Lecture 14

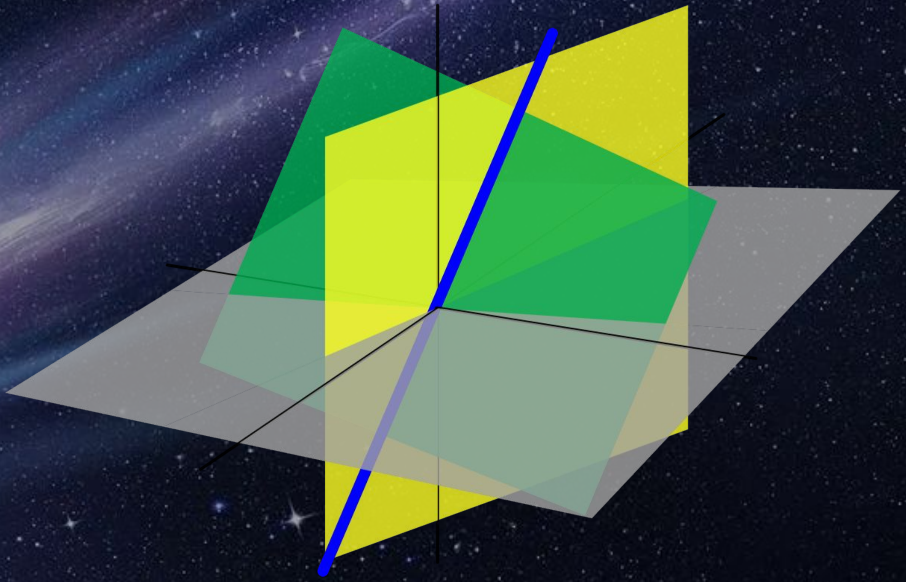
Linear Algebra in Python

What is Linear Algebra?

- Linear Algebra is its own branch of mathematics concerned with linear equations & maps

- In linear algebra, we represent equations as vectors and matrices

- These objects have their own mathematical rules that can help make our lives a lot easier



Vectors

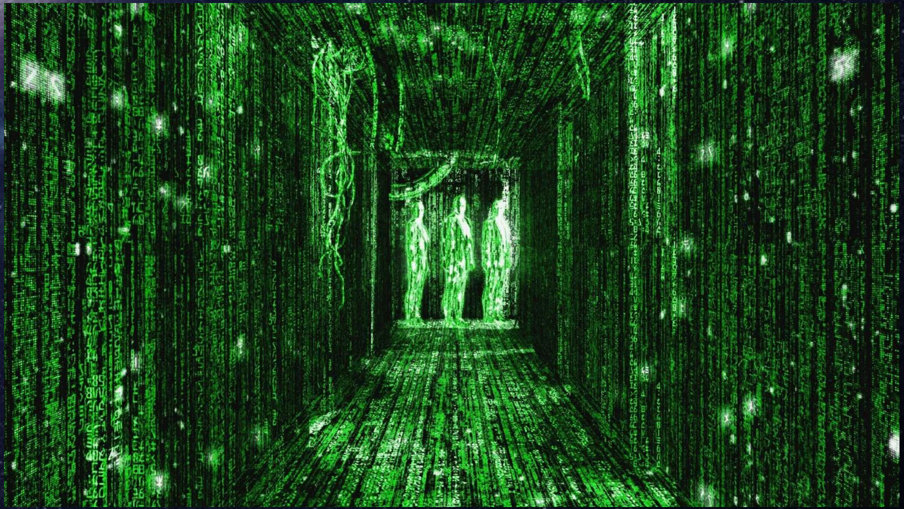
- A vector is any value with both direction and magnitude
- It's opposite is a scalar, which does not have direction
- Examples: Velocity, Acceleration, Momentum, etc.
- For our purposes, we can represent vectors as a matrix

Matrices

- Matrices are just an array of numbers with some dimensions

$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ \vdots \\ m
\end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & \dots & n \end{array} \\
\begin{bmatrix}
a_{11} & a_{12} & \dots & a_{1n} \\
a_{21} & a_{22} & \dots & a_{2n} \\
a_{31} & a_{32} & \dots & a_{3n} \\
\vdots & \vdots & \vdots & \vdots \\
a_{m1} & a_{m2} & \dots & a_{mn}
\end{bmatrix}
\end{array}
$$

- We've been working with matrices all along we just haven't been calling them that



- They can be used and manipulated to represent and calculate a number of scientific concepts

Determinants

- A scalar value calculated from the elements of a square matrix
- Has applications in Physics as they pertain to eigenvalues (lots of uses in quantum mechanics)
- For our purposes, we can use the numpy function: np.linalg.det

DETERMINANT OF MATRIX

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$$

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$= aei + bfg + cdh - ceg - bdi - afh$$

Dot Product & Cross Product

- Two different methods of matrix multiplication used for a number of different scenarios in physics
- Dot product results in a scalar value
- Cross product results in another vector perpendicular to both other vectors
- In Python, these are easy:
  - np.dot
  - np.cross

Lengh of vector **u**,**v**

Symbol for inner product

Angle between **u** and **v**

$$\mathbf{u} \bullet \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos(\theta) \qquad (1)$$

$$= x_1 \times x_2 + y_1 \times y_2 \qquad (2)$$

Vector Cross Product Formula

$$\vec{a} \times \vec{b} = |\vec{a}|\ |\vec{b}|\ \sin\theta\ \hat{n}$$

$$\vec{a} \times \vec{b} = i\,(a_2 b_3 - a_3 b_2) + j\,(a_3 b_1 - a_1 b_3) + k\,(a_1 b_2 - a_2 b_1)$$

Solving Linear Equations

- A linear equation is simply an equation which, when plotted, results in a line
- We can easily solve these systems of equations using matrices
- By representing the coefficients in one matrix and the results of the equations in another, we can solve for the variables

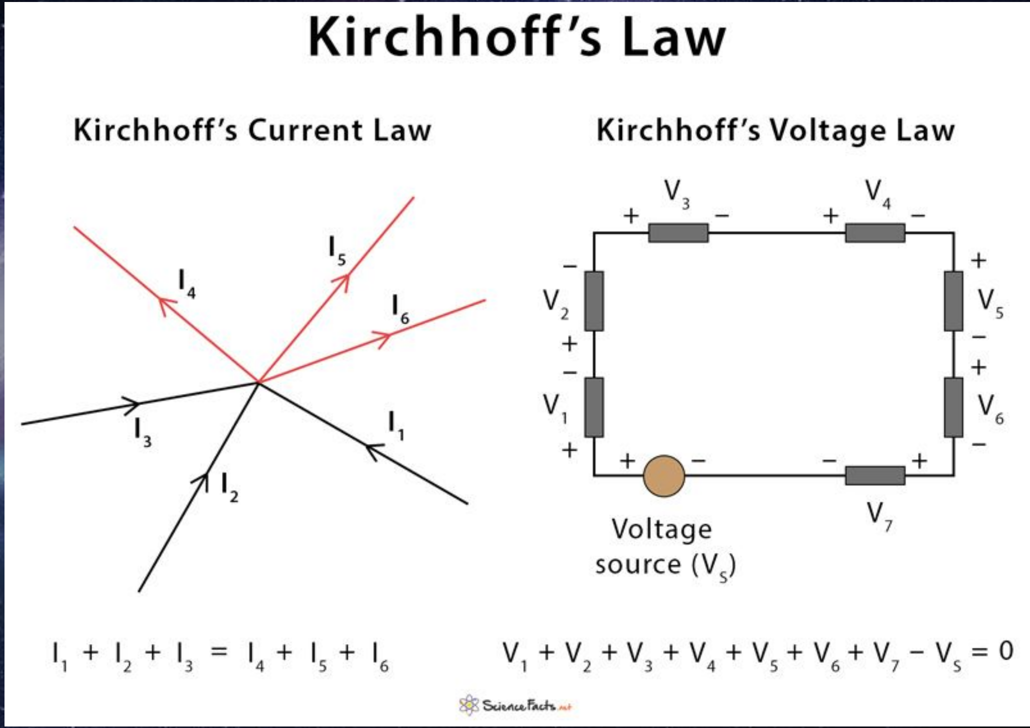
System of linear equations

$$ax + by = c$$

$$a_1x + b_1y = c_1$$

Solving Linear Equations Example: Kirchhoff's Law

- This is just one example of where you may need to solve a system of equations when studying physics
- Using Kirchoff's Law, we can define a system of equations for the current going through the nodes of an electrical circuit
- Those equations can then be solved using matrices (you'll do this in the workshop)



np.linalg.inv().dot()

Using inverse matrices to solve systems of equations

$$x + 2y + 2z = 5$$
$$3x - 2y + z = -6$$
$$2x + y - z = -1$$

$$\underbrace{\begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 2 & 1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{v} = \underbrace{\begin{bmatrix} 5 \\ -6 \\ -1 \end{bmatrix}}_{B}$$

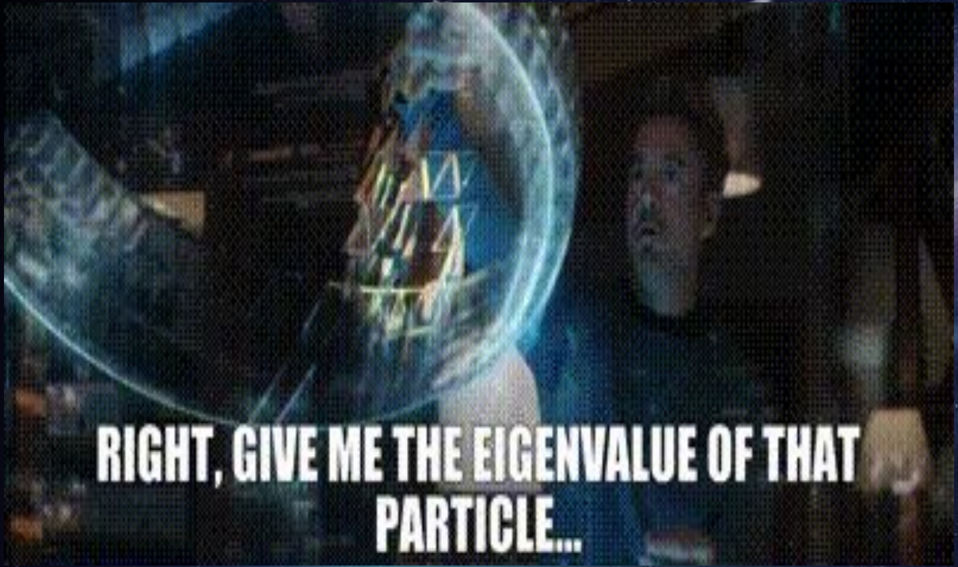
$$Av = B$$

$$Av = B$$
$$A^{-1}Av = A^{-1}B$$
$$v = A^{-1}B$$

$$A^{-1} = \text{inverse of matrix } A = \frac{adjoint(A)}{det(A)}$$

- One of our new best friends solving linear equations in python
- Takes the math on the left here and turns it into just one line of code
- Matrices can just be multiplied and divided like numbers, we need to introduce a new object; the inverse matrix
- Like dividing a number by itself a matrix multiplied by its inverse will equal 1

Eigenvalues & Eigenvectors



- These are special values associated with matrices
- Any matrix, multiplied by its own eigenvector, will return the same eigenvector multiplied by its associated eigenvalue
- $Av=\lambda v$
- These values are very useful in the study of quantum mechanics

Linear Algebra in Python Coding Demo