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In [1]: load('g2_motives.sage')
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In [2]: #in this notebook, we show how to use the code to compute Fourier coefficients  
#in particular, we compute Fourier coefficients of the unique level one cusp
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In [4]: X1,Y1=list_to_oct_pair_vec([],[],[],Xoct,Yoct);  
%time my_coefficient_dict=G2_FC_dict_I(3,[2], X1, Y1)  
#this will test if the Theta_I lift for X1,Y1 in weight 6  
#has a nonzero Fourier coefficient of small discriminant
```

```
CPU times: user 217 ms, sys: 2.74 ms, total: 220 ms  
Wall time: 220 ms
```

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In [5]: my_coefficient_dict
```

```
Out[5]: {(0, -1, 0): (-336), (1, -1, 0): (720)}
```

```
In [6]: initialize_dict4(7,1)  
#this will show us how many Fourier coefficients we will compute for the par
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```
Out[6]: {(0, -3, -1): (0),  
        (0, -3, 0): (0),  
        (0, -3, 1): (0),  
        (0, -2, -1): (0),  
        (0, -2, 0): (0),  
        (0, -2, 1): (0),  
        (0, -1, 0): (0),  
        (1, -3, -3): (0),  
        (1, -3, -2): (0),  
        (1, -3, -1): (0),  
        (1, -3, 0): (0),  
        (1, -3, 1): (0),  
        (1, -2, -2): (0),  
        (1, -2, -1): (0),  
        (1, -2, 0): (0),  
        (1, -1, 0): (0)}
```

```
In [7]: #some of the above triples (b,c,d) correspond to binary cubics in the same G  
#so there is a bit of redundancy in the computation
```

```
In [8]: %time my_coefficient_dict_7=G2_FC_dict_I(7,[2], X1, Y1)
```

```
CPU times: user 55min 59s, sys: 7.78 s, total: 56min 7s  
Wall time: 1h 5min 46s
```

```
In [9]: my_coefficient_dict_7
```

```
Out[9]: {(0, -3, -1): (2332800),
        (0, -3, 0): (77760),
        (0, -3, 1): (2332800),
        (0, -2, -1): (720),
        (0, -2, 0): (80640),
        (0, -2, 1): (720),
        (0, -1, 0): (-336),
        (1, -3, -3): (-483840),
        (1, -3, -2): (1227600),
        (1, -3, -1): (1382400),
        (1, -3, 0): (-71280),
        (1, -3, 1): (80640),
        (1, -2, -2): (-1440),
        (1, -2, -1): (604800),
        (1, -2, 0): (-3024),
        (1, -1, 0): (720)}
```

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In [10]: gcd(336,720)
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Out[10]: 48
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In [ ]: #let's scale the Fourier coefficients by 48 to get nicer numbers
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In [11]: my_coefficient_dict_7_scaled={}
         for key in my_coefficient_dict_7:
             my_coefficient_dict_7_scaled.update({key:my_coefficient_dict_7[key]/48})
```

```
In [12]: my_coefficient_dict_7_scaled
```

```
Out[12]: {(0, -3, -1): (48600),
        (0, -3, 0): (1620),
        (0, -3, 1): (48600),
        (0, -2, -1): (15),
        (0, -2, 0): (1680),
        (0, -2, 1): (15),
        (0, -1, 0): (-7),
        (1, -3, -3): (-10080),
        (1, -3, -2): (25575),
        (1, -3, -1): (28800),
        (1, -3, 0): (-1485),
        (1, -3, 1): (1680),
        (1, -2, -2): (-30),
        (1, -2, -1): (12600),
        (1, -2, 0): (-63),
        (1, -1, 0): (15)}
```

```
In [ ]:
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