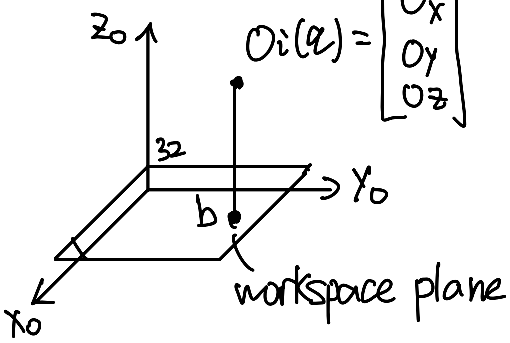
2.



$$O_i(q)=\begin{bmatrix}O_x\\O_y\\O_z\end{bmatrix}$$

$$\rho(O_i(q)) = \|O_i(q)-b\| = O_z - 32$$

$$O_i(q)-b = O_i(q)-\begin{bmatrix}O_x\\O_y\\32\end{bmatrix} = \begin{bmatrix}0\\0\\O_z-32\end{bmatrix}$$

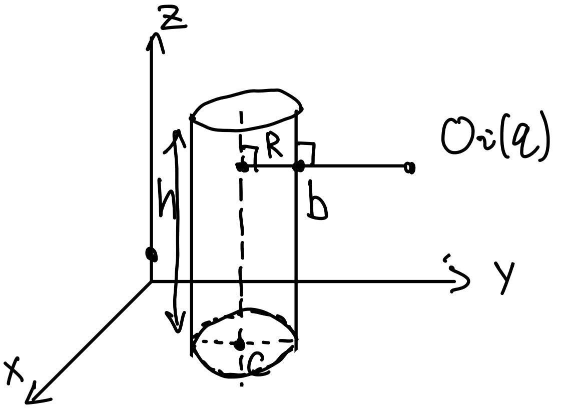
$$\nabla\rho(O_i(q)) = \frac{O_i(q)-b}{\|O_i(q)-b\|} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$F_{rep,i}(q) = \eta_i\left(\frac{1}{\rho(O_i(q))}-\frac{1}{\rho_0}\right)\frac{1}{\rho^2(O_i(q))}\nabla\rho(O_i(q))$$

$$= \eta_i\left(\frac{1}{O_z-32}-\frac{1}{\rho_0}\right)\frac{1}{(O_z-32)^2}\begin{bmatrix}0\\0\\1\end{bmatrix}, \quad \text{for } O_z \geq 32\text{ mm}$$

$$F_{rep,i}(q) = \eta_i\left(\frac{1}{32-O_z}-\frac{1}{\rho_0}\right)\frac{1}{(32-O_z)^2}\begin{bmatrix}0\\0\\-1\end{bmatrix}, \quad \text{for } O_z < 32\text{ mm}$$

Let R be the Radius of the cylinder, c be the center of the bottom of the cylinder



$$C=\begin{bmatrix}C_x\\C_y\\32\end{bmatrix}, \quad O_i(q)=\begin{bmatrix}O_x\\O_y\\O_z\end{bmatrix}$$

For $\sqrt{(O_x-C_x)^2+(O_y-C_y)^2} > R$, $0 < O_z \leq h$

$$\rho(O_i(q)) = \|O_i(q)-b\| = \sqrt{(O_x-C_x)^2+(O_y-C_y)^2} - R$$

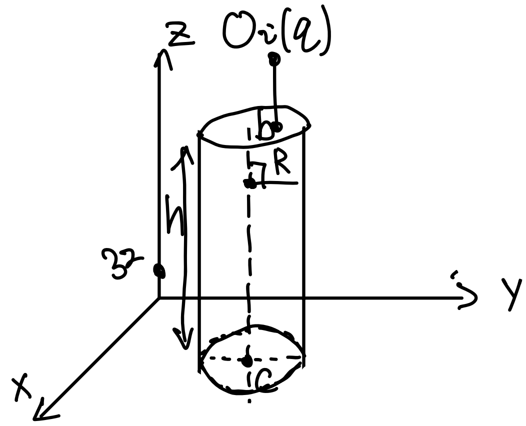
$$O_i(q)-b = \frac{O_i(q)-[C_x\ C_y\ O_z]^T}{\|O_i(q)-[C_x\ C_y\ O_z]^T\|}\cdot\left(\sqrt{(O_x-C_x)^2+(O_y-C_y)^2}-R\right)$$

$$\nabla\rho(O_i(q)) = \frac{O_i(q)-b}{\|O_i(q)-b\|} = \frac{O_i(q)-[C_x\ C_y\ O_z]^T}{\|O_i(q)-[C_x\ C_y\ O_z]^T\|}$$

$$F_{rep,i}(q) = \eta_i\left(\frac{1}{\rho(O_i(q))}-\frac{1}{\rho_0}\right)\frac{1}{\rho^2(O_i(q))}\nabla\rho(O_i(q))$$

$$= \eta_i\left(\frac{1}{\sqrt{(O_x-C_x)^2+(O_y-C_y)^2}}-\frac{1}{\rho_0}\right)\frac{1}{(O_x-C_x)^2+(O_y-C_y)^2}\,\frac{O_i(q)-[C_x\ C_y\ O_z]^T}{\|O_i(q)-[C_x\ C_y\ O_z]^T\|}$$

For $\sqrt{(O_x-C_x)^2+(O_y-C_y)^2} < R$ and $O_z > h$



$$\rho(O_i(q)) = \|O_i(q) - b\| = O_z - (h+32) = O_z - h - 32$$

$$O_i(q) - b = \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix} - \begin{bmatrix} O_x \\ O_y \\ h \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ O_z - h \end{bmatrix}$$

$$\nabla\rho(O_i(q)) = \frac{O_i(q) - b}{\|O_i(q) - b\|} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$F_{rep,i}(q) = \eta_i\left(\frac{1}{\rho(O_i(q))} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2(O_i(q))}\nabla\rho(O_i(q))$$

$$= \eta_i\left(\frac{1}{O_z - h} - \frac{1}{\rho_0}\right)\frac{1}{(O_z - h)^2}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

For $\sqrt{(O_x-C_x)^2+(O_y-C_y)^2} < R$ and $O_z < 0$

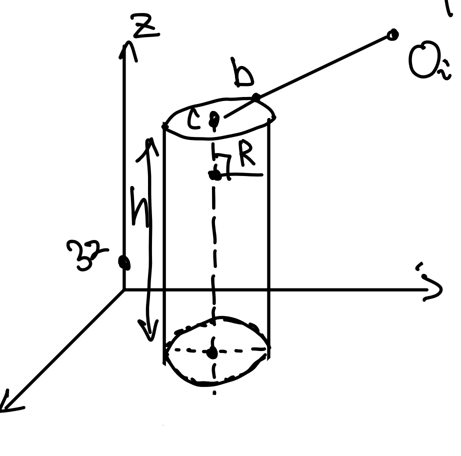
$$\rho(O_i(q)) = \|O_i(q) - b\| = -O_z$$

$$O_i(q) - b = \begin{bmatrix} O_x \\ O_y \\ O_z \end{bmatrix} - \begin{bmatrix} O_x \\ O_y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ O_z \end{bmatrix}$$

$$\nabla\rho(O_i(q)) = \frac{O_i(q) - b}{\|O_i(q) - b\|} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$F_{rep,i}(q) = \eta_i\left(\frac{1}{\rho(O_i(q))} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2(O_i(q))}\nabla\rho(O_i(q))$$

$$= \eta_i\left(\frac{1}{-O_z} - \frac{1}{\rho_0}\right)\frac{1}{(-O_z)^2}\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$



For $\sqrt{(O_x-C_x)^2+(O_y-C_y)^2} > R$, but $O_z > h$ or $O_z < 0$

$$O_i - C = \begin{bmatrix} O_x - C_x \\ O_y - C_y \\ O_z - h \end{bmatrix} \qquad C = \begin{bmatrix} C_x \\ C_y \\ h \end{bmatrix}$$

$$b = C + \begin{bmatrix} O_x - C_x \\ O_y - C_y \\ 0 \end{bmatrix} \cdot \frac{1}{\sqrt{(O_x-C_x)^2+(O_y-C_y)^2}} \cdot R$$

$$\rho(O_i(q)) = \|O_i(q) - b\| \qquad \nabla\rho(O_i(q)) = \frac{O_i(q) - b}{\|O_i(q) - b\|}$$

$$F_{rep,i}(q) = \eta_i\left(\frac{1}{\rho(O_i(q))} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2(O_i(q))}\nabla\rho(O_i(q))$$