$$Q(z) = P(z)$$

$$Q_N(z) = P_N(z)$$

$$P(N+1) = P(N) + \sum_{k=0}^{N+1}\binom{N+1}{k} z^{N+1-k}\cdot z_0^k\, a_{N+1}$$

$$A(N) = \underbrace{\sum_{n=0}^{N} z^n \sum_{k=0}^{N-n} a_{n+k}\binom{n+k}{k}\cdot z_0^k}_{Q(N)} = \underbrace{\sum_{n=0}^{N}\sum_{k=0}^{n}\binom{n}{k} z^{n-k}\cdot z_0^k\, a_n}_{P(N)}$$

$$\Rightarrow Q(N+1) = \sum_{n=0}^{N+1} z^n \sum_{k=0}^{N+1-n} a_{n+k}\binom{n+k}{k}\cdot z_0^k$$

$$= \sum_{n=0}^{N} z^n\left(\sum_{k=0}^{N-n} a_{n+k}\binom{n+k}{k}\cdot z_0^k + \underbrace{a_{N+1}\binom{N+1}{N+1-n}\cdot z_0^{N+1-n}}_{k=N+1-n}\right) + \overbrace{z^{N+1}\cdot a_{N+1}\binom{N+1}{0}}^{n=N+1}$$

$$= \underbrace{\sum_{n=0}^{N} z^n \sum_{k=0}^{N-n} a_{n+k}\binom{n+k}{k}\cdot z_0^k}_{P(N)} + z^n a_{N+1}\binom{N+1}{N+1-n} z_0^{N+1-n} + z^{N+1} a_{N+1}\binom{N+1}{0}$$

$$= \underbrace{\sum_{n=0}^{N}\sum_{k=0}^{n}\binom{n}{k} z^{n-k}\cdot z_0^k\, a_n}_{P(N)} + z^n a_{N+1}\binom{N+1}{N+1-n} z_0^{N+1-n}$$

$$P(N) + \sum_{n=0}^{N} z^n a_{N+1}\binom{N+1}{N+1-n} z_0^{N+1-n} + z^{N+1} a_{N+1}\binom{N+1}{N+1-N-1}$$

$$= P(N) + \sum_{n=0}^{N+1} z^n a_{N+1}\binom{N+1}{N+1-n}\cdot z_0^{N+1-n}$$

$$= P(N) + \sum_{k=0}^{N+1} z^k a_{N+1} z_0^{N+1-k}\binom{N+1}{N+1-k}$$

$$= P(N) + \sum_{k=0}^{N+1} z^{N+1-k} a_{N+1} z_0^k\binom{N+1}{k}$$

$$= \sum_{n=0}^{N}\sum_{k=0}^{n}\binom{n}{k} z^{n-k}\cdot z_0^k\cdot a_n + \sum_{k=0}^{N+1}\binom{N+1}{k} z^{N+1-k} a_{N+1} z_0^k$$

$$= P(N+1)$$

$$P(N+1) = P(N) + \sum_{k=0}^{N+1}\binom{N+1}{k} z^{N+1-k}\cdot z_0^k\, a_{N+1}$$