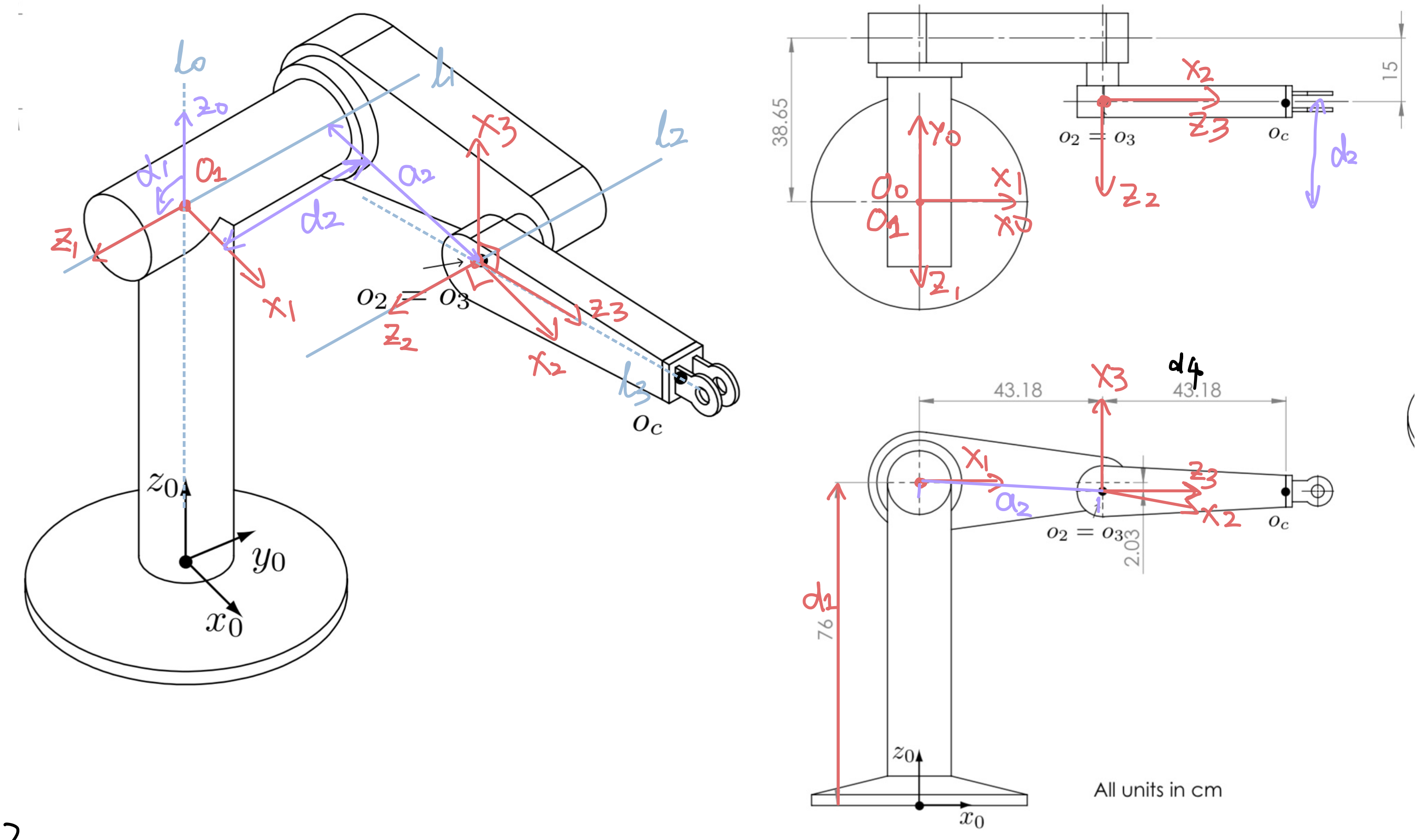
3.1



3.2

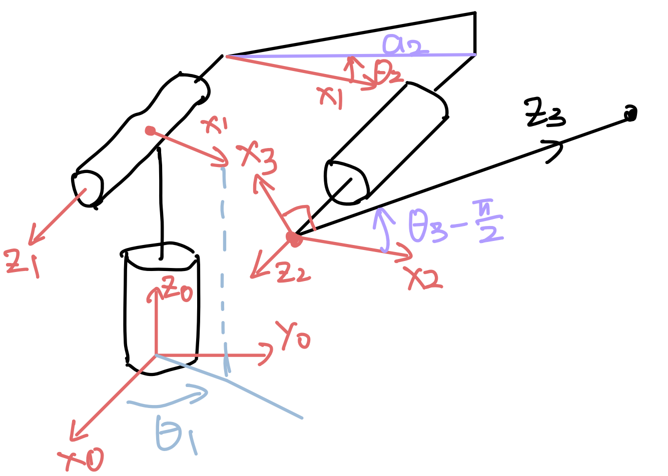
DH Table

Link	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	76	θ_1
2	43.23	0	23.5	θ_2
3	0	$\pi/2$	0	θ_3
4	0	$-\pi/2$	43.18	θ_4
5	0	$\pi/2$	0	θ_5
6	0	0	d_6	θ_6

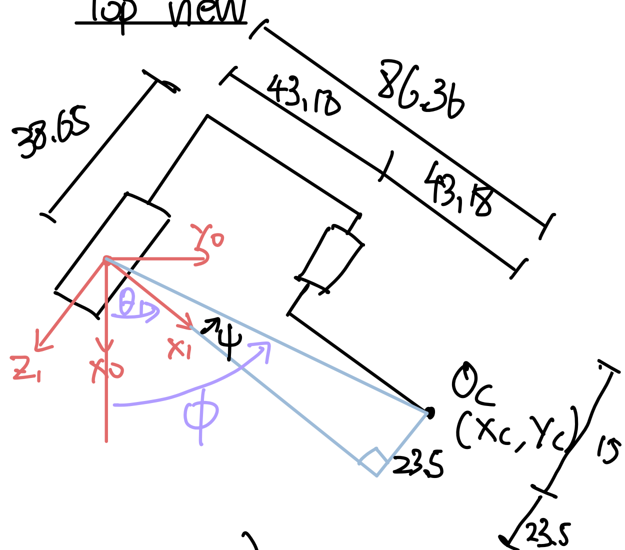
$$a_2 = \sqrt{43.18^2 + 2.03^2} = 43.23$$

3.3

Given $O_c^0 = O_d - d_6 R_d \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$



Top view

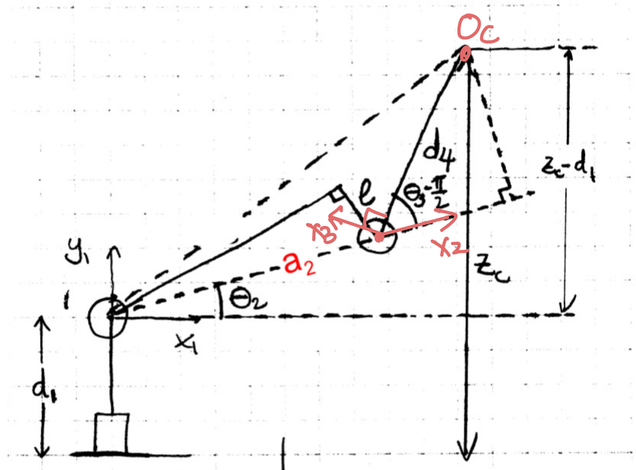


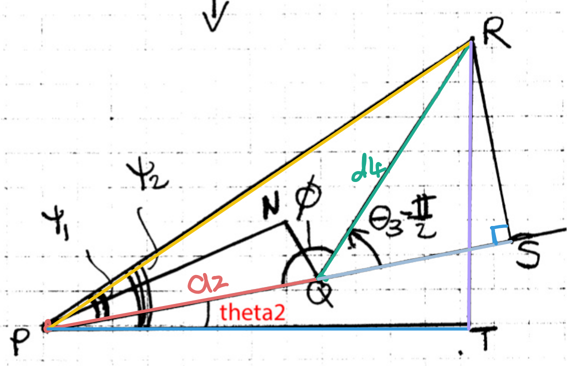
$\phi = \operatorname{atan2}(y_c, x_c)$

$\psi = \operatorname{atan2}(\sin\psi, \cos\psi) = \operatorname{atan2}(23.5, 86.36)$

$$\boxed{\theta_1 = \phi - \psi = \operatorname{atan2}(y_c, x_c) - \operatorname{atan2}(23.5\ ,\ 86.36)}$$

Side View: (graph copied from hints)





$\overline{RT} = z_c - d_1$

$\overline{PT} = \sqrt{x_c^2 + y_c^2 - d_2^2}$

$\overline{PR} = \sqrt{(z_c - d_1)^2 + x_c^2 + y_c^2 - d_2^2}$

$\overline{PQ} = a_2$

$\overline{QR} = d_4$

1) law of cosine to $\triangle PQR$

$$\overline{PR}^2 = \overline{PQ}^2 + \overline{QR}^2 - 2\,\overline{PQ}\,\overline{QR}\cos(\phi)$$

$$(z_c - d_1)^2 + x_c^2 + y_c^2 - d_2^2 = a_2^2 + d_4^2 - 2a_2 d_4 \cos(\phi)$$

$$\boxed{\cos(\phi) = \frac{a_2^2 + d_4^2 - (z_c - d_1)^2 - x_c^2 - y_c^2}{2a_2 d_4}}$$

$$\phi = \operatorname{atan2}\left(\sqrt{1 - \cos(\phi)^2}\ ,\ \cos(\phi)\right)$$

$\cos(\phi)$

$$\phi = \pi - \left(\theta_3 - \frac{\pi}{2}\right) = \frac{\pi}{2} - \theta_3$$

$$\boxed{\theta_3 = \frac{\pi}{2} - \phi = \frac{\pi}{2} - \operatorname{atan2}\left(\sqrt{1 - \cos(\phi)^2}\ ,\ \cos(\phi)\right)}$$

With θ_3 known, we can calculate $\overline{QS}$

$$\cos\left(\theta_3 - \frac{\pi}{2}\right) = \frac{\overline{QS}}{\overline{QR}} = \frac{\overline{QS}}{d_4}$$

$$\overline{QS} = d_4 \cos\left(\theta_3 - \frac{\pi}{2}\right)$$

$$\overline{PS} = \overline{PQ} + \overline{QS} = a_2 + d_4\cos\left(\theta_3 - \frac{\pi}{2}\right)$$

$$\overline{RS} = d_4 \sin\left(\theta_3 - \frac{\pi}{2}\right)$$

$$\psi_1 = \operatorname{atan2}(\overline{RS}, \overline{PS})$$

$$\boxed{\psi_1 = \operatorname{atan2}\left(d_4\sin\left(\theta_3 - \frac{\pi}{2}\right),\ a_2 + d_4\cos\left(\theta_3 - \frac{\pi}{2}\right)\right)}$$

$$\psi_2 = \operatorname{atan2}(\overline{RT}, PT)$$

$$\boxed{\psi_2 = \operatorname{atan2}\left(z_c - d_1\ ,\ \sqrt{x_c^2 + y_c^2 - d_2^2}\right)}$$

$$\theta_2 = \psi_2 - \psi_1$$

$$\boxed{\theta_2 = \operatorname{atan2}\left(z_c - d_1\ ,\ \sqrt{x_c^2 + y_c^2 - d_2^2}\right) - \operatorname{atan2}\left(d_4\sin\left(\theta_3 - \frac{\pi}{2}\right),\ a_2 + d_4\cos\left(\theta_3 - \frac{\pi}{2}\right)\right)}$$

Solve $\theta_4, \theta_5, \theta_6$ for the spherical wrist:

We know from lecture that $\theta_4, \theta_5, \theta_6$ are precisely the ZYZ Euler angles where $\theta_4 = \phi$, $\theta_5 = \theta$, $\theta_6 = \psi$

We need to solve $\quad R_3^0(\theta_1, \theta_2, \theta_3)\, R_6^3(\theta_4, \theta_5, \theta_6) = R_d$

$$R_6^3(\theta_4, \theta_5, \theta_6) = R_3^0(\theta_1, \theta_2, \theta_3)^T R_d = M$$

let M_{ij} be the entries of M

Using the formula for Euler angles, we can find:

$$\theta_4 = \operatorname{atan2}(m_{23}, m_{13}) \qquad\qquad \theta_4 = \operatorname{atan2}(-m_{23}, -m_{13})$$
$$\theta_5 = \operatorname{atan2}(\sqrt{1-m_{33}^2}, m_{33}) \qquad \text{OR} \qquad \theta_5 = \operatorname{atan2}(-\sqrt{1-m_{33}^2}, m_{33})$$
$$\theta_6 = \operatorname{atan2}(m_{32}, -m_{31}) \qquad\qquad \theta_6 = \operatorname{atan2}(-m_{32}, +m_{31})$$

where $m_{13}^2 + m_{23}^2 \neq 0$