

You finished this assignment

Grade received 100%

Go to next item

Module 6 Quiz

Latest Submission Grade 100%

1. Prompt 1: Suppose 30 numbers are selected at random from the interval  $[0, 1]$ . That is,  $X_i \sim U[0, 1]$  for  $i = 1, 2, \dots, 30$ . Given  $\bar{X} = (1/30) \sum_{i=1}^{30} X_i$ , estimate  $P(0.5 \leq \bar{X} \leq 0.6)$ . (Round your answer to three decimal places.)

1/1 point

0.471

Correct

$$P\left(\frac{0.5 - \frac{1}{2}}{\sqrt{\frac{1}{360}}} \leq Z \leq \frac{0.6 - \frac{1}{2}}{\sqrt{\frac{1}{360}}}\right)$$

$$\Phi\left(\frac{0.1}{\sqrt{\frac{1}{360}}}\right) - \Phi(0) = 0.9711 - 0.5 = 0.4711$$

2. Prompt 2: A company wants to compare the efficiency of two types of fuel for a car. 5 identical cars will be driven 500 kilometers each, two with the first type of fuel and three with the second type. Let  $X_1$  and  $X_2$  be the observed fuel efficiency for the first type and  $Y_1, Y_2,$  and  $Y_3$  be the efficiency for the second type. Suppose these variables are independent and  $X_i \sim N(20, 4)$  for  $i = 1, 2$  and  $Y_j \sim N(18, 9)$  for  $j = 1, 2, 3$ . Define a new random variable:  $W = (X_1 + X_2)/2 - (Y_1 + Y_2 + Y_3)/3$ .

1/1 point

Find  $E(W)$ .

Correct

$$E(W) = \frac{1}{2}(20 + 20) - \frac{1}{3}(18 + 18 + 18) = 20 - 18 = 2$$

3. Prompt 2: A company wants to compare the efficiency of two types of fuel for a car. 5 identical cars will be driven 500 kilometers each, two with the first type of fuel and three with the second type. Let  $X_1$  and  $X_2$  be the observed fuel efficiency for the first type and  $Y_1, Y_2,$  and  $Y_3$  be the efficiency for the second type. Suppose these variables are independent and  $X_i \sim N(20, 4)$  for  $i = 1, 2$  and  $Y_j \sim N(18, 9)$  for  $j = 1, 2, 3$ . Define a new random variable:  $W = (X_1 + X_2)/2 - (Y_1 + Y_2 + Y_3)/3$ .

1/1 point

Find  $Var(W)$ .

Correct

$$Var(W) = \frac{1}{4}Var(X_1) + \frac{1}{4}Var(X_2) + \frac{1}{9}Var(Y_1) + \frac{1}{9}Var(Y_2) + \frac{1}{9}Var(Y_3)$$

$$= \frac{1}{4}(4 + 4) + \frac{1}{9}(9 + 9 + 9) = 2 + 3 = 5$$

4. Prompt 2: A company wants to compare the efficiency of two types of fuel for a car. 5 identical cars will be driven 500 kilometers each, two with the first type of fuel and three with the second type. Let  $X_1$  and  $X_2$  be the observed fuel efficiency for the first type and  $Y_1, Y_2,$  and  $Y_3$  be the efficiency for the second type. Suppose these variables are independent and  $X_i \sim N(20, 4)$  for  $i = 1, 2$  and  $Y_j \sim N(18, 9)$  for  $j = 1, 2, 3$ . Define a new random variable:  $W = (X_1 + X_2)/2 - (Y_1 + Y_2 + Y_3)/3$ .

1/1 point

Find  $P(W \geq 0)$ . Round answer to four decimal places.

0.8145

Correct

$$P(W \geq 0) = \text{round}(1 - \text{pnorm}(-2/\sqrt{5}), 0, 1), 4) = 0.8145$$

$$\bar{X} \sim N\left(\frac{1}{2}, \frac{1}{12 \times 30}\right)$$

$$E(x) = (a+b)/2$$

$$V(x) = (b-a)^2/12$$

$$E(W) = \frac{1}{2}(X_1 + X_2) - \frac{1}{3}(Y_1 + Y_2 + Y_3)$$

$$= \frac{1}{2}(E(X_1) + E(X_2)) - \frac{1}{3}(E(Y_1) + E(Y_2) + E(Y_3))$$

$$Var(W) = Var\left(\frac{1}{2}X_1 + \frac{1}{2}X_2 - \frac{1}{3}Y_1 - \frac{1}{3}Y_2 - \frac{1}{3}Y_3\right)$$

$$W \sim N(2, 5)$$

X and Y indep.