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Empirical Orthogonal Functions (EOFs)

- EOFs <=> Principal Components (PCs)
- widely used statistical technique
- eigenvectors of covariance matrix between grid pts (stations)
- not based on physical principals
- used to explore data
- let x => f(Time, Space)
 - example: slp(time,lat,lon), T=>time, S=lat,lon
- partitioned/decomposed into orthogonal patterns/modes
 - efficient representation of 'system variance'
 - linear combinations that compress the data
 - 1st linear combination explains largest variance
- spatial patterns & time series of each pattern's amplitude
- may/may-not have explainable physical info

EOF: functions

NCL has two functions:

eofunc_Wrap: calculates orthogonal patterns/modes
eofunc_ts_Wrap: calculates pattern/mode amplitudes

eofunc_Wrap

- expects 'time' dimension to be rightmost dimension
 - may have to reorder using named dimensions
 - x should be weighted to reflect spatial extent
 - user specifies number of EOFs (rarely more than 4)

EOF: eofunc Calculation Details

Examines the **S**patial & **T**emporal sizes (**S**,**T**) of input **x**

- may do a linear transformation to yield smallest COV(x)
 - generally, T << S; hence, TxT in sym. storage mode</p>
- if linear transformation performed; reverse transform

anomaly covariance matrix created (or correlation matrix)

- covariance between the i^{th} and j^{th} locations over time (N)
- $cov(xa)_{i,j} = [\Sigma(x_{n,i} X_i) (x_{n,j} X_j)]/(N 1)$
 - $-\mathbf{X}_{i}$, \mathbf{X}_{i} are temporal means of \mathbf{x} at each location
 - xa is the anomaly covariance matrix

EOFs (patterns/modes): LAPACK's "dspevx"

- user specifies number of EOFs to return (K)
- returns eigenvalues; % variance explained

ts: amplitude time series:

- for each EOF_k : $ts_{k,n} = \Sigma(EOF_{i,i,k} * ax_{i,i,n})$

EOF: eofunc returned info

- EOFs (spatial)
- % variance explained by each EOF
- eigenvalues of the covariance matrix
 - if applicable, eigenvalues of transformed matrix also

EOFS: Simple Example (1)

```
load "$NCARG ROOT/lib/ncarg/nclscripts/csm/gsn code.ncl"
load "$NCARG ROOT/lib/ncarg/nclscripts/csm/gsn csm.ncl"
load "$NCARG ROOT/lib/ncarg/nclscripts/csm/contributed.ncl"
                                                 ; rectilinear
f = addfile("erai 1989-2009.mon.msl psl.nc","r") ; open file
p = f->SLP(::12, \{0:90\}, :)
                                                 ; (21,61,240)
                                                 ; spatial weighting
w = sqrt(cos(0.01745329*p&latitude))
                                                 ; weights(61)
                                                  ; wp(21,61,240)
wp = p*conform(p, w, 1)
copy_VarCoords(p, wp)
      = wp(latitude|:,longitude|:,time|:)
                                                  ; reorder wgt data
X
neof = 4
                                                  ; user specify
eof = eofunc_Wrap(x, neof, False)
eof ts = eofunc_ts_Wrap (x, eof, False)
printVarSummary( eof )
                                                ; examine EOF variables
printVarSummary( eof_ts )
```

EOFS: Simple Example (1)

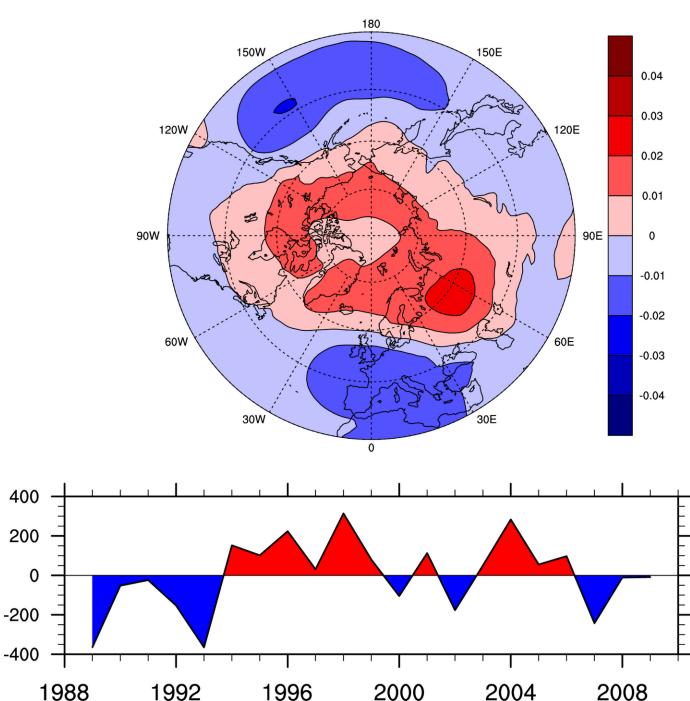
```
Variable: eof
                                              "printVarSummary" Output
Number of Dimensions: 3
Dimensions and sizes: [evn | 4] x [latitude | 61] x [longitude | 240]
Coordinates:
           evn: [1..4]
           latitude: [ 0..90]
           longitude: [ 0...358.5]
Number Of Attributes: 6
  eval transpose: (47.2223, 32.42917, 21.44406, 15.27389)
  eval: (34519.5, 23705.72, 15675.61, 11165.21)
 pcvar: (26.83549, 18.42885, 12.18624, 8.679848)
 matrix: covariance
 method : transpose
 _FillValue : 1e+20
Variable: eof ts
Number of Dimensions: 2
Dimensions and sizes: [evn | 4] x [time | 21]
Coordinates:
           evn: [1..4]
           time: [780168..955488]
Number Of Attributes: 3
 ts mean: (3548.64, 18262.12, 20889.75,10387.08)
 matrix: covariance
  FillValue: 1e+20
```

EOF: write a NetCDF file

```
; Create netCDF: no define mode [simple approach]
system("/bin/rm -f EOF.nc") ; rm any pre-existing file
fout = addfile("EOF.nc", "c") ; new netCDF file
fout@title = "EOFs of SLP 1989-2009"
fout->EOF = eof
fout->EOF_TS = eof_ts
```

Graphics: http://www.ncl.ucar.edu/Applications/Scripts/eof_2.ncl

SLP: 1989-2009: EOF 1: % Variance=26.8

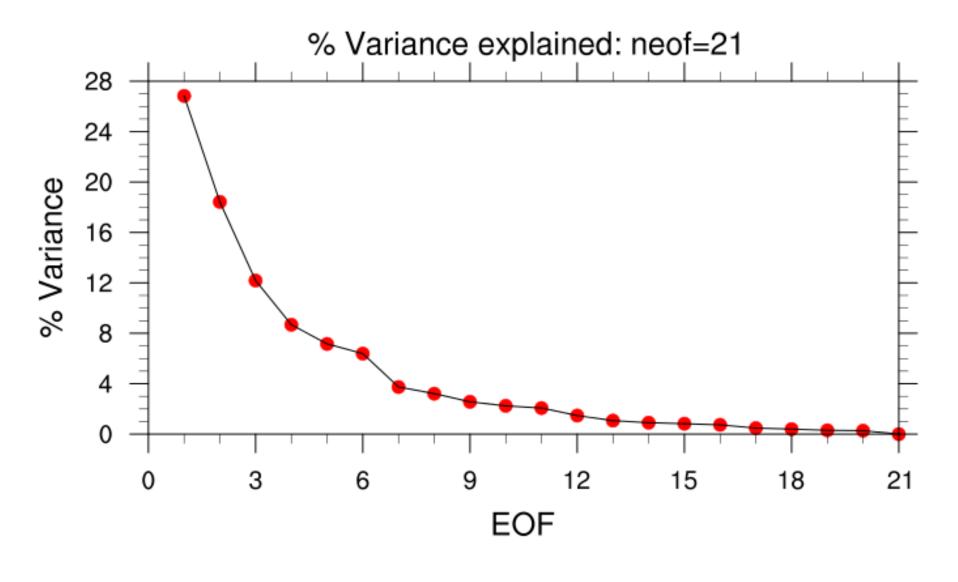


EOF: significance

- successive eigenvalues should be distinct
 - if not, the eigenvalues and associated patterns are noise
 - 1 from 2, 2 from 1 and 3, 3 from 2 and 4, etc
 - North et. al (MWR, July 1982: eq 24-26) provide formula
 - http://dx.doi.org/ 10.1175/1520-0493(1982)110<0699:SEITEO>2.0.CO;2
 - Quadrelli et. Al (JClimate, Sept, 2005) more information
 - http://dx.doi.org/10.1175/JCLI3500.1

NOTE: patterns are domain dependent

EOF: Sample % Variance Distribution



Shape is 'red'

EOF: North (1982): eigenvalue separation: function

North et al (1982): eqn 22: this is an 'objective approximation'

$$\delta \lambda \approx \lambda \left(\sqrt{\frac{2}{N}} \right)$$

```
undef ("eval north")
function eval_north( eval[*]:numeric, ntim[1]:integer, prinfo[1]:logical)
local neval, dlam, low, high, sig, n
begin
 neval = dimsizes(eval)
 dlam = eval * sqrt(2.0/ntim) ; eq 22 of North et al. (1982): Mon. Wea. Rev
 low = eval - dlam
 high = eval + dlam
 sig
       = new(neval, logical)
 sig
       = False
                                   ; default is not significantly separated
```

EOF: function eigenvalue separation

```
; take care of 1<sup>st</sup> and last special cases
                                            ; 1st eigenvalue (index 0)
 if (eval(0).gt.high(1)) then
   sig(0) = True
 end if
 if (eval(neval-1).lt.low(neval-2)) then ; last eigenvalue (index 'neval-1')
   sig(neval-1) = True
 end if
 do n=1,neval-2
                                            ; loop over all other eigenvalues
   if (eval(n).lt.low(n-1) .and. eval(n).gt.high(n+1)) then
     sig(n) = True
   end if
 end do
 if (prinfo) then
   print(dlam+" "+low+" "+eval+" "+high+" "+sig)
 end if
 sig@long_name = "eval significantly separated"
 return(sig)
end
```

EOF: eigenvalue separation: script output

North et al (1982) test: $\delta \lambda \approx \lambda \left(\sqrt{\frac{2}{N}}\right)$

```
prinfo = True
sig = eval_north(eof@eval, ntim, prinfo)
```

eval					
index	dlam	low	eval	high	sig
(0)	10652.9	23866.6	34519.5	45172.4	True
(1)	7315.8	16390	23705.7	31021.5	True
\ /	4837.6	10838	15675.6	20513.2	True
(3)	3445.7	7719.6	11165.2	14610.9	False
(4)	2841.2	6365.3	9206.5	12047.6	False
[snip]					

EOF: Rotation via Varimax

- rotates EOFs via Kaiser varimax criterion
 - rotated EOFs will be orthogonal
 - time series will be correlated (not orthogonal)
- how many EOFs should be used? No objective method!
- my opinion
 - you should (? must ?) know what you are doing
 - use when no significant EOFs were derived
 - rotation may reduce noise and yield interpretable info
 - however, if some are distinct and some are not then performing a rotation will mix the results

```
eof_vmax = eofunc_varimax_Wrap(eof, 1)
eofunc_varimax_reorder(eof_vmax)
```

EOF: Principal Oscillation Pattern (POP) Analysis

- uses EOFs and much more!
- http://www.ncl.ucar.edu/Applications/prn_osc_pat.shtml

```
Gehne, M. (2014): Irregularity and decadal variation in ENSO:
```

a simplified model based on Principal Oscillation Patterns.

Climate Dynamics: Dec 2014, Volume 43, 12, pp 3327-3350

http://dx.doi.org/10.1007/s00382-014-2108-6

von Storch, H. et al (1995): Principal Oscillation Patterns: A Review.

J. Climate, 8, 377-400.

http://dx.doi.org/10.1175/1520-0442(1995)008<0377:POPAR>2.0.CO;2

Compositing

```
t1 = (/15, 37, 95, 88, 90 /)
                                   ; cd calendar, ind, get1Dindex
  t2 = (1, 22, 31, 97, 100, 120)
  f = addfile("01-50.nc", "r")
 T1 = f -> T(t1, ..., ...)
                                   ; T(time,lev,lat,lon)
 T2 = f - T(t2, ..., ...)
                                   ; composite averages
 T1avg = dim_avg_n_Wrap(T1, 0); (lev,lat,lon)
 T2avg = dim avg n Wrap(T2, 0)
  Tdiff = T2avg
                                   ; trick to transfer meta data
  Tdiff = T2avg - T1avg
                                  ; difference
  Tdiff@long name = T2@long name + ": composite difference"
Also use coordinate subscripting: let "time" have units yyyymm
 t1 = (/ 190401, 191301, 192001, ....., 200301/)
 T1 = \hat{f} - T(\{t1\}, ..., ...)
```

Compositing: temporal

Compositing: combining data from different periods that satisfy some common criteria

```
: Climate Prediction Center
fnam = "ElNino LaNina.txt"
                                       ; contains seasonal SST anomalies
nrow = numAsciiRow(fnam)
data = readAsciiTable(fnam, 13, "float", 2) ; ncol=13, nskip=2
year = data(:,0)
                                            : DJF seasonal values
sea = data(:,1)
nyren = ind(sea .gt. 0.5)
                                            ; indices for El Nino > 0.5
                                                      La Nina < -0.5
nyrla = ind(sea .lt.-0.5)
                                            ; January of El Nino year
YYYY01_en = year(nyren)*100+1
YYYY01_la = year(nyrla)*100+1
                                           ; January of La Nina years
```

Compositing: temporal

```
f
    = addfile("air.sig995.mon.mean.nc","r") ; near surface temperatures
YYYYMM = cd_calendar(f->time, -1) ; all times on 'f'
      = get1Dindex(YYYYMM, YYYY01_en); indices of YYYYMM
ien
      = get1Dindex(YYYYMM, YYYY01_la)
ila
ten = \frac{\text{short2flt}(f-\text{sir}(ien,:,:))}{\text{[time | 21]}} \times [lat | 91] \times [lon | 180]
tla = short2flt(f->air(ila,;;;))
                                     ; [time | 19] x [lat | 91] x [lon | 180]
ten_avg = dim_avg_n_Wrap(ten,0); [lat | 91] x [lon | 180]
tla_avg = dim_avg_n_Wrap(tla,0)
tdif = ten_avg - tla_avg ; temp range (El Nino – La Nina)
copy_VarCoords(ten_avg, tdif)
```

Composite: Result

