In [1]:
```
load('g2_motives.sage')
```

In [2]:
```
#in this file, we give an example of using the G2^c-Sp(6) lift
#we also verify the claims about such G2 lifts made in the "G2 computations'
```

In [3]:
```
#Tsplit_list=make_T_list_a1_vecs_split([1,b,c,d,e,f])
#a_Sp6_g_FC(Tsplit_list,k1,k2,[r1,r2,r3,r4,r5,r6]).expand()
```

In [5]:
```
%time Tsplit_list=make_T_list_a1_vecs_split([1,1,1,1,1,1])
```

```
CPU times: user 601 ms, sys: 8.9 ms, total: 610 ms
Wall time: 609 ms
```

In [6]:
```
len(Tsplit_list)
```

Out[6]:
```
1512
```

In [7]:
```
#thus, there are 1512 rank one T in J_R with projection T0=[1,1,1,1,1,1]
```

In [11]:
```
def poly_to_coef_list(poly,k1,k2):
    #this function converts a homogeneous polynomial to a list of its coeffi
    coef_list=[]
    for a1 in range(k1+1):
        for b1 in range(k1+1-a1):
            c1=k1-a1-b1
            for a2 in range(k2+1):
                for b2 in range(k2+1-a2):
                    c2=k2-a2-b2
                    temp=poly.coefficient(v1^(a1)*v2^(b1)*v3^(c1)*w1^(a2)*w2
                    coef_list.append(temp)
    return coef_list
```

In [12]:
```
def quick_dim(k1,k2):
    #this function takes three random-ish null pairs,
    #computes the T0=[1,1,1,1,1,1] Fourier coefficient of the weight (k1,k2)
    #then computes the dimension of the span of these Fourier coefficients
    #it thus gives a lower bound on the dimension of the space of G2 lifts o
    FC1=a_Sp6_g_FC(Tsplit_list,k1,k2,[0,0,0,0,0,0]).expand()
    FC2=a_Sp6_g_FC(Tsplit_list,k1,k2,[0,3,0,1,-1,2]).expand()
    FC3=a_Sp6_g_FC(Tsplit_list,k1,k2,[1,1,0,-1,0,0]).expand()
    clist1=poly_to_coef_list(FC1,k1,k2)
    clist2=poly_to_coef_list(FC2,k1,k2)
    clist3=poly_to_coef_list(FC3,k1,k2)
    M=matrix([clist1,clist2,clist3])
    return M.rank()
```

In [13]:
```
%time quick_dim(0,4)
```

```
CPU times: user 3.66 s, sys: 21.4 ms, total: 3.68 s
Wall time: 3.64 s
```

Out[13]: 1

In [14]:
```
%time quick_dim(2,4)
```

```
CPU times: user 6.93 s, sys: 80.6 ms, total: 7.01 s
Wall time: 6.97 s
```

Out[14]: 1

In [15]:
```
%time quick_dim(3,3)
```

```
CPU times: user 7.18 s, sys: 76.3 ms, total: 7.26 s
Wall time: 7.22 s
```

Out[15]: 1

In [16]:
```
%time quick_dim(0,6)
```

```
CPU times: user 4.13 s, sys: 31.5 ms, total: 4.17 s
Wall time: 4.13 s
```

Out[16]: 2

In [17]:
```
%time quick_dim(3,4)
```

```
CPU times: user 9.03 s, sys: 98.5 ms, total: 9.13 s
Wall time: 9.09 s
```

Out[17]: 1

In [18]:
```
%time quick_dim(6,2)
```

```
CPU times: user 10.1 s, sys: 113 ms, total: 10.2 s
Wall time: 10.1 s
```

Out[18]: 1

In [19]:
```
%time quick_dim(5,3)
```

```
CPU times: user 11.2 s, sys: 123 ms, total: 11.3 s
Wall time: 11.3 s
```

Out[19]: 1

In [20]:
```
%time quick_dim(4,4)
```

```
CPU times: user 11.9 s, sys: 137 ms, total: 12 s
Wall time: 12 s
```

Out[20]: 1

In [21]:
```
%time quick_dim(7,2)
```

```
CPU times: user 11.9 s, sys: 138 ms, total: 12 s
Wall time: 12 s
```

Out[21]:
```
1
```

In [22]:
```
%time quick_dim(9,1)
```

```
CPU times: user 12.8 s, sys: 129 ms, total: 13 s
Wall time: 12.9 s
```

Out[22]:
```
1
```

In [23]:
```
%time quick_dim(6,3)
```

```
CPU times: user 14 s, sys: 153 ms, total: 14.2 s
Wall time: 14.1 s
```

Out[23]:
```
2
```

In [24]:
```
%time quick_dim(8,2)
```

```
CPU times: user 14.4 s, sys: 167 ms, total: 14.6 s
Wall time: 14.5 s
```

Out[24]:
```
2
```

In [25]:
```
#As explained to us by Chenevier, there is one-dimensional space of G2^c for
#however, the Arthur Multiplicity Conjecture predicts that this form should
#thus, when we lift to Sp6, we had better get 0 for any Fourier coefficient

%time quick_dim(0,7)
```

```
CPU times: user 4.4 s, sys: 29.7 ms, total: 4.43 s
Wall time: 4.38 s
```

Out[25]:
```
0
```

In [ ]:
```

```