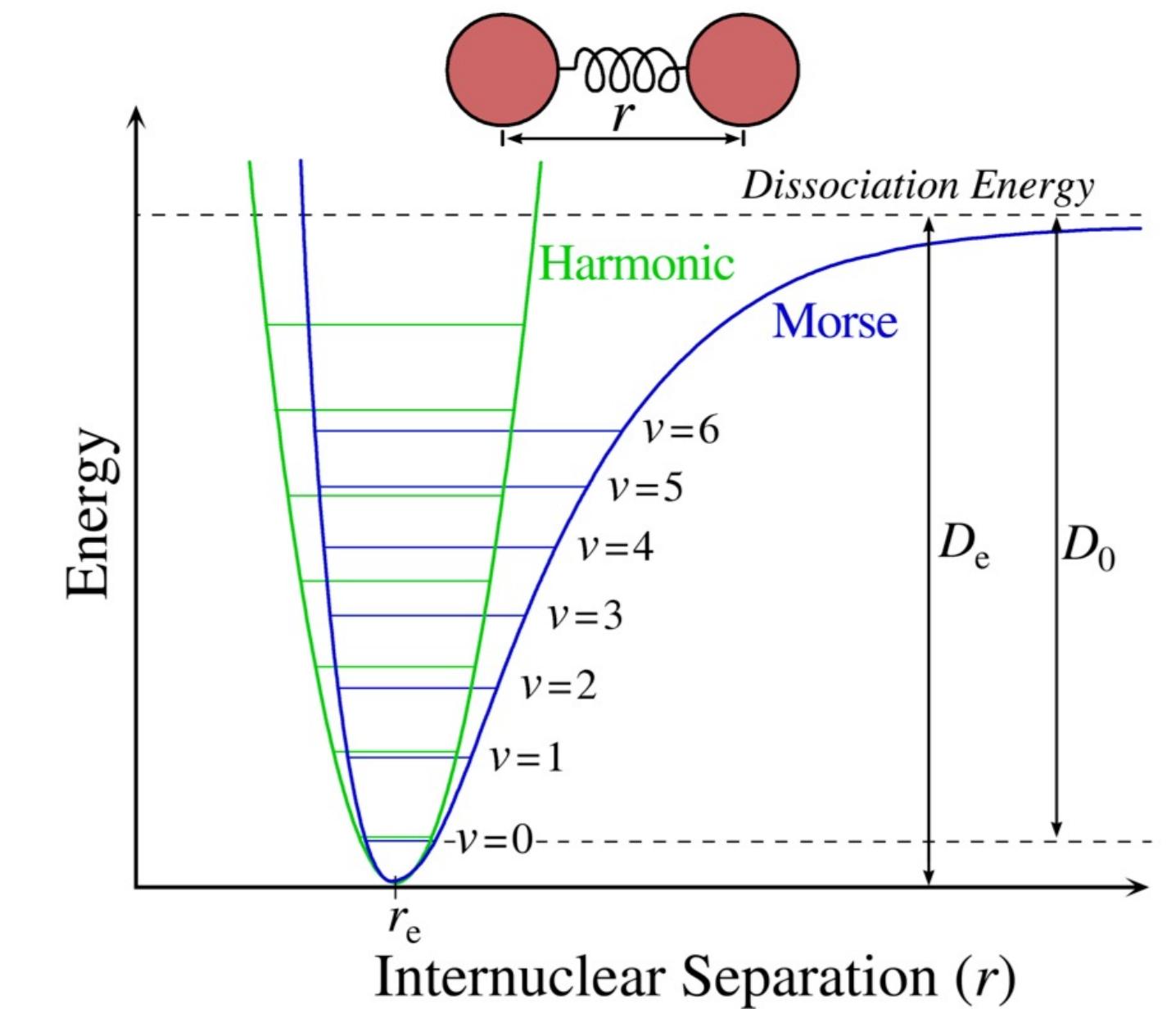


VIBRATIONAL STATE-TO-STATE THERMOCHEMICAL MODELING OF HIGH-TEMPERATURE OXYGEN FLOWS

Aaron Larsen

Kyle Hanquist



ARIZONA RESEARCH
CENTER FOR
HYPERSONICS

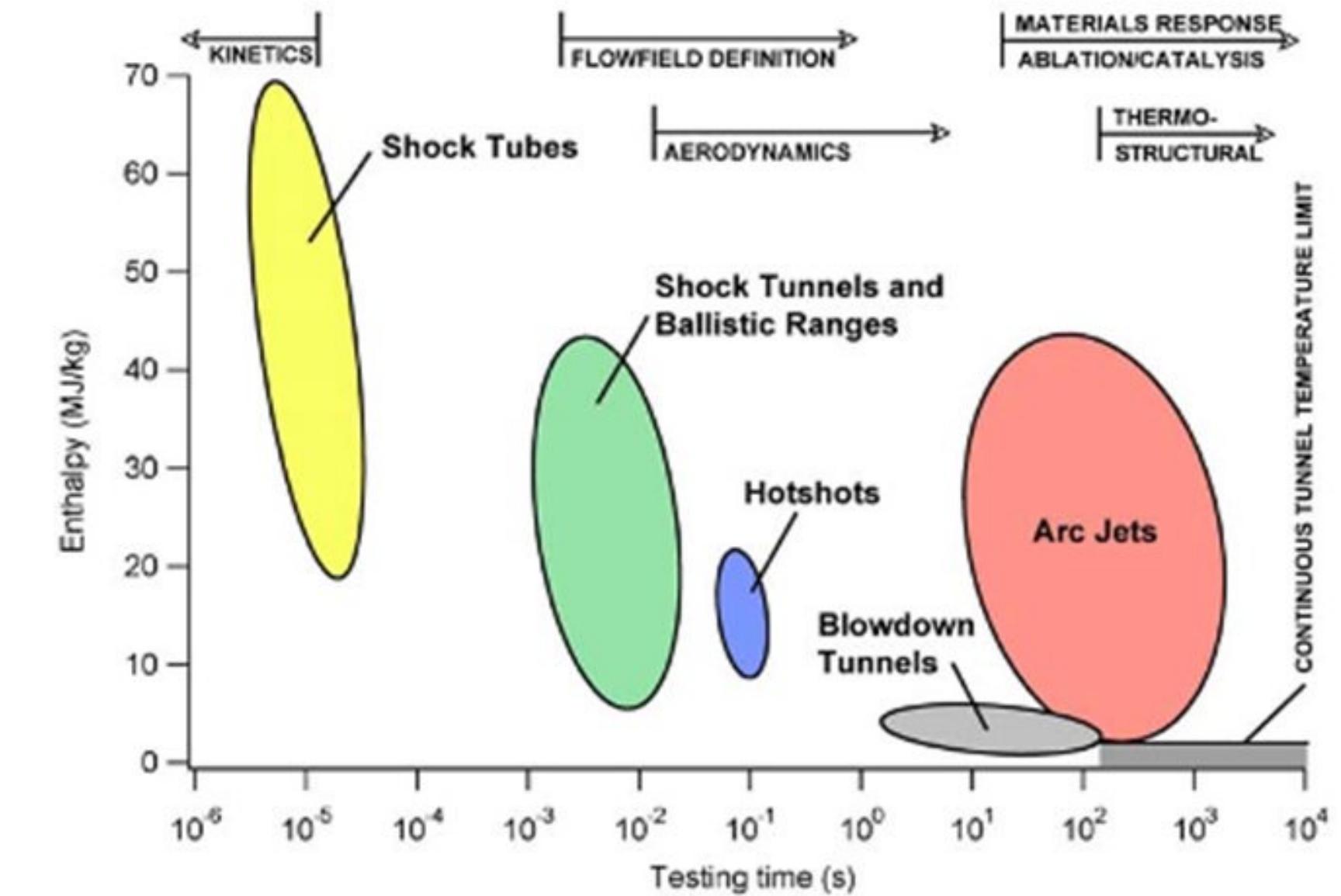
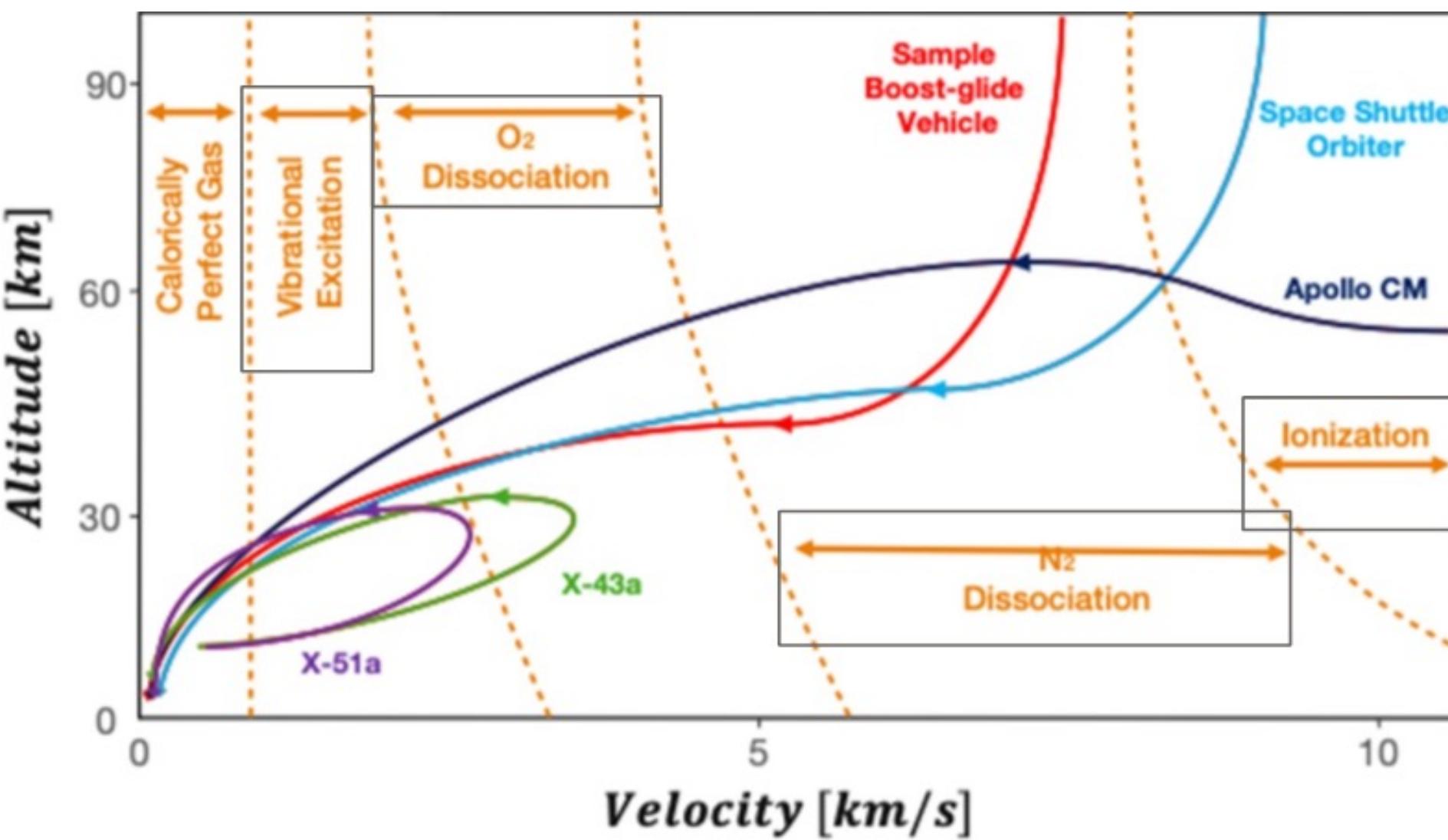


Outline

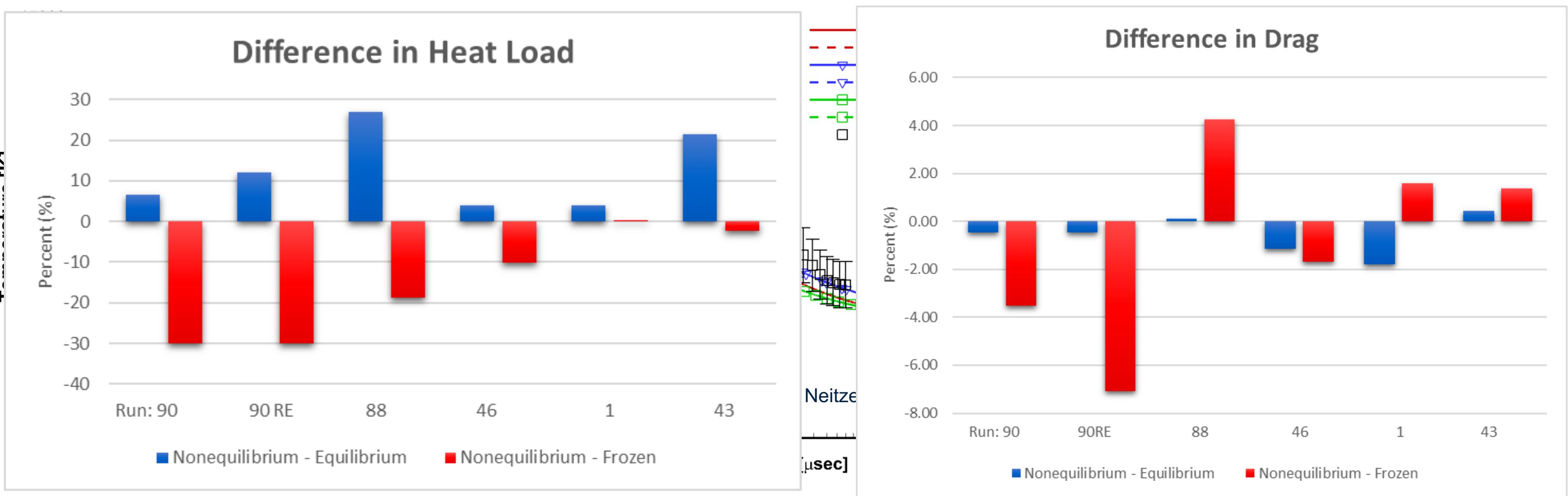
- Motivation
- Background
- Approaches
 - Mapping approaches
 - State-to-State / Mutation++
 - Multi-Temperature
- Results
 - Mapping
 - Contours
 - State-to-State simulations of oxygen and oxygen diluted in argon
- Conclusions and Future Work



Motivation



Background - Nonequilibrium Flows

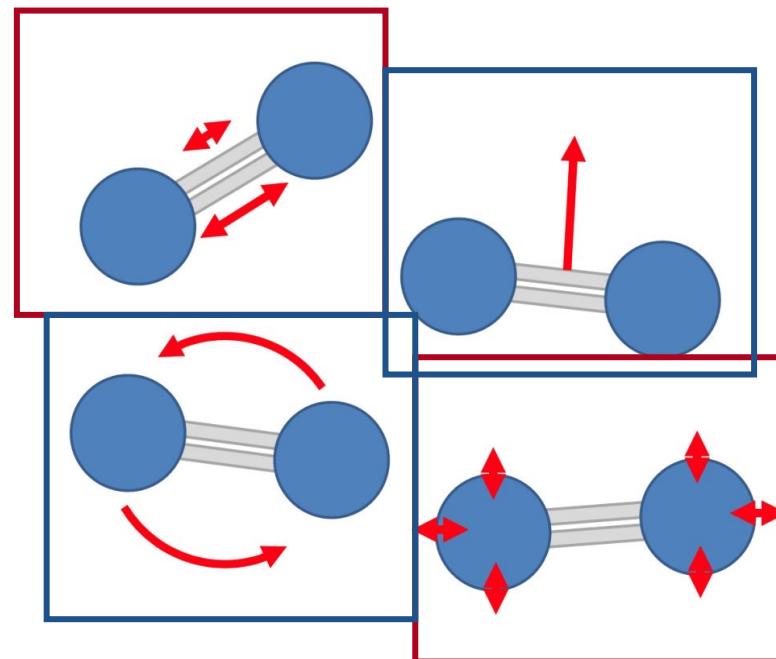


Neitzel, Andrienko, and Boyd, "Aerothermochemical Nonequilibrium Modeling for Oxygen Flows," JTHT, 2017.

Holloway, Hanquist, and Boyd, "Assessment of Thermochemistry Modeling for Hypersonic Flow over a Double Cone," JTHT, 2020.



Background - Nonequilibrium Flows – Multi-Temperature



Two-temperature (2T)

Energy Transfer

$$T_{tr} = T_{rot} \leftrightarrow T_{vib} = T_{ee}$$

$$\frac{\partial(\rho e_{vib})}{\partial x} = \rho_{O_2} \frac{e_{vib}^* - e_{vib}}{\tau_{vib}}$$

Millikan-White (MW)

$$p\tau_{vib} = \exp(A(T^{-1/3} - B) - 18.42) \quad [atm - sec]$$

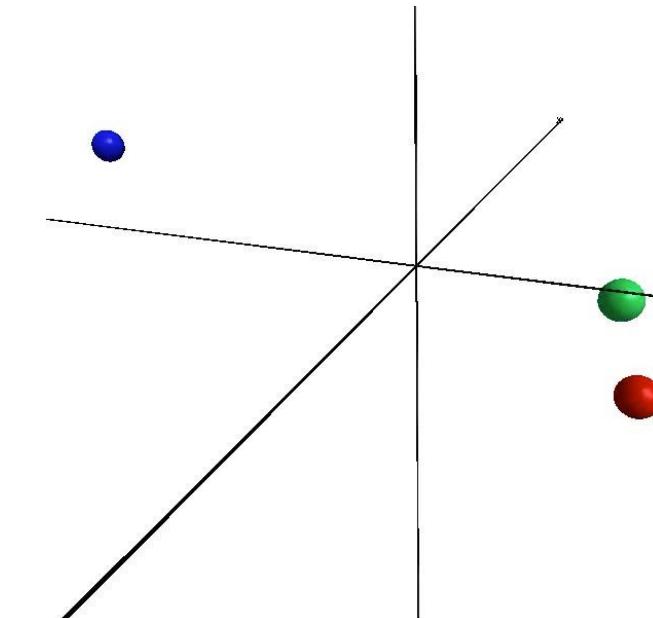
Park's High Temperature Correction (HTC)

$$p\tau_{Park} = \frac{1}{n\sigma_{vib}c} \quad [atm - sec]$$

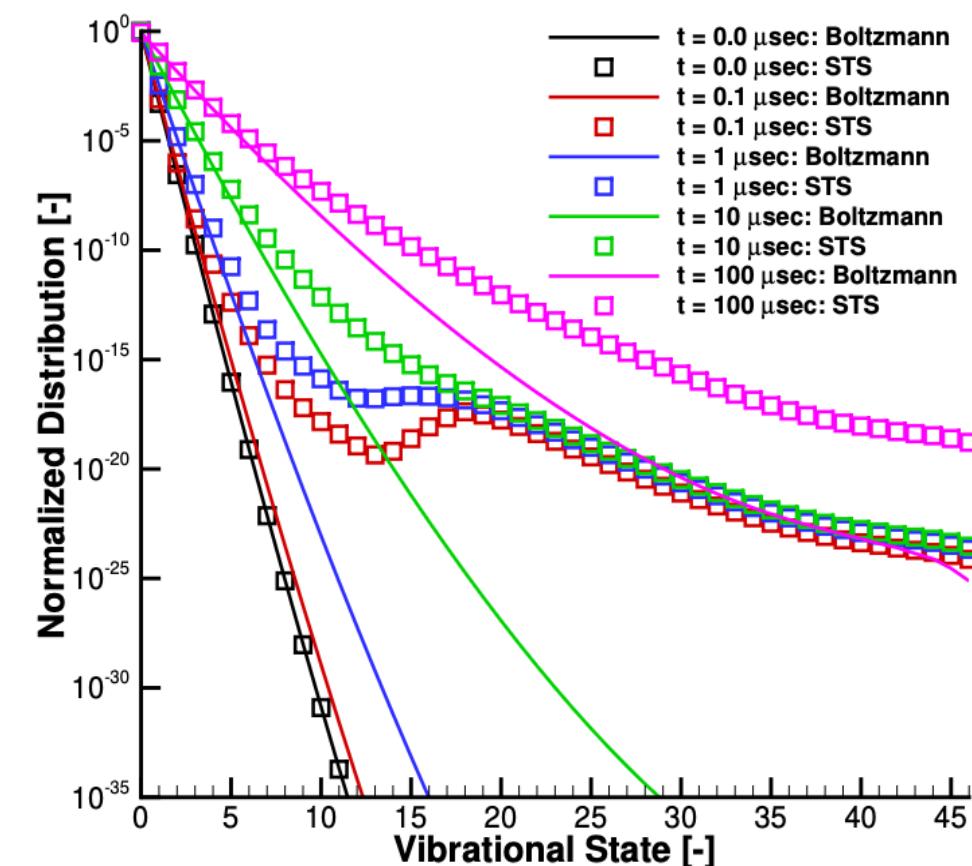
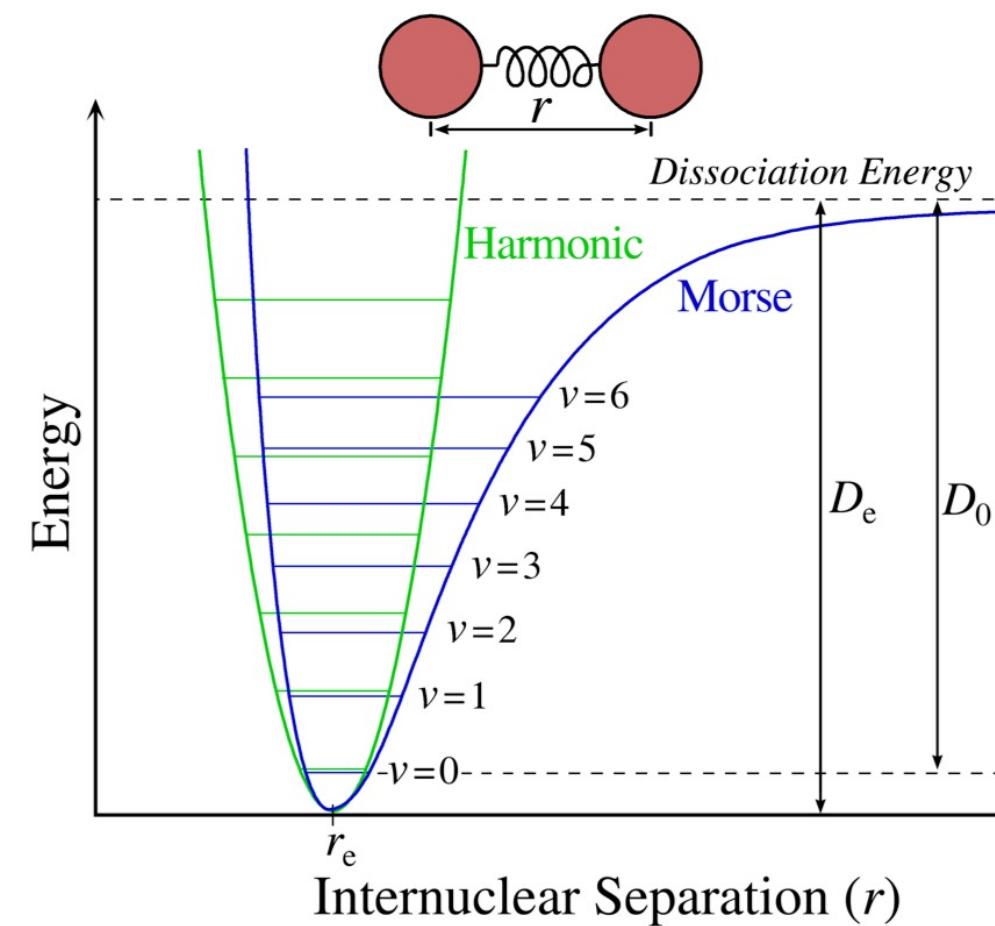
Chemistry



$$k_f = A \cdot T^\alpha \exp(-E/T)$$



Background – Nonequilibrium Flows – State Resolved

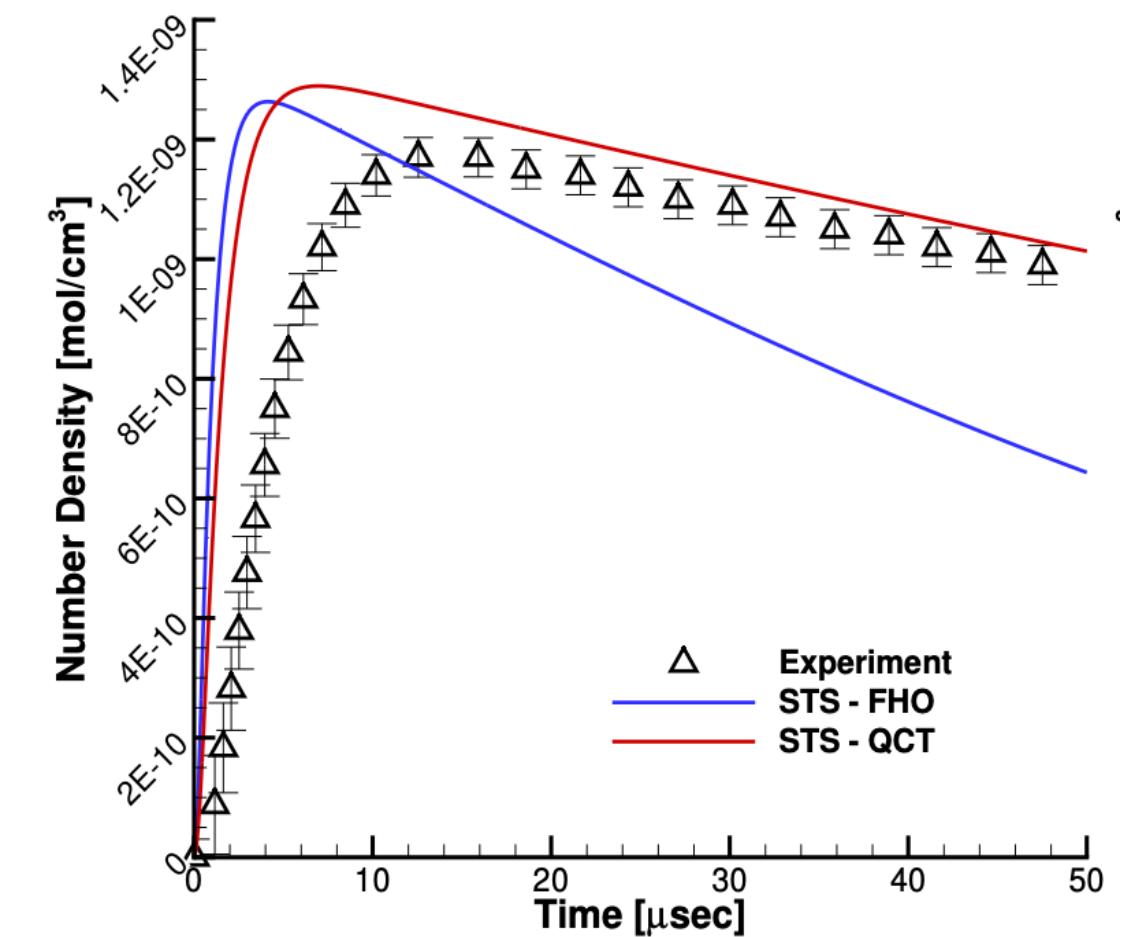


Non-Boltzmann distribution in
Mach 5 flow

Bound-Bound Transitions (rates)
Bound-Free Transitions (rates)

Analytical theory (e.g., Forced Harmonic Oscillator (FHO))
Quantum chemistry (e.g., Quasi-classical Trajectory (QCT))

$$\frac{dn_v}{dx} = \sum_s (R_{v,s} n_O^2 n_s - D_{v,s} n_v n_s) + \sum_s (K_{v',v} n'_v n_s - K_{v,v'} n_v n_s), \quad v = 0, \dots, v_{max}$$



4th vibrational state of O_2

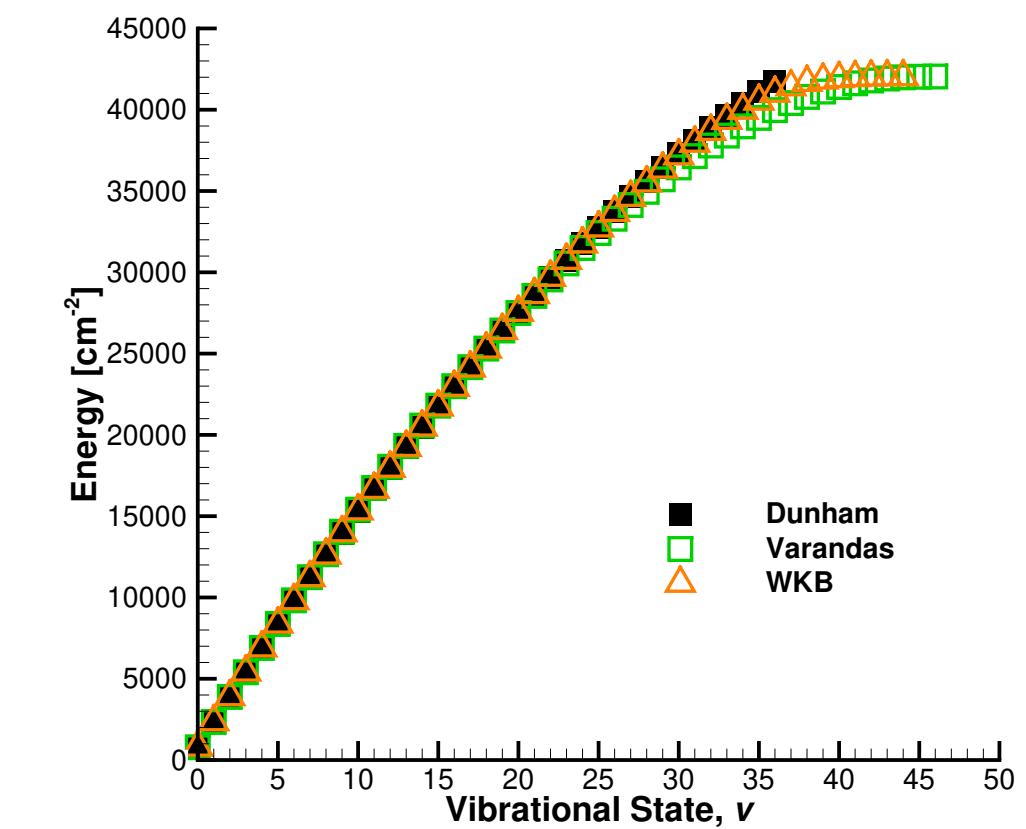
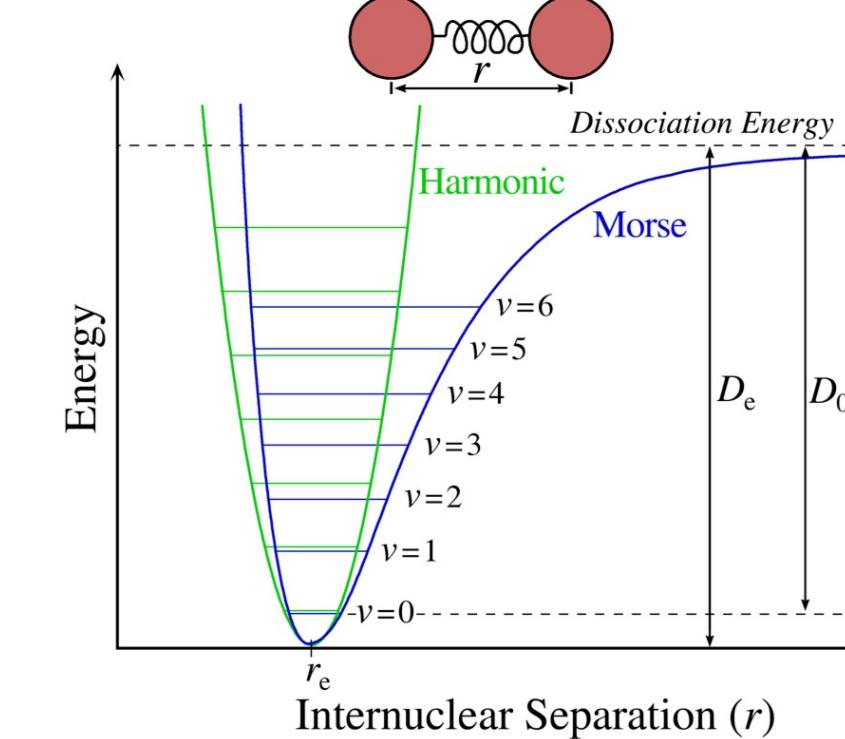
Motivation

Bound-Bound Transitions (rates)
Bound-Free Transitions (rates)



Analytical theory (e.g., Forced Harmonic Oscillator (FHO))
Quantum chemistry (e.g., Quasi-classical Trajectory (QCT))

- Rates from quantum chemistry are expensive and often done on different vibrational energy ladders
- This creates a modeling challenge for complex mixtures
- Goal: develop a consistent mapping between vibrational ladders



Motivation - Example

Assess chemical kinetics with shock tube data

Shock-tube measurements of coupled vibration-dissociation time-histories and rate parameters in oxygen and argon mixtures from 5000 K to 10 000 K 

Cite as: Phys. Fluids 32, 076103 (2020); doi: 10.1063/5.0012426

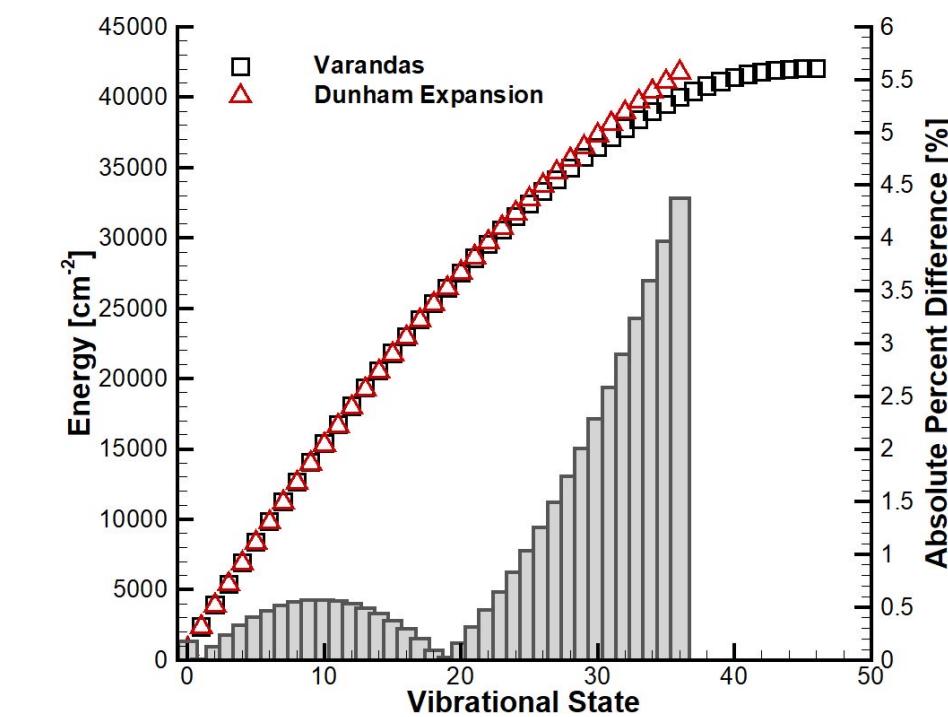
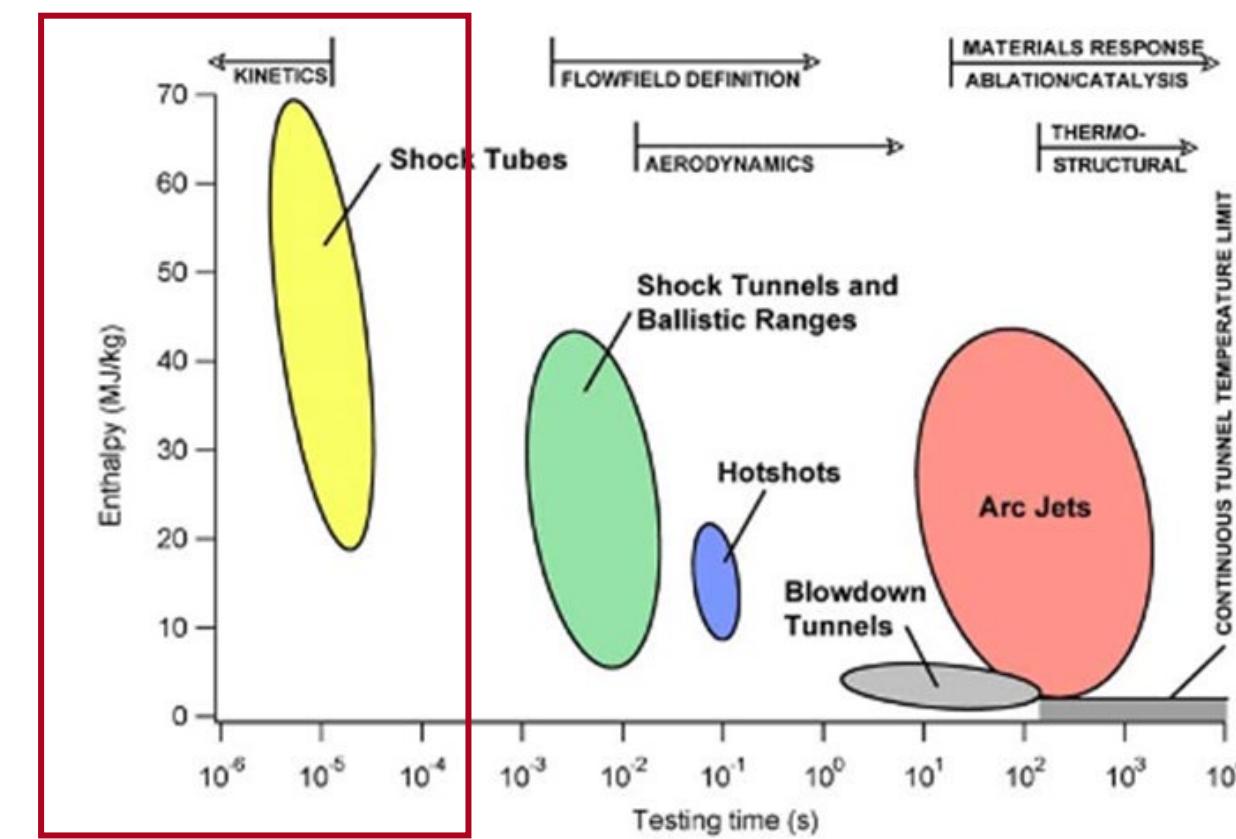


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Published Online: 1 July 2020

Jesse W. Streicher,^{1,a)}  Ajay Krish,¹  Ronald K. Hanson,¹  Kyle M. Hanquist,²  Ross S. Chaudhry,³  and Iain D. Boyd³ 

- O₂-O (Varandas – 0 → 46 vibrational energy levels)
- O₂-O₂ (Varandas – 0 → 46 vibrational energy levels)
- O₂-Ar (Dunham – 0 → 36 vibrational energy levels)



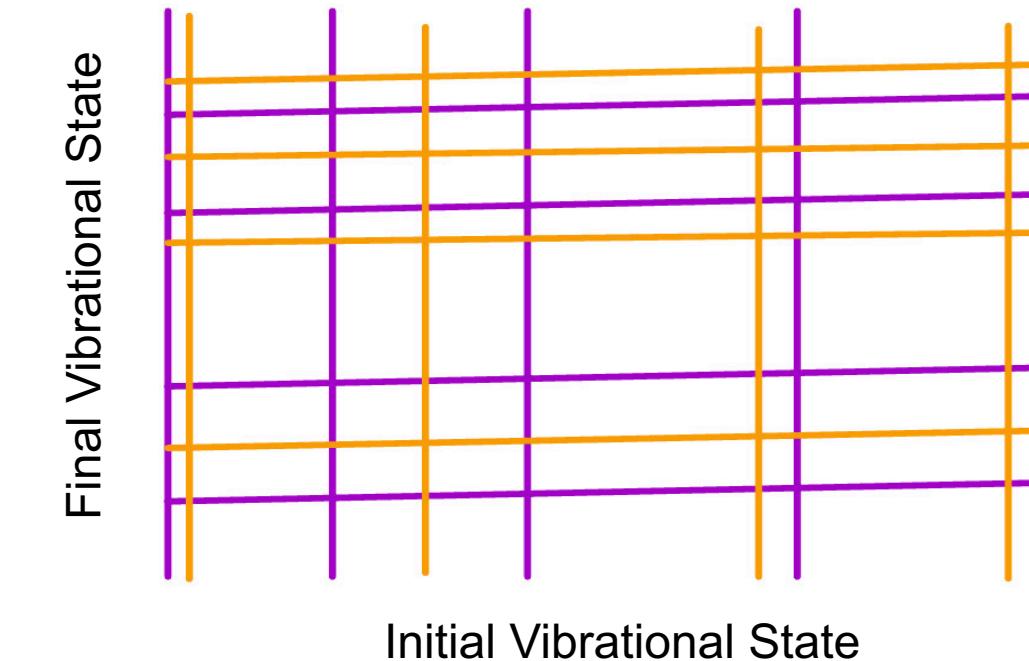
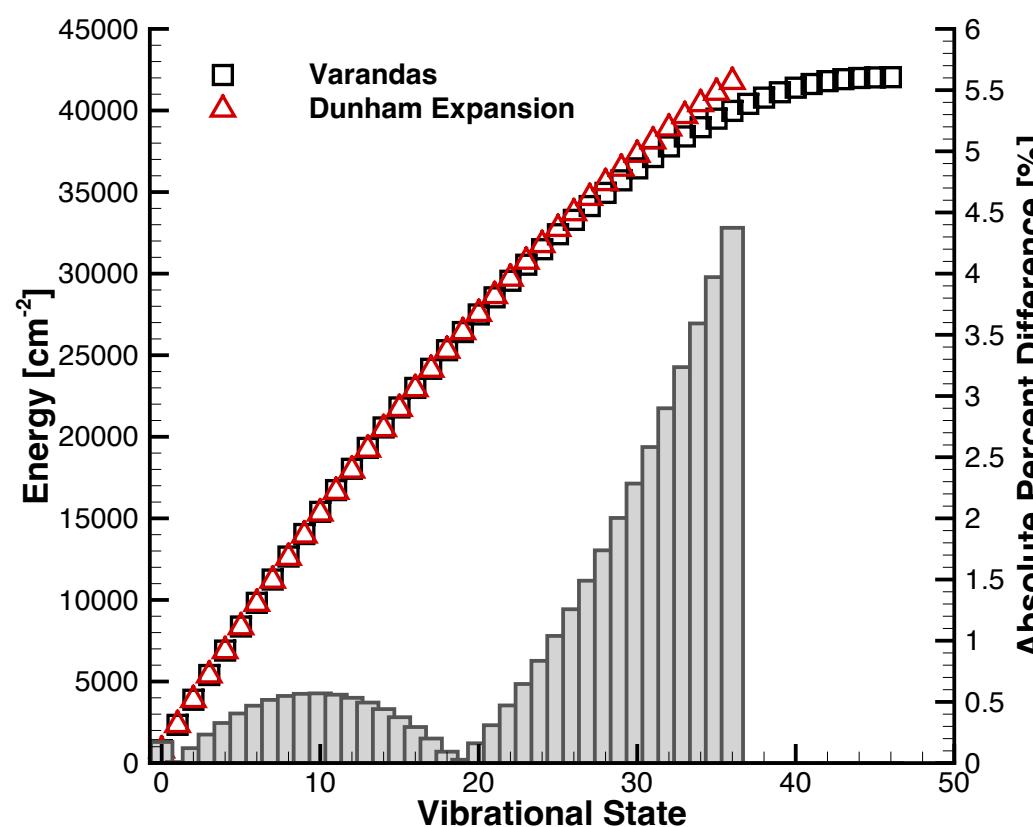
OBJECTIVE 1: MAP RATES FROM ONE VIBRATIONAL LADDER TO ANOTHER



Approach – Mapping – Taylor Series Expansion

$$\begin{bmatrix} \sum \xi_k & \sum (x_k - x_0) \xi_k & \sum (y_k - y_0) \xi_k \\ \sum (x_k - x_0) \xi_k & \sum (x_k - x_0)^2 \xi_k & \sum (x_k - x_0)(y_k - y_0) \xi_k \\ \sum (y_k - y_0) \xi_k & \sum (x_k - x_0)(y_k - y_0) \xi_k & \sum (y_k - y_0)^2 \xi_k \end{bmatrix} \begin{bmatrix} K|_0 \\ \partial K / \partial x |_0 \\ \partial K / \partial y |_0 \end{bmatrix} = \begin{bmatrix} \sum K_k \xi_k \\ \sum (x_k - x_0) K_k \xi_k \\ \sum (y_k - y_0) K_k \xi_k \end{bmatrix}$$

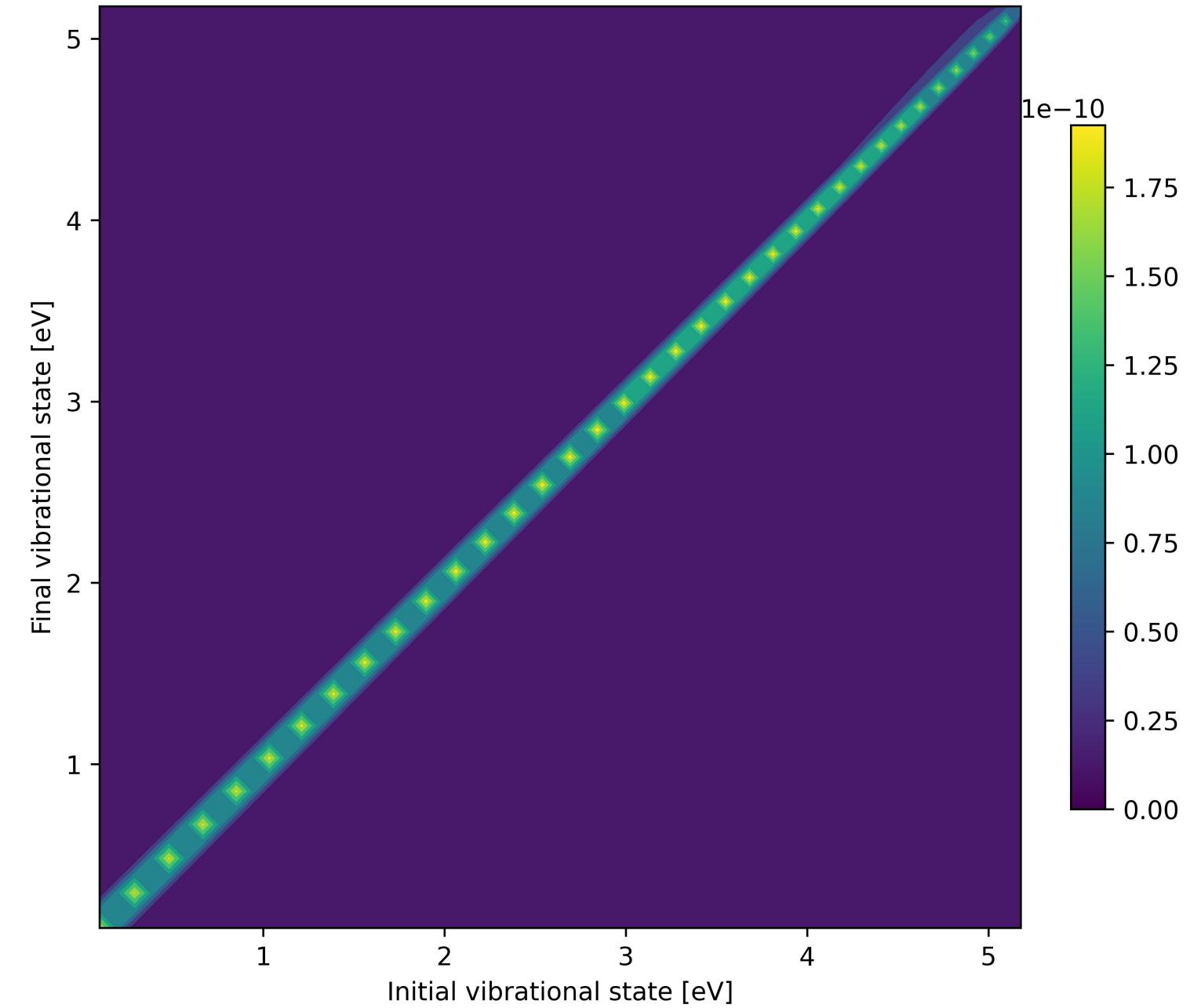
$$\xi_k = \frac{1}{(x_k - x_0)^4 + (y_k - y_0)^4}$$



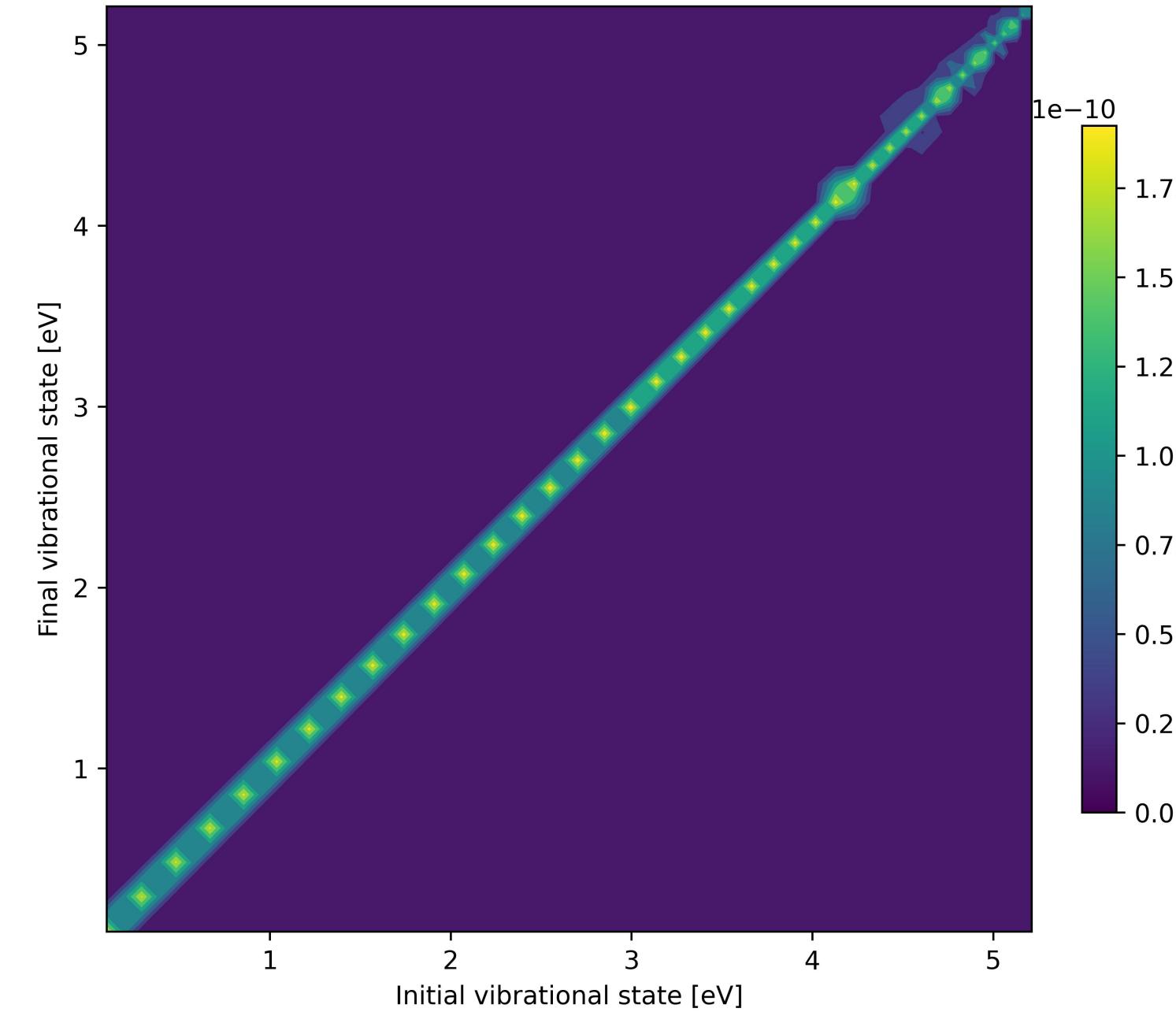
*Andrienko and Boyd, "Kinetics of O₂-N₂ collisions at hypersonic temperatures," AIAA Paper 2018-3438.

**Mavriplis, "Revisiting the least-squares procedure for gradient reconstruction on unstructured meshes," AIAA Paper 2003-3986.

Results – Mapping – Taylor Series Expansion



Original/Known - Dunham (36 state ladder)

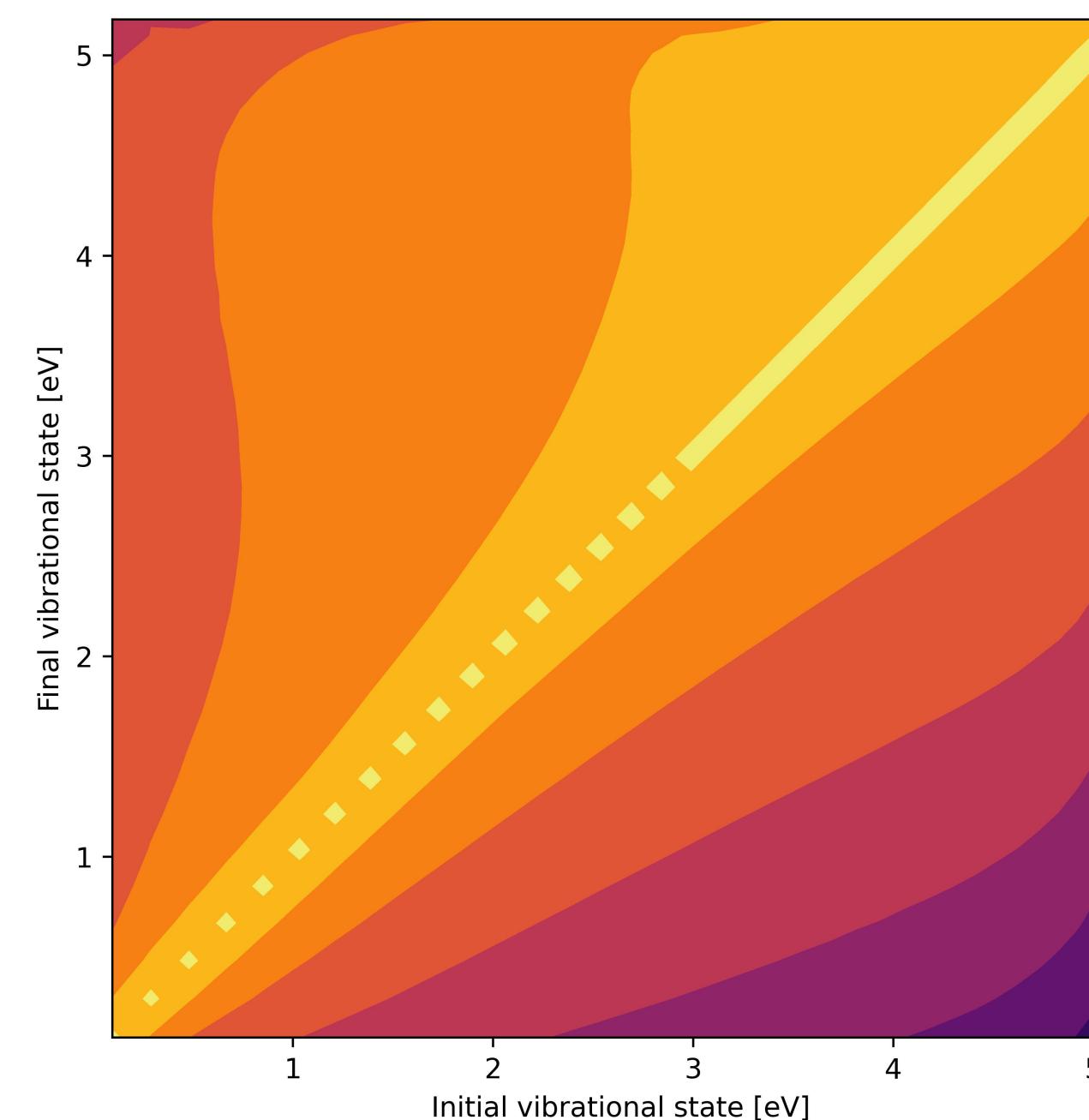


Mapped/Unknown - Varandas (46 state ladder)

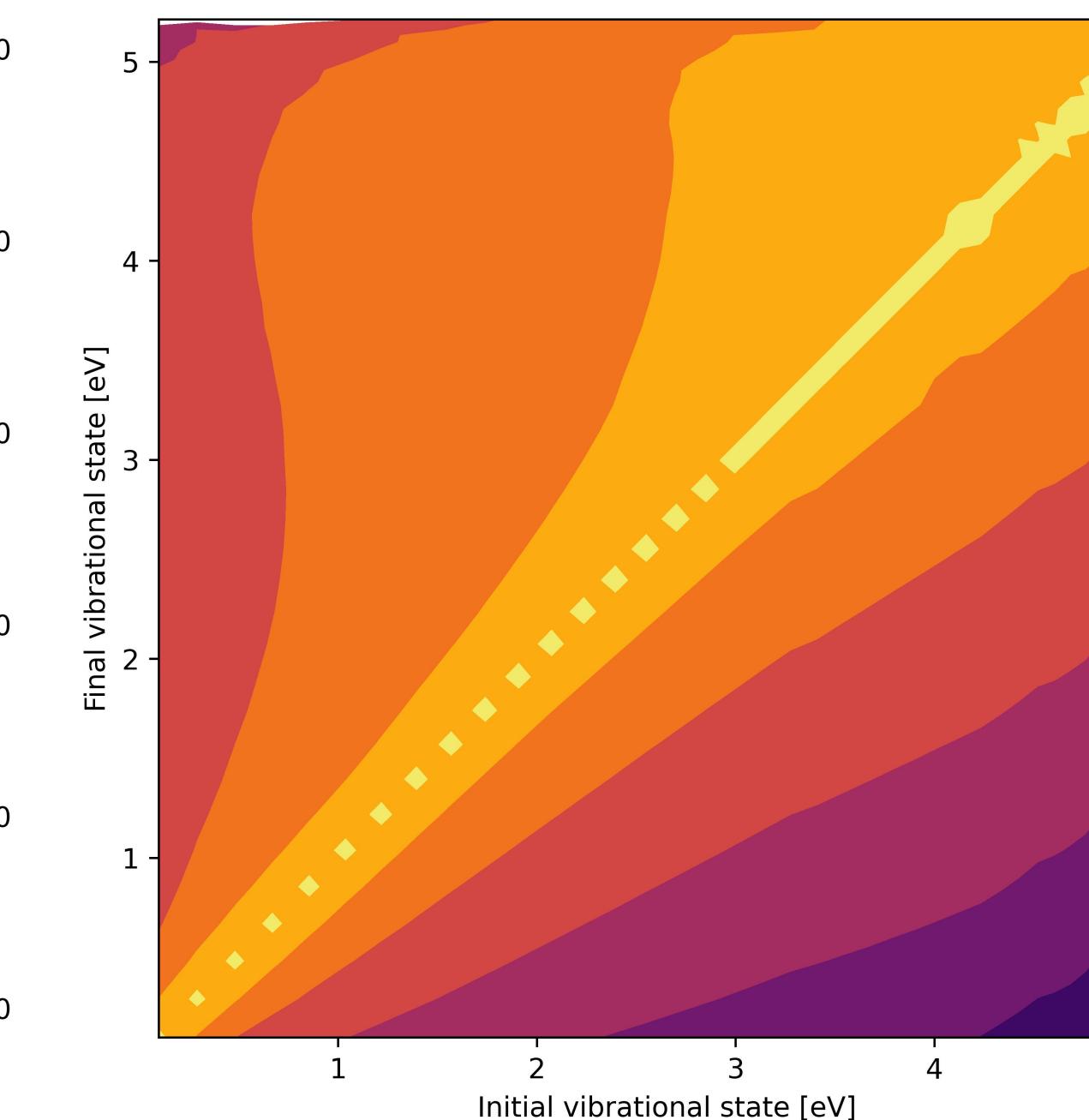
O_2 -Ar bound-bound rates at 3000 K



Results – Mapping – Taylor Series Expansion



Original/Known - Dunham (36 state ladder)

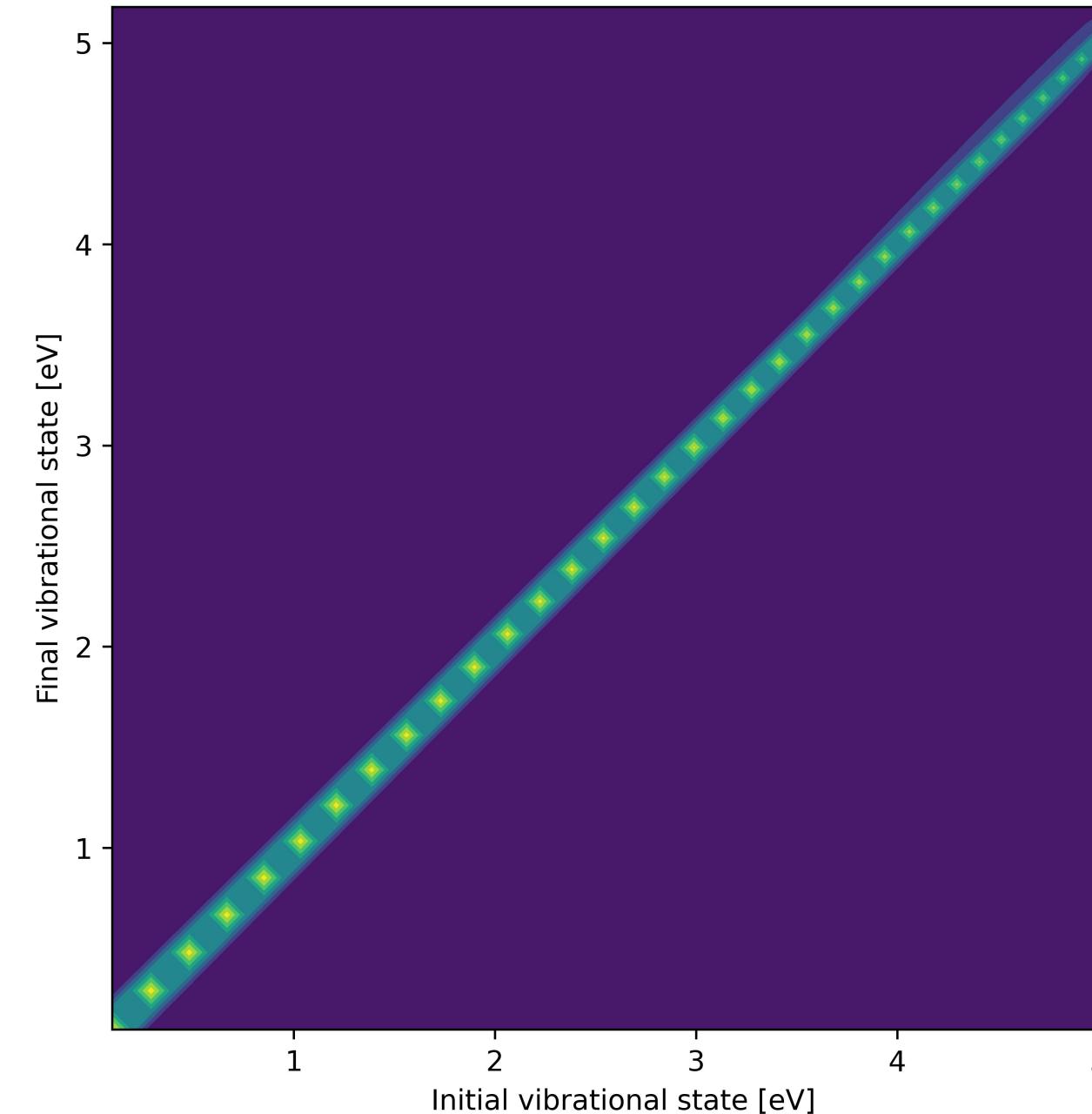


Mapped/Unknown - Varandas (46 state ladder)

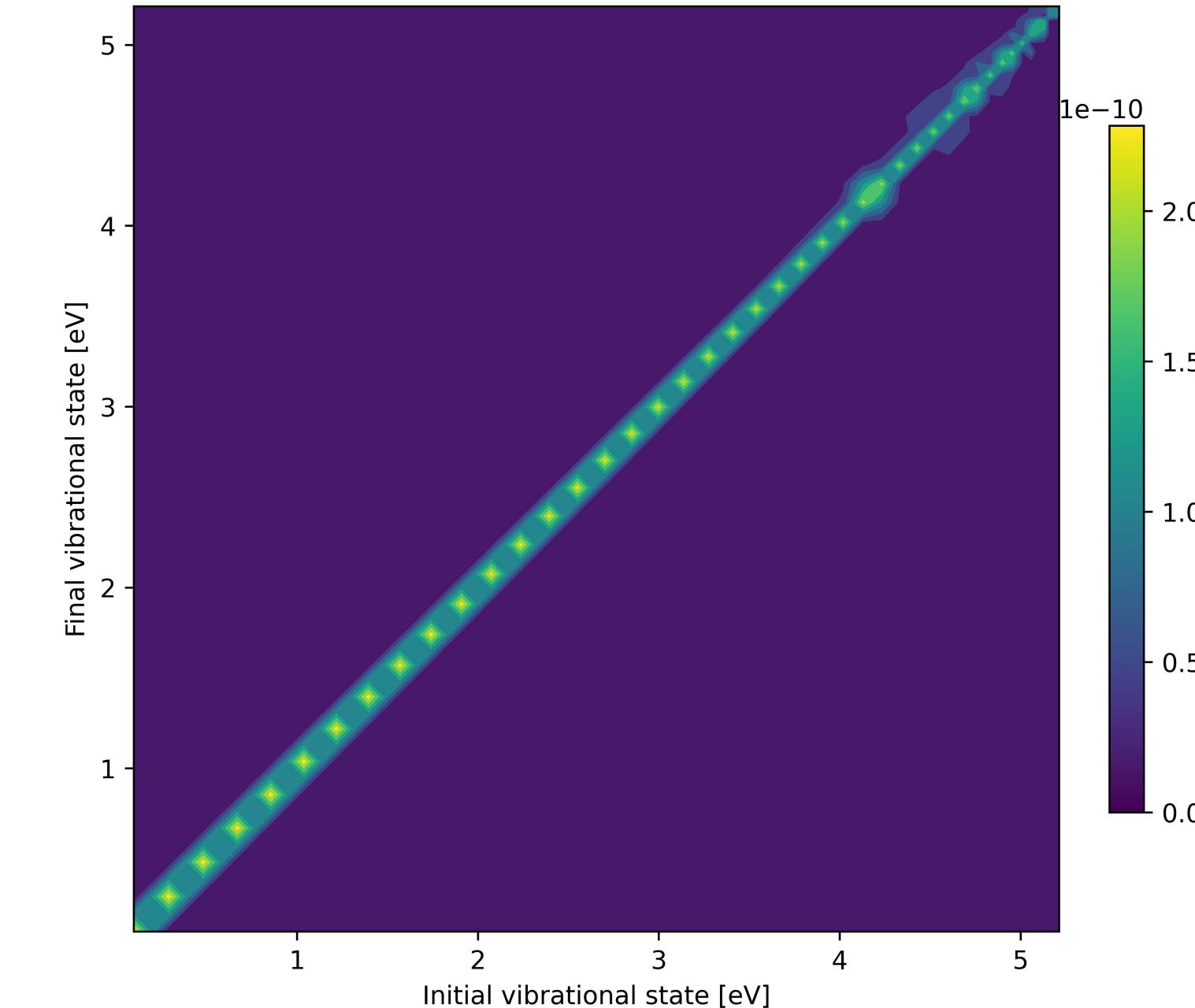
O₂-Ar bound-bound **log(rates)** at 3000 K



Results – Mapping – Taylor Series Expansion



Original/Known - Dunham (36 state ladder)

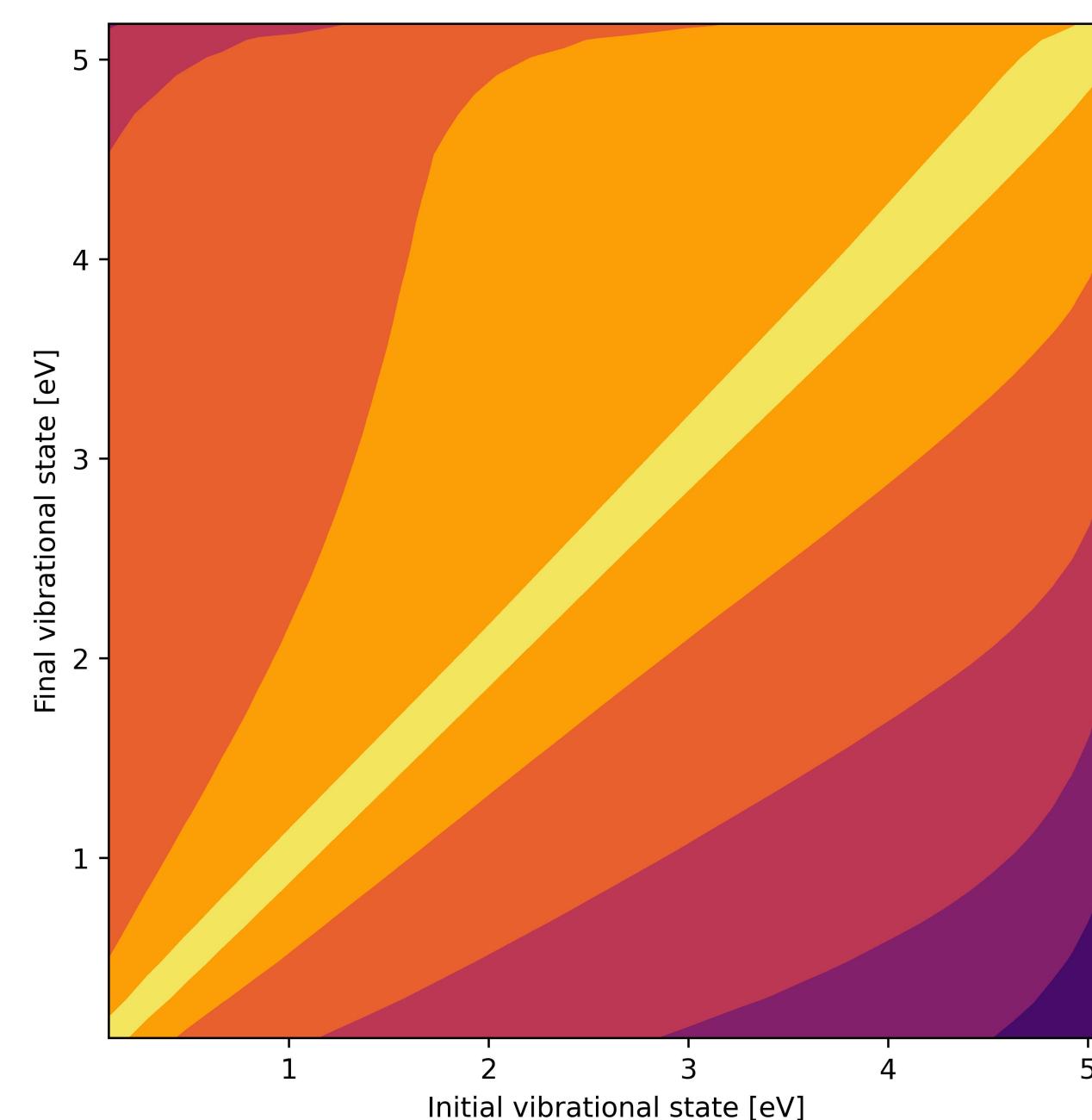


Mapped/Unknown - Varandas (46 state ladder)

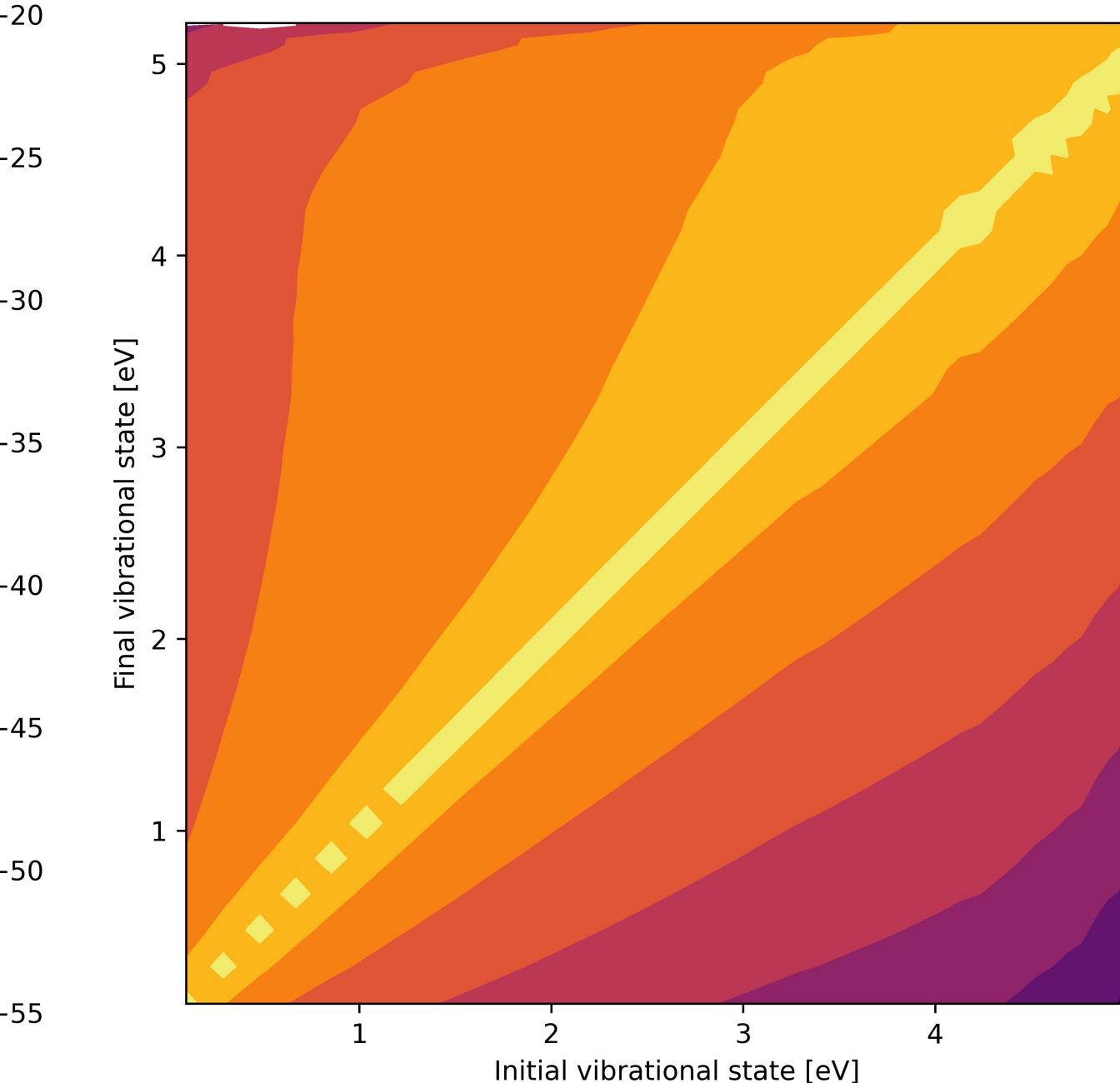
O_2 -Ar bound-bound rates at 5000 K



Results – Mapping – Taylor Series Expansion



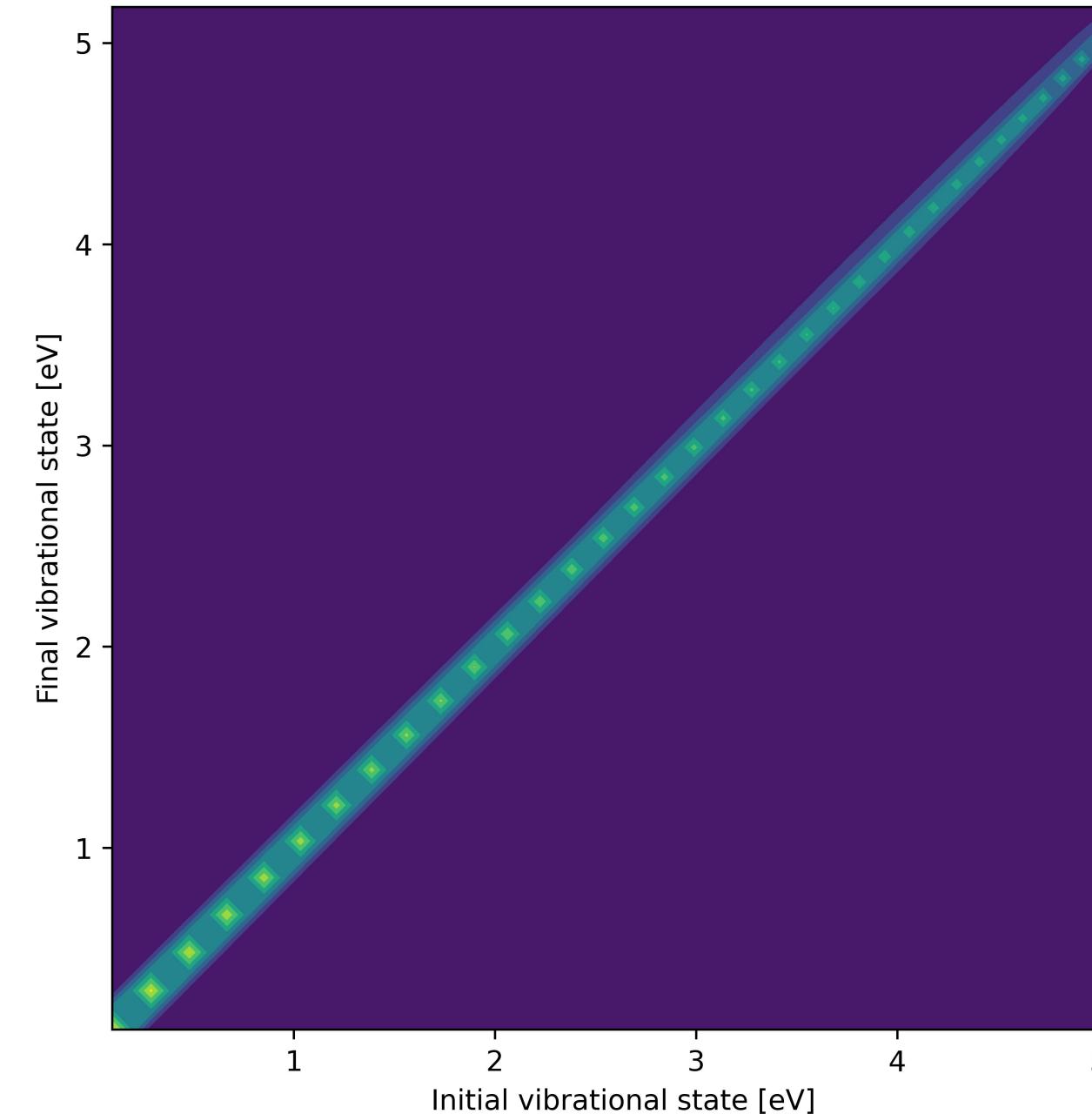
Original/Known - Dunham (36 state ladder)



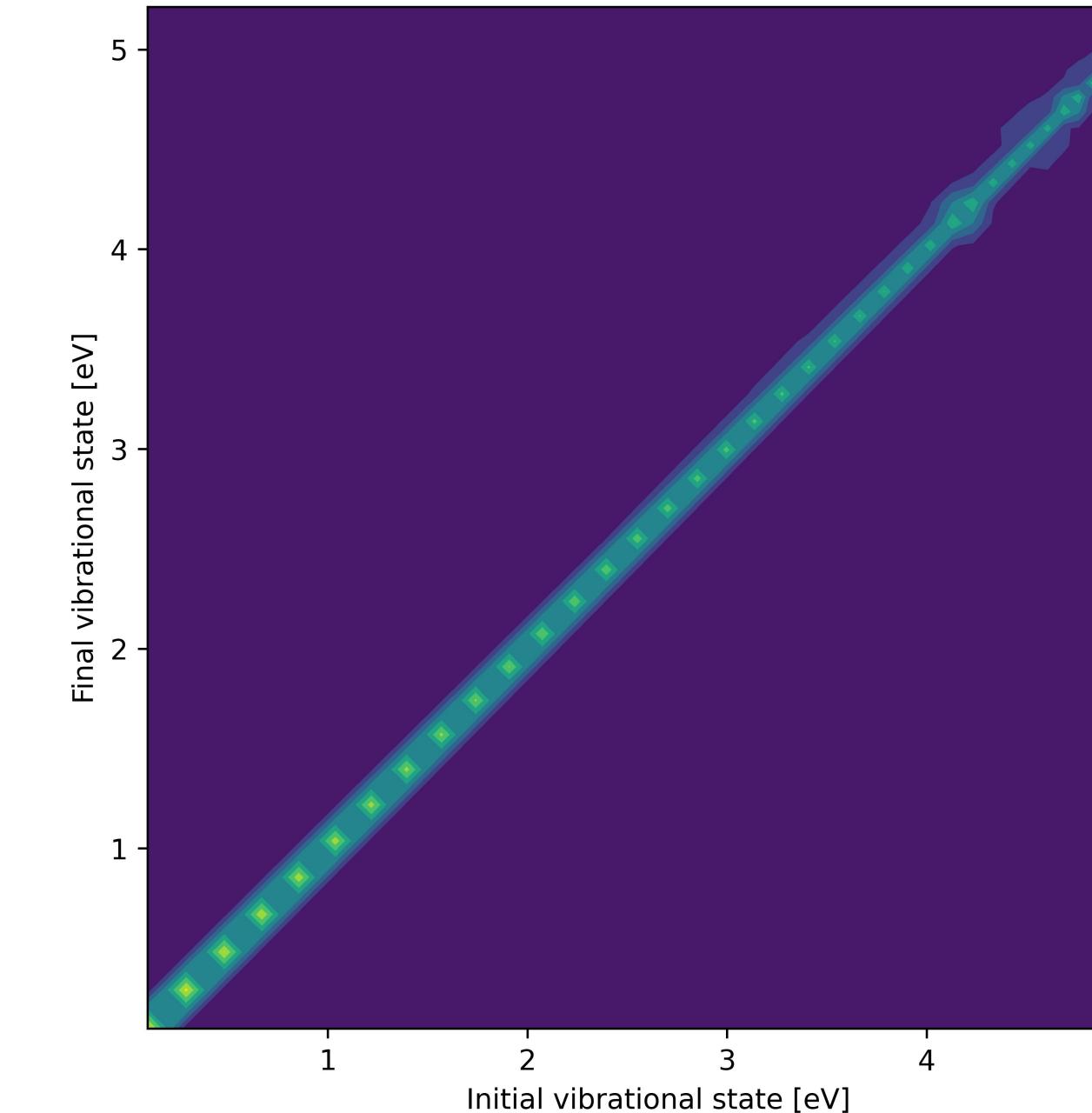
Mapped/Unknown - Varandas (46 state ladder)

$\text{O}_2\text{-Ar}$ bound-bound **log(rates)** at 5000 K

Results – Mapping – Taylor Series Expansion



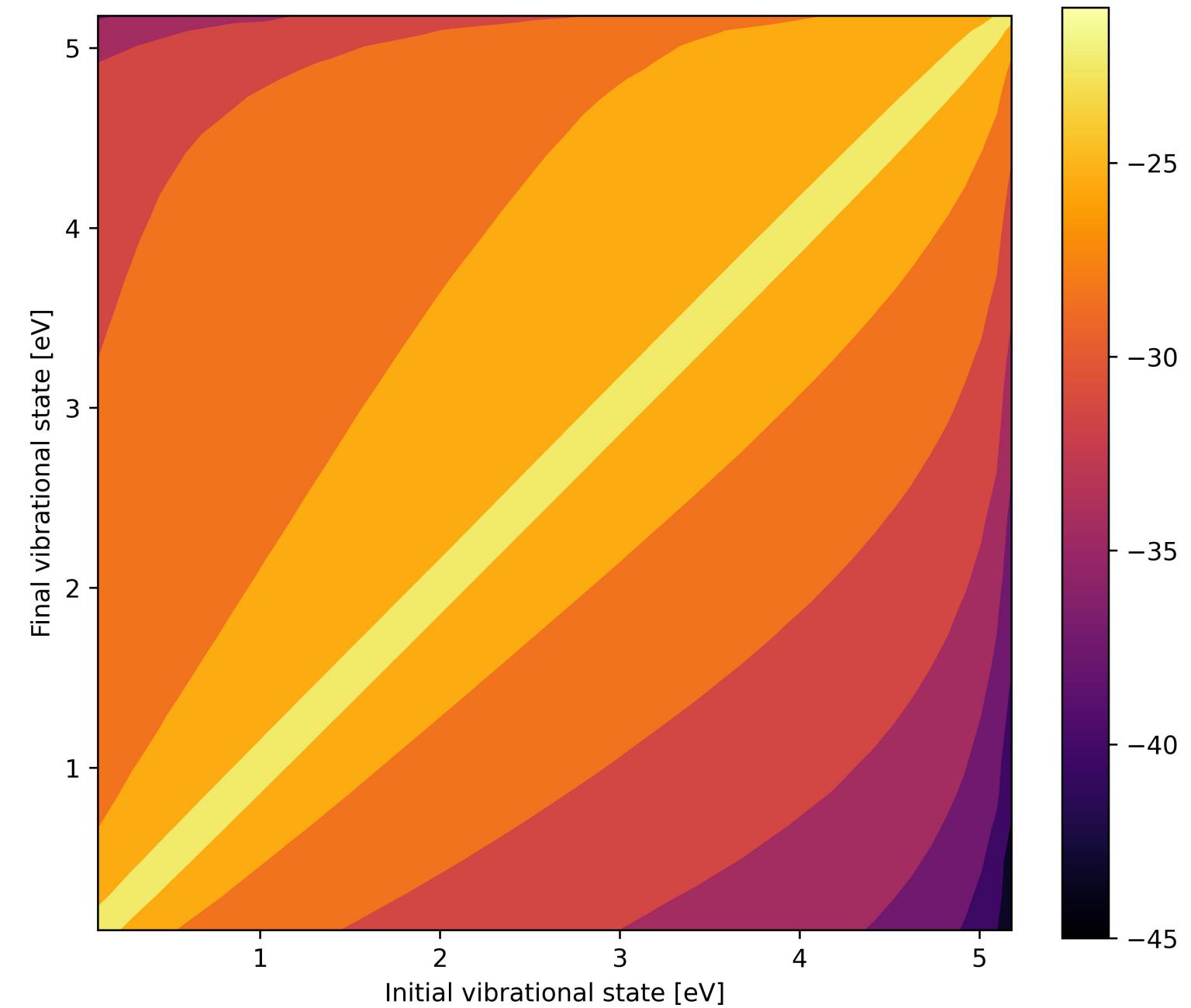
Original/Known - Dunham (36 state ladder)



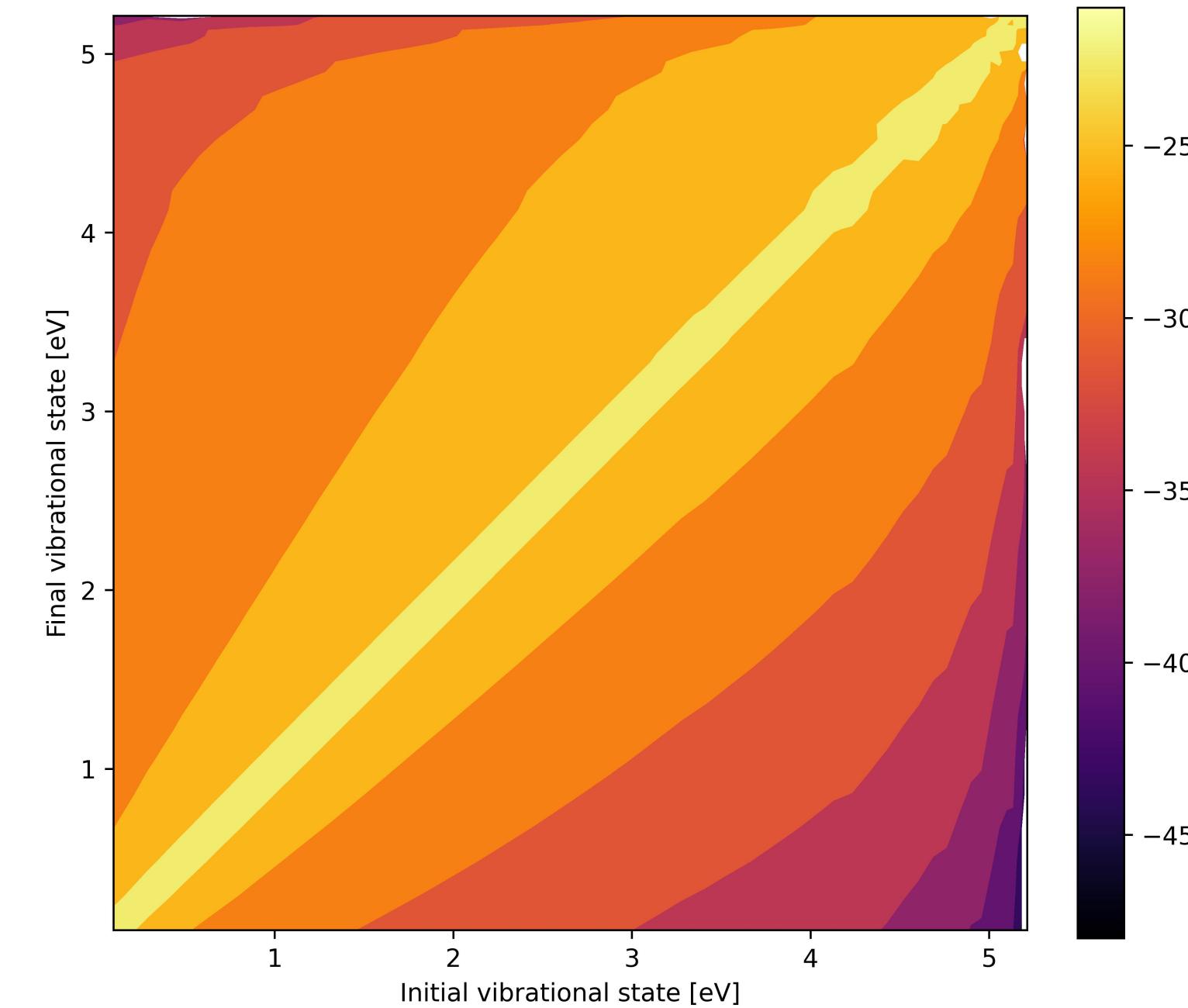
Mapped/Unknown - Varandas (46 state ladder)

O_2 -Ar bound-bound rates at 10000 K

Results – Mapping – Taylor Series Expansion



Original/Known - Dunham (36 state ladder)



Mapped/Unknown - Varandas (46 state ladder)

O₂-Ar bound-bound **log(rates)** at 10000 K

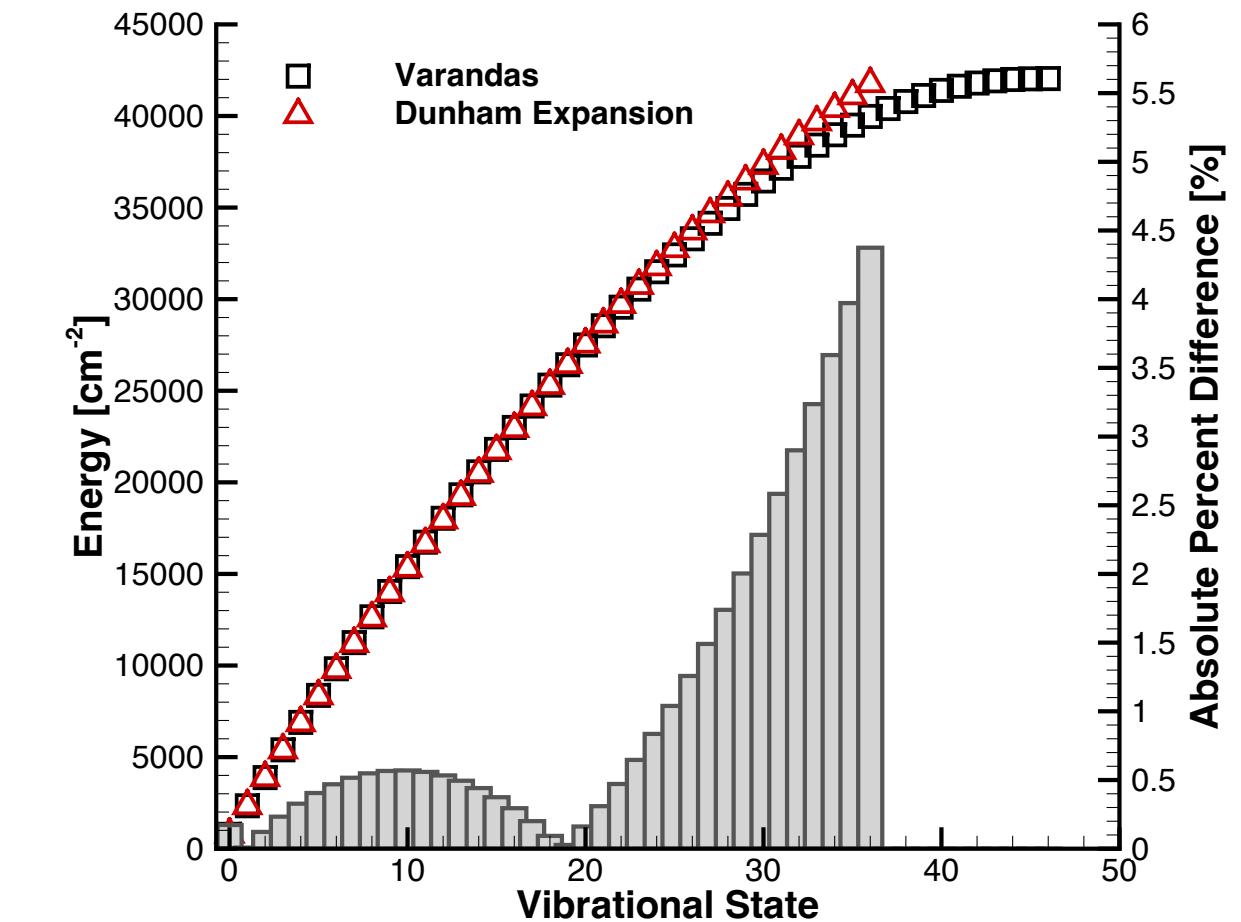


Mapping Approaches – Radial Basis Functions (RBF)

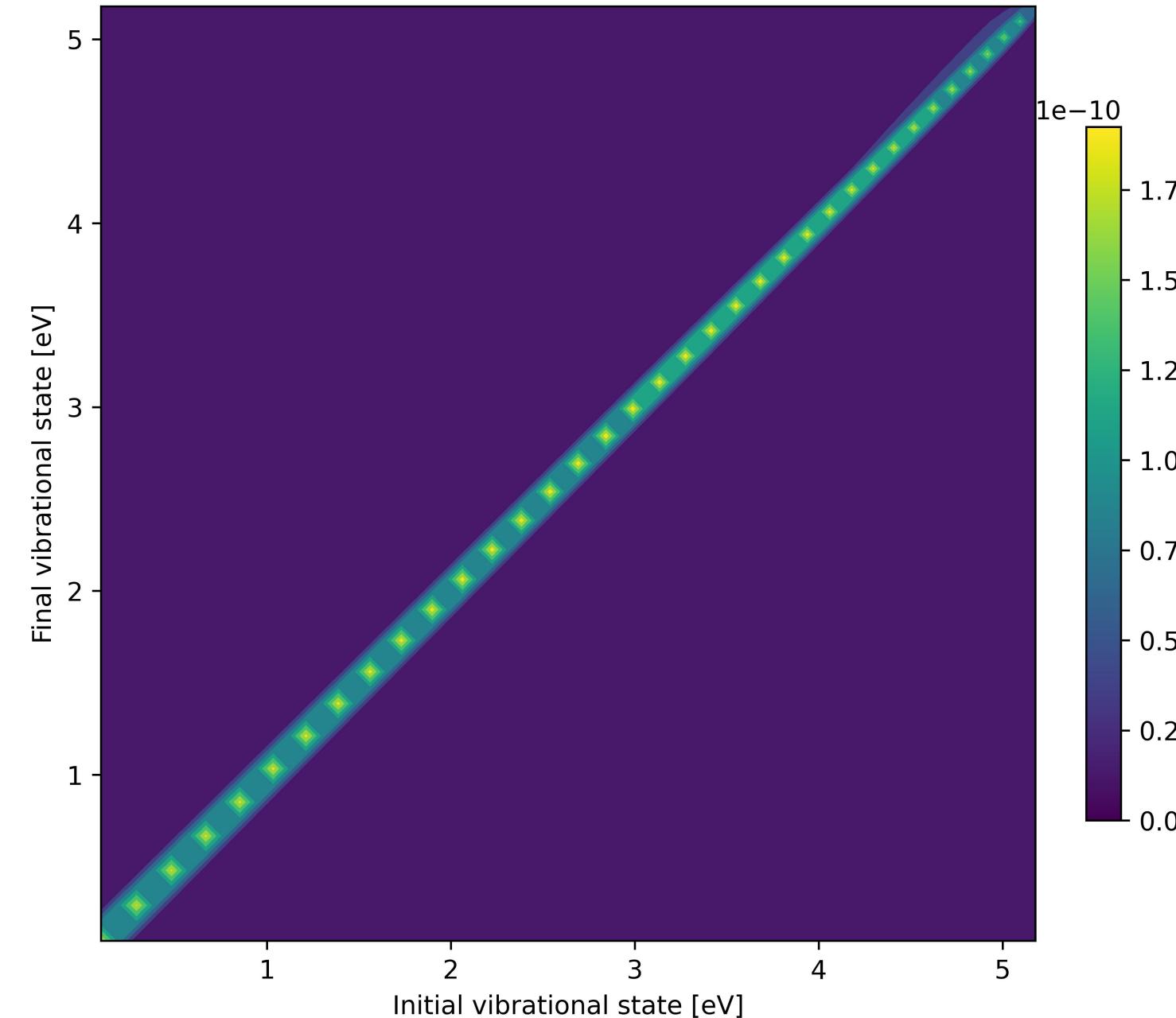
$$\phi(r) = r^2 \log(r) \quad r = \|\mathbf{v} - \mathbf{d}_i\|$$

$$f(\mathbf{v}) = \sum_{i=1}^N w_i \phi (\|\mathbf{v} - \mathbf{d}_i\|)$$

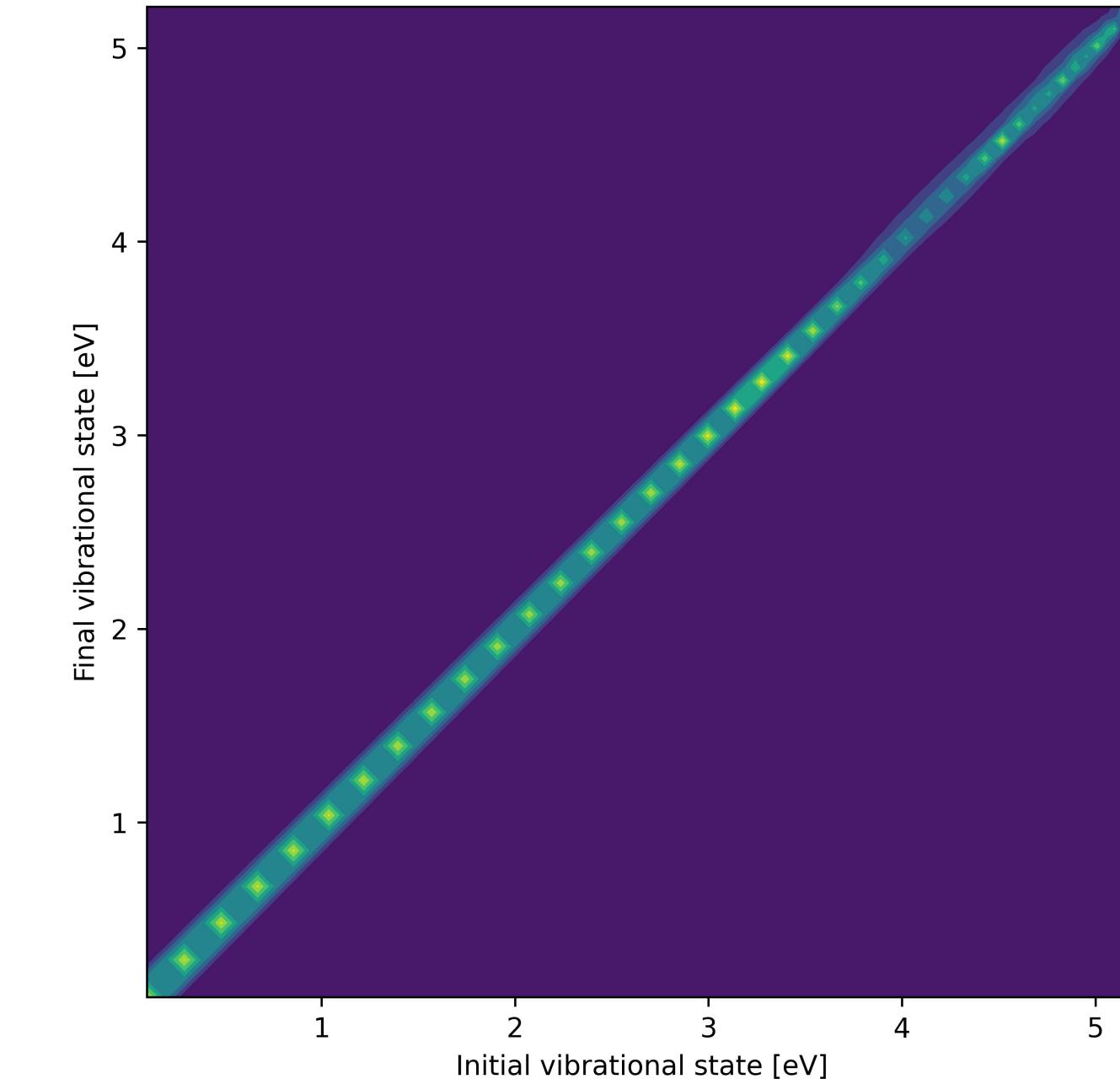
$$\begin{bmatrix} \phi (\|\mathbf{d}_1 - \mathbf{d}_1\|) & \cdots & \phi (\|\mathbf{d}_1 - \mathbf{d}_n\|) \\ \phi (\|\mathbf{d}_2 - \mathbf{d}_1\|) & \cdots & \phi (\|\mathbf{d}_2 - \mathbf{d}_n\|) \\ \vdots & \ddots & \vdots \\ \phi (\|\mathbf{d}_n - \mathbf{d}_1\|) & \cdots & \phi (\|\mathbf{d}_n - \mathbf{d}_n\|) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$



Results – Mapping – Radial Basis Functions



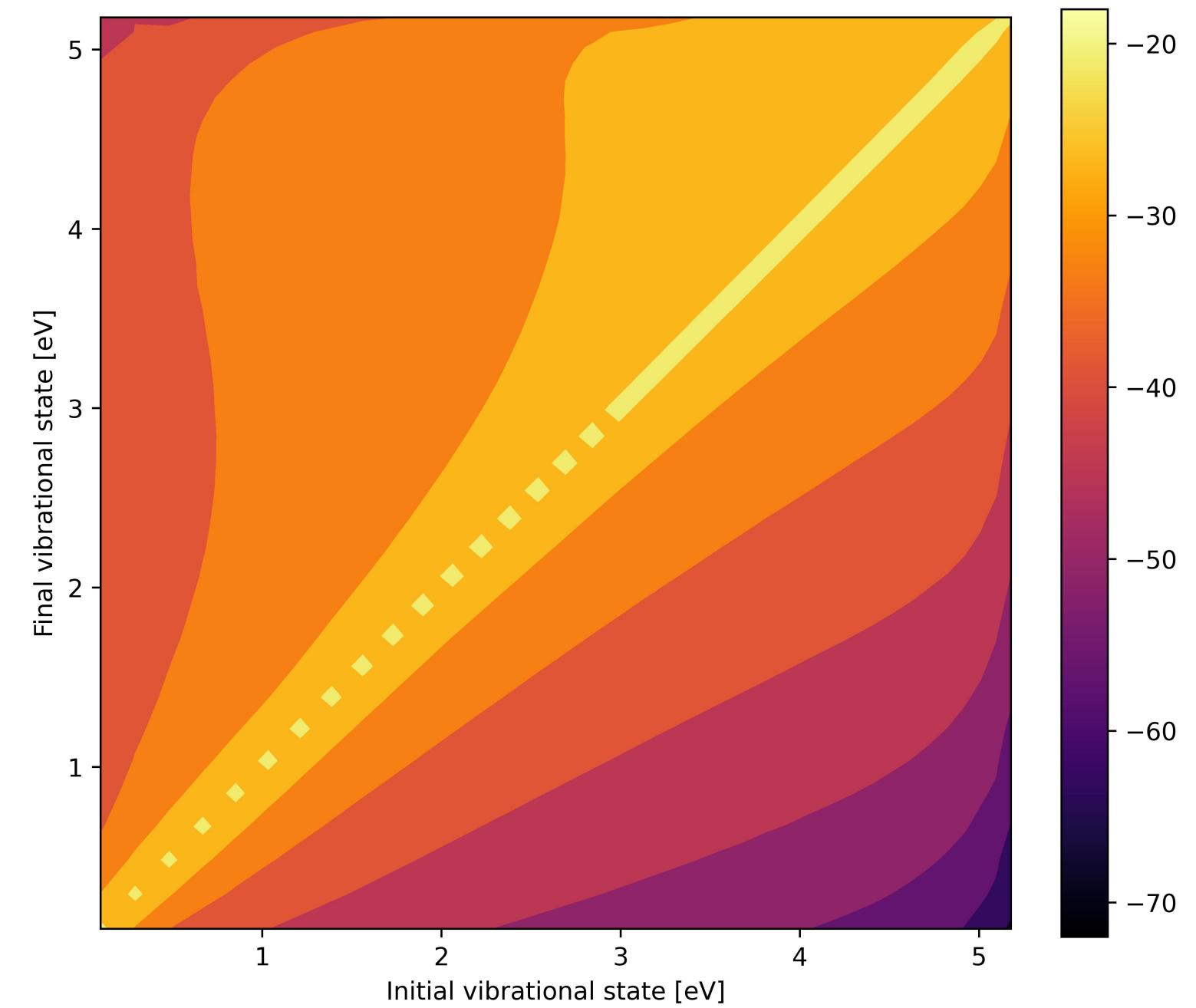
Original/Known - Dunham (36 state ladder)



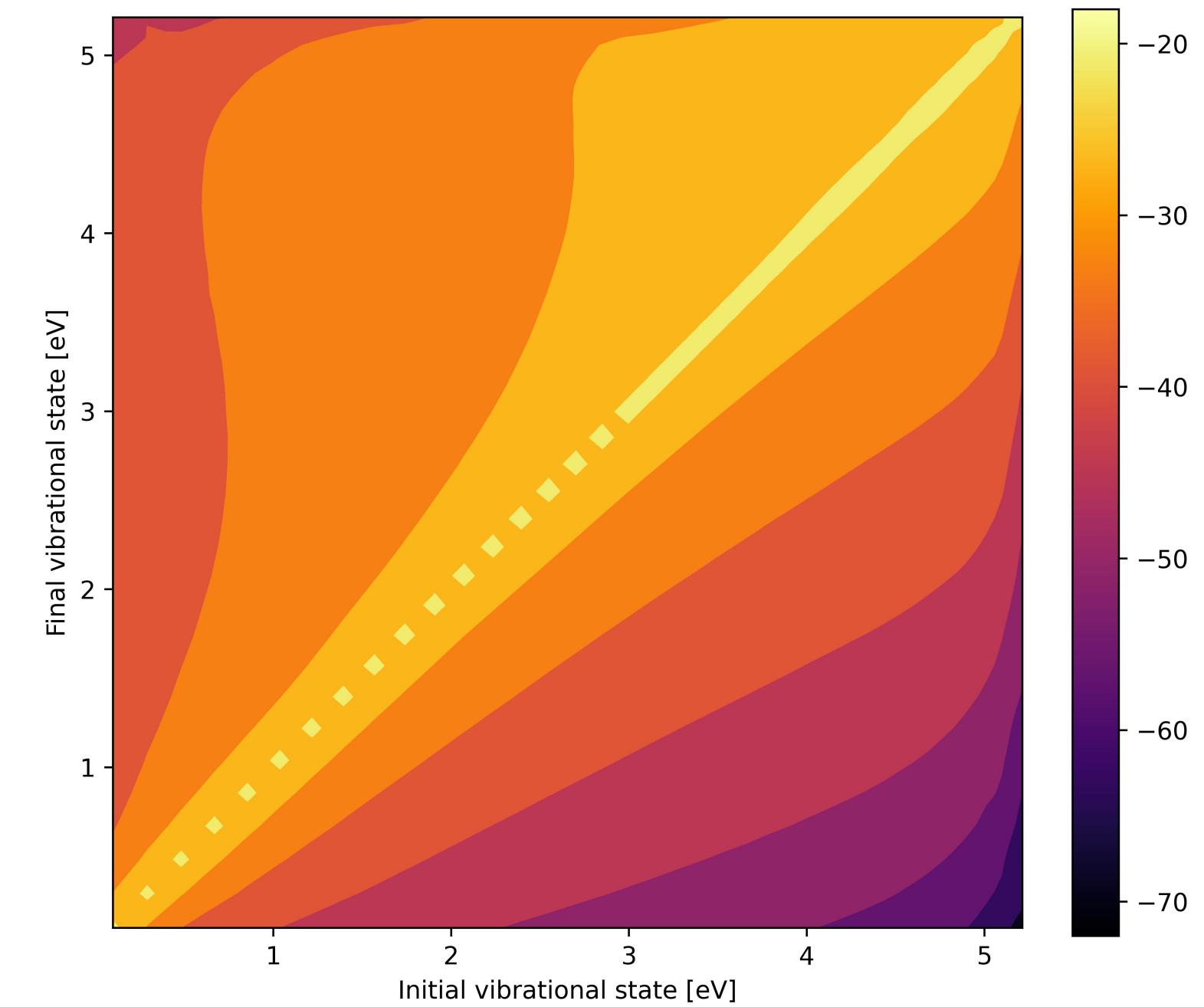
Mapped/Unknown - Varandas (46 state ladder)

O_2 -Ar bound-bound rates at 3000 K

Results – Mapping – Radial Basis Functions



Original/Known - Dunham (36 state ladder)

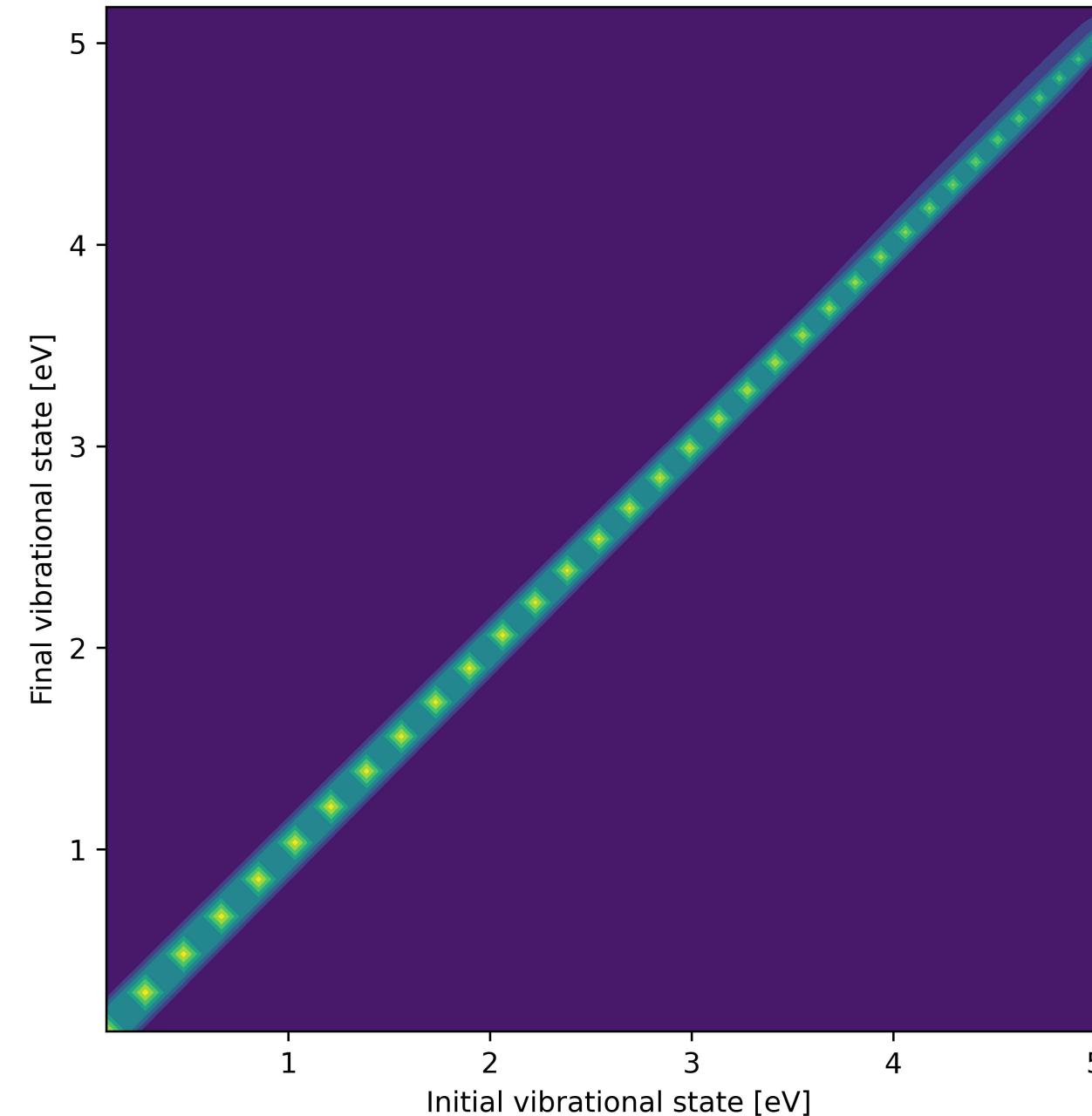


Mapped/Unknown - Varandas (46 state ladder)

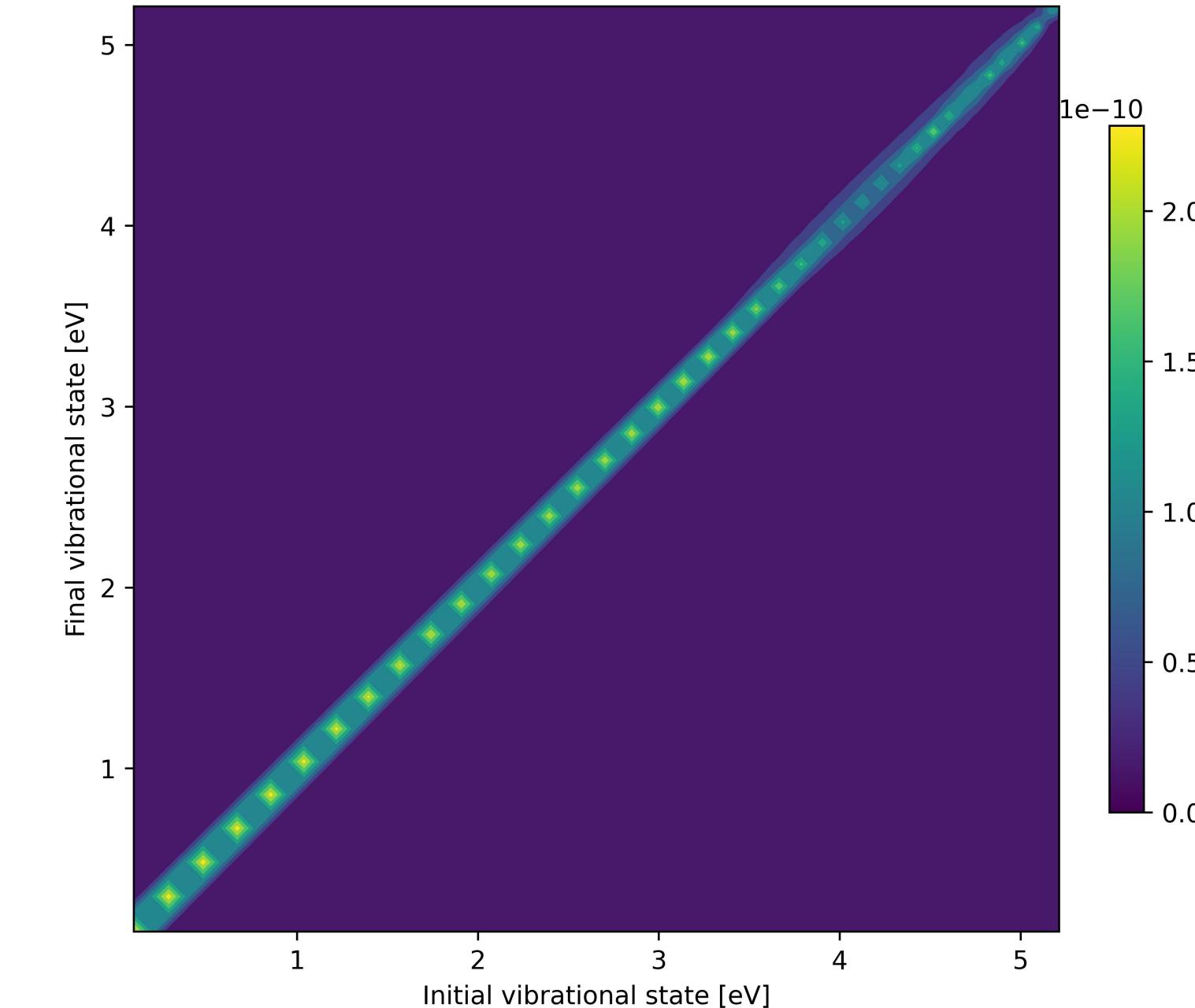
$\text{O}_2\text{-Ar}$ bound-bound **log(rates)** at 3000 K



Results – Mapping – Radial Basis Functions



Original/Known - Dunham (36 state ladder)

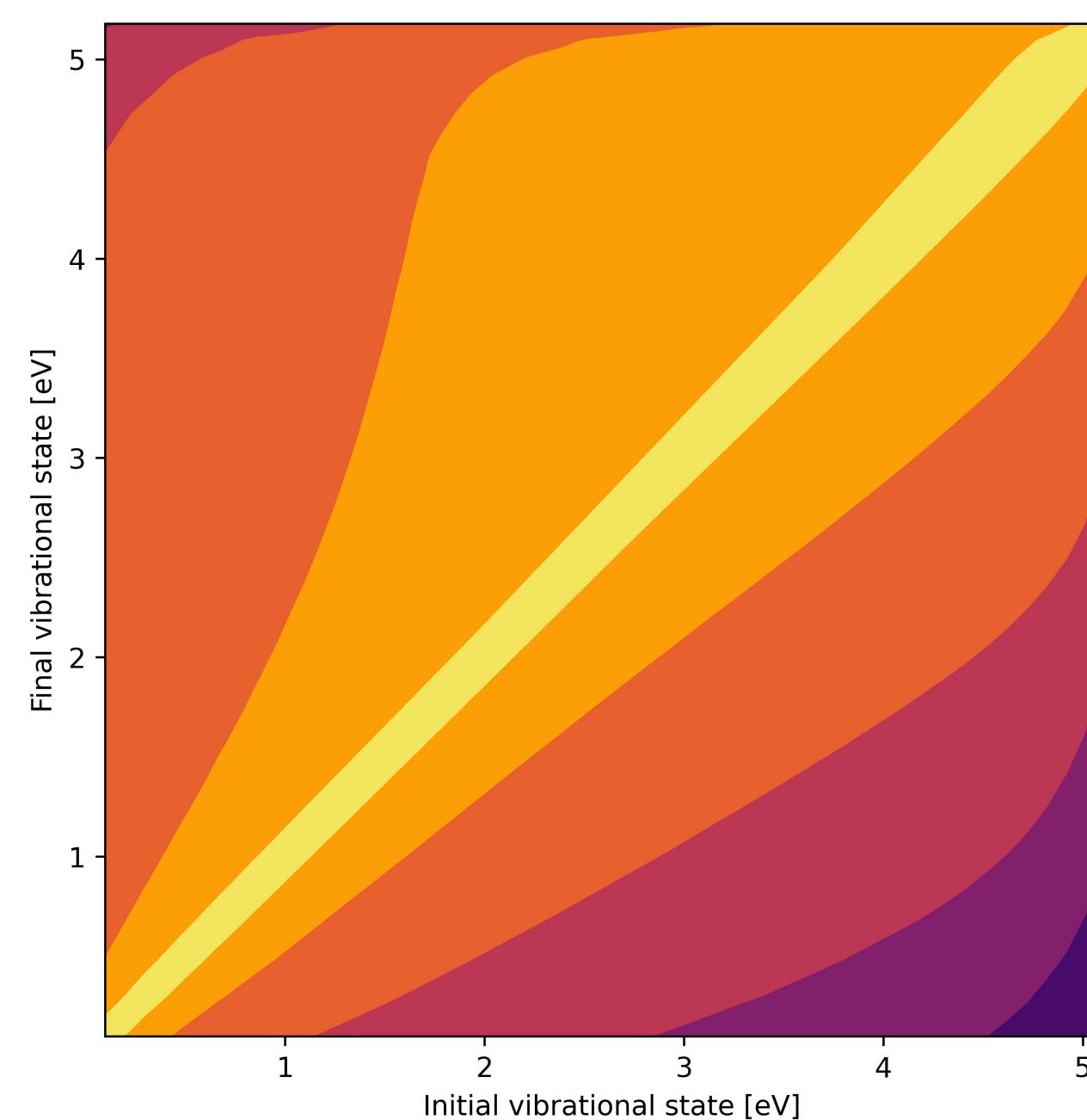


Mapped/Unknown - Varandas (46 state ladder)

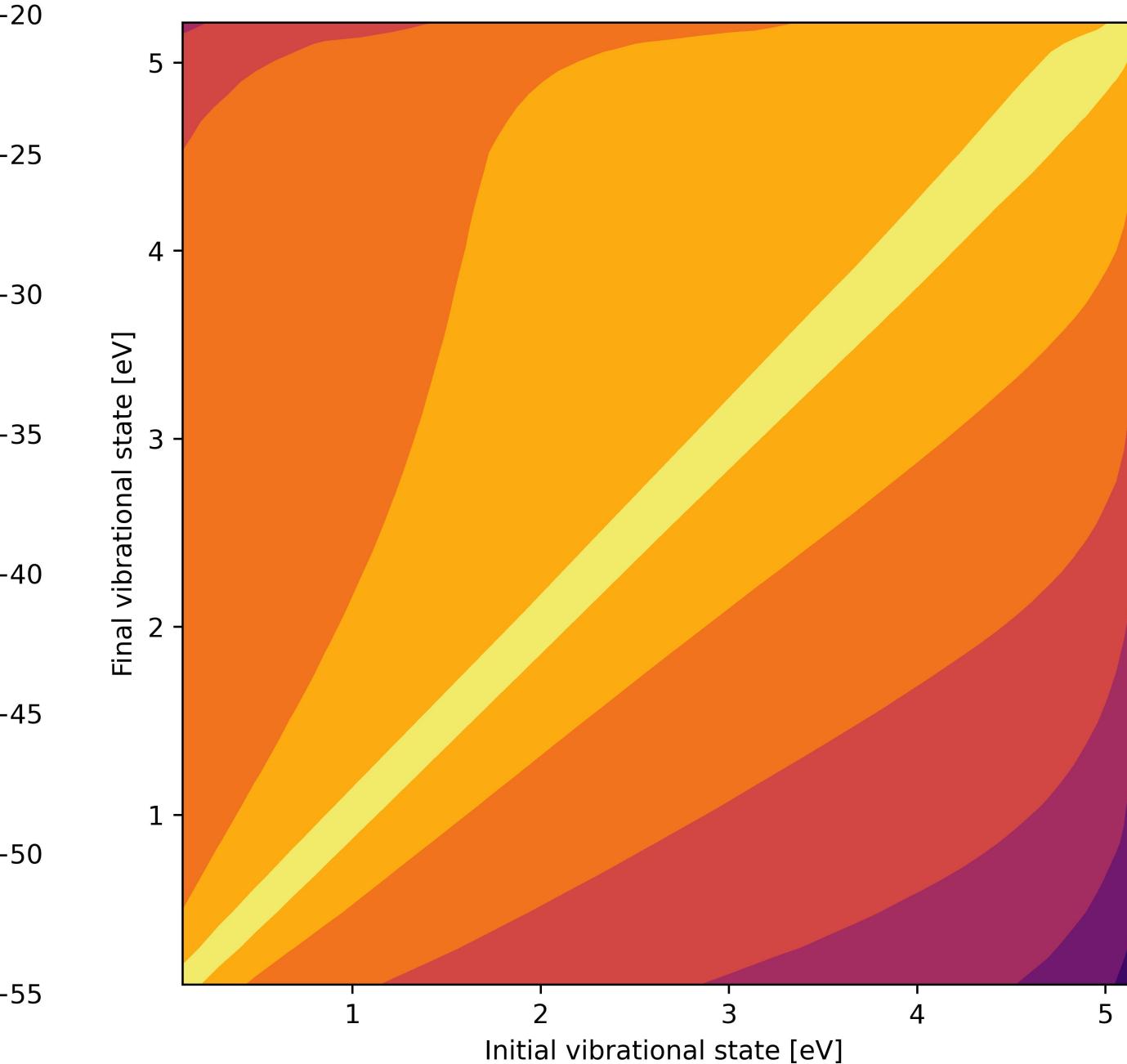
O_2 -Ar bound-bound rates at 5000 K



Results – Mapping – Radial Basis Functions



Original/Known - Dunham (36 state ladder)

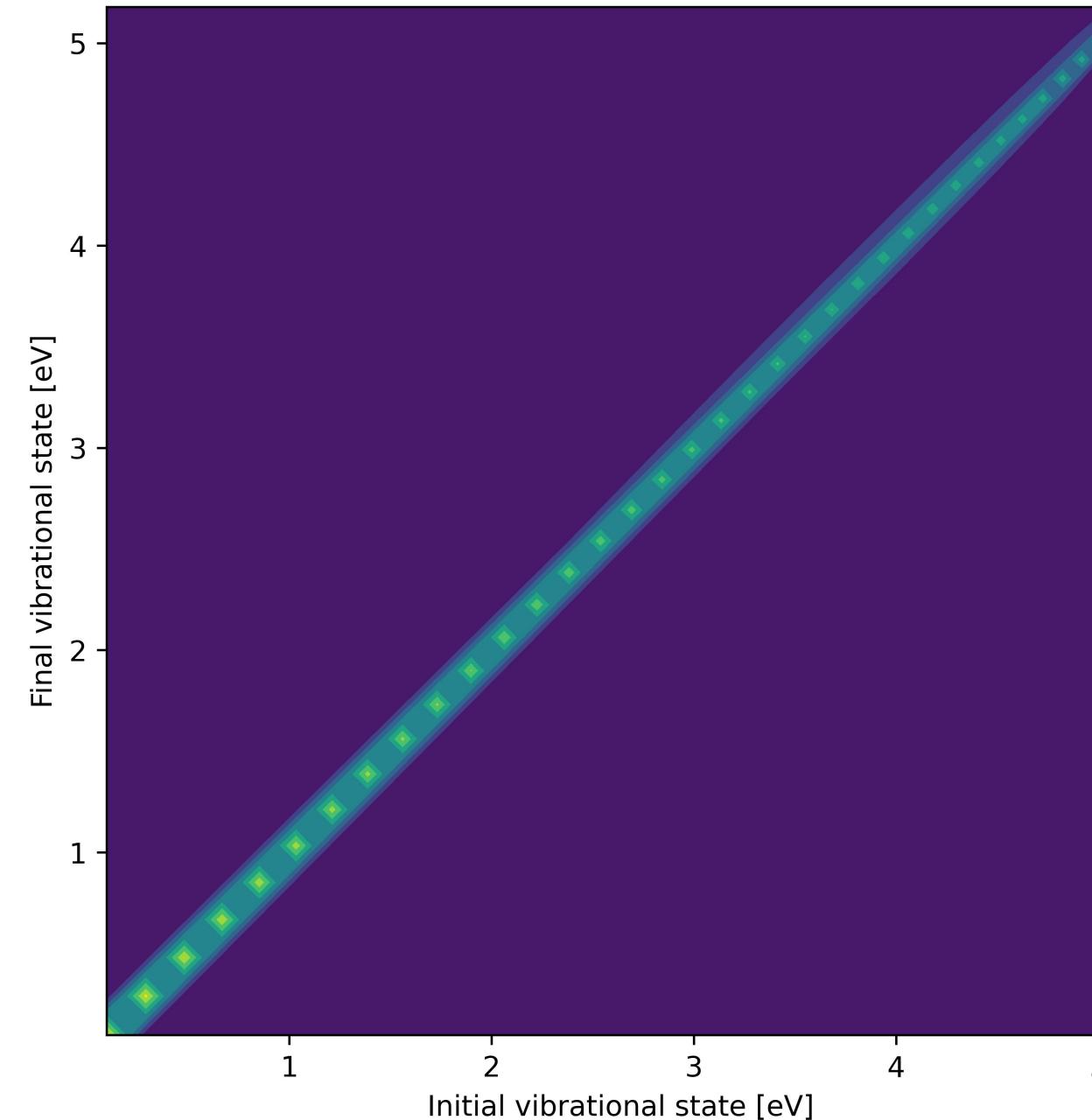


Mapped/Unknown - Varandas (46 state ladder)

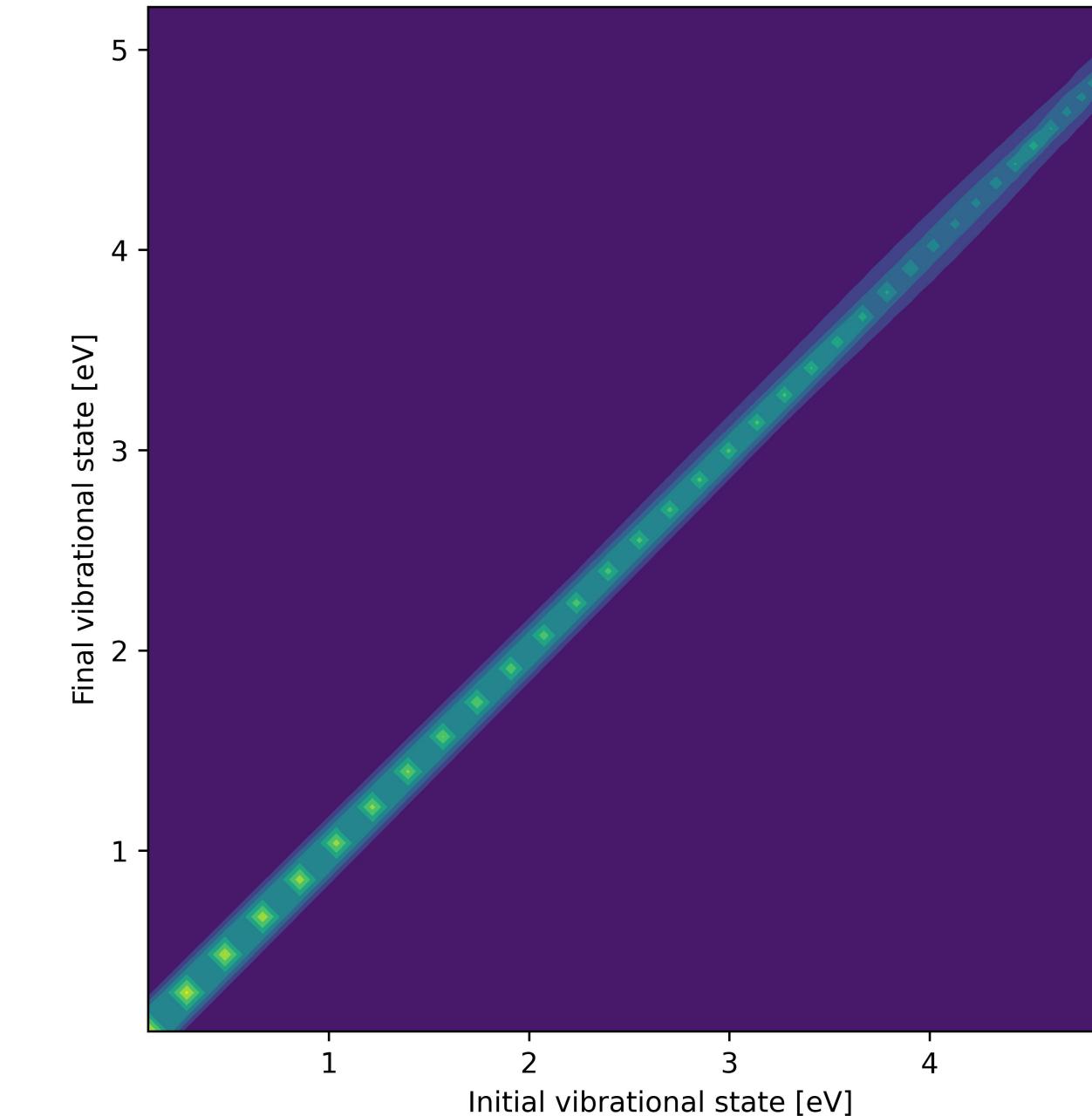
O₂-Ar bound-bound **log(rates)** at 5000 K



Results – Mapping – Radial Basis Functions



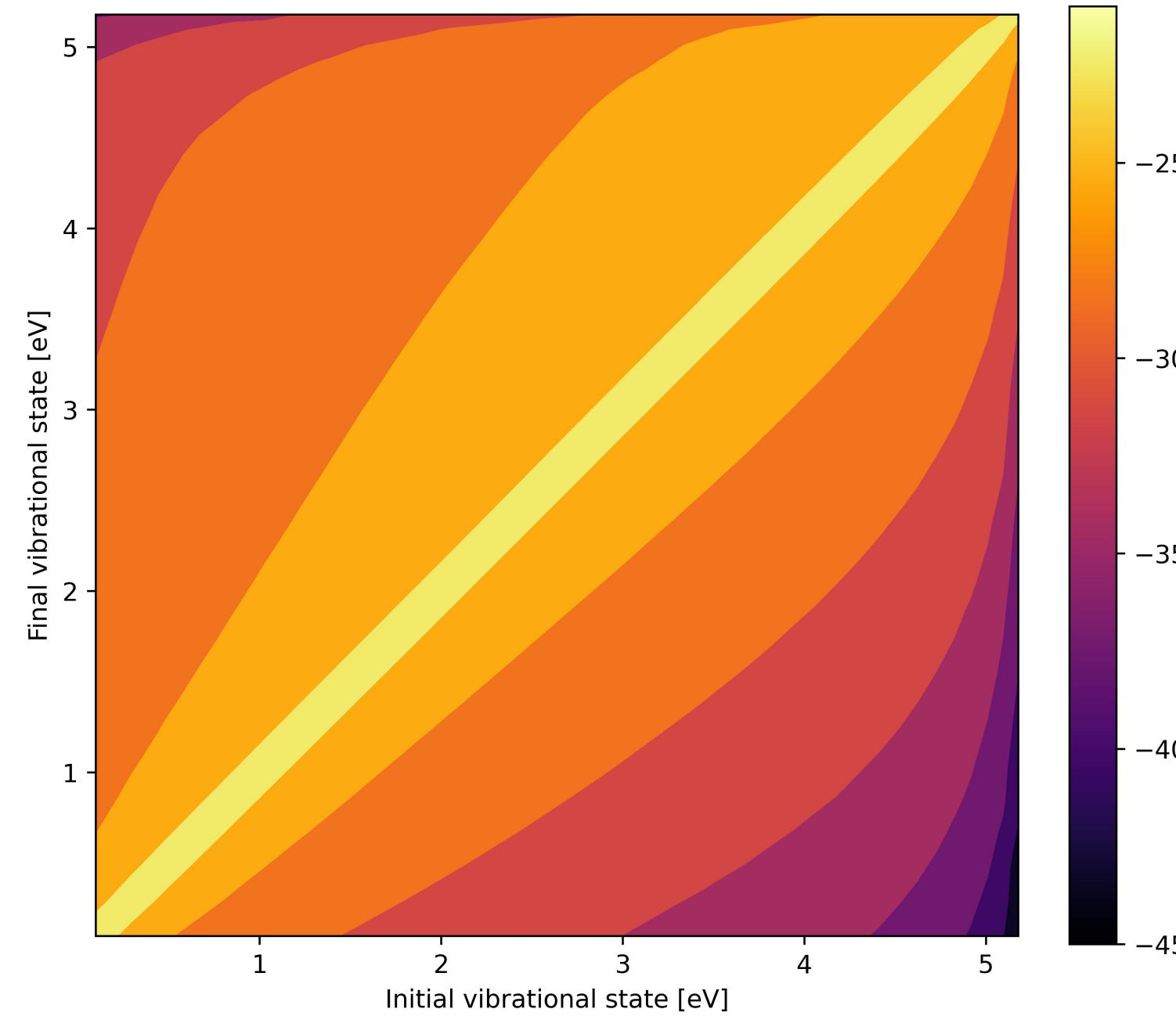
Original/Known - Dunham (36 state ladder)



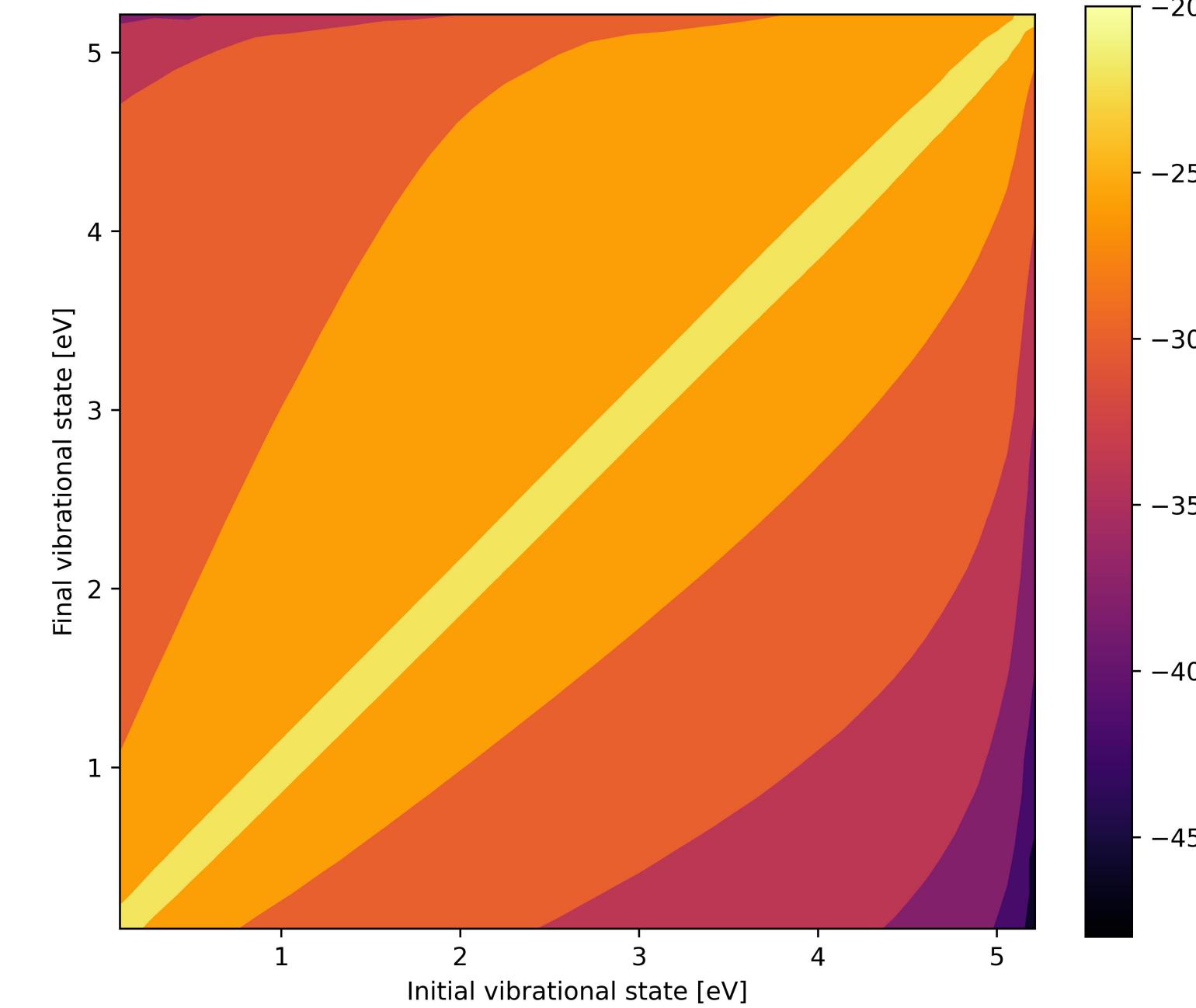
Mapped/Unknown - Varandas (46 state ladder)

O₂-Ar bound-bound rates at 10000 K

Results – Mapping – Radial Basis Functions



Original/Known - Dunham (36 state ladder)



Mapped/Unknown - Varandas (46 state ladder)

O₂-Ar bound-bound **log(rates)** at 10000 K



OBJECTIVE 2: PERFORM STATE-TO-STATE SIMULATIONS OF OXYGEN-ARGON FLOWS



Approach – State-to-State Modeling

- Approach 1: perform state-to-state simulations in 1D code designed for argon-oxygen mixtures¹
 - It works!
 - Difficult to test different configurations/mixtures

$$\frac{dn_v}{dx} = \sum_s (R_{v,s} n_O^2 n_s - D_{v,s} n_v n_s) + \sum_s (K_{v',v} n'_v n_s - K_{v,v'} n_v n_s), \quad v = 0, \dots, v_{max}$$

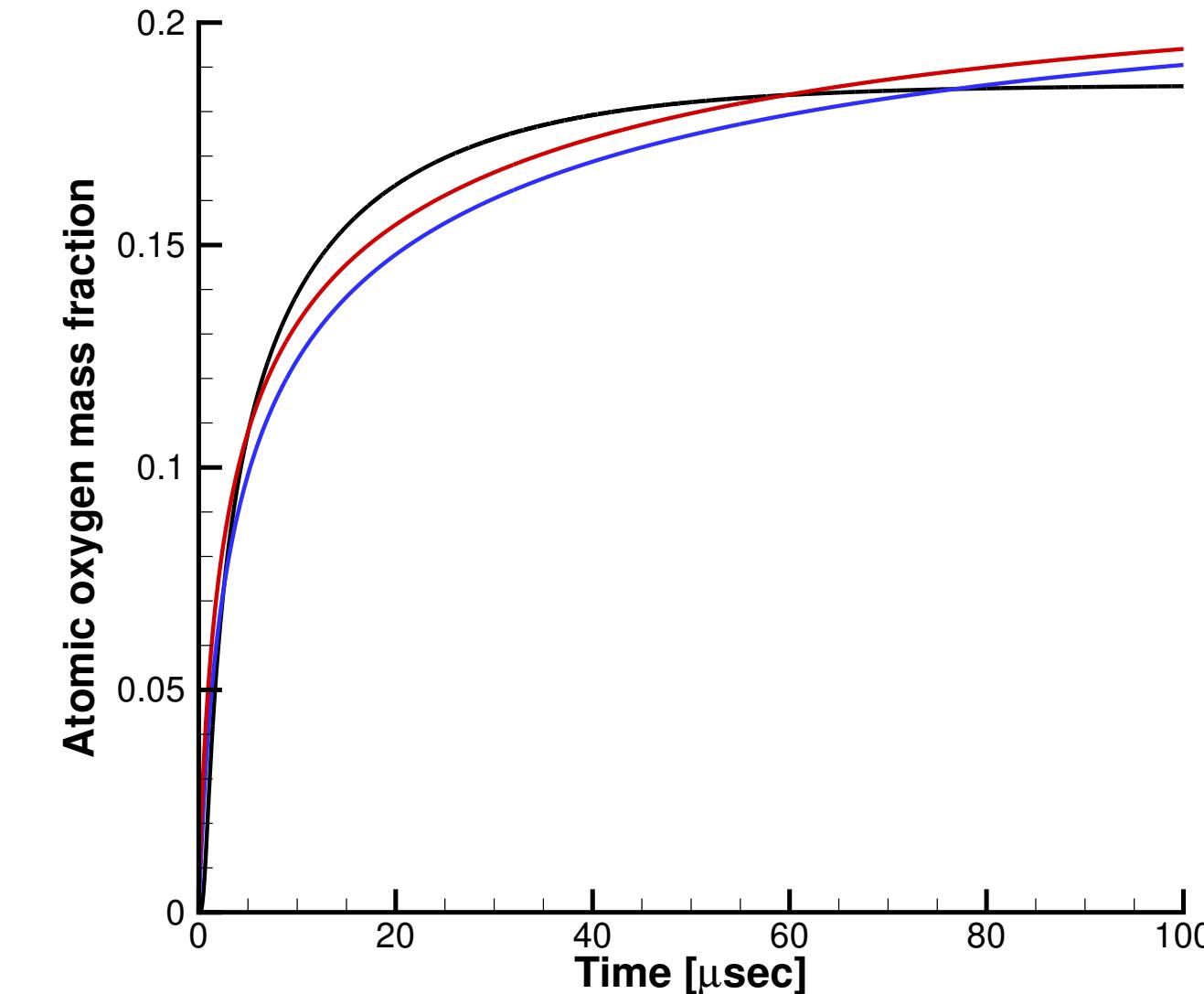
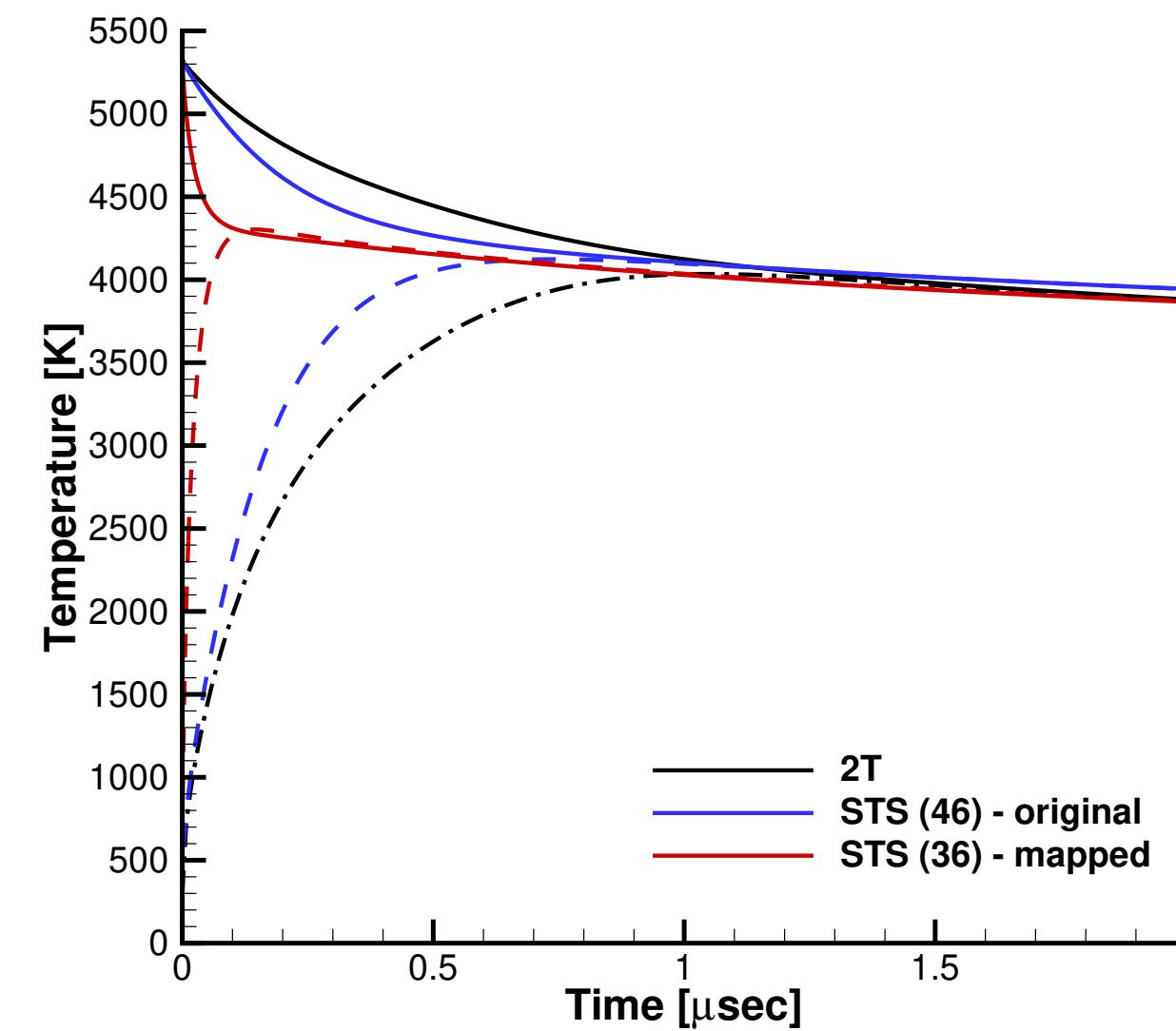
- Approach 2: implement state-to-state capabilities into Mutation++, which is coupled to existing CFD codes
 - In development
 - More easily extended to different configurations/mixtures

¹Hanquist, Chaudhry, Boyd, Streicher, Krish, and Hansen, “Detailed Thermochemical Modeling of O₂-Ar Mixtures in Reflected Shock Tube Flows,” AIAA Paper 2020-3275.

²Scoggins, Leroy, Bellas-Chatzigeorgis, Dias, and Magin. “Mutation++: MUlticomponent Thermodynamic And Transport properties for IONized gases in C++”. SoftwareX 12, 2020.

Results – Assess RBF mapping approach

- Mach 9.44 oxygen flow at \sim 40 km altitude ¹
- O₂-O₂ and O₂-O rates²

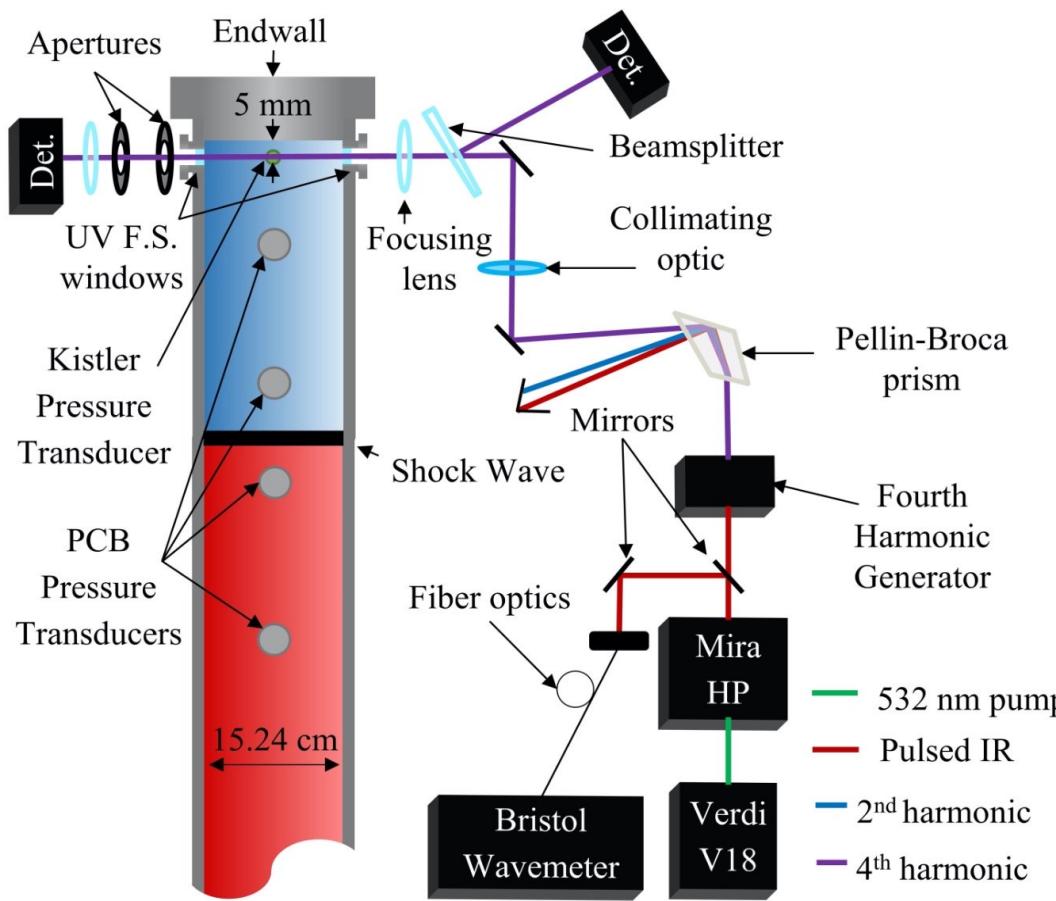


¹Ibraguimova, Sergievskaya, Levashov, Shatalov, Tunik, and Zabelinkii, The Journal of Chemical Physics, 2013.

²Andrienko and Boyd, Physics of Fluids, 2015; Chemical Physics, 2017; The Journal of Chemical Physics, 2016.

Approach 1 – State-to-State Modeling

- Oxygen diluted in argon flows¹
- O₂-O₂ and O₂-O rates²
- O₂-Ar rates³



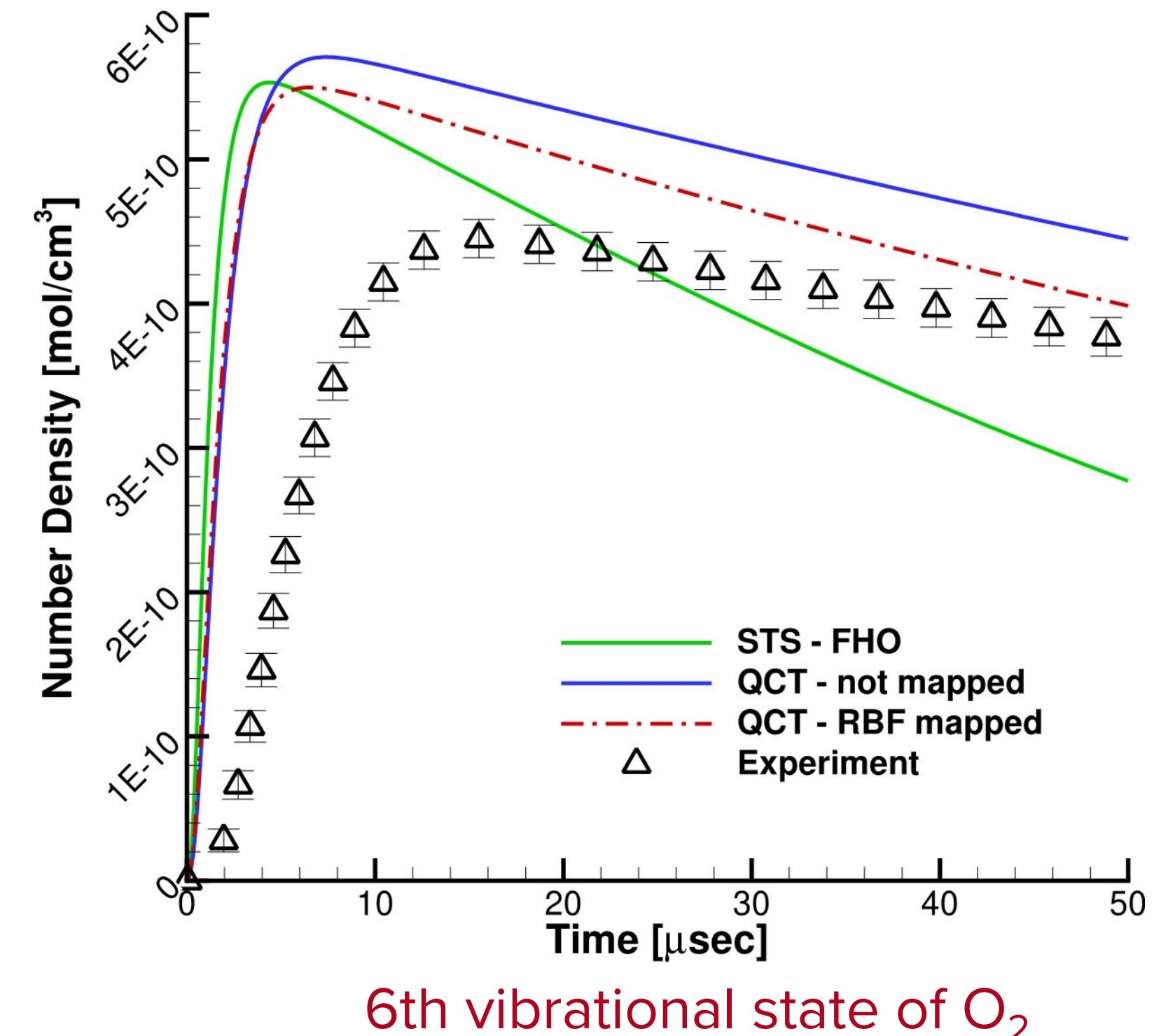
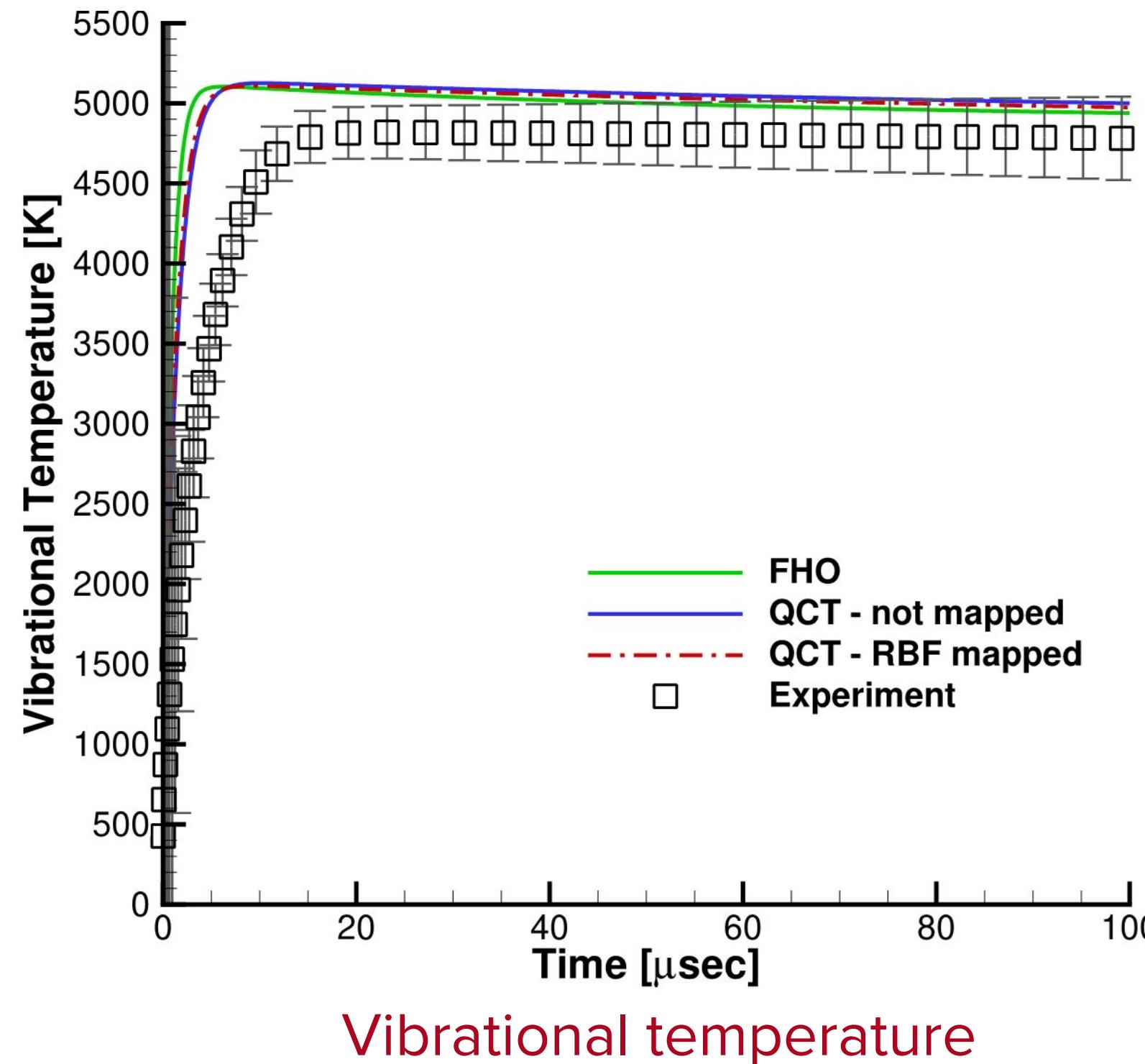
Case	5280 K Case	10710 K Case
<i>Shock Velocity</i>		
Incident [km/s]	1.55	2.24
Reflected [km/s]	0.79	1.09
<i>Before Incident Shock</i>		
Composition [mol fraction O ₂]	0.02	0.05
Pressure [torr]	2.25	0.08
Temperature [K]	296	296
<i>Between Incident/Reflected Shock</i>		
Composition [mol fraction O ₂]	0.02	0.05
Pressure [torr]	65.4	4.56
Temperature [K]	2,400	4,700
<i>After Reflected Shock</i>		
Pressure [torr]	344	28
Temperature [K]	5,300	10,700

¹Streicher, Krish, Hanson, Hanquist, Chaudhry, and Boyd, “Shock-tube measurements of coupled vibration-dissociation time-histories and rate parameters in oxygen and argon mixtures from 5,000-10,000 K,” Physics of Fluids, 2020.

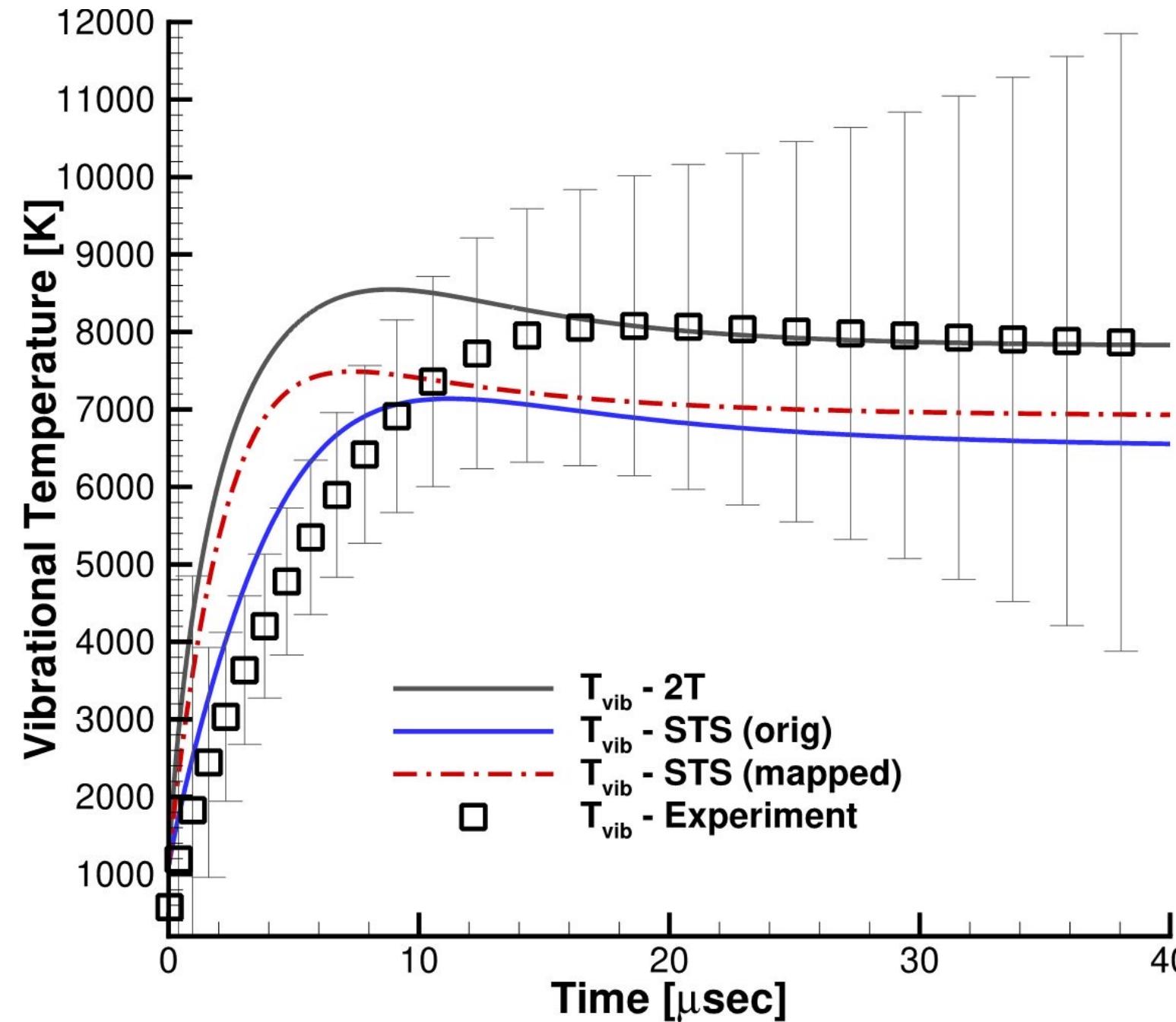
²Andrienko and Boyd, Physics of Fluids, 2015; Chemical Physics, 2017; The Journal of Chemical Physics, 2016.

³Kim and Boyd, Chemical Physics, 2014.

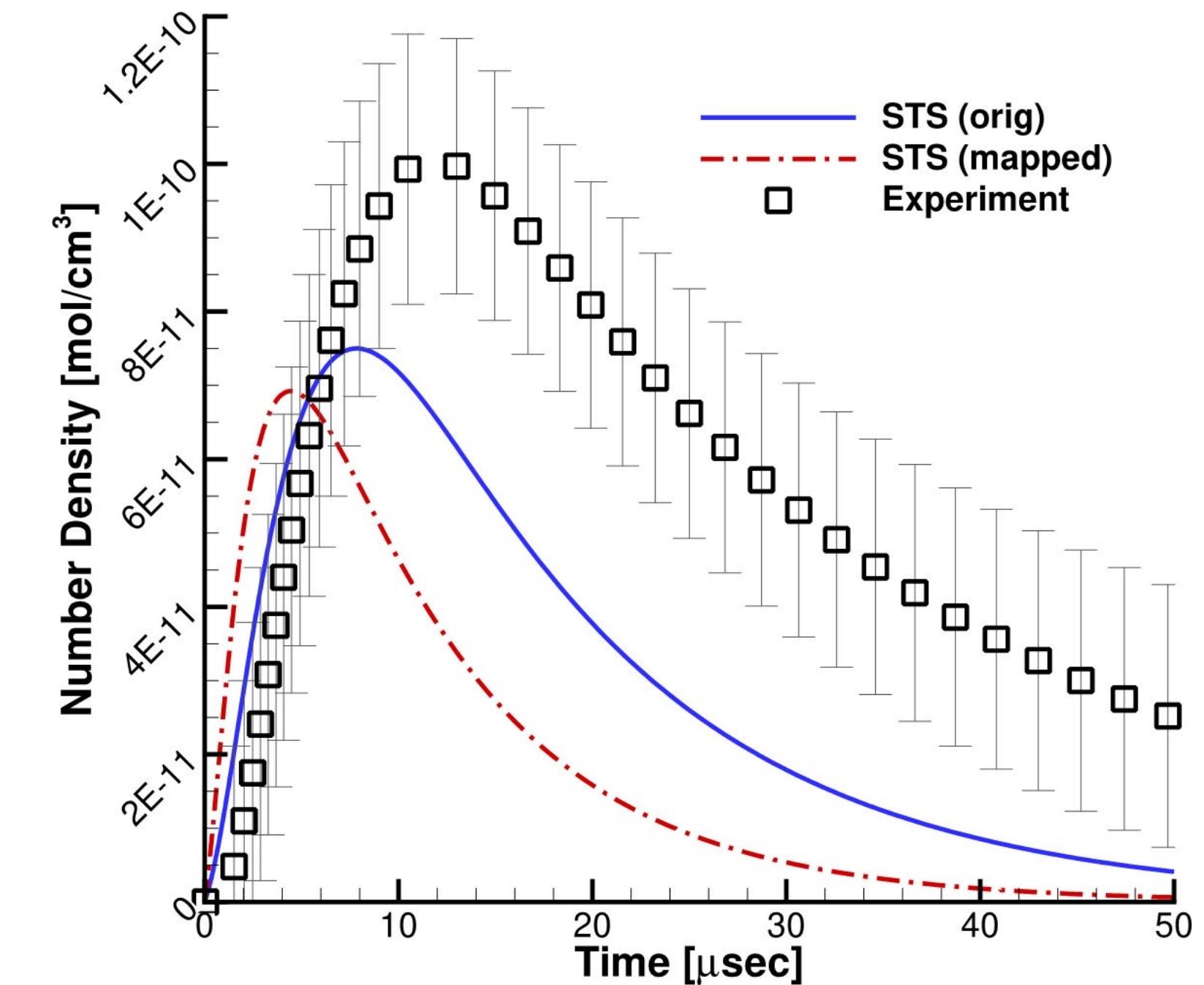
Results – Diluted Oxygen Flows – 5280 K case



Results – Diluted Oxygen Flows – 10,710 K case

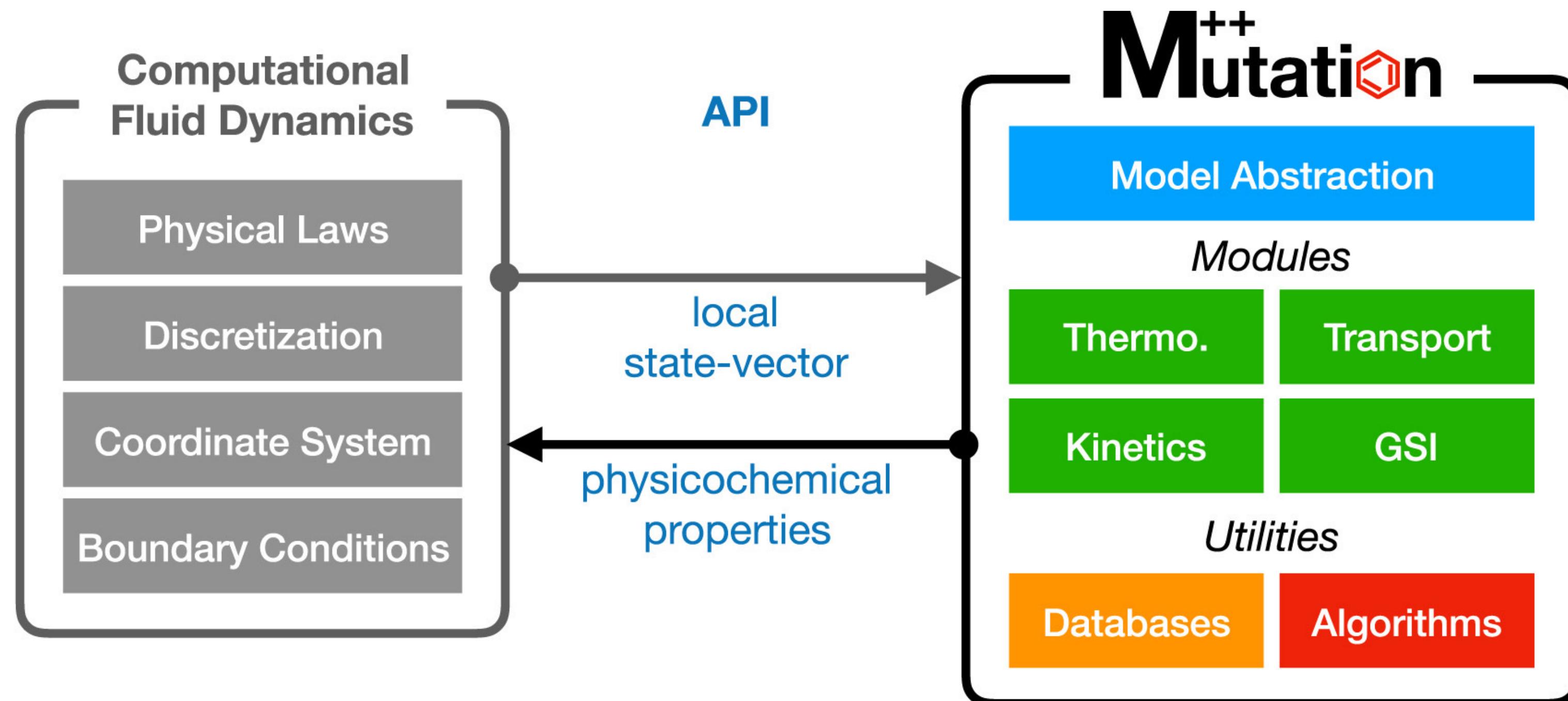


Vibrational temperature



6th vibrational state of O_2

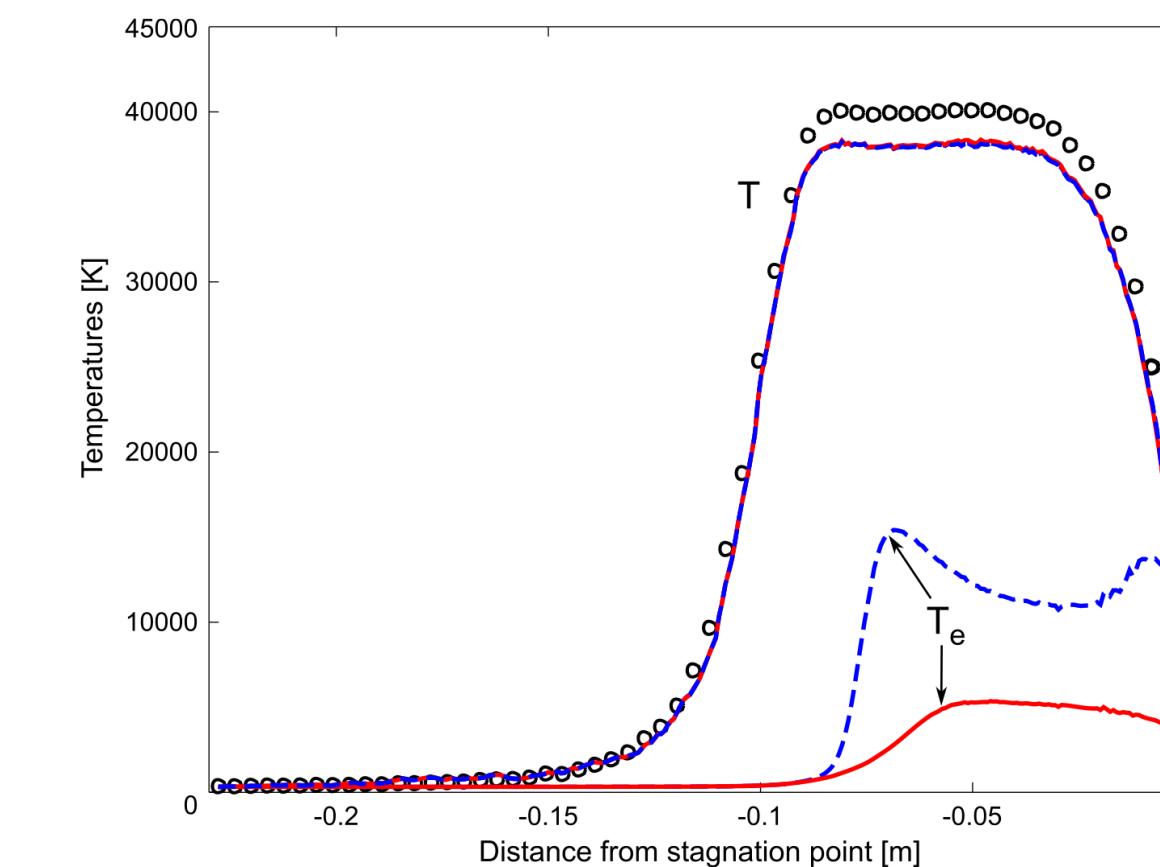
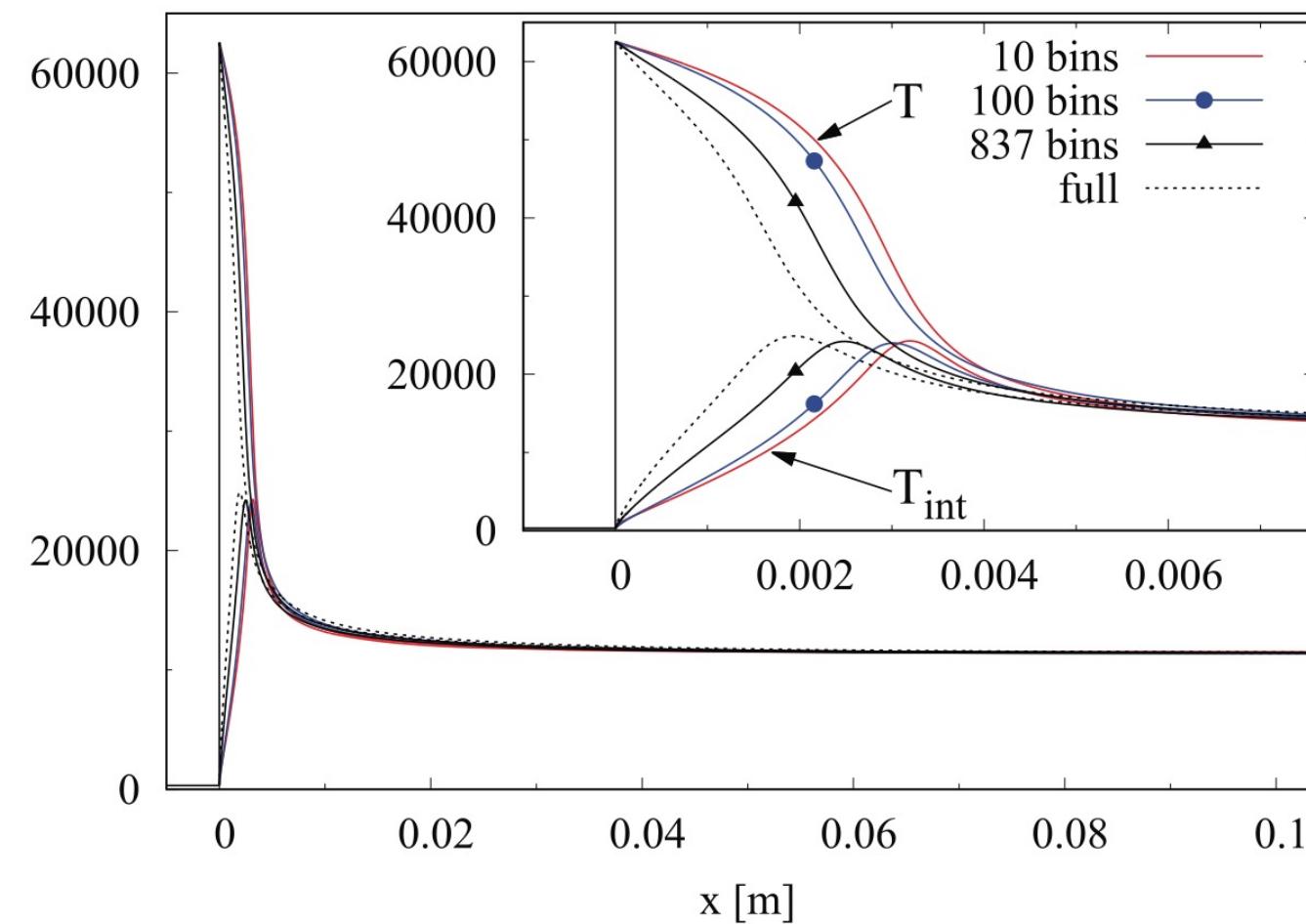
Approach 2 – State-to-State and Mutation++



Scoggins, Leroy, Bellas-Chatzigeorgis, Dias, and Magin. "Mutation++: MULTicomponent Thermodynamic And Transport properties for IONized gases in C++". SoftwareX 12, 2020.

Approach 2 – State-to-State and Mutation++

- We are not the first to perform state-to-state in Mutation++
 - Rovibrational binning of nitrogen - Torres et al.
 - Collisional radiative of electronically excited argon flows - Boccelli et al.
 - *But we are trying to create a generic state-resolved framework in Mutation++*



¹Torres, Bellas-Chatzigeorgis, and Magin, “How to build coarse-grain transport models consistent from the kinetic to fluid regimes ,” Physics of Fluids, 2020.

²Boccelli, Bariselli, Dias, and Magin, “Lagrangian diffusive reactor for detailed thermochemical computations of plasma flows ,” Plasma Sources Sci. Technol., 2019.

Approach 2 – State-to-State and Mutation++

- Mutation++ requires:
 - Mixtures ✓
 - Rates ✓
 - Thermodynamic database
 - Specific Heat
 - Enthalpy
 - Entropy
 - Gibbs Free Energy

Approach 2 – State-to-State and Mutation++

$$s_{\text{tr}} = NK \left[1 + \ln \left(\frac{Q}{N} \right) + T \partial_T [\ln(Q)] \right]$$

$$e_{\text{tr}} = RT^2 \partial_T [\ln(Q)]$$

$$h_{\text{tr}} = e_{\text{tr}} + pv$$

$$(c_p)_{\text{tr}} = \left(\frac{\partial e_{\text{tr}}}{\partial T} \right)_V + R$$

$$g_{\text{tr}} = h_{\text{tr}} - Ts_{\text{tr}}$$

$$s_{\text{int}} = NK \left[\ln \left(\frac{Q}{N} \right) + T \partial_T [\ln(Q)] \right]$$

$$e_{\text{int}} = RT^2 \partial_T [\ln(Q)]$$

$$h_{\text{int}} = e_{\text{int}}$$

$$(c_p)_{\text{int}} = \left(\frac{\partial e_{\text{int}}}{\partial T} \right)_P$$

$$g_{\text{int}} = h_{\text{int}} - Ts_{\text{int}}$$

Approach 2 – State-to-State and Mutation++

Translational Mode

$$Q_{\text{tr}} = \sum_{a \in \mathcal{A}} \exp\left(-\frac{(\varepsilon_{\text{tr}})_a}{kT}\right) = \sum_{n_1, n_2, n_3} \exp\left(-\frac{h^2}{8mkT} \left[\frac{n_1^2}{a_1^2} + \frac{n_2^2}{a_2^2} + \frac{n_3^2}{a_3^2} \right]\right)$$

$$\begin{aligned}s_{\text{tr}} &= Nk \left[1 + \ln\left(\frac{Q_{\text{tr}}}{N}\right) + T\partial_T[\ln(Q_{\text{tr}})] \right] \\ &= R \left[\frac{5}{2} \ln(kT) - \ln(P) + \ln\left(\left[\frac{2\pi m}{h^2}\right]^{3/2}\right) + \frac{5}{2} \right]\end{aligned}$$

$$h_{\text{tr}} = e_{\text{tr}} + pv = RT^2\partial_T[\ln(Q_{\text{tr}})] + RT = \frac{5}{2}RT$$

$$(c_p)_{\text{tr}} = \left(\frac{\partial e_{\text{tr}}}{\partial T}\right)_V + R = \frac{3}{2}R + R = \frac{5}{2}R$$

Approach 2 – State-to-State and Mutation++

Rotational Mode

$$Q_{\text{rot}} = \sum_{j=0}^{\infty} (2J+1) \exp\left(-\frac{\theta_{\text{rot}} J(J+1)}{T}\right) \approx \int_0^{\infty} (2J+1) \exp\left[-\frac{\theta_{\text{rot}}}{T} J(J+1)\right] dJ = \frac{T}{\sigma \theta_{\text{rot}}}$$

$$s_{\text{rot}} = -\frac{\partial}{\partial T} [-NkT \ln [Q_{\text{rot}}]] = R \left[\ln\left(\frac{T}{\sigma \theta_{\text{rot}}}\right) + 1 \right]$$

$$h_{\text{rot}} = RT^2 \frac{\partial}{\partial T} \left[\ln\left(\frac{T}{\theta_{\text{rot}}}\right) \right] = RT$$

$$(c_p)_{\text{rot}} = \left(\frac{\partial e_{\text{rot}}}{\partial T} \right)_V = R$$

Approach 2 – State-to-State and Mutation++ Vibrational Mode

$$Q_{\text{vib}} = \exp\left(-\frac{\varepsilon_i}{kT}\right)$$

$$s_{\text{vib}} = -\frac{\partial}{\partial T} \left[-NkT \ln \left[\exp\left(-\frac{\varepsilon_i}{kT}\right) \right] \right] = -\frac{\partial}{\partial T} \left[-NkT \left[-\frac{\varepsilon_i}{kT} \right] \right] = 0$$

$$h_{\text{vib}} = RT^2 \frac{\partial}{\partial T} \left[\ln \left(\exp\left(-\frac{\varepsilon_i}{kT}\right) \right) \right] = RT^2 \left(\frac{\varepsilon_i}{kT^2} \right) = R\varepsilon_i$$

$$(c_p)_{\text{vib}} = \left(\frac{\partial e_{\text{vib}}}{\partial T} \right)_V = 0$$

Approach 2 – State-to-State and Mutation++

Electronic Mode

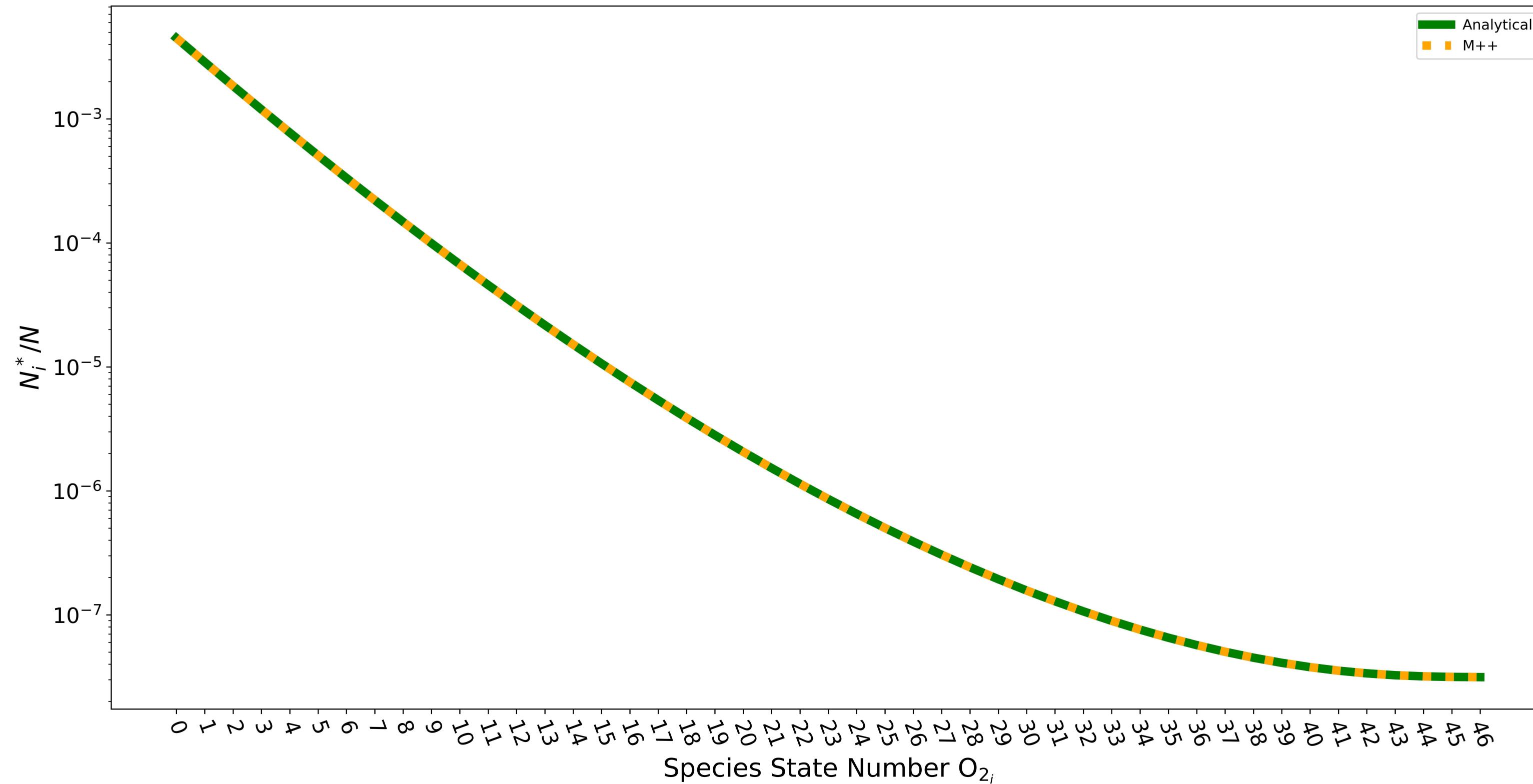
$$Q_{\text{el}} = \sum_i g_i \exp\left(-\frac{\theta_i}{kT}\right)$$

$$\begin{aligned} s_{\text{el}} &= -\frac{\partial}{\partial T} \left[-NkT \ln \left[g_0 \exp\left(-\frac{\theta_0}{T}\right) + g_1 \exp\left(-\frac{\theta_1}{T}\right) \right] \right] \\ &= R \left[\ln(g_0) + \ln \left\{ 1 + \frac{g_1}{g_0} \exp\left(-\frac{\theta_1}{T}\right) \right\} + \frac{(g_1/g_0)(\theta_1)(T) \exp(-\theta_1/T)}{1 + (g_1/g_0) \exp(-\theta_1/T)} \right] \end{aligned}$$

$$h_{\text{el}} = RT^2 \frac{\partial}{\partial T} \left[g_0 + g_1 \exp\left(-\frac{\theta_1}{T}\right) \right] = RT^2 \left[-\frac{g_1 \theta_1 \left(-\frac{1}{T^2} \exp\left(-\frac{\theta_1}{T}\right) \right)}{g_0 + g_1 \exp\left(-\frac{\theta_1}{T}\right)} \right] = \frac{R\theta_1 \left(\frac{g_1}{g_0} \right) \exp\left(-\frac{\theta_1}{R}\right)}{1 + \frac{g_1}{g_0} \exp\left(-\frac{\theta_1}{T}\right)}$$

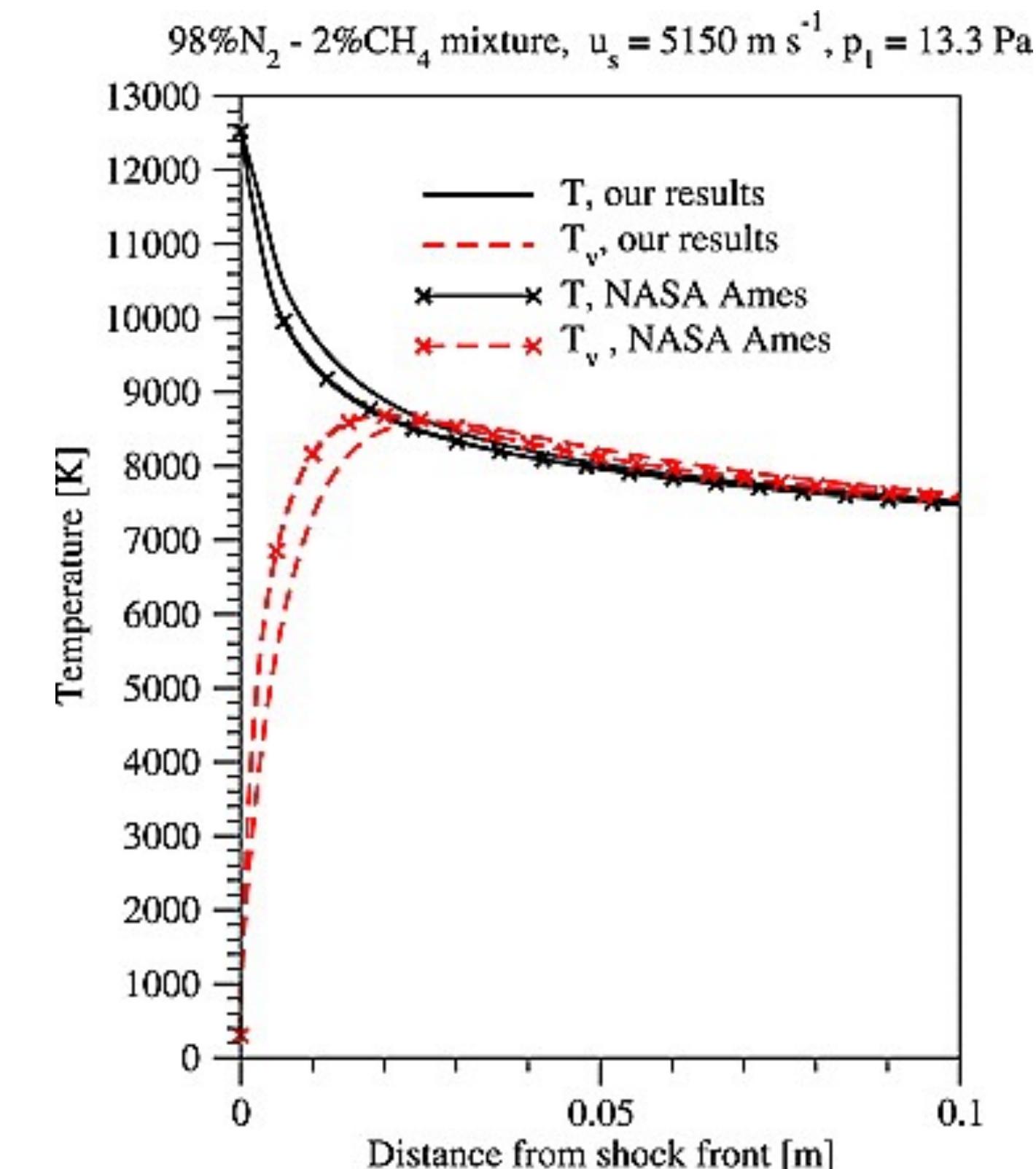
$$(c_p)_{\text{el}} = \left(\frac{\partial e_{\text{el}}}{\partial T} \right)_V = R \left(\frac{\theta_1}{T} \right)^2 \frac{\frac{g_1}{g_0} \exp\left(-\frac{\theta_1}{T}\right)}{\left[1 + \frac{g_1}{g_0} \exp\left(-\frac{\theta_1}{T}\right) \right]^2}$$

Boltzmann Distribution at Equilibrium using Thermodynamic Database at 5000 K



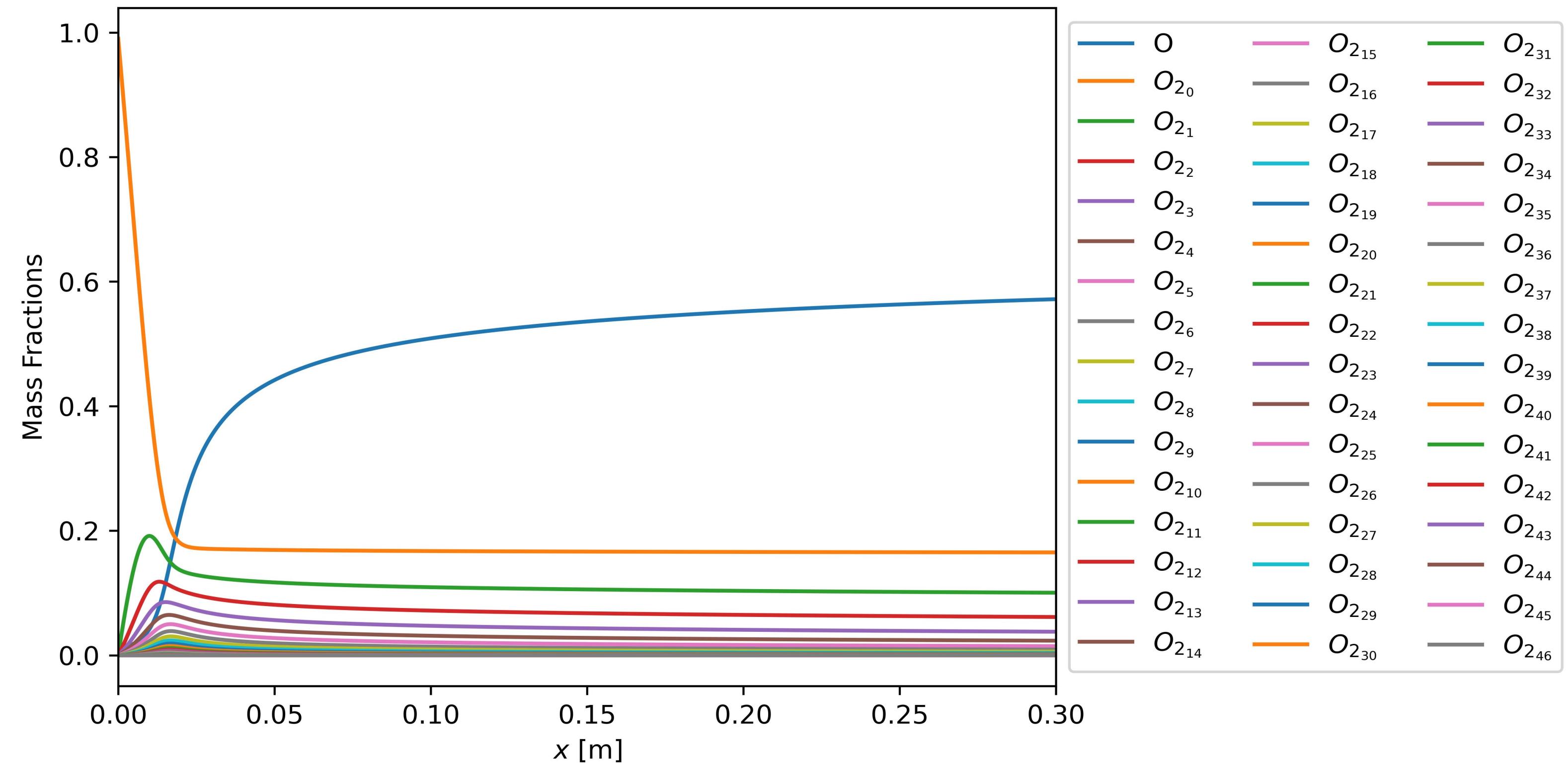
brODErs

- brODErs is a collection of ODE solvers for chemically reacting hypersonic flows developed at the von Karman Institute for Fluid Dynamics
- The downstream flow field is computed by solving one-dimensional conservation equations of mass, momentum, global energy, as well as conservation of vibrational energy of the
- Problem Setup:
 - Freestream Pressure = 31.18 Pa
 - Freestream Temperature = 250 K
 - Freestream Velocity = 5255 m/s

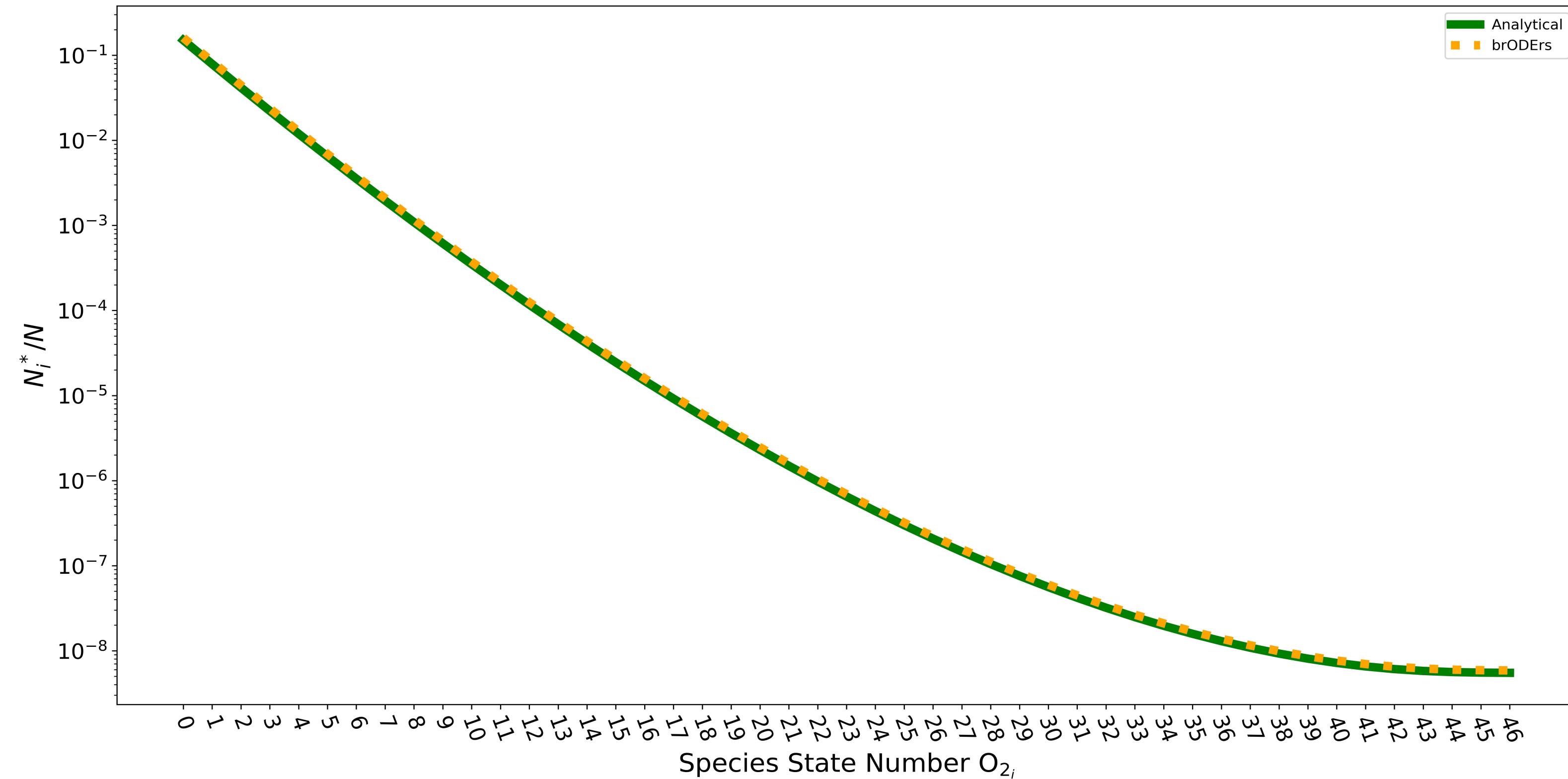


Magin, T. E., Caillault, L., Bourdon, A., & Laux, C. O. (2006). Nonequilibrium radiative heat flux modeling for the Huygens entry probe. *Journal of Geophysical Research: Planets*, 111(E7).

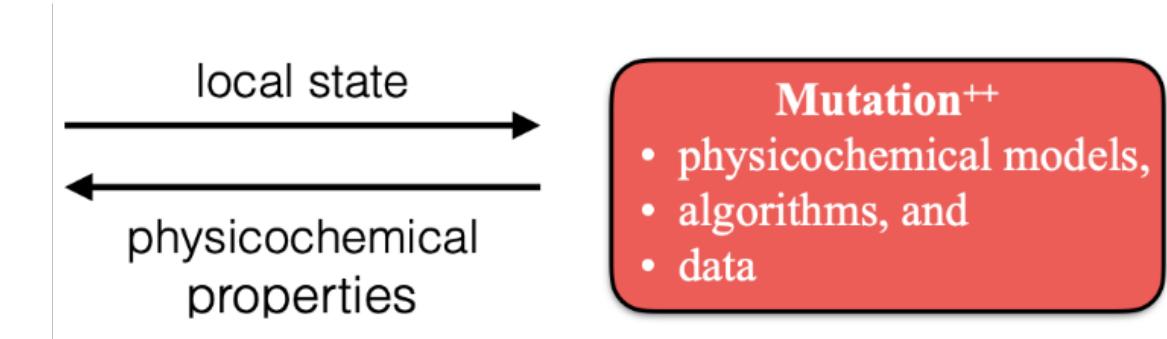
brODErs



brODErs

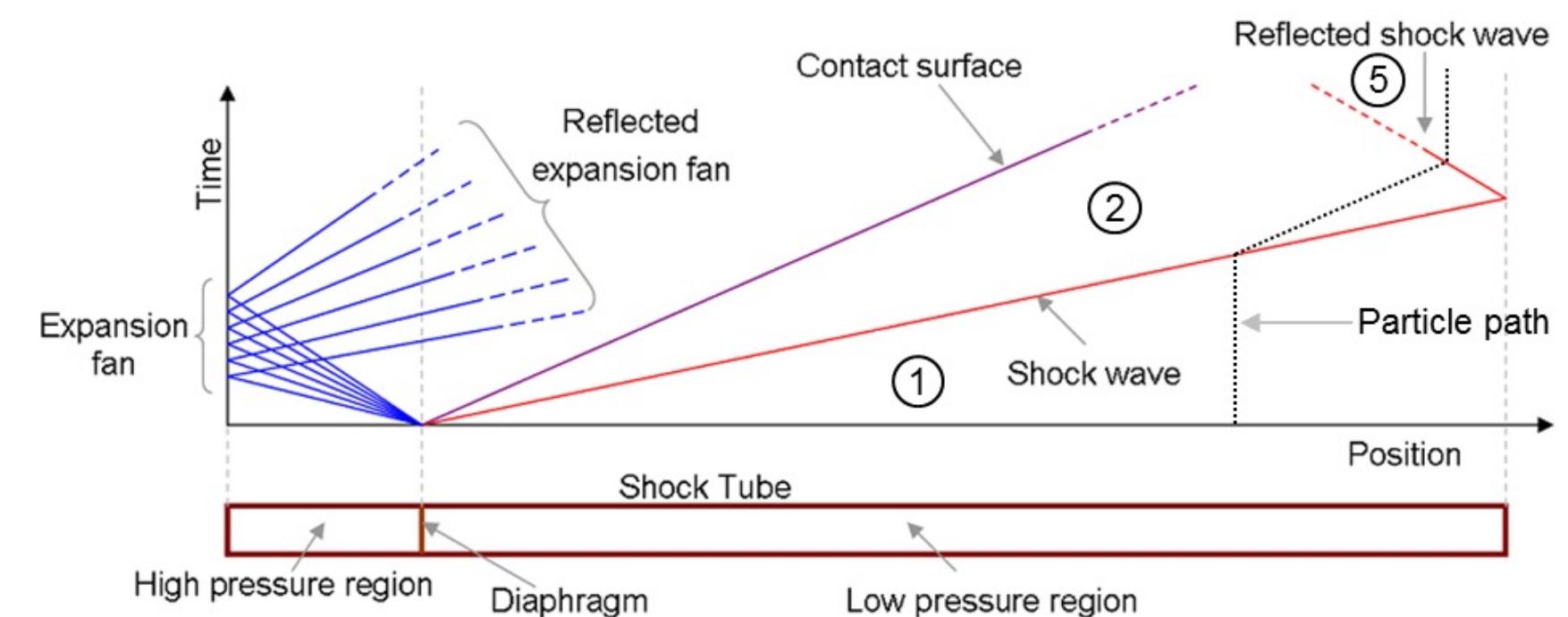


CFD – Unsteady

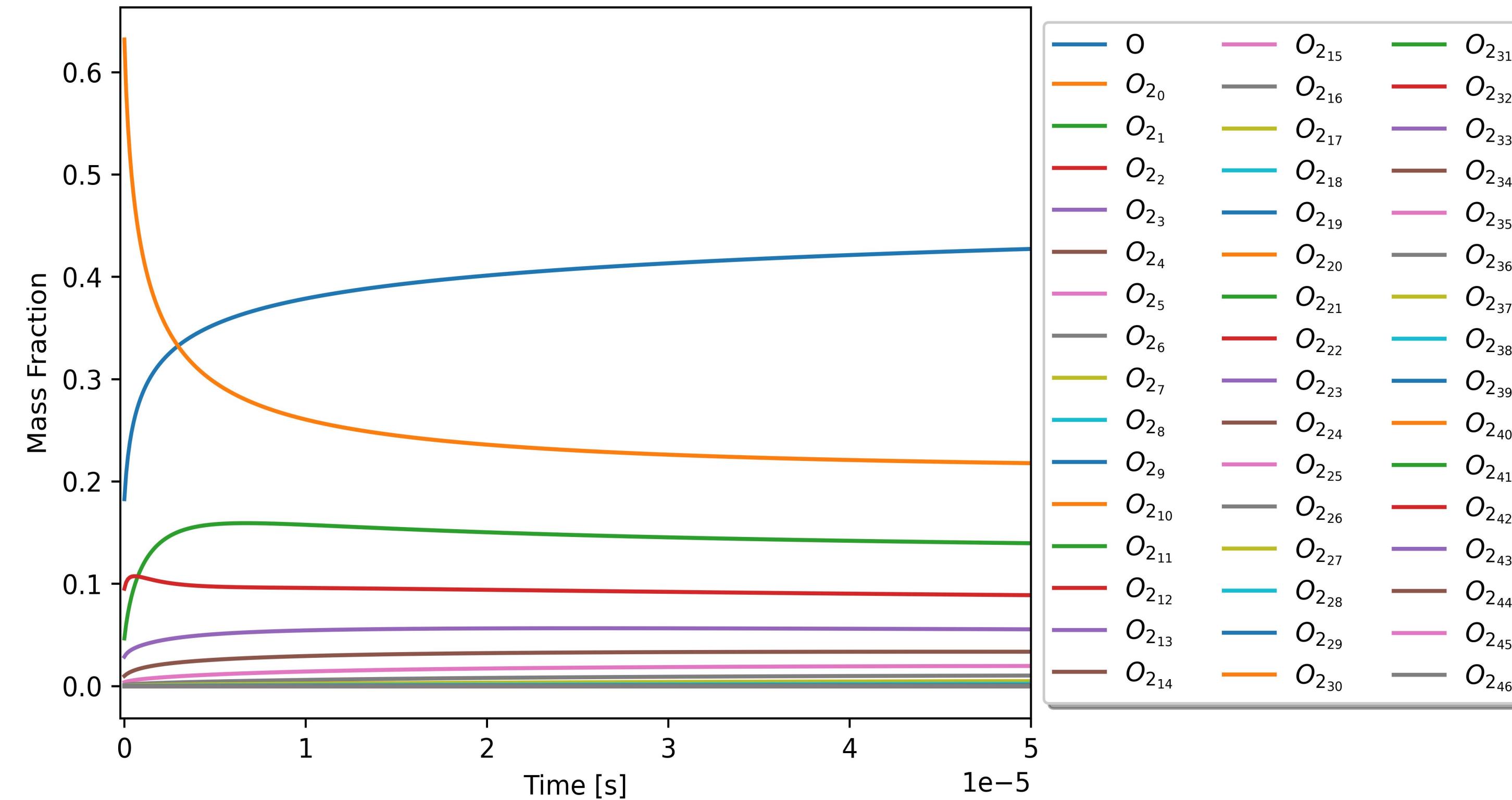


- Euler Solver
- Mesh: 5x5 Grid
- Boundary Conditions
 - Symmetry walls (no boundary layer effects)
- SU2-NEMO CFD Code
- Test Case: Heat Bath (section 5 of reflected shock tube)
- $T = 15,000 \text{ K}$, $P = 11040 \text{ Pa}$

Maier, et al., AIAA Paper 2023-3488

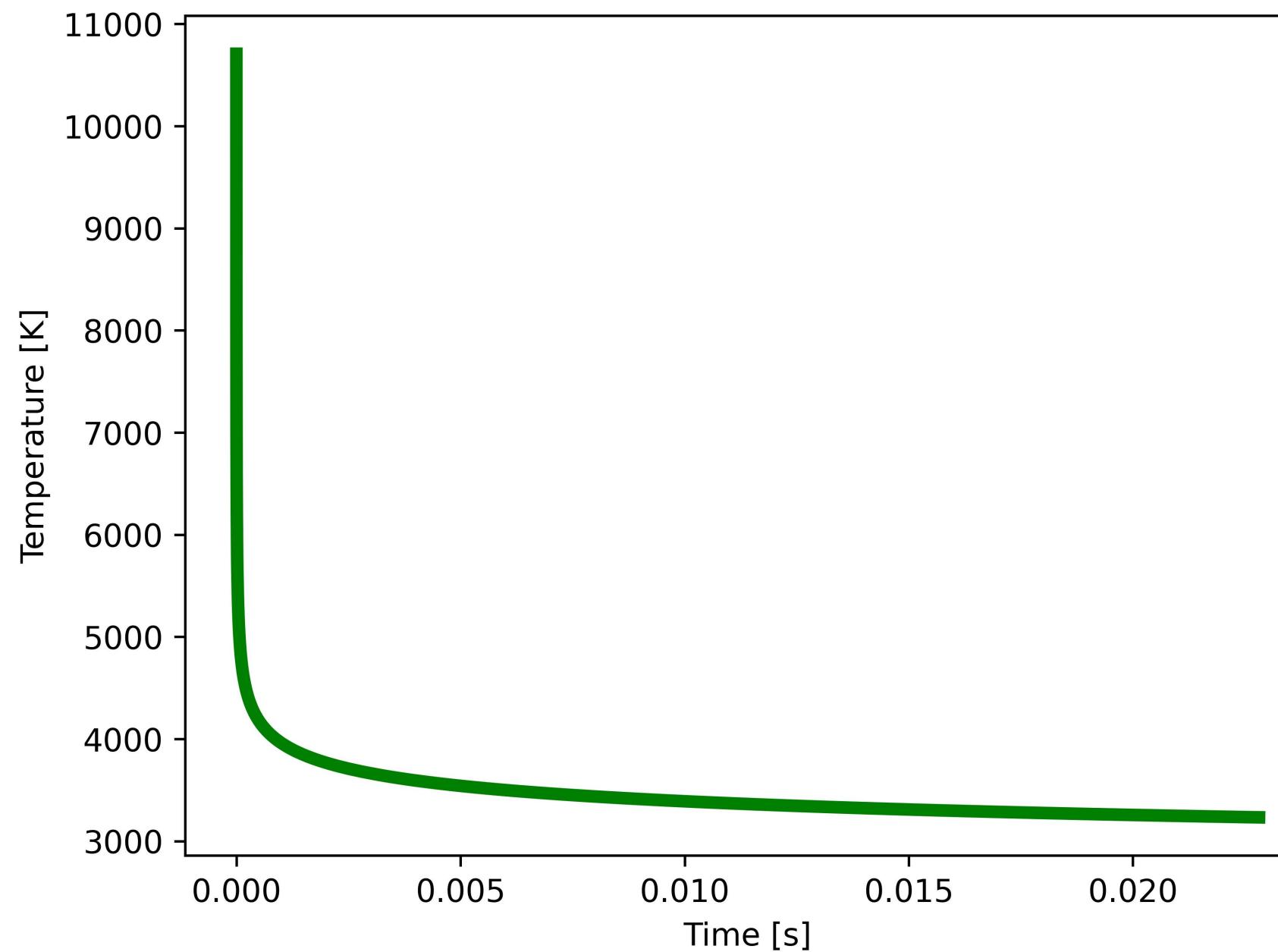


CFD – Unsteady

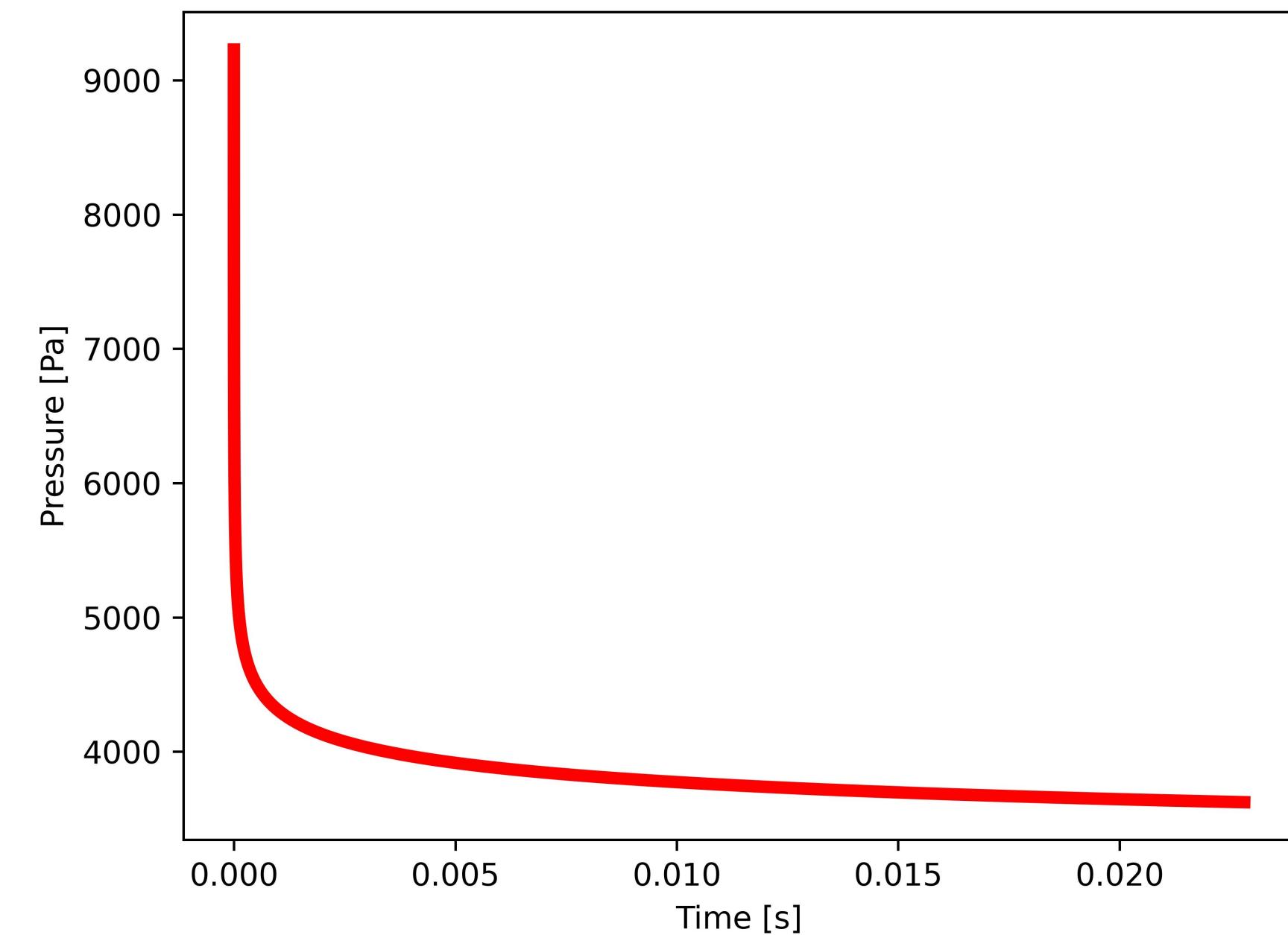


CFD – Unsteady

Translational Temperature

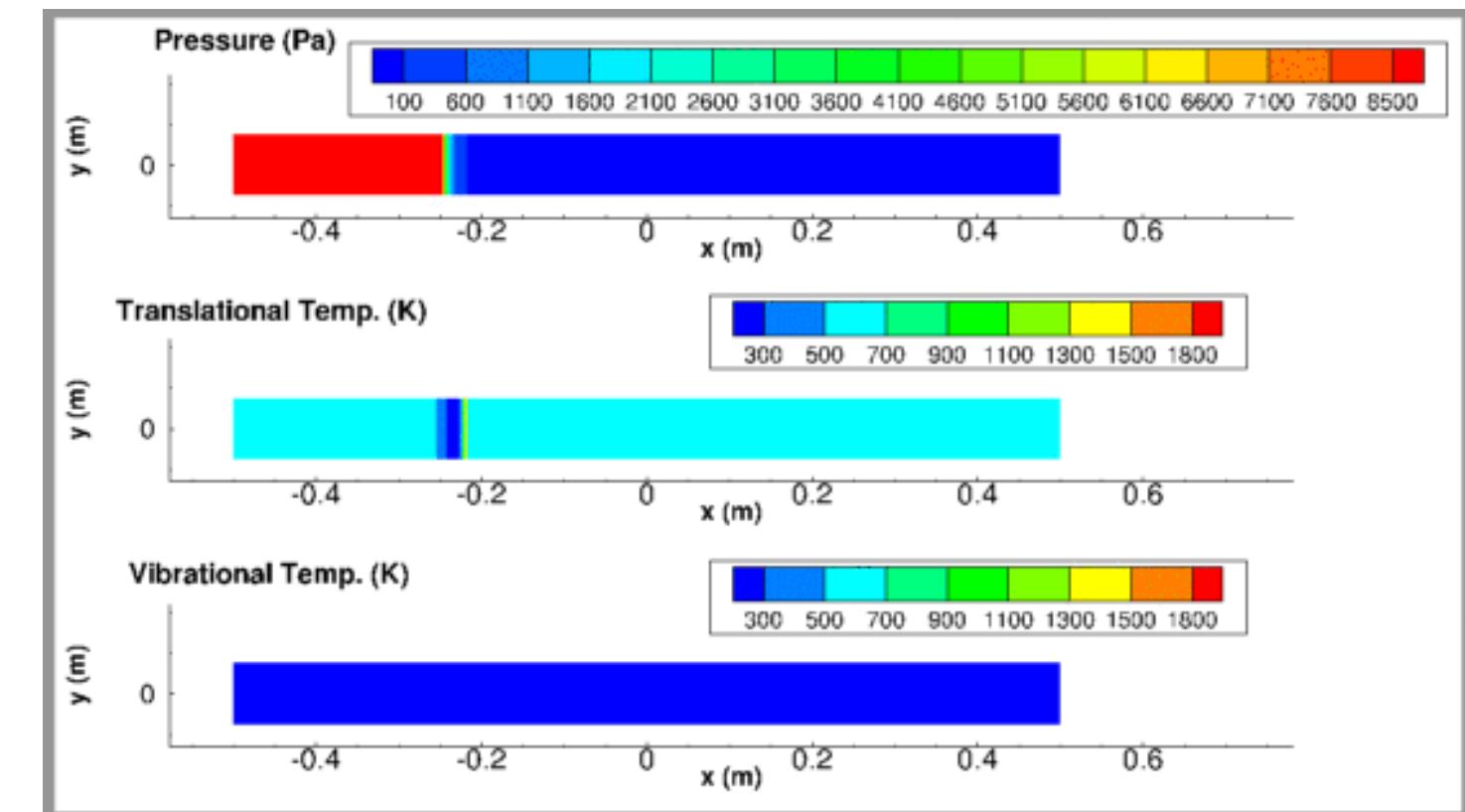


Pressure



Future work

- Refine mapping approach
 - Binning approach (10 state?)
 - Machine learning/physics informed for rates
- Run in shock tube to assess thermal bath assumption
 - Are we not matching experimental data due to rates or assumptions (e.g., heat bath)?
 - Our talk on Monday
- Assess modeling approach with recent Hansen group data



Coupled vibration-dissociation time-histories and rate measurements in shock-heated, nondilute O₂ and O₂-Ar mixtures from 6000 to 14 000 K 

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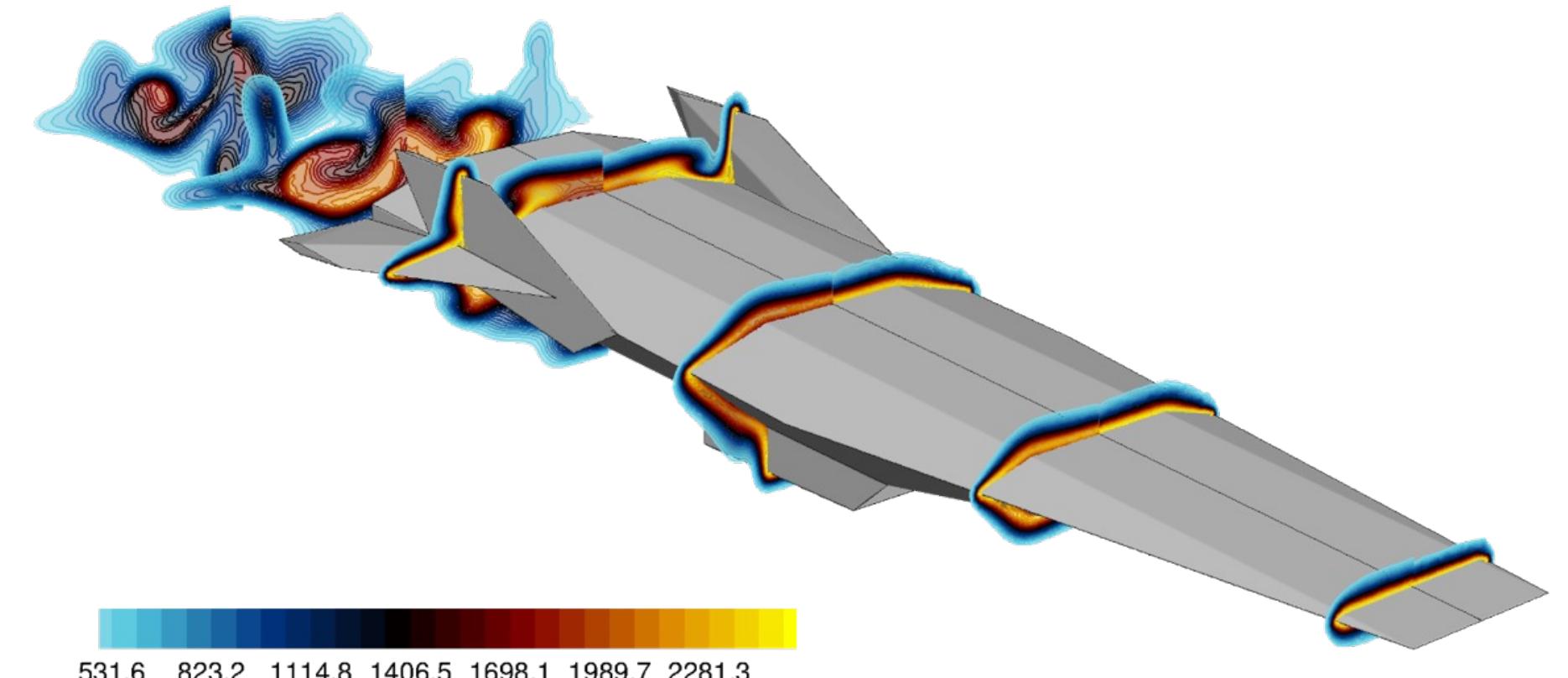
Jesse W. Streicher,^{a)}  Ajay Krish,  and Ronald K. Hanson 



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QUESTIONS/DISCUSSION

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