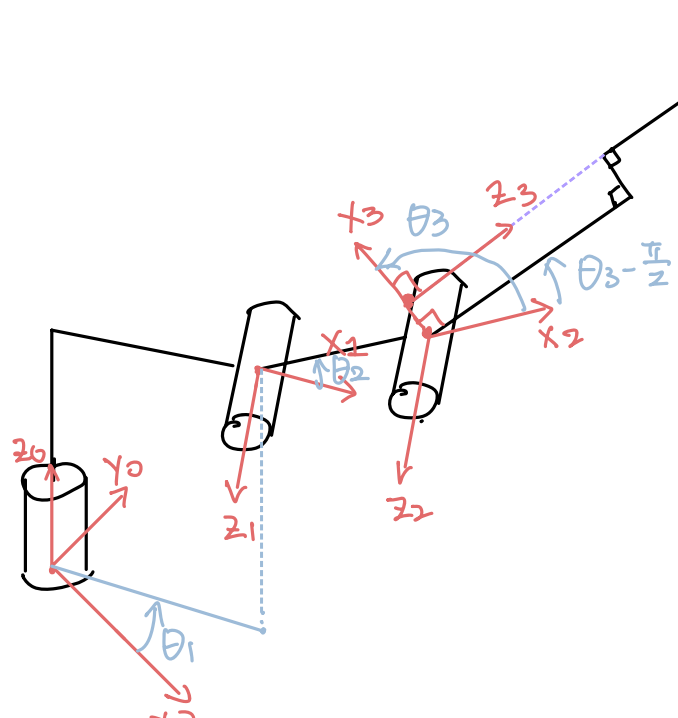


DH Table 1

Link	a_i	d_i	d_i	θ_i
1	25	$\pi/2$	400	θ_1
2	315	0	0	θ_2
3	35	$\pi/2$	0	θ_3
4	0	$-\pi/2$	365	θ_4
5	0	$\pi/2$	0	θ_5
6	-296.23	0	161.44	θ_6

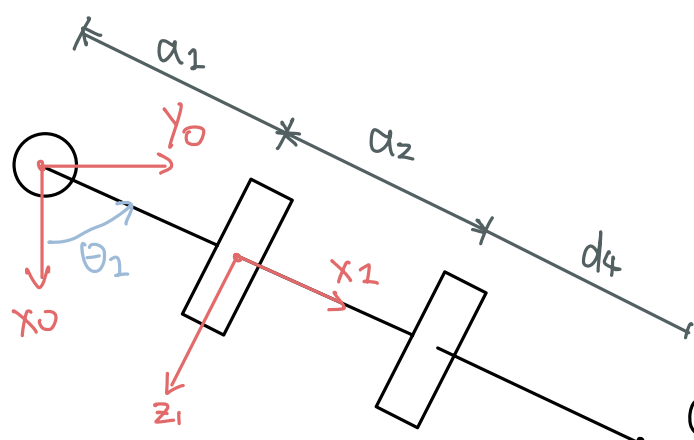
in mm



$$O_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = O_d^0 - R_d \begin{bmatrix} -d_6 \\ 0 \\ d_6 \end{bmatrix}$$

Find θ_1 :

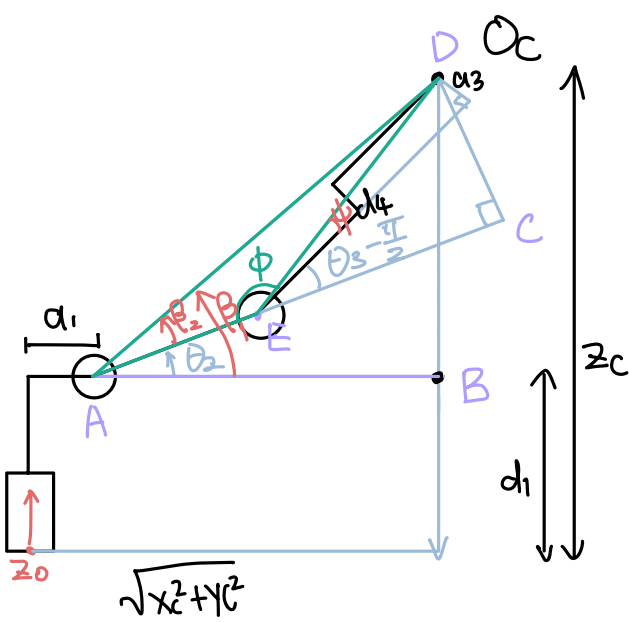
Top view:



$$\theta_1 = \text{atan2}(y_c, x_c)$$

$$O_c = \begin{bmatrix} x_c \\ y_c \end{bmatrix}$$

Find θ_2, θ_3



$$\begin{aligned} \overline{AE} &= a_2 & \overline{DE} &= \sqrt{a_2^2 + d_4^2} \\ \overline{AB} &= \sqrt{x_c^2 + y_c^2} - a_1 & \overline{DB} &= z_c - d_1 \\ \overline{AD}^2 &= \overline{AB}^2 + \overline{DB}^2 = (\sqrt{x_c^2 + y_c^2} - a_1)^2 + (z_c - d_1)^2 \end{aligned}$$

$$\overline{AD}^2 = \overline{AE}^2 + \overline{DE}^2 - 2\overline{AE}\overline{DE}\cos\phi$$

$$(\sqrt{x_c^2 + y_c^2} - a_1)^2 + (z_c - d_1)^2 = a_2^2 + a_3^2 + d_4^2 - 2a_2\sqrt{a_3^2 + d_4^2}\cos\phi$$

$$D = \cos\phi = \frac{a_2^2 + a_3^2 + d_4^2 - (\sqrt{x_c^2 + y_c^2} - a_1)^2 - (z_c - d_1)^2}{2a_2\sqrt{a_3^2 + d_4^2}}$$

$$\phi = \text{atan2}(\sqrt{1 - D^2}, D)$$

$$\psi = \text{atan2}(a_3, d_4)$$

$$\theta_3 - \frac{\pi}{2} = \pi - \phi - \psi$$

$$\theta_3 = \frac{3}{2}\pi - \phi - \psi = \frac{3}{2}\pi - \text{atan2}(\sqrt{1 - D^2}, D) - \text{atan2}(a_3, d_4)$$

$$\overline{CE} = \overline{DE}\cos(\theta_3 - \frac{\pi}{2} + \psi)$$

$$= \sqrt{a_3^2 + d_4^2}\cos(\theta_3 - \frac{\pi}{2} + \psi)$$

$$\overline{CD} = \overline{DE}\sin(\theta_3 - \frac{\pi}{2} + \psi)$$

$$= \sqrt{a_3^2 + d_4^2}\sin(\theta_3 - \frac{\pi}{2} + \psi)$$

$$\overline{AC} = \overline{AE} + \overline{CE} = a_2 + \sqrt{a_3^2 + d_4^2}\cos(\theta_3 - \frac{\pi}{2} + \psi)$$

$$\beta_2 = \text{atan2}(\overline{CD}, \overline{AC})$$

$$\beta_2 = \text{atan2}(\sqrt{a_3^2 + d_4^2}\sin(\theta_3 - \frac{\pi}{2} + \psi), a_2 + \sqrt{a_3^2 + d_4^2}\cos(\theta_3 - \frac{\pi}{2} + \psi))$$

$$\beta_1 = \text{atan2}(\overline{AB}, \overline{DB})$$

$$= \text{atan2}(z_c - d_1, \sqrt{x_c^2 + y_c^2} - a_1)$$

$$\theta_2 = \beta_1 - \beta_2$$

$$\theta_2 = \text{atan2}(z_c - d_1, \sqrt{x_c^2 + y_c^2} - a_1) - \text{atan2}(\sqrt{a_3^2 + d_4^2}\sin(\theta_3 - \frac{\pi}{2} + \psi), a_2 + \sqrt{a_3^2 + d_4^2}\cos(\theta_3 - \frac{\pi}{2} + \psi))$$

$$\text{Let } (R_3^0)^T R_d = M,$$

$$\theta_4 = \text{atan2}(m_{23}, m_{13})$$

$$\theta_5 = \text{atan2}(\sqrt{1 - m_{33}^2}, m_{33})$$

$$\theta_6 = \text{atan2}(m_{32}, -m_{31})$$

Elbow down

OR

$$\theta_4 = \text{atan2}(-m_{23}, -m_{13})$$

$$\theta_5 = \text{atan2}(-\sqrt{1 - m_{33}^2}, m_{33})$$

$$\theta_6 = \text{atan2}(-m_{32}, +m_{31})$$

Elbow up

