Introduction to Computational Finance 2021, Homework 2

Due April 7, 2021

1/22/2021

Readings

- EZ lecture notes on review of probability theory and random variables.
- Ruppert, chapter 2 (Returns) and Appendix (probability review)
- A Beginner's Guide to R, chapters 3-7.
- R Cookbook, chapters 8 (probability) and 10 (graphics)

Programs and Data

- `probReview.xls` (Moodle)
- `probReview.r` (Moodle)

Instructions

In this homework you will become more familiar with random variables and probability distributions. Try to do all of the calculations and plots in R. You can also do everything in Excel too. You will find the examples in `probReview.R` and `probReview.xls` to be helpful for some of the exercises that follow.

Exercises

1. Suppose X is a normally distributed random variable with mean 0.05 and variance $(0.10)^2$. Compute the following

- $\Pr(X > 0.10)$
- $\Pr(X < -0.10)$
- $\Pr(-0.05 < X < 0.15)$
- 1% quantile, $q_{0.01}$
- 5% quantile, $q_{0.05}$
- 95% quantile $q_{0.95}$
- 99% quantile , $q_{0.99}$

Hint: you can use the R functions `pnorm()` and `qnorm()` to answer these questions.

2. Let X denote the monthly return on Microsoft Stock and let Y denote the monthly return on Starbucks stock. Assume that $X \sim N(0.05, (0.10)^2)$ and $Y \sim N(0.025, (0.05)^2)$.

- Using a grid of values between -0.25 and 0.35, plot the normal curves for X and Y. Make sure that both normal curves are on the same plot.
- Comment on the risk-return tradeoffs for the two stocks.

3. Let R denote the simple monthly return on Microsoft stock and let W_0 denote initial wealth to be invested over the month. Assume that $R \sim N(0.04, (0.09)^2)$ and that $W_0 = \$100,000$.

- Determine the 1% and 5% value-at-risk (VaR) over the month on the investment. That is, determine the loss in investment value that may occur over the next month with 1% probability and with 5% probability.

4. Let r denote the continuously compounded monthly return on Microsoft stock and let W_0 denote initial wealth to be invested over the month. Assume that $r \sim$ iid $N(0.04, (0.09)^2)$ and that $W_0 = \$100,000$.

- Determine the 1% and 5% value-at-risk (VaR) over the month on the investment. That is, determine the loss in investment value that may occur over the next month with 1% probability and with 5% probability. (Hint: compute the 1% and 5% quantile from the Normal distribution for r and then convert continuously compounded return quantile to a simple return quantile using the transformation)
- Determine the 1% and 5% value-at-risk (VaR) over the year on the investment. Hint: to answer this question, you must determine the normal distribution that applies to the annual (12 month) continuously compounded return. This was done as an example in class.

5. In this question, you will examine the chi-square and Student's t distributions.

- On the same graph, plot the probability curves of chi-squared distributed random variables with 1, 2, 5 and 10 degrees of freedom. Use different colors and line styles for each curve.
- On the same graph, plot the probability curves of Student's t distributed random variables with 1, 2, 5 and 10 degrees of freedom. Also include the probability curve for the standard normal distribution. Use different colors and line styles for each curve.

6. Consider the following joint distribution of X and Y:

X-Y	1	2	3
1	0.1	0.2	0
2	0.1	0	0.2
3	0	0.1	0.3

- Find the marginal distributions of X and Y. Using these distributions, compute $E[X]$, $var(X)$, $SD(X)$, $E[Y]$, $var(Y)$ and $SD(Y)$.
- Compute COV(X, Y) and CORR(X, Y)
- Are X and Y independent? Fully justify your answer.

7. Consider a one month investment in two Northwest stocks: Amazon and Costco. Suppose you buy Amazon and Costco at the end of September at $P_{A,t-1} = \$38.23, P_{C,t-1} = \$41.11$ and then sell at the

end of the October for $P_{A,t} = \$41.29, P_{C,t} = \$41.74$. (Note: these are actual closing prices for 2004 taken from Yahoo!)

- What are the simple monthly returns for the two stocks?
- What are the continuously compounded returns for the two stocks?
- Suppose Costco paid a \$0.10 per share cash dividend at the end of October. What is the monthly simple total return on Costco? What is the monthly dividend yield?
- Suppose the monthly returns on Amazon and Costco from question (a) above are the same every month for 1 year. Compute the simple annual returns as well as the continuously compounded annual returns for the two stocks.
- At the end of September, 2004, suppose you have $\$10,000$ to invest in Amazon and Costco over the next month. If you invest $\$8,000$ in Amazon and $\$2,000$ in Costco, what are your portfolio shares, x_A and x_C. (Assume partial share purchases are possible)
- Continuing with the previous question, compute the monthly simple return and the monthly continuously compounded return on the portfolio. Assume that Costco does not pay a dividend.

8. Consider an investment in a foreign stock (e.g. a stock trading on the London stock exchange) by a U.S. national (domestic investor). The domestic investor takes U.S. dollars, converts them to the foreign currency (e.g. British Pound) via the exchange rate (price of foreign currency in U.S. dollars) and then purchases the foreign stock using the foreign currency. When the stock is sold, the proceeds in the foreign currency must then be converted back to the domestic currency. To be more precise, consider the information in the table below:

Time	Cost of 1 Pound	Value of UK Shares	Value in U.S. \$
0	\$1.50	£40	1.5*40 = 60
1	\$1.30	£45	1.3*45 = 58.5

- Compute the simple rate of return, Re, from the prices of foreign currency.
- Compute the simple rate of return, Ruk, from the UK stock prices.
- Compute the simple rate of return, Rus, from the prices in US dollars.
- What is the relationship between Rus, Ruk and Re?

9. Let R_t denote the simple monthly return and assume that $R_t \sim N(\mu, \sigma^2)$. Consider the 2-period simple return $R_t(2) = (1 + R_t)(1 + R_{t-1}) - 1$.

- Assuming that $\text{COV}(R_t, R_{t-1}) = 0$, show that $E[R_t R_{t-1}] = \mu^2$. Hint: Use $\text{COV}(R_t, R_{t-1}) = E[R_t R_{t-1}] - E[R_t]E[R_{t-1}]$.
- Show that $E[R_t(2)] = (1 + \mu)^2 - 1$
- Is $R_t(2)$ normally distributed? Why or why not?