Gompertz Processes

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April 19, 2020

Abstract

Motivated by considering infinitesimal stochastic accelerations of time, we outline a theory of Gompertz processes, Poisson processes subordinated by integrated Geometric Brownian motion.

1 Motivation

Integrated geometric Brownian motion occurs in the asymptotic limit of infinitesimal stochastic accelerations. In the context of accelerated failure times we are observing a fixed absorbing event. We do not have access to metabolic time, which integrated geometric Brownian motion models. By Lévy-Khintchine this is the only representation that is Lévy and jump free.

Definition 1 (Gompertz Process)**.** A Gompertz process G_t is a subordinated Poisson process N_t, with rate λ, where the subordinating process is integrated geometric Brownian motion Y_t, with drift μ and diffusion σ:

$$G_t = N_{Y_t} \tag{1}$$

given:

$$\begin{aligned} Y_t &= \int_0^t X_s ds \qquad (2) \\ &= \int_0^t e^{\mu s + \sigma W_s} ds \qquad (3) \end{aligned}$$

2 Preliminaries

Lemma 1 (Acceleration Lemma)**.** *Induction on powers of geometric Brownian motion, conditioned on integrated geometric Brownian motion.*

Lemma 2 (Recursion Convolution Lemma)**.** *Powers of integrated geometric Brownian motion.*

3 Martingale

Two point conditional expectation

4 Lévy Process

Triangular matrix embedding resulting in independent multiplicative increments

5 Fokker-Planck

Two dimensional stochastic differential equation

6 Hazard Rate

Partial differential equation of Gompertz process hazard rate.