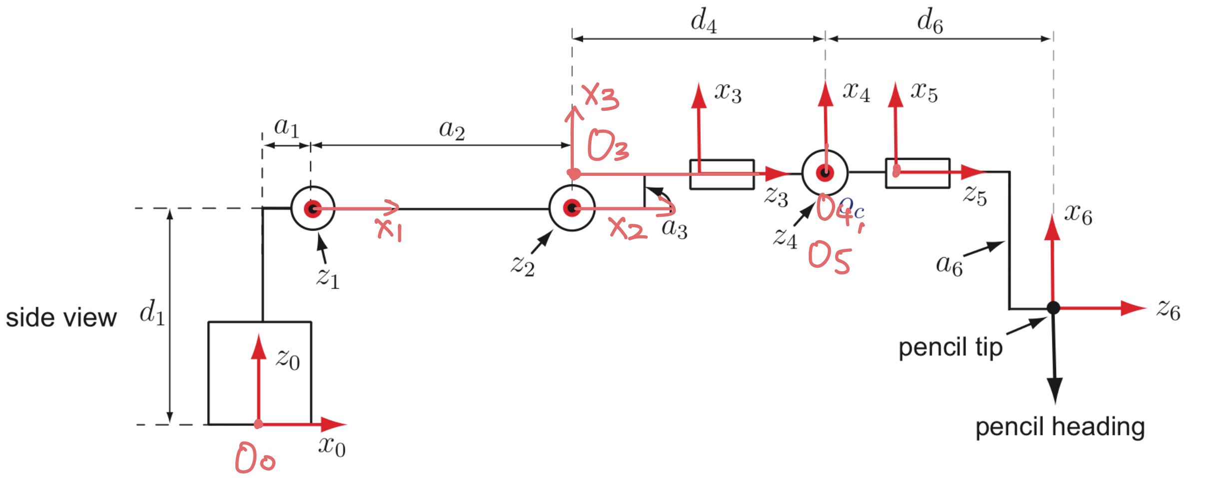
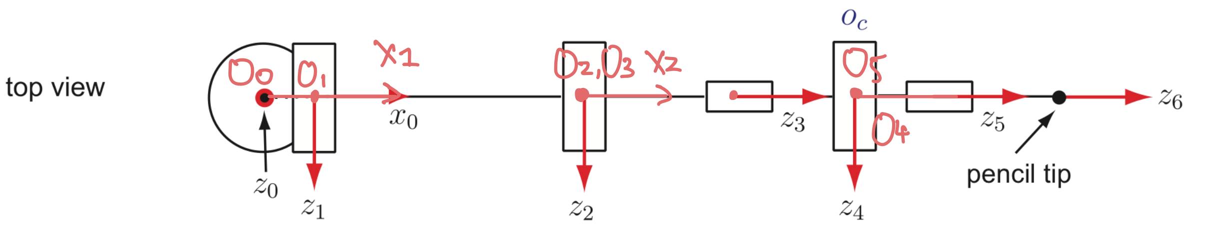
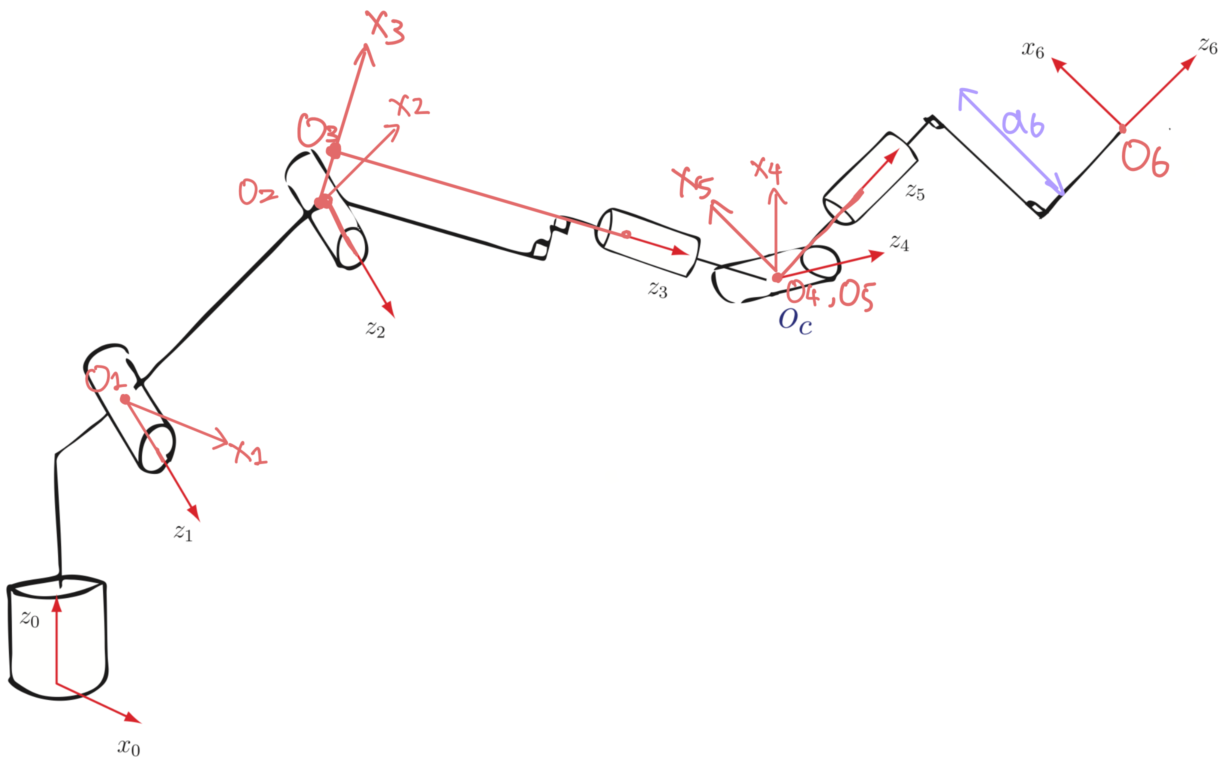
1.

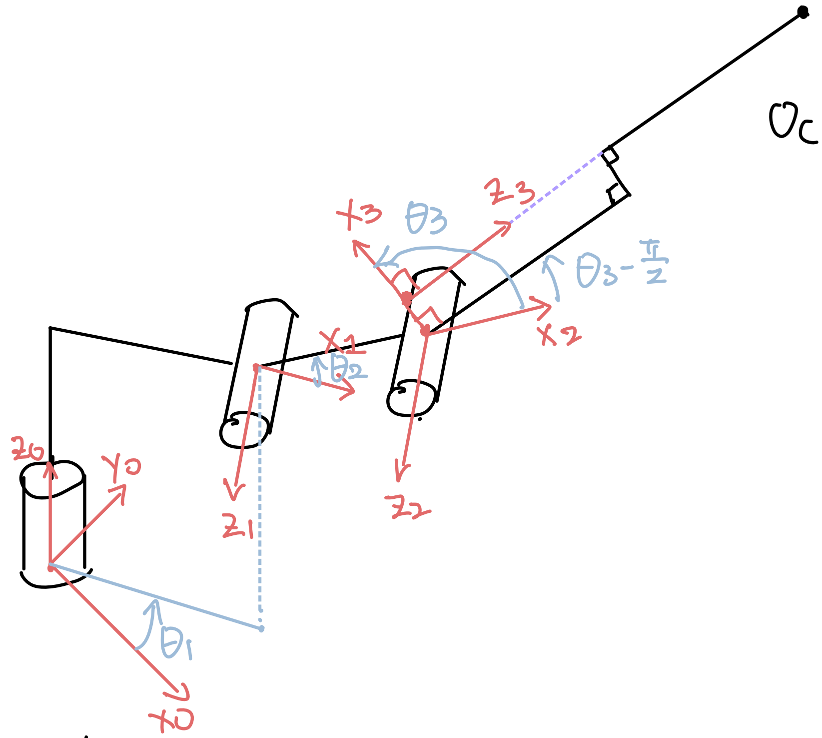






DH Table: in mm

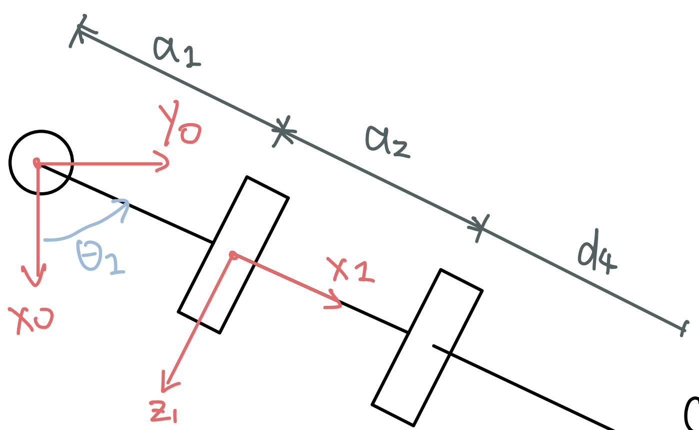
Link	a_i	α_i	d_i	θ_i
1	25	$\pi/2$	400	θ_1
2	315	0	0	θ_2
3	35	$\pi/2$	0	θ_3
4	0	$-\pi/2$	365	θ_4
5	0	$\pi/2$	0	θ_5
6	-296.23	0	161.44	θ_6



$$O_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = O_d^0 - R_d \begin{bmatrix} -d_6 \\ 0 \\ d_6 \end{bmatrix}$$

Find θ_1:

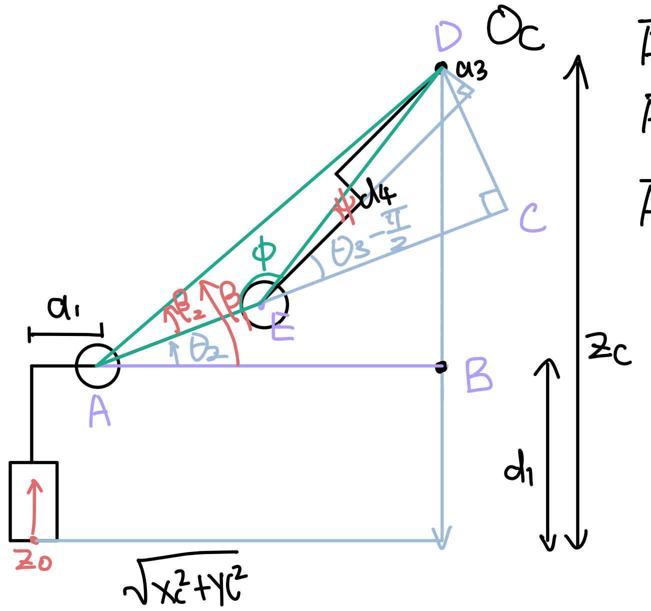
Top view:



$$O_c = \begin{bmatrix} x_c \\ y_c \end{bmatrix}$$

$$\boxed{\theta_1 = \operatorname{atan2}(y_c, x_c)}$$

Find θ_2, θ_3



$\overline{AE} = a_2 \qquad \overline{DE} = \sqrt{a_3^2 + d_4^2}$

$\overline{AB} = \sqrt{x_c^2 + y_c^2} - a_1 \qquad \overline{DB} = z_c - d_1$

$$\overline{AD}^2 = \overline{AB}^2 + \overline{DB}^2 = \left(\sqrt{x_c^2 + y_c^2} - a_1\right)^2 + \left(z_c - d_1\right)^2$$

$$\overline{AD}^2 = \overline{AE}^2 + \overline{DE}^2 - 2\,\overline{AE}\,\overline{DE}\cos\phi$$

$$\left(\sqrt{x_c^2+y_c^2}-a_1\right)^2+\left(z_c-d_1\right)^2 = a_2^2+a_3^2+d_4^2-2a_2\sqrt{a_3^2+d_4^2}\cos\phi$$

$$\boxed{\cos\phi = \frac{a_2^2+a_3^2+d_4^2-\left(\sqrt{x_c^2+y_c^2}-a_1\right)^2+\left(z_c-d_1\right)^2}{2a_2\sqrt{a_3^2+d_4^2}}}$$

$$\phi = \operatorname{atan2}\left(\sqrt{1-\cos\phi^2},\ \cos\phi^2\right)$$

$$\psi = \operatorname{atan2}(a_3,\ d_4)$$

$$\theta_3 - \frac{\pi}{2} = \pi - \phi - \psi$$

$$\boxed{\theta_3 = \frac{3}{2}\pi - \phi - \psi = \frac{3}{2}\pi - \operatorname{atan2}\left(\sqrt{1-\cos\phi^2},\ \cos\phi^2\right) - \operatorname{atan2}(a_3, d_4)}$$

$$\overline{CE} = \overline{DE}\cos\left(\theta_3-\frac{\pi}{2}+\psi\right) = \sqrt{a_3^2+d_4^2}\cos\left(\theta_3-\frac{\pi}{2}+\psi\right)$$

$$\overline{CD} = \overline{DE}\sin\left(\theta_3-\frac{\pi}{2}+\psi\right) = \sqrt{a_3^2+d_4^2}\cos\left(\theta_3-\frac{\pi}{2}+\psi\right)$$

$$\overline{AC} = \overline{AE}+\overline{CE} = a_2+\sqrt{a_3^2+d_4^2}\cos\left(\theta_3-\frac{\pi}{2}+\psi\right)$$

$$\beta_2 = \operatorname{atan2}\left(\overline{CD},\ \overline{AC}\right)$$

$$\beta_2 = \operatorname{atan2}\left(\sqrt{a_3^2+d_4^2}\sin\left(\theta_3-\frac{\pi}{2}+\psi\right),\ a_2+\sqrt{a_3^2+d_4^2}\cos\left(\theta_3-\frac{\pi}{2}+\psi\right)\right)$$

$$\beta_1 = \operatorname{atan2}\left(\overline{AB},\ \overline{DB}\right) = \operatorname{atan2}\left(z_c-d_1,\ \sqrt{x_c^2+y_c^2}-a_1\right)$$

$$\theta_2 = \beta_1 - \beta_2$$

$$\boxed{\begin{aligned}\theta_2 = {} & \operatorname{atan2}\left(z_c-d_1,\ \sqrt{x_c^2+y_c^2}-a_1\right) \\ & - \operatorname{atan2}\left(\sqrt{a_3^2+d_4^2}\sin\left(\theta_3-\frac{\pi}{2}+\psi\right),\ a_2+\sqrt{a_3^2+d_4^2}\cos\left(\theta_3-\frac{\pi}{2}+\psi\right)\right)\end{aligned}}$$

Let $(R_3^0)^T R_d = M$,

$$\theta_4 = \operatorname{atan2}(m_{23},\ m_{13})$$
$$\theta_5 = \operatorname{atan2}\left(\sqrt{1-m_{33}^2},\ m_{33}\right)$$
$$\theta_6 = \operatorname{atan2}(m_{32},\ -m_{31})$$

Elbow down

OR

$$\theta_4 = \operatorname{atan2}(-m_{23},\ -m_{13})$$
$$\theta_5 = \operatorname{atan2}\left(-\sqrt{1-m_{33}^2},\ m_{33}\right)$$
$$\theta_6 = \operatorname{atan2}(-m_{32},\ +m_{31})$$

Elbow up