SuSi: the dark Unverse
SNOLAB, Summer 2024
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Exercise 1

Measuring the expansion rate: The 2011 Nobel Prize went to two groups, the Supernova Cosmology project and the high-z supernova project, for demonstrating the accelerating expansion of the Universe using Type Ia supernovae as standard candles. Let's see what that looks like. Distances are normally reported in terms of the *distance modulus*:

$$\mu = (m - M) \equiv 5 \log_{10} \frac{d_L}{10\text{pc}}, \tag{1}$$

where d_L is the luminosity distance. We will be using recent supernova data compiled by the SN cosmology project (data.dat). The first column is the redshift z of each supernova, and the second is μ. The third is the error on the modulus. Start by making a scatter plot of μ vs z to make sure you know what you are doing (we'll use this later).

1. We will assume a *flat* universe that contains matter and dark energy. Write down the (first) Friedmann equation as a function of the cosmological parameters: H_0, Ω_m, Ω_r and Ω_Λ. Explain why it will probably be ok to leave out radiation.

2. Let's begin by determining the expansion rate today, H_0.

   (a) If we interpret $1+z$ as a Doppler shift, determine the expression for the recession velocity as a function of redshift for $v \ll 1$.

   (b) Determine the expression for the luminosity distance for $z \ll 1$.

   (c) Next take the μ and z data from the file above, and select only the data with redshift $z < 0.03$. **Make a scatter plot of speed versus distance**. Express speed in km/s, and distance in megaparsecs.

   (d) Perform a linear fit on your data to infer a value for H_0 based on the expressions you found. Remember to set the intercept to $v = 0$ when $z = 0$.

3. Now we can look at the full data set. Determine the expression for the luminosity distance given the Friedmann equation you wrote down in part a) (do not attempt to perform the integral analytically).

4. On top of your scatter plot of the full data set (μ vs z), and using your value of H_0, plot the curve for μ that you obtain 1) for a matter-dominated universe; 2) a dark energy-dominated universe; 3) a radiation-dominated universe.

   Note that in evaluating the integral over z (or a) to obtain d_L, it is sufficient to make a grid of z values and perform a trapezoidal integral. If you're unfamiliar, look up trapz and cumtrapz, which are routines available in both Matlab and scipy and are very easy to deal with.

5. Now construct a chi-squared statistic:

$$\chi^2 = \sum_i \frac{(\hat{\mu}(z_i) - \mu_i)^2}{\sigma_i^2} \tag{2}$$

where the μ_i are the data points, σ_i are their corresponding errors, and $\hat{\mu}(z_i)$ is the theoretical distance modulus at the redshift z_i of that particular supernova, and the sum is over all the data points in the sample. If $\Omega = 1$ and we ignore radiation, this model has two free parameters, H_0 and Ω_m. Perform a fit to find the combination of H_0 and Ω_m that best fits the data. Draw the 1 and 2 sigma ($\Delta\chi^2 = 2.3$ and 4.16, respectively) contours.