Hypothesis Testing.

$H_0 = \mu = \mu_0$

Problem-1

1.) Hypothesis statement

$H_0: \mu = 50$

$H_a: \mu \neq 50$

2. Choose test method, significance level & check assumptions

- Two tailored test $\alpha = 0.05$

$df = n-1 = 40-1 = 39$

Critical value.

$t_{0.025, 39} \approx 2.0227$

$$SE = \frac{s}{\sqrt{n}} = \frac{12}{\sqrt{40}} = \frac{12}{6.346} \approx 1.8974$$

Confidence Interval.

$$ME = t \times SE = 2.0227 \times 1.8974$$
$$\approx 3.838$$

$$\bar{X} + ME = 52 \pm 3.838$$
$$= (55.838,\ 48.162)$$

The null values lies ~~as~~ inside 95% CI (48.16, 55.838) Therefore fail to reject H_0 at $\alpha = 0.05$.

Problem: 3

→ Sol^n:-

Given:

Sample $(n) = 500$

No. of success $(X) = 290$

Sample proportion $(\hat{p}) = \frac{290}{500} = 0.58$

$\alpha = 0.05$

$$SE = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= \sqrt{\frac{6.5\,(1-0.58)}{500}}$$

$$= 0.022$$

Critical value at 95% confidence

$$Z_{0.025} \approx 1.9$$

$$ME = Z \times SE$$
$$= 1.9 \times 0.22$$
$$= 0.418$$

Confidence Interval

$$CI = \hat{p} \pm ME$$
$$= 0.58 + 0.418$$
$$= (0.998, 0.162)$$

The null value lies inside 95% CI
Therefore fail to reject H_0 at $\alpha = 0.05$.

2.2 Problem 2: Delivery time (one-sided)

Sol^n:-

Hypothesis statement

$H_0: \mu = 4$ vs $H_a: \mu < M$

Choose test method significance level & check assumptions

use t- distribution, $\alpha = 0.05$

$df = n-1 = 25-1 = 24$

Find critical value:

$$SE = \frac{s}{\sqrt{n}} = \frac{0.9}{\sqrt{25}} = 0.18$$

For one sided test at $\alpha = 0.05$

$$t_{0.05,\ 24} \approx 1.7109$$

Compute Confidence Interval.

$$CI = \bar{X} \pm ME$$
$$= (4.107,\ 3.493)$$

Decision Rule:

Since $4 < 4.108$, the null value is inside the C fail to regret H_0.

3.3 Problem A. Retail Price Test

Soln:-

Given

$n = 12$

Sample mean $(\bar{x}) = 79.8$

Sample Standard deviation $(s) = 9.6$

$\alpha = 0.05$

$H_0 : \mu = 75$

$H_a : \mu \neq 75$

$df = n - 1 = 12 - 1 = 11$

Critical value,

$t_{0.025,\,11} \approx 2.201$

$$SE = \frac{s}{\sqrt{n}} = \frac{9.6}{\sqrt{12}} \approx 2.77$$

$M.E = t \times S.E = 2.201 \times 2.77 \approx 0.10.$

Confidence Interval

$$C.I = \bar{X} \pm ME$$
$$= 79.8 \pm 6.10$$
$$= (73.7,\ 85.9)$$

Hypothesis mean 75 is inside the CI so we fail to reject H_0.

$$t = \frac{\lambda - \mu_0}{S/\sqrt{n}}$$

$$= \frac{79.8 - 75}{2.77}$$

$$= 1.732.$$

Decision Rule:

If $|t| > t_{critical}$, reject H_0

Here $|1.732| < 2.201$, fail to reject H_0.

Problems :-

⇒ Solⁿ :-

$H_0 : \mu \le 2.5$

$H_1 : \mu > 2.5$

Given

$n = 16$

Sample mean $(\bar{X}) = 2.8$

Sample standard deviation $(S) = 0.9$

$\alpha : 0.10$

Compute t-statistics

$$t = \frac{\bar{X} - \mu_0}{S\sqrt{n}} = \frac{2.8 - 2.5}{0.9\sqrt{16}} = \frac{0.3}{0.225} = 1.33$$

Critical Value

$df = 16 - 1 = 15$

$t_{0.9, 15} \approx 1.3406$

$$S.E = \frac{S}{\sqrt{n}} = \frac{0.9}{\sqrt{16}} = 0.225$$

$ME = t \times SE$
$= 1.3400 \times 0.225$
$= 0.3016.$

Confidence Interval.

$C.I = ME \pm \bar{x}$
$= 2.8 \pm 0.3016$
$= (3.1016, 2.4924)$

Hypothesis mean of 2.5 lies between 3.1016 & 2.4984 so we fail to reject μ_0.

Decision Rule.

If ~~Rule~~ $|t| >$ t critical, reject H_0
$|1.33| < 1.3406$ so fail to reject μ_0.

Problem C: Email Checking Through.

Hypothesis Statement

$H_0 : p = 0.08$

$H_1 : p \neq 0.08$

Given, $\alpha = 0.05$

Sample proportion:

$$\hat{p} = \frac{78}{900} = 0.0867$$

Compute Test Statistics

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.0867 - 0.08}{\sqrt{\frac{0.08 \times 0.92}{900}}} = \frac{0.0067}{0.00906}$$

$$\approx 0.74.$$

Compute critical value:

For a two-tailed test at $\alpha = 0.05$

$Z_{0.975} \approx \pm 1.96$

Decision Rule:

If $|z| > 1.96$, reject H_0

Here $|z| = 0.74 < 1.96$, fail to reject H_0.

There is not enough evidence at the 5% significance level to conclude that the true CTR differs from 8%.

Problem: D: Delivery Reliability

→ Sol^n:

Given

$n = 1200$

No. of success $(x) = 1100$

Sample proportion $(\hat{p}) = \frac{1100}{1200} = 0.9167$.

$\alpha = 0.01$

Hypothesis statement

$H_0 : p = 0.95$

$H_1 : p \neq 0.95$

compute test statistics.

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.9167 - 0.95}{\sqrt{\frac{0.95 \times 0.05}{1200}}}$$

$$= \frac{0.0333}{0.00629} \approx -5.29$$

Compute critical value.

For a two tailored test at $\alpha = 0.01$

$Z_{0.995} \approx \pm 2.576$

Decision Rule:

If $|z| > 2.570$ reject H_0

Here, $|z| = |-5.29| > 2.576$, so reject H_0.

There is a strong statistical evidence at the 1% level that the true on delivery rate differs from 95%. Since the sample proportion is lower than this suggests underperformance relative to the KPI.

Problem: E A/B Test Revenue lift.

⇒ Soln:-

Given

For group - Control (A):

$n_1 = 45, \quad \bar{x}_1 = 24.50 \ ; \ S_1 = 7.2$

For Group- Treatment (B):

$n_2 = 50, \quad \bar{x}_2 = 27.10, \quad S_2 = 8.0$

Hypothesis statement

$H_0 : \mu_2 = \mu_1 \quad vs \quad H_a : \mu_2 > \mu_1$

one right tailed test, with significance level of $\alpha =$ ~~0.05~~

compute Test statistic (welch - statistic)

$$SE = \sqrt{\frac{S_1^2}{n} + \frac{S_2^2}{n_2}} = \sqrt{\frac{7.2^2}{4.5} + \frac{8.0^2}{50}}$$

$\simeq 1.55949$

Degree of freedom - Welch - Satterthwaite :

$$df \approx \frac{\left(s_1^2/n + s_2^2/n\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}} \approx 93.0$$

Compute critical value.

Two sides 95%, CI $t_{0.975, 93} \approx 1.661$

Decision by critical value approach

$|t| > t_{critical}$ i.e $1.6672 > 1.661$, so reject H_0

Verified by p-value : Compute p-value :-

p-value $= P(T \geq |t_{observed}|)$

using python & scipt stats :

1 - stats . t . cdf (t-statistics, df)

Since p-value $\leq \alpha$, we reject null (H_0)

p-value $= 0.05 \leq \alpha = 0.05$

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```
[5] import numpy as np
    import scipy.stats as st
```

4.4 Exercises: Problem A–Which CreativeWins? Comparing Mean CTR Across Ads: Fresh-mart tested four ad creatives in an emailcampaign. For each group, you collected CTR (%) from equal sub- samples. Given (Summary Stats:- Group A: n1 =30, x1=5.2,s1 = 0.9- GroupB: n2 =30, x2 =5.7,s2 =1.0- Group C: n3 30, x3=5.1,s3 =0.8- Group D: n4 =30, x4=5.9,s4 =1.1• Test at a=0.05 using one -way ANOVA.

```
# Sample sizes for each group
n = [30, 30, 30, 30]

# Sample means for each group
means = [5.2, 5.7, 5.1, 5.9]

# Sample standard deviations for each group
stds = [0.9, 1.0, 0.8, 1.1]

# Total number of observations
N = sum(n)

# Number of groups
k = len(n)

# Step 1: Compute the grand mean (weighted average of group means)
grand_mean = sum([n[i] * means[i] for i in range(k)]) / N
print(f"Grand Mean: {grand_mean:.3f}")

# Step 2: Compute the Sum of Squares Between (SSB)
# Measures variability between group means and the grand mean
SSB = sum([n[i] * (means[i] - grand_mean) ** 2 for i in range(k)])
```

```
print(f"SSB: {SSB:.3f}")

# Step 3: Compute the Sum of Squares Within (SSW)
# Measures variability within each group using standard deviations
SSW = sum([(n[i] - 1) * stds[i] ** 2 for i in range(k)])
print(f"SSW: {SSW:.3f}")

# Step 4: Degrees of freedom
df_between = k - 1          # for SSB
df_within = N - k           # for SSW
print(f"df_between: {df_between}, df_within: {df_within}")

# Step 5: Compute Mean Squares
MSB = SSB / df_between      # Mean Square Between
MSW = SSW / df_within       # Mean Square Within
print(f"MSB: {MSB:.3f}, MSW: {MSW:.3f}")

# Step 6: Compute F-statistic
F = MSB / MSW
print(f"F-statistic: {F:.3f}")

# Step 7: Compute p-value using F-distribution
p_value = stats.f.sf(F, df_between, df_within)
print(f"p-value: {p_value:.5f}")

# Step 8: Decision at alpha = 0.05
alpha = 0.05
if p_value < alpha:
    print("Decision: Reject H0 - At least one group mean differs.")
else:
    print("Decision: Fail to reject H0 - No significant difference between group means.")
```

```
Grand Mean: 5.475
```

```
Grand Mean: 5.475
SSB: 13.425
SSW: 106.140
df_between: 3, df_within: 116
MSB: 4.475, MSW: 0.915
F-statistic: 4.891
p-value: 0.00308
Decision: Reject H0 - At least one group mean differs.
```

Problem B -Regional Sales Performance A retailer collected the following average order values ($)from three regions:North (15 stores) South (12 stores) West (18 stores) 82.50 76.80 88.10 85.30 74.20 90.50 79.80 78.50 85.70 83.10 75.90 92.30 81.40 77.30 87.60 Tasks:Region number of sample ni Mean xi Standard Deviation si North 15 stores South 12 storesWest 18 stores

1. Complete the summarystatistics table:
2. Perform ANOVA at a=0.01

```
[3] import numpy as np
    import scipy.stats as stats

    # Step 0: Given data (only first 5 values provided; assuming these are the sample for each region)
    north = np.array([82.50, 85.30, 79.80, 83.10, 81.40])
    south = np.array([76.80, 74.20, 78.50, 75.90, 77.30])
```

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```python
# Step 0: Given data (only first 5 values provided; assuming these are the sample for each region)
north = np.array([82.50, 85.30, 79.80, 83.10, 81.40])
south = np.array([76.80, 74.20, 78.50, 75.90, 77.30])
west = np.array([88.10, 90.50, 85.70, 92.30, 87.60])

# Step 1: Summary statistics (mean and std)
means = [np.mean(north), np.mean(south), np.mean(west)]
stds = [np.std(north, ddof=1), np.std(south, ddof=1), np.std(west, ddof=1)]
ns = [15, 12, 18]  # given sample sizes

print("Region-wise summary statistics:")
for region, n_i, mean_i, std_i in zip(["North", "South", "West"], ns, means, stds):
    print(f"{region}: n={n_i}, mean={mean_i:.2f}, std={std_i:.2f}")

# Step 2: Compute Grand Mean
N = sum(ns)
grand_mean = sum([means[i] * ns[i] for i in range(3)]) / N
print(f"\nGrand Mean: {grand_mean:.3f}")

# Step 3: Compute SSB (Sum of Squares Between)
SSB = sum([ns[i] * (means[i] - grand_mean) ** 2 for i in range(3)])
print(f"SSB: {SSB:.3f}")

# Step 4: Compute SSW (Sum of Squares Within, from sample stds)
SSW = sum([(ns[i] - 1) * stds[i] ** 2 for i in range(3)])
print(f"SSW: {SSW:.3f}")

# Step 5: Degrees of Freedom
df_between = 3 - 1
df_within = N - 3
print(f"df_between: {df_between}, df_within: {df_within}")

# Step 6: Mean Squares
MSB = SSB / df_between
```

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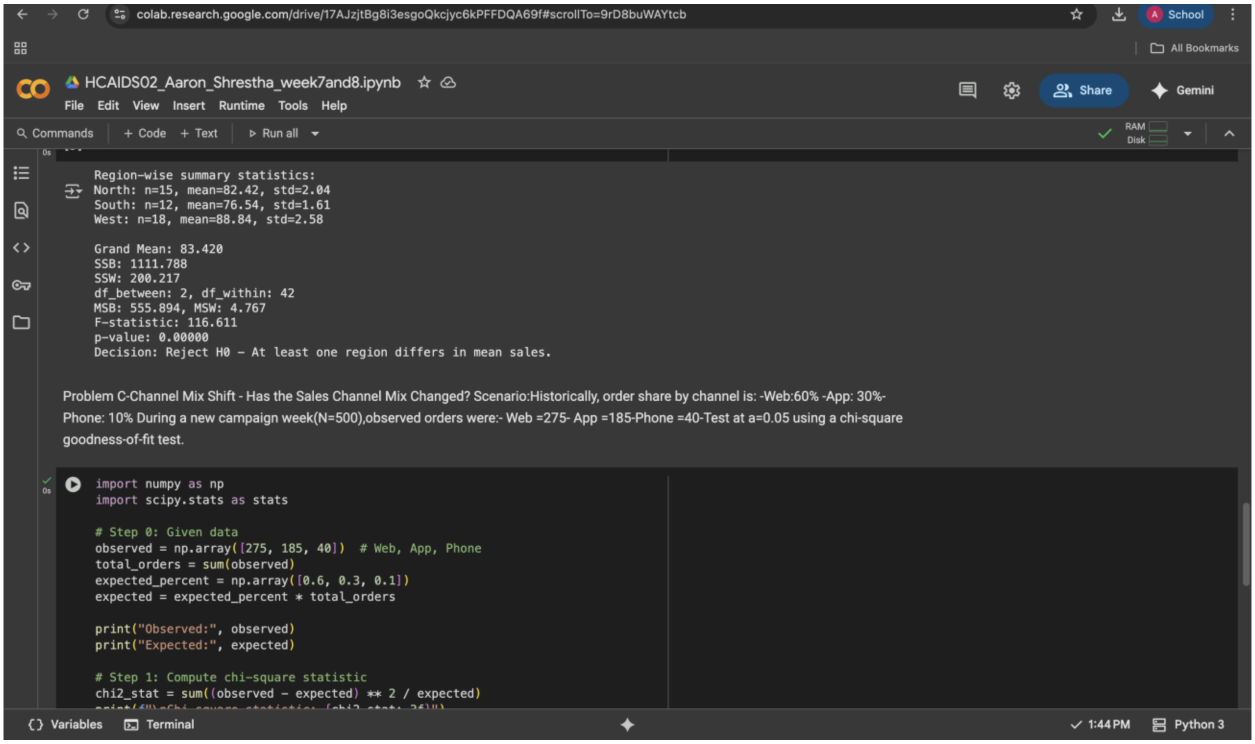
```python
print(f"SSW: {SSW:.3f}")

# Step 5: Degrees of Freedom
df_between = 3 - 1
df_within = N - 3
print(f"df_between: {df_between}, df_within: {df_within}")

# Step 6: Mean Squares
MSB = SSB / df_between
MSW = SSW / df_within
print(f"MSB: {MSB:.3f}, MSW: {MSW:.3f}")

# Step 7: F-statistic
F = MSB / MSW
print(f"F-statistic: {F:.3f}")

# Step 8: p-value
p_value = stats.f.sf(F, df_between, df_within)
print(f"p-value: {p_value:.5f}")

# Step 9: Decision at alpha = 0.01
alpha = 0.01
if p_value < alpha:
    print("Decision: Reject H0 - At least one region differs in mean sales.")
else:
    print("Decision: Fail to reject H0 - No significant difference between regions.")
```

```
Region-wise summary statistics:
North: n=15, mean=82.42, std=2.04
South: n=12, mean=76.54, std=1.61
West: n=18, mean=88.84, std=2.58

Grand Mean: 83.420
SSB: 1111.788
```


colab.research.google.com/drive/17AJzjtBg8i3esgoQkcjyc6kPFFDQA69f#scrollTo=9rD8buWAYtcb
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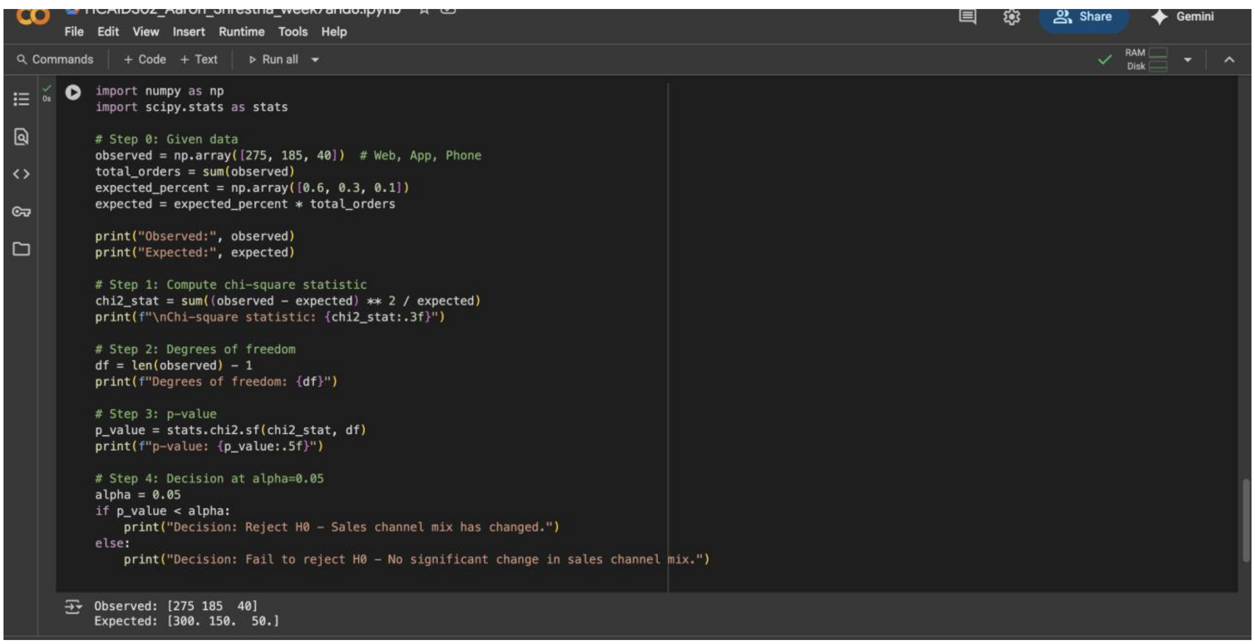
Region-wise summary statistics:
North: n=15, mean=82.42, std=2.04
South: n=12, mean=76.54, std=1.61
West: n=18, mean=88.84, std=2.58
Grand Mean: 83.420
SSB: 1111.788
SSW: 200.217
df_between: 2, df_within: 42
MSB: 555.894, MSW: 4.767
F-statistic: 116.611
p-value: 0.00000
Decision: Reject H0 – At least one region differs in mean sales.
Problem C-Channel Mix Shift - Has the Sales Channel Mix Changed? Scenario:Historically, order share by channel is: -Web:60% -App: 30%-Phone: 10% During a new campaign week(N=500),observed orders were:- Web =275- App =185-Phone =40-Test at a=0.05 using a chi-square goodness-of-fit test.
import numpy as np
import scipy.stats as stats
# Step 0: Given data
observed = np.array([275, 185, 40]) # Web, App, Phone
total_orders = sum(observed)
expected_percent = np.array([0.6, 0.3, 0.1])
expected = expected_percent * total_orders
print("Observed:", observed)
print("Expected:", expected)
# Step 1: Compute chi-square statistic
chi2_stat = sum((observed - expected) ** 2 / expected)


import numpy as np
import scipy.stats as stats
# Step 0: Given data
observed = np.array([275, 185, 40]) # Web, App, Phone
total_orders = sum(observed)
expected_percent = np.array([0.6, 0.3, 0.1])
expected = expected_percent * total_orders
print("Observed:", observed)
print("Expected:", expected)
# Step 1: Compute chi-square statistic
chi2_stat = sum((observed - expected) ** 2 / expected)
print(f"\nChi-square statistic: {chi2_stat:.3f}")
# Step 2: Degrees of freedom
df = len(observed) - 1
print(f"Degrees of freedom: {df}")
# Step 3: p-value
p_value = stats.chi2.sf(chi2_stat, df)
print(f"p-value: {p_value:.5f}")
# Step 4: Decision at alpha=0.05
alpha = 0.05
if p_value < alpha:
print("Decision: Reject H0 – Sales channel mix has changed.")
else:
print("Decision: Fail to reject H0 – No significant change in sales channel mix.")
Observed: [275 185 40]
Expected: [300. 150. 50.]

```
Observed: [275 185  40]
Expected: [300. 150.  50.]

Chi-square statistic: 12.250
Degrees of freedom: 2
p-value: 0.00219
Decision: Reject H0 – Sales channel mix has changed.
```