CS 180 Homework 2

Due *Friday, July* 16, 2021

Problem. Arrange the following functions in the ascending order of growth rate. Explain the suggested ordering.

$$\log_2^2 n$$

$$\log_{10} 2^n$$

$$\log_2 n^3$$

$$\log_2^2 n^n$$

$$\log_2^2 n + 2n$$

$$\log_{10}(10n)$$

$$a\log_{10}^2 n + b\log_{10} n + c$$

$$\log_2^2 (n + 2n)$$

Hwk #2

$(\log_2 n)^2$

$\log_2{}^2 n \Rightarrow \log_2 \log_2 n \rightarrow O(\log n^2)$

$\log_{10} 2^n \Rightarrow n \cdot \log_{10} 2 \Rightarrow O(n)$

$\log_2 n^3 \Rightarrow 3 \log_2 n = O(3 \log n) = O(\log n)$

$\log_2{}^2 n^n \Rightarrow \log_2 \log_2 n^n$ ← greatest, $n^n \rightarrow$ exponential

$\log_2{}^2 n + 2n \Rightarrow \log_2 \log_2 n + 2n \Rightarrow O((\log n)^2) + O(n)$

$\log_{10}(10n) \Rightarrow 1 + \log_{10}(n)$ ← fastest $= O(\log n)$

$a \log_{10}{}^2 n + b \log_{10} n + c$ ←

$\log_2{}^2 (n+2n) \Rightarrow \log_2{}^2 (3n) \Rightarrow (\log_2 (3n))^2$

Ascending Order:

fastest →

0. $a \log_{10}{}^2 n + b \log_{10} n + c$ where $a=0, b=0, c \neq 0$ case #3

1. $\log_{10}(10n)$

same = ↓

2. $a \log_{10}{}^2 n + b \log_{10} n + c$ where $a \neq 0, b \neq 0, c \neq 0$ case #2

3. $\log_2 n^3$

4. $\log_2{}^2 n$

5. $\log_2{}^2 (n+2n)$

6. $a \log_{10}{}^2 n + b \log_{10} n + c$ where $a \neq 0, b \neq 0, c \neq 0$ case #1

7. $\log_{10} 2^n$

8. $\log_2{}^2 n + 2n$

9. $\log_2{}^2 n^n$

largest ↗

- Case #1:
  - $\{a>0, b>0, c>0\}$
  - $O((\log n)^2)$
  - It would be equally ranked as #5
- Case #2:
  - $\{a=0, b \neq 0, c \neq 0\}$
  - $O(\log n)$
  - It would be equally ranked as #1
- Case #3:
  - $\{a=0, b=0, c \neq 0\}$
  - $O(1)$ constant
  - It would be the fastest.