Part 3:

$[0]_\sim = \{0, 6, 12, \ldots\}$

$[1]_\sim = \{1, 7, 13, \ldots\}$

$\vdots$

$[5]_\sim = \{5, 11, 17, \ldots\}$

Now consider ~~$F_3(n) = n$~~ $F_3(n) = 2^n$

Is $\hat{F}$ single-valued?

recall: $\hat{F}$ def: $\forall t\,(t \in \hat{F} \leftrightarrow \exists x\,(x \in \omega \wedge t = \langle [x]_\sim, [2^x] \rangle))$

$\omega = \{0, 1, 2, \ldots\}$

$\sim$ def: $\forall t\,(t \in \sim \leftrightarrow \exists m, n\,(m, n \in \omega \wedge t = \langle m, n \rangle \wedge \exists k\,(k \in \mathbb{Z} \wedge m - n = 6k)))$

Show $\forall m\,(m \in \operatorname{dom} \hat{F} \rightarrow \exists_1 y\,\langle m, y \rangle \in \hat{F})$

Let m be a set

$m \in \operatorname{dom} \hat{F}$

$\rightarrow \exists y\,(\langle m, y \rangle \in \hat{F})$

$\rightarrow \langle m, y_1 \rangle \in \hat{F}$ (EI / y_1)

$\rightarrow$ let $x_2 \in \omega \wedge \langle m, y_1 \rangle = \langle [x_2]_\sim, [2^{x_2}]_\sim \rangle$

$m = [x_2]_\sim$

$y_1 = [2^{x_2}]_\sim$

Show $\forall z\,(\langle m, z \rangle \in \hat{F} \rightarrow z = y_1)$

Let z be a set

$\langle m, z \rangle \in \hat{F}$

$\rightarrow$ let $x_3 \in \omega \wedge \langle m, z \rangle = \langle [x_3]_\sim, [2^{x_3}]_\sim \rangle$

$m = [x_3]_\sim$, $z = [2^{x_3}]_\sim$

So, $[x_2]_\sim = [x_3]_\sim$

$\leftrightarrow \langle x_2, x_3 \rangle \in \sim$

$\rightarrow \exists k\,(k \in \mathbb{Z} \wedge x_2 - x_3 = 6k)$

$\rightarrow k_1 \in \mathbb{Z} \wedge x_2 - x_3 = 6k_1$ *

want $[2^{x_2}] = [2^{x_3}]$

ie. $2^{x_2} - 2^{x_3} = 6k_2$ for some k_2

Is this possible?

~~$x_2 - x_3 = 6k_1\ (k_1 \in \mathbb{Z})$~~

$2(2^{x_2 - 1} - 2^{x_3 - 1})$

Find counter example!

$[1] = [7]$

$[2^1] = [2]$

$[2^7] = [128]$

$128 - x = 6k$

$x = 128 - 6k$

$x = 128 - 6(21)$

$x = 2$ ✓

$[5] = [11]$

$[2^5] \mid [2^{11}]$

$[2] \mid [2]$

$[0] = [6]$

$[2^0] = [1]$

$[2^6] = [64] = [4]$

but $[1] \neq [4]$ ✗