Problem 1

First, we observe that L can be transformed into a normally distributed random variable with mean 0 and standard deviation 1. We do this by creating a new random variable $Z = \frac{L-\mu}{\sigma}$. The probability $P(L > 12)$ can now be expressed as $P(Z > \frac{12-0}{\sigma})$. We can use the R function qnorm to find the value c for which $P(Z < c) = 1 - p$ where p is 2.5%, as given in the problem. Thus, calling `qnorm( 1 - 0.025 )` gives us 1.44 as the value for c and solving for σ in the equation $c = \frac{12}{\sigma}$ yields $\sigma \approx 6.123$. We can now plug this value into `pnorm()` to find $P(L > 12)$ when σ increases by 10%:

```
1 - pnorm( 12, 0, 1.1 * ( 12 / qnorm( 0.975 ) ) )
```

This produces a value of approximately 3.74%, so flooding increased from 2.5% of days to 3.74% of days.

Problem 2

$P(X > 3.0)$ can be easily derived by integrating the density function over all $X > 3.0$.

$$\begin{aligned} P(X > 3.0) &= \int_3^\infty \frac{1}{t^2} dt \\ &= -\frac{1}{t}\bigg|_3^\infty \\ &= \frac{1}{3} \end{aligned}$$

To find $E(X^{0.5})$, we need to multiply $t^{0.5}$ by the probability for each t and sum them all together (i.e. integrate).

$$\begin{aligned} E(X^{0.5} &= \int_1^\infty t^{0.5} \cdot \frac{1}{t^2} dt \\ &= \int_1^\infty \frac{1}{t^{1.5}} dt \\ &= -\frac{2}{t^{0.5}}\bigg|_1^\infty \\ &= 2 \end{aligned}$$

Simulation verifies both those calculations. To perform the simulation, we needed to find a way to generate random numbers from the distribution. From the book, we know that we can take the inverse of the cdf and then substitute random numbers from the uniform distribution for t. First, we find the cdf:

$$\begin{aligned} F_X(t) &= \int_1^t \frac{1}{t^2}\ dt \\ &= -\frac{1}{t}\bigg|_1^t \\ &= 1 - \frac{1}{t} \end{aligned}$$

Then, the inverse can be found by swapping F_X and t, substituting F_X^{-1} for F_X, and solving for F_X^{-1}.

$$\begin{aligned} t &= 1 - \frac{1}{F_X^{-1}} \\ \Rightarrow F_X^{-1} &= \frac{1}{1-t} \end{aligned}$$

Problem 3

There were two steps involved in proving $Cov(V) = c^2(A'A)^{-1}$.

First, simplify V:

$$\begin{aligned} V &= (A'A)^{-1}A'U \\ &= A^{-1}A'^{-1}A'U \\ &= A^{-1}U \end{aligned}$$

Next, plug the simplified V into the $Cov(V)$ formula:

$$\begin{aligned} Cov(V) &= Cov(A^{-1}U) \\ &= A^{-1}Cov(U)A'^{-1} \\ &= A^{-1}(c^2I)A'^{-1} \\ &= c^2A^{-1}IA'^{-1} \\ &= c^2A^{-1}A'^{-1} \\ &= c^2(A'A)^{-1} \end{aligned}$$