Info: Cf = .001, Cd = 1 for sphere assumption

Source: Introduction to Flight

EXAMPLE 8.11

Consider two bodies in circular orbit around the earth at an altitude of 800 km above the surface of the earth. Each body has a mass of 1800 kg. One body is a slender cone with a total vertex angle of 10°. The other body is a sphere. For the cone, the pressure drag coefficient at hypersonic Mach numbers is 0.017 and the skin friction drag coefficient is 0.01. For the sphere, the pressure drag coefficient is 1.0 and the friction drag coefficient is 0.001. Calculate and compare the total aerodynamic heating input to each body during atmospheric entry.

■ **Solution**

The entry velocity of both bodies from orbit is obtained from Eq. (8.57), where $r = r_e + h_G$ and r_e is the radius of the earth, $r_e = 6.4 \times 10^6$ m, and h_G is the geometric altitude above sea level, $h_G = 800$ km $= 0.8 \times 10^6$ m.

$$V_E = \sqrt{\frac{k^2}{r}} = \sqrt{\frac{3.986\times10^{14}}{(6.4+0.8)\times10^6}} = 0.789\times10^4 \text{ m/sec}$$

The total heat input is given by Eq. (8.155), repeated here:

$$Q_{\text{total}} = \frac{1}{2}\frac{C_f}{C_D}\left(\frac{1}{2}m V_E^2\right) \qquad (8.155)$$

where

$$\frac{1}{2}m V_E^2 = \frac{1}{2}(1800)(0.789\times10^4)^2 = 5.60\times10^{10} \text{ joule}$$

a. For the cone:

$$C_D = C_{D_p} + C_f = 0.017 + 0.01 = 0.027$$

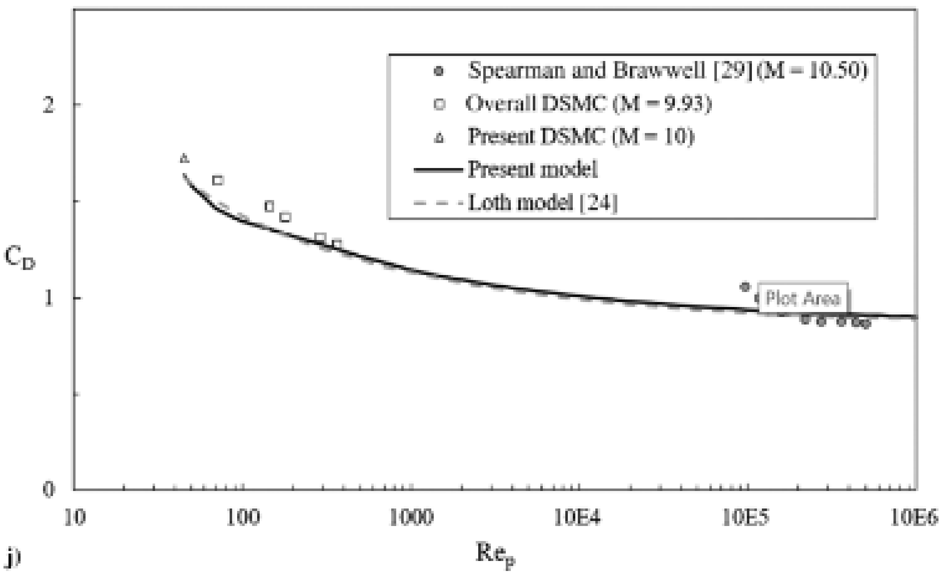
$$\frac{C_f}{C_D} = \frac{0.01}{0.027} = 0.37$$

From Eq. (8.155),

$$Q_{\text{total}} = \frac{1}{2}(0.37)(5.6\times10^{10}) = \boxed{1.036\times10^{10} \text{ joule}} \text{ (cone)}$$

Info: Cd vs Re @ mach 10+ graph. Suggests Cd approaches 1 like example problem gave. Validation.

Source: Supersonic and Hypersonic Drag Coefficients for a Sphere https://doi.org/10.2514/1.J060153

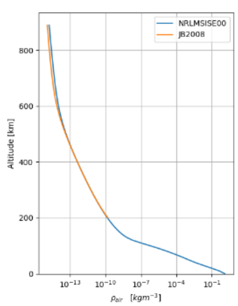


j)

Fig. 7 (Continued)

Info: Re range from 800 to 50 km yields Cd 1.5 to 1
800: (10^-14 kg/m^3)*(945 m/s)*(3.5 m)/(1.825*10^-5 kg/m-s) = 1.812E-6
50: 10^(-3)*(945)*(3.5)/(6.63*10^-5) = 4.989E4
Locks in example problem giving 1 as Cd

Source: Google Images Graph for density
https://www.engineersedge.com/physics/viscosity_of_air_dynamic_and_kinematic_14483.htm



Info: Heat equation. Q = 1.17717E6 J

Source: Introduction to Flight

8.16 Entry Heating 725

Equating Eqs. (8.153) and (8.151),

$$\frac{dQ}{dV_\infty}\left(-\frac{1}{2m}\rho_\infty V_\infty^2 S C_D\right) = \frac{1}{4}\rho_\infty V_\infty^3 S C_f$$

or

$$\frac{dQ}{dV_\infty} = -\frac{1}{2} m V_\infty \frac{C_f}{C_D}$$

or

$$dQ = -\frac{1}{2} m \frac{C_f}{C_D}\frac{dV_\infty^2}{2} \tag{8.154}$$

Integrate Eq. (8.154) from the beginning of entry, where $Q = 0$ and $V_\infty = V_E$, and the end of entry, where $Q = Q_{\text{total}}$ and $V_\infty = 0$:

$$\int_0^{Q_{\text{total}}} dQ = -\frac{1}{2}\frac{C_f}{C_D}\int_{V_E}^{0} d\left(m\frac{V_\infty^2}{2}\right)$$

$$\boxed{Q_{\text{total}} = \frac{1}{2}\frac{C_f}{C_D}\left(\frac{1}{2} m V_E^2\right)} \tag{8.155}$$

Equation (8.155) is the desired result for total heat input to the entry vehicle. It is an important relation—examine it closely. It reflects two vital conclusions:

1. The quantity $\frac{1}{2} m V_E^2$ is the initial kinetic energy of the vehicle as it first enters

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Info: Q to the 6th J is a reasonable amount. Validated.

Source: Intro to Flight

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■ Solution

The entry velocity of both bodies from orbit is obtained from Eq. (8.57), where $r = r_e + h_G$ and r_e is the radius of the earth, $r_e = 6.4 \times 10^6$ m, and h_G is the geometric altitude above sea level, $h_G = 800$ km $= 0.8 \times 10^6$ m.

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8.17 Lifting Entry, with Application to the Space Shuttle 729

b. For the sphere:

$$C_D = C_{D_p} + C_f = 1.0 + 0.001 = 1.001$$

$$\frac{C_f}{C_D} = \frac{0.001}{1.001} = 0.999\times10^{-4}$$

From Eq. (8.155),

$$Q_{\text{total}} = \frac{1}{2}(0.999\times10^{-4})(5.6\times10^{10}) = \boxed{2.8\times10^6 \text{ joule}} \text{ (sphere)}$$

As expected, the sphere, being a much blunter body, experiences a much smaller total heat input compared to the slender cone.

- Online can find unit of heat per area material can absorb

- Use found Q to calculate thickness and area

- Now you can find gradient of temperature independent of full craft. Check inner layer meets requirements

Dragonfly: https://ntrs.nasa.gov/api/citations/20190028683/downloads/20190028683.pdf

2.5E2 W/cm^2 = 2.5E6 W/m^2

.0366 m thickness

3.75 m diameter

Allowable: https://ntrs.nasa.gov/api/citations/20160006481/downloads/20160006481.pdf

4E2 W/cm^2

Info: peak heat flux of 11.3 MW/m$_2$,
heat load of 209 MJ/m$_2$
Used those ^ numbers to make relationship with dragonfly

- Entry heat pulse: 250 sec.
  - Peak heat flux 250 W/cm^2 margined

Got dragonfly expected heat load and divided by expected time of descent 250 s to get average heat load of 184,000
Source:
https://www.tsijournals.com/articles/ablative-heat-shield-design-for-a-sample-return-vehicle.pdf

https://ntrs.nasa.gov/api/citations/20190028683/downloads/20190028683.pdf