Part I

$\hat{F}$-def: $\forall t(t \in \hat{F} \leftrightarrow \exists x(x \in \omega \wedge t = \langle [x]_\sim, [F(x)]_\sim \rangle))$

$\omega = \{0, 1, 2, \ldots\}$

$\sim$-def: $\forall t(t \in \sim \leftrightarrow \exists m,n(m,n \in \omega \wedge t = \langle m,n \rangle \wedge \exists k(k \in \mathbb{Z} \wedge m - n = 6k))$

want: $x_2^2 - x_3^2 = 6k_2$

consider: $F_2(n) = n^2$

Is F_2 single-valued?

show $\forall m(m \in \mathrm{dom}\, F \rightarrow \exists_1 y(\langle m,y \rangle \in F))$

let m be a set

$m \in \mathrm{dom}\, F$

$\rightarrow \exists y(\langle m,y \rangle \in \hat{F})$

$\rightarrow \langle m, y_1 \rangle \in \hat{F}$ (EI)

$\rightarrow$ let $x_2 \in \omega \wedge \langle m, y_1 \rangle = \langle [x_2]_\sim, [x_2^2]_\sim \rangle$

$m = [x_2]_\sim$

$y_1 = [x_2^2]_\sim$

show $\forall z(\langle m,z \rangle \in \hat{F} \rightarrow z = y_1)$

let z be set $\wedge \langle m,z \rangle \in \hat{F}$

let $x_3 \in \omega \wedge \langle m,z \rangle = \langle [x_3]_\sim, [x_3^2]_\sim \rangle$

$m = [x_3]_\sim$

$z = [x_3^2]_\sim$

$[x_2] = [x_3]$

$\langle x_2, x_3 \rangle \in \sim$

$x_2 - x_3 = 6k_1$