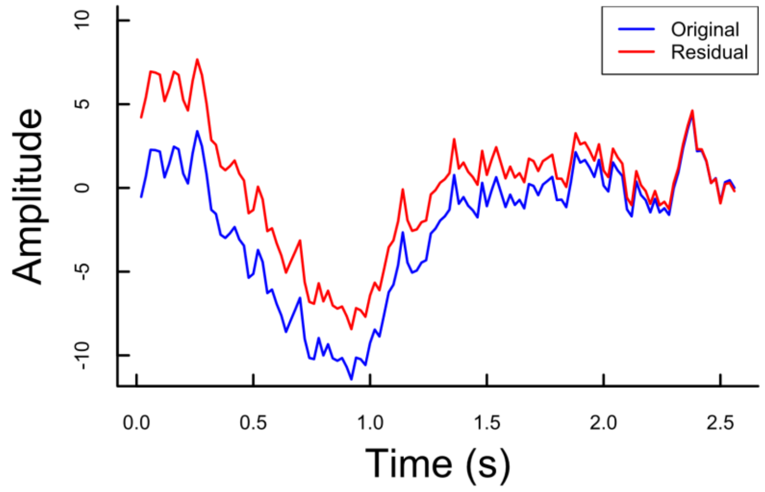
Step 2

- Divide the series into a sequence of non-overlapping windows.
- In this example, the time series was separated into 4 windows (Top).

→ Zooming in

DFA ANALYSIS STEP BY STEP





Step 3

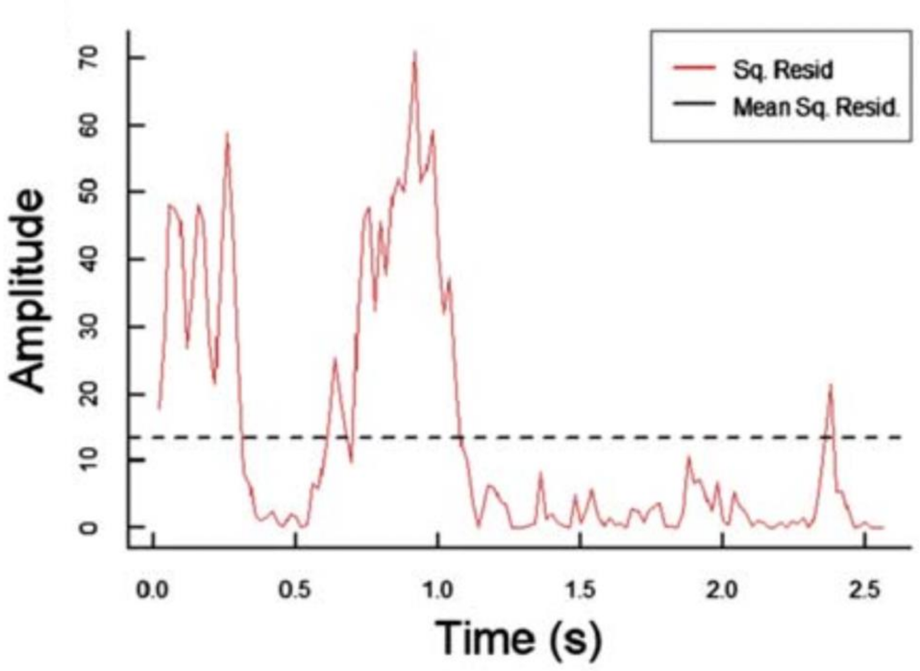
Detrending stage:

- This means fitting a regression line within each of the windows (Top)
- Then, we subtract the fitted trend line from the data in each window (Bottom)

DFA ANALYSIS STEP BY STEP

Step 4

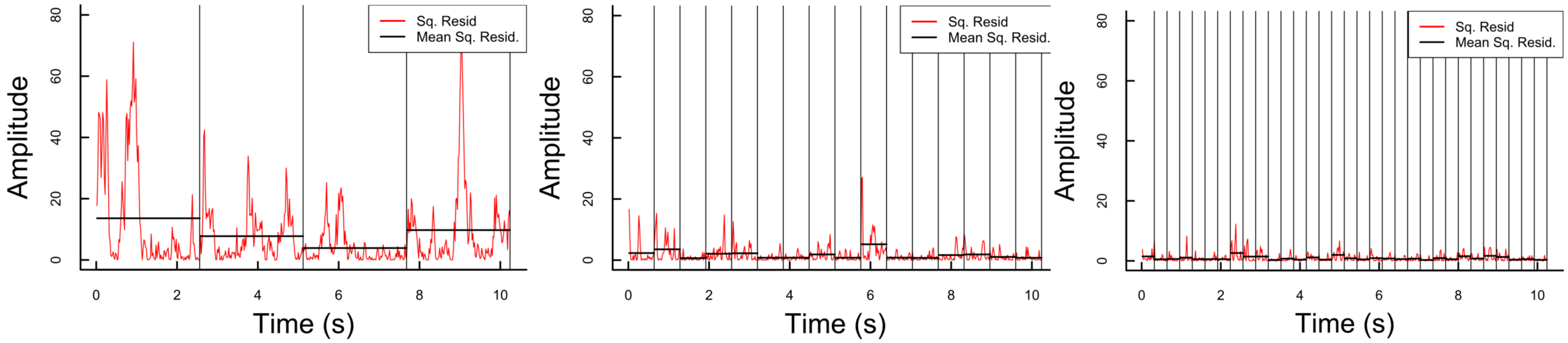
- Find the average fluctuation F(s) by taking the root mean square of the local residuals within each scale.



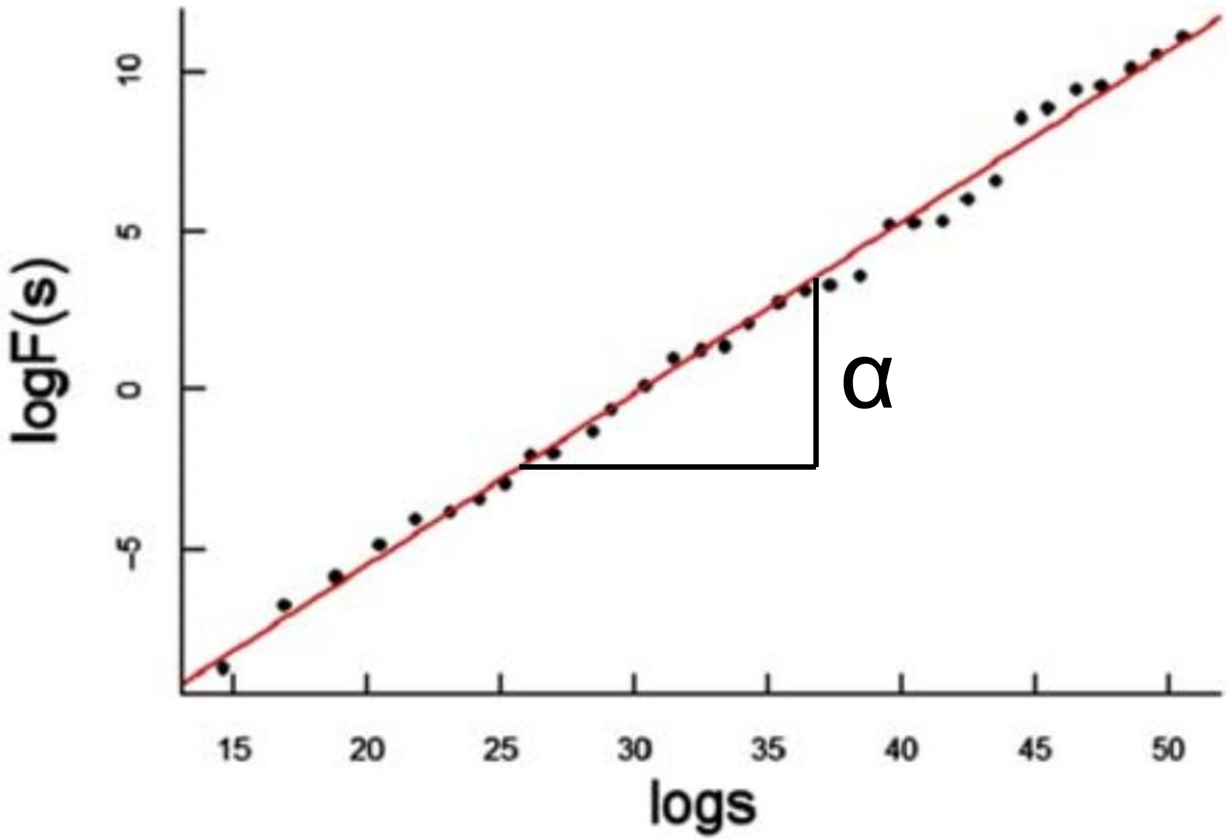
DFA ANALYSIS STEP BY STEP

Step 4

- Find the average fluctuation F(s) by taking the root mean square of the local residuals within each scale.



DFA ANALYSIS STEP BY STEP



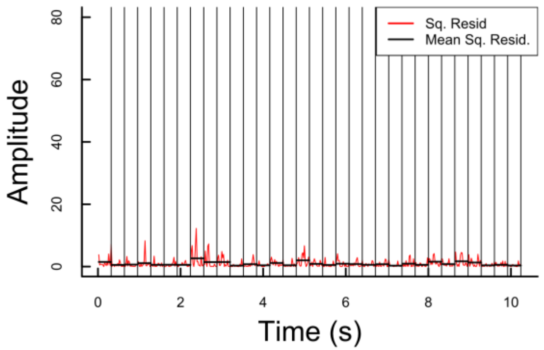
Step 5

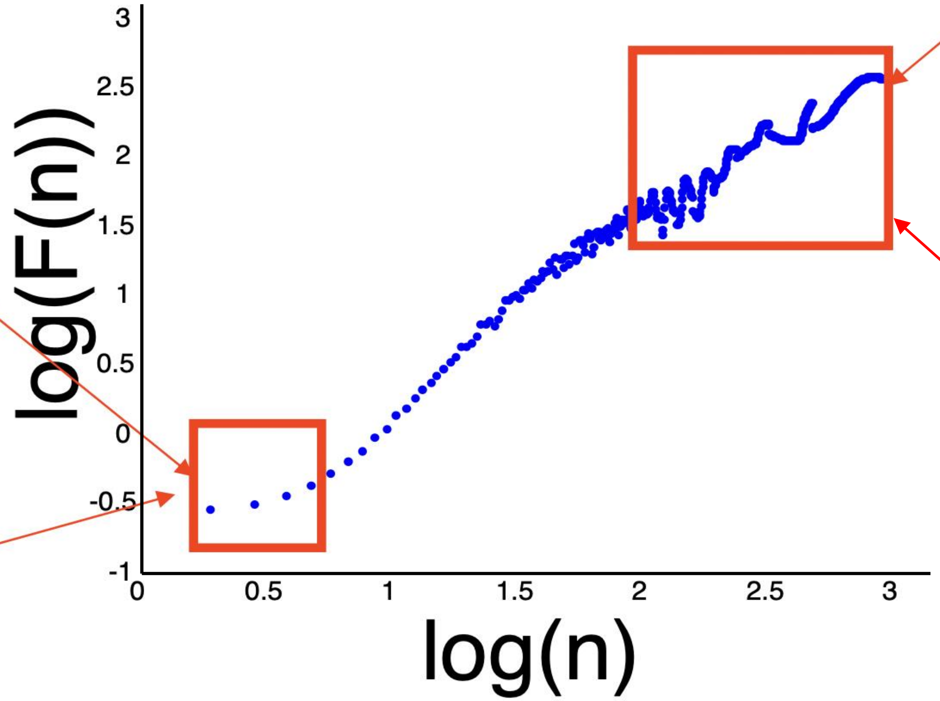
- Perform a regression to estimate the scaling exponent α
- α is a measure of how fast the standard deviation changes as a function of timescale

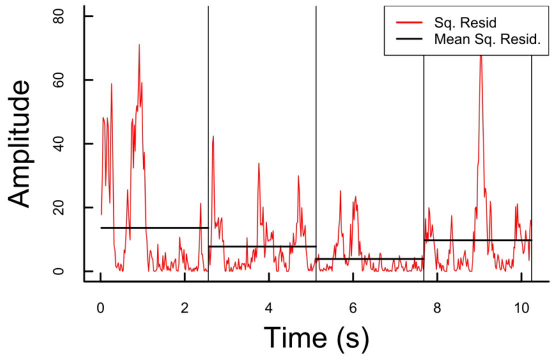
DFA ANALYSIS Interpretation

- Many small windows
- SD's computed from only a few points
- Statistically less reliable

- Few large windows
- Fewer windows to compute average fluctuations
- More variable



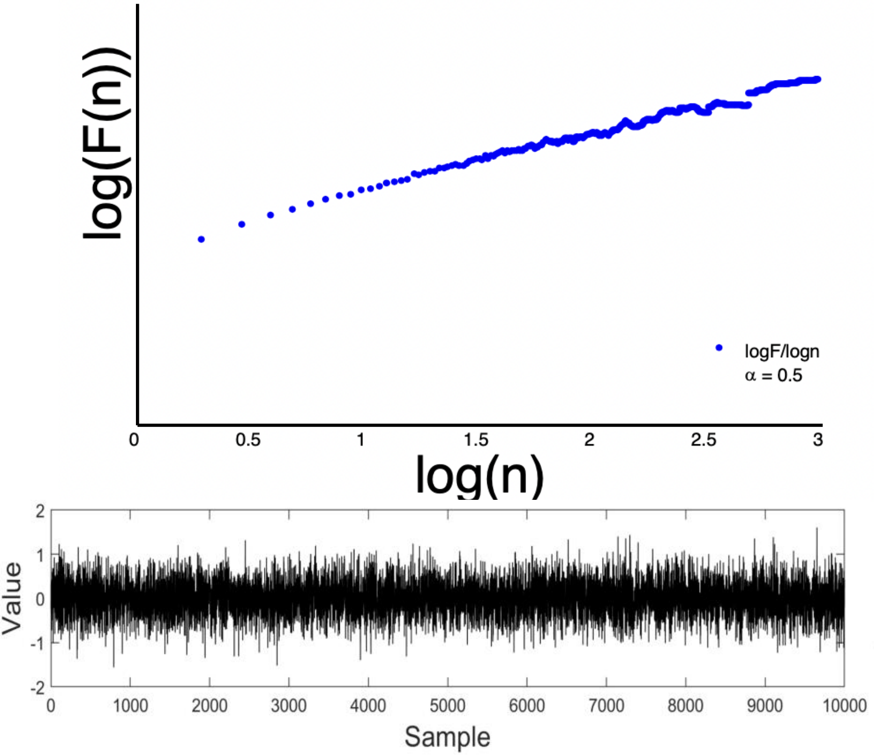




DFA ANALYSIS Interpretation

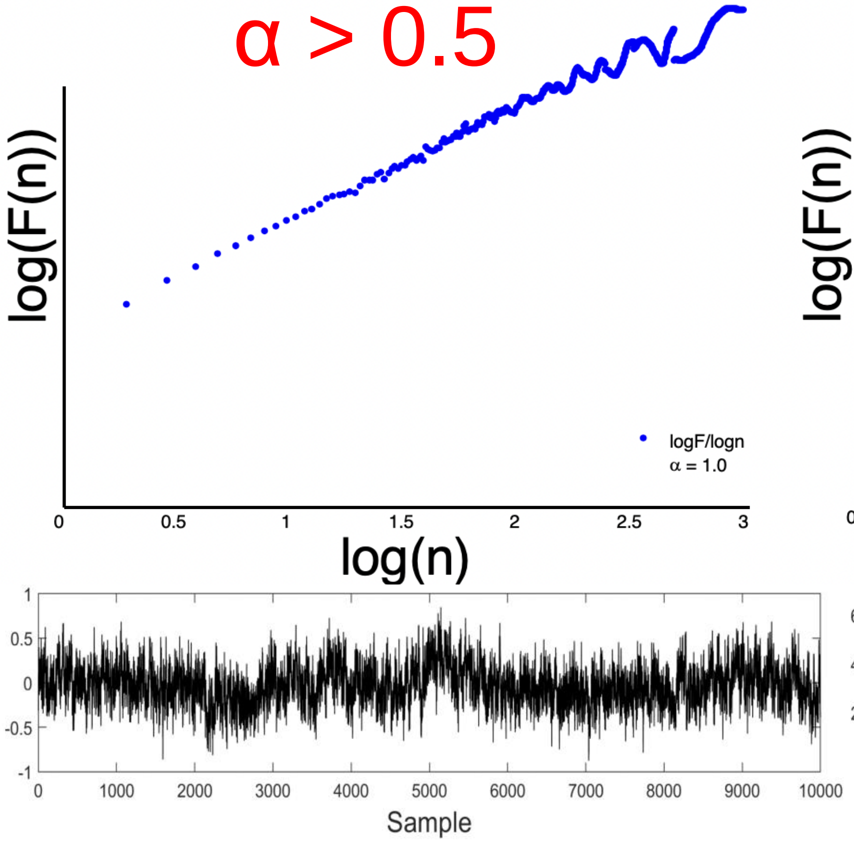
Negatively Correlated

$\alpha < 0.5$



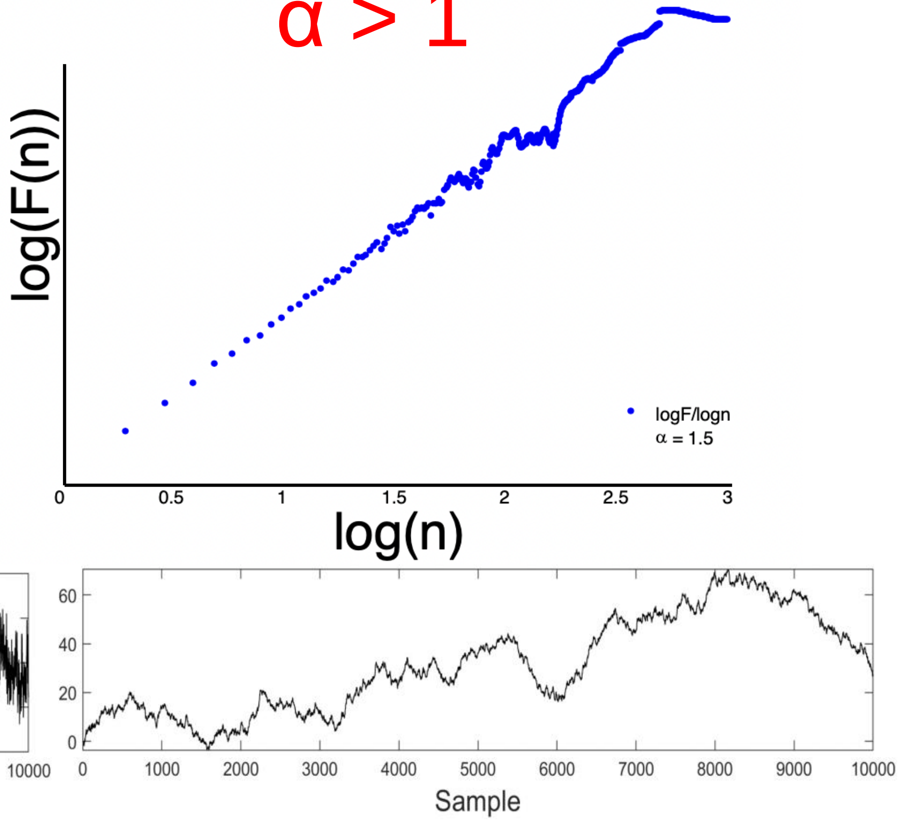
Positively Correlated

$\alpha > 0.5$



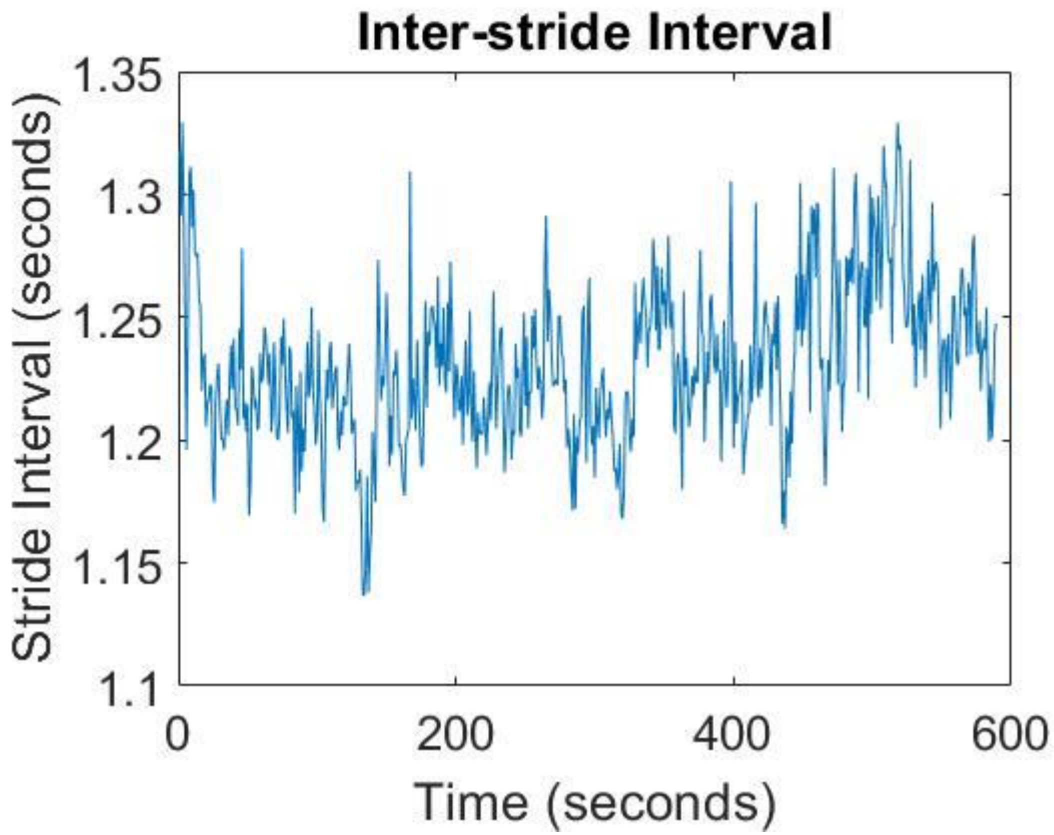
Nonstationary/Unbounded

$\alpha > 1$



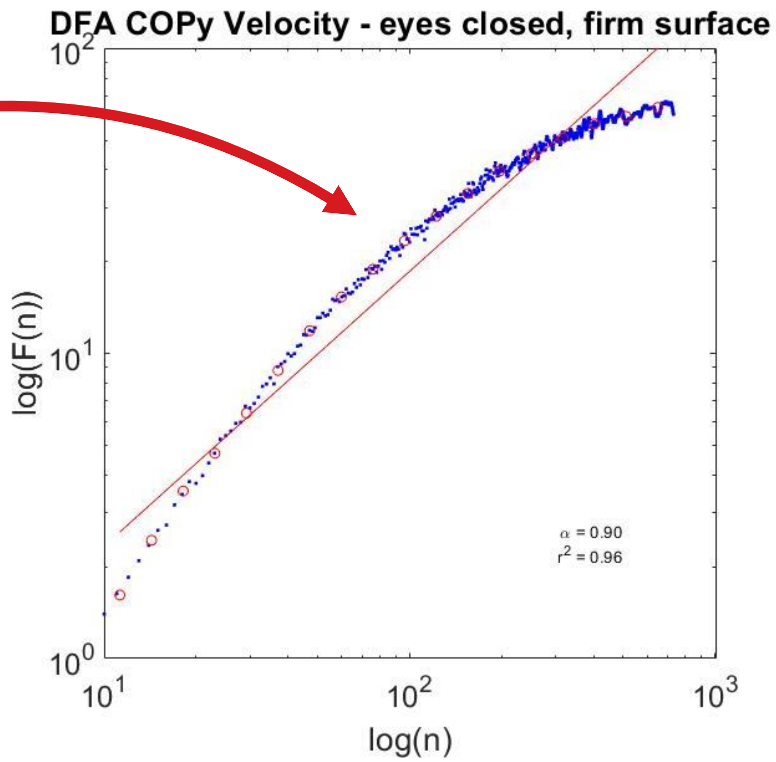
DFA Best Practices

- Plot your data
  - Visually inspect for:
    - ↑ or ↓ trends
    - Structure or roughness
- Time series length
  - More is almost always better
  - ≥ 500 datapoints for stable α estimates
    - i.e. 500 steps, taps, beats
    - Options exist for shorter time series



DFA Best Practices

- Investigating crossovers
  - Crossovers of short/long-term scaling behaviors
  - Begin DFA at lowest order
    - Inspect logF(n) x log(n) plot for crossover
      - Repeat DFA with higher order if crossover exists
- Timescales
  - Minimum = 16 data points
  - Maximum = $\frac{Length\ of\ series}{9}$



*******5 min Break*******

Github link:

https://github.com/aaronlikens/NACOB-2022-Multifractal-Methods-in-Movement-Science.git

MATLAB VERSION

There are no known incompatibilities using MATLAB version R2019a or later.

MATLAB Toolboxes Required:

Statistics and Machine Learning Toolbox

Signal Processing Toolbox

Image Processing Toolbox

Images

- By Created by Wolfgang Beyer with the program Ultra Fractal 3. - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=321973
- https://phys.org/news/2017-03-fractal-patterns-nature-art-aesthetically.html
- https://larryriddle.agnesscott.org/ifs/kcurve/kcurve.htm
- By Beojan Stanislaus, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=8862246
- https://www.researchgate.net/figure/The-first-stages-of-the-construction-of-von-Koch-curve-and-a-random-version-of-the-curve_fig1_329695536
- https://isquared.digital/visualizations/2020-06-15-koch-curve/
- https://iternal.us/what-is-a-fractal/
- https://webvision.med.utah.edu/tag/retinal-vasculature/
- By Ivar Leidus - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=97556224

Papers on polynomial order

- Kantelhardt, J. W., Koscielny-Bunde, E., Rego, H. H. A., Havlin, S., & Bunde, A. (2001). Detecting long-range correlations with detrended fluctuation analysis. Physica A: Statistical Mechanics and Its Applications, 295(3), 441454. Available from https://doi.org/10.1016/S0378-4371(01)00144-3.
- Likens, A. D., Fine, J. M., Amazeen, E. L., & Amazeen, P. G. (2015). Experimental control of scal- ing behavior: What is not fractal? Experimental Brain Research, 233(10), 28132821.

Peng paper on DFA

- Peng, C.-K., S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley, and A. L. Goldberger. "Mosaic Organization of DNA Nucleotides." *Physical Review E* 49, no. 2 (February 1, 1994): 1685–89. https://doi.org/10.1103/PhysRevE.49.1685.

Papers on time series length

- Delignieres, D., Ramdani, S., Lemoine, L., Torre, K., Fortes, M., & Ninot, G. (2006). Fractal analyses for 'short' time series: A re-assessment of classical methods. Journal of Mathematical Psychology, 50(6), 525544. Available from https://doi.org/10.1016/j.jmp.2006.07.004.
- Stroe-Kunold, E., Stadnytska, T., Werner, J., & Braun, S. (2009). Estimating long-range dependence in time series: An evaluation of estimators implemented in R. Behavior Research Methods, 41(3), 909923. Available from https://doi.org/10.3758/BRM.41.3.909.
- Marmelat, V., & Meidinger, R. L. (2019). Fractal analysis of gait in people with Parkinson's disease: Three minutes is not enough. Gait & Posture, 70, 229234. Available from https://doi.org/10.1016/j.gaitpost.2019.02.023.
- Yuan, Q., Gu, C., Weng, T., & Yang, H. (2018). Unbiased detrended fluctuation analysis: Long-range correlations in very short time series. Physica A: Statistical Mechanics and Its Applications, 505, 179189. Available from https://doi.org/10.1016/j.physa.2018.03.043.

Papers on time scales

- Almurad, Z. M. H., & Delignières, D. (2016). Evenly spacing in detrended fluctuation analysis. Physica A: Statistical Mechanics and Its Applications, 451, 6369. Available from https://doi.org/10.1016/j.physa.2015.12.155.
- Likens, A. D., Fine, J. M., Amazeen, E. L., & Amazeen, P. G. (2015). Experimental control of scaling behavior: What is not fractal? Experimental Brain Research, 233(10), 28132821.
- Yuan, Q., Gu, C., Weng, T., & Yang, H. (2018). Unbiased detrended fluctuation analysis: Long-range correlations in very short time series. Physica A: Statistical Mechanics and Its Applications, 505, 179189. Available from https://doi.org/10.1016/j.physa.2018.03.043.

Part II: MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS

Anaelle E. Charles

Outline:

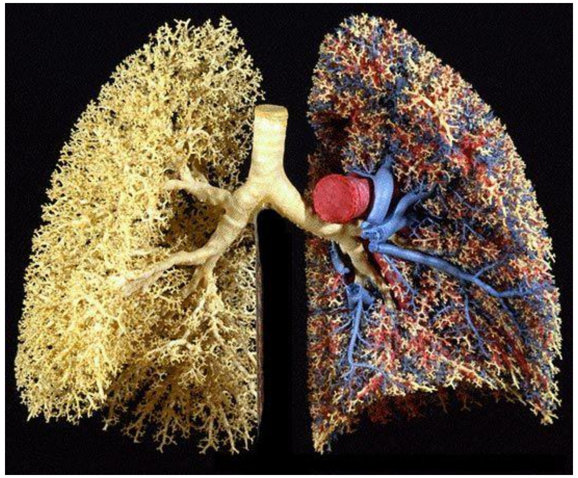
Part II: Multifractal Analysis

i. From monofractal to multifractal analysis
ii. Multifractal Detrended Fluctuations Analysis step by step
iii. MATLAB Tutorial

Brief Recap on Fractals

Aloe Vera plant

Fractals are ubiquitous in nature



Many natural structures exhibit ***self-similarity***, where their structure repeat itself over many scales

Romanesco broccoli

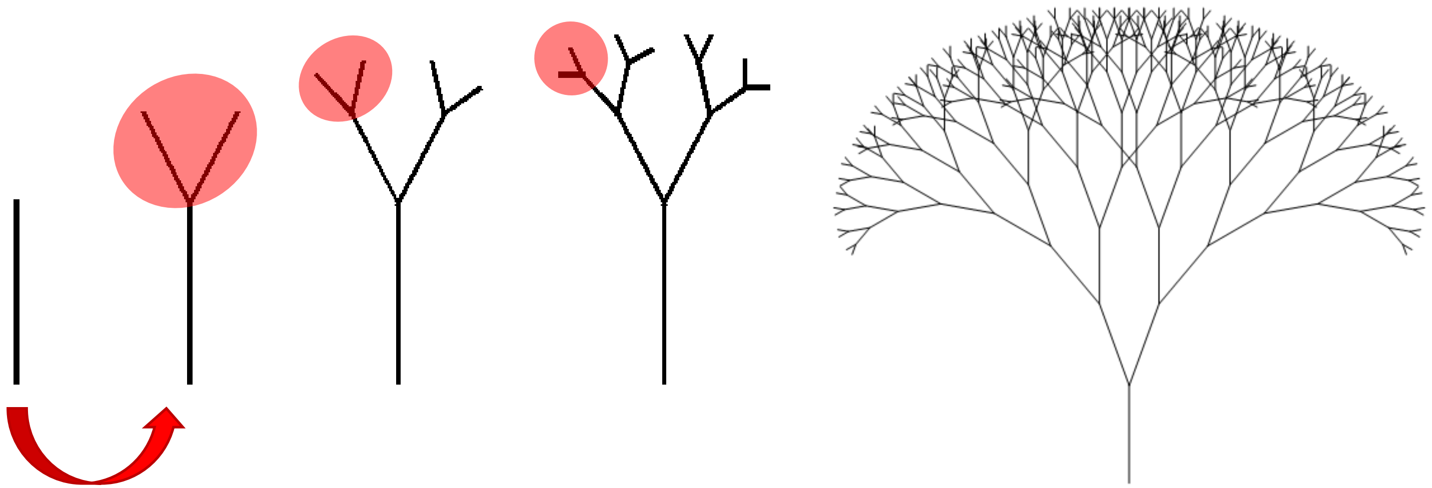
Brief Recap on Fractals

In fact, ***self-similarity*** can be easily illustrated through very simple mathematical rules.

From a very simple set of rules, you get a complex structure

RULES:

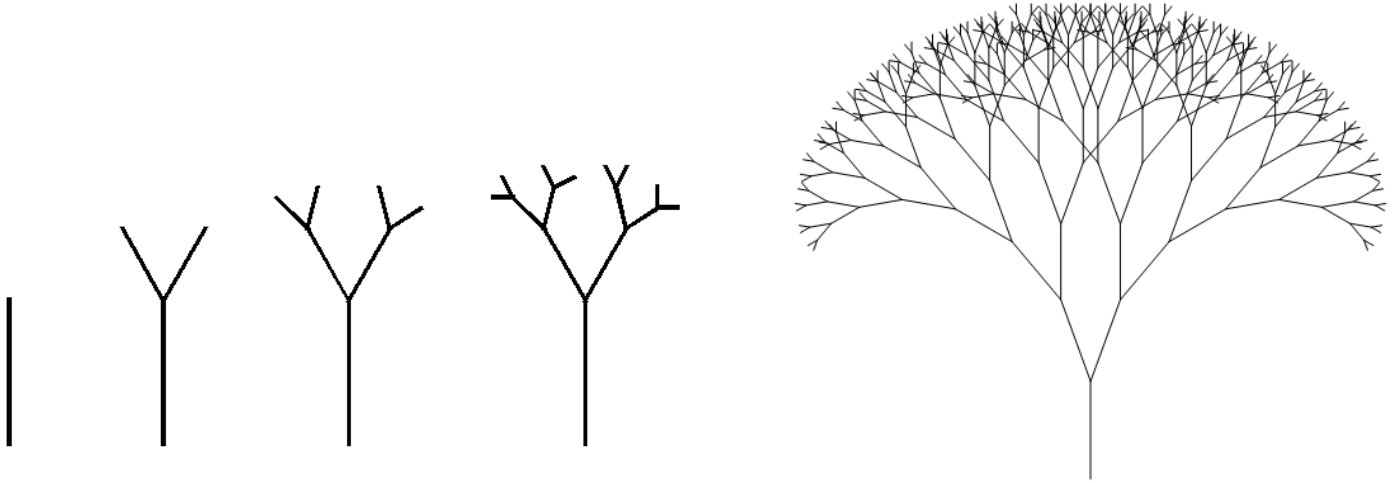
1. **Create a vertical line**
2. **Break that line in half and add it on top**
3. **Repeat**



Fractal properties: *self-similarity*, *scale invariance*, *roughness*.

Brief Recap on Fractals

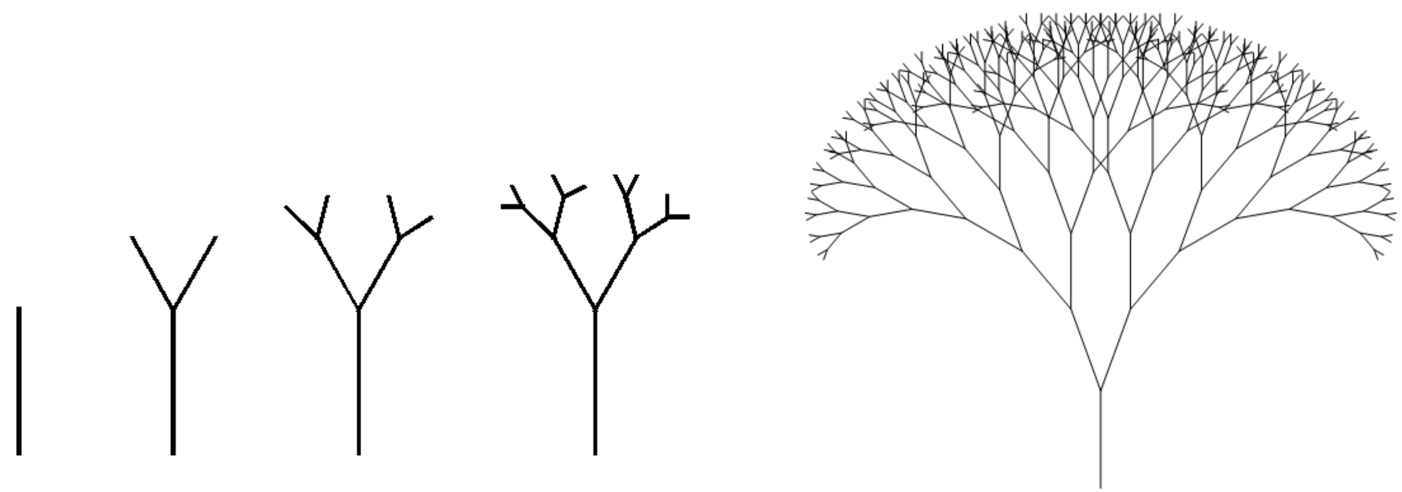
What are differences that you observe between those two pictures?



?

Brief Recap on Fractals

What are differences that you observe between those two pictures?

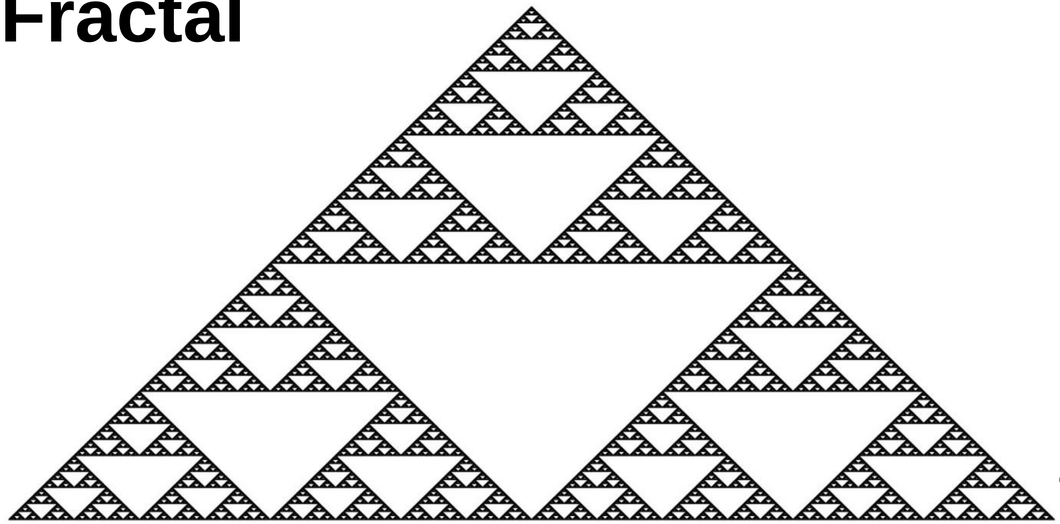


- There are some similarities, but nature is not quite perfect
- There is a physiological limit: trees don't grow forever

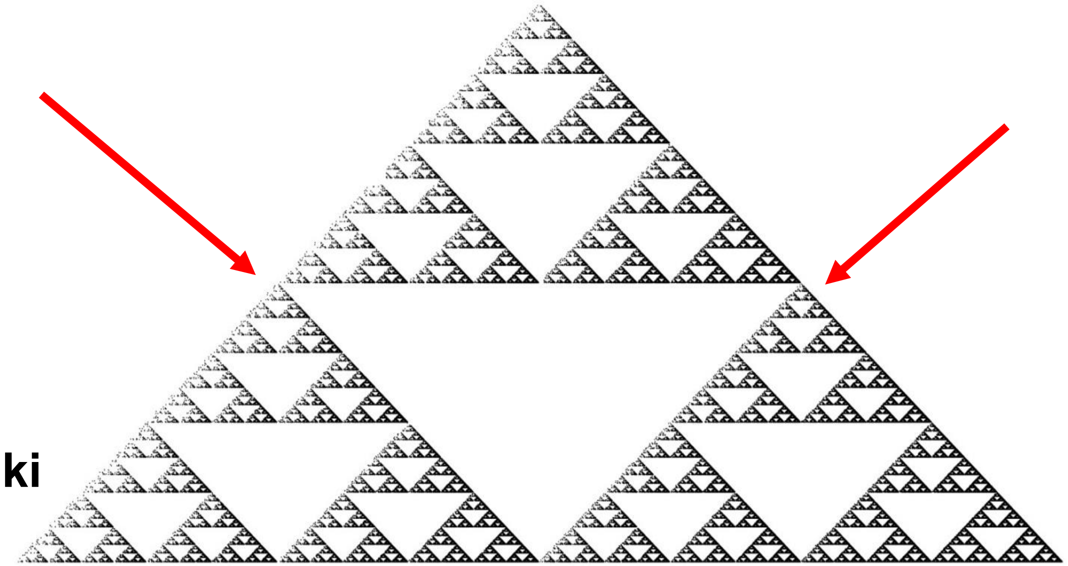
MULTIFRACTALS ARE SELF-AFFINE*

Here the scaling behavior is the same everywhere
→ **Fractal**

Here the scaling behavior depends on where you look
→ **Multifractal**



Sierpiński triangle



Multifractals have more than one scaling relationship, more than one scaling exponent

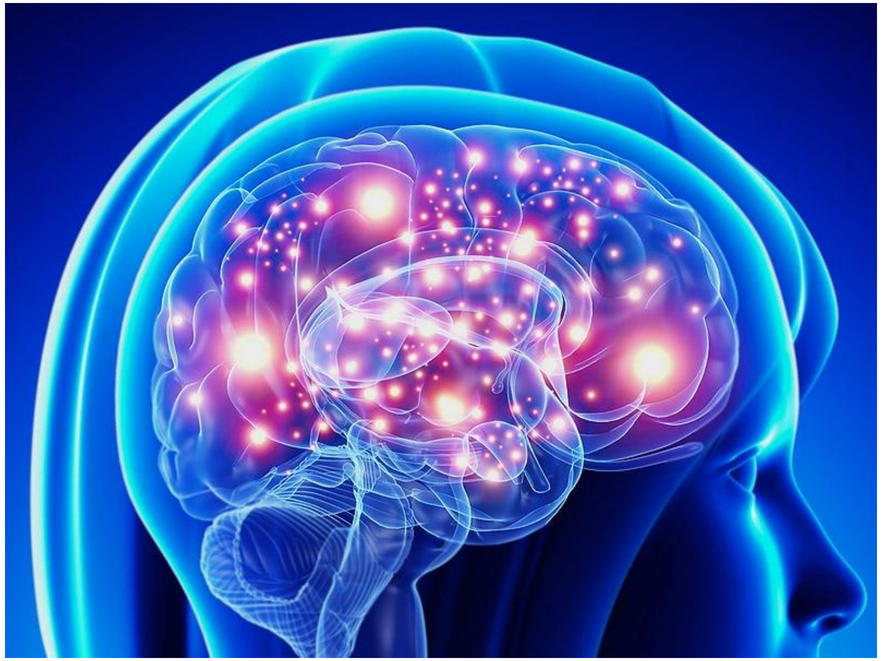
JUST LIKE THE EXAMPLE ABOVE, OUR MOVEMENT IS NEVER PERFECTLY SIMILAR…

The surface you walk on is not always flat and stable

Your movement patterns constantly adapt to the environment

Movement patterns also vary from task to task (i.e., walk, run, …)

So…Does it really make sense to only have a single strategy?



Monofractals are overly restrictive!!!

MULTIFRACTALS

- Multifractals are **important** because they characterize many aspects of human variability:

- Entail coordination of multiple time scales
- Reflect the time varying demands and adaptability

- **Human gait**
- **Upright posture**
- Heart rate
- Neural activity
- Haptic perception
- Reaction times
- Speech production
- Time estimation
- Visual perception

Cavanaugh et al., 2017; Harrison & Stergiou, 2015; Ihlen (2013); Ihlen & Vereijken (2010); Ivanov et al., (2001); Likens et al., 2014; Palatinus et al., 2014; West & Scafetta (2003); Scafetta, Griffin, & West, (2003)

DFA vs. MFDFA

DFA

DFA is used to measure how **variance (2nd Statistical moment)** changes as a function of different time scales

MFDFA

Multifractal Detrended Fluctuation Analysis (**MFDFA**) is used to measure how **variance** and **other statistical moments** change as a function of different time scales

*Before we go further into the analysis, let’s refresh our memory on a few earlier statistical concepts…

Who remembers the four main statistical moments from introductory statistics?

QUICK RECALL

Just like there are higher derivative orders:

$y = f(x)$	Position (Displacement)
$\frac{dy}{dx} = y' = f'(x)$	?
$\frac{d^2y}{dx^2} = y'' = f''(x)$	
$\frac{d^3y}{dx^3} = y''' = f'''(x)$	
$\frac{d^4y}{dx^4} = y^{(4)} = f^{(4)}(x)$	
$\frac{d^5y}{dx^5} = y^{(5)} = f^{(5)}(x)$	
$\frac{d^6y}{dx^6} = y^{(6)} = f^{(6)}(x)$	

Etc.

There are higher statistical moments:

1. ?
2.
3.
4.
5.
6. …

QUICK RECALL

Just like there are higher derivative orders:

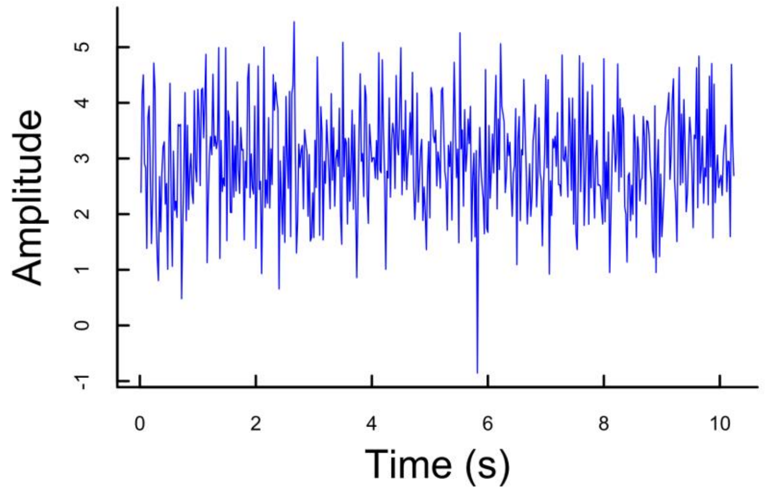
$y = f(x)$	Position (Displacement)
$\frac{dy}{dx} = y' = f'(x)$	Velocity
$\frac{d^2y}{dx^2} = y'' = f''(x)$	Acceleration
$\frac{d^3y}{dx^3} = y''' = f'''(x)$	Jerk
$\frac{d^4y}{dx^4} = y^{(4)} = f^{(4)}(x)$	Snap (Jounce)
$\frac{d^5y}{dx^5} = y^{(5)} = f^{(5)}(x)$	Crackle (Flounce)
$\frac{d^6y}{dx^6} = y^{(6)} = f^{(6)}(x)$	Pop (Pounce)

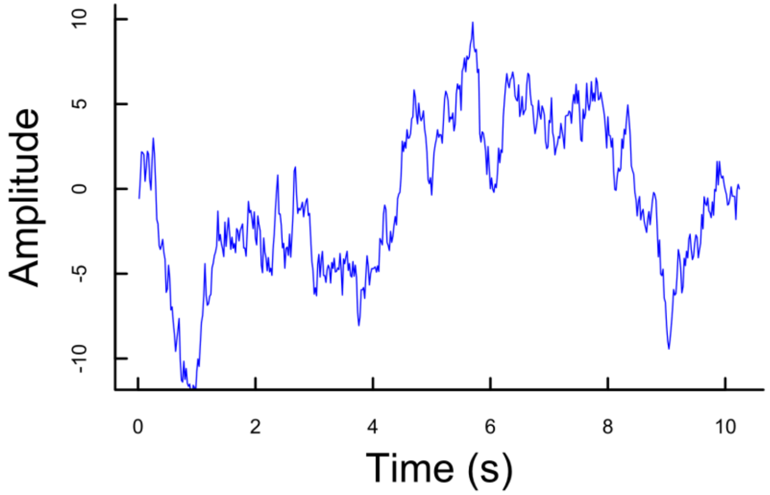
Etc.

There are higher statistical moments:

1. Mean
2. Variance
3. Skewness
4. Kurtosis
5. …
6. …

MFDFA ANALYSIS STEP BY STEP



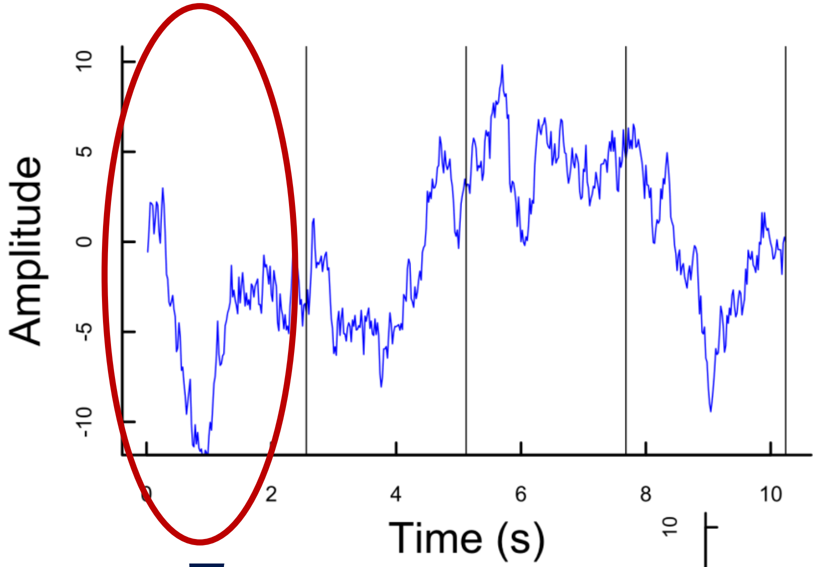


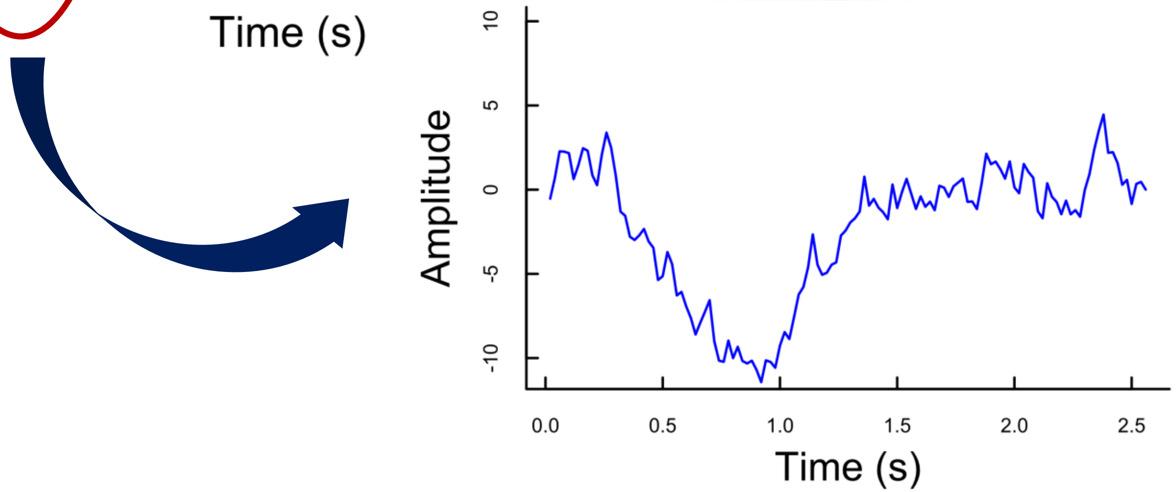
Step 1

- Create a profile of a time series:
  - subtracting its mean from each data point
  - Integrate the time series

- This step allows the conversion of the time series to random walk-like process that meets the theoretical assumptions of DFA

MFDFA ANALYSIS STEP BY STEP





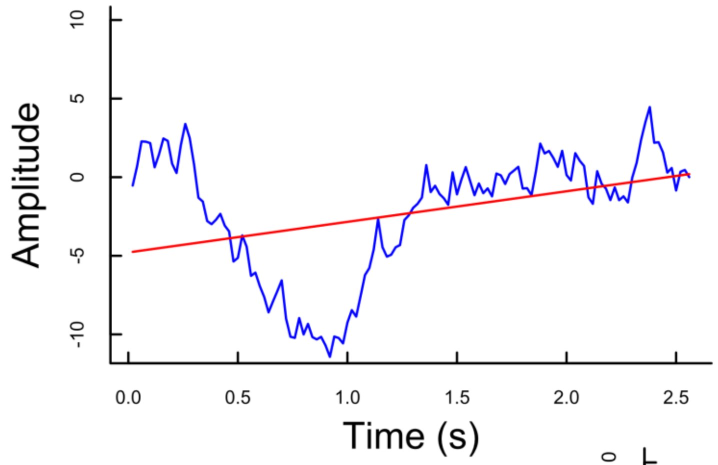
<u>Step 2</u>

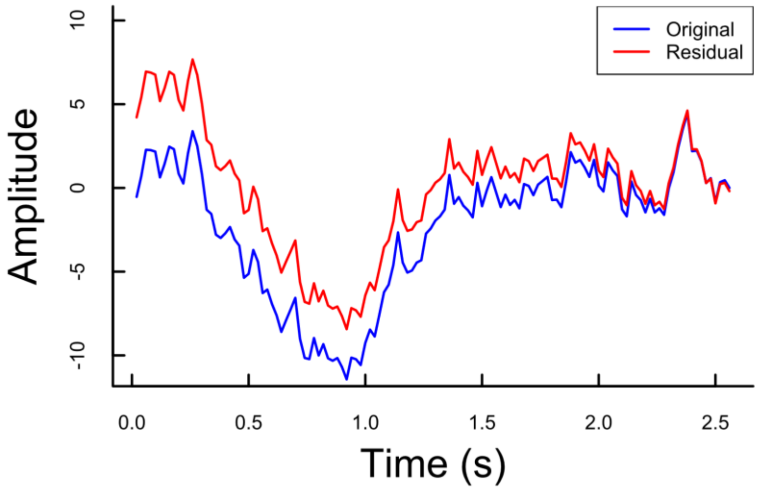
- Divide the series into a sequence of nonoverlapping windows.
- In this example, the time series was separated into 4 windows* (Top).

→ Zooming in

*In practice, 4 windows is not enough to obtain unbiased estimates of fluctuation but for the sake of this tutorial it will do

MFDFA ANALYSIS STEP BY STEP





<u>Step 3</u>

Detrending stage:

- This means fitting a regression line within each of the windows (Top).
- Then, we subtract the fitted trend line from the data in each window (Bottom).

MFDFA ANALYSIS STEP BY STEP

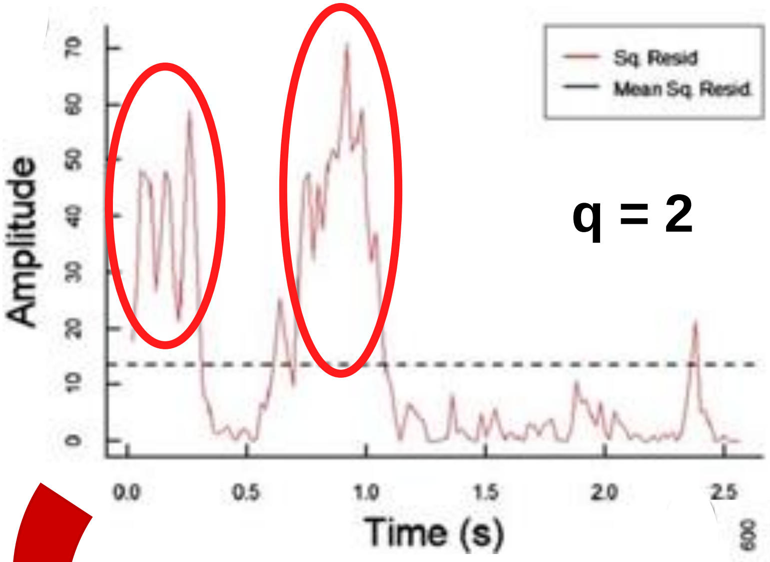
<u>Step 4</u>

- Multifractal analysis involves analyzing several statistical moments (q-order)
- MFDFA focuses on analyzing the scaling behavior of these statistical moments.
- ***Monofractality*** (single power law) vs. ***Multifractality*** (multiple power law).

Squared the residuals q = 2

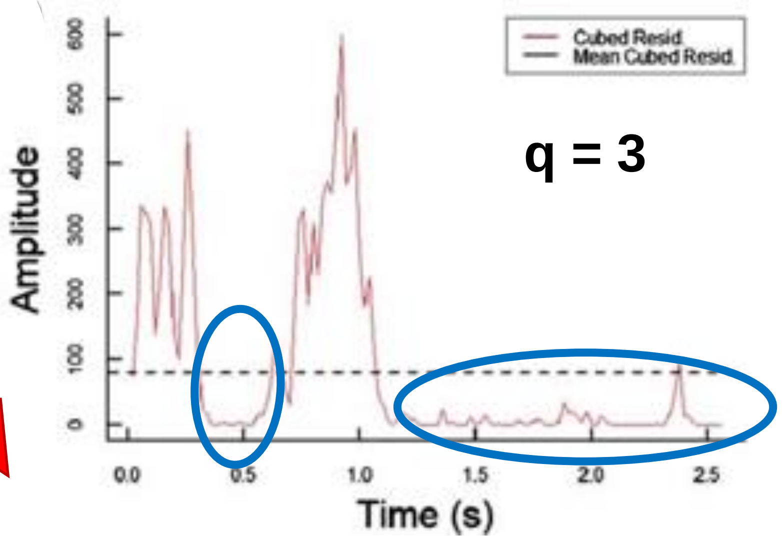
Multiple q-order

MFDFA ANALYSIS STEP BY STEP



→ Represents the squared residuals obtained in the 4th DFA step within a single window

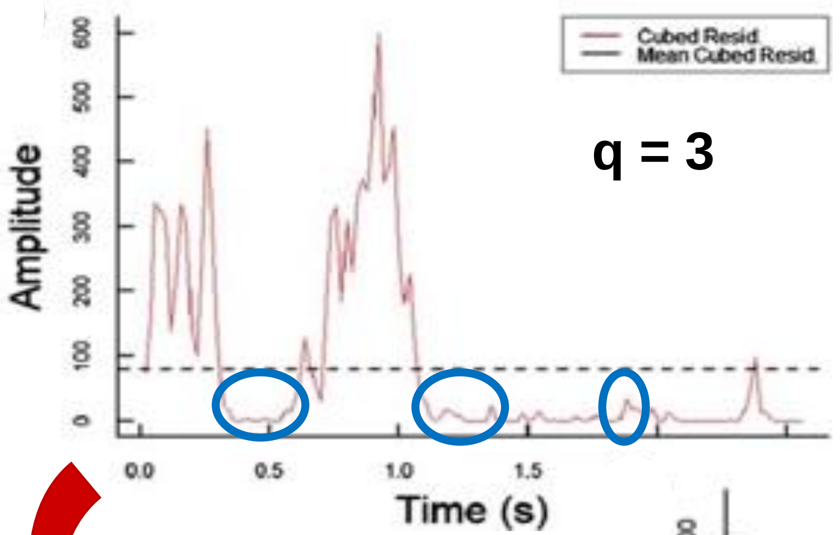
Note: Range is significantly larger



<u>Step 4</u> (continued)

→ In MFDFA we cubed the residuals instead of squaring them. This **magnifies large fluctuations** like those between 0 and 1.0s, while **minimizing smaller fluctuations**.

MFDFA ANALYSIS STEP BY STEP

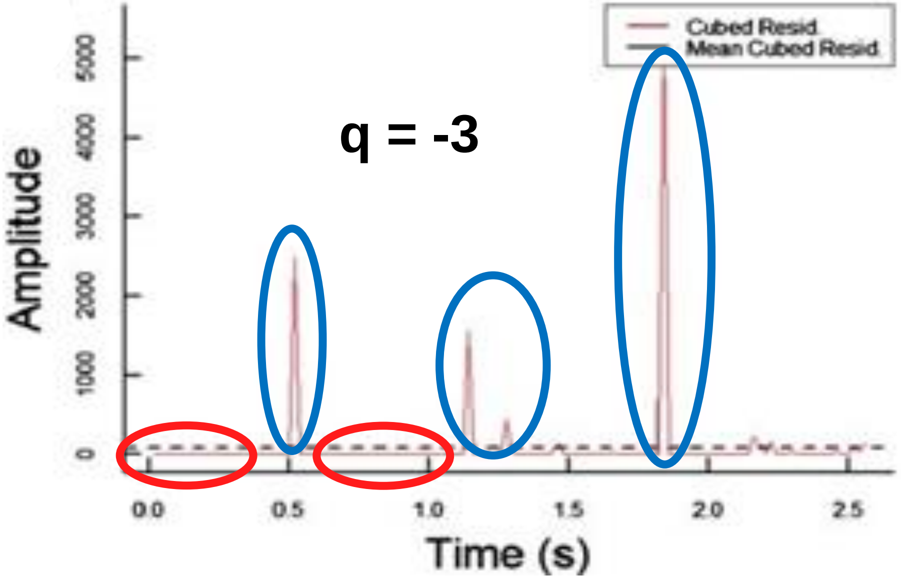


→ Represents the cubed residuals, raised to the **<u>3rd power</u>**

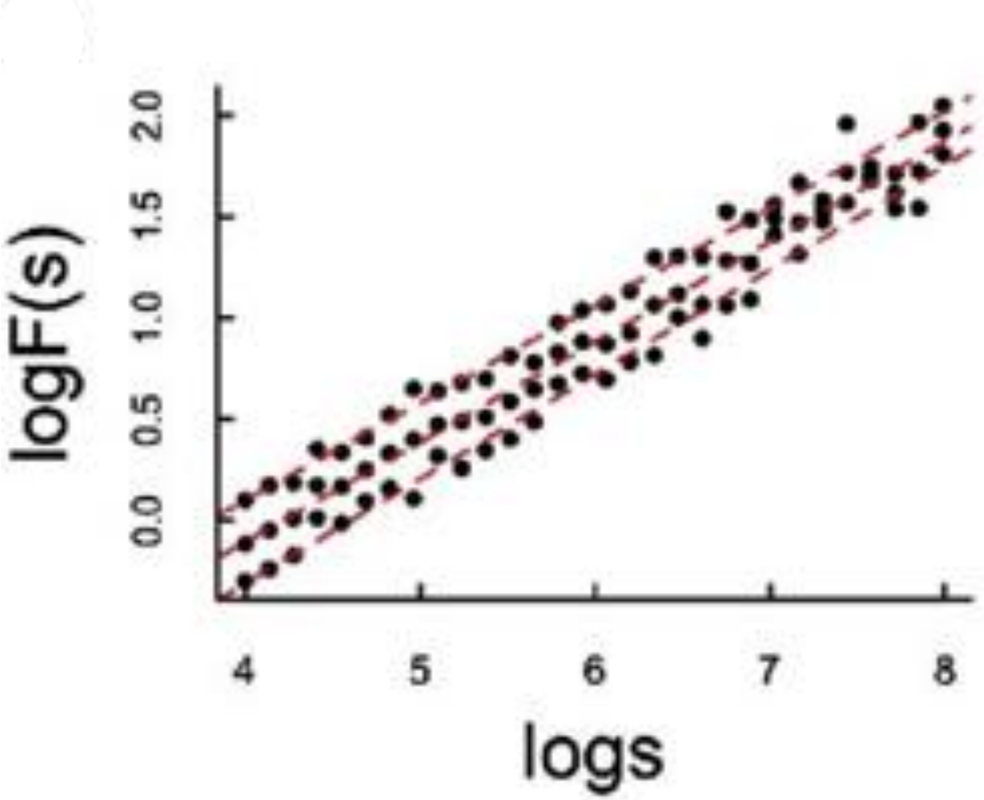
<u>Step 4</u> (*continued*)

→ Residuals raised to a negative value, **<u>-3rd power,</u>** **emphasize small fluctuations** and **minimize larger fluctuations**.

Note: Range is significantly larger



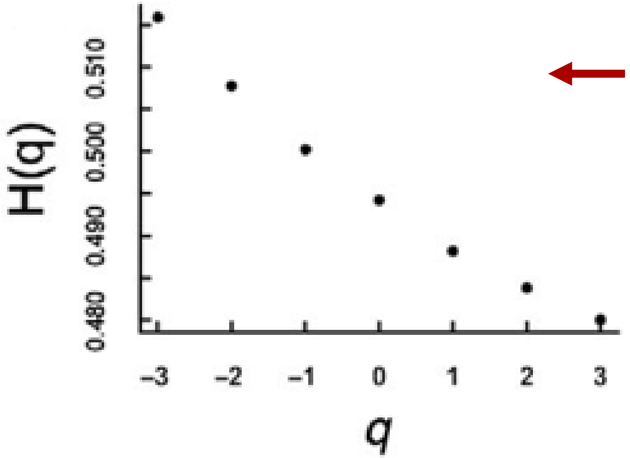
MFDFA ANALYSIS STEP BY STEP



Step 5

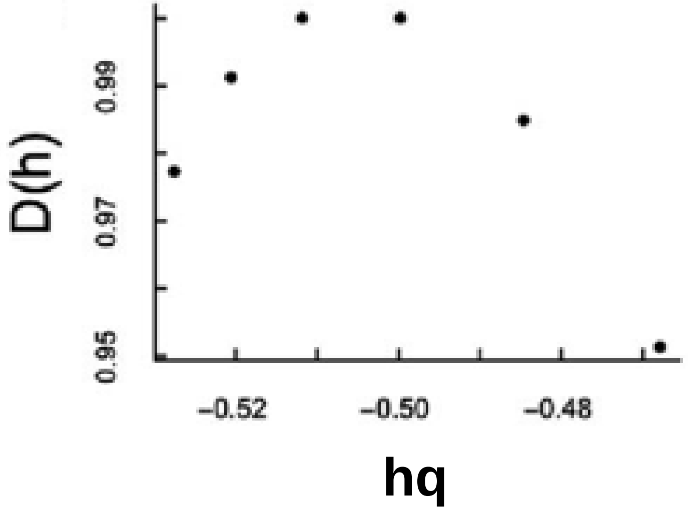
Now we can measure how $F^{(q)}(s)$ varies as a function of scale, ‘s’, via ordinary least squares regression.

MFDFA ANALYSIS STEP BY STEP



Spectrum generalized Hurst exponents, H(q)

Alternative representation of the spectrum– Normalized probability distribution obtained with Legendre Transform



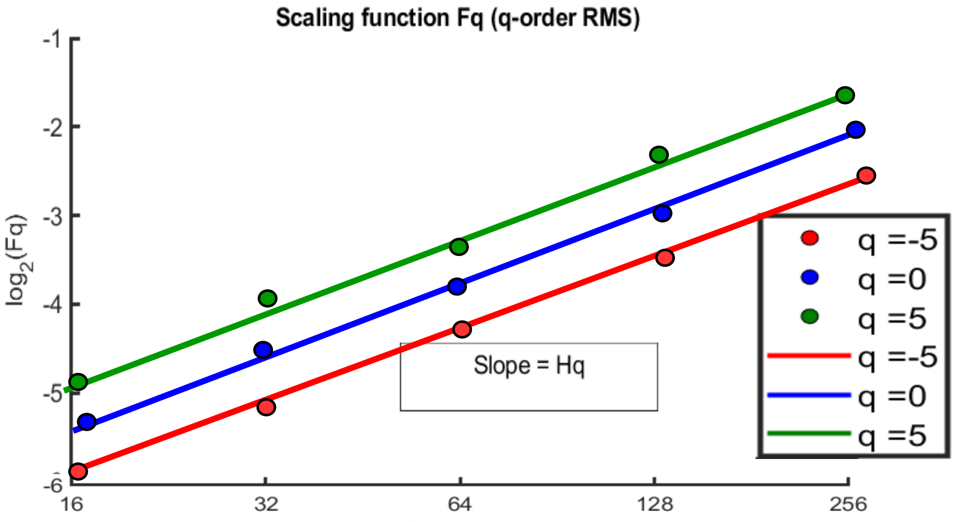
Step 5 (continued)

- The multifractal exponent provides a summary of the time-varying scaling exponents present in the data known as the ***generalized exponent***, H(q).

Monofractals VS. Multifractals

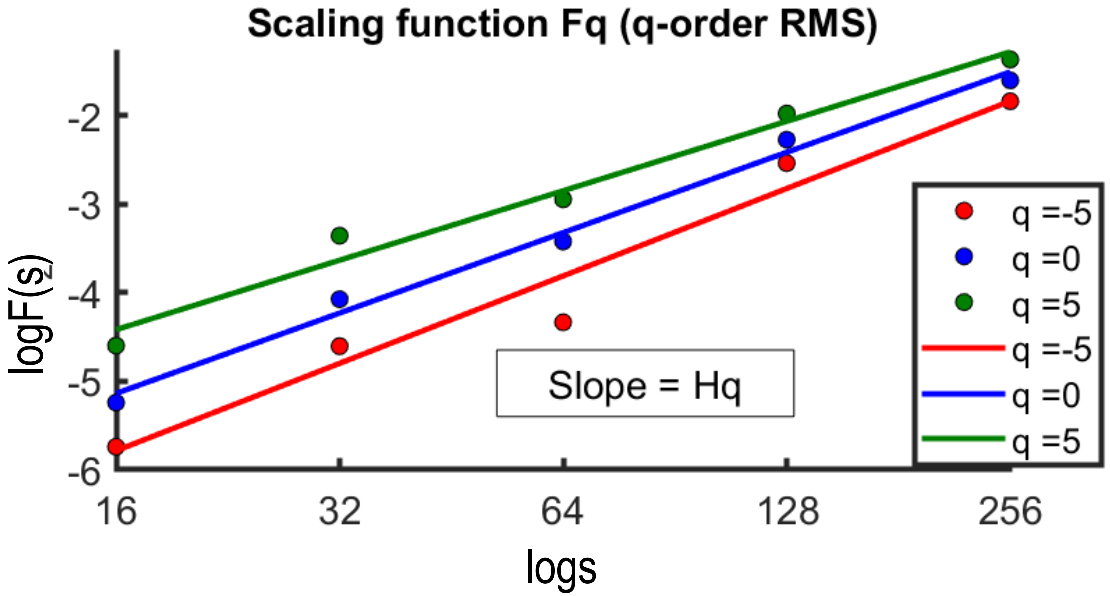
Monofractals

- All regression lines are:
  - Parallel, single exponent
  - Constant (relatively)



Multifractals

- q-order regression lines have different slopes, reflecting differing scaling behavior for different statistical moments



MATLAB TUTORIAL PREVIEW

- MFDFA Functions quick run through:

[Hq, tq, hq, Dq, Fq] = MFDFA (signal, scale, q, m, Fig)

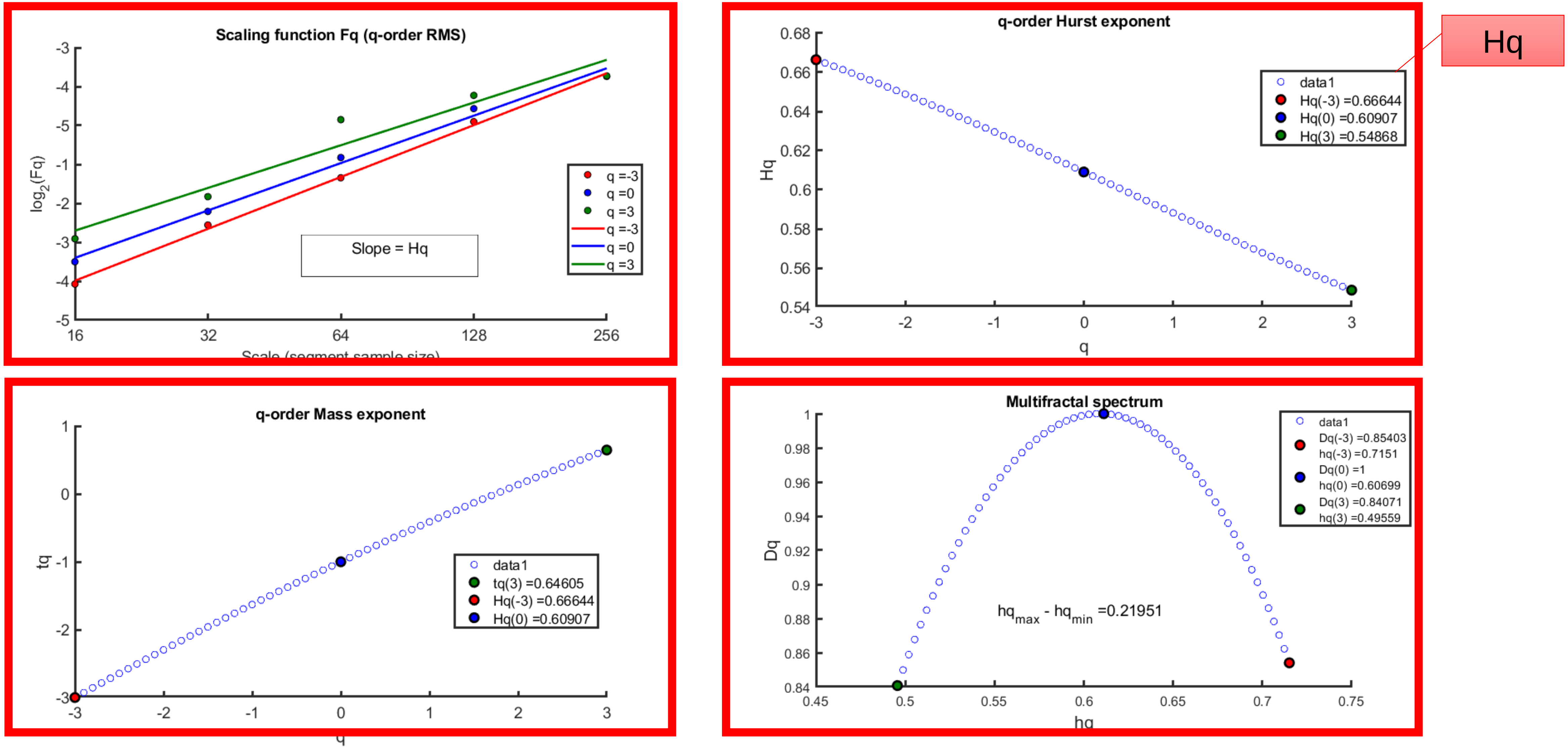
OUTPUT VARIABLES:

- ***Hq:*** q-order Hurst exponent
- ***tq:*** q-order mass exponent
- ***hq:*** q-order singularity exponent
- ***Dq:*** q-order dimension
- ***Fq:*** q-order scaling function

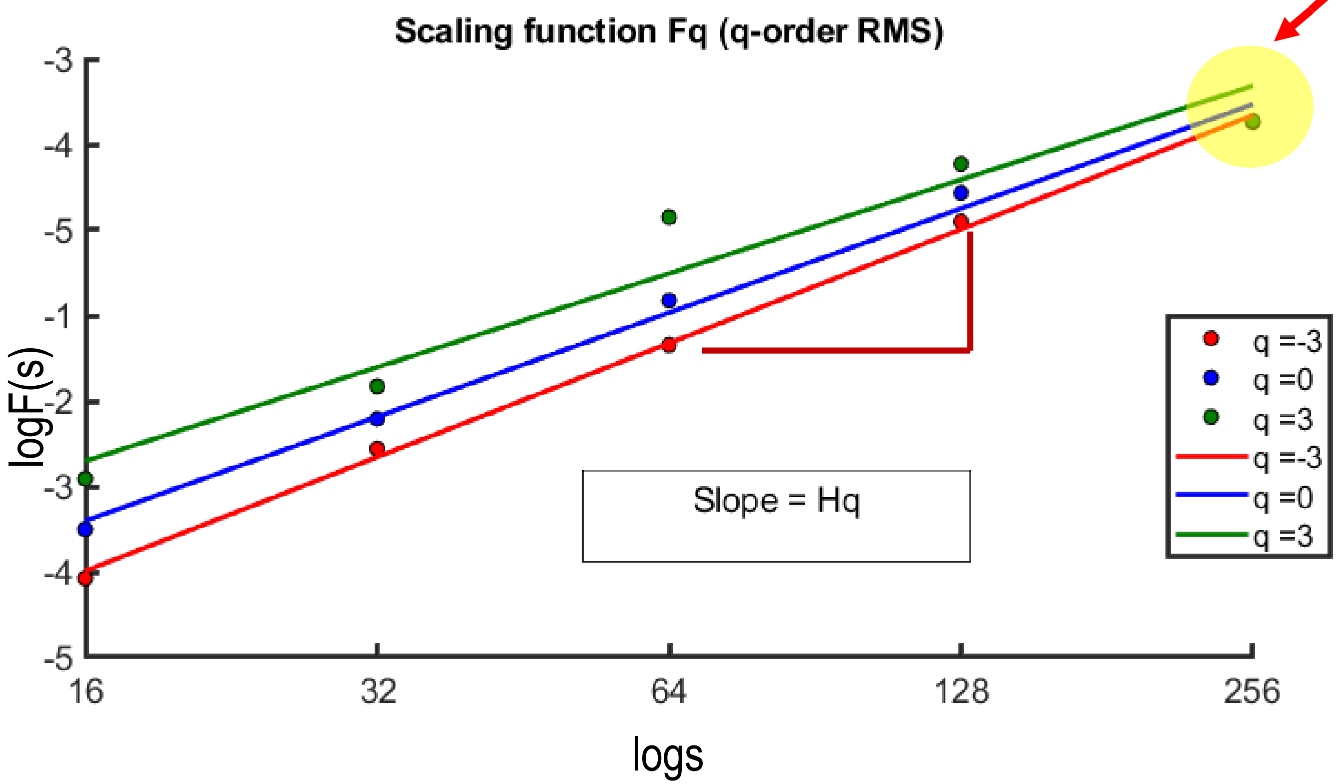
INPUT PARAMETERS:

- ***signal***: input signal
- ***scale***: vector of scales
- ***q-order:*** weights the local variations
- ***m***: polynomial order for the detrending
- ***Fig***: 1 or 0 binary for output plot

ANALYSIS OUTPUT- What to look for?



ANALYSIS OUTPUT- What to look for?

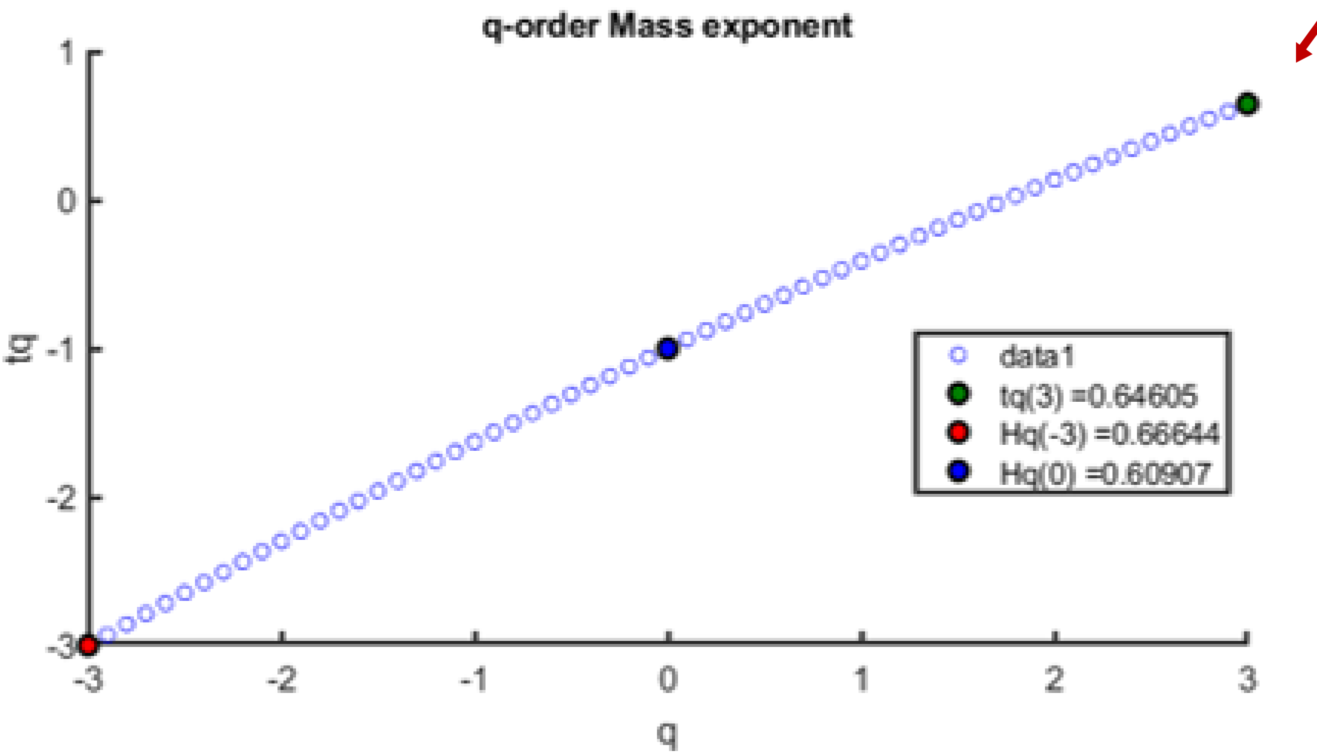


What should I look for?

You should have a linear relationship.

- **Monofractal** = All regression lines are parallel and do not **converge**.
- **Multifractal** = q-order regression lines have different slopes, reflecting differing scaling behavior for different statistical moments.

ANALYSIS OUTPUT- What to look for?



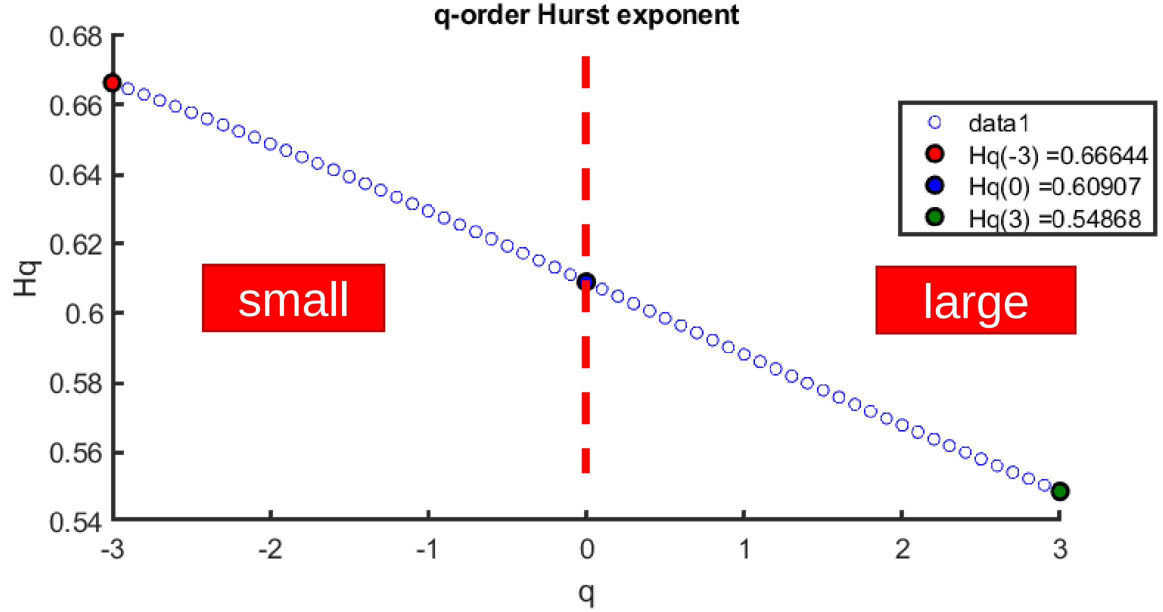
This graph represents a different scaling exponent.

What should I look for?

- Nonlinearity: different slope on either side of q = 0
- Increasing function

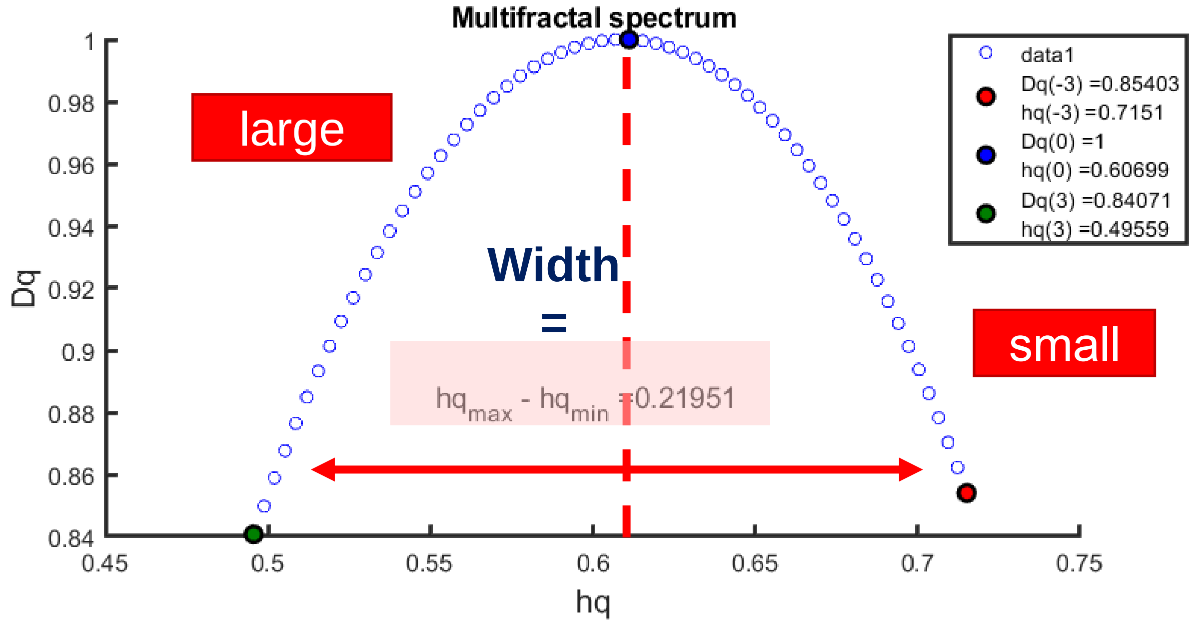
ANALYSIS OUTPUT- What to look for?

Spectrum generalized Hurst exponents, H(q)



- Should be a **monotonically decreasing function** (negative slope and no change of direction)
- Values **< q** correspond to **smaller** fluctuations
- Values **> q** correspond to **larger** fluctuations

Alternative representation of the spectrum



- **Inverted "U"**
- **hq- x-axis:** large values correspond to small fluctuations; small values correspond to large fluctuations
- **Width** will increase as the q-order increases
- **Look for asymmetry**

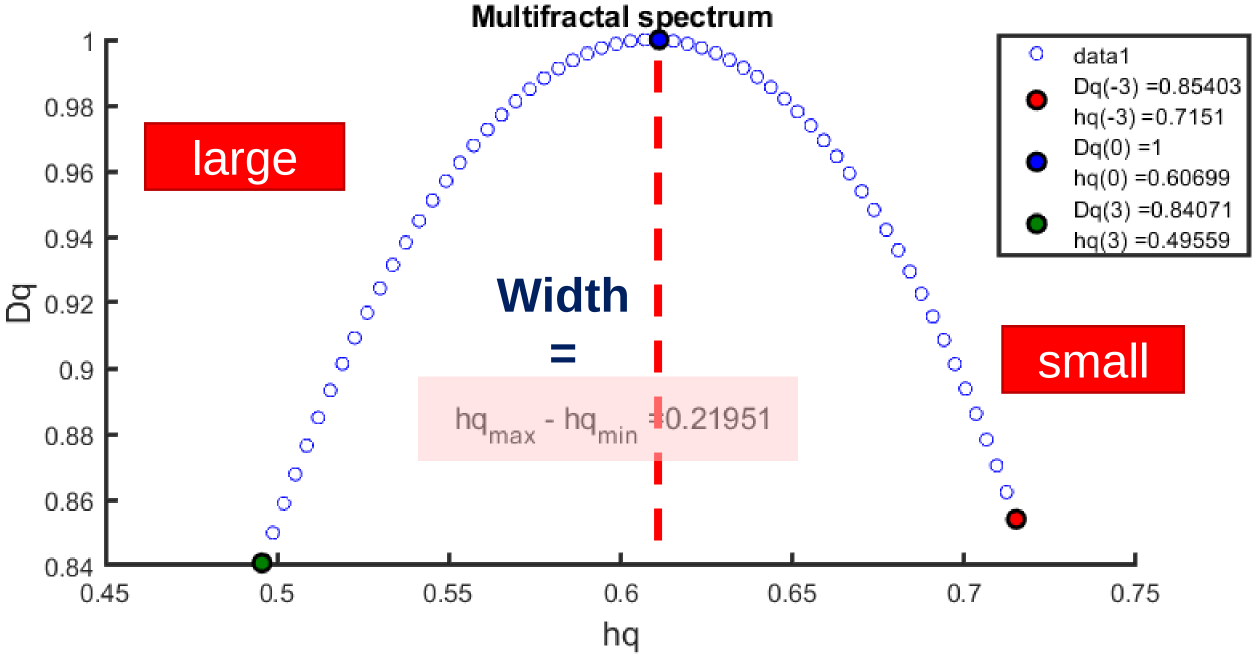
INTERPRETATION

How do I interpret the multifractal width?

The ***width*** and ***shape*** of the multifractal spectrum reflect the temporal variation of the local *Hurst exponent*, to put it simply, it **reflects the change in scaling exponents overtime**.

Larger width can be interpreted as greater number of patterns (i.e., more behavioral strategies) in a time series.

Alternative representation of the spectrum

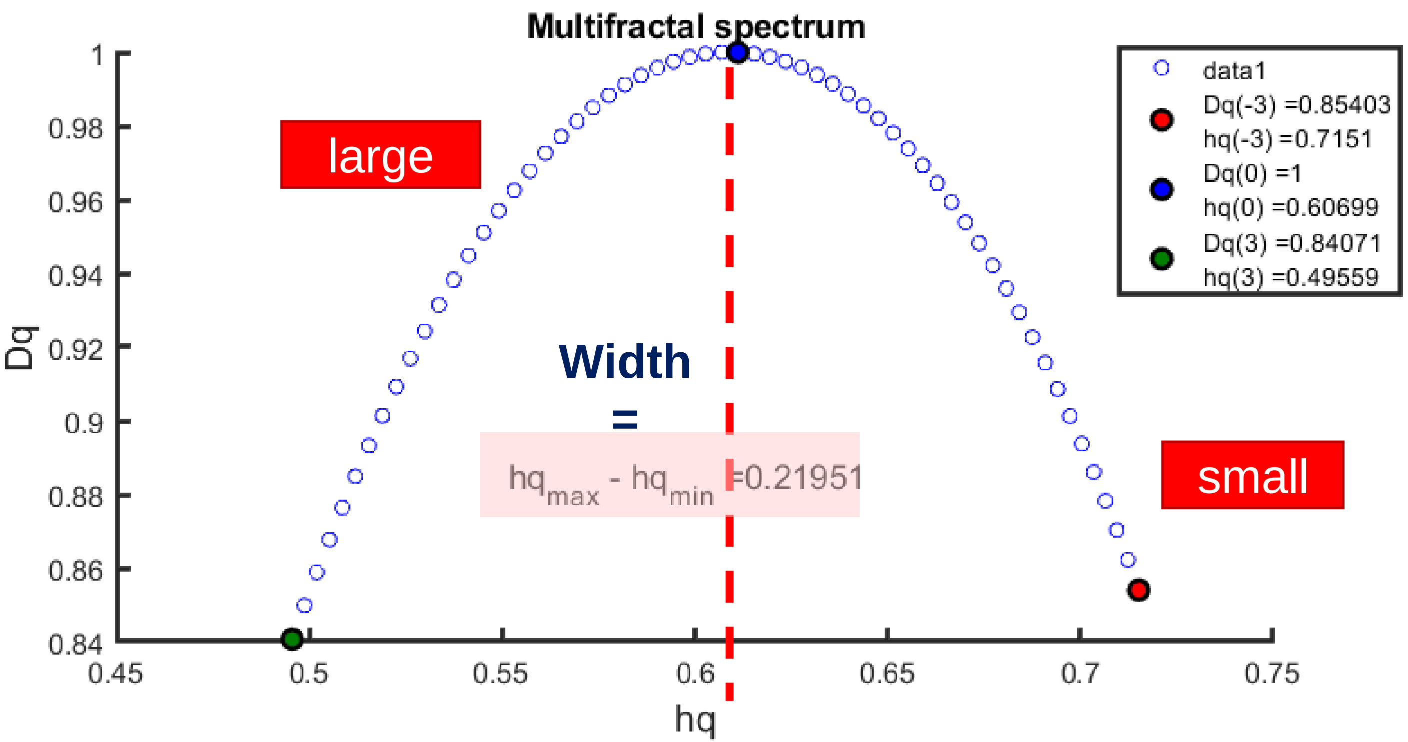


INTERPRETATION

What does the inverted "U" mean if it is asymmetric?

Indicates more variability in the larger asymmetric side. For instance, let's say that the left side (**large fluctuation**) is larger than the right side. It would mean that for this trial and task the participant exhibited more variability in the large fluctuations compared to the small fluctuations.

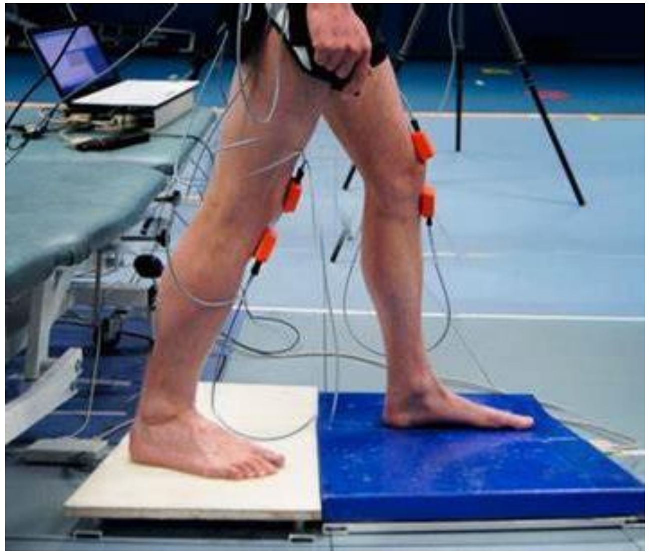
Alternative representation of the spectrum



MFDFA Best Practices

What should my data look like to perform MFDFA?

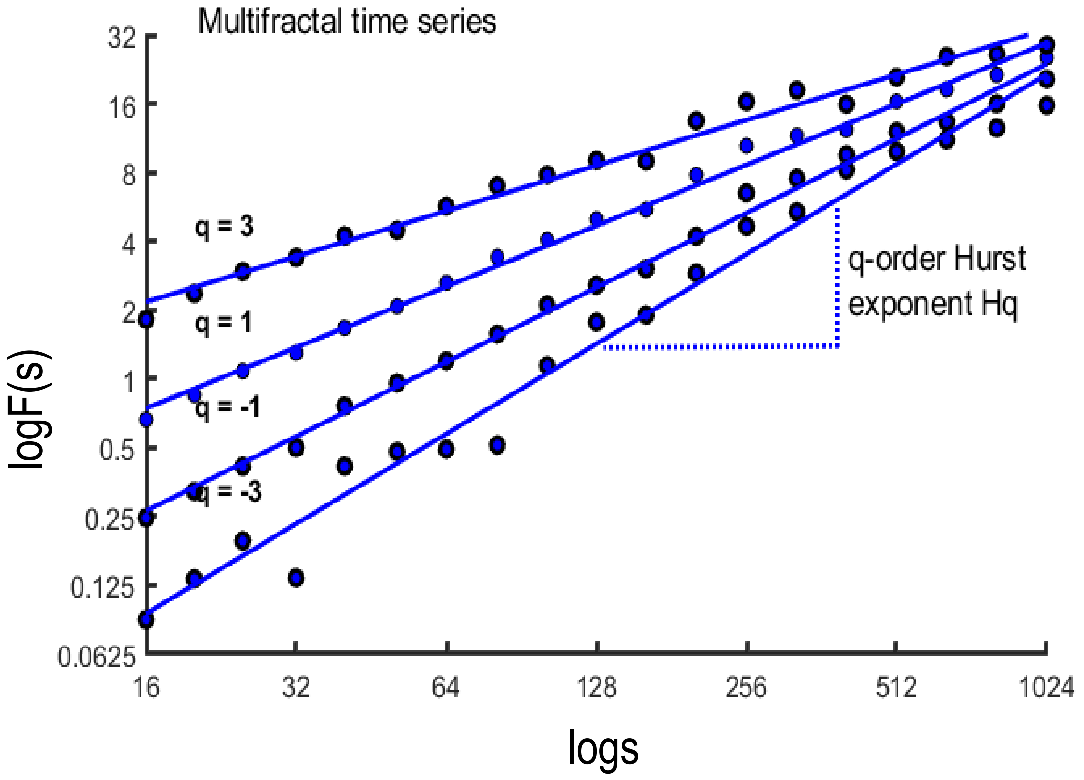
1. Structurally your data should be a ***time-series similar to noise***, meaning your time series should NOT be smooth. *Examples of noisy data:* stride intervals, velocity,… If your time series is smooth (i.e., COP), raw data will require some transformation in order to run this analysis.
2. Local fluctuations within the time series ***cannot be close to zero.***
3. Time series should be ***scale-invariant*** within the predefined range of scales



Ilhen, 2012

MFDFA Best Practices

- **Check for scale-invariance -** *examine the logscale by log(fq) plot for linearity*
- **Series Length -** 500 time points minimum
- **Scales -** [16 < scale < N/9]
- **q range/magnitude -** more extreme values of q require longer data sets. Recommend q = -3:3
- **Detrending order, m -** linear detrending has been used in movement science
  - Selecting a min scale is recommended to avoid overfitting
  - See Kantelhardt et al. (2001)



MATLAB TUTORIAL

TIME TO PRACTICE WHAT WE HAVE LEARNED!!!!!!

You can use your own data or you can practice on the data set provided in **Github**: https://github.com/aaronlikens/NACOB-2022-Multifractal-Methods-in-Movement-Science.git

DOWNLOAD MFDFA FOLDER

This folder contains the following folders: **ANALYSIS OUTPUT** (figures, summary tables), **DATA** (Stride Intervals for each trials and participants), **FUNCTION** (MFDFA)

OPEN MATLAB

Open the script called 'J.12 MultifractalsTutorial'

********5 min Break*******

Github link:
https://github.com/aaronlikens/NACOB-2022-Multifractal-Methods-in-Movement-Science.git

MATLAB VERSION:
There are no known incompatibilities using MATLAB version R2019a or later.

MATLAB Toolboxes Required:
Statistics and Machine Learning Toolbox
Signal Processing Toolbox
Image Processing Toolbox

Part III: Surrogate Analysis

Aaron D. Likens

Introduction

What are the goals of time series analysis?

↓

To understand the underlying mechanism that generates different dynamics for different time series

↓ Random

↓ Deterministic

What kind of dynamics?

Nonlinearity

Does a time series contain nonlinear structures?

↓

Apply nonlinear tools

↓

Spurious results simply due to the reflection of practical limitations

Important to establish the evidence of nonlinearity in a time series first.

Nonlinear Systems

1) A nonlinear system is a system which does not have a linear origin.

2) For our discussion, a nonlinear system will be limited only to a stationary time series and will not include non-stationary stochastic process.

What is surrogation?

surrogate verb

sur·ro·gate | \ ˈsər-ə-ˌgāt , ˈsə-rə- \

surrogated; **surrogating**

Definition of *surrogate* (Entry 2 of 2)

transitive verb

: to put in the place of another:

a : to appoint as successor, deputy, or substitute for oneself

b : SUBSTITUTE

Merriam-Webster Dictionary
https://www.merriam-webster.com/dictionary/surrogate

Identifying Nonlinearity

Direct application of nonlinear measures

- Examples:
- Largest LyE
- Correlation Dimension
- Spurious results are possible due to limitations in data
- Subjective judgments are involved

Surrogate methods

- Can be used to compensate the weakness of the first approach
- An indirect approach to identify the evidence of nonlinearity by excluding a linear origin as a null hypothesis

General Surrogate Method

Propose a null hypotheses (H_0).

↓

Generate surrogate time series consistent with null hypothesis.

↓

Apply some test statistic.

↙ ↘

Null rejected

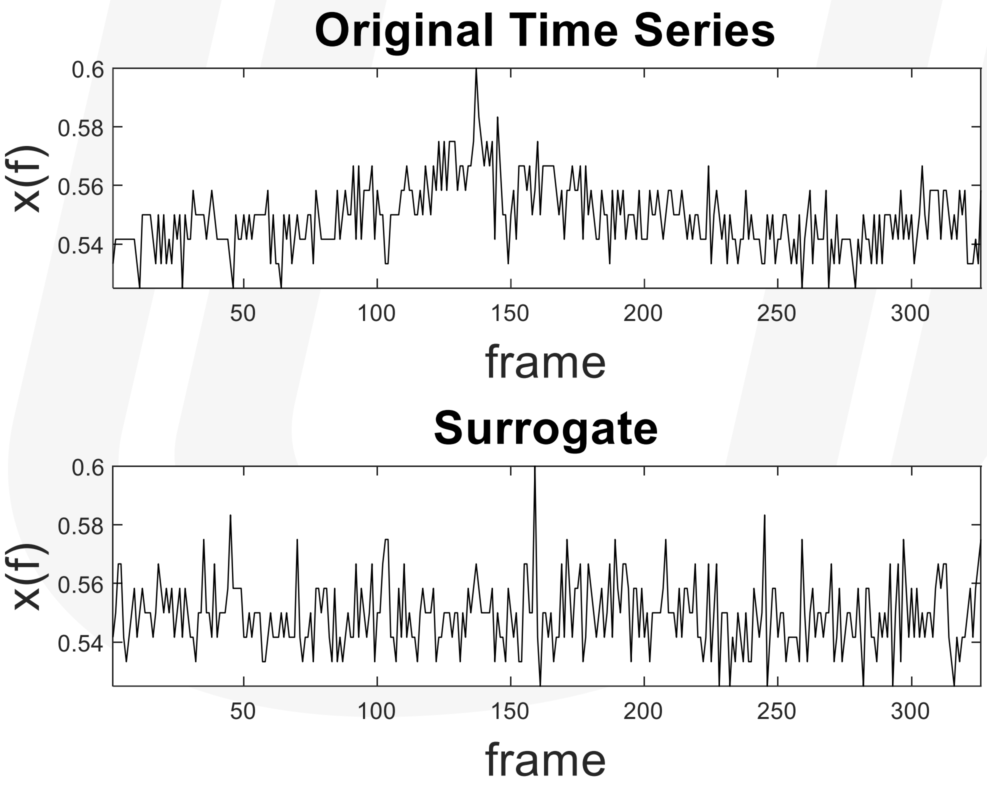
Null not rejected

LyE

Compare

LyE

H_o: The time series is generated by a linear stochastic process.



Discriminating Statistics

There are many different discriminating statistics.

Examples:

- Correlation dimension.
- Largest Lyapunov exponent.
- Approximate Entropy.

Discriminating Statistics

In general . . .

Null Hypothesis is true

- Consistent results for both surrogates and original time series

Null Hypothesis is not true

(reject Null)

- The discriminating statistics of the original time series should be different from the distribution of the discriminating statistics for its surrogates.

Discriminating Statistics

Two different views

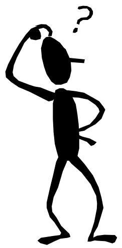
All nonlinear methods should be able to detect the presence of nonlinearity by rejecting the null hypothesis at different significance levels.

The mismatch between a surrogate algorithm and discriminating statistics can lead to a spurious result.

The use of <u>multiple discriminating statistics</u> is encouraged to establish the evidence of nonlinearity in a time series.

Rank order

What criteria should we use to determine whether the null hypothesis should be rejected or not ?



Parametric Statistics

- Let μ_H be the mean and σ_H be the standard deviation of discriminating statistics for surrogate data.
- Let Q_D be the statistic for the original data.
- The rejection of the null hypothesis at 95% level of confidence is indicated by "significance" of about 2 'sigmas (S)'.

$$\underbrace{Q_1 \; Q_2 \; Q_3 \; \ldots \; Q_N}_{\mu_H \text{ and } \sigma_H}$$

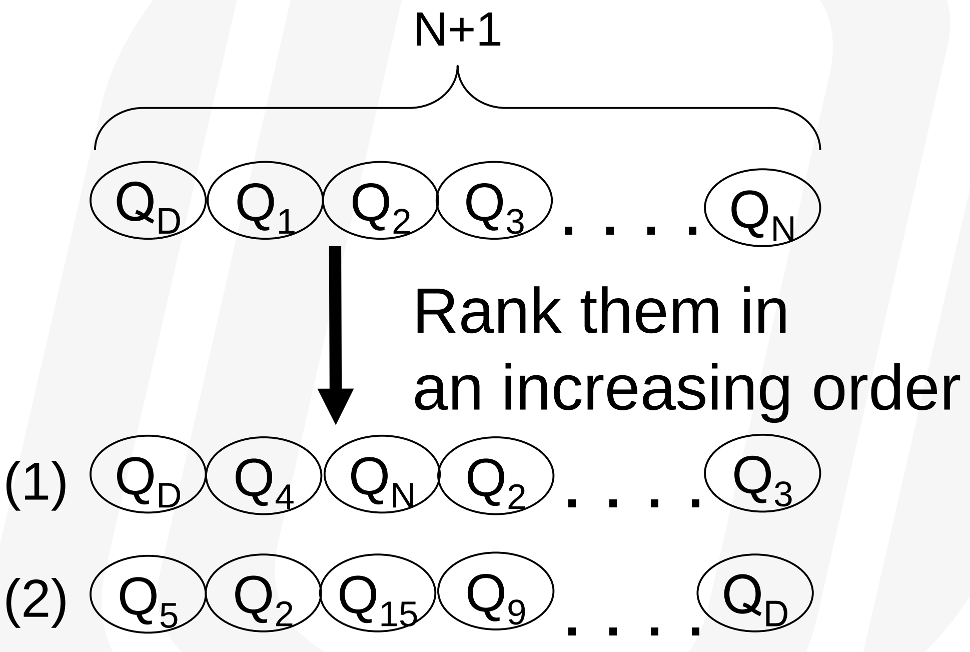
$$\boxed{S = \frac{|Q_D - \mu_H|}{\sigma_H}}$$

Assumption: The distribution of discriminating statistics is Gaussian.

Rank-Order Criterion

Examines the ranks of discriminating statistics of an original time series and surrogates.

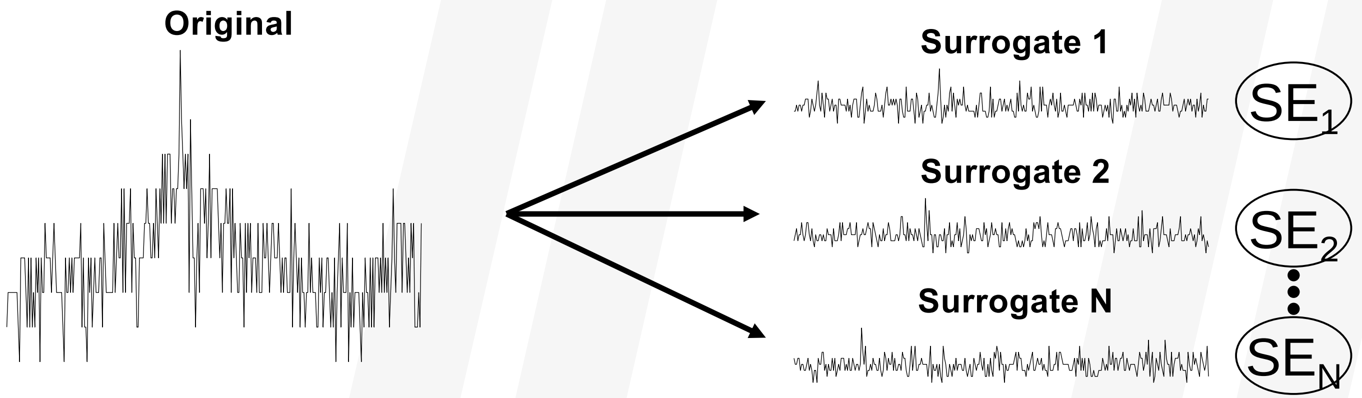
- Rank all discriminating statistics in an ascending order.
- The null hypothesis is rejected when discriminating statistics of an original time series is the smallest (1) or the largest values (2).
- For a one sided test, *1/(N+1)* is regarded as a false rejection rate while *2/(N+1)* for a two-sided test.



If the original time series is generated by a process consistent with the null hypothesis, the probability of Q_D to be the smallest or the largest will be *1/(N+1).*

Example with Sample Entropy

Let SE be the Sample Entropy calculated from N surrogates.
$SE_0 < SE_{1...N}$ Null hypothesis rejected
$SE_0 \nless SE_{1...N}$ Null hypothesis rejected



Smallest → Largest

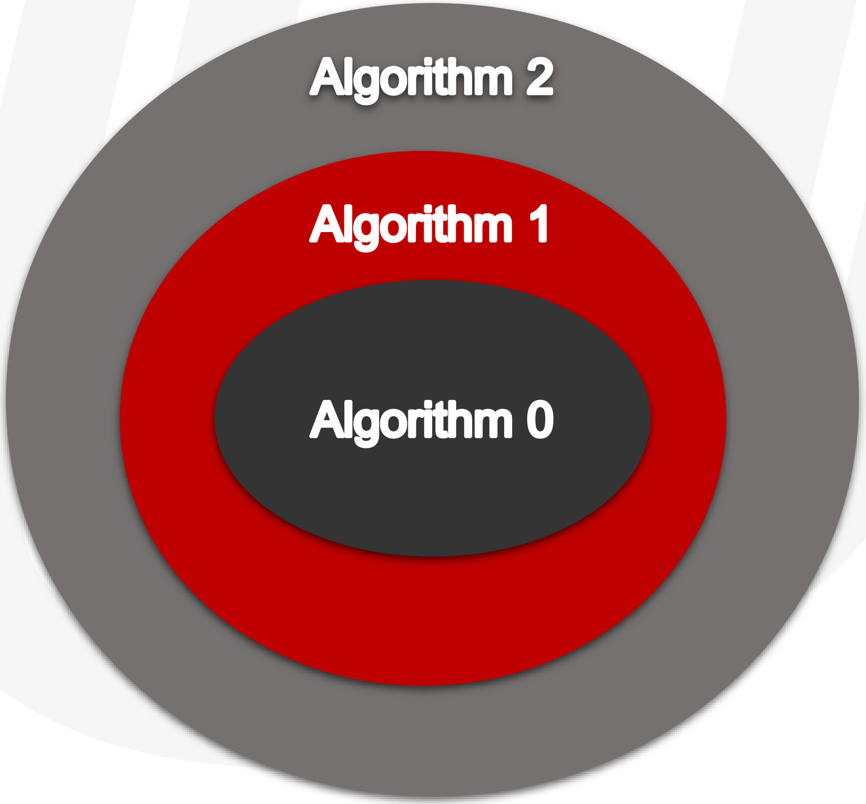
Null Rejected: SE SE_5 SE_4 SE_2 SE_8 SE_{15} SE_{12} SE_{16} SE_6 SE_1 SE_9 SE_{10} SE_{11} SE_3 SE_{13} SE_{14} SE_7

Null Not Rejected: SE_5 SE_4 SE_2 SE_8 SE_{15} SE_{12} SE_{16} SE_6 SE SE_1 SE_9 SE_{10} SE_{11} SE_3 SE_{13} SE_{14} SE_7

Linear Surrogation

Linear Surrogation Methods

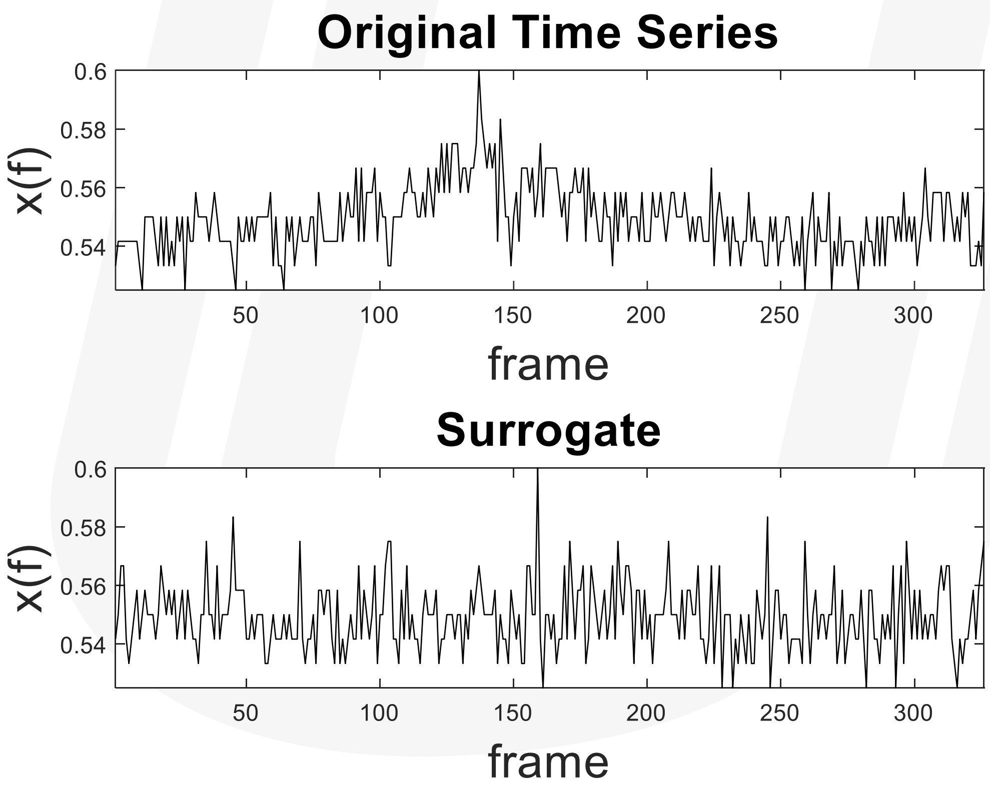
- Most commonly applied surrogate algorithms originally developed by Theiler et al (1992).
- Applied to a stationary irregular time series without any long-term trend or periodicity.
- Hierarchical composite hypotheses.
  - Algorithm 0.
  - Algorithm 1 (Fourier transform surrogate).
  - Algorithm 2 (Amplitude Adjusted Fourier Transform, AAFT).



Linear Surrogate Methods

Algorithm 0

- H_o: Independent and identically Distributed (IID)
- Original time series is visually different from the randomly shuffles A0 surrogate

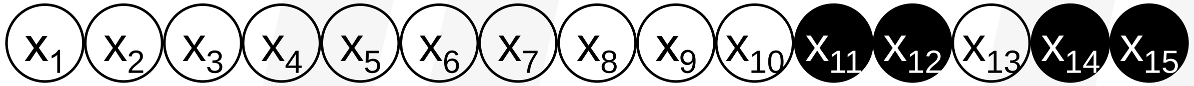


Linear Surrogate Methods

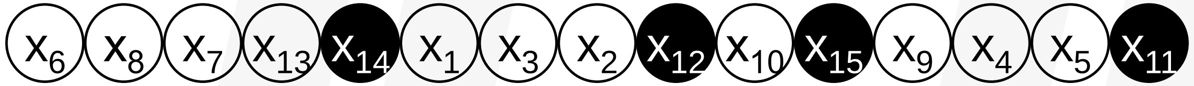
<u>Algorithm 0</u>

- Original time series and surrogate have the <u>*same*</u> probability distribution

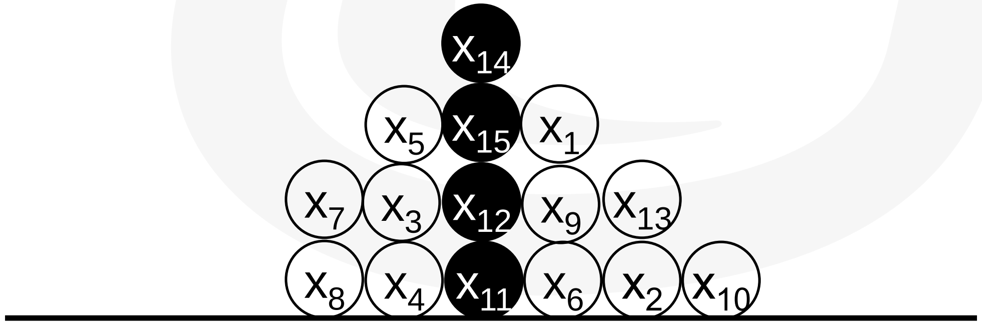
Original Time Series



Surrogate Time Series: Randomly Shuffled



The distribution in both is preserved



The rejection of the null hypothesis of linearly filtered noise

↓

May be an indication of the presence of more complex structure such as nonlinearity in the time series

↙ ↘

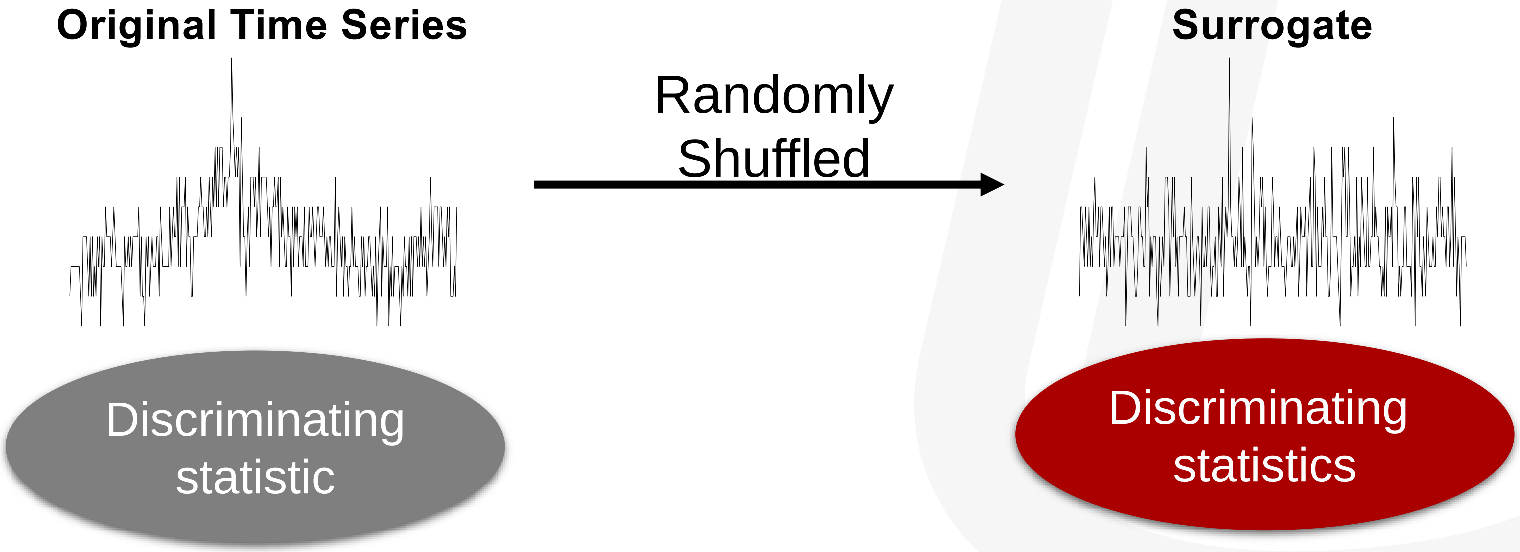
~~Underlying dynamics are nonlinear~~

Due to a measurement distortion. Underlying dynamics are linear stochastic

Linear Surrogate Methods

Algorithm 0

- Null Hypothesis: IID noise



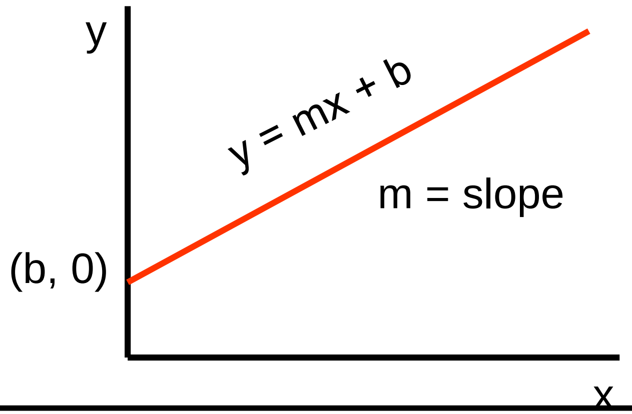
Reject the hypothesis of independence.
Some kind of correlations among data points

Linear Surrogate Methods

Algorithm 1:

- Used to test if a time series is generated from a linear Gaussian stochastic process.
- H_o: A linear filter of IID noise.

Linear system: **y = mx + b**



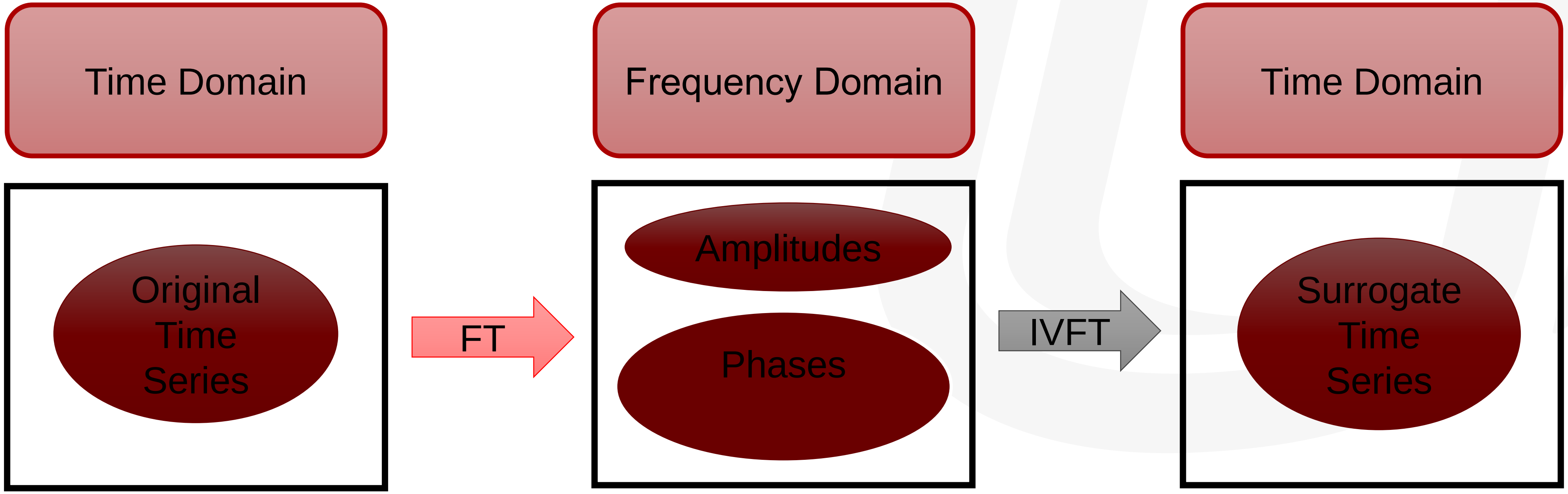
Linear Gaussian stochastic process

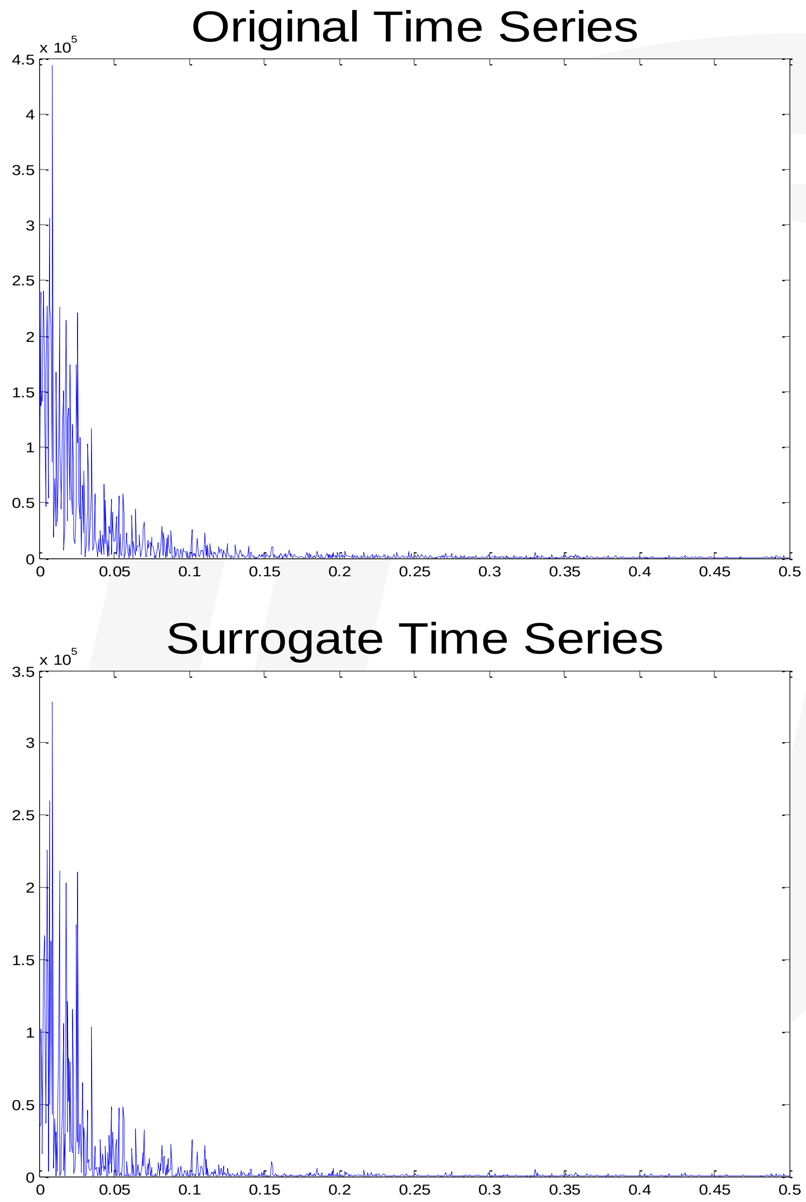
$$y_n = \sum_{i=0}^{M} a_i y_{n-i} + \sum_{i=0}^{N} b_i \eta_{n-i} = a_0 y_{n-1} + b_0 \eta_{n-1} + a_1 y_{n-2} + b_2 \eta_{n-2} + a_2 y_{n-3} + b_3 \eta_{n-3}$$

Linear Surrogate Methods

Algorithm 1

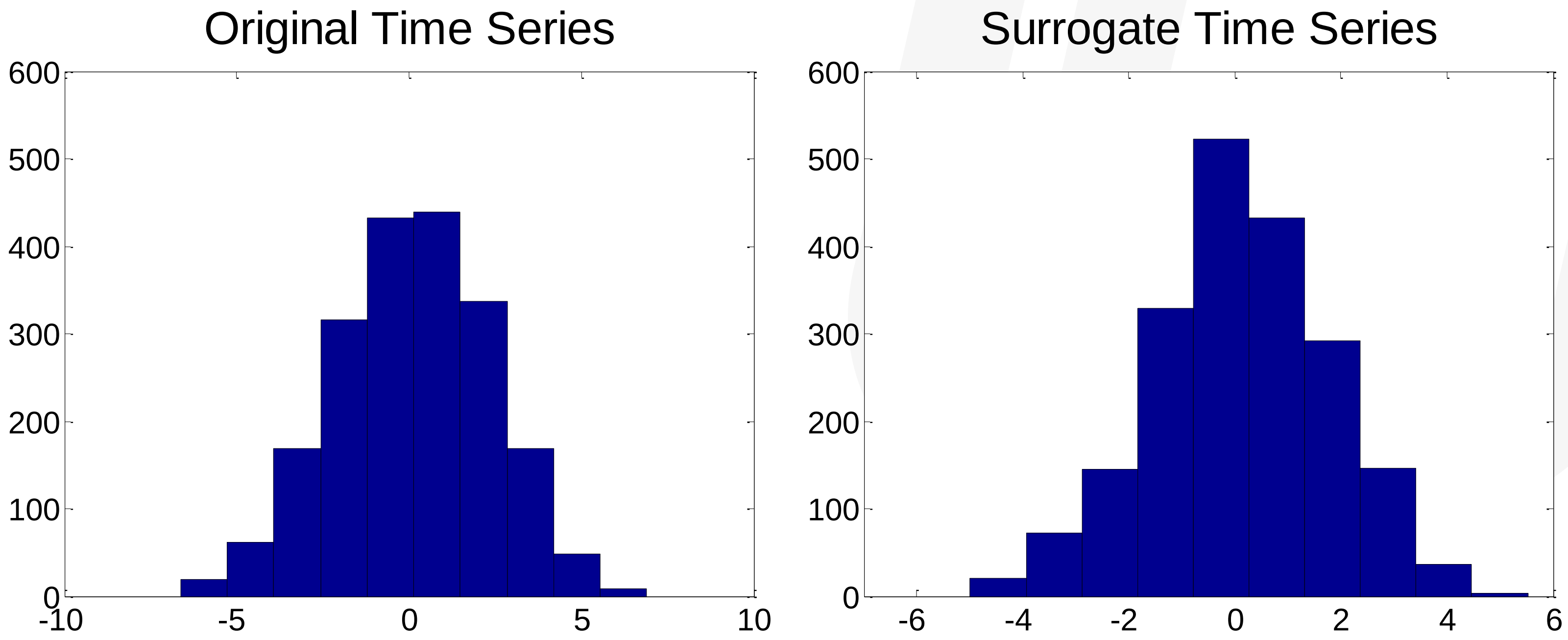
- Data is transformed into the frequency domain, shuffled by phases, and inverse transformed back into the time domain





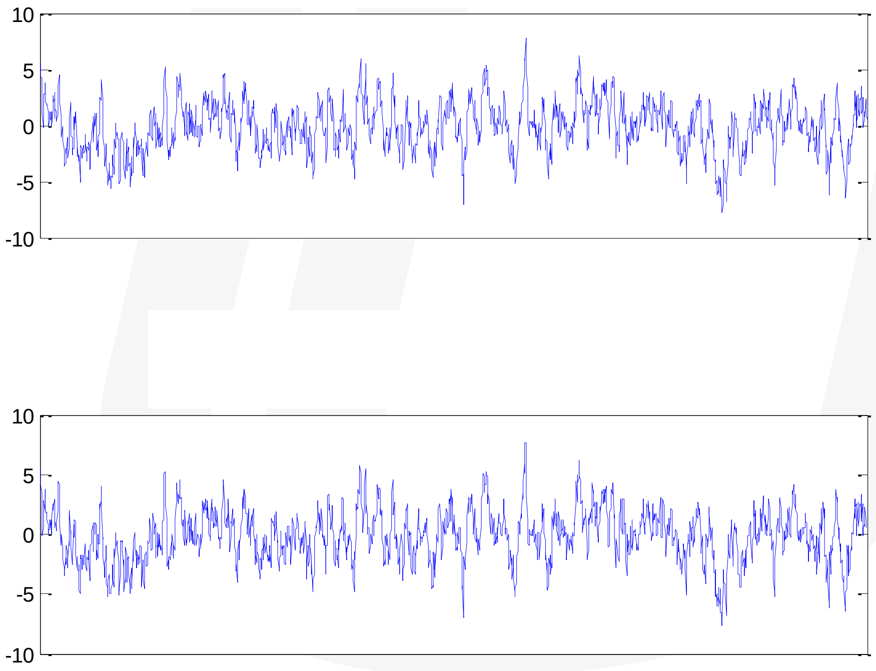
- Surrogate time series generated by Algorithm 1 preserves the linear correlations, the discrete Fourier power spectrum as the original time series.
- Any additional structure should be destroyed.

- Surrogate time series does not preserve the probability distribution of the original data.
- A false rejection of the null hypothesis may occur (especially with a coarsely grained time series).

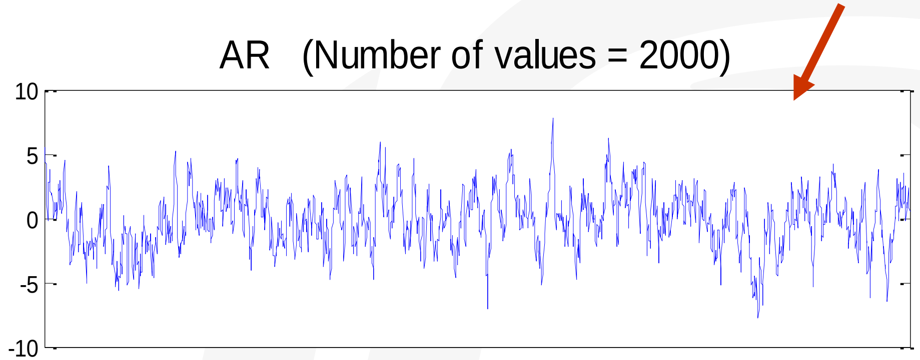


Example

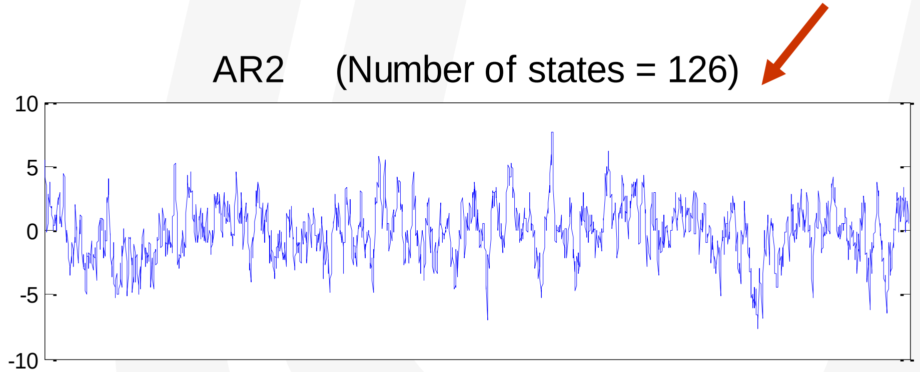
What's the difference between these two time series?



- AR is generated by the linear Gaussian stochastic process, which is consistent with the null hypothesis of Algorithm 1.
- AR2 is another time series created by making AR coarse (removing the precision).
- AR has higher resolutions while AR2 has lower resolutions.



ApEn = 1.42



ApEn = 1.39

e.g.

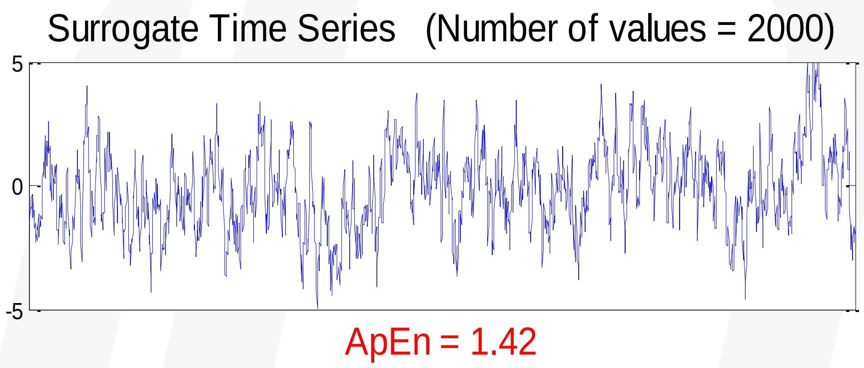
AR: 1.2345 → AR has more states.

AR2: 1.2 → AR2 has fewer states than AR.

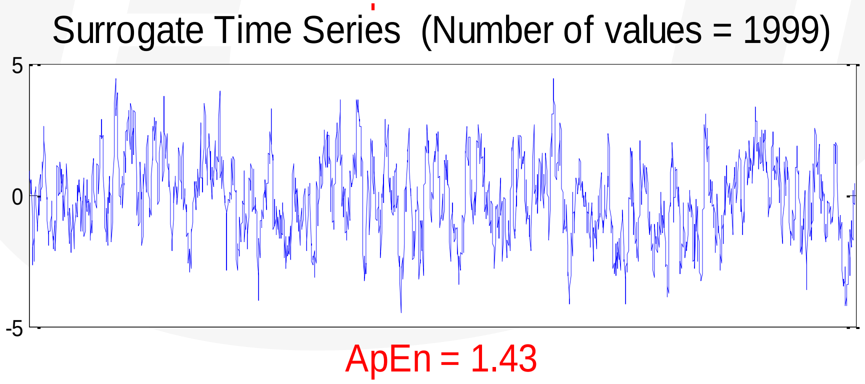
Example

- 19 surrogate time series are generated from AR and AR2.
- Visually, it’s hard to differentiate between the surrogates generated from AR and ones generated from AR2.

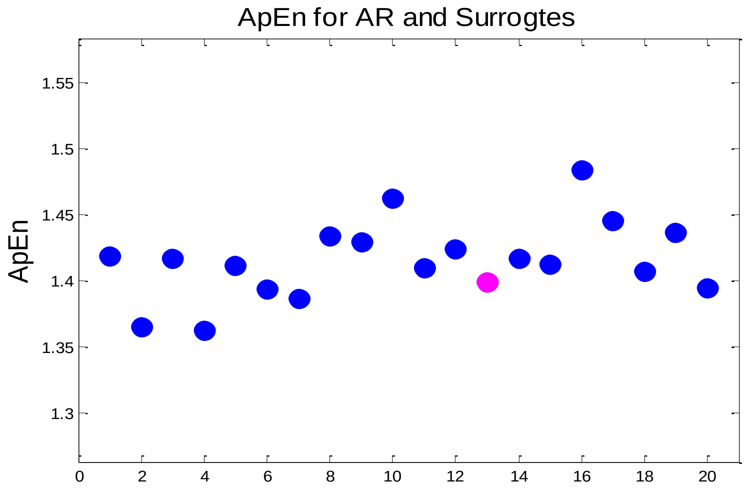
AR



AR2

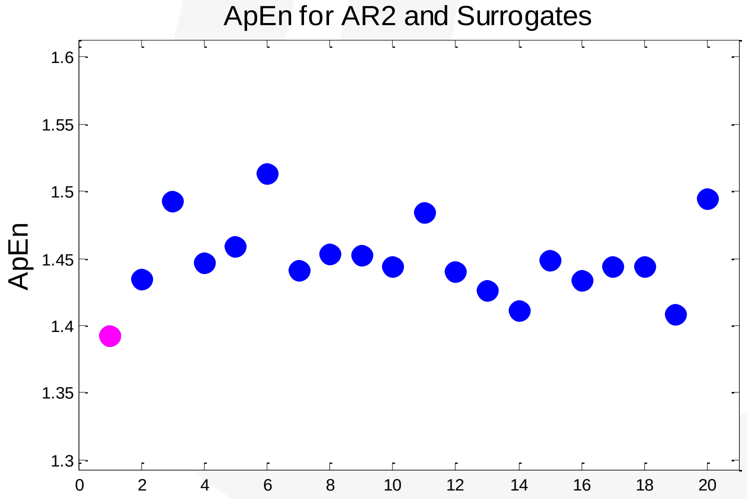


- However, the results of the null hypothesis testing using the ApEn as a discriminating statistics are different between AR and AR2.
- A pink dot represent the ApEn for the original time series while blue dots represent ApEn's for the surrogate time series.



Expected result

H_o should not be rejected



False rejection of H_o

How did this happen?

- The discrepancy in the results between AR and AR2 hypothesis testing seems to be due to an increase in the number of values in surrogate time series generated from AR2.
- The process of generating surrogate time series using the Fourier transform increases the number of unique values in surrogate time series generated from AR2.

	# of states	ApEn
AR	2000	1.42
Surrogates	2000	1.42

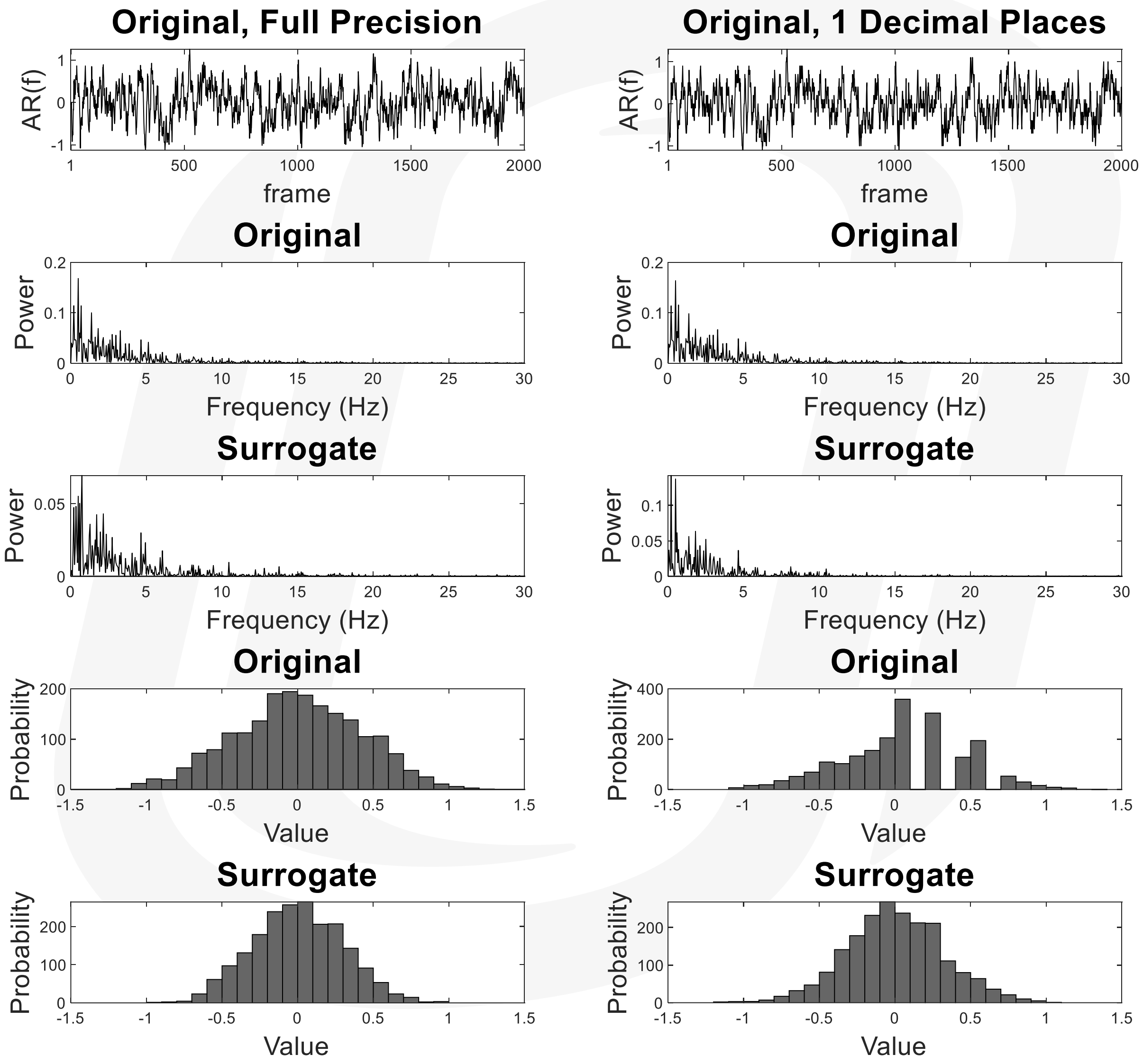
	# of states	ApEn
AR2	126	1.39
Surrogates	1999	1.43

The increase in the number of values in the surrogate time series is reflected in the value of ApEn of surrogates generated from AR2.

Linear Surrogate Methods

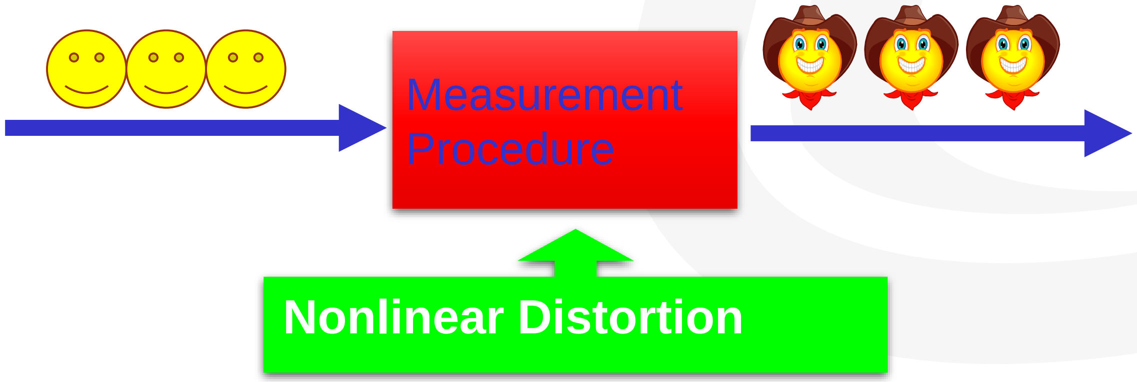
<u>Algorithm 1</u>

- Frequency distribution in AR and in AR2 are preserved.
- The probability distribution in AR and in AR2 <u>*is not*</u> preserved.
- **A false rejection of the null hypothesis may occur**



Algorithm 2 (AAFT)
(Amplitude Adjusted Fourier Transform)

- H_o: A static monotonic nonlinear filter.
- A measurement procedure is regarded as a nonlinear filter which enhances fluctuations in a system with linear dynamics as it goes through the filter.



Linear Surrogate Methods

Algorithm 2 or AAFT

H_o: static and monotonic nonlinear filter

1. Original time series
2. Rank-order the original $\{x_n\}$ time series
3. Generate a random time series and call it $\{y_n\}$
4. Sort $\{y_n\}$ in ascending order and call it Sorted $\{y_n\}$
5. Re-order Sorted $\{y_n\}$ according to original rank and call it Re-ordered $\{y_n\}$
6. Apply Algorithm 1 on Re-ordered $\{y_n\}$ and call it Surrogate $\{y_n\}$
7. Rank order Surrogate $\{y_n\}$ and call it Rank Surrogate $\{y_n\}$
8. Re-order Rank Surrogate $\{y_n\}$ and call it Final Reorder
9. Arrange $\{x_n\}$ according to the Final Reorder

A result of testing with Algorithm 1 → not generated from a linear Gaussian stochastic process.

$$O_n = f(U_n)$$

Test to see if it's generated from a linear Gaussian stochastic process with Algorithm 2

Linear Surrogate Methods

Algorithm 2 or AAFT

1. Original time series
2. Rank-order the original $\{x_n\}$ time series
3. Generate a random time series and call it $\{y_n\}$
4. Sort $\{y_n\}$ in ascending order and call it Sorted $\{y_n\}$

Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8	Step 9
125.00	5	1.5	-2.43	0.23	1.07	5	9	188.22
67.37	4	-0.28	-1.50	-0.28	1.19	4	10	276.97
46.33	1	0.50	-0.52	-2.34	-0.55	3	3	49.83
49.64	2	1.03	-0.28	-1.50	-1.06	9	2	49.64
49.83	3	-1.5	0.23	-0.52	-1.48	8	1	46.33
172.36	8	-2.34	0.26	0.50	-0.06	6	6	138.77
188.22	9	0.26	0.32	1.03	0.17	7	7	157.40
157.40	7	-0.52	0.50	0.32	-0.12	10	5	125.00
138.77	6	0.23	1.03	0.26	-0.28	1	4	67.37
276.97	10	0.32	1.50	1.50	0.30	2	8	172.36

Linear Surrogate Methods

Algorithm 2 or AAFT

5. Re-order Sorted $\{y_n\}$ according to original rank and call it Re-ordered $\{y_n\}$

6. Apply Algorithm 1 on Re-ordered $\{y_n\}$ and call it Surrogate $\{y_n\}$

7. Rank order Surrogate $\{y_n\}$ and call it Rank Surrogate $\{y_n\}$

Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8	Step 9
125.00	5	1.5	-2.43	-0.52	1.07	9	9	188.22
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138.77	6	0.23	1.03	0.32	-0.28	4	4	67.37
276.97	10	0.32	1.50	1.50	0.30	8	8	172.36

Linear Surrogate Methods

Algorithm 2 or AAFT

8. Re-order Rank Surrogate $\{y_n\}$ and call it Final Reorder

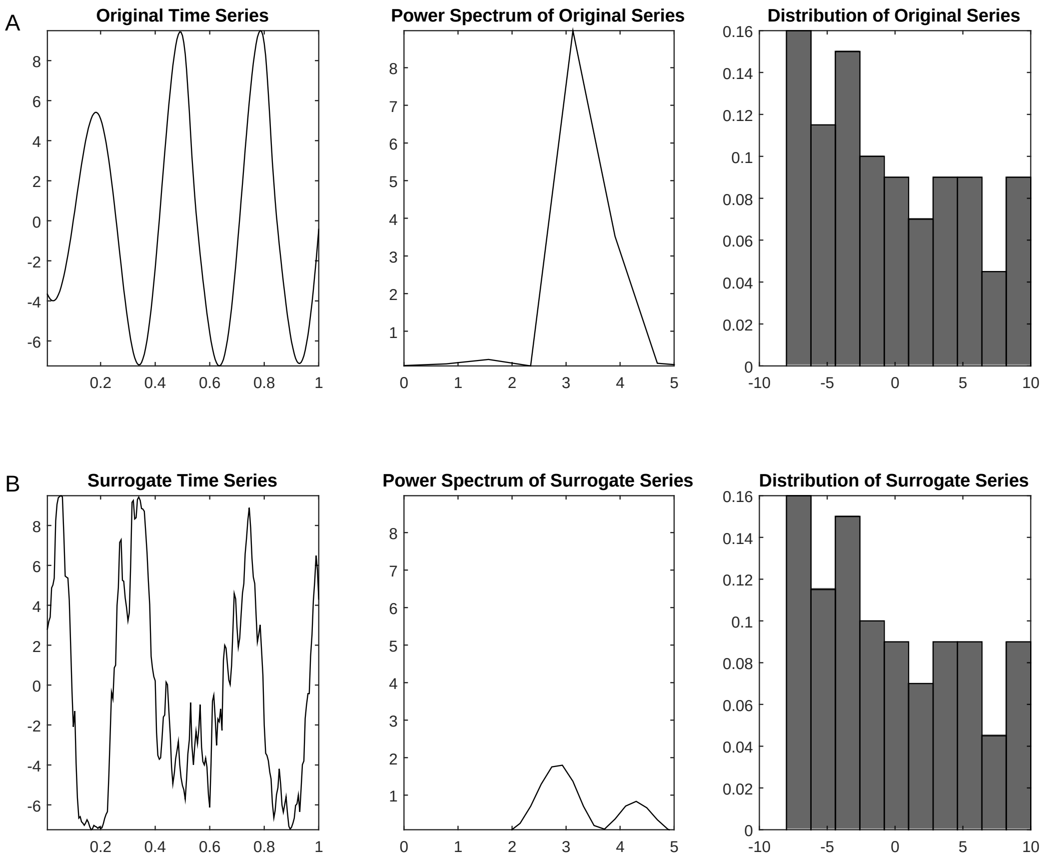
9. Arrange $\{x_n\}$ according to the Final Reorder

Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8	Step 9
125.00	5	1.5	-2.43	-0.52	1.07	9	9	188.22
67.37	4	-0.28	-1.50	-0.28	1.19	10	10	276.97
46.33	1	0.50	-0.52	0.23	-0.55	3	3	49.83
49.64	2	1.03	-0.28	1.50	-1.06	2	2	49.64
49.83	3	-1.5	0.23	-2.43	-1.48	1	1	46.33
172.36	8	-2.34	0.26	1.03	-0.06	6	6	138.77
188.22	9	0.26	0.32	0.50	0.17	7	7	157.40
157.40	7	-0.52	0.50	0.26	-0.12	5	5	125.00
138.77	6	0.23	1.03	0.32	-0.28	4	4	67.37
276.97	10	0.32	1.50	1.50	0.30	8	8	172.36

Linear Surrogate Methods

Algorithm 2

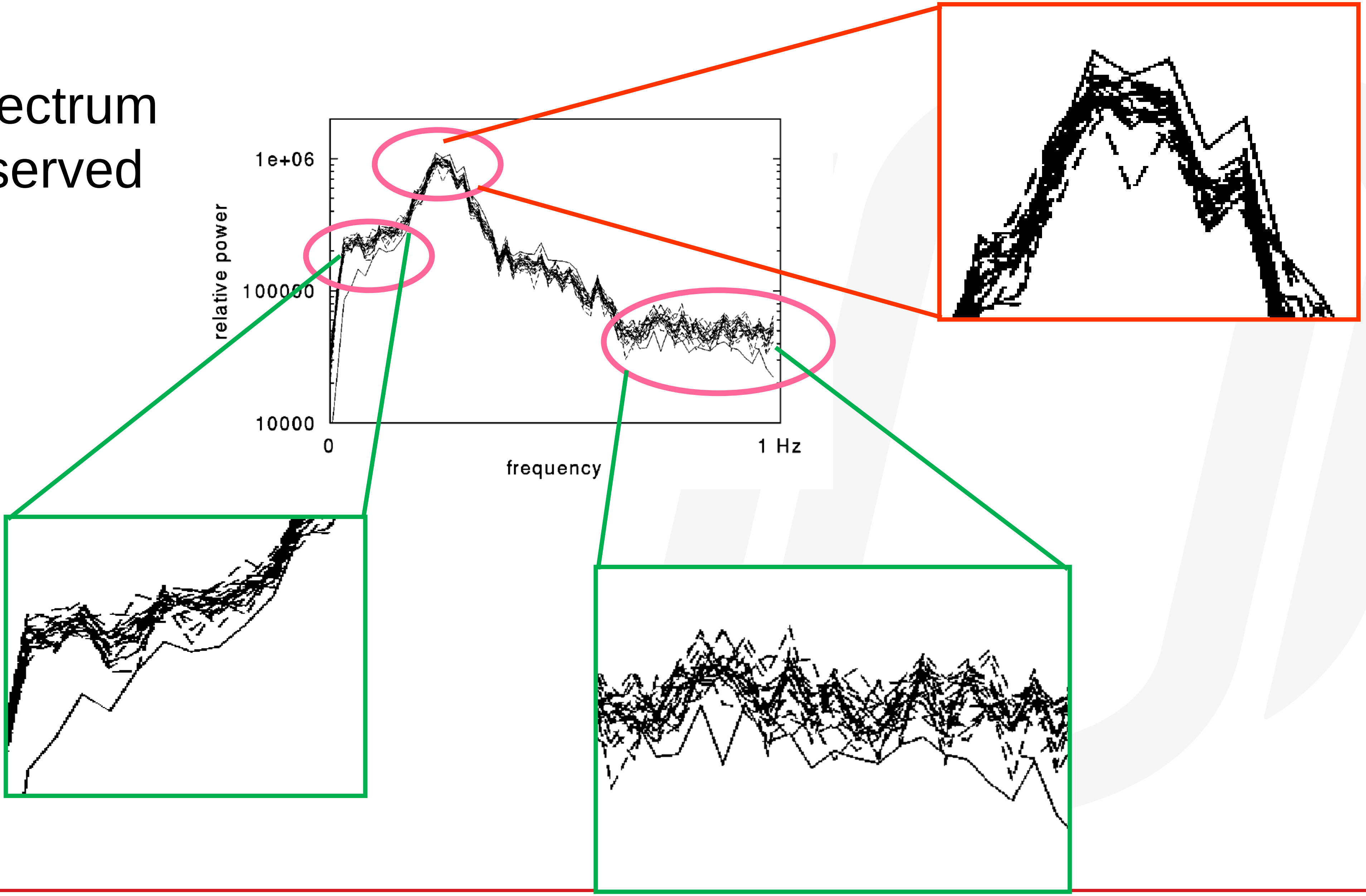
- Algorithm 2 preserves the amplitude distribution and power spectrum of the original time series (top) except during short and strongly correlated series (bottom)



When a time series is short and strongly correlated, rescaling of the inverse Fourier transformed data can change the linear correlations of the time series

A discrepancy between the power spectrum of the original time series and surrogate time series.

Power Spectrum
is not preserved



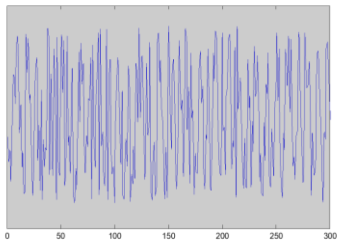
Linear Surrogate Methods

Iterated AAFT (IAAFT)

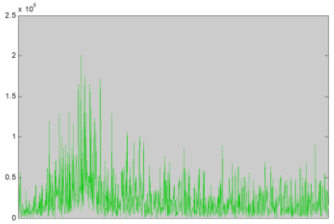
Proposed by Schreiber et al (1996) as an improvement to AAFT

Let $\{x_n\}$be the original time series where $n=1,2,\ldots,N$.

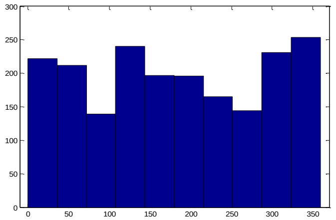
1. Sort $\{x_n\}$ in an ascending order and store this sorted time series Sorted_$\{x_n\}$.
2. Take the Fourier transform of $\{x_n\}$ and store the squared amplitudes of the Fourier transform of $\{x_n\}$, X_k^2.
3. Shuffle $\{x_n\}$ and take the Fourier transform and call it FTRandomized$\{x_n\}$.
4. To adjust power spectrum, replace the squared amplitudes of FTRandomized$\{x_n\}$ by $\{X_k^2\}$. The phases are kept unchanged. Then transform back by taking the inverse Fourier transform.
5. The procedure at Step 4 will change the amplitude distribution. Therefore, adjust the amplitudes by ranking the values of this time series and replacing them by the values of Sorted_$\{x_n\}$.
6. However, again the procedure at Step 5 may alter the power spectrum, so Step 4 and Step 5 are repeated until some convergence is achieved.



Original Time Series



Power spectrum



Distribution

Shuffle the original time series

FT

Adjust power spectrum

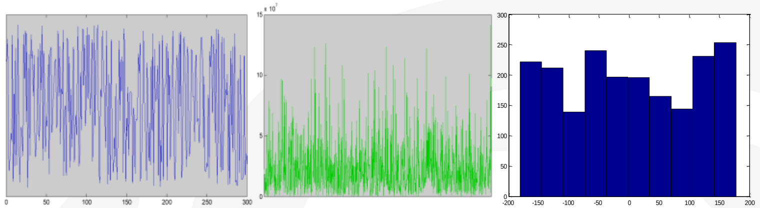
IVFT

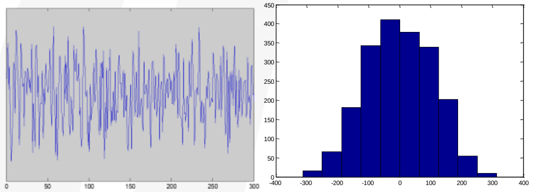
Adjust amplitude distribution

Converged?

NO

YES





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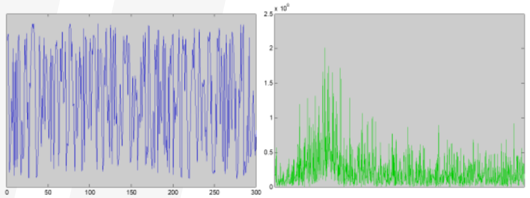
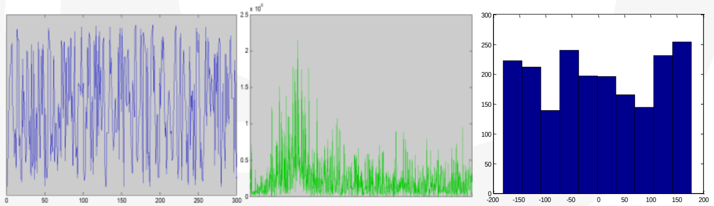


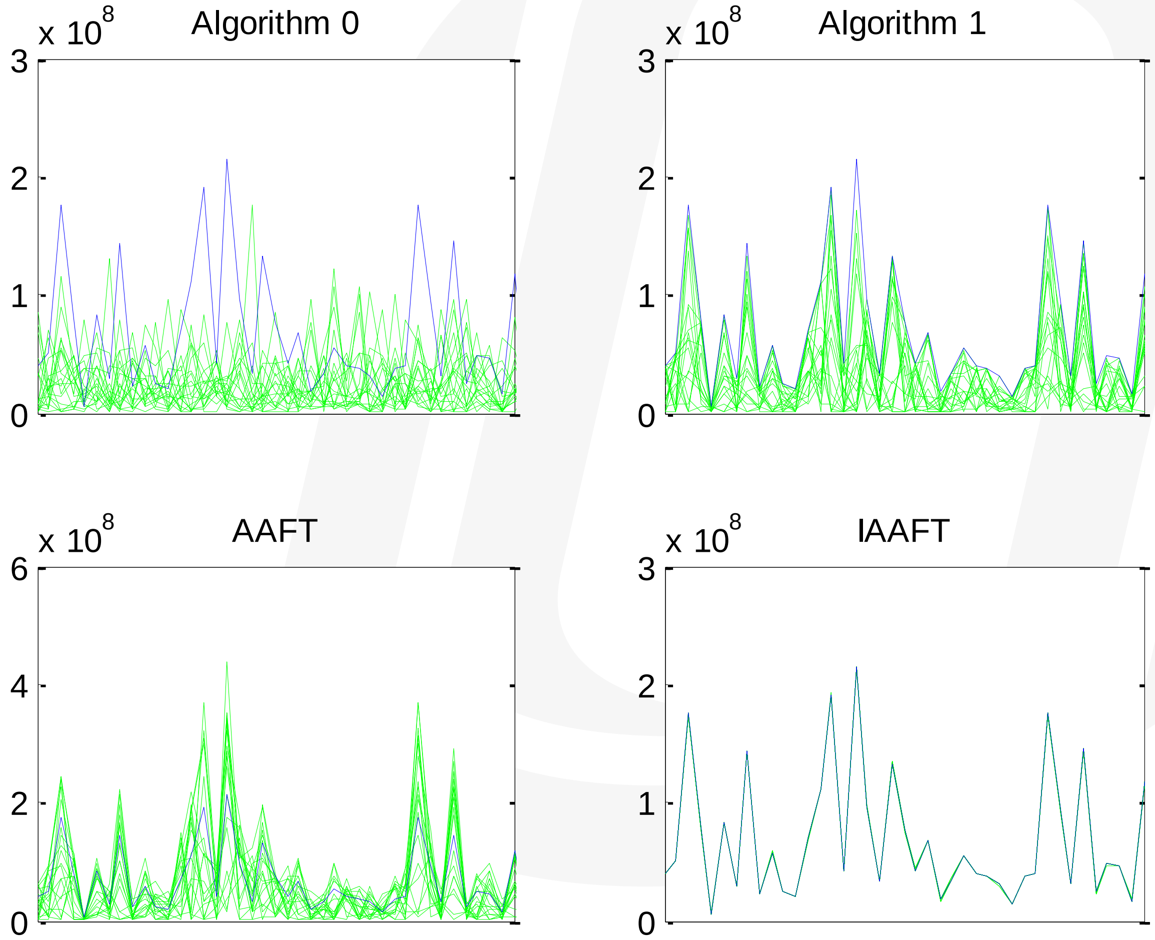
4



5

Iterated AAFT (IAAFT)

A surrogate time series generated by IAAFT preserves the power spectrum of an original time series much better than the other surrogate algorithms.



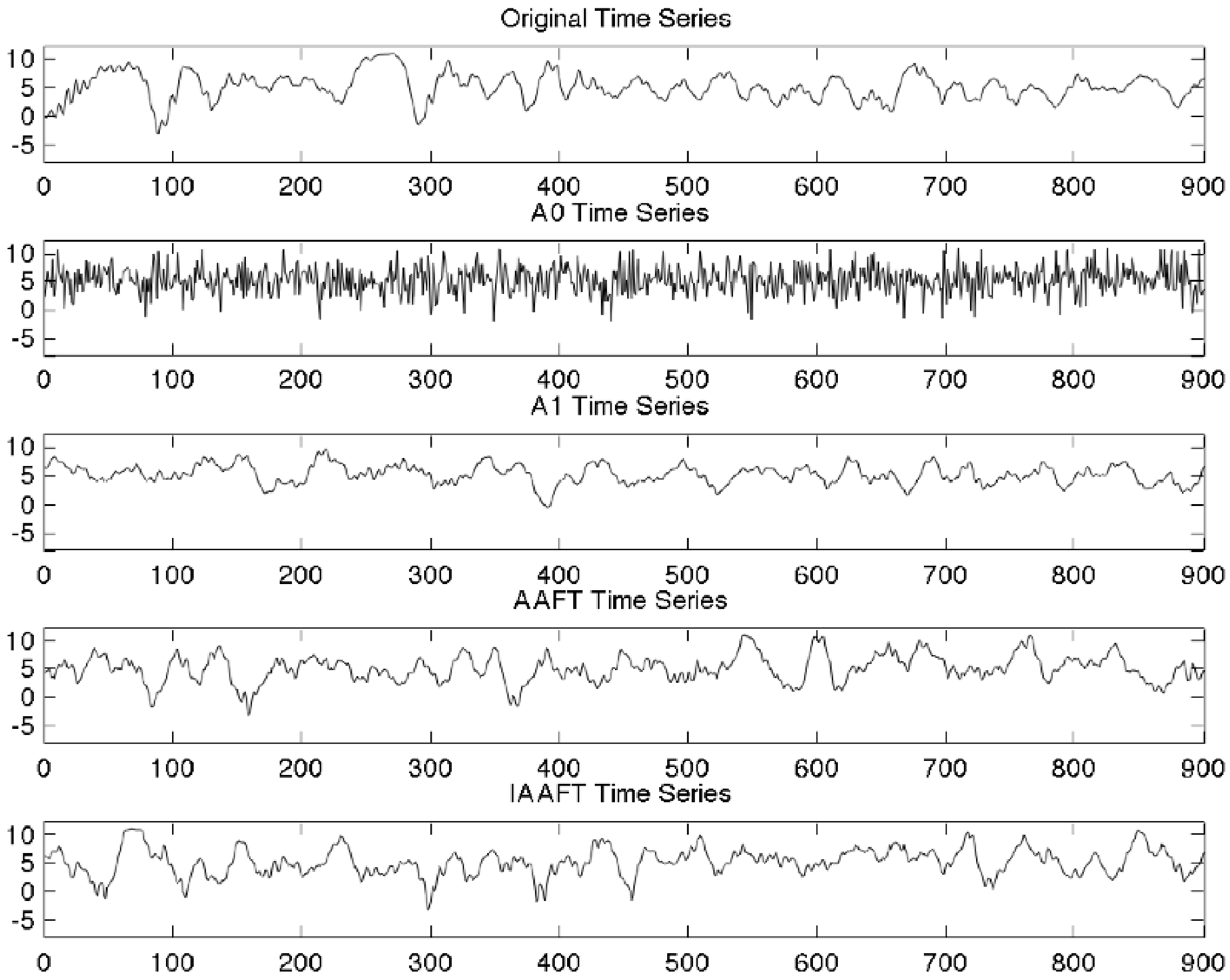
Green: power spectrum of surrogates.

Blue: power spectrum of original time series.

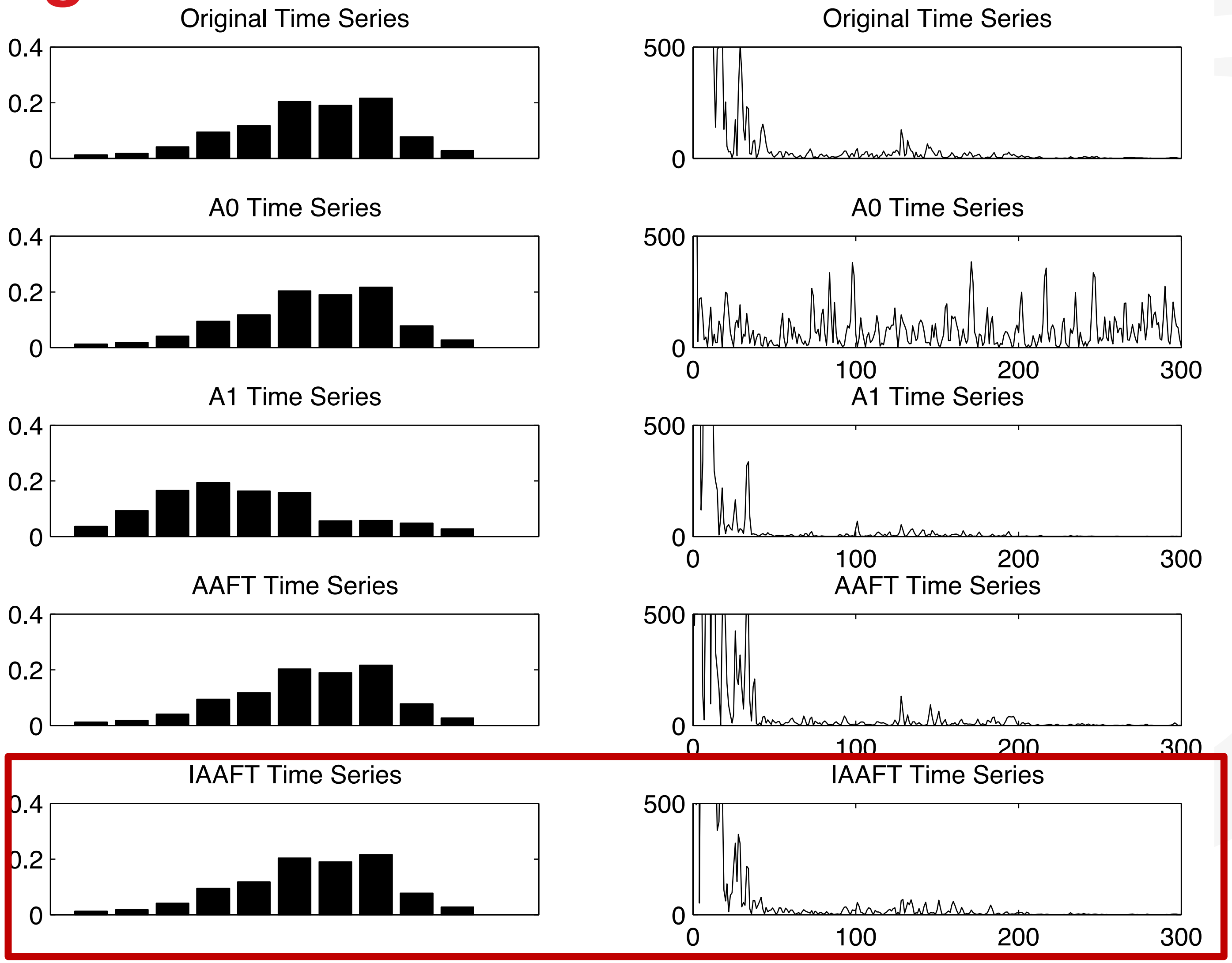
Limitations of IAAFT

- There is no guarantee that iterations will eventually converge.
- There is also a concern that surrogate time series generated by IAAFT for a data with short length may not have enough randomization, which makes hypothesis testing against a specific system rather than a general class of the system.

Linear Surrogate Methods



Linear Surrogate Methods



Rejection of Null Hypothesis

- Rejecting the H_0:
  - Indicates more complex dynamics than linear dynamics
  - Indicates the underlying dynamics are not consistent with the null hypothesis
- May not conclude the original and surrogate are from the same population
  - May be due to inadequate statistics
- As with all nonlinear methods, multiple nonlinear tools should be used
- Techniques described thus far are not appropriate for time series with inherent periodicity
  - False rejection of H_0 because structure of time series has changed (Algorithm 0 and Algorithm 1) or the geometric structure has changed (Algorithm 2 and IAAFT)

Summary

- Surrogate methods are used with hypothesis testing.
- Each algorithm generates a surrogate time series which is consistent with a specific null hypothesis.
- The linear surrogate methods are applied to a stationary irregular time series without any long term trend or periodicity.
- Surrogate methods are used as an indirect approach to identify the nature of a time series. They try to narrow down the possibility of what a time series is by eliminating the possibilities of what a time series is not.
- A surrogate method alone cannot decide what the time series is, but it may be a helpful tool when it is used with other nonlinear tools.

Acknowledgements

- NONAN would like to thank and recognize the following past employees for their contributions in putting together this presentation:
  - Mr. Joel Sommerfeld
  - Mr. Tyler Wiles
  - Dr. Jenna Yentes
  - Mr. Ben Senderling
  - Mr. Cory Fredrick

Thank you for attending our workshop!!!

Please fill out the Post-Workshop Survey.

We would really appreciate it! Thank you!

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- Assistance in experimental design
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- Interpretation and dissemination of results
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- The Core is also actively exploring and validating new techniques and algorithms for the next generation of nonlinear analysis.

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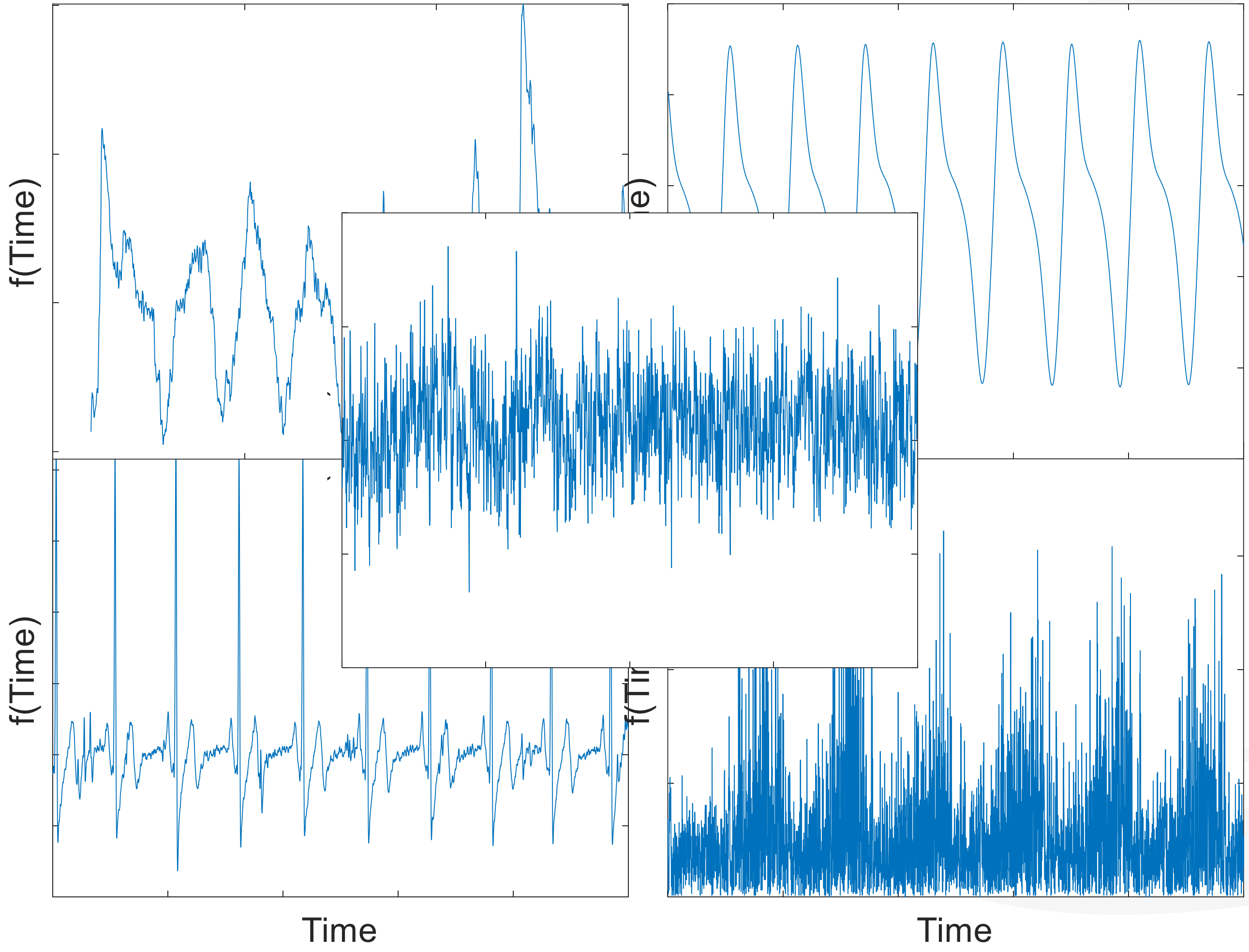
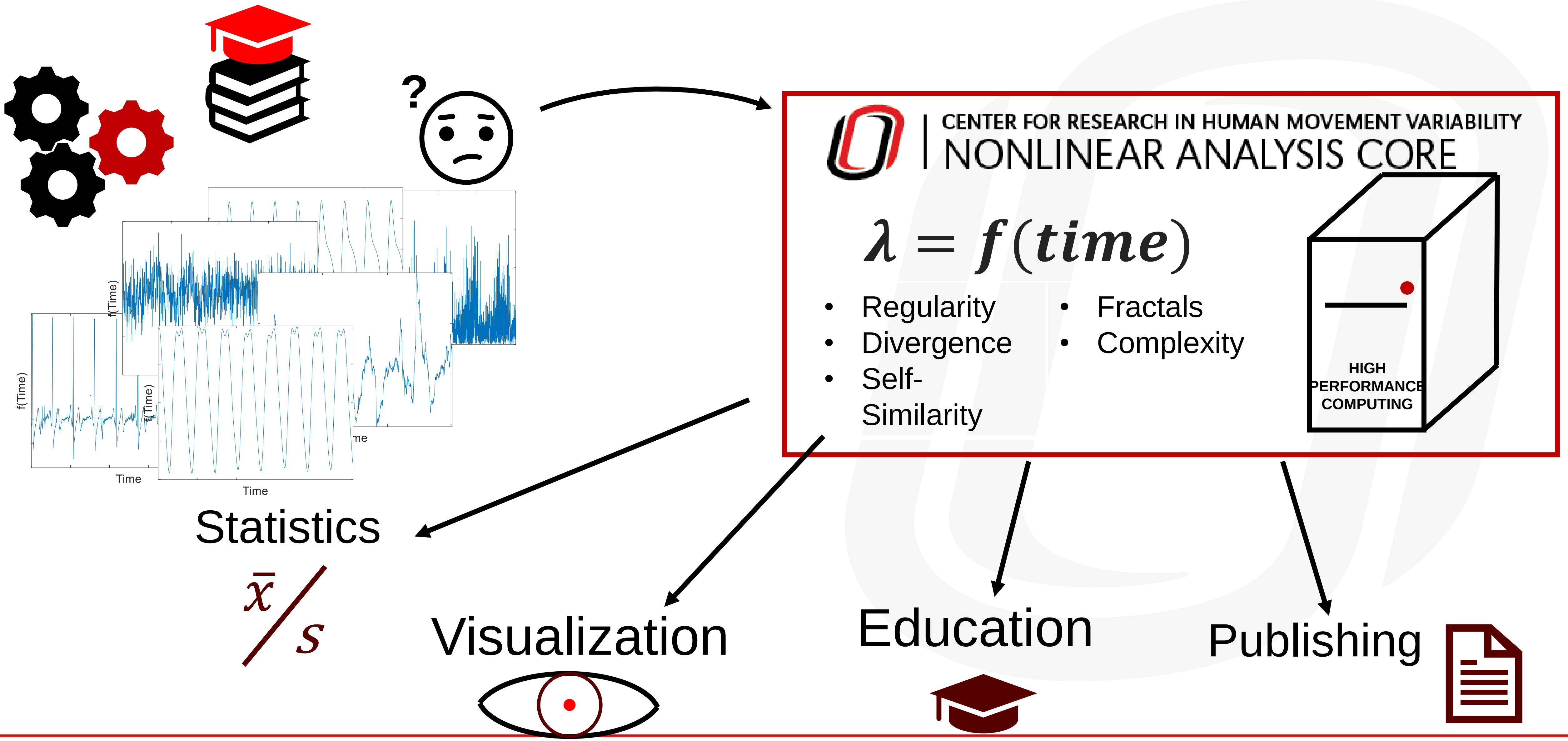
Does your data look this?
You can see the patterns but not the next step?



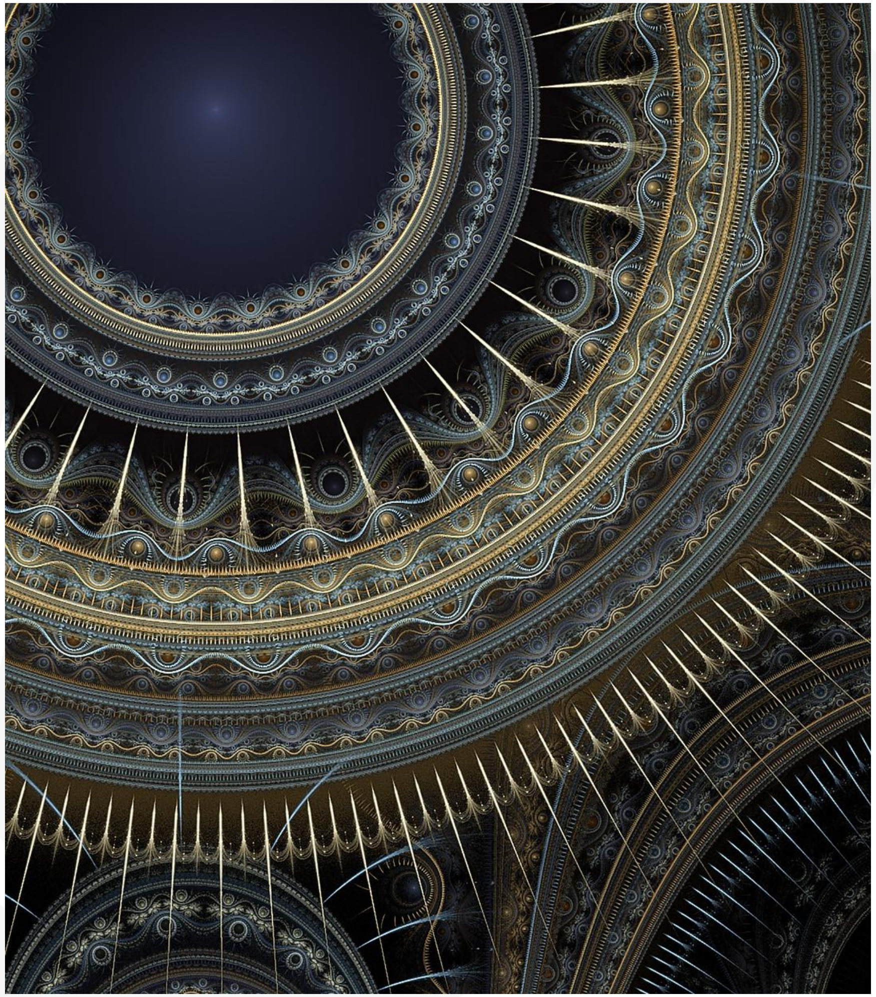

?
f(Time)
Time
CENTER FOR RESEARCH IN HUMAN MOVEMENT VARIABILITY
NONLINEAR ANALYSIS CORE
$\lambda = f(time)$
Regularity
Divergence
Self-Similarity
Fractals
Complexity
HIGH PERFORMANCE COMPUTING
Statistics
$\bar{x}/s$
Visualization
Education
Publishing

What do we do?

- NONAN provides expertise and resources for innovative nonlinear analyses.
- Provide:
  - Consultation
  - Data processing
  - Education
  - Interpretation
  - Dissemination
  - Quality assurance

Analysis Methods

- Detrended Fluctuation Analysis
- Entropy
- Largest Lyapunov Exponent
- Recurrence Quantification Analysis
- Relative Phase
- Surrogation



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