



WHAT'S WRONG WITH THESE EQUATIONS?

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A major impediment to writing physics gracefully comes from the need to embed in the prose many large pieces of raw mathematics. Nothing in freshman composition courses prepares us for the literary problems raised by the use of displayed equations. Our knowledge is acquired implicitly by reading textbooks and articles, most of whose authors have also given the problem no thought. When I was a graduate teaching assistant in a physics course for non-scientists, I was struck by the exceptional clumsiness with which extremely literate students who lacked the exposure even to such dubious examples treated mathematics in their term papers. The equations stood out like droppings on a well-manicured lawn. They were invariably introduced by the word "equation," as in "Pondering the problem of motion, Newton came to the realization that the key lay in the equation

$$F = ma, \quad (1)$$

To these innocents equations were objects, gingerly to be pointed at or poked, not inseparably integrated into the surrounding prose.

Clearly people are not born knowing how to write mathematics. The implicit tradition that has taught us what we do know contains both good strands and bad. One of my defects of character being a preference for form over substance, I have worried about this over the years, collecting principles that ought to govern the marriage of equations to readable prose. I present a few of them here, emphasizing that the list makes no claim to be complete. We are constantly as-

saulted by so many egregious violations of even these simple precepts that I offer them in the hope that a few sinners—not only writers, but copy editors, publishers of journals and even the authors of the mathematics subsections of literary style manuals—may read them and repent the error of their ways, or even be inspired to further beneficial studies of the sadly neglected field of mathematico-grammatics.

Rule 1 (Fisher's rule). This rule, named after the savant who reprimanded me for abusing it when I was young and foolish, simply enjoins one to *number all displayed equations*. The most common violation of Fisher's rule is the misguided practice of numbering only those displayed equations to which the text subsequently refers back. I call this heresy Occam's rule. Back in the days of pens, pencils and typewriters, use of Occam's rule was kept under control by the pain of having to renumber everything by hand whenever it was deemed wise to add a reference to a hitherto-unremarked-upon equation. One often encountered papers displaying the results of the ungainly Fisherian-Occamite compromise: Number all displayed equations that you think you *might* want to refer to. Now that automatic equation numbering macros can act upon symbolic names, the barrier to full Occamism has been removed, and it is necessary to state emphatically that Fisher's rule is for the benefit not of the author, but the reader.

For although you, dear author, may have no need to refer in your text to the equations you therefore left unnumbered, it is presumptuous to assume the same disposition in your readers. And though you may well have acquired the solipsistic habit of writing under the assumption that you will have no readers at all, you are wrong. There is always the referee. The referee may desire to make reference to equations that you did

not. Beyond that, should fortune smile upon you and others actually have occasion to mention your analysis in papers of their own, they will not think the better of you for forcing them into such locutions as "the second equation after (3.21)" or "the third unnumbered equation from the top in the left-hand column on p. 2485." Even should you solipsistically choose to publish in a journal both unrefereed and unread, you might subsequently desire (just for the record) to publish an erratum, the graceful flow of which could only be ensured if you had adhered to Fisher's rule in your original manuscript.

Rule 2 (Good Samaritan rule). A Good Samaritan is compassionate and helpful to one in distress, and there is nothing more distressing than having to hunt your way back in a manuscript in search of Eq. (2.47) not because your subsequent progress requires you to inspect it in detail, but merely to find out what it is *about* so you may know the principles that go into the construction of Eq. (7.38). The Good Samaritan rule says: *When referring to an equation identify it by a phrase as well as a number.* No compassionate and helpful person would herad the arrival of Eq. (7.38) by saying "inserting (2.47) and (3.51) into (5.13)..." when it is possible to say "inserting the form (2.47) of the electric field E and the Lindhard form (3.51) of the dielectric function ϵ into the constitutive equation (5.13)..." To be sure, it's longer this way. Consistent use of the Good Samaritan rule might well increase the length of your paper by a few percent. But admit it. Your paper is probably already too long by at least 30% because you were in such a rush to get it out that you didn't really take enough care putting it all together. So prune elsewhere, but don't force your poor readers—you really *must* assume you will have some, or it is madness to go on writing—to go leafing back when a few words from you would save them the trouble.

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Admittedly sometimes an equation is buried so deep in the guts of an argument, so contingent on context, so ungainly in form that no brief phrase can convey to a reader even a glimmer of what it is about, and anybody wanting to know why it was invoked a dozen pages further on cannot do better than to wander back along the trail and gaze at the equation itself, all glowering and menacing in its lair. Even here, the mere attempt to apply the Good Samaritan rule can have its benefits. If the nature of the equation is inherently uncharacterizable in a compact phrase, is the cross-reference really necessary? Indeed, is the equation itself essential? Or is it the kind of nasty and fundamentally uninteresting intermediate step that readers would either skip over or, if seriously interested, work out for themselves, in neither case needing to have it appear in your text? If so, drop it. You will then have to revise the argument that referred back to it, but the chances are good that the argument will gain in clarity from not having an uncharacterizable monster of an equation at its heart.

Rule 3 (Math Is Prose rule). The Math Is Prose rule simply says: *End a displayed equation with a punctuation mark.* It is implicit in this statement that the absence of a punctuation mark is itself a degenerate form of punctuation that, like periods, commas or semicolons, can be used *provided it makes sense*. For unlike the figures and tables in your article, unlike droppings on a lawn, the equations you display are embedded in your prose and constitute an inseparable part of it. The detailed theory of how equations are to be viewed as prose need not concern us here. Sometimes they function as subordinate clauses, the equals sign being the verb; sometimes they appear as substantive phrases, like a list of the contents of a room; sometimes, regrettably, they must merely be presented to the reader as objects like quotations (but with the convention that quotation marks are not required [except in the rare case that Math Is Prose requires it, as in Eq. (1) above (which I never dreamed I would be referring back to when I first put it into this essay)]).

Regardless, however, of the often subtle question of how to parse the equation internally, certain things are clear to anyone who understands the equation and the prose in which it is embedded. Thus the end of the equation may or may not coincide with the end of the sentence in which it stands. If it does, then the equation

should end with a period or, rarely, if the equation terminates an interrogative sentence, it should end with a question mark. (Having now succeeded in publishing an equation requiring a quotation mark, it remains my dream to publish an article with an equation that requires a question mark; somehow I haven't got around to it.) If the equation terminates a clause or is part of an extended list, then it should end with a comma or semicolon. Only infrequently is no punctuation required, as, for example, in "Only when

$$\sum_{i=1}^N f(x_i) = 0 \quad (2)$$

is it impermissible to divide by this sum."

We punctuate equations because they are a form of prose (they can, after all, be read aloud as a sequence of words) and are therefore subject to the same rules as any other prose. To decree that every sentence should end in a period *unless* the sentence terminates in a displayed equation is grotesque. (If you disagree, try the rule that every opening quotation mark must be followed by a closing one unless the quotation terminates in an equation.) But one does not punctuate equations only because it is ugly not to; more importantly, punctuation makes them easier to read and often clarifies the discussion in which they occur. Acquiring the habit of viewing an equation not as a grammatically irrelevant blob, but as a part of the text fully deserving of punctuation, can only improve the fluency and grace of one's expository mathematical prose.

Most journals punctuate their equations, even if the author of the manuscript did not, but a sorry few don't, removing all vestiges of the punctuation carefully supplied by the author. This unavoidably weakens the coupling between the math and the prose, and often introduces ambiguity and confusion. I'm sorry to say that PHYSICS TODAY is guilty of this practice. To be sure, its use of equations is sufficiently light that this does not inflict substantial hardship on readers, but it greatly undermines the role PHYSICS TODAY so commendably plays in other respects as a model of good writing about hard science. May the appearance of Eq. (1)

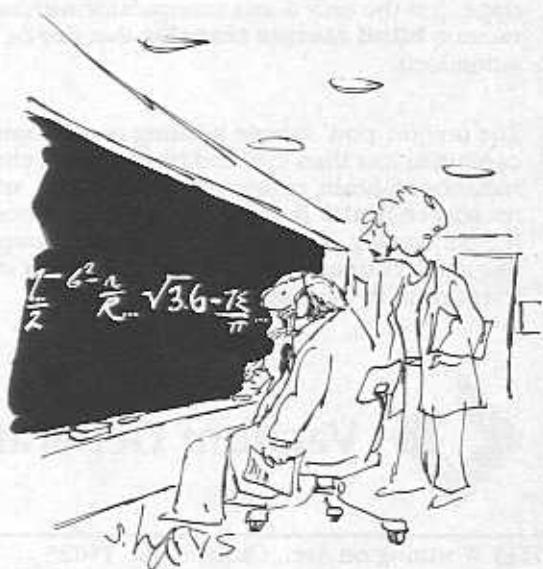
above signal the start of a new and better tradition.

We should strive, more generally, to make errant journals mend their ways. It is easier than you might think. One of my students and I once did a piece of work that required us to lead the reader (or at least, we know for a fact, the referee) through unavoidably dense thickets of equations. Unfortunately the otherwise obvious journal for our paper systematically violated the Math Is Prose rule, so in our letter of submission we emphasized that the punctuation in our equations was essential for the comprehensibility of our argument. The letter of acceptance, however, informed us that the publisher adhered in this and all its other journals, as well as in its books, to a firm policy of never punctuating equations. In that case, we wrote back, just return the manuscript and we'll send it somewhere else. After a long pause we were informed that at a meeting of the board of directors of the publishing firm a special dispensation had been granted to our paper, and indeed, it appeared with punctuated equations.¹

Fortunately Fisher's rule and the Good Samaritan rule don't require assent from boards of directors, so you have nobody to blame but yourself if your papers don't observe them; you can mend your ways right now. At a minimum you will make life much easier for an overworked referee, and with luck you might even have a few happily undistressed readers.

Reference

1. A. Garg, N. D. Mermin, *Found. Phys.* **14**, 1 (1984). ■



"THEN I SAY TO MYSELF, 'WHAT'S THE USE? THERE ISN'T ANY NOBEL PRIZE FOR MATH.'"