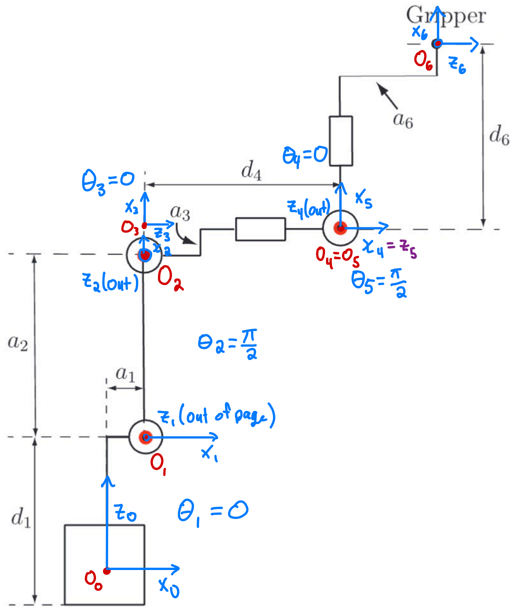


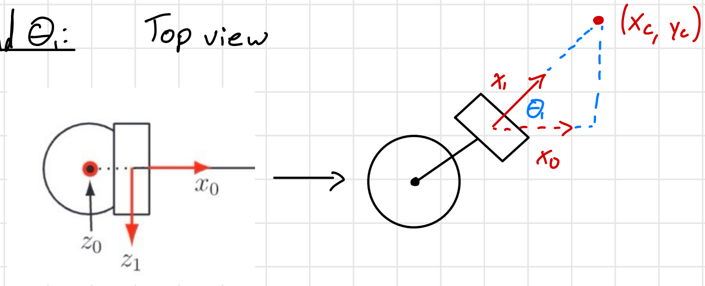
DH Table

l	a	α	d	θ
1	25	$\frac{\pi}{2}$	400	θ_1
2	315	0	0	θ_2
3	35	$\pi/2$	0	θ_3
4	0	$-\frac{\pi}{2}$	365	θ_4
5	0	$\pi/2$	0	θ_5
6	296.25	0	161.44	θ_6

(All in mm)

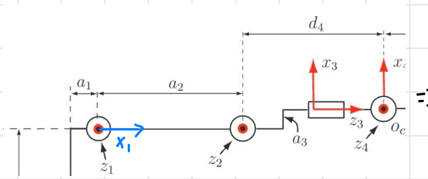
Find θ_1:

Top view

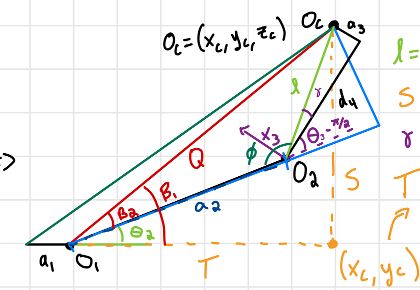


$$\theta_1 = \text{atan}2(y_c, x_c)$$

θ_3, θ_2



=>



$$l = \sqrt{d_4^2 + a_3^2}$$

$$s = z_c - d_1$$

$$\gamma = \text{atan}2(a_3, d_4)$$

$$T = \sqrt{x_c^2 + y_c^2} - a_1$$

$$Q = \sqrt{(z_c - d_1)^2 + \left(\sqrt{x_c^2 + y_c^2} - a_1\right)^2}$$

$$\phi = \pi - \gamma - \left(\theta_3 - \frac{\pi}{2}\right)$$

$$= \frac{3\pi}{2} - \theta_3 - \text{atan}2(a_3, d_4)$$

$$Q^2 = l^2 + a_2^2 - 2 l a_2 \cos\phi$$

$$\cos\left(\frac{3\pi}{2} - \theta_3 - \operatorname{atan2}(a_3, d_4)\right) = \frac{d_4^2 + a_3^2 + a_2^2 - (z_c - d_1)^2 + \left(\sqrt{x_c^2 + y_c^2} - a_1\right)^2}{2\left(\sqrt{d_4^2 + a_3^2}\right)(a_2)} \Rightarrow D$$

$$-\sin\left(\theta_3 + \operatorname{atan2}(a_3, d_4)\right) = \quad \downarrow \quad = D$$

$$-\sin(\theta_T) = D$$

$$\theta_T = \operatorname{atan2}\left(D, \sqrt{1 - D^2}\right)$$

$$\theta_3 = \operatorname{atan2}\left(D, \sqrt{1 - D^2}\right) - \operatorname{atan2}(a_3, a_4)$$

$\underline{\theta_2}$.

$$\beta_1 = \operatorname{atan2}\left(z_c - d_1, \sqrt{x_c^2 + y_c^2} - a_1\right)$$

$$\beta_2 = \operatorname{atan2}\left(l \sin\left(\operatorname{atan2}(a_3, d_4) + \theta_3 - \frac{\pi}{2}\right), l \cos\left(\operatorname{atan2}(a_3, d_4) + \theta_3 - \frac{\pi}{2}\right)\right)$$

$$= \operatorname{atan2}\Big(\sqrt{d_4^2 + a_3^2}\, \sin\left[\operatorname{atan2}(a_3, d_4) + \left(\operatorname{atan2}\left(D, \sqrt{1-D^2}\right) - \operatorname{atan2}(a_3, a_4)\right) - \frac{\pi}{2}\right],$$
$$\sqrt{d_4^2 + a_3^2}\, \cos\left[\operatorname{atan2}(a_3, d_4) + \left(\operatorname{atan2}\left(D, \sqrt{1-D^2}\right) - \operatorname{atan2}(a_3, a_4)\right) - \frac{\pi}{2}\right]\Big)$$

$$\theta_2 = \beta_1 - \beta_2$$

$\underline{\theta_4, \theta_5, \theta_6}$

$$R_3^0(\theta_1, \theta_2, \theta_3)\, R_6^3(\theta_4, \theta_5, \theta_6) = R_d$$

$$R_6^3(\theta_4, \theta_5, \theta_6) = \underbrace{\left[R_3^0(\theta_1, \theta_2, \theta_3)\right]^T}_{\text{from forward kin.}} \overbrace{R_d}^{\text{Given}} = M \rightarrow \text{with components } m_{xy}$$

$\Rightarrow$ Applying Euler Angle formulas

$$\theta_4 = \operatorname{atan2}(m_{23}, m_{13}) \quad \rightarrow \text{elbow up config.}$$

$$\theta_5 = \operatorname{atan2}\left(\sqrt{1 - m_{33}^2}, m_{33}\right)$$

$$\theta_6 = \operatorname{atan2}(m_{32}, -m_{31})$$