SuSi: the Dark Universe
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Exercise 2

Dark matter detection The great experimental physicists at the POCO experiment have recently discovered how to use krypton, a noble gas, to perform a background-free spin-independent search for dark matter! They ran the experiment, consisting of 100 kg of natural Kr (i.e. krypton with natural isotopic abundances), for 200 days and observed no events. They have asked you, phenomenologist extraordinaire, to derive their exclusion limits on the spin-independent DM-nucleon scattering cross section σ_p. The goal here is thus to produce a plot of the excluded σ_p as a function of mass.

The total number of observed events for a given isotope A should be:

$$N = TM_{T,A}\frac{\rho_\chi}{m_\chi}\frac{A^2}{2\mu_p^2}\sigma_p\int_{E_{th}}^{\infty} dE_R\eta(v_{min}, v_{esc})F^2(q) \tag{1}$$

Where T is the exposure time, M_T is the total target mass, m_N is the mass of the nucleus, $\rho_\chi = 0.4$ GeV/cm^3 is the local DM density, μ_p is the DM-proton reduced mass, and $E_{th} = 5$ keV is the energy threshold of the experiment (the smallest energy that can be recorded). We are neglecting the energy efficiency here and for simplicity. $\eta(v_{min})$ encodes the integral over the DM speed distribution $f(v)/v$.

To produce the limits, we are looking for the cross section σ_p for each mass that leads to an expected number of events $N = 2.3$. This is just the expected number for which the probability of observing no events is less than 10% assuming a poisson process (i.e. a 90% exclusion).

1. (15) When producing your final numbers, think carefully about the units that go into each factor of dR/dE_R, and multiply by the appropriate factors of $\hbar$ and c. Your final plot should be of σ_p in cm^2 and m_χ in GeV. Plot your result on a log scale, for $m_\chi = 1$ to 10^4 GeV.

2. (5) on the same plot, show the exclusion curves that you would instead obtain with 1) a 1 keV threshold energy; and 2) another 200 days of exposure. Briefly explain why your curves change in the way that they do.

3. (1) Why is using krypton for such an experiment actually a really bad idea?

A little help: instead of computing η from scratch, you may use the following result for $\eta(v_{min}, v_{esc})$

$$\eta = N\left(\frac{\mathrm{Erf}(a_+) - \mathrm{Erf}(a_-)}{2v_{\mathrm{lag}}} - \frac{1}{\sqrt{\pi}v_{\mathrm{lag}}}(a_+ - a_-)e^{-(v_{\mathrm{esc}}/\sigma_v)^2/2}\right), \tag{2}$$

where

$$N = \left(\mathrm{Erf}(a_{\mathrm{esc}}) - \sqrt{\frac{2}{\pi}}\left(\frac{v_{\mathrm{esc}}}{\sigma_v}\right)e^{-a_{\mathrm{esc}}^2}\right)^{-1} \tag{3}$$

$$a_\pm = \min\left(v_{\mathrm{min}} \pm v_{\mathrm{lag}}, v_{\mathrm{esc}}\right)/\sqrt{2}\sigma_v \tag{4}$$

$$a_{\mathrm{esc}} = v_{\mathrm{esc}}/\sqrt{2}\sigma_v \tag{5}$$

Here, $v_{lag} = 220$ km/s, $\sigma_v = 156$ km/s and $v_{esc} = 550$ km/s. For the form factor, you may use the following:

$$F(q) = e^{-y} \tag{6}$$

where $y = (bq/2)^2$, $q = |\vec{q}|$, and

$$b = 32.58/\sqrt{45A^{-1/3} - 25A^{-2/3}} \text{ GeV}^{-1}. \tag{7}$$