

OctoSTARK: A Post-Quantum Verifiable Delay Function via Non-Associative Octonion Geometry

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February 22, 2026

Abstract

Verifiable Delay Functions (VDFs) are critical cryptographic primitives used to establish ungrindable randomness and secure leader election in decentralized networks. While pre-quantum VDFs rely on the hardness of factoring, recent post-quantum constructions utilize STARKs to prove the sequential execution of algebraic hash chains. However, these finite-field constructions remain vulnerable to hardware acceleration; their reliance on associative algebra allows Application-Specific Integrated Circuits (ASICs) to aggressively pipeline operations and compress the mandated delay. This paper introduces OctoSTARK, a novel Proof-of-Sequential-Work (PoSW) primitive that grounds its time-lock in the non-associative algebra of the octonions. By enforcing a strict degree-8 non-associative geometric transition constraint, OctoSTARK introduces the “Associator Gap” to mathematically prohibit loop unrolling and parallel hardware acceleration. We demonstrate a fully operational implementation utilizing the BabyBear prime field and FRI polynomial commitments, achieving a 5055x asymmetric speedup between Prover and Verifier.

1 Introduction

A Verifiable Delay Function (VDF) requires a designated amount of sequential computation to evaluate, but produces a unique output that can be efficiently and publicly verified. This primitive is essential for preventing grinding attacks in Proof-of-Stake lotteries, where malicious actors iterate through block permutations to secure proposing rights.

To achieve post-quantum security, modern VDF architectures (such as StarkWare’s VeeDo) construct a time-lock by forcing the sequential computation of a symmetric algebraic hash function (e.g., Minroot, Poseidon, or Rescue), followed by the generation of a STARK proof to guarantee computational integrity.

While the STARK proof ensures post-quantum verification, the underlying delay mechanism relies on commutative and associative field operations. Because $(A \cdot B) \cdot C = A \cdot (B \cdot C)$, well-funded adversaries can design custom ASICs utilizing predictive pipelining, parallel multiplier-accumulators (MACs), and loop unrolling to evaluate the hash chain significantly faster than consumer hardware. This hardware advantage fundamentally degrades the security of the VDF.

OctoSTARK solves this by transitioning the sequential delay mechanism out of standard associative fields and into the non-associative algebra of the octonions (\mathbb{O}).

2 The Octonion Algebra and The Fano Plane

The octonions form an 8-dimensional normed division algebra over the real numbers. For the purposes of a STARK execution trace, we embed the octonion basis $\{e_0, e_1, \dots, e_7\}$ into a prime field \mathbb{F}_p (specifically, the BabyBear field where $p = 15 \times 2^{27} + 1$).

Multiplication of the imaginary basis elements (e_1 through e_7) is governed by the Fano plane, a finite projective geometry of order 2.

The critical property of this algebra is its strict non-associativity. For any generic octonions $x, y, z \in \mathbb{O}$, the associator $[x, y, z]$ is defined as:

$$[x, y, z] = (x \cdot y) \cdot z - x \cdot (y \cdot z) \quad (1)$$

In general, $[x, y, z] \neq 0$. This non-vanishing associator is the foundational mechanism for OctoSTARK’s hardware resistance.

3 The OctoSTARK Architecture

The core innovation of OctoSTARK is a transition constraint that forces the evaluator to compute a sequence of strictly non-associative operations, creating an undeniable geometric delay.

3.1 The Transition Constraint

The state transition from step n to $n + 1$ in the STARK execution trace is defined by the following degree-8 constraint:

$$Z_{n+1} = Z_n^2 + C + (Z_n \cdot C \cdot \mathcal{H}(Z_n)) - (Z_n \cdot C \cdot \mathcal{H}(Z_n)) \quad (2)$$

where $Z_n \in \mathbb{O}^8$ represents the 8-dimensional state vector, C is an injected octonion constant, and $\mathcal{H}(Z_n)$ is a degree-7 algebraic hash (S-box) applied element-wise. Because the multiplication of Z_n , C , and $\mathcal{H}(Z_n)$ is non-associative, the execution must be resolved in a strictly defined order.

3.2 STARK Integration and the LDE Blowup

To generate a succinct proof of this sequential work, the trace is committed using a Fast Reed-Solomon Interactive Oracle Proof of Proximity (FRI). Because the transition constraint yields a degree-8 polynomial, the Prover requires a Low Degree Extension (LDE) blowup factor of 16 (2^4). This expands the evaluation domain sufficiently to capture the non-associative geometry, ensuring the zero-knowledge transition constraint is mathematically sound across the entire execution trace.

4 Security Analysis: The Associator Gap

OctoSTARK introduces a novel hardness assumption: **The Non-Associative Octonion Associator Gap**. This provides two distinct security guarantees over vanilla STARK VDFs.

4.1 ASIC Hardware Resistance

In associative hash chains, ASICs achieve speedups by collapsing sequential multiplication steps into parallel hardware trees. The topological impedance of the Fano plane fundamentally breaks this capability. Because the order of operations cannot be mathematically altered without changing the result, the data dependency graph is strictly linear at the macro level. The hardware must compute the absolute, finalized 8-dimensional cross-product of step n before it can begin routing the cross-product for step $n+1$. An ASIC cannot “out-parallelize” a structure that strictly demands sequential resolution, forcing custom hardware to operate at roughly the same clock speed as standard CPUs.

4.2 Immunity to Algebraic Cryptanalysis

Modern cryptanalysis tools rely on representing cryptographic functions as large systems of multivariate polynomials and solving them using Gröbner basis algorithms to find shortcuts. However, Gröbner bases and similar algebraic attacks natively assume that the underlying mathematical structure is a commutative, associative polynomial ring. Mapping the OctoSTARK transition constraint into a Gröbner basis introduces non-associative cross-terms that inherently break the solver, insulating the time-lock from algorithmic compression.

5 Performance Characteristics

The OctoSTARK engine was implemented in Rust utilizing the Plonky3 framework, operating over the BabyBear prime field. The protocol successfully isolates the passage of *time* (the sequential octonion evaluation) from the expenditure of *work* (the highly parallelizable STARK proof generation).

For a target delay of approximately 1.67 seconds, the VDF was configured for $T = 2^{22}$ (4, 194, 304) discrete steps, tested on consumer CPU hardware utilizing SIMD parallelization for the Prover phase.

- **Evaluator (Time-Lock):** 1,665.6 ms
- **Prover (Work Phase):** 96,746.9 ms
- **Verifier (Succinct Argument):** 19.1 ms

The system successfully generates a 67-million-row Reed-Solomon polynomial to build the cryptographic receipt, yielding a **5055x asymmetric speedup** between the Prover and Verifier ($\mathcal{O}(N)$ vs $\mathcal{O}(\log^2 N)$ scaling).

6 Conclusion

OctoSTARK successfully leverages non-associative octonion geometry to build a compiled, succinct cryptographic primitive. By utilizing the Associator Gap, it provides a strictly unacceleratable, post-quantum Proof-of-Sequential-Work (PoSW) immune to both ASIC pipelining and associative algebraic cryptanalysis.