

The background of the slide is a deep space photograph, likely from the Hubble Space Telescope, showing a dense field of galaxies and stars. The galaxies are of various shapes and sizes, some appearing as bright, fuzzy clouds, others as more distinct, elongated structures. The stars are small, bright points of light, some with visible diffraction spikes. The overall color palette is dominated by black, with highlights of white, yellow, orange, and blue from the celestial objects.

Lecture 15

Differential Equations in Python

The background of the slide is a deep-field astronomical image, likely from the Hubble Space Telescope. It shows a vast field of galaxies and distant stars against a black background. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. Some are bright and clear, while others are faint and distant. The stars appear as small, bright points of light, some with visible diffraction spikes. The overall scene is a dense field of celestial objects, representing the large-scale structure of the universe.

Workshop 5 Review

Differential equations (DiffEQs)...

- Any equation with a derivative can be described as a differential equation
- You have already seen some and you will see more in the future

$$\frac{dx}{dt} = v(t)$$

ODEs

$$\frac{d^2 \mathbf{r}}{dt^2} = \frac{\mathbf{F}(\mathbf{r})}{m}$$

$$\frac{\partial^2 f(x, t)}{\partial t^2} = c^2 \frac{\partial^2 f(x, t)}{\partial x^2}$$

PDE

The background of the entire image is a deep space photograph showing a vast field of galaxies, stars, and cosmic dust. The galaxies are of various shapes and sizes, some appearing as bright, distinct structures while others are fainter. The stars are scattered throughout, with some showing prominent diffraction spikes. The overall color palette is dominated by dark blues and blacks, with highlights of yellow, orange, and white from the celestial bodies.

ODE = EASYish

What we will
cover today

PDE = HARD

Save these for an
upper division
calculus class

Solving ordinary differential equations (ODEs) Analytically

- When these equations are simple enough, we can actually just solve them
- By using some math trickery and the calculus we learned last week, we can turn this into an easy to solve problem
- Unfortunately, not every differential equation can be solved like this, so we need some approximation methods

$$dy/dx = x$$

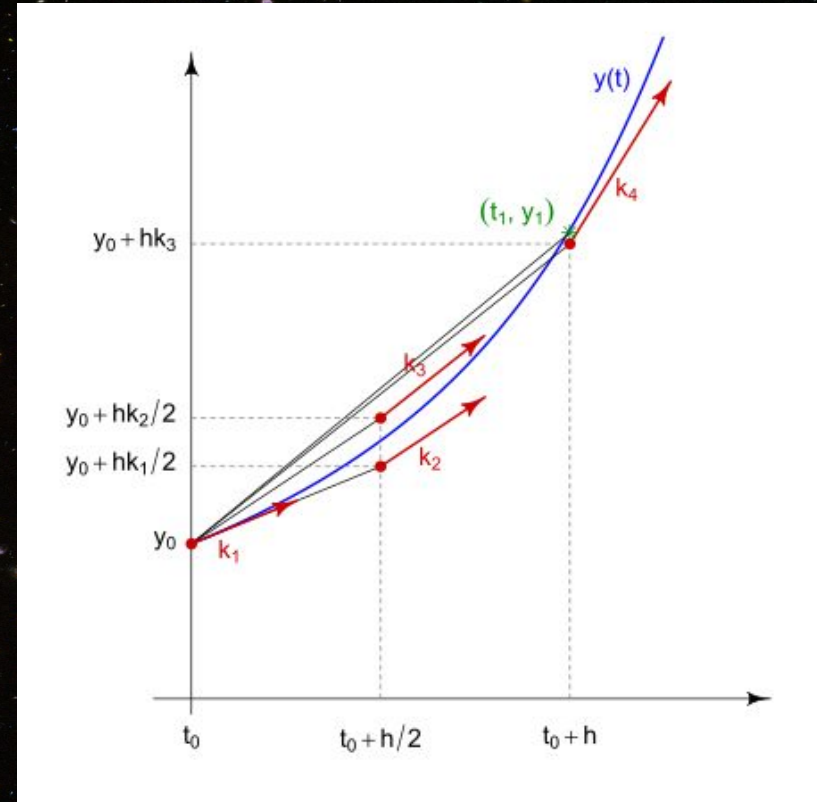
$$dy = x \cdot dx$$

$$\int dy = \int x \, dx$$

$$y = \frac{1}{2} x^2$$

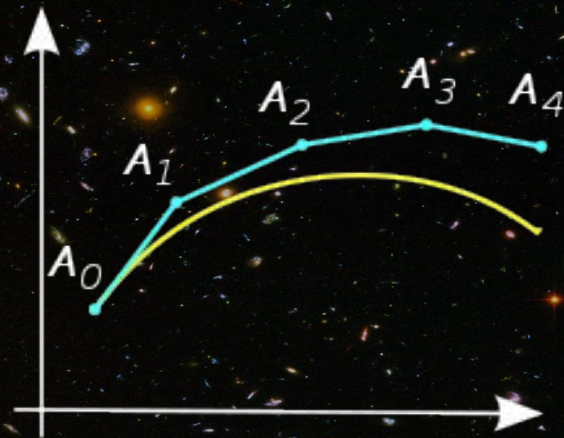
Solving ODEs: Runge Kutta Methods

- A family of different iterative methods (including the Euler Method) used for solving nonlinear equations
- We have some initial conditions (y_0) that we then iterate bit by bit to solve the equation
- Similar idea to the integration approximation methods we studied last week



Solving ODEs by Euler

- **Euler's method** (very similar to numerical integration)
 - Break down to segments
 - Specifying initial condition
 - You need to start from somewhere
 - Order of diff eq. = # of initial conditions
 - Need to specify your step size
- Assign! Not Append!



Solving ODEs...

$$\frac{dy}{dx} = xy$$

$$(x_0, y_0) = (2, 3)$$

```
dx = 0.01
```

```
x0, y0 = 2, 3
```

```
def derivative(y,x):  
    return x*y
```

```
x_arr = np.linspace(x0,5,301)
```

```
y_arr = np.zeros(301)
```

```
y_arr[0] = y0
```

```
for i in range(1, 301):  
    dy = derivative(y_arr[i-1],  
                    x_arr[i-1]) * dx  
    y = y_arr[i-1]+dy  
    y_arr[i] = y
```


Scipy does it better...

<https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.odeint.html>

```
from scipy.integrate import odeint
```

`odeint(func, y0, t)`

`def deriv(y,t):
 return ...`

Your function, make sure
y is the first variable

Initial condition on y

Time point at which you
would like to evaluate
the function for

Equivalent to x_arr

The background of the slide is a deep-field astronomical image, likely from the Hubble Space Telescope. It shows a vast field of galaxies, including spiral, elliptical, and irregular shapes, scattered across a black cosmic background. The galaxies are concentrated in a cluster, with many appearing as bright, glowing objects. The text "Final Coding Demo" is centered in the middle of the image in a white, sans-serif font.

Final Coding Demo