Design and construction of a 1.045 MHz FM receiver

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Abstract

We present and detail the design of a 1.045 MHz FM receiver with bandwidth 200 kHz. The filter is composed of a bandpass RLC filter, diode envelope detector, and common-emitter transistor amplifier with gain 2. Input and output impedances are not matched to appropriate loads or sources but may be matched by use of emitter-followers or appropriate resistors. The diode detector as presented and tested is unusable, but we present a possible solution as well as an improvement for the common-emitter amplifier. We additionally provide a simple procedure to characterize random noise using the central limit theorem.

1 Introduction

Modulation of radio frequency (RF) electromagnetic waves is a simple and effective method for long distance information transmission, as exemplified by amplitude/frequency modulated radio. The generation, transmission, and reception of modulated signals are interesting engineering problems which are also accessible to interested amateurs and beginning students of electronics. Simple modulators and demodulators may be built with a minimum of passive and active components (i.e., enough to fit on a small breadboard). RF circuits hold great educational potential for undergraduate students.

Here we present the design and construction of an FM receiver (1.045 MHz with bandwidth 200 kHz). The only non-linear components necessary are a diode and a transistor. The primary purpose of this circuit is for self-edification, but the layout may also serve as a starting point for further exploration of RF circuit design.

The envelope detector as presented is unusable and requires an additional resistor to ground. We present simple improvements for the envelope detector and amplifier, and provide a procedure to characterize receiver noise.

2 FM receiver design

The receiver is comprised of a passive RLC bandpass filter, diode envelope detector, and a common-emitter amplifier chained between antenna input and speaker output. Blocking capacitors between different functional sections enable voltage biasing.

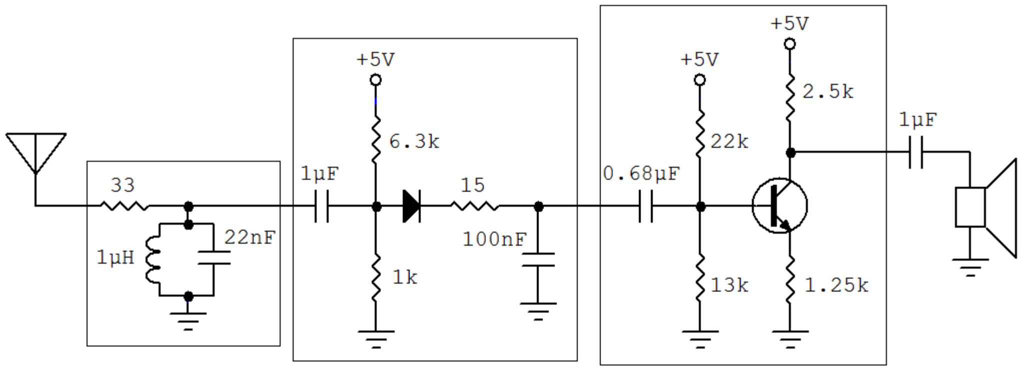

Figure 1. FM receiver circuit diagram with component values.
Boxes demarcate functional components (see corresponding sections in text).

2.1 Bandpass RLC filter

The input signal is first passed through an RLC filter centered at approximately $1.045 MHz$ with the -3 dB point at ± 200 kHz on each side. For component values R, L, C (resistor, inductor, capacitor) and input signal component with angular frequency ω, the gain of this filter is given by the equation:

$$\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \left[1 + (RC)^2 \left(\frac{\omega_0^2 - \omega^2}{\omega}\right)^2\right]^{-1/2}$$

Here $\omega_0 \equiv 1/\sqrt{LC}$, emphasizing that this filter has unity gain at $\omega = \omega_0$. The -3 dB roll-off frequency occurs when the gain is equal to $1/\sqrt{2}$, or when:

$$(RC)\frac{\omega_0^2 - \omega^2}{\omega} = 1$$

Let the bandwidth be denoted by Δf, with corresponding cut-offs at $f = f_0 \pm \Delta f/2$, where $f_0 = w_0/(2\pi)$. This gives:

$$2\pi f_0 RC \approx \frac{f_0}{\Delta f}$$

as a prescription for our cut-off. Our component values $R = 33\ \Omega, L = 1\ \mu\text{H}, C = 22$ nF give $f_0 = 1.073$ MHz with bandwidth $\Delta f = 220$ kHz. Figure 2 plots the frequency-dependent gain along with the transmission band of interest at 1.045 MHz.

2.2 Diode envelope detector

We use a diode in series with a low-pass RC filter as a rudimentary envelope detector. Prior to a diode, we use a blocking capacitor and voltage divider to apply a bias voltage of 0.68 V to the input signal; this allows the positive half of the signal to be passed mostly unattenuated by the diode.

The blocking capacitor value is set by the behavior of the capacitor and the biasing voltage divider as a high pass filter. Looking into the voltage divider, the resistors are seen in parallel and so the input impedance is about 860 Ω. To allow our carrier signal (~ 1 MHz) to pass, the blocking capacitance must be at least 0.2 nF, which is satisfied by our 1 μF capacitor.

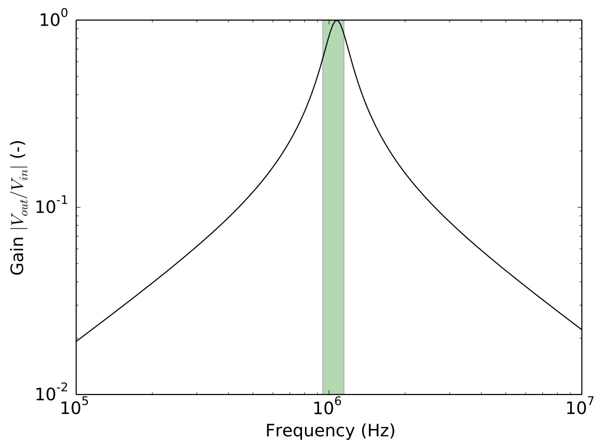

Figure 2. RLC filter gain against input frequency f (Hz).
FM signal bandwidth plotted in light green.

Our diode circuit is expected to obtain a signal envelope as follows. The diode extracts only the positive half of an input signal, with the diode's knee voltage compensated for by our voltage biasing. However, when the carrier signal peaks and begins to decrease, the RC circuit prevents the maximum imposed voltage from decaying immediately; it is forced to decrease over a timescale RC. The voltage is only "unpinned" once the next carrier cycle passes through the diode.

The diode is placed in series with a low-pass RC filter of cutoff frequency ~ 110 kHz. In actuality, the cutoff frequency should have been closer to 1 MHz for envelope detection. Our envelope detector did not operate correctly anyways, which we discuss further below. We plot the gain of the low-pass filter in Figure 3, to show that audible frequencies will not be attenuated in a correctly designed circuit.

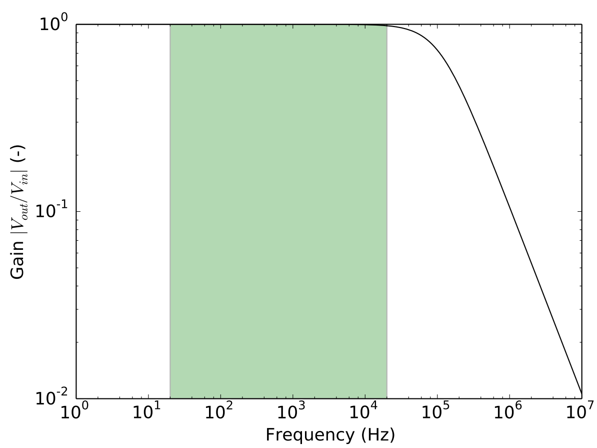

Figure 3. Low-pass RC filter gain as a function of input frequency f (Hz). Approximate human hearing range plotted in light green.

2.3 Common-emitter amplifier

We pass the extracted envelope curve from the diode demodulator to a common-emitter transistor amplifier. The transistor is a 2N5089 NPN bipolar junction transistor. Our choice of component values and following explanation of circuit operation are guided in large part by *Horowitz and Hill* [1989].

The transistor's quiescent base voltage is set by a biasing voltage divider, which controls emitter voltage. For appropriate choice of collector and emitter resistors, the collector voltage amplifies the base voltage signal. The gain is given as $-R_c/R_e$ (where R_c, R_e denote collector and emitter resistors respectively).

For maximal voltage swing, the quiescent collector voltage should be 2.5 V (or, $V_{\mathrm{cc}}/2$). We choose a quiescent current of 1 mA and hence set collector resistor to 2.5 kΩ. We set the emitter resistor at 1.25 kΩ for a gain of 2. The quiescent emitter voltage is then 1.25 V, and the quiescent base voltage should be ~ 1.85 V. We bias the base voltage using 22 kΩ and 13 kΩ resistors.

In actual practice, we used slightly different values ($R_E = 1.5$ kΩ, $R_C = 2.7$ kΩ, voltage divider resistors 22 kΩ and 12 kΩ). However, our results are not significantly affected.

The blocking capacitor prior to the amplifier biasing circuit acts as a high-pass filter. The input impedance, looking into the transistor base, is ~ 8 kΩ (set primarily by the two divider resistors in parallel). We choose 0.68 μF as our capacitor, which gives a cut-off frequency of 30 Hz, at the lower end of the audible range (cf. Figure 3).

2.4 Receiver impedances

External loads or sources (antennae, transmission lines, speakers) connected to our receiver should be impedance matched to maximize power throughput.

The output impedance of the amplifier sets the receiver's output impedance, as the amplifier transistor buffers any lower impedance upstream. In our circuit, the output impedance is the parallel impedance of the transistor and collector resistor, or roughly 2.5 kΩ. This is much larger than the 50 Ω impedance of transmission lines, or the 8 Ω impedance of our speaker.

An emitter-follower circuit may be placed after the amplifier, then, to decrease the receiver's output impedance by a factor of $\sim 1/\beta$; β is the transistor amplification factor. We may make further adjustments by placing resistors in parallel with the follower output, or adding resistors in series with the speaker at the end of the line (although, I'm not sure what is common practice).

The input impedance appears more difficult to calculate. For signals far from the bandpass, the LC tank circuit provides a low-impedance path to ground; for signals within the bandpass (i.e., where the tank circuit is resonant and impedes flow to ground), we would have to consider the impedance of our diode circuit.

3 Results

To test the RLC bandpass filter, we input a sine wave of varying frequencies and measured the output across the LC tank circuit. We obtained a maximum gain of 0.83 at input frequency

1.05 MHz (caveat: we may have used incorrect termination procedure for this measurement, but this only impacts our gain measurement). The circuit qualitatively showed the correct roll-off behavior.

When we input an FM signal at 1.045 MHz and observed the filter output on an oscilloscope, the signal was correctly converted into an AM signal with envelope code matching output audio. We also confirmed that the voltage divider added a bias of approximately 0.7 V. However, the diode envelope detector appeared to flatline and output a roughly fixed voltage.

When testing the demodulator circuit, with or without amplifier, the speaker appeared to output audio when connected to the circuit output. However, we found that the audio level was independent of whether the circuit was powered, or whether the FM signal was even connected to the circuit input.

The common-emitter amplifier begins clipping at low voltages with input of 1.4 V_{pp}, or output of 2.34 V_{pp}. At this input, there appeared to be a very small voltage difference between collector and emitter. For smaller voltages at 10 kHz it functioned correctly.

4 Discussion

4.1 Envelope detector design

The fatal flaw in our envelope detector is, simply, that there is no path to ground. After a cycle maximum, the voltage at the back of the diode decreases, but the voltage at the front of the diode is held constant by the filter and blocking capacitors, which can only discharge through diode leakage current at sufficiently low input frequencies.

The traditional resolution to this problem is to neglect the series resistor (in the low-pass RC filter) and instead place a resistor in parallel to ground. As before, the characteristic time RC should be comparable with the timescale of carrier wave oscillations (i.e., $(1\ \mathrm{MHz})^{-1}$). Unfortunately, we neglected this and so our circuit design is inoperable.

4.2 Common-emitter amplifier design

Without the bypass capacitor, the amplifier frequency bounds are set by 1. high pass filter of blocking capacitor and bias divider, and 2. stray circuit and/or transistor capacitance. We did not collect measurements to characterize frequency dependence of amplification in our actual circuit, however.

Although effective, the amplifier design presented can only tolerate so much gain. As the emitter resistor is made smaller, the current flow is increasingly influenced by the temperature dependent transistor resistance (r_e), as described by the transconductance model [*Horowitz and Hill*, 1989]. The biasing circuit exerts less control on the emitter voltage and so the circuit is less reliable.

Both gain and stability may be improved by adding an emitter capacitor (C_e) in parallel with the resistor capacitor. The new gain is:

$$\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \left[\left(\frac{R_c}{R_e}\right)^2 + \omega^2 R_c^2 C_e^2\right]^{1/2}$$

By choosing $C_e = 50$ nF, we were able to observe 10x gain at input frequency 10 kHz. However, the gain with capacitor preferentially amplifies higher frequencies, and we have neglected transistor resistance r_e.

We may address both issues by placing an emitter resistor in series with a bypassed emitter resistor (Figure 4). The following analysis, again, is largely informed by *Horowitz and Hill* [1989].

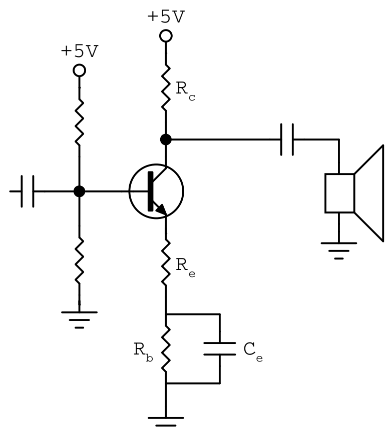

Figure 4. Circuit schematic of common-emitter amplifier with partially-bypassed emitter resistor.

The gain of this circuit (not bothering to take an absolute value) is

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_c}{R_e + R_b/(1 + j\omega R_b C_e)}$$

where j is the imaginary unit, and we have denoted the bypassed resistor by R_b. We have neglected transistor resistance r_e, which may be included via R_e if desired. For sufficiently large ω, the term $R_b/(1 + j\omega R_e C_e)$ becomes unimportant and the gain reduces to R_c/R_e. However, the quiescent voltage is set by $R_e + R_b$, which improves stability (e.g., reduces likelihood of current saturation).

To obtain a stable gain over the frequency band of interest, we need only consider the lowest frequency ω_{min} and impose the condition

$$\frac{R_b^2}{1 + \omega_{\text{min}}^2 R_b^2 C_e^2} << R_e^2$$

For $R_c = 2.5$ kΩ and $\omega_{\text{min}} = 20$ Hz, we might pick $R_e = 250$, $R_b = 1$ kΩ and $C_e = 32$ μF. This would give a stable gain of 10 for all frequencies above ~ 20 Hz – a most desirable state of affairs.

4.3 Noise characterization

We attempted to quantify the Johnson-Nyquist noise in a resistor and in our receiver. We sought to drive our receiver with a 39 Ω resistor and estimate the noise figure of the receiver by amplifying and measuring the receiver's output noise. In the process, we discovered that a majority of available commercial amplifiers were dysfunctional and that we could not obtain any amplification. Nevertheless, we present some theoretical calculations, simulations, and a procedure for estimating the noise figure of our receiver.

The procedure is as follows. We would connect a 39 Ω resistor across the input of our receiver, with some form of impedance matching, and drive the resistor at variable power (0–2 W). By plotting output temperature T_{out} against resistor temperature T_R, we would perform a linear least squares fit to estimate our two unknowns: receiver gain G and receiver temperature T; this is known as the Y-factor method. From this we could estimate the noise figure of our receiver.

We consider the Johnson–Nyquist noise of a 50 Ω resistor in the bandwidth 25–35 MHz (B = 10 MHz) at room temperature. The root mean square (RMS) noise voltage is given as $V_{\text{rms}} = (4k_B TBR)^{1/2}$, where k_B is the Boltzmann constant, T is resistor temperature in Kelvin, B is bandwidth, and R is resistance. We find $V_{\text{rms}} = 2.9\ \mu\text{V}$.

We used two Cougar Components AC1068C amplifiers, powered with $V_{\text{cc}} = +15$ V, to attempt to amplify the noise. Each amplifier has a stated gain of 24.5 dB for frequencies below 1.2 GHz and a stated noise figure of 3.7 dB. Two amplifier stages give a 282x voltage gain, which should amplify 2.9 μV to 0.81 mV. In comparison, we found that the noise floor of our oscilloscope was approximately 0.3–0.5 mV, depending on termination; if successful, the amplified noise should be measurable.

We also wish to check the highest resistor temperature that can be achieved at ~ 2 W. We drive a 39 Ω resistor with 8.83 V from a DC voltage supply and measure its temperature to be 300 ± 5 degrees Fahrenheit, or 422 ± 10 K, using an infrared thermometer. In comparison, we predict a temperature of 700 K for surface area 1 cm^2 and emissivity 1 using the Stefan-Boltzmann law. A few effects may explain this discrepancy: 1. power is being dissipated by non radiative means (conduction through leads, advective cooling by air); the blackbody power dissipated at 422 K is about 0.1 W, and 2. the DC power supply may be unable to supply the desired current; we noted the power supply typically read a current draw of 20–30 mA.

We would then, using our resistor temperature measurement(s), drive our receiver with the resistor's Johnson-Nyquist noise, feed the receiver output through two amplifier stages and a bandpass filter, and attempt to measure the RMS voltage using an oscilloscope. As noted above, we do not do so due to dysfunctional amplifiers.

4.4 Variance test of simulated noise

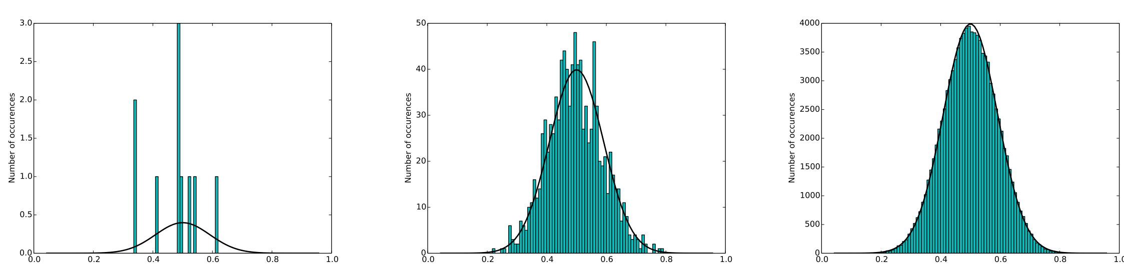

Figure 5. Histograms of $10^1, 10^3, 10^5$ 10-sample means with theoretical prediction (black line) of appropriately scaled Gaussian). As we add more sample means, we converge towards a normal distribution. Plot labels are catastrophically small but not really necessary anyways.

As an additional exercise, we would also wish to demonstrate that the output receiver noise was indeed random. To this end, we could sample the output noise for varying amounts of time and demonstrate that the standard deviation of the mean of the noise decreases as the square root of the sampling time; i.e., the number of samples averaged.

The central limit theorem states that m-sample means drawn from any random distribution will be normally distributed around the underlying mean, and the normal distirbution will have standard deviation $\sigma/\sqrt{m}$, where σ is the standard deviation of the underlying random distribution. In Figure 5, we show that for an increasing collection of m-sample means, the histogram converges to a normal distribution ($m = 10$ fixed).

Figure 6 considers the standard deviation of 10^5 m-sample means for varying m. We show that the standard deviation indeed falls off as $\sigma/\sqrt{m}$, which we plot as our theoretical prediction.

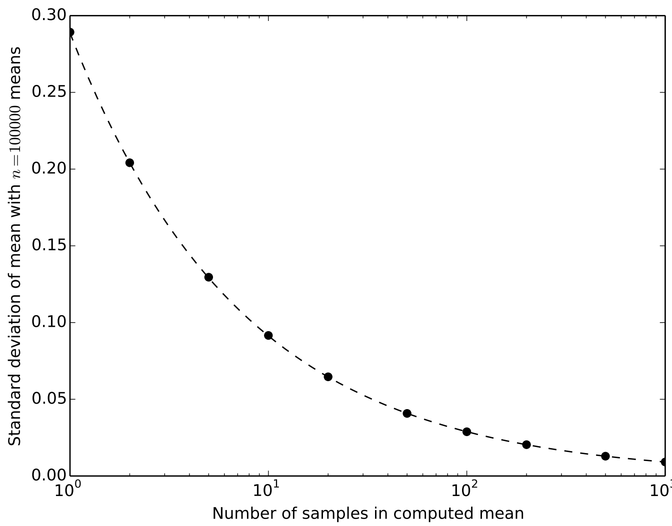

Figure 6. Standard deviation of 10^5 m-sample means scales as $\sigma/\sqrt{m}$, where $\sigma = 1/sqrt12$ for the underlying uniform distribution on $[0, 1)$.

The extension to characterization of receiver noise is non-trivial, but the conceptual idea is the same. Sampled receiver noise should have decreasing spread with increasing averaging time. Code used for simulations is available at https://github.com/aarontran/ay121/lab1/scripts/central_limit.py.

5 Conclusion

Although our receiver design is flawed, we have outlined a number of simple fixes and improvements that would enable a better receiver to be constructed. We expect that an improved circuit would produce output audibly stronger than that due to antenna-like pickup. We are missing a substantial number of error estimates and measurements to characterize receiver behavior, but we have provided the necessary theoretical calculations and demonstrated that, at the correct operating frequencies (1 MHz carrier or 10 kHz audio), our circuit is mostly functional.

6 Acknowledgments

Thanks to Patrick and Kyle for much patience and time in lab. Much thanks to Baylee, Karto, and Prof. Parsons for answering our questions and helping us root out some mistakes (though not all, as evidenced here). Other lab members are thanked for discussions of variable seriousness and positive utility.

7 References

Horowitz, P. and W. Hill (1989), *The Art of Electronics*, Cambridge Univ. Press, Cambridge, UK.