Bootstrapping Confidence Intervals (with Confidence!)

Aaron Kaufman

November 16, 2018

Previously: Calculating Confidence Intervals

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Statistics: science of uncertainty,

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95% Confidence Interval:

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95% Confidence Interval: If we construct 100 such intervals, 95% will contain the truth

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1. Collect a sample of data from the population

Previously: Calculating Confidence Intervals

Statistics: science of uncertainty, and CIs are our most important tool

95% Confidence Interval: If we construct 100 such intervals, 95% will contain the truth

Procedure:

1. Collect a sample of data from the population
2. Calculate a quantity of interest, **X**

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95% Confidence Interval: If we construct 100 such intervals, 95% will contain the truth

Procedure:

1. Collect a sample of data from the population
2. Calculate a quantity of interest, **X**
3. Calculate a standard deviation, s

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95% Confidence Interval: If we construct 100 such intervals, 95% will contain the truth

Procedure:

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2. Calculate a quantity of interest, **X**
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4. Calculate a standard **error** σ: $\frac{s}{\sqrt{n}}$

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6. For 95%: $[X - 1.96 \times \sigma, X + 1.96 \times \sigma]$

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Data:

Example:

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Data: 2016 American National Election Study (4,142 US adults)

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1. **Average:** 85.7%
2. **Standard Deviation:** 35.0%

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1. **Average:** 85.7%
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Interpretation?

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This is a pain!

An Alternative Approach: The Bootstrap

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Bootstrap: A computational solution to confidence intervals

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Why sample with replacement?

- We need variation in the quantity of interest
- Sampling without replacement $\rightsquigarrow$ identical samples $\rightsquigarrow$ same QOI every time!

The Bootstrap: Terminology

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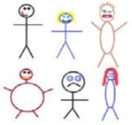
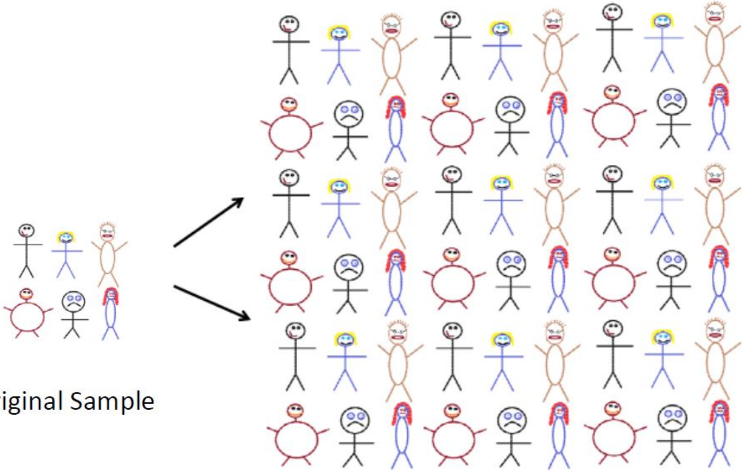


Original Sample

The Bootstrap: Terminology

What is the CI for the average height?

The Bootstrap: Terminology



Original Sample

A simulated “population” to sample from

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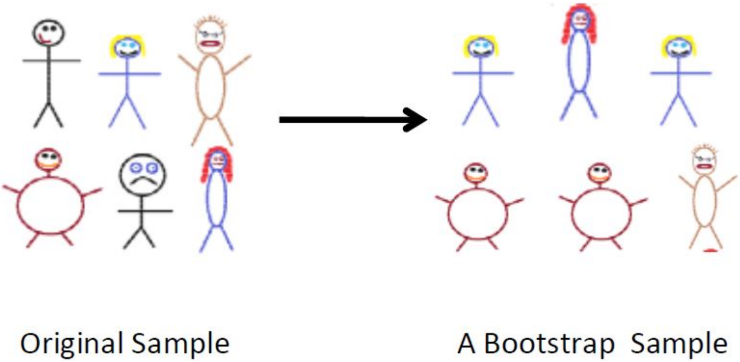

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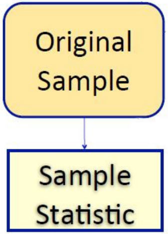

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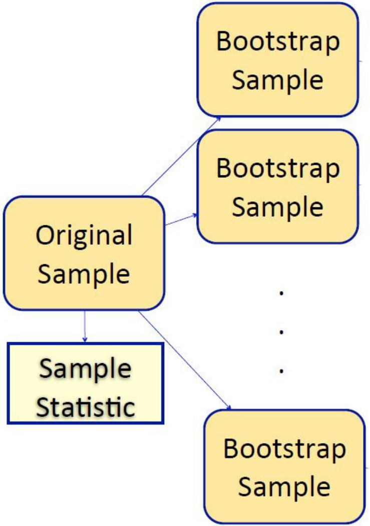

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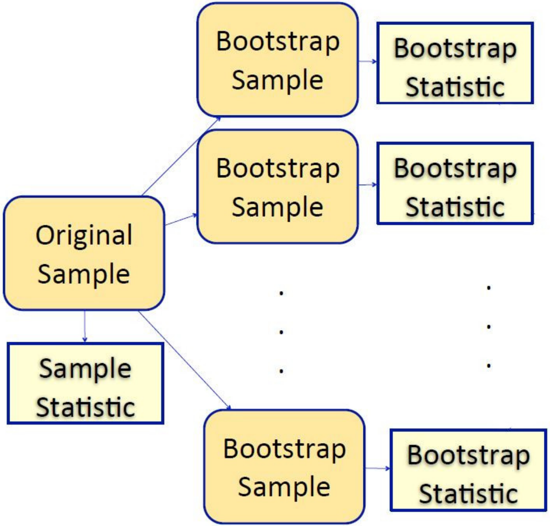

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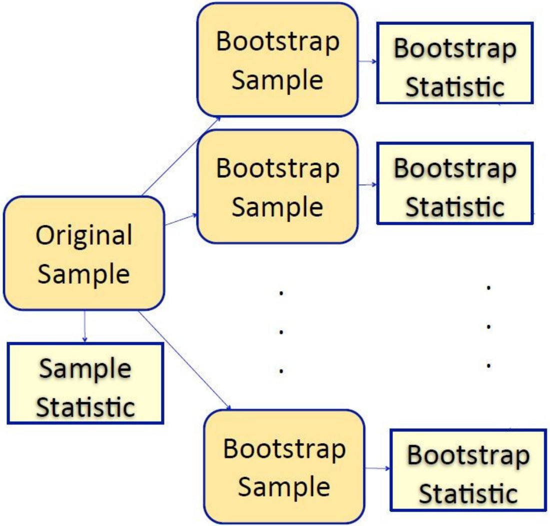

86.3%

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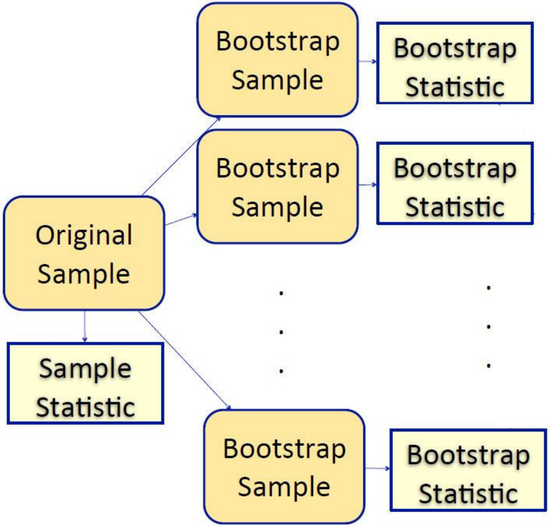
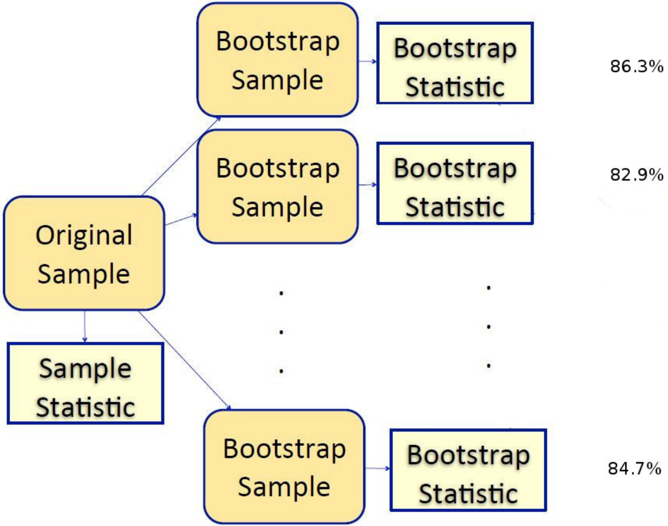


86.3%

82.9%

The Bootstrap: Terminology



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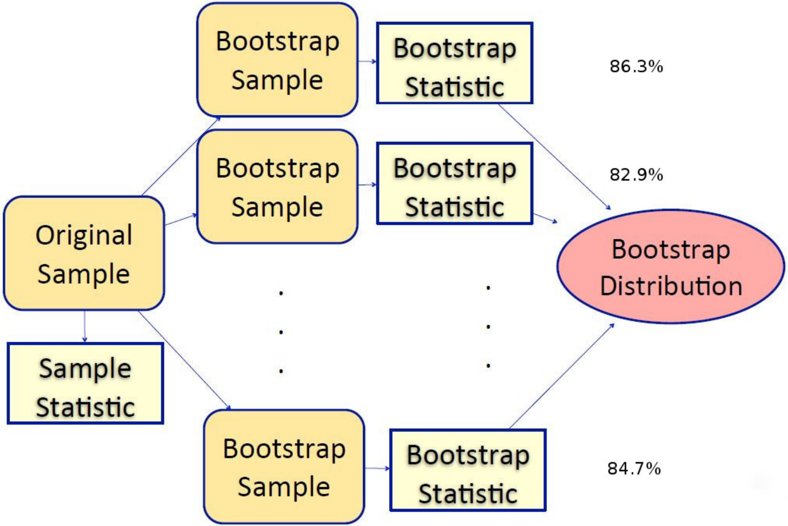

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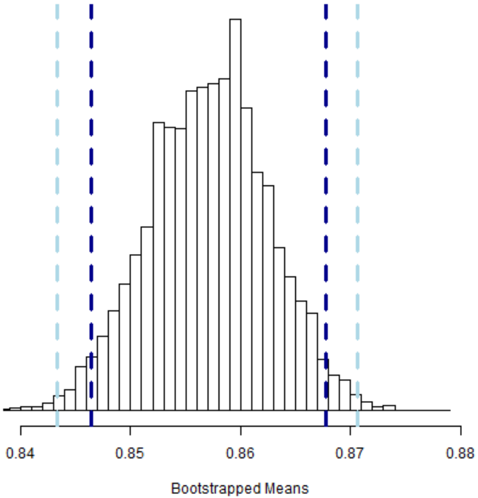

R Coding: The American National Election Study

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```
X = mean(anes16$registered)
X # 0.857
```

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X = mean(anes16$registered)
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SE = SD/sqrt(nrow(anes16))
SE # 0.005
```

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```
X = mean(anes16$registered)
X # 0.857

SD = sd(anes16$registered)
SD # 0.350

SE = SD/sqrt(nrow(anes16))
SE # 0.005

X + 1.96*SE # 0.868
X - 1.96*SE # 0.848
```

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```
bootstrapped.sample = mosaic::shuffle(anes16, replace=TRUE)
mean(bootstrapped$registered)
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bootstrapped.distribution = replicate(100, {
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quants = quantile(bootstrapped.distribution, c(0.025, 0.975))
quants # 0.846, 0.867
```

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```

How many bootstrapped samples do we generate?

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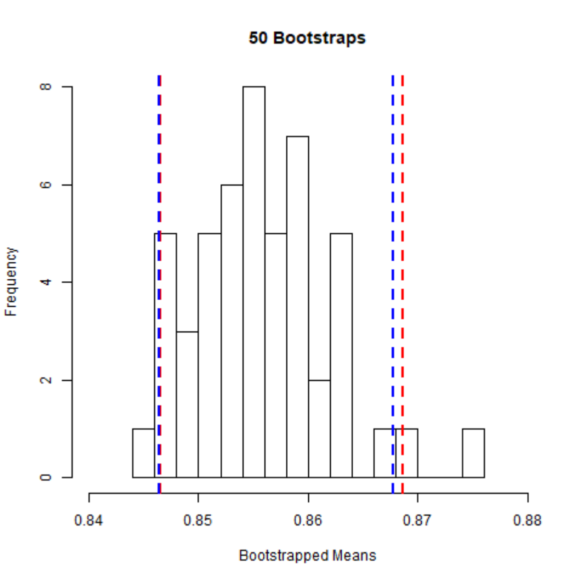

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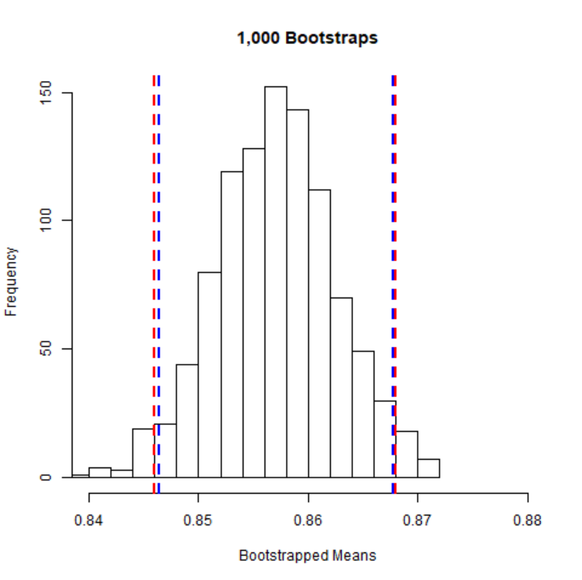

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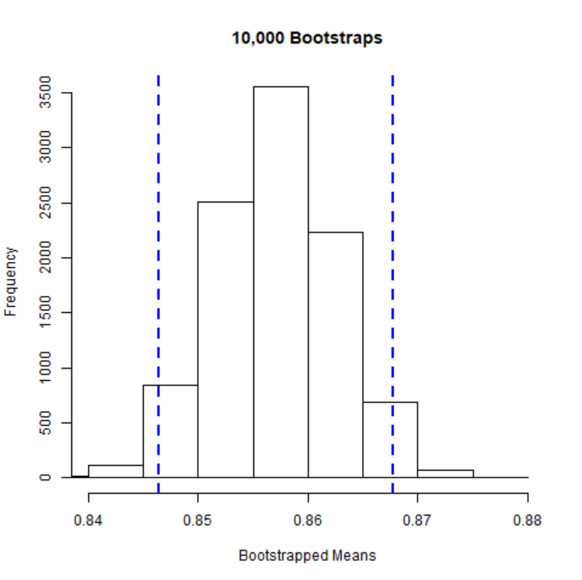

R Coding: The American National Election Study

But we can also bootstrap more complicated stuff!

```
cor(anes16$age, anes16$registered) # 0.199
```

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bootstrapped.sample = mosaic::shuffle(anes16,replace=TRUE)
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   cor(bootstrapped.sample$registered, bootstrapped.sample$age)
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corrs = quantile(bootstrapped.corr, c(0.025, 0.975))
corrs # 0.173, 0.230
```

Even more complicated stuff

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How do they “call” an election? How do they call elections with only 1% of the data?

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- **Bootstrap!**

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When can we "call" an election?

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When can we "call" an election? When 99% CI only contains one winner

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$$\text{bootstrapped.corr} = \text{replicate}(100, \{$$

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bootstrapped.corr = replicate(100, {
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- Use the `shuffle()` function from the `mosaic` library:

Summary & Coding tips

The bootstrap...

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2. Is useful when analytic approaches fail (or when we are feeling lazy)
3. Works for complicated sample statistics too: correlations, predictions, modeling accuracy...

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- Calculate quantiles: `quantile(bootstrapped.avgs, c(0.025, 0.975))`

Thank you!

LaTeX, R code, and data at:

`http://www.github.com/aaronrkaufman/bootstrap`