A(x) $T_A(x)$ – time complexity of algorithm A applied to x

$|x| = n$, when size of input is = to some number n.

$T_A(n) = \max(T_A(x);|x|=n)$ called worst-case time complexity

$T_{Aa}(n) = \frac{1}{K}\Sigma_{(|x|=n)}T_A(x)$ average-case time complexity

There are k inputs with size n.

$T_{Ab}(n) = \min(T_A(x);|x|=n)$ best-case time complexity

Asymptotically stable upper bound

- Some function g(n) is/=/E O(f(n))
- There exists #c > O $\exists$ n_oE…
- $g(n) \leq cf(n)$

let's say h(n) > g(n)

$g(n) = 5n^2 + 10n + 100$ $f(n) = n^2$

g(n)

Asymptotically stable lower bound

- $g(n) = \Omega(h(n))$
- $\exists \# d > O \exists n_o \ldots$
- $g(n) \geq dh(n)$

Asymptotically stable tight bound

- $g(n) = \Theta(f(n)) \Leftrightarrow g(n) = O(f(n))$ & $g(n) = \Omega(f(n))$, when g(n) equals upper and lower, its called tight bound

Pr.1.

If $g = O(f)$, then then $f = \Omega(g)$

Proof: $\exists c > O \exists n_o E\ N \forall n > n_0\ (g(n) \leq cf(n))$

$d = \frac{1}{c} = c^{-1}$ $f(n) \geq dg(n)$

Cor.1. If $g=O(f)$ & $f=O(g)$, then $g=\Theta(f)$

Pr.2.

$g=\Theta(f) \Leftrightarrow f=\Theta(g)$

Symmetry

Pr.3.

a) $g=O(f)$ & $f=O(h) \Rightarrow g=O(h)$
b) $g=\Omega(f)$ & $f=\Omega(h) \Rightarrow g=\Omega(h)$
c) $g=\Theta(f)$ & $f=\Theta(h) \Rightarrow g=\Theta(h)$

Proof of a:

$\exists c > O \exists n_o E\ N \forall n > n_0\ (g(n) \leq cf(n))$

$\exists d > O \exists m_o E\ N \forall n > m_0\ (f(n) \leq dh(n))$

$g(n) \le c(dh(n)) = (cd)h(n), (c+d)=k$

$\exists k>0\ \exists r_0 E\ N \forall n>r_0 \quad (r_0 = \max(n_0, m_0))$

Pr.4.

a) $g=O(f)$ & $h=O(f) \Rightarrow g+h=O(f)$
b) $g=\Omega(f)$ & $h=\Omega(f) \Rightarrow g+h=\Omega(f)$
c) $g=\Theta(f)$ & $h=\Theta(f) \Rightarrow g+h=\Theta(f)$

Proof of a:

$\exists c>0 \exists n_0 E\ N \forall n>n_0\ (g(n) \le cf(n))\ \ (g(n) \le cf(n))$

$\exists d>0 \exists m_0 E\ N \forall n>m_0\ (h(n) \le df(n))$

$g(n)+h(n) \le cf(n)+df(n)= (c+d)f(n), (c+d)=k$

$\exists k>0\ \exists r_0 E\ N \forall n>r_0 \quad (r_0 = \max(n_0, m_0))$

Pr.5.

a) $g=O(f) \Rightarrow \forall k>0\ (Kg = O(f))$
b) $g=\Omega(f) \Rightarrow \forall k>0\ (Kg = \Omega(f))$
c) $g=\Theta(f) \Rightarrow \forall k>0\ (Kg = \Theta(f))$

Pr.6.

a) $g=O(f)$ & $f \le h \Rightarrow g=O(h)$
b) $g=\Omega(f)$ & $l \le f \Rightarrow g=\Omega(l)$
c) $g=\Theta(f)$ & $t \le g \Rightarrow t=O(f)$
d) $g=\Omega(f)$ & $q \ge g \Rightarrow q=\Omega(f)$

Pr.7.

$g=O(f) \Rightarrow \forall k > 0\ (g=O(f+k))$