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Module 4 Quiz

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1. **Prompt 1:** A skilled worked requires at least 10 minutes, and no more than 20 minutes, to complete a certain task. The completion time X is a continuous random variable with density function $f(x) = c/x^2$ for $10 \leq x \leq 20$ and $f(x) = 0$ for all other values of x. **1 / 1 point**

What is the value of c?

20

Correct

$$c\int_{10}^{20} \frac{1}{x^2}\,dx = c\int_{10}^{20} x^{-2} = c \cdot -\frac{1}{x}\Big|_{10}^{20}$$

$$= c \cdot \left(-\frac{1}{20} + \frac{1}{10}\right) = 1$$

$$c \cdot \frac{1}{20} = 1$$

$$c = 20$$

2. **Prompt 1:** A skilled worked requires at least 10 minutes, and no more than 20 minutes, to complete a certain task. The completion time X is a continuous random variable with density function $f(x) = c/x^2$ for $10 \leq x \leq 20$ and $f(x) = 0$ for all other values of x. **1 / 1 point**

Find the probability that the worker completes the task in 15 minutes or less. Round your answer to three decimal places.

0.667

Correct

$$f(x) = \frac{20}{x^2}$$

$$20\int_{0}^{15} \frac{1}{x^2}\,dx = 20 \cdot -\frac{1}{x}\Big|_{10}^{15} = 20 \cdot \left(-\frac{1}{15} + \frac{1}{10}\right) = $$

$$= 0.667$$

3. **Prompt 1:** A skilled worked requires at least 10 minutes, and no more than 20 minutes, to complete a certain task. The completion time X is a continuous random variable with density function $f(x) = c/x^2$ for $10 \leq x \leq 20$ and $f(x) = 0$ for all other values of x. **1 / 1 point**

Find the expected time expected time for the worker to complete the task. Round your answer to three decimal places.

13.863

Correct

$$E(x) = \int_{10}^{20} x \cdot f(x) \cdot dx = 20\int_{10}^{20} 1/x\,dx = 20 \cdot \log(x)\Big|_{10}^{20}$$

$$= 20\,(\log(20) - \log(10)) = 13.863$$

4. **Prompt 2:** The number of eggs laid on a tree leaf by an insect of a certain type is a Poisson random variable, X, with parameter c, where $E(X) = c$. However, such a random variable can only be observed if it is positive, since if it is 0, then we cannot know that such an insect was on the leaf. If we let Y denote the observed number of eggs, then $P(Y = i) = P(X = i | X > 0)$, where X is Poisson with parameter c. **1 / 1 point**

Find $E(Y)$.

(Note 1: your answer will be a mathematical expression with c. When you type in your answer, the preview box will display what the mathematical expression looks like.

Note 2: Coursera uses the capital letter "E" to represent the mathematical constant "e" equivalent to approximately 2.72.)

$\frac{c}{1-e^{-c}}$

c/(1-E^-c)

Correct

$$\frac{c}{1-e^{-c}}$$

5. **Prompt 3:** Suppose X is a random variable $X \sim N(12, 4)$. **1 / 1 point**

Find the probability that X is within 1.5 standard deviations of the mean. Round your answer to four decimal places.

0.8664

Correct
$P(9 \leq X \leq 15) = P(-1.5 \leq Z \leq 1.5) = \Phi(1.5) - \Phi(-1.5) \approx .9332 - .0668 = .8664$

pnorm(1.5, mean=0, sd=1) − pnorm(−1.5, mean=0, sd=1)
= .8664

6. **Prompt 3:** Suppose X is a random variable $X \sim N(12, 4)$. **1 / 1 point**

Find k such that $P(X > k) = 0.10$. Round your answer to two decimal places.

14.56

Correct
$P(X > k) = P(Z > (k-12)/2) = .1$ implies $P(Z \leq (k-12)/2) = 0.9$. Using the normal tables, this gives $(k-12)/2 = 1.282$. So, $k = 14.564$.

$$P(X > k) = 0.10$$

$$P\left(Z > \frac{k-12}{2}\right) = 0.1$$

$$P\left(Z \leq \frac{k-12}{2}\right) = 0.9$$

$$\Phi\left(\frac{k-12}{2}\right) = 0.9$$

$$\frac{k-12}{2} = \text{qnorm}(0.9, \text{mean}=0, \text{sd}=1)$$

$$k = 2\,\text{qnorm}(0.9, 0, 1) + 12 = 14.56$$

Problem-2 : Here, X is a random variable denoting the number of eggs laid on a tree leaf by an insect.

$X \sim \text{Poisson}(c)$ and $E(X)$.

$\therefore$ PMF of X, is,

$$P[X=i] = e^{-c}\frac{c^i}{i!}, \quad i=0,1,2,\ldots \quad c>0 \qquad \text{--- } (*)$$

Now, Y denotes the observed number of eggs.
and

$$P(Y=i) = P(X=i \mid X>0), \quad i=0,1,2\ldots$$

$$= \frac{P(X=i, X>0)}{P(X>0)}$$

$$= \frac{P(X=i)}{1-P(X=0)}, \quad i=1,2,\ldots$$

$$= \frac{e^{-c}\cdot\frac{c^i}{i!}}{1-e^{-c}} \quad \left[\text{from } (*),\ P[X=i]=e^{-c}\frac{c^i}{i!},\ P[X=0]=e^{-c}\right]$$

$$\therefore E(Y) = \sum_{i=1}^{\infty} i P(Y=i)$$

$$= \sum_{i=1}^{\infty} i \cdot \frac{e^{-c}\frac{c^i}{i!}}{1-e^{-c}}$$

$$= \frac{c}{1-e^{-c}}\sum_{i=1}^{\infty} e^{-c}\cdot\frac{c^{i-1}}{(i-1)!}$$

$$= \frac{ce^{-c}}{1-e^{-c}}\sum_{i=1}^{\infty}\frac{c^{i-1}}{(i-1)!} = \frac{c}{1-e^{-c}}$$