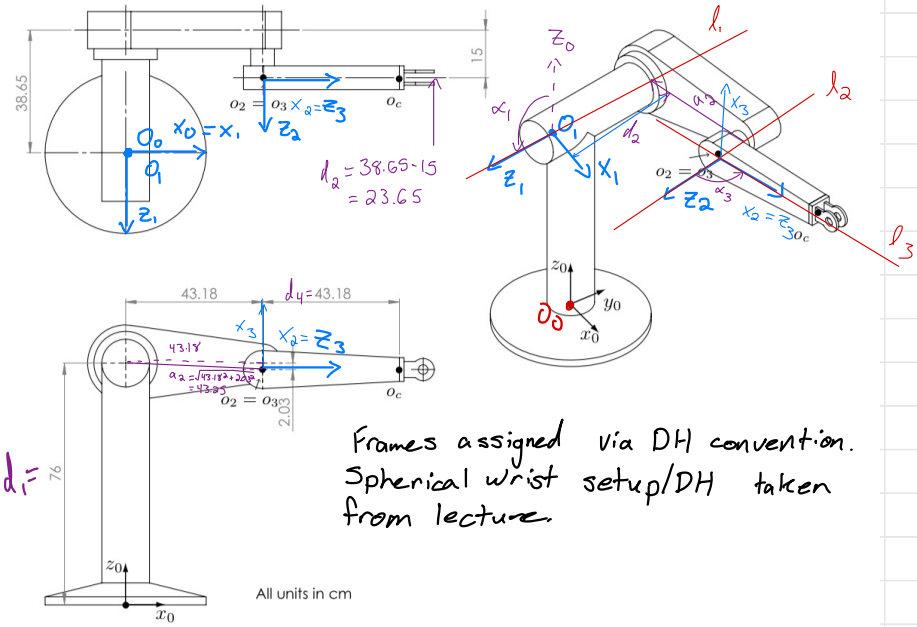


Frames assigned via DH convention. Spherical wrist setup/DH taken from lecture.

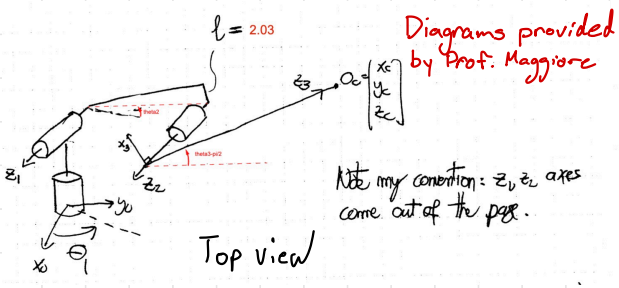
DH Table

L	a_i	α_i	d_i	θ_i
1	0	$\pi/2$	76	θ_1
2	43.23	0	-23.65	θ_2
3	0	$\pi/2$	0	θ_3
4	0	$-\frac{\pi}{2}$	43.18	θ_4
5	0	$\pi/2$	0	θ_5
6	0	0	20	θ_6

$x_4 = -z_3 \times z_4$

$\Rightarrow \alpha_4 = -\frac{\pi}{2}$

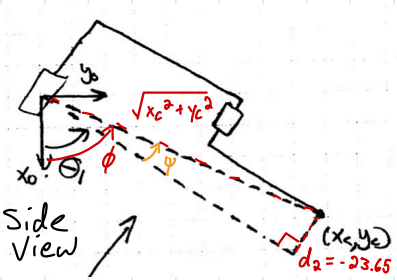
Inverse Kinematic Equations



Diagrams provided by Prof. Maggiore

Note my convention: z_0, z_2 axes come out of the page.

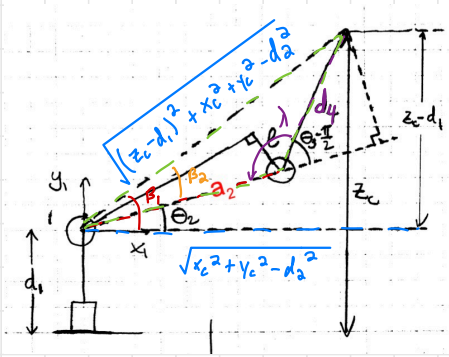
Top view



Side View

$$\phi = \operatorname{atan2}(y_c, x_c)$$
$$\psi = \operatorname{atan2}\left(-d_2, \sqrt{x_c^2 + y_c^2 - (-d_2)^2}\right)$$

$$\Rightarrow \theta_1 = \phi - \psi = \operatorname{atan2}(y_c, x_c) - \operatorname{atan2}\left(-d_2, \sqrt{x_c^2 + y_c^2 - d_2^2}\right)$$

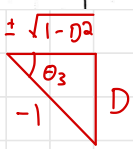


$$\lambda = \pi - (\theta_3 - \pi/2) = \frac{3\pi}{2} - \theta_3$$

$$(z_c - d_1)^2 + x_c^2 + y_c^2 - d_2^2 = a_2^2 + d_4^2 - 2a_2 d_4 \cos\lambda$$

$$\cos\left(\frac{3\pi}{2} - \theta_3\right) = \frac{\left[(z_c - d_1)^2 + x_c^2 + y_c^2 - d_2^2\right] - a_2^2 - d_4^2}{2a_2 d_4} = D$$

$$-\sin(\theta_3) = D$$



$$\Rightarrow \theta_3 = \operatorname{atan2}\left(D, \sqrt{1 - D^2}\right)$$

$$\beta_1 = \operatorname{atan2}\left(z_c - d_1, \sqrt{x_c^2 + y_c^2 - d_2^2}\right)$$
$$\beta_2 = \operatorname{atan2}\left(d_4 \sin\left(\theta_3 - \tfrac{\pi}{2}\right), a_2 + d_4 \cos\left(\theta_3 - \tfrac{\pi}{2}\right)\right)$$

$$\theta_2 = \operatorname{atan2}\left(z_c - d_1, \sqrt{x_c^2 + y_c^2 - d_2^2}\right) - \operatorname{atan2}\left(d_4 \sin\left(\theta_3 - \tfrac{\pi}{2}\right), a_2 + d_4 \cos\left(\theta_3 - \tfrac{\pi}{2}\right)\right)$$

Note: Spherical wrist follows ZYZ parameterization, $\theta_4, \theta_5, \theta_6 = \phi, \theta, \psi$

$$R_3^0(\theta_1, \theta_2, \theta_3)\, R_6^3(\theta_4, \theta_5, \theta_6) = R_d$$

$$R_6^3(\theta_4, \theta_5, \theta_6) = \left[R_3^0(\theta_1, \theta_2, \theta_3)\right]^T R_d = M \rightarrow \text{with components } m_{xy}$$

(R_d: Given; R_3^0: from forward kin.)

→ Applying Euler Angle formulas

$$\theta_4 = \operatorname{atan2}(m_{23}, m_{13}) \qquad \theta_4 = \operatorname{atan2}(-m_{23}, -m_{13})$$
$$\theta_5 = \operatorname{atan2}\left(\sqrt{1 - m_{33}^2}, m_{33}\right) \qquad \theta_5 = \operatorname{atan2}\left(-\sqrt{1 - m_{33}^2}, m_{33}\right)$$
$$\theta_6 = \operatorname{atan2}(m_{32}, -m_{31}) \qquad \theta_6 = \operatorname{atan2}(-m_{32}, m_{31})$$