Homework 03

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1 Monday 1/27

Section 4

16. Solve the differential equation $yy'^2 + 2xy' - y = 0$ by changing from variables y, x, to r, x, where $y^2 = r^2 - x^2$; then $yy' = rr' - x$.

$$yy'^2 + 2xy' - y = 0$$
$$y^2y'^2 + 2xyy' - y^2 = 0$$
$$y^2y'^2 + 2xyy' + x^2 - y^2 - x^2 = 0$$
$$(yy' + x)^2 - (y^2 + x^2) = 0$$
$$(rr')^2 - r^2 = 0$$
$$r'^2 - 1 = 0$$
$$r' = \pm 1$$
$$\int r' dx = \int \pm 1 dx$$
$$r = \pm x + C$$
$$r^2 = x^2 \pm 2xC + C^2$$
$$r^2 - x^2 = \pm 2xC + C^2$$
$$y^2 = \pm 2xC + C^2$$
$$y = \sqrt{\pm 2xC + C^2}$$

Section 5

1. $y'' + y' - 2y = 0$

$$(D^2 + D - 2)y = 0$$
$$\text{Auxillary Equation: } (D^2 + D - 2) = 0$$
$$\implies (D + 2)(D - 1) = 0 \implies \text{root(s): } -2, 1$$
$$\text{General Solution: } y = C_1e^{-2x} + C_2e^{x}$$

2. $y'' - 4y' + 4y = 0$

$$(D^2 - 4D + 4)y = 0$$
$$\text{Auxillary Equation: } (D^2 - 4D + 4) = 0$$
$$\implies (D-2)^2 = 0 \implies \text{root(s): } 2$$
$$\text{General Solution: } y = e^{2x}(Ax + B)$$

5. $(D^2 - 2D + 1)y = 0$

$$\text{Auxillary Equation: } (D^2 - 2D + 1) = 0$$
$$\implies (D-1)^2 = 0 \implies \text{root(s): } 1$$
$$\text{General Solution: } y = e^{x}(Ax + B)$$

8. $(D)(D+5)y = 0$

$$\text{Auxillary Equation: } (D)(D+5) = 0$$
$$\implies \text{root(s): } 0, -5$$
$$\text{General Solution: } y = C_1 + C_2 e^{-5x}$$

10. $y'' - 2y' = 0$

$$(D^2 - 2D)y = 0$$
$$\text{Auxillary Equation: } (D^2 - 2D) = 0$$
$$\implies (D-2)D = 0 \implies \text{root(s): } 0, 2$$
$$\text{General Solution: } y = C_1 + C_2 e^{2x}$$

5. $(2D^2 + D - 1)y = 0$

$$\text{Auxillary Equation: } (2D^2 + D - 1) = 0$$
$$\implies (2D-1)(D+1) = 0 \implies \text{root(s): } -1, \frac{1}{2}$$
$$\text{General Solution: } y = C_1 e^{-x} + C_2 e^{\frac{x}{2}}$$

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Section 5

3. $y'' + 9y = 0$

$$(D^2 + 9)y = 0$$
$$\text{Auxillary Equation: } (D^2 + 9) = 0$$
$$\implies (D + 3i)(D - 3i) = 0 \implies \text{root(s): } \pm 3i$$
$$\text{General Solution: } y = Ae^{3ix} + Be^{-3ix}$$

6. $(D^2+16)y=0$

$$(D^2+16)y=0$$
$$\text{Auxillary Equation: } (D^2+16)=0$$
$$\implies (D+4i)(D-4i)=0 \implies \text{root(s): } \pm 4i$$
$$\text{General Solution: } y=Ae^{4ix}+Be^{-4ix}$$

9. $(D^2-4D+13)y=0$

$$(D^2-4D+13)y=0$$
$$\text{Auxillary Equation: } (D^2-4D+13)y=0$$
$$\implies (D-(2+3i))(D-(2-3i))=0$$
$$\implies \text{root(s): } 2\pm 3i \implies \alpha=2;\ \beta=3$$
$$\text{General Solution: } y=e^{2x}(C_1\cos(3x)+C_2\sin(3x))$$

Section 6

1. $y''-4y=10$

First Find General Solution to homogenous equation:

$$y_c''-4y_c=0 \implies (D^2-4)y_c=0$$
$$\text{Auxillary Equation: } (D^2-4)=0$$
$$\implies (D+2)(D-2)=0 \implies \text{root(s): } \pm 2$$
$$\text{General Solution: } y_c=Ae^{2x}+Be^{-2x}$$

Now Find particular Solution:

$$\text{Because this is a constant we know, } -4y_p=10 \implies y_p=-\frac{5}{2}$$

Combine:

$$y=y_c+y_p$$
$$y=Ae^{2x}+Be^{-2x}-\frac{5}{2}$$

2. $(D-2)^2 y = 16$

First Find General Solution to homogenous equation:

$$(D-2)^2 y_c = 0$$
$$\text{Auxillary Equation: } (D-2)^2 = 0 \implies \text{root(s): } 2$$
$$\text{General Solution: } y_c = e^{2x}(Ax+B)$$

Rewrite as

$$y'' - 2y' + 4y = 16$$

Now Find particular Solution:

$$\text{Because this is a constant we know,} 4y_p = 16 \implies y_p = 4$$

Combine:

$$y = y_c + y_p$$
$$y = e^{2x}(Ax+B) + 4$$