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Module 3 Quiz

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$X \sim Bin(2, 0.9)$

~~$P(1) = 0.9^1$~~

$P(2) = 0.9^2$

1. **Prompt 1:** A critical system has 5 different components, each of which works with probability 0.90. Assume each component works independently of the others. Let X be the number of components that are working. **2 / 2 points**

   Find $P(X = 2)$. Round your answer to four decimal places.

   0.0081

   ~~$0.9 \times 0.9 = 0.081$~~

   Correct

   dbinom(2, size = 5, prob = 0.9) = 0.081

2. **Prompt 1:** A critical system has 5 different components, each of which works with probability 0.90. Assume each component works independently of the others. Let X be the number of components that are working. **1 / 1 point**

   Find $E(X)$.

   4.5

   Correct

   $E(X) = np = 5 \times 0.9 = 4.5$

   ~~$E(X) = 1 \times 0.9^4 + 2 \times 0.9^6 + 3 \times 0.9^8 + 4 \times 0.9^8 + 5 \times 0.9^8 = 15 \times 0.9 =$~~ 5

3. **Prompt 1:** A critical system has 5 different components, each of which works with probability 0.90. Assume each component works independently of the others. Let X be the number of components that are working. **1 / 1 point**

   Find $V(X)$. Round your answer to two decimal places.

   0.45

   Correct

   $V(X) = np(1-p) = 5 \times 0.9(1 - 0.9) = 0.45$

4. **Prompt 2:** A certain type of item produced by a factory has a 6% chance of being defective. Draw a random sample until you get the first defective. Let X be the number of items that are drawn. **1 / 1 point**

   Find $P(X = 2)$. Round your answer to four decimal places.

   0.0564

   Correct

   $(1 - 0.06)^{2-1} \cdot 0.06 = 0.0564$

5. **Prompt 2:** A certain type of item produced by a factory has a 6% chance of being defective. Draw a random sample until you get the first defective. Let X be the number of items that are drawn. **1 / 1 point**

   Find $E(X)$. Round answer to two decimal places.

   16.67

   Correct

   $E(X) = \frac{1}{p} = \frac{1}{0.06} = 16.67$

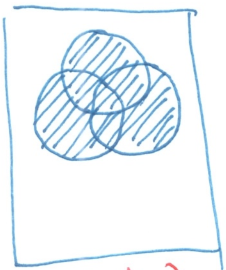
6. **Prompt 2:** A certain type of item produced by a factory has a 6% chance of being defective. Draw a random sample until you get the first defective. Let X be the number of items that are drawn. **1 / 1 point**

   Find $V(X)$. Round answer to two decimal places.

   261.11

   Correct

   $V(X) = \frac{1-p}{p^2} = \frac{1-0.06}{0.06^2} = 261.11$



7. **Prompt 3:** A certain system can experience three different types of defects. Let $A_i, i = 1, 2, 3$ be the event that the system has a defect of type i. Suppose that $P(A_1) = 0.17, P(A_2) = 0.07, P(A_3) = 0.13, P(A_1 \cup A_2) = 0.18, P(A_2 \cup A_3) = 0.18, P(A_1 \cup A_3) = 0.19$, and $P(A_1 \cap A_2 \cap A_3) = 0.01$. Let the random variable X be the number of defects that are present. **1 / 1 point**

   Calculate $P(X = 0)$. No defects = 1 − P(at least 1 defect)

   0.81

   Correct

   $$\boxed{\begin{aligned} P = (A_1 \cup A_2 \cup A_3) &= P(A_1) + P(A_2) + P(A_3) \\ &- P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3) \end{aligned}}$$

   $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$

8. **Prompt 3:** A certain system can experience three different types of defects. Let $A_i, i = 1, 2, 3$ be the event that the system has a defect of type i. Suppose that $P(A_1) = 0.17, P(A_2) = 0.07, P(A_3) = 0.13, P(A_1 \cup A_2) = 0.18, P(A_2 \cup A_3) = 0.18, P(A_1 \cup A_3) = 0.19$, and $P(A_1 \cap A_2 \cap A_3) = 0.01$. Let the random variable X be the number of defects that are present. **1 / 1 point**

   ~~$0.17 + 0.07 + 0.13$~~

   ~~$-0.18 - 0.18 - 0.19$~~

   ~~$+0.01$~~

   Calculate $P(X = 1)$. $= P(\text{only 1 defect})$

   0.03

   ~~$P(A_1$~~

   $P(X=1) = P(A_1 \cap A_2' \cap A_3') + P(A_1' \cap A_2 \cap A_3') + P(A_1' + A_2' + A_3)$

0.02

Correct

9. **Prompt 3:** A certain system can experience three different types of defects. Let A_i, i = 1, 2, 3 be the event that the system has a defect of type i. Suppose that $P(A_1) = .17, P(A_2) = 0.07, P(A_3) = 0.13, P(A_1 \cup A_2) = 0.18, P(A_2 \cup A_3) = 0.18, P(A_1 \cup A_3) = 0.19$, and $P(A_1 \cap A_2 \cap A_3) = .01$. Let the random variable X be the number of defects that are present.

1 / 1 point

Calculate $P(X = 2)$

$P(X=2) = P(A_1 \cap A_2 \cap A_3') + P(A_1 \cap A_2' \cap A_3) + P(A_1' \cap A_2 \cap A_3)$

0.16

Correct

10. **Prompt 3:** A certain system can experience three different types of defects. Let $A_i, i = 1, 2, 3$ be the event that the system has a defect of type i. Suppose that $P(A_1) = 0.17, P(A_2) = 0.07, P(A_3) = 0.13, P(A_1 \cup A_2) = 0.18, P(A_2 \cup A_3) = 0.18, P(A_1 \cup A_3) = 0.19$, and $P(A_1 \cap A_2 \cap A_3) = 0.01$. Let the random variable X be the number of defects that are present.

1 / 1 point

Calculate $P(X = 3)$ $= P(3 \text{ defects}) = P(A_1 \cap A_2 \cap A_3)$

$= 0.01$

0.01

Correct

11. **Prompt 3:** A certain system can experience three different types of defects. Let A_i, i = 1, 2, 3 be the event that the system has a defect of type i. Suppose that $P(A_1) = .17, P(A_2) = 0.07, P(A_3) = 0.13, P(A_1 \cup A_2) = 0.18, P(A_2 \cup A_3) = 0.18, P(A_1 \cup A_3) = 0.19$, and $P(A_1 \cap A_2 \cap A_3) = .01$. Let the random variable X be the number of defects that are present.

1 / 1 point

Find $E(X)$

0.37

Correct

12. **Prompt 3:** A certain system can experience three different types of defects. Let A_i, i = 1, 2, 3 be the event that the system has a defect of type i. Suppose that $P(A_1) = .17, P(A_2) = 0.07, P(A_3) = 0.13, P(A_1 \cup A_2) = 0.18, P(A_2 \cup A_3) = 0.18, P(A_1 \cup A_3) = 0.19$, and $P(A_1 \cap A_2 \cap A_3) = .01$. Let the random variable X be the number of defects that are present.

1 / 1 point

Find $V(X)$. Give four decimal places for your answer.

0.6131

Correct

13. **Prompt 3:** A certain system can experience three different types of defects. Let A_i, i = 1, 2, 3 be the event that the system has a defect of type i. Suppose that $P(A_1) = .17, P(A_2) = 0.07, P(A_3) = 0.13, P(A_1 \cup A_2) = 0.18, P(A_2 \cup A_3) = 0.18, P(A_1 \cup A_3) = 0.19$, and $P(A_1 \cap A_2 \cap A_3) = .01$. Let the random variable X be the number of defects that are present.

1 / 1 point

Find σ, the standard deviation of X. Round your answer to three decimal places.

0.783

Correct

solution: (7) A certain system can experience three different types of defects. Let $A_i; i=1,2,3$ be the event that the system has a defect type i.

→ We have, $P(A1) = 0.17$, $P(A2) = 0.07$, $P(A_3) = 0.13$, $P(A1 \cup A2) = 0.18$, $P(A1 \cup A3) = 0.19$, $P(A2 \cup A3) = 0.18$, $P(A1 \cap A2 \cap A3) = 0.01$.

→ Let the random variable X be the number of defects that are present.

⇒ We have to calculate $P(X=0)$.

$P(X=0) = P(\text{No defects})$ (A_1', A_2' and A_3' are complementary events showing non-defect)

$\therefore P(X=0) = P(A_1' \cap A_2' \cap A_3')$ ← (formula: I, $P(A_1' \cap A_2' \cap A_3') = 1 - P(A_1 \cup A_2 \cup A_3)$)

$\therefore P(X=0) = 1 - P(A_1 \cup A_2 \cup A_3) \longrightarrow (1)$

formula: (i) $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

(ii) $P(A_1 \cap A_2) = P(A_1) + P(A2) - P(A_1 \cup A_2)$

(iii) $P(A_1 \cap A_3) = P(A_1) + P(A_3) - P(A_1 \cup A3)$

(iv) $P(A_2 \cap A_3) = P(A2) + P(A_3) - P(A2 \cup A_3)$

$\therefore$ (ii) ⇒ $P(A_1 \cap A_2) = 0.17 + 0.07 - 0.18 = \underline{\underline{0.06}}$

(iii) ⇒ $P(A_1 \cap A_3) = 0.17 + 0.13 - 0.19 = \underline{\underline{0.11}}$

(iv) ⇒ $P(A_2 \cap A_3) = 0.07 + 0.13 - 0.18 = \underline{\underline{0.02}}$

$P(A_1) = 0.17 \qquad P(A_2) = 0.07 \qquad P(A_3) = 0.13$

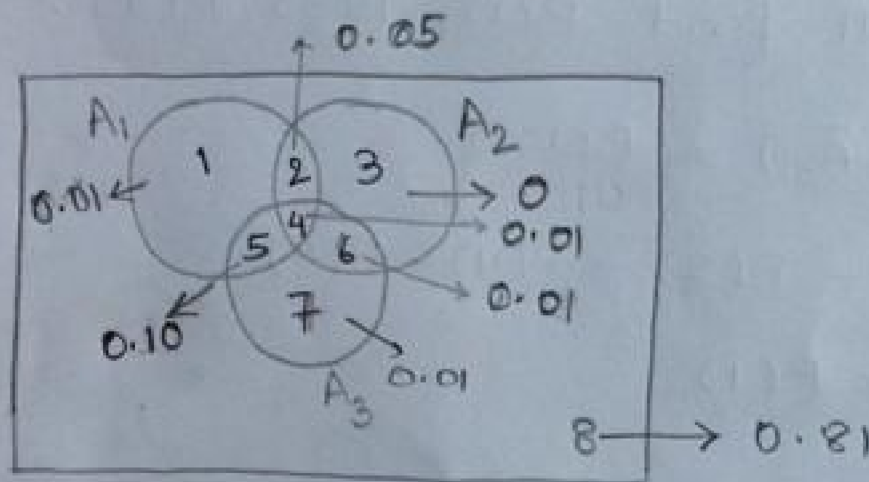
$P(A_1 \cup A_2) = 0.18 \qquad P(A_2 \cup A_3) = 0.18 \qquad P(A_1 \cap A_3) = 0.19$

$P(A_1 \cap A_2 \cap A_3) = 0.01$

X = no. of defects. So, X can take values 0, 1, 2, 3.

$P(X = n)$ is the probability that X has n defects.
$n = 0, 1, 2, 3$.

(#) Let's draw the Venn diagram :-



$P(4) = P(A_1 \cap A_2 \cap A_3) = 0.01$

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$\Rightarrow 0.18 = 0.17 + 0.07 - P(A_1 \cap A_2)$

$\Rightarrow P(A_1 \cap A_2) = 0.06$

$P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = P(A_1 \cap A_2 \cap \bar{A_3})$

$\Rightarrow P(A_1 \cap A_2 \cap \bar{A_3}) = 0.06 - 0.01$

$\Rightarrow P(A_1 \cap A_2 \cap \bar{A_3}) = 0.05 = P(2)$

In a similar way, we have to find $P(5), P(6)$.

$P(A_2 \cup A_3) = P(A_2) + P(A_3) - P(A_2 \cap A_3)$

$\Rightarrow P(A_2 \cap A_3) = 0.02$

$\Rightarrow P(A_2 \cap A_3 \cap \bar{A_1}) = P(A_2 \cap A_3) - P(A_1 \cap A_2 \cap A_3)$
$= 0.01 = P(6)$

$$P(A_1 \cap A_3) = P(A_1) + P(A_3) - P(A_1 \cup A_3)$$
$$= 0.11$$

$$P(A_1 \cap \bar{A}_2 \cap A_3) = P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)$$
$$= 0.10 = P(5)$$

Now, we will find $P(1), P(3), P(7)$:-

$$P(A_1 \cap \bar{A}_2 \cap \bar{A}_3) = P(1)$$
$$= P(A_1) - P(2) - P(4) - P(5)$$
$$= 0.01 = P(1)$$

$$P(\bar{A}_1 \cap A_2 \cap \bar{A}_3) = P(3)$$
$$= P(A_2) - P(2) - P(4) - P(6)$$
$$= 0$$

$$P(\bar{A}_1 \cap \bar{A}_2 \cap A_3) = P(7)$$
$$= P(A_3) - P(4) - P(5) - P(6)$$
$$= 0.01$$

$$P(8) = 1 - (P(1) + P(2) + \dots + P(7))$$
$$= 1 - 0.19$$
$$= 0.81 = P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3)$$

$X = 0$, i.e., X has 0 defects

$$P(X=0) = P(8) = P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3)$$
$$= 0.81$$

$X = 1$, i.e., X has 1 defect (exactly)

$$P(X=1) = P(1) + P(3) + P(7)$$
$$= 0.01 + 0 + 0.01$$
$$= 0.02$$

$X = 2$, i.e., X has exactly two defects

$$P(X=2) = P(2) + P(5) + P(6)$$
$$= 0.05 + 0.10 + 0.01$$
$$= 0.16$$

$X = 3$, i.e., X has exactly three defects

$$P(X=3) = P(4)$$
$$= 0.01$$

X:	0	1	2	3
$P(X=x)$:	0.81	0.02	0.16	0.01

ii) $E(X) = \sum_x x\, P(X=x)$

$$= 0 * 0.81 + 1 * 0.02 + 2 * 0.16 + 3 * 0.01$$
$$= 0.37$$

$$E(X^2) = \sum_x x^2 P(X=x)$$

$$= 0 * 0.81 + 1 * 0.02 + 4 * 0.16 + 9 * 0.01$$

$$= 0.75$$

12) $\mathrm{Var}(X) = E(X^2) - \{E(X)\}^2$

$$= 0.75 - (0.37)^2$$

$$= 0.6131$$

13) $sd(X) = \sigma = \sqrt{\mathrm{Var}(X)}$

$$= 0.783$$