Antenna gain in a given direction (θ, ϕ) can be defined as

$$G = \frac{4\pi U(\theta, \phi)}{P_{\text{in}}}, \tag{1}$$

where $U(\theta, \phi)$ is the radiation intensity in the direction (θ, ϕ) and P_{in} is the power input to the antenna. The radiation intensity is defined in terms of far field Electric field values as

$$U(\theta, \phi) \approx \frac{r^2}{2\eta}\left[|E_\theta(\theta, \phi)|^2 + |E_\phi(\theta, \phi)|^2\right], \tag{2}$$

where r is the distance from the antenna (which we can set to $r = 1$), η is the impedance of free space, and $E_\theta(\theta, \phi)$ and $E_\phi(\theta, \phi)$ are the far field Electric field values in the θ and ϕ directions, respectively.

A pyramidal horn antenna can be treated as an aperture antenna with aperture surface currents given by

$$J_y(x, y) = -\frac{E_0}{\eta}\cos\left(\frac{\pi}{a_1}x\right)e^{-j\left[k\left(x^2/\rho_2 + y^2/\rho_1\right)/2\right]} \tag{3}$$

$$M_x(x, y) = E_0\cos\left(\frac{\pi}{a_1}x\right)e^{-j\left[k\left(x^2/\rho_2 + y^2/\rho_1\right)/2\right]}, \tag{4}$$

where values are defined in Balanis, "Antenna Theory: Analysis and Design." From these, we can see that the radiation intensity from a horn will be proportional to $|E_0|^2$. When defining the aperture fields, E_0 is a free parameter, but physically it contains information about the input power to the antenna and the antenna efficiency. Therefore, to get physically accurate values of gain (equivalently, physically accurate relationship between P_{in} and P_{out}), we need to intelligently set E_0.

One solution is to arbitrarily set E_0 to 1, calculate the far-field radiation intensity and corresponding gain numerically, then scale E_0 as necessarily to achieve the correct gain value.

Assuming an initial (arbitrary) E_0 value, define the corresponding gain value as

$$G_0 = \frac{4\pi U_0}{P_{in}} = \frac{4\pi\beta|E_0|^2}{P_{in}}, \tag{5}$$

where β is a proportionality constant. Then, assume we can scale E_0 by a factor γ to correct the gain value to the desired value G:

$$G = \frac{4\pi\beta|\gamma E_0|^2}{P_{in}} \tag{6}$$

γ is then given by

$$\gamma = \sqrt{\frac{GP_{in}}{\beta 4\pi|E_0|^2}} = \sqrt{\frac{G}{G_0}}. \tag{7}$$

So the corrected E_0 value can be found by multiplying the initial E_0 value by γ to achieve the desired gain of G.