Math16B – Week 3 Homework

Instructor: Assaf Bar-Natan, Goldsmith 112

Contact: safibn@brandeis.edu

Learning Goals

The following are the learning goals for this week. These are a good benchmark for testing your understanding, and these goals will be used to create the evaulations in this course.

Please submit your code as a .py file using the template on LATTE.

Coding Skills

- Break up a large problem into smaller problems
- Use lists, dictionaries, and flow control to define functions
- Write comments on code that explain what the code is doing

Linear Algebra

- Check if two vectors are scalar multiples of each other
- Recall formulas for orthogonal projections

Problems

New: Code documentation: Submissions must be submitted with documentation docstring. Your documentation does not need to conform to any particular standard, but your function names, and descriptions need to be sufficiently clear that anyone who is encountering dictionaries for the first time will be able to understand what you are doing!

Each problem will be graded out of **two points:**

- 1 point for autograder (running all the tests successfully)
- 1 point for writing code that is readable, correct, and is well-documented

For these problems, you may not use any built-in Python libraries

(1) (2 points) Write a function that checks if two vectors, v_1 and v_2 are linearly independent.

(2) (2 points) You are given a list of vectors, but unfortunately, it contains duplicates! Instead of deleting the duplicates, you want to simplify the data, and create a dictionary whose keys are the vectors, and whose values are the number of times that vector appears in the list. *Your code should work for any objects in the list, not just vectors.*[1]

(3) (2 points) After creating the dictionary in the last question, you decide to do the reverse - to make a dictionary whose entries are numbers, and whose values are lists of vectors that appear that many times in the original list. Implement the function `reverse_dictionary`, whose input is a dictionary d, and whose output is the *reversed* dictionary, whose keys are the values of d, and whose values are the list of *keys* of d which have that value. For example, if `d = {1 : a, 2: b, 3: b}`, then `reverse_dictionary(d)` is `{a : 1, b: [2,3]}`

[1]Normally, lsits cannot be used as keys in a dictionary because they are mutable. For this problem, the vectors will be given as tuples.

(4) (2 point) Write a function that normalizes a nonzero vector. That is, given some $v \neq 0$, your function should return the vector $\frac{v}{\|v\|}$.

(5) (2 points) Write a function that computes orthogonal projection: given v and w, your function should return two vectors: x and y, such that:

- $v = x + y$
- x is parallel to w
- y is perpendicular to w ($y \cdot w = 0$)

You may assume that $w \neq 0$.