Probabilistic Quantum Algorithm for comparing two integers

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March 2023

1 Introduction

There is already a way to use a part of Shor's algorithm to compare two integers by encoding them to basis states. But here, I made a new probabilistic way entirely different from the previous shor's approach. It encodes our two integers in the probability amplitude. And then, by measuring the state in the computational basis, we can know which is the bigger number.

2 Method

Step 1: Add $(1 - ((a + b)/2)$ to both integers 'a' and 'b'.

a⟶a+(1-((a+b)/2)) = ((a+2-b)/2) = x
a⟶b+(1-((a+b)/2)) = ((-a+2+b)/2) = y

This makes the larger integer greater than 1 and the smaller integer less than 1 for any a,b∈Z.

Step 2: Take 'nth power' of both 'x' and 'y'. So it becomes,

$$x^n = ((a + 2 - b)/2)^n$$
$$y^n = ((-a + 2 + b)/2)^n$$

'n' is odd here and can be varied as large as you want, the larger the 'n' more probable and accurate our result will be.

Step 3: Map both x^n, y^n to the sigmod function $(1/(1 + e^{-z}))$.

$$x^n \longrightarrow (1/(1+\mathrm{e}^{-x^n}))$$
$$y^n \longrightarrow (1/(1+\mathrm{e}^{-y^n}))$$

This maps x^n,y^n between 0 and 1 such that larger value is kept larger after the mapping.

Step 4: Encode in probability amplitudes.

$$|\psi\rangle = \frac{1}{1+e^{-x^n}}|0\rangle + \frac{1}{1+e^{-y^n}}|1\rangle$$

Normalize this vector

$$|\psi'\rangle = \frac{\frac{1}{1+e^{-x^n}}}{\sqrt{\left(\frac{1}{1+e^{-x^n}}\right)^2+\left(\frac{1}{1+e^{-y^n}}\right)^2}}|0\rangle + \frac{\frac{1}{1+e^{-y^n}}}{\sqrt{\left(\frac{1}{1+e^{-x^n}}\right)^2+\left(\frac{1}{1+e^{-y^n}}\right)^2}}|1\rangle$$

Step 5: Now measure this in the computational basis. For 'n' sufficiently large, we can make our probabilities arbitrarily close to 0 or 1 as desired:

1. If $a > b$ then the probability of collapsing to $|0\rangle \approx 1$.
2. If $a < b$ then the probability of collapsing to $|1\rangle \approx 1$.

Hence by checking what basis state our final state is collapsing to we can know which number in bigger. This can more directly be seen by the mapping:

$$a \longrightarrow x \longrightarrow x^n \longrightarrow (1/(1+\mathrm{e}^{-x^n})) \longrightarrow (1/(1+e^{-x^n}))/Normalizingfactor$$

$$b \longrightarrow y \longrightarrow y^n \longrightarrow (1/(1+\mathrm{e}^{-y^n})) \longrightarrow (1/(1+e^{-y^n}))/Normalizingfactor$$

These are our probability amplitudes corresponding to $|0\rangle$ and $|1\rangle$ basis states, respectively.

3 Conclusion

Hence if 'a' is bigger, it will collapse to $|0\rangle$ with probability arbitrarily close to one and hence mostly always. or if 'b' is bigger it will collapse to $|1\rangle$ with probability arbitrarily close to one and hence mostly always.

By choosing 'n' to be large we can make our chances more favorable by making our arbitrarily close to 1 and hence our results are more accurate even with less number of shots.