

CS 180 Homework 1

Due Friday, July 2, 2021

Problem 1 (25 pts). What is the difference between an algorithm and a program?

Problem 2 (75 pts). Men, women and yupi live on the planet Alphaomega. Their family pattern is a triple that consists of a man, a woman and a yupi. Three sets are given: M includes n men, W includes n women and Y includes n yupi. A matching is a set H of ordered triples of the form (m, w, y) with the property that each member of M , each member of W and each member of Y appears in at most one triple from H . A matching H is called *perfect* if each member of M , each member of W and each member of Y appears exactly in one triple from H .

Assume that each man ranks all women and all yupi, each woman ranks all men and all yupi, and each yupi ranks all women and all men.

Two triples (m, w, y) and (m', w', y') form an instability in a matching H if one of the following conditions is true:

- (1) m prefers w' to w and w' prefers m to m'
- (2) m prefers y' to y and y' prefers m to m'
- (3) y prefers w' to w and w' prefers y to y'

A matching H is called *stable* if it does not have instabilities.

Decide whether the following statement is true or false.

There is an algorithm that solves the Stable Matching Problem for every instance of this problem.

If it is true, design an algorithm for building a stable perfect matching. Note that when you design an algorithm, you have to prove that it solves the necessary problem

If it is false, give a counterexample.

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Aaron Lu

#1 Solution:

An algorithm can be viewed as a systemic/accurate approach to accomplish a specific task or solve a specific problem. It is written in a form of design. A program is a sequence of lines (codes, for example) to run/execute in a machine through syntax and compiler. It is written in a form of implementation.

To speak in a general scheme, algorithm lies between mathematics and computing. Though, a program doesn't solve anything, it can serve as an implementation of algorithms.

#2 Solution:

False. Counter example:

(next page)

False

Counterexample:

Men's Preference Profile:

$$m \begin{array}{|c|c|} \hline w' & w \\ \hline y' & y \\ \hline \end{array}$$

$$m' \begin{array}{|c|c|} \hline w' & w \\ \hline y' & y \\ \hline \end{array}$$

Women's Preference Profile:

$$w \begin{array}{|c|c|} \hline m & m' \\ \hline y & y' \\ \hline \end{array}$$

$$w' \begin{array}{|c|c|} \hline m & m' \\ \hline y & y' \\ \hline \end{array}$$

Yupi's Preference Profile:

$$y \begin{array}{|c|c|} \hline m & m' \\ \hline w' & w \\ \hline \end{array}$$

$$y' \begin{array}{|c|c|} \hline m & m' \\ \hline w' & w \\ \hline \end{array}$$

All possible instances:

	$m \begin{array}{ c c } \hline w' & y' \\ \hline m' & w' \\ \hline \end{array}$ (unstable)	$m \begin{array}{ c c } \hline w & y' \\ \hline m' & w' & y' \\ \hline \end{array}$ (unstable)	$m \begin{array}{ c c } \hline w' & y' \\ \hline m' & w' & y' \\ \hline \end{array}$ (unstable)	$m \begin{array}{ c c } \hline w & y' \\ \hline m' & w' & y' \\ \hline \end{array}$ (unstable)
Pair	(w', y) unstable	(m, w') unstable	(m, y') unstable	(m, w') unstable
	$w' \text{ prefers } y \text{ \& } y \text{ prefers } w'$	$m \text{ prefers } w' \text{ \& } w' \text{ prefers } m$	$m \text{ prefers } y' \text{ \& } y' \text{ prefers } m$	$m \text{ prefers } w' \text{ \& } w' \text{ prefers } m$

\therefore All instances are unstable.