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POLYNOMIAL AND NON LINEAR REGRESSION

Ariana Villegas

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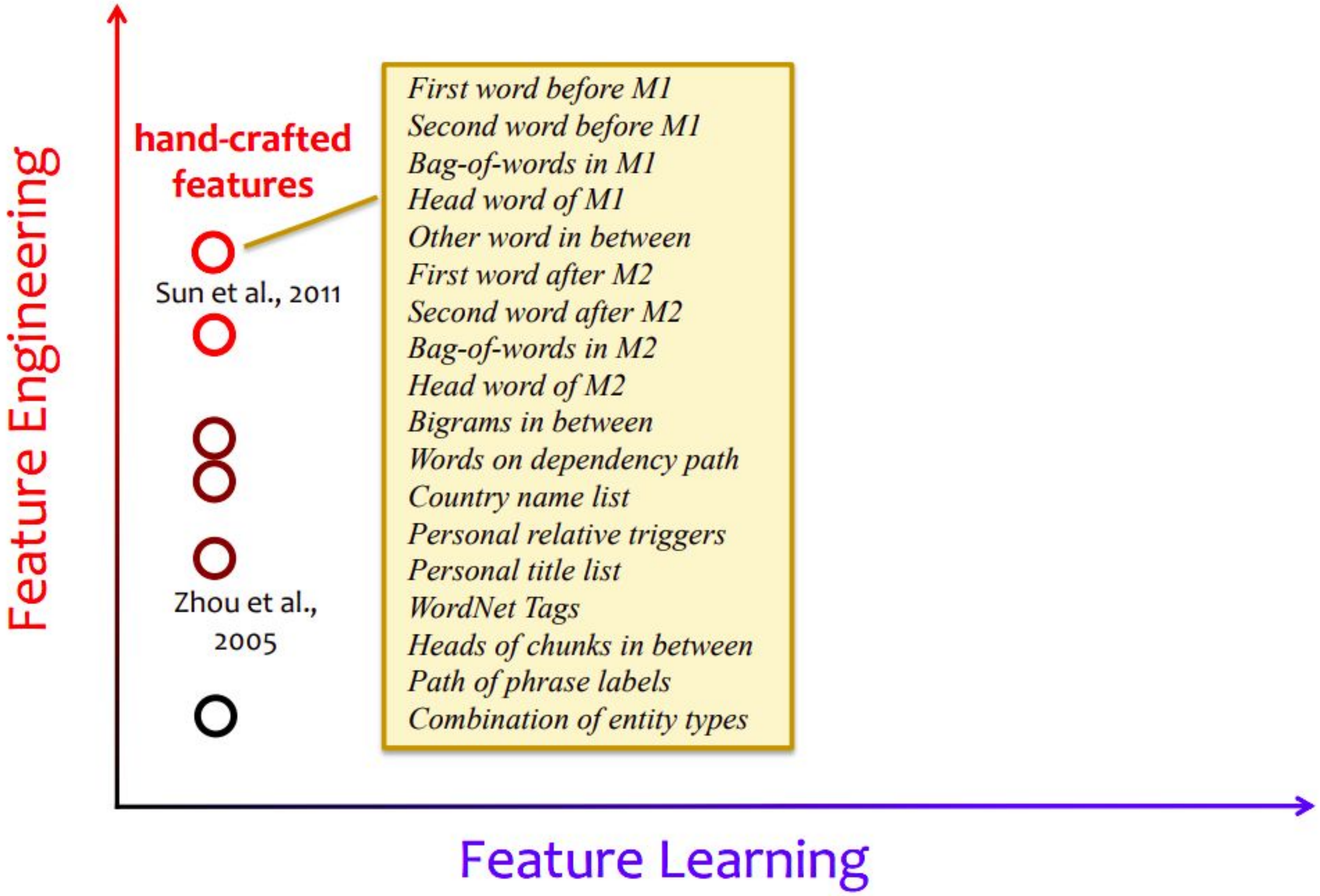
1.

Feature Engineering
NLP & CV

Where do features come from?



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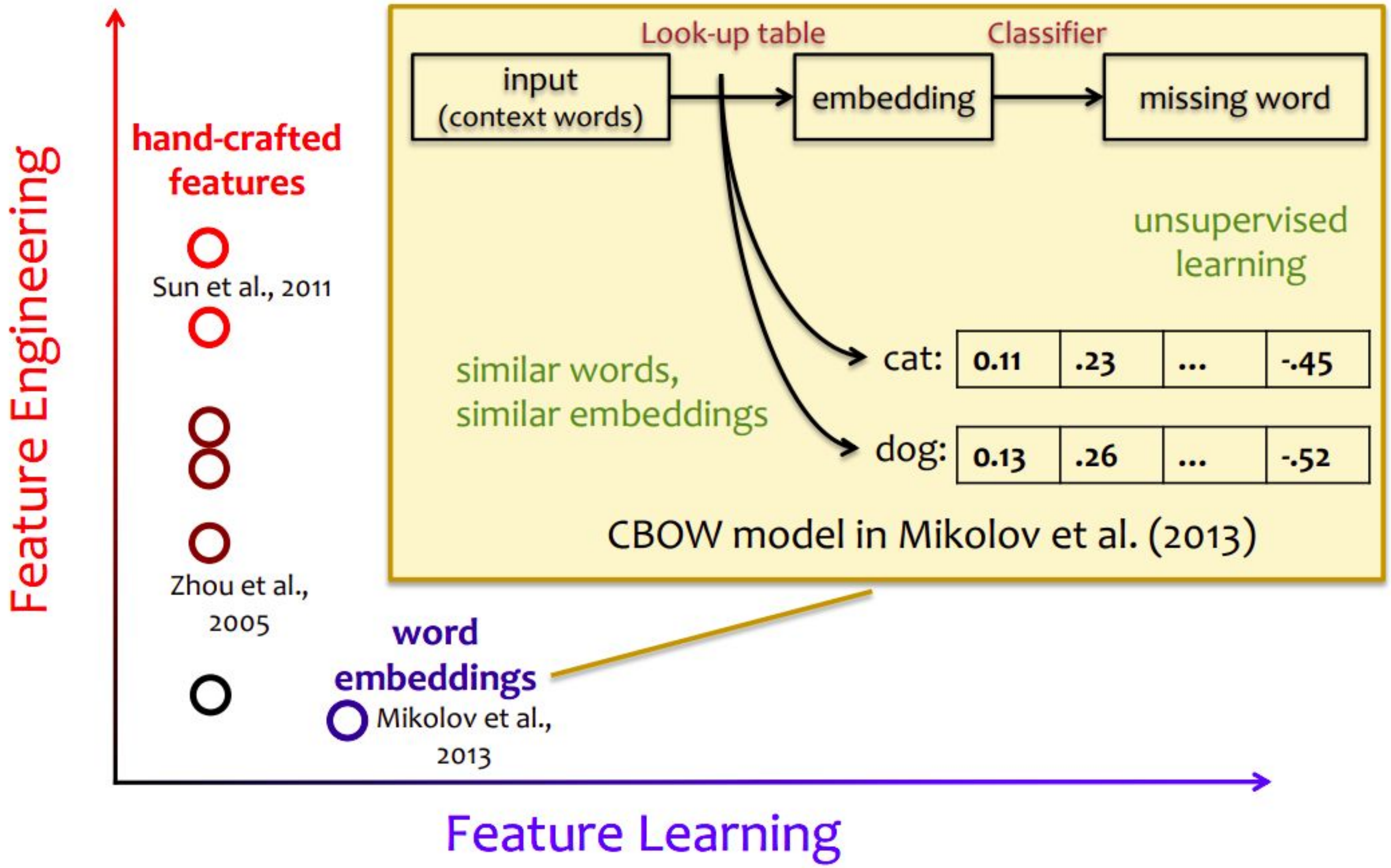

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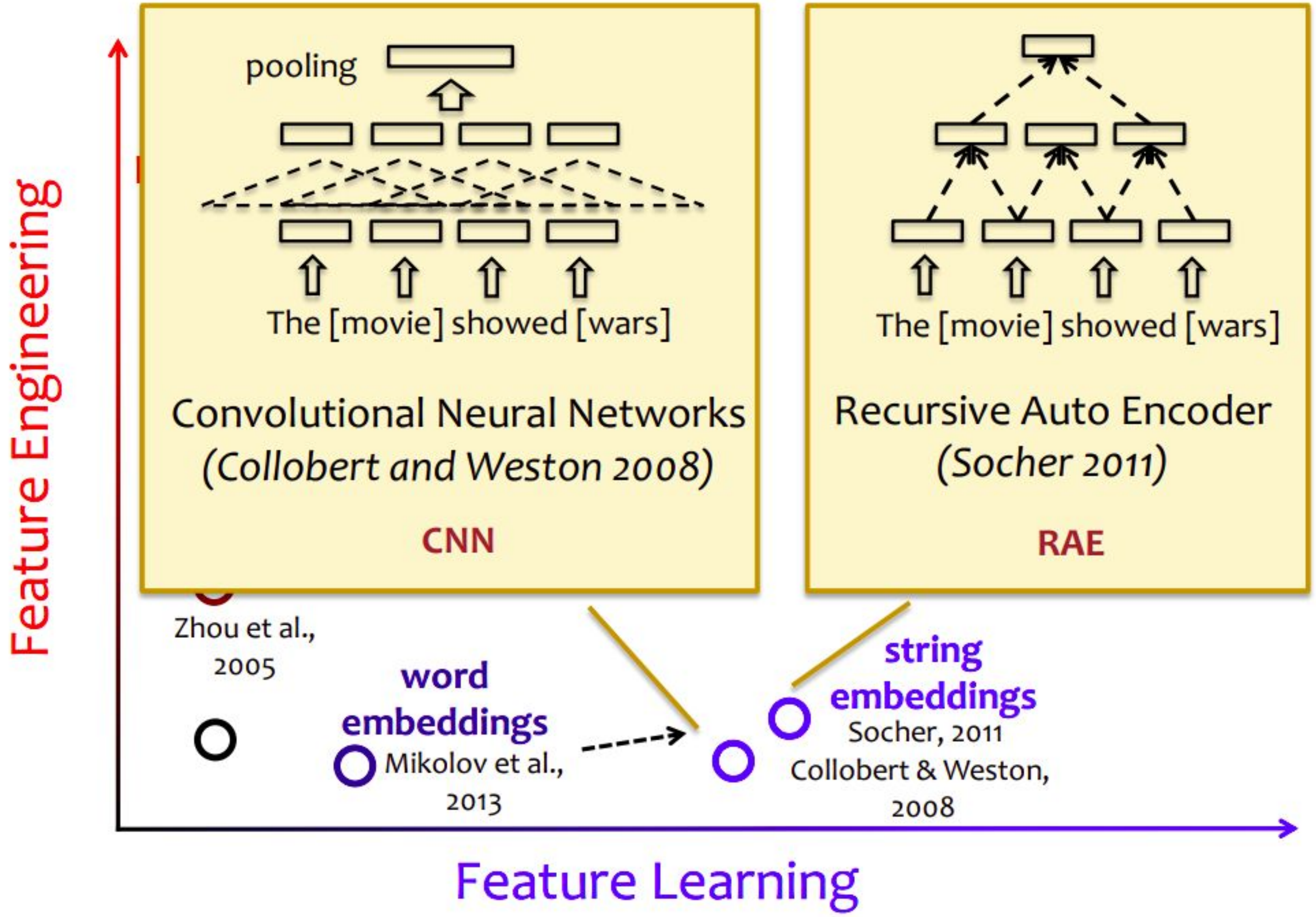

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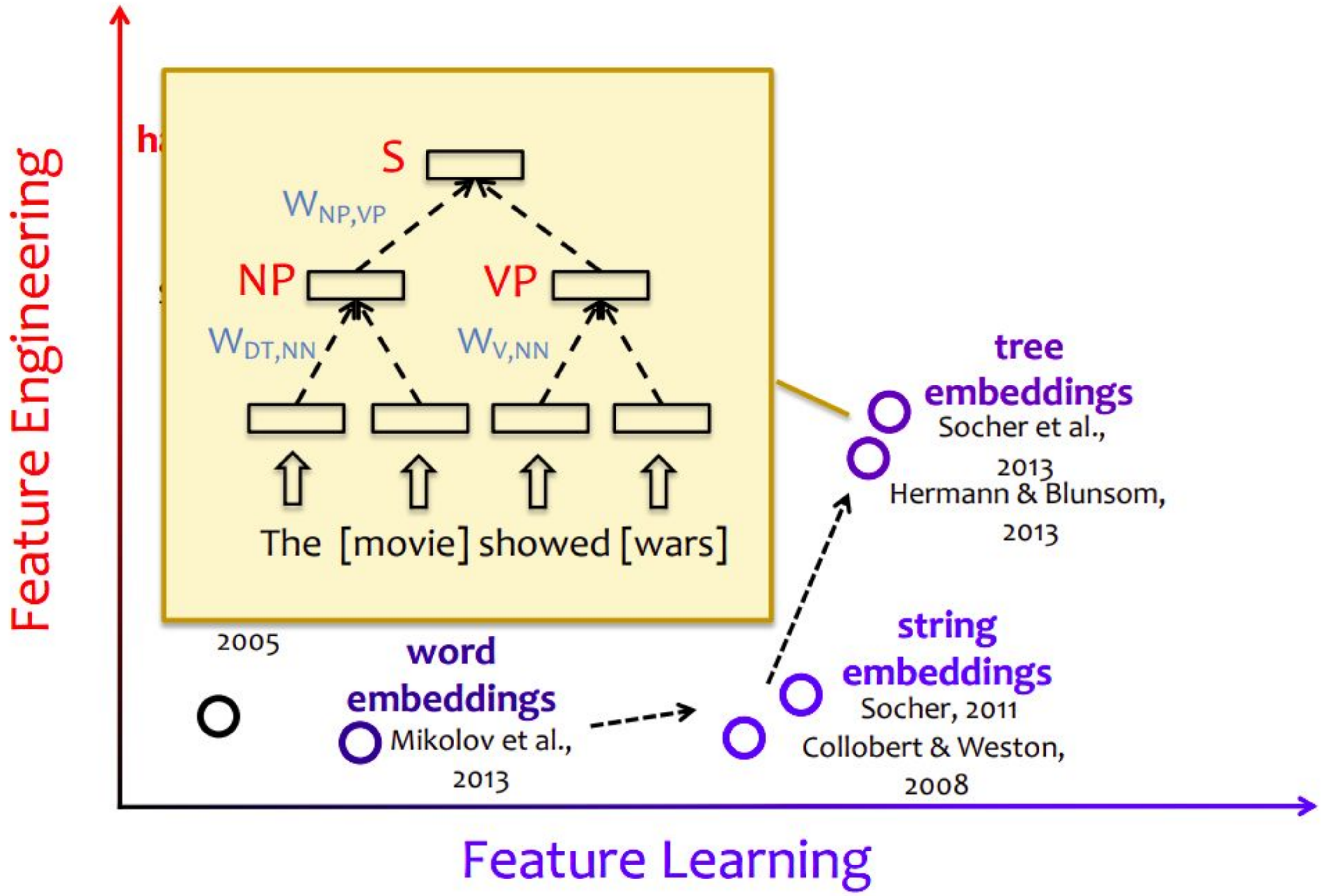

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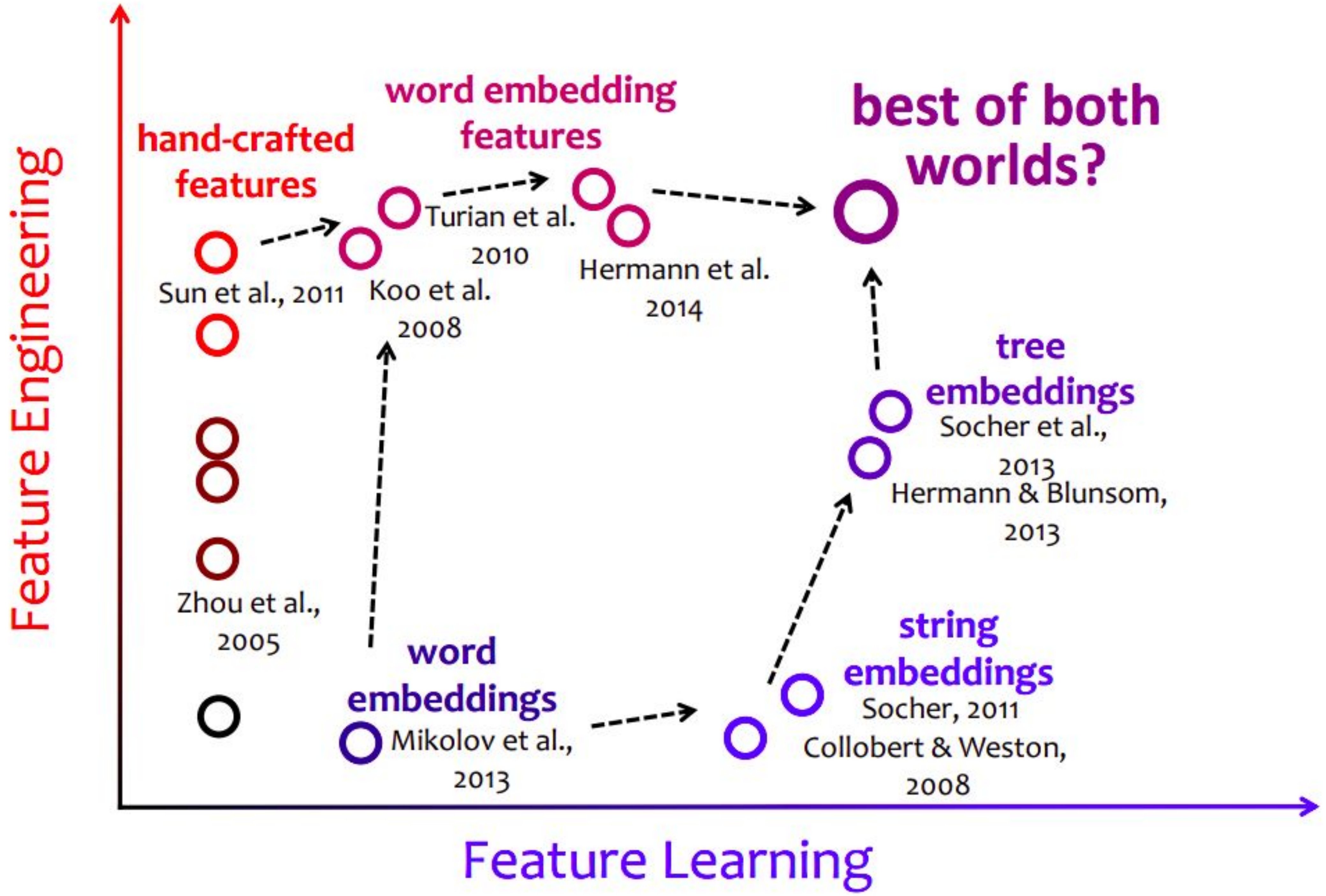

What features should you use for text?

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Per-word Features:

	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	$x^{(4)}$	$x^{(5)}$	$x^{(6)}$
is-capital(w_i)	1	0	1	0	0	0
endswith(w_i,"e")	1	1	0	0	0	1
endswith(w_i,"d")	0	0	0	1	1	0
endswith(w_i,"ed")	0	0	0	1	1	0
w_i == "aardvark"	0	0	0	0	0	0
w_i == "hope"	0	0	0	0	0	1
...	...	...	...	...	...	...

deter.	noun	noun	verb	verb	noun
The	*movie*	*I*	*watched*	*depicted*	*hope*

What features should you use for text?

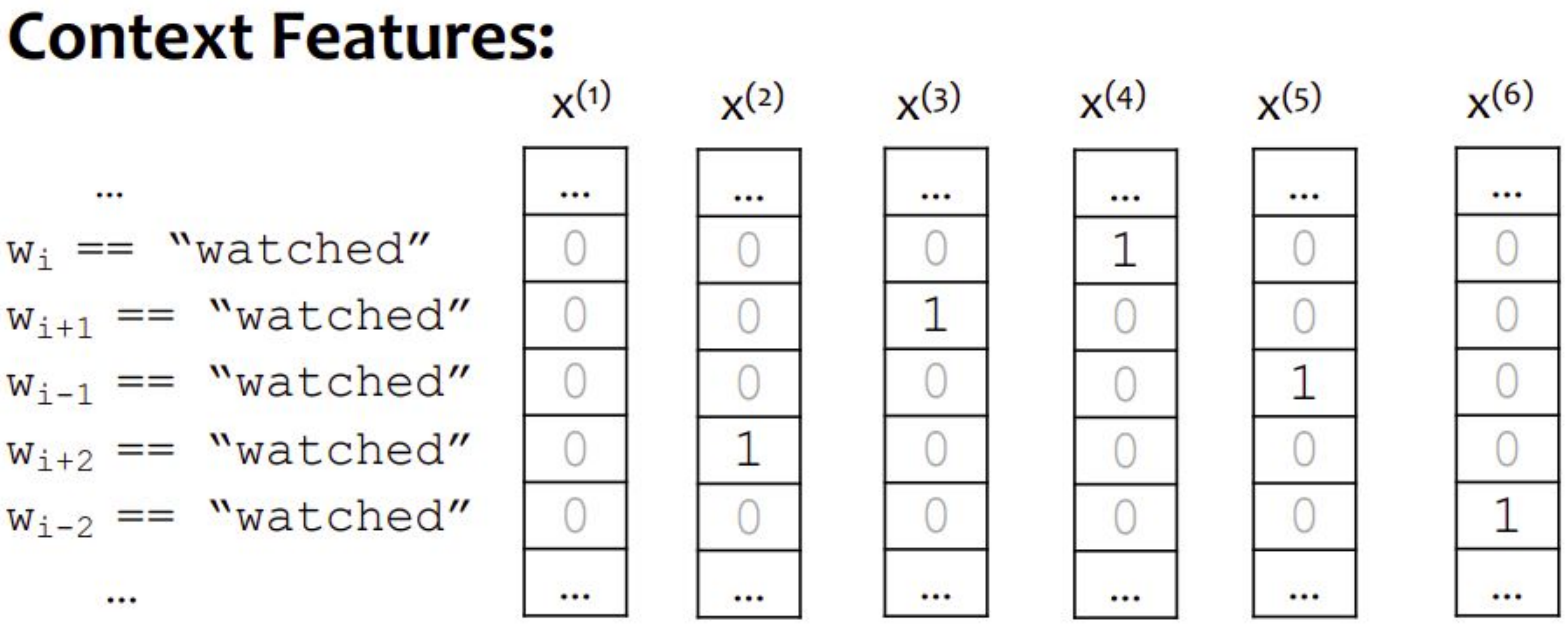

Table 3. Tagging accuracies with different feature templates and other changes on the *WSJ* 19-21 development set.

Model	Feature Templates	# Feats	Sent. Acc.	Token Acc.	Unk. Acc.
3GRAMMEMM	See text	248,798	52.07%	96.92%	88.99%
NAACL 2003	See text and [1]	460,552	55.31%	97.15%	88.61%
Replication	See text and [1]	460,551	55.62%	97.18%	88.92%
Replication′	+rareFeatureThresh = 5	482,364	55.67%	97.19%	88.96%
5W	$+\langle t_0, w_{-2}\rangle, \langle t_0, w_2\rangle$	730,178	56.23%	97.20%	89.03%
5WSHAPES	$+\langle t_0, s_{-1}\rangle, \langle t_0, s_0\rangle, \langle t_0, s_{+1}\rangle$	731,661	56.52%	97.25%	89.81%
5WSHAPESDS	+ distributional similarity	737,955	56.79%	97.28%	90.46%

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Word Embeddings

One-hot vectors

- Standard representation of a word in NLP: 1-hot vector (aka. a string)
- Vectors representing related words share nothing in common

	a	and	be	cat	dog		you	zebra
cat:	0	0	0	1	0	...	0	0
dog:	0	0	0	0	1	...	0	0

Word embeddings

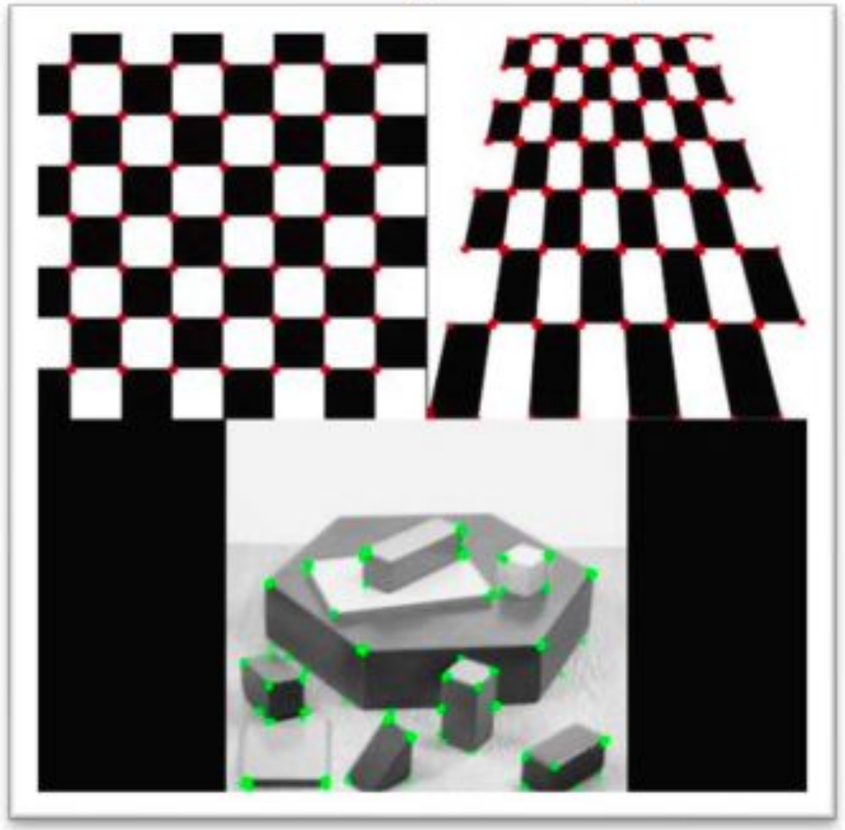
- Word embedding: real-valued vector representation of a word in *M* dimensions
- Related words have similar vectors
- Long history in NLP: Term-doc frequency matrices, Reduce dimensionality with {LSA, NNMF, CCA, PCA}, Brown clusters, Vector space models, Random projections, Neural networks / deep learning

cat:	0.13	.26	...	-.52
dog:	0.11	.23	...	-.45

What features should you use for images?

Edge detection (Canny)

Corner Detection (Harris)



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2.

Non-linear Regression
Definition & Examples

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What if our linear model is not good? How can we create a more complicated model?

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Fitting a Polynomial

We can create a more complicated model by defining input variables that are **combinations of components of x**

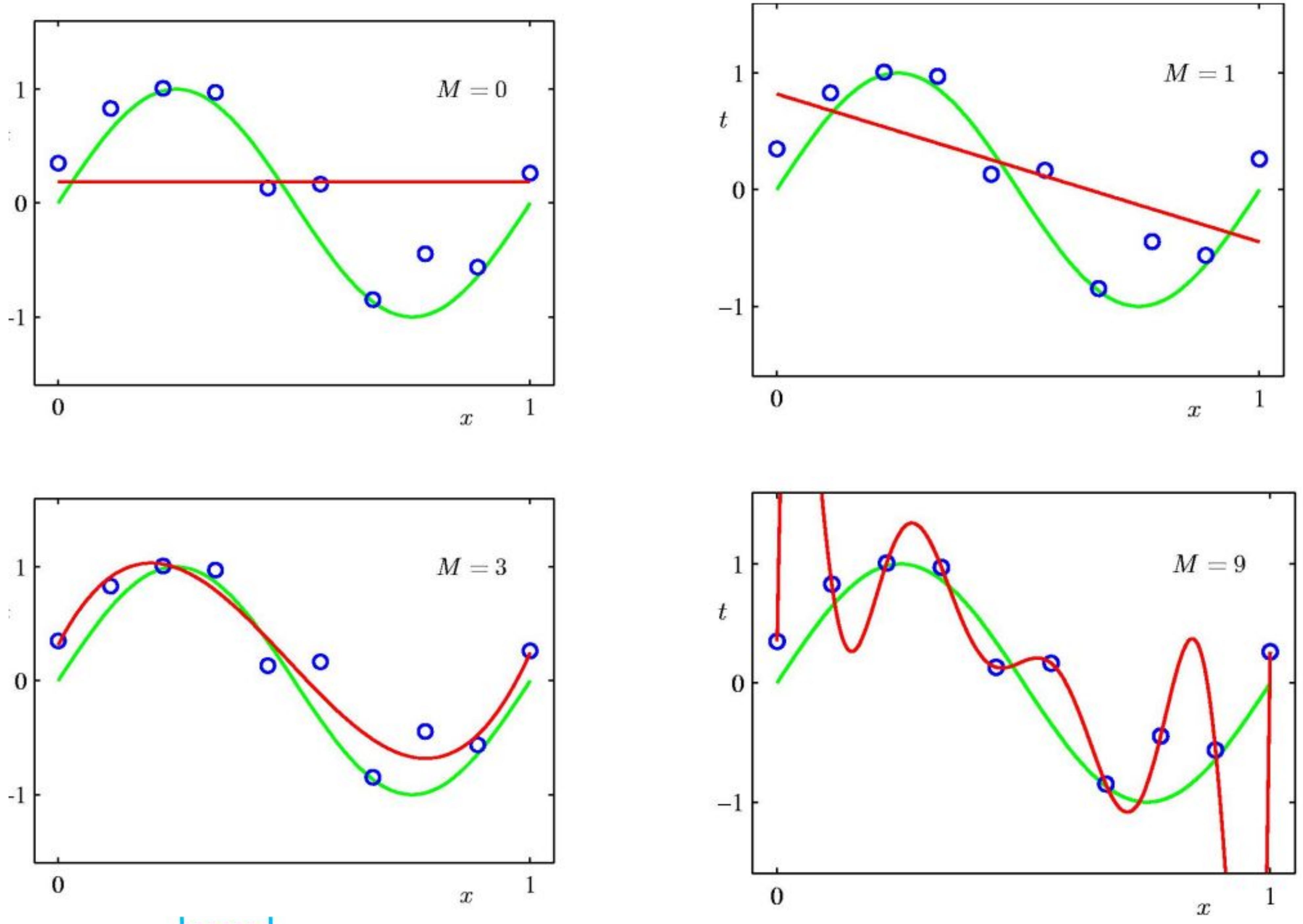
Example: an **M-th order polynomial** function of **one dimensional** feature x:

$$y(x, \mathbf{w}) = w_0 + \sum_{j=1}^{M} w_j x^j$$

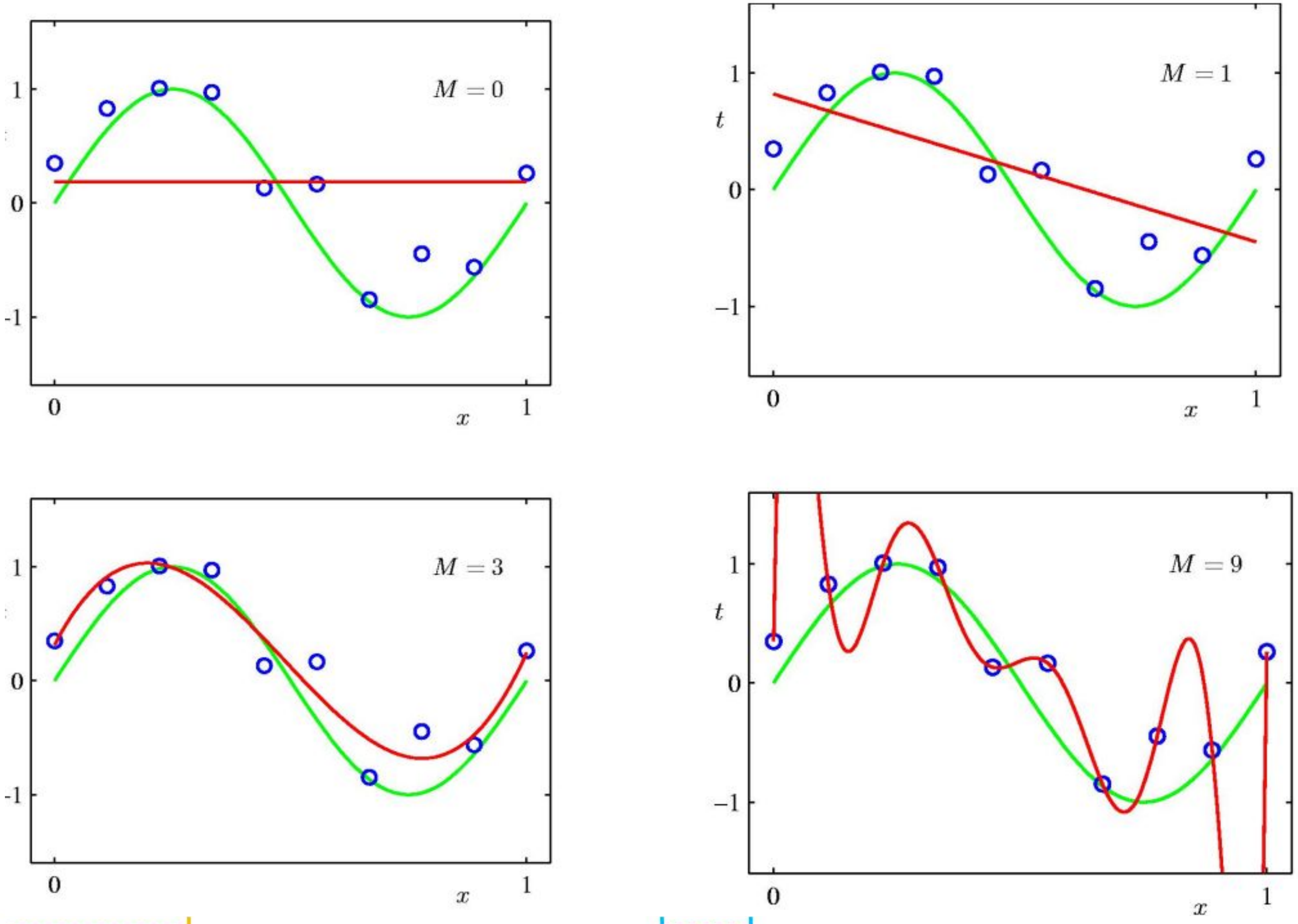
where $\mathbf{x}^j$ is the j-th power of x

We can use the same approach to optimize for the **weights w**

Fitting a Polynomial



Fitting a Polynomial

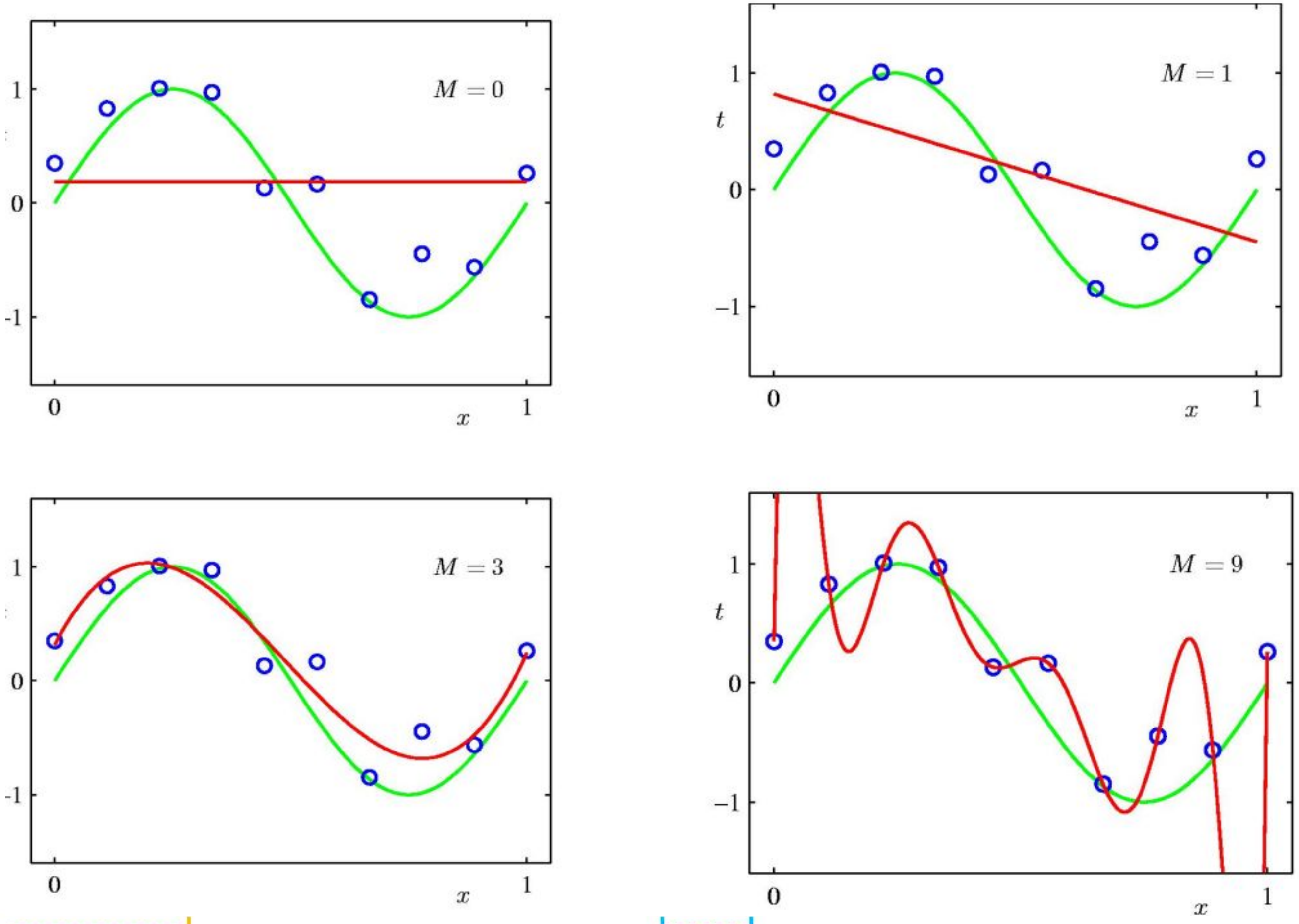


Error

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - f(x))^2$$

Fitting a Polynomial



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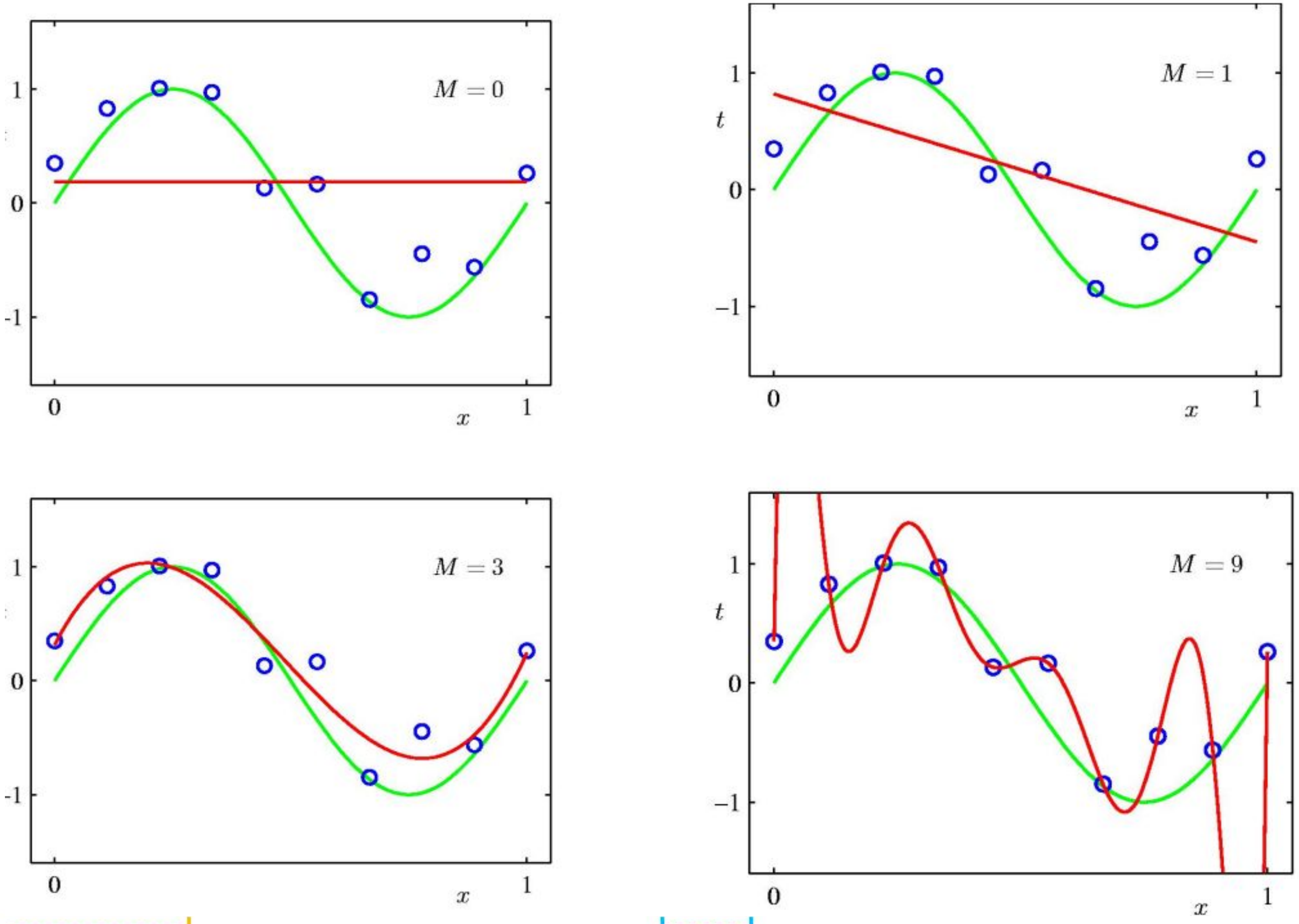
$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - (wx+b))^2$$

Fitting a Polynomial



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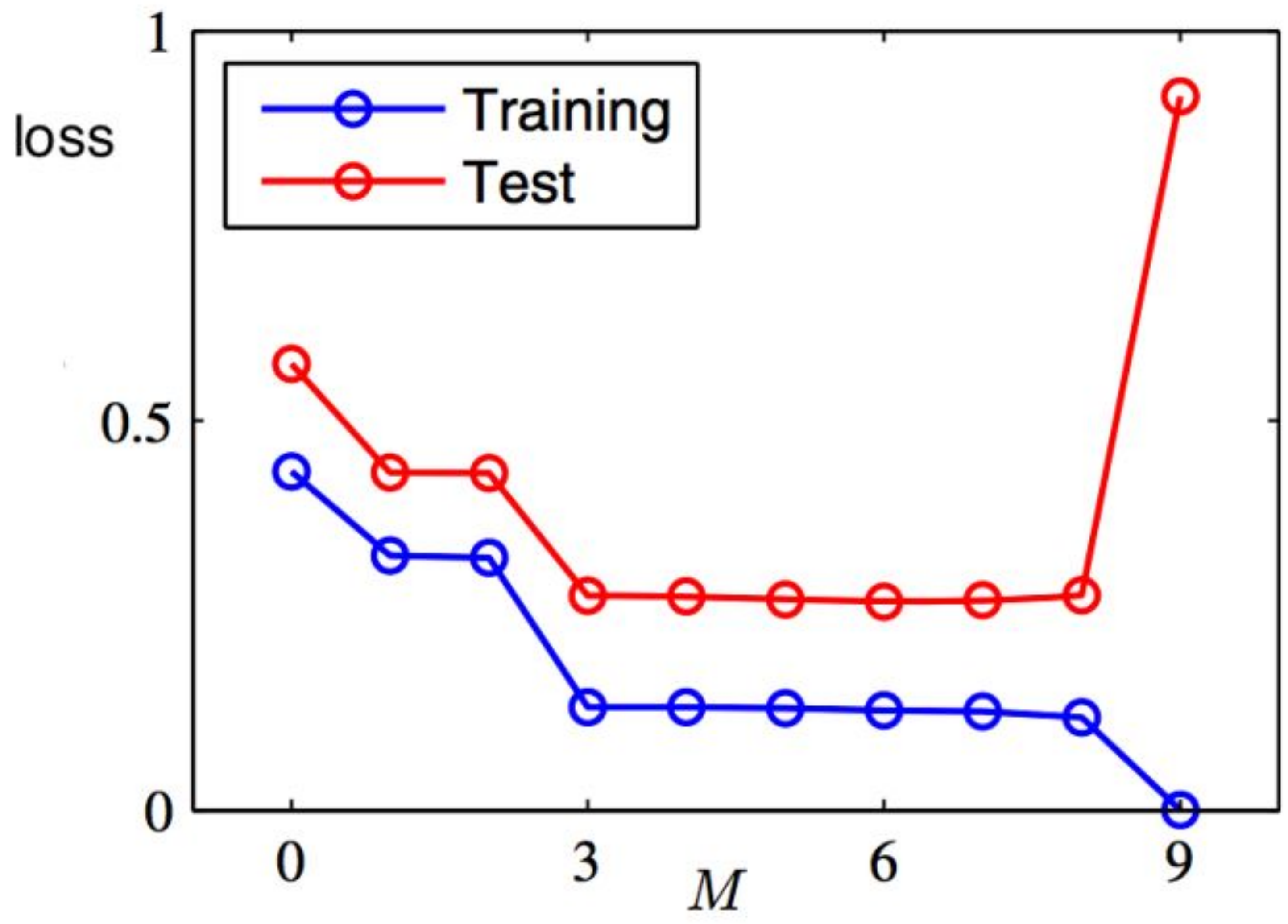
$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - (wx + b))^2$$

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - (w_0x^2 + w_1x + b))^2$$

Generalization

Generalization = model's **ability** to predict the **held out data**

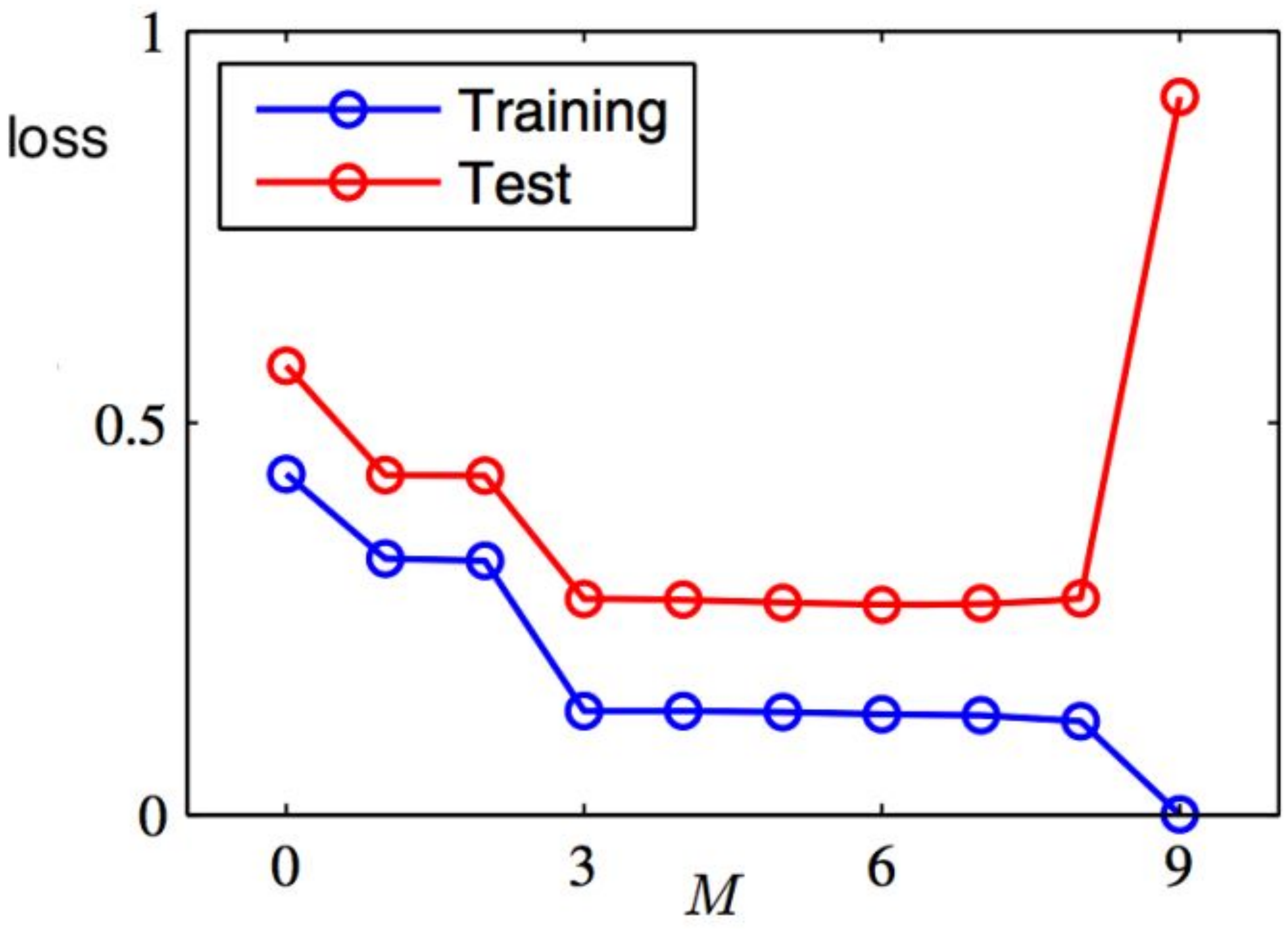
What is happening?



Generalization

Generalization = model's **ability** to predict the **held out data**

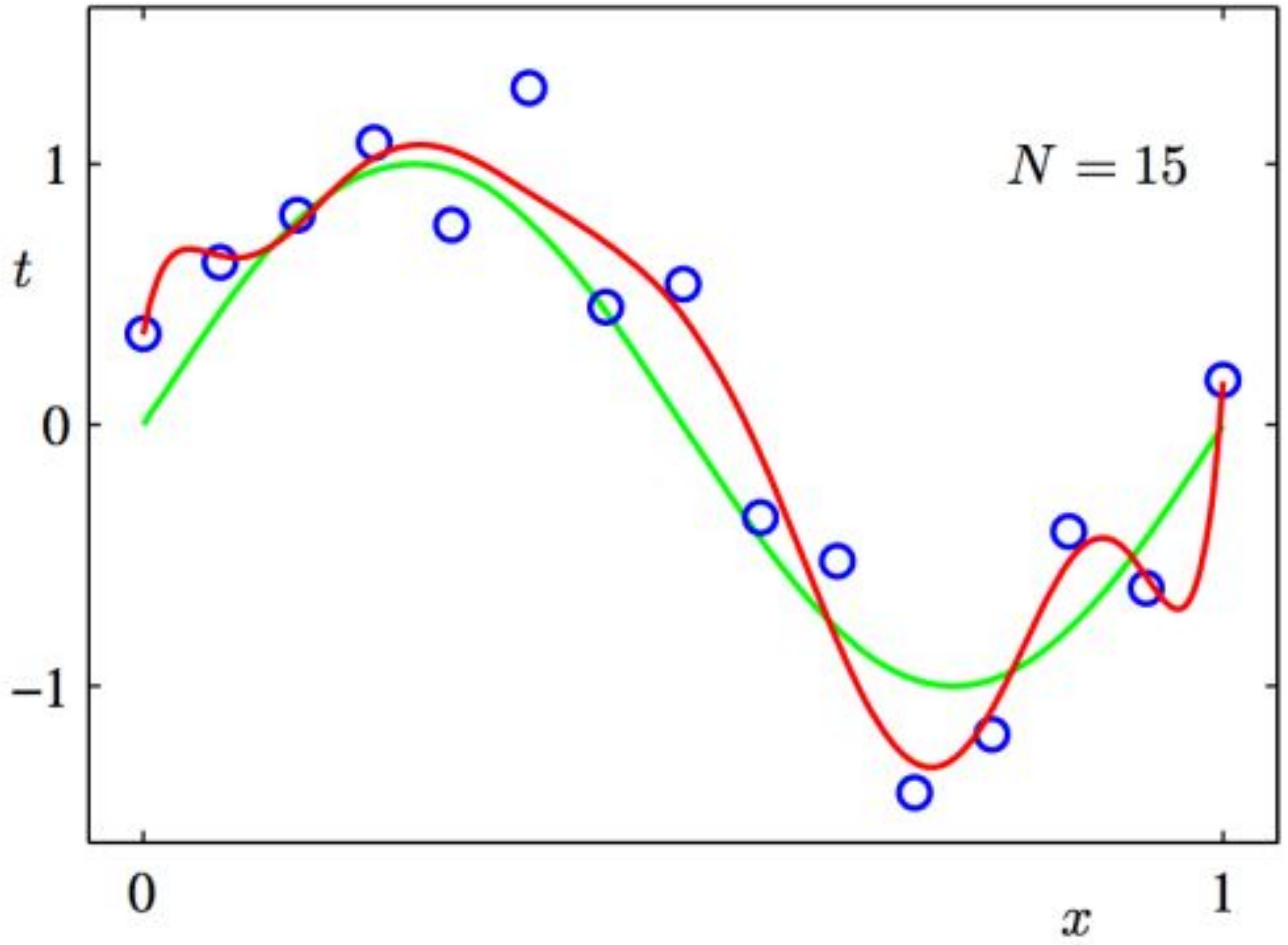
What is happening?

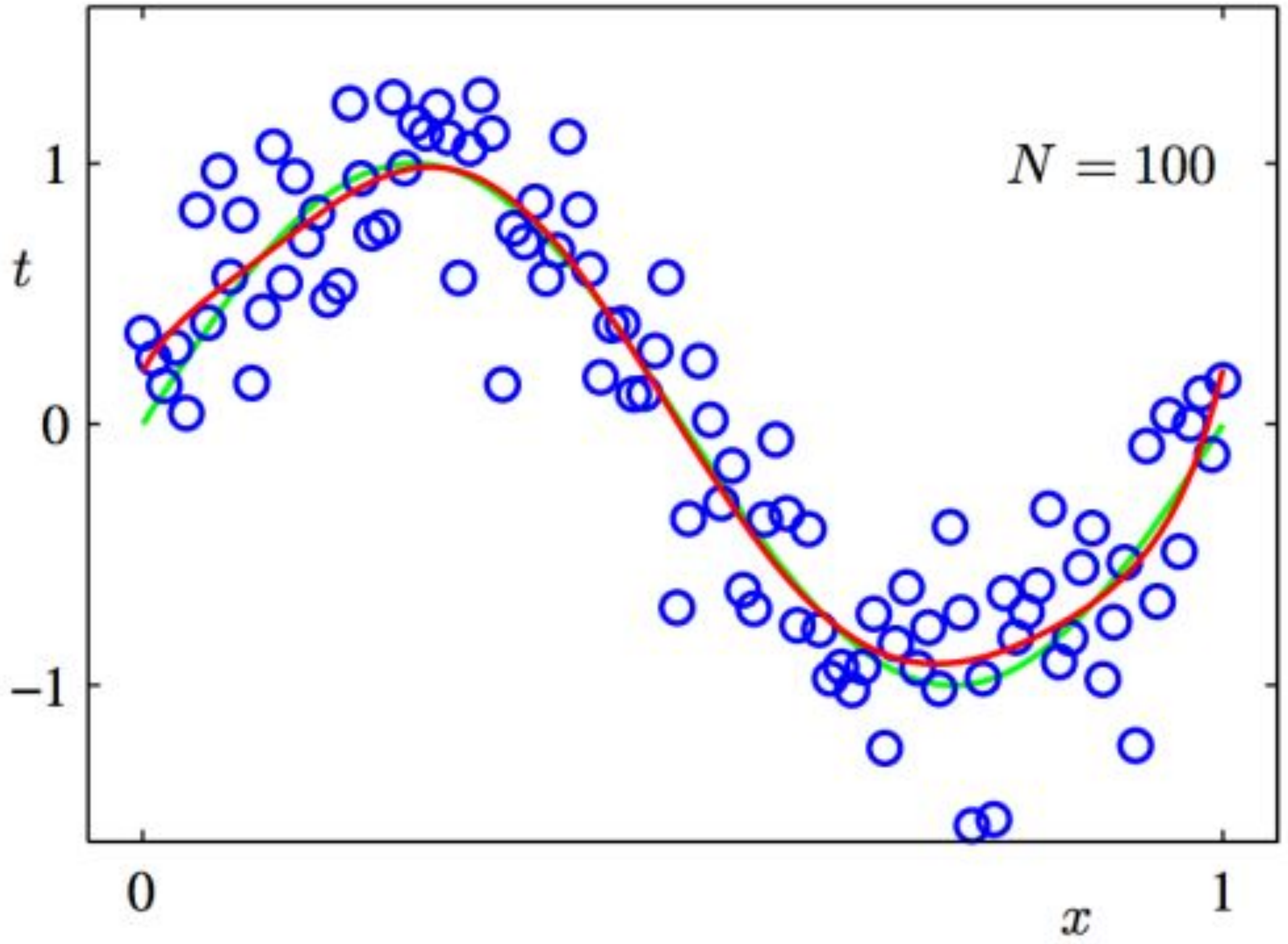


Our model with M = 9 **overfits** the data (it models also noise)

Generalization

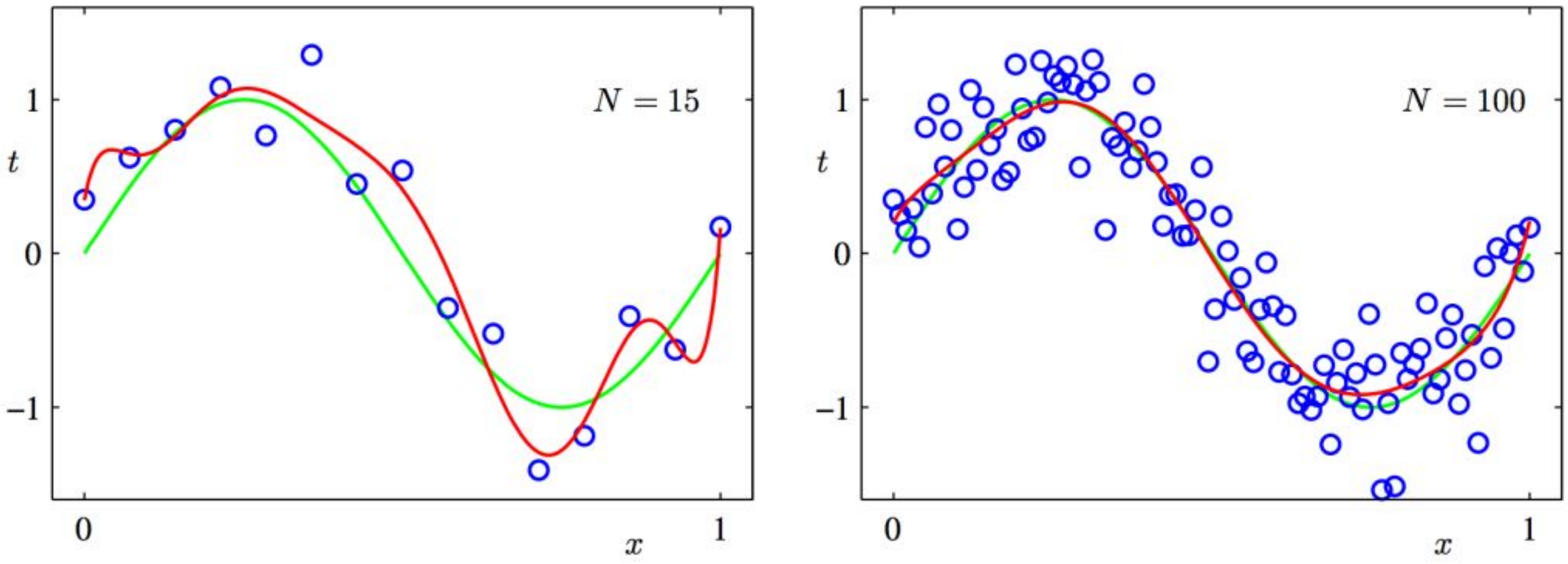
What can we do?





Generalization

What can we do? Not a problem if we have lots of training examples



Generalization

Let's look at the **estimated weights** for **various M** in the case of fewer examples

	$M=0$	$M=1$	$M=6$	$M=9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		-1.27	7.99	232.37
$w_2^\star$			-25.43	-5321.83
$w_3^\star$			17.37	48568.31
$w_4^\star$				-231639.30
$w_5^\star$				640042.26
$w_6^\star$				-1061800.52
$w_7^\star$				1042400.18
$w_8^\star$				-557682.99
$w_9^\star$				125201.43

Generalization

Let's look at the **estimated weights** for **various M** in the case of fewer examples

The weights are becoming huge to compensate for the noise

	$M = 0$	$M = 1$	$M = 6$	$M = 9$
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How can we deal with this?

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3.

Regularization

L0 & L1 & L2

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Regularization

Occam's Razor: prefer the simplest hypothesis

What does it mean for a hypothesis (or model) to be simple?

1. small number of features **(model selection)**

2. small number of "important" features **(shrinkage)**

Regularization

Given objective function: J(θ)

Goal is to find

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, J(\boldsymbol{\theta}) + \lambda r(\boldsymbol{\theta})$$

Key idea: Define regularizer r(θ) s.t. we tradeoff between fitting the data and keeping the model simple

Choose form of regularizer

Common choice: p-norm

$$r(\boldsymbol{\theta}) = ||\boldsymbol{\theta}||_p = \left[\sum_{m=1}^{M} |\theta_m|^p\right]^{\left(\frac{1}{p}\right)}$$

Regularization

Given objective function: J(θ)

Goal is to find

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, J(\boldsymbol{\theta}) + \lambda r(\boldsymbol{\theta}) \qquad r(\boldsymbol{\theta}) = ||\boldsymbol{\theta}||_p = \left[\sum_{m=1}^{M} |\theta_m|^p\right]^{\left(\frac{1}{p}\right)}$$

p	$r(\boldsymbol{\theta})$	**yields parameters that are...**	**name**	**optimization notes**
0	$\lVert\boldsymbol{\theta}\rVert_0 = \sum \mathbb{1}(\theta_m \neq 0)$	zero values	L0 reg.	no good computational solutions
1	$\lVert\boldsymbol{\theta}\rVert_1 = \sum \lvert\theta_m\rvert$	zero values	L1 reg.	subdifferentiable
2	$(\lVert\boldsymbol{\theta}\rVert_2)^2 = \sum \theta_m^2$	small values	L2 reg.	differentiable

L2+L1 Regularization

L2 and L1 regularization can be **combined**

$$R(\mathbf{w}) = \lambda_2 ||\mathbf{w}||^2 + \lambda_1 ||\mathbf{w}||_1$$

- Also called **ElasticNet**
- Can work better than either type alone
- Can **adjust hyperparameters** to control which of the two penalties is more important

4.

More non linear

Non polynomial

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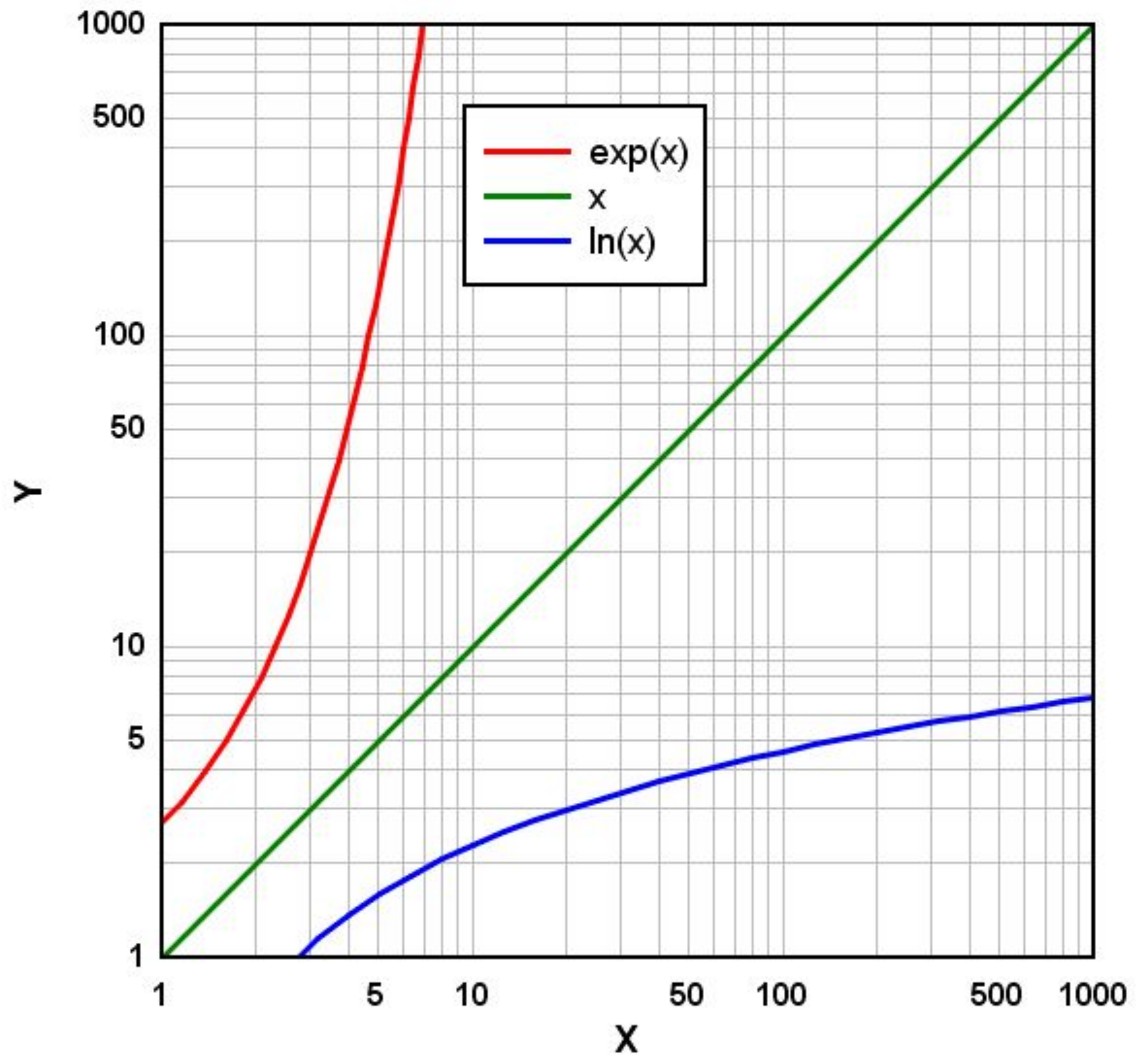
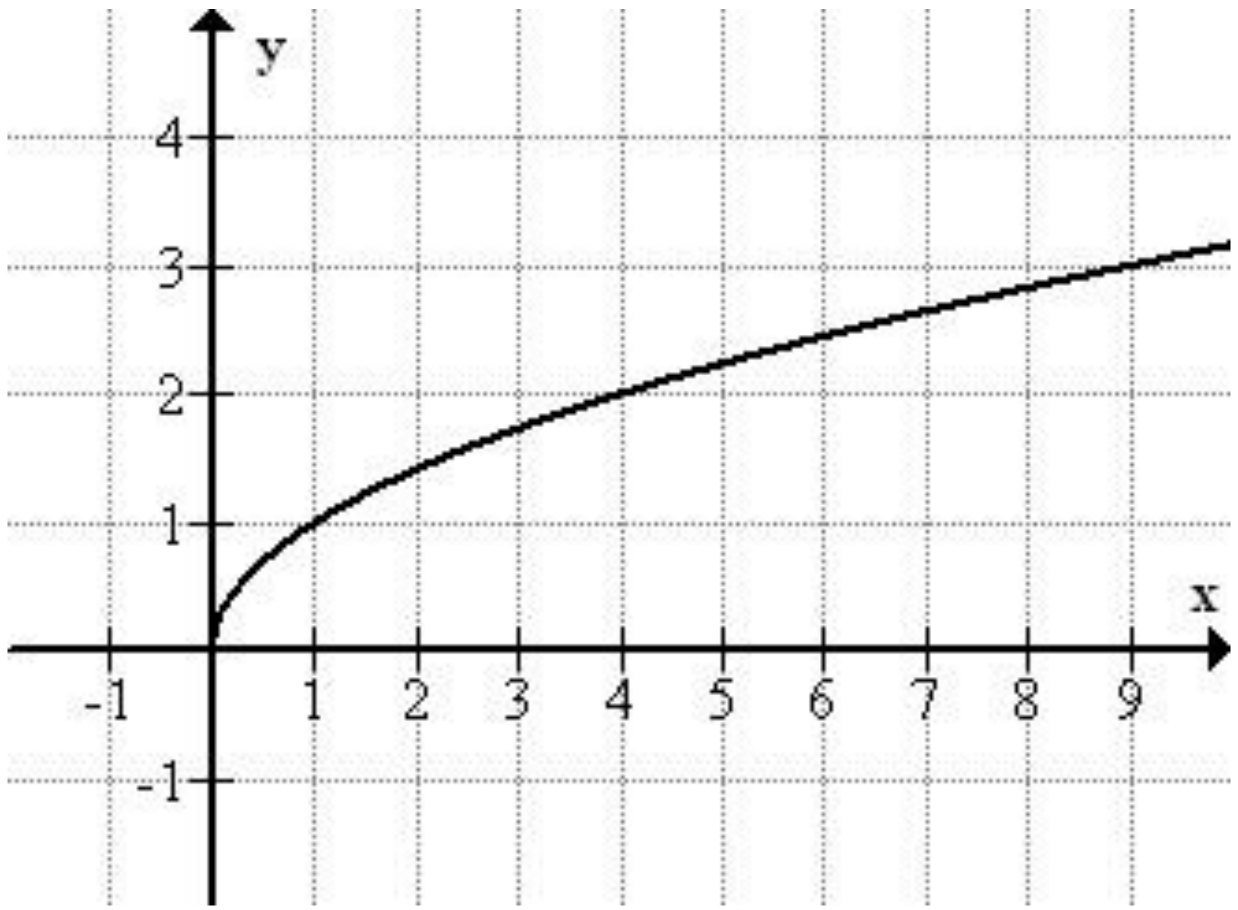
More non linear functions...

Logarithmic and exponential function



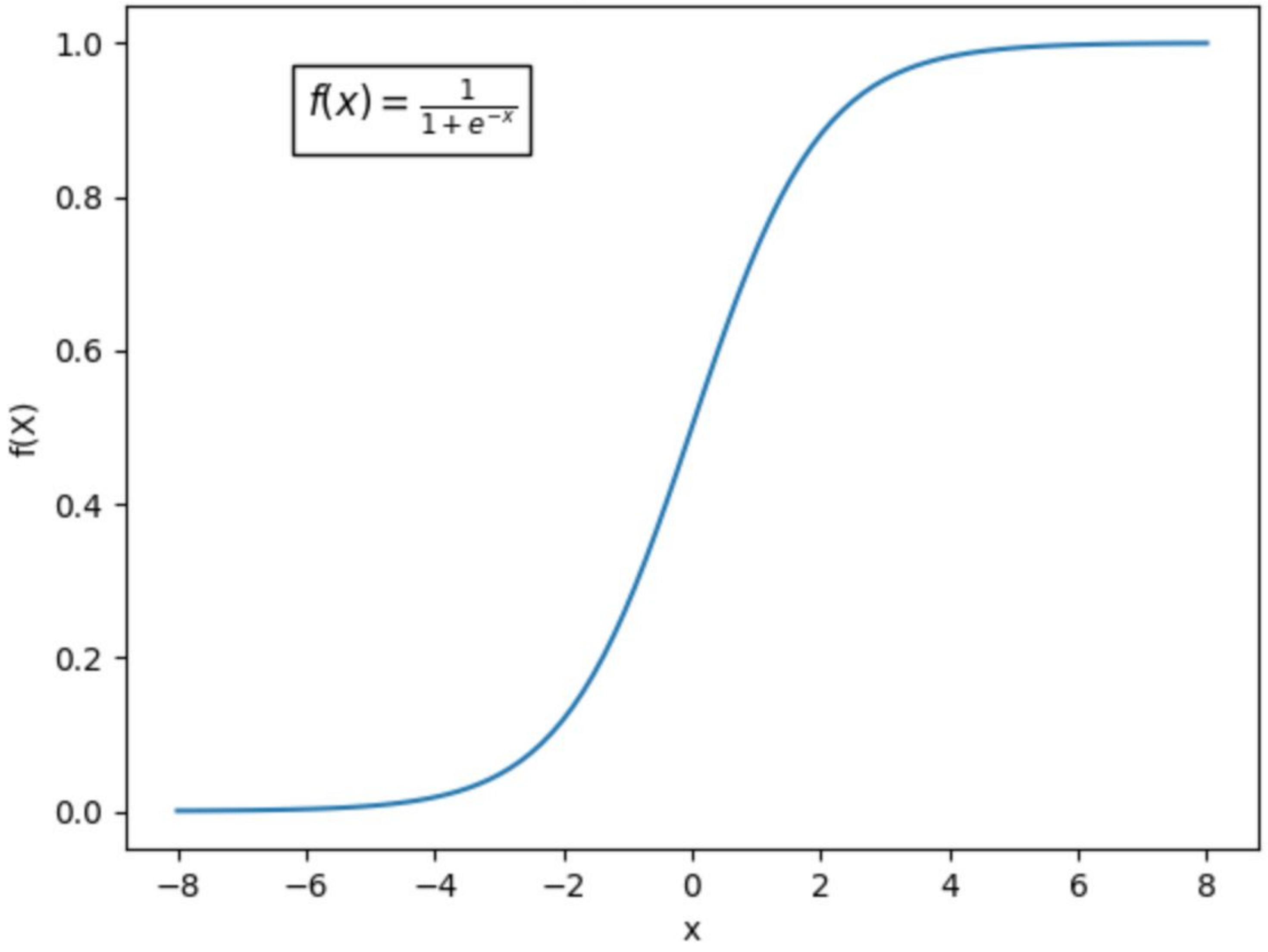
More non linear functions...

Square root function



More non linear functions...

Sigmoid function



What about more dimensions?

$$f(x) = w_0 + x_1 w_1 + x_2 w_2$$

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$$f(x) = w_0 + x_1w_1 + x_1^2w_2 + x_1^3w_3 + x_2^2w_4 + x_2w_5$$

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$$f(x) = w_0 + x_1w_1 + x_1^2w_2 + x_1^3w_3 + x_2^2w_4 + x_2w_5 + x_1x_2w_6$$

What about more dimensions?

$$f(x) = w_0 + x_1 w_1 + x_2 w_2$$

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$$f(x) = w_0 + x_1 w_1 + x_1^2 w_2 + x_1^3 w_3 + x_2^2 w_4 + x_2 w_5 + x_1 x_2 w_6$$

$$f(x) = w_0 + \exp(x_1) w_1 + x_2 w_2$$

What about more dimensions?

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$$f(x) = w_0 + x_1w_1 + x_1^2w_2 + x_2w_3$$

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$$f(x) = w_0 + x_1w_1 + x_1^2w_2 + x_1^3w_3 + x_2^2w_4 + x_2w_5 + x_1x_2w_6$$

$$f(x) = w_0 + \exp(x_1)w_1 + x_2w_2$$

$$f(x) = w_0 + \exp(x_1)w_1 + x_2^2w_2 + x_2w_3$$

What about more dimensions?

$$f(x) = w_0 + x_1w_1 + x_2w_2$$

$$f(x) = w_0 + x_1w_1 + x_1^2w_2 + x_2w_3$$

$$f(x) = w_0 + x_1w_1 + x_1^2w_2 + x_1^3w_3 + x_2^2w_4 + x_2w_5$$

$$f(x) = w_0 + x_1w_1 + x_1^2w_2 + x_1^3w_3 + x_2^2w_4 + x_2w_5 + x_1x_2w_6$$

$$f(x) = w_0 + \exp(x_1)w_1 + x_2w_2$$

$$f(x) = w_0 + \exp(x_1)w_1 + x_2^2w_2 + x_2w_3$$

$$\vdots$$

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5.

Non Linear Regression
Example

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**https://colab.research.google.com
/drive/1-ZJqtjlM_PwAdN2vMmYzD
RDv-DH9ILdj?usp=sharing**

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More Exercises?

MACHINE LEARNING WORKOUT: Chapter 5 and 6

Kahoot Time!


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