In [1]:
```
load('g2_motives.sage')
```

In [2]:
```
#in this file, we give examples of theta lifting to split G2
#specifically, we verify the claims made about such lifts in the "G2 computa
```

In [3]:
```
pari.allocatemem(40737418240)
```

```
PARI stack size set to 40737418240 bytes, maximum size set to 40737423360
```

In [4]:
```
n=5 #this parameter dictates how many Fourier coefficients we will compute
```

In [5]:
```
%time short_vecs=J_E_2.short_vector_list_up_to_length(n+1, up_to_sign_flag=F
```

```
CPU times: user 23.5 s, sys: 759 ms, total: 24.3 s
Wall time: 24.4 s
```

In [6]:
```
all_short_vecs=flatten(short_vecs,max_level=1)
```

In [7]:
```
len(all_short_vecs)
```

Out[7]:
```
1618815
```

In [9]:
```
initialize_dict4(n,1)
#this will show us in advance which triples (b,c,d) we will compute Fourier
```

Out[9]:
```
{(0, -2, -1): (0),
 (0, -2, 0): (0),
 (0, -2, 1): (0),
 (0, -1, 0): (0),
 (1, -2, -2): (0),
 (1, -2, -1): (0),
 (1, -2, 0): (0),
 (1, -1, 0): (0)}
```

In [10]:
```
#thus to compute Fourier coefficients for the 8 binary cubics above
#we will need to use about 1.6 million elements T in J_R
```

In [11]:
```
%time my_splitting_dictionary=vec_list_to_dict2(all_short_vecs)
```

```
CPU times: user 30.6 s, sys: 399 ms, total: 31 s
Wall time: 31.1 s
```

In [12]:
```
X1_E,Y1_E=list_to_oct_pair_E([1,1,-1],[],[-2,1],Xoct,Yoct)
#I just typed in some random-ish lists of length at most 8,
#to give me a random-ish singular pair
```

In [13]:
```
%time G2_FC_dict_E(my_splitting_dictionary,X1_E,Y1_E,n,[1,3,5,7])
#G2_FC_dict_E is the main function for computing Fourier coefficients of lif
#we did some necessary precomputation above using the function vec_list_to_c
#the list [1,3,5,7] corresponds to modular form of weight [5,7,9,11] (I adde
#so the function will compute Fourier coefficients of some random-ish cusp f
```

Out[13]:
```
CPU times: user 6min 34s, sys: 2.19 s, total: 6min 36s
Wall time: 6min 36s
{(0, -2, -1): (0, 0, 0, 0),
 (0, -2, 0): (0, 0, 0, 0),
 (0, -2, 1): (0, 0, 0, 0),
 (0, -1, 0): (0, 0, 0, 0),
 (1, -2, -2): (0, 0, 0, 0),
 (1,
  -2,
  -1): (0, 0, -54182510828775/256*t + 2462914684125/32, 846359765655833835/3
2768*t + 242318988110602295555/65536),
 (1, -2, 0): (0, 0, 0, 0),
 (1, -1, 0): (0, 0, 0, 0)}
```

In [14]:
```
#the above computation proves that the Theta_E cusp form is nonzero for some
```

In [ ]:
```

```